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Conditions for anti-Unruh effect

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Abstract Conditions for anti-Unruh effect in (1+1)-dimensional spacetime are obtained. For detectors with Gaussian switching functions, the analytical results are similar to previous ones, indicating that anti-Unruh effect occurs when the energy gap matches the characteristic time scale. However, this conclusion does not hold for detectors with square wave switching functions, in which case the condition turns out to depend on both the energy gap and the characteristic time scale in some nontrivial way. We also show analytically that there is no anti-Unruh effect for detectors with Gaussian switching functions in (3+1)-dimensional spacetime.

1 Introduction

It is well known that a uniformly accelerated observer views the Minkowski vacuum as a thermal state with the temperature T proportional to the observer's acceleration a, $T = a/2\pi$, usually called the Unruh effect [1–3]. To give a coordinate-invariant characterization of Unruh effect, people often employ the so-called Unruh–DeWitt detector and study how it "tinkles" when accelerated [1,4]. The simplest Unruh–DeWitt detector is a two-level system, and it is expected that when an accelerating detector interacts with some quantum field, there is a transition from the initial ground state to the excited state, with the probability increasing with the increase of the acceleration [5].

However, in recent years it was found that under some circumstances the transition probability decreases with the increase of the acceleration on the contrary [6], which seems to imply that the detector gets cooler when the acceleration increases. This effect is called the anti-Unruh effect. Since the anti-Unruh effect is defined according to the behavior of detectors, it is, unlike the Unruh effect, highly dependent on the types of detectors.

The anti-Unruh effect may lead to the enhancement of the entanglement between accelerating Unruh–DeWitt detectors [7–10]. Moreover, the results can be applied to black holes [11–15] and other thermal systems [16,17]. However, despite some discussions on the mechanism of anti-Unruh phenomena [6,18], its physical reason remains unclear. To provide some insight on the physical nature of the anti-Unruh effect, an analytical approach is useful. Especially, in some cases, the condition for the anti-Unruh effect might be complicated and is hard to be fully determined by merely trying different parameters, e.g. in the case of square wave switching function. In such cases, the analytic approach is advantageous.

In this paper, we analytically derive the conditions of the anti-Unruh effect for detectors with Gaussian and square wave switching functions, by considering the transition probability of one Unruh–DeWitt detector coupled with a scalar field. Unlike approaches relying on numerical integration, our derivation is analytical, and the numerical integration only serves as an assisting tool to verify the derivation.

This paper mainly deals with (1+1)-dimensional spacetime, as the analytical treatment to the case of (3+1)-dimensional spacetime is very difficult. In (1+1)-dimensional spacetime, for Gaussian switching functions, the anti-Unruh effect appears when $\Omega\sigma < 1/\sqrt{2}$, while for square wave switching functions, the anti-Unruh effect appears when $\left(2(\Omega\sigma)^2 - 3\right)\cos(2\Omega\sigma) - 4\Omega\sigma\sin(2\Omega\sigma) + 3 < 0$, where Ω and σ are the energy gap and the characteristic switching time respectively. Some other switching functions, such as the Dirac delta function, can either directly follows from or be well approximated by these two functions.

For the case of (3+1)-dimensional spacetime, we find that no anti-Unruh effect exists, at least for Gaussian switching

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functions. However, results for detectors with square wave function can be obtained using the methods in [19]. It can be shown that the absence of anti-Unruh effect also occurs for such switching functions. The method in [19] depends on numerical integration, however, the conclusion regarding the Gaussian functions there agrees with the results in this paper.

We expect our analytical calculations and results be useful in revealing the physical reason of the anti-Unruh effect. This paper is organized as the following. In Sect. 2 we review the basic model for the anti-Unruh effect in (1+1)-dimensional and (3+1)-dimensional spacetimes. We present and analyze our main results in Sect. 3. Section 4 is the summary.

2 Model

In this section, we review the simplest model for this effect [6]. First, we consider a uniformly accelerated two-level Unruh–DeWitt detector with the energy gap Ω in (1+1)-dimensional Minkowski spacetime. The detector interacts with a massive scalar field φ, with the interaction Hamiltonian

$$H_I = \lambda \chi(\tau, \sigma)\mu(\tau)\varphi\left(x\left(\tau\right), t\left(\tau\right)\right), \tag{1}$$

where λ is the strength of the coupling and τ is the proper time along the detector's worldline,

$$\mu(\tau) = \exp\left(i\Omega\tau\right)\sigma^{+} + \exp\left(-i\Omega\tau\right)\sigma^{-} \tag{2}$$

is the monopole operator, χ is the switching function, which we can, for example, choose as the Gaussian type

$$\chi(\tau, \sigma) = e^{-\frac{\tau^2}{2\sigma^2}}, \tag{3}$$

with σ being the characteristic time.

Suppose the initial state is $|g\rangle|0\rangle$, where $|g\rangle$ refers to the ground state of the detector and $|0\rangle$ refers to the vacuum state of the scalar field in the Minkowski spacetime. The evolution of the system is

$$U|g\rangle|0\rangle = \left(1 - i\int d\tau H(\tau) + \cdots\right)|g\rangle|0\rangle, \tag{4}$$

where we have used the perturbation expansion. Given the monopole operator $\mu(\tau)$ and the mode expansion of the massive scalar field in (1+1)-dimensional Minkowski spacetime

$$\varphi(x, t) = \int \frac{dk}{\sqrt{4\pi\omega}}\left[a(k)e^{-i(\omega t - kx)} + a^{\dagger}(k)e^{i(\omega t - kx)}\right], \tag{5}$$

where $\omega = \sqrt{k^2 + m^2}$ (m is the mass of the scalar field), we obtain the final state of the system,

$$|g\rangle|0\rangle - i\lambda\int d\tau\chi(\tau, \sigma)e^{i\Omega\tau}\int\frac{dk}{\sqrt{4\pi\omega}}e^{i(\omega t - kx)}|e\rangle|1\rangle_k, \tag{6}$$

where $|e\rangle$ is the excited state of the detector and $|1\rangle_k$ is the one-particle state of the field in mode k. The typical trajectory of a uniformly accelerated detector can be given as $x(\tau) = a^{-1}\left(\cosh\left(a\tau\right) - 1\right)$ and $t(\tau) = a^{-1}\sinh\left(a\tau\right)$, where a is the acceleration. Therefore the transition probability is

$$P(\Omega, a, \sigma, m) = \int_{-\infty}^{+\infty} dk|I_k|^2, \tag{7}$$

with

$$I_k = \frac{\lambda}{\sqrt{4\pi\omega}}\int_{-\infty}^{+\infty} d\tau\chi(\tau, \sigma)\exp \times\left(i\Omega\tau + i\frac{\omega}{a}\sinh a\tau - i\frac{k}{a}\left(\cosh a\tau - 1\right)\right). \tag{8}$$

Similar results hold for anti-Unruh effect in (3+1)-dimensional spacetime. In such a case, the scalar field mode expansion is

$$\varphi(\vec{x}, t) = \int\frac{d^3\vec{k}}{\sqrt{(2\pi)^3 2\omega}} \times\left[a(\vec{k})e^{-i(\omega t - \vec{k}\cdot\vec{x})} + a^{\dagger}(\vec{k})e^{i(\omega t - \vec{k}\cdot\vec{x})}\right]. \tag{9}$$

Suppose the detector accelerates along x axis ($y = z = 0$). Then following similar calculations, we obtain the final state

$$|g\rangle|0\rangle - i\lambda\int d\tau\chi(\tau, \sigma)e^{i\Omega\tau} \times\int\frac{d^3\vec{k}}{\sqrt{(2\pi)^3 2\omega}}e^{i(\omega t - \vec{k}\cdot\vec{x})}|e\rangle|1\rangle_{\vec{k}}, \tag{10}$$

and thus the transition probability

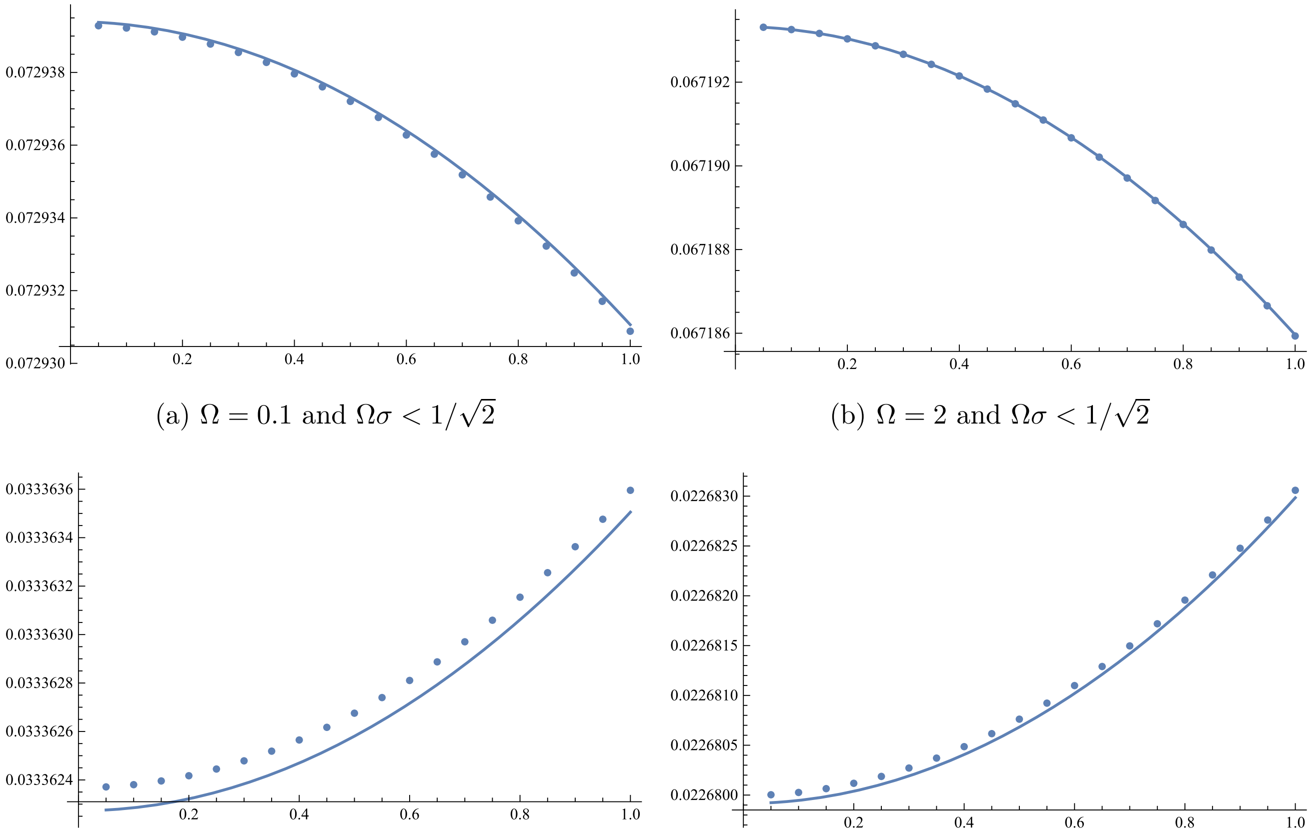
$$P(\Omega, a, \sigma, m) = \int_{-\infty}^{+\infty} d^3\vec{k}|I_{\vec{k}}|^2, \tag{11}$$

with

$$I_{\vec{k}} = \frac{\lambda}{\sqrt{(2\pi)^3 2\omega}}\int_{-\infty}^{+\infty} d\tau\chi(\tau, \sigma)\exp \times\left(i\Omega\tau + i\frac{\omega}{a}\sinh a\tau - i\frac{k_x}{a}\left(\cosh a\tau - 1\right)\right). \tag{12}$$

3 Results

In this section, we present the conditions for anti-Unruh effects analytically calculated in (1+1)-dimensional and (3+1)-dimensional spacetimes. The details of the calculation are given in the Appendix.



(a) $\Omega = 0.1$ and $\Omega\sigma < 1/\sqrt{2}$

(b) $\Omega = 2$ and $\Omega\sigma < 1/\sqrt{2}$

(c) $\Omega = 8$ and $\Omega\sigma > 1/\sqrt{2}$

(d) $\Omega = 10$ and $\Omega\sigma > 1/\sqrt{2}$

Fig. 1 Comparison of the numerical results $2\pi\sigma^2 P_\pm$ and the analytical results $P^{(G)}_{LO} + P^{(G)}_{NLO}$ when $\sigma = 0.1$ and $m = 0.01$. We performed numerical integrals with cutoff $\int_{-30}^{30} dk \left| \int_{-1}^{1} d\tau I(\tau, k) \right|^2$

3.1 (1+1)-dimensional spacetime

In the case of $D = 1 + 1$, we focus on Unruh–DeWitt detectors with Gaussian or square wave switching functions, which can be written as

$$\chi^{(G)}(\tau, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\tau^2}{2\sigma^2}},$$
$$\chi^{(S)}(\tau, \sigma) = \frac{1}{2\sigma} H(\sigma - \tau) H(\sigma + \tau), \tag{13}$$

where H is the Heaviside step function. Note that the Fourier transforms of the switching functions are

$$\tilde{\chi}^{(G)}(\omega, \sigma) = \frac{e^{-\frac{\omega^2\sigma^2}{2}}}{\sqrt{2\pi}},$$
$$\tilde{\chi}^{(S)}(\omega, \sigma) = \frac{\sin(\sigma\omega)}{\sqrt{2\pi}\sigma\omega}, \tag{14}$$

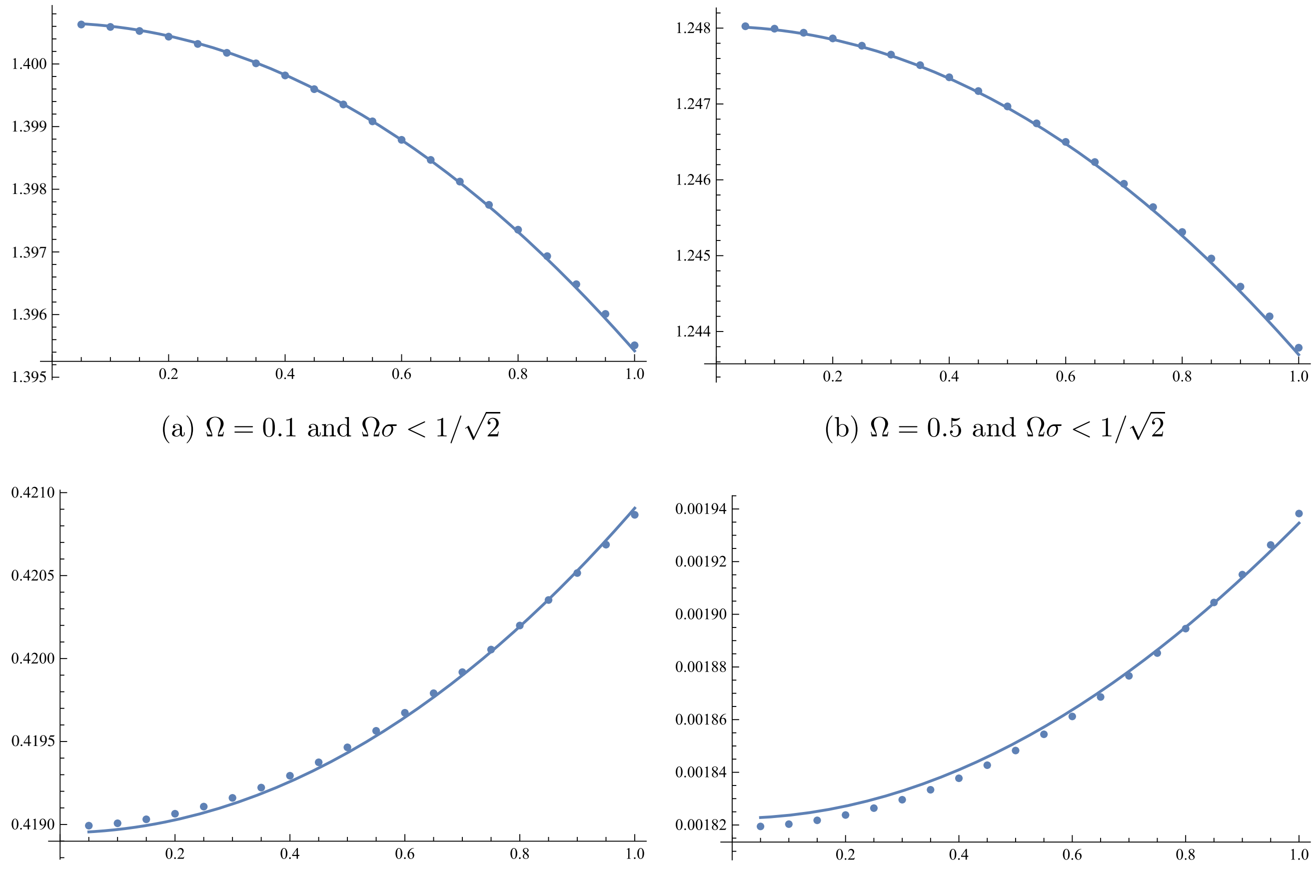
and they are both square integrable,

$$\int_{-\infty}^{\infty} d\omega \left| \tilde{\chi}^{(G)}(\omega, \sigma) \right|^2 = \frac{1}{2\sqrt{\pi}\sigma},$$
$$\int_{-\infty}^{\infty} d\omega \left| \tilde{\chi}^{(S)}(\omega, \sigma) \right|^2 = \frac{1}{2\sigma}. \tag{15}$$

3.1.1 *Gaussian switching function*

We start with Gaussian switching functions. As shown in Eqs. (A17) and (A22), we obtain the analytical expression for the transition probability in the small mass limit,

$$P^{(G)}_{\pm} = \frac{1}{2\pi\sigma^2} \left(P^{(G)}_{LO} + P^{(G)}_{NLO} \right) + \mathcal{O}(m^3),$$
$$P^{(G)}_{LO} = \sigma^2 e^{-\Omega^2\sigma^2} \left\{ \left(\Omega^2\sigma^2 \, {}_2F_2\left(\left. \begin{matrix} 1, 1 \\ \frac{3}{2}, 2 \end{matrix} \right| \Omega^2\sigma^2 \right) - \log\frac{m\sigma}{2} - \frac{\pi}{2}\operatorname{erfi}(\Omega\sigma) - \frac{\gamma_E}{2} \right) - \frac{a^2\sigma^2}{12} \left(1 - 2\Omega^2\sigma^2 - \frac{a^2\sigma^2}{60} \left(4\Omega^4\sigma^4 - 12\Omega^2\sigma^2 + 3 \right) \right) \right\} + \mathcal{O}(a^6\sigma^8),$$



(a) $\Omega = 0.1$ and $\Omega\sigma < 1/\sqrt{2}$

(b) $\Omega = 0.5$ and $\Omega\sigma < 1/\sqrt{2}$

(c) $\Omega = 2$ and $\Omega\sigma > 1/\sqrt{2}$

(d) $\Omega = 5$ and $\Omega\sigma > 1/\sqrt{2}$

Fig. 2 Comparison of the numerical results $2\pi\sigma^2 P_\pm$ and the analytical results $P_{LO}^{(G)} + P_{NLO}^{(G)}$ when $\sigma = 0.5$ and $m = 0.01$. We performed numerical integrals with cutoff $\int_{-10}^{10} dk \left|\int_{-6}^{6} d\tau I(\tau, k)\right|^2$

$$P_{NLO}^{(G)} = \frac{2m^2\sigma^4}{\sqrt{\pi}} \left(2\Omega\sigma \left(\frac{d}{dx}\, {}_1F_1\left(\left. \begin{matrix} x \\ \frac{3}{2} \end{matrix} \right| -\Omega^2\sigma^2 \right) \right) \Bigg|_{x=2} - \left(\left(2\Omega^2\sigma^2 - 1 \right) F(\Omega\sigma) - \Omega\sigma \right) \times (2\log(m\sigma) + \pi + \gamma_E - 1) \right), \tag{16}$$

where ${}_pF_q$ are generalized hypergeometric function, as defined in Eq. (A6). Note that although infrared divergence is encountered in $\log(m\sigma)$, the result is still valid for small mass.

At the leading order of a^2, the coefficient of a^2 is $1 - 2\Omega^2\sigma^2$. Therefore the anti-Unruh effect can be found at

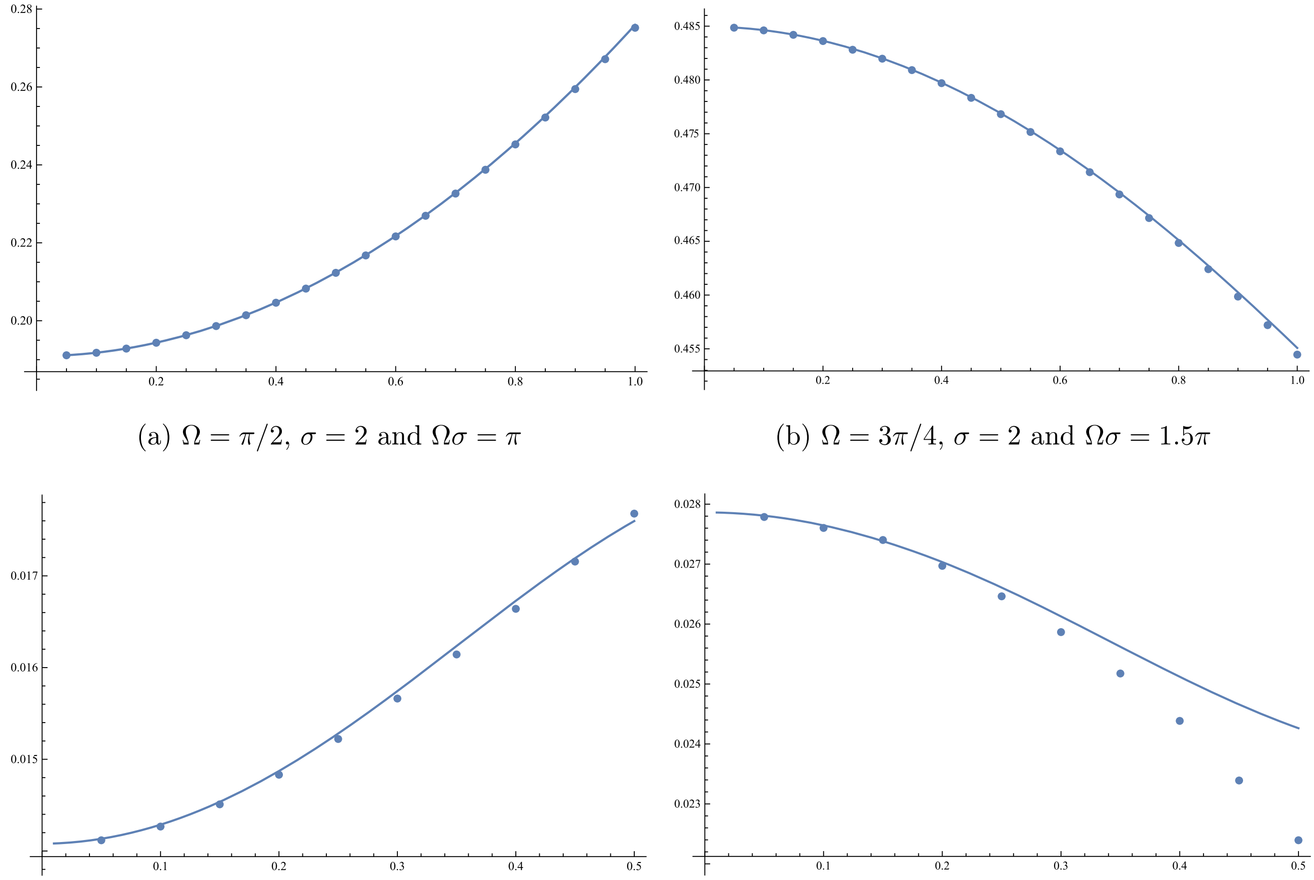
$$\Omega\sigma < \frac{1}{\sqrt{2}}. \tag{17}$$

This is in agreement with the original statement of anti-Unruh effect which claims the interaction time interval to be finite $\sigma \sim \Omega^{-1}$ [6]. The comparison of the analytical and numerical results is shown in Figs. 1 and 2.

3.1.2 Square wave switching function

For detectors with square wave switching functions, as shown in Eq. (A25), the transition probability is given as

$$\begin{aligned} P_\pm^{(S)} &= \frac{1}{2\sigma}\left(P_{LO}^{(S)} + P_{NLO}^{(S)}\right) + \mathcal{O}(m^3), \\ P_{LO}^{(S)} &= \frac{1}{2\pi\Omega^2}\Bigg\{ -2\mathrm{Ci}(2\Omega\sigma) - 2\log\left(\frac{1}{m\sigma}\right)\cos(2\Omega\sigma) \\ &+2\log\left(\frac{2\Omega}{m}\right) + 4\Omega s\,\mathrm{Si}(2\Omega\sigma) - 2\pi\Omega\sigma \\ &-2 + \pi\sin(2\Omega\sigma) + 2\gamma\cos(2\Omega\sigma) + 2\cos(2\Omega\sigma) \\ &+a^2\frac{2\Omega^2\sigma^2\cos(2\Omega\sigma) - 4\Omega\sigma\sin(2\Omega\sigma) - 3\cos(2\Omega\sigma) + 3}{6\Omega^2} \\ &+\frac{a^4}{180\Omega^4}\Big(-2\Omega^4\sigma^4\cos(2\Omega\sigma) + 8\Omega^3\sigma^3\sin(2\Omega\sigma) \\ &+18\Omega^2\sigma^2\cos(2\Omega\sigma) - 24\Omega\sigma\sin(2\Omega\sigma) \\ &-15\cos(2\Omega\sigma) + 15\Big) + \mathcal{O}\left(\frac{\sigma^8 a^6}{\Omega^2}\right)\Bigg\}, \end{aligned}$$



(a) $\Omega=\pi/2$, $\sigma=2$ and $\Omega\sigma=\pi$

(b) $\Omega=3\pi/4$, $\sigma=2$ and $\Omega\sigma=1.5\pi$

(c) $\Omega=10$, $\sigma=2\pi$ and $\Omega\sigma=20\pi$

(d) $\Omega=10$, $\sigma=41\pi/20$ and $\Omega\sigma=20.5\pi$

Fig. 3 Comparison of the numerical results $2\sigma P_{\pm}$ and the analytical results $P_{LO}^{(S)}+P_{NLO}^{(S)}$ with $m=0.01$. We performed numerical integrals with cutoff $\int_{-100}^{100} dk\left|\int_{-\sigma}^{\sigma} d\tau\, I(\tau,k)\right|^2$. Note that when σ is large, the analytical result is only accurate when a is small

$$\begin{aligned}
P_{NLO}^{(S)} = \frac{m^2}{4\Omega^4\pi}\Bigg\{&\log\left(\frac{64\Omega^6}{m^6}\right) - 6\mathrm{Ci}\,(2\Omega\sigma) \\
&+\sin(2\Omega\sigma)\Big(8\Omega\sigma\log(m\sigma) - 2\pi\Omega^2\sigma^2 \\
&+4(2\gamma_E-1)\Omega\sigma + 3\pi\Big) + \cos(2\Omega\sigma)\Big(\Big(6-4\Omega^2\sigma^2\Big)\log(m\sigma) \\
&-4(\gamma_E-1)\Omega^2\sigma^2 - 4\pi\Omega\sigma + 6\gamma_E + 5\Big) \\
&+4\Omega\sigma\,\mathrm{Si}(2\Omega\sigma) - 2\pi\Omega\sigma - 5\Bigg\}, \qquad (18)
\end{aligned}$$

where Ci and Si are cosine and sine integral functions, as defined in Eq. (A26). Likewise, we assume the mass of the scalar field to be small though nonzero. At the leading order of a^2, the coefficient of a^2 is $\left(2(\Omega\sigma)^2-3\right)\cos(2\Omega\sigma) - 4\Omega\sigma\sin(2\Omega\sigma)+3$. Therefore the condition for anti-Unruh effect can be written in "closed form" as

$$\left(2(\Omega\sigma)^2-3\right)\cos(2\Omega\sigma) - 4\Omega\sigma\sin(2\Omega\sigma) + 3 < 0. \qquad (19)$$

The comparison of the analytical and numerical results is shown in Fig. 3.

It can be checked easily that the condition of anti-Unruh effect for detectors with square-wave switching functions is quite different from that for detectors with Gaussian switching functions. The anti-Unruh effect can be found not as $\Omega\sigma \to 0$ but at, for example, $\Omega\sigma = 20.5\pi$. This means that anti-Unruh effects occur even when the interaction time is long (with the energy gap fixed). Therefore our results support the argument that anti-Unruh effect are not due to non-equilibrium transient effects [6], since the KMS condition [20,21] is satisfied [6,22]. Furthermore, the condition Eq. (19) depends on $\Omega\sigma$ in the form of sine and cosine function, which can be naturally expected from the Fourier transform of the square wave function Eq. (14). In particular, this means that for some given energy gap, anti-Unruh effect can be found from time to time with the increase of σ, which is a surprising result.

3.2 (3+1)-dimensional spacetime

We conclude this section by displaying expressions for the transition probability of Unruh–DeWitt detectors with Gaussian switching functions in (3+1)-dimensional spacetime. As shown in Eq. (A31), the result can be obtained as

$$P_{\pm}^{D=3+1} = \frac{1}{2\pi\sigma^2}\left(P_a^{(0)} + P_a^{(2)}\right) + \mathcal{O}(a^4),$$
$$P_a^{(2)} = \frac{a^2\sigma^2}{24\pi}e^{-\Omega^2\sigma^2} + \mathcal{O}(m). \tag{20}$$

Note that $P_a^{(0)}$ is UV divergent; however $P_a^{(0)} \sim \mathcal{O}(a^0)$ and is therefore of little concern to us. The dependence of $P_a^{(2)}$ on a shows that when a is small there is no anti-Unruh effect in the small mass limit. The same result for massless scalar field can be obtained by a different method in [19], where the transition probability is calculated with numerical integration. Furthermore, it can be shown using the method in [19] (simply by replacing the Fourier transform of the Gaussian functions with the square wave ones) that in (3+1)-dimensional spacetime, the absence of anti-Unruh effect also occurs with the square wave switching functions. Together with the discovery in [19,23] that there is no anti-Unruh effect for detectors coupled quadratically with a scalar field or linearly with a spinor field in (3+1)-dimensional spacetime, we conjecture that the increase of spacetime dimensions plays a negative role in the possibility of anti-Unruh effect.

4 Summary

We obtain analytically the conditions for anti-Unruh effect in (1+1)-dimensional spacetime. The product of the detector's energy gap Ω and the interaction time σ is the characteristic quantity in the conditions. We show that for detectors with Gaussian switching functions, the condition is $\Omega\sigma < \frac{1}{\sqrt{2}}$. However, for detectors with square wave switching functions, anti-Unruh effect could happen when $\Omega\sigma$ is large. Furthermore, for a fixed energy gap, whether anti-Unruh effect occurs or not depends on the interaction time non-monotonically. Our results support the argument that anti-Unruh effect is in accordance with the KMS condition and is therefore not a transient effect. We hope that our calculations would provide some insight on the physical nature of anti-Unruh effect. Finally we show that for detectors with Gaussian switching functions there is no anti-Unruh effect in (3+1)-dimensional spacetime.

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Data availability This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical paper and contains no experimental data.]

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Appendix A: The analytical results with small mass

In general, the integral to be calculated can be written as

$$P_{\pm} = \int d^d k \left|\int_{-\infty}^{\infty} d\tau \frac{1}{\sqrt{4\pi\omega}}\chi(\tau,\sigma)\exp\right.$$
$$\left.\times\left(i\Omega\tau + i\frac{\omega}{a}\sinh(a\tau) - i\frac{k_x}{a}(\cosh(a\tau)-1)\right)\right|^2. \tag{A1}$$

where $\omega \equiv \sqrt{k^2+m^2}$, $\chi(\tau,\sigma)$ is the switching function, and Ω is defined as $\pm\Omega_0$ for $P_{\pm}$.

A.1 The case of D = 1 + 1

It is convenient to integrate over k first. The integral can be written in a somewhat symmetric form as

$$P_{\pm} = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \chi(\tau_1,\sigma)\chi(\tau_2,\sigma)e^{i\Omega(\tau_2-\tau_1)}$$
$$\times\left(P_k(A,B) + P_k(A,-B)\right),$$
$$P_k(A,B) = \int_0^{\infty} dk \frac{1}{4\pi\sqrt{m^2+k^2}}$$
$$\times\exp\left(i\left(A\sqrt{m^2+k^2} - Bk\right)\right),$$
$$A = \frac{\sinh(a\tau_2)-\sinh(a\tau_1)}{a},$$
$$B = \frac{\cosh(a\tau_2)-\cosh(a\tau_1)}{a}. \tag{A2}$$

In the case of small mass, one have

$$P_k(A,B) = \int_0^{\infty} dk \frac{1}{4\pi\sqrt{m^2+k^2}}$$
$$\times\left(e^{i(A-B)k} + \frac{iAm^2}{2k}e^{i(A-B)k} + \mathcal{O}\left(\frac{Am^4}{k^3}\right)\right). \tag{A3}$$

The leading-order term of Eq. (A3) can be integrated out as

$$\begin{aligned}&\int_0^\infty dk \frac{1}{\sqrt{m^2+k^2}} \exp(iCk))\\ &= -\hat{F}\left(1, \frac{C^2m^2}{4}\right) - \frac{1}{2} i\pi \boldsymbol{L}_0(Cm)\\ &\quad + \log\left(-\frac{1}{2}iCm\right)(-I_0(Cm))\\ &= -\log\left(-\frac{1}{2}iCm\right) - \gamma_E - iCm\\ &\quad - \frac{C^2m^2}{4}\left(\log(-\frac{1}{2}iCm) - \gamma_E + 1\right) + \mathcal{O}(m^3), \end{aligned} \tag{A4}$$

where $\gamma_E \approx 0.57721$ is the Euler constant and I_0 is modified Bessel function of the first kind. $\boldsymbol{L}_0$ is modified Struve function, and $\hat{F}$ is defined as

$$\hat{F}(a,z) \equiv \frac{d}{da'}\left(\frac{{}_0F_1(a'|z)}{\Gamma(a')}\right)\Bigg|_{a'=a}, \tag{A5}$$

where ${}_pF_q$ is the generalized hypergeometric function defined as

$${}_pF_q\left(\begin{matrix} a_1, a_2, \ldots, a_p \\ b_1, b_2, \ldots, b_q \end{matrix} \middle| x\right) = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^p (a_i)_n}{\prod_{j=1}^q (b_j)_n} \frac{x^n}{n!}. \tag{A6}$$

Verified by numerical results, we conclude when mass is small,

$$\begin{aligned}&\int_0^\infty dk \frac{1}{\sqrt{m^2+k^2}} \exp\left(i(A\sqrt{k^2+m^2} - Bk)\right)\\ &= -\log\left(-\frac{1}{2}i(A-B)m\right) - \gamma_E + \mathcal{O}(m). \end{aligned} \tag{A7}$$

Next we can calculate the next-to-leading order term. Note that the integral can be written as

$$P_k(A,B) = P_k(0,B) + \int_0^A dA' \frac{\partial P_k(A',B)}{\partial A'}, \tag{A8}$$

where the first term $P_k(0, B)$ is already known in Eq. (A4) as

$$\begin{aligned}P_k(0,B) &= -\log\left(-\frac{1}{2}iBm\right) - \gamma_E - iBm\\ &\quad - \frac{B^2m^2}{4}\left(\log(-\frac{1}{2}iBm) - \gamma_E + 1\right)\\ &\quad + \mathcal{O}(m^3). \end{aligned} \tag{A9}$$

We define the integrand of the second term as $p(m)$

$$p(m) \equiv \frac{\partial P_k(A',B)}{\partial A'} = \frac{i}{4\pi}\int_0^\infty dk e^{i(A'\sqrt{m^2+k^2} - Bk)}, \tag{A10}$$

and similarly,

$$p(m) = p(0) + \frac{i}{4\pi}\int_0^m dm' \int_0^\infty dk \frac{\partial e^{i(A'\sqrt{m'^2+k^2} - Bk)}}{\partial m'}. \tag{A11}$$

The first term can be integrated out, while the second term is

$$\begin{aligned}&\frac{i}{4\pi}\int_0^\infty dk \frac{\partial e^{i(A'\sqrt{m'^2+k^2} - Bk)}}{\partial m'}\\ &= -\frac{A'm'}{4\pi}\int_0^\infty dk \frac{1}{\sqrt{m'^2+k^2}} e^{i(A'\sqrt{m'^2+k^2} - Bk)}, \end{aligned} \tag{A12}$$

with the leading-order term also already known in Eq. (A7). Therefore we have

$$\begin{aligned}p(m) &= -\frac{1}{A'-B} - \frac{A'm^2}{2}\left(-\log\left(-\frac{i}{2}(A'-B)m\right)\right.\\ &\quad \left. -\gamma_E + \frac{1}{2}\right) + \mathcal{O}(m^3) \end{aligned} \tag{A13}$$

and using Eqs. (A2), (A8–A10) and (A13),

$$\begin{aligned}P_\pm &= \frac{1}{4\pi}\int d\tau_1 d\tau_2 \chi(\tau_1,\sigma)\chi(\tau_2,\sigma)\exp(i\Omega(\tau_2-\tau_1))\\ &\quad \times\left(2\log\frac{2a}{m} - 2\log\left(2i\sinh\left(\frac{a(\tau_1-\tau_2)}{2}\right)\right)\right.\\ &\quad -2\gamma_E + \frac{1}{4}m^2(\tau_1-\tau_2)^2(2\log(im(\tau_1-\tau_2))\\ &\quad -2 + 2\gamma_E - \log(4))) + \mathcal{O}(m^3). \end{aligned} \tag{A14}$$

A.1.1 The Gaussian switching function

The Gaussian switching function can be written as

$$\chi^{(G)}(\tau,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\tau^2}{2\sigma^2}}. \tag{A15}$$

Using

$$T = \frac{\tau_1+\tau_2}{2}, \quad t = \tau_1 - \tau_2, \tag{A16}$$

and integrating over T first, we get

$$\begin{aligned}P_\pm^{(G)} &= \frac{1}{2\pi\sigma^2}\left(P_{LO}^{(G)} + P_{NLO}^{(G)}\right) + \mathcal{O}(m^3),\\ P_{LO}^{(G)} &= \sigma^2 e^{-\Omega^2\sigma^2}\left\{\Omega^2\sigma^2\, {}_2F_2\left(\begin{matrix}1,1\\ \frac{3}{2},2\end{matrix}\middle|\Omega^2\sigma^2\right)\right.\\ &\quad \left. -\log\frac{m\sigma}{2} - \frac{\pi}{2}\mathrm{erfi}(\Omega\sigma) - \frac{\gamma_E}{2}\right\} + I_t,\\ P_{NLO}^{(G)} &= \frac{2m^2\sigma^4}{\sqrt{\pi}}\left(2\Omega\sigma\left(\frac{d}{dx}\,{}_1F_1\left(\begin{matrix}x\\ \frac{3}{2}\end{matrix}\middle| -\Omega^2\sigma^2\right)\right)\right|_{x=2}\\ &\quad -\left(\left(2\Omega^2\sigma^2 - 1\right)F(\Omega\sigma) - \Omega\sigma\right)\\ &\quad \times(2\log(m\sigma) + \pi + \gamma_E - 1)), \end{aligned} \tag{A17}$$

where I_t is defined as

$$I_t = -\frac{\sqrt{\pi}\sigma}{\pi}\int_0^\infty dt \exp\left(-\frac{t^2}{4\sigma^2}\right)\cos(\Omega t) \times \log\left(\frac{2\sinh\frac{at}{2}}{at}\right). \tag{A18}$$

Considering only the case in which $a < 1$, we have

$$\log\left(\frac{2\sinh\frac{at}{2}}{at}\right) = \frac{1}{24}a^2t^2 - \frac{1}{2880}a^4t^4 + \frac{1}{181440}a^6t^6 + \mathcal{O}(a^8), \tag{A19}$$

therefore

$$\begin{aligned} I_t &= \frac{1}{24}I_t^1 - \frac{1}{2880}I_t^2 + \frac{1}{181440}I_t^3 + \mathcal{O}(a^8\sigma^{10}),\\ I_t^n &= -\frac{\sqrt{\pi}\sigma}{\pi}\int_0^\infty dt \exp\left(-\frac{t^2}{4\sigma^2}\right)\cos(\Omega t)\, a^{2n}t^{2n}\\ &= -\pi^{\frac{1}{4}}2^n\sigma^2 e^{-\frac{1}{2}\Omega^2\sigma^2}\sqrt{\frac{(2n)!}{\Omega}}(ia\sigma)^{2n}\\ &\quad\times \phi_{2n}(\Omega,\sigma) \sim \mathcal{O}(a^{2n}\sigma^{2n+2}), \end{aligned} \tag{A20}$$

where $\phi_{2n}(\Omega\sigma)$ is the wave function of harmonic oscillator defined as

$$\phi_n(\Omega,\sigma) \equiv \frac{\left(\frac{\Omega^2}{\pi}\right)^{\frac{1}{4}}}{\sqrt{2^n n!}}e^{-\frac{\Omega^2\sigma^2}{2}}H_n(\Omega\sigma), \tag{A21}$$

and $H_n(x)$ is the Hermit polynomial.

We keep the result to order $\mathcal{O}(a^4\sigma^6)$ and obtain

$$\begin{aligned} P_{LO}^{(G)} &= \sigma^2 e^{-\Omega^2\sigma^2}\left\{\left(\Omega^2\sigma^2\,{}_2F_2\left(\left.\begin{matrix}1,1\\ \frac{3}{2},2\end{matrix}\right|\Omega^2\sigma^2\right)\right.\right.\\ &\quad\left. -\log\frac{m\sigma}{2} - \frac{\pi}{2}\mathrm{erfi}(\Omega\sigma) - \frac{\gamma_E}{2}\right)\\ &\quad -\frac{a^2\sigma^2}{12}\left(1 - 2\Omega^2\sigma^2 - \frac{a^2\sigma^2}{60}\right.\\ &\quad\left.\left.\times\left(4\Omega^4\sigma^4 - 12\Omega^2\sigma^2 + 3\right)\right)\right\} + \mathcal{O}(a^6\sigma^8). \end{aligned} \tag{A22}$$

A.1.2 The square wave switching function

The square wave switching function can be written as

$$\chi^{(S)}(\tau,\sigma) = \frac{1}{2\sigma}H(\sigma-\tau)H(\sigma+\tau). \tag{A23}$$

where $H(x)$ is the Heaviside step function.

Also using Eq. (A14) and the variable substitution in Eq. (A16), we can easily integrate T out and obtain

$$\begin{aligned} P_\pm^{(S)} &= \frac{1}{\sigma}\mathrm{Re}\left[\frac{1}{4\pi}\int_0^{2\sigma} dt\,(2\sigma - t)\exp(-i\Omega t)\right.\\ &\quad\times\left(2\log\frac{2a}{m} - 2\log\left(2i\sinh\left(\frac{at}{2}\right)\right) - 2\gamma_E + \frac{1}{4}m^2t^2\right.\\ &\quad\left.\left.(2\log(imt) - 2 + 2\gamma_E - \log(4))\right)\right] + \mathcal{O}(m^3). \end{aligned} \tag{A24}$$

Similarly, we use the expansion in Eq. (A19) and find

$$\begin{aligned} P_\pm^{(S)} &= \frac{1}{2\sigma}\left(P_{LO}^{(S)} + P_{NLO}^{(S)}\right) + \mathcal{O}(m^3),\\ P_{LO}^{(S)} &= \frac{1}{2\pi\Omega^2}\left\{-2\mathrm{Ci}(2\Omega\sigma) - 2\log\left(\frac{1}{m\sigma}\right)\cos(2\Omega\sigma)\right.\\ &\quad +2\log\left(\frac{2\Omega}{m}\right) + 4\Omega s\mathrm{Si}(2\Omega\sigma) - 2\pi\Omega\sigma\\ &\quad -2 + \pi\sin(2\Omega\sigma) + 2\gamma\cos(2\Omega\sigma) + 2\cos(2\Omega\sigma)\\ &\quad +a^2\frac{2\Omega^2\sigma^2\cos(2\Omega\sigma) - 4\Omega\sigma\sin(2\Omega\sigma) - 3\cos(2\Omega\sigma) + 3}{6\Omega^2}\\ &\quad +\frac{a^4}{180\Omega^4}\left(-2\Omega^4\sigma^4\cos(2\Omega\sigma) + 8\Omega^3\sigma^3\sin(2\Omega\sigma)\right.\\ &\quad +18\Omega^2\sigma^2\cos(2\Omega\sigma) - 24\Omega\sigma\sin(2\Omega\sigma)\\ &\quad\left.\left. -15\cos(2\Omega\sigma) + 15\right) + \mathcal{O}\left(\frac{\sigma^8a^6}{\Omega^2}\right)\right\},\\ P_{NLO}^{(S)} &= \frac{m^2}{4\Omega^4\pi}\left\{\log\left(\frac{64\Omega^6}{m^6}\right) - 6\mathrm{Ci}\,(2\Omega\sigma)\right.\\ &\quad +\sin(2\Omega\sigma)\left(8\Omega\sigma\log(m\sigma) - 2\pi\Omega^2\sigma^2\right.\\ &\quad +4(2\gamma_E - 1)\Omega\sigma + 3\pi) + \cos(2\Omega\sigma)\left(\left(6 - 4\Omega^2\sigma^2\right)\log(m\sigma)\right.\\ &\quad -4(\gamma_E - 1)\Omega^2\sigma^2 - 4\pi\Omega\sigma\\ &\quad\left. +6\gamma_E + 5) + 4\Omega\sigma\mathrm{Si}(2\Omega\sigma) - 2\pi\Omega\sigma - 5\right\}, \end{aligned} \tag{A25}$$

where Ci and Si are cosine and sine integral functions defined as

$$\mathrm{Ci}(z) \equiv -\int_z^\infty dt\frac{\cos(t)}{t}, \quad \mathrm{Si}(z) \equiv \int_0^z dt\frac{\sin(t)}{t}. \tag{A26}$$

A.2 The case of D = 3 + 1

In the case of $D = 3 + 1$, the integral is UV divergent. However, we can still extract how $P_\pm$ depends on a with small mass and small a. Expanding the integrand over a, we obtain

$$\begin{aligned} &\exp\left(i\frac{i\omega}{a}\sinh(a\tau) - \frac{ik_x}{a}(\cosh(a\tau) - 1)\right)\\ &= e^{i\tau\omega} - \frac{1}{2}ie^{i\tau\omega}k_xt^2a\\ &\quad - \frac{1}{24}a^2\left(\tau^3e^{i\tau\omega}\left(3k_x^2\tau - 4i\omega\right)\right) + \mathcal{O}(a^3). \end{aligned} \tag{A27}$$

We integrate over τ using Gaussian switching function, and find

$$
\begin{aligned}
I \equiv & \left| \int_{-\infty}^{\infty} dt e^{-\frac{t^2}{2\sigma^2}} e^{i\Omega t} \exp \right. \\
& \left. \times \left(i \frac{i\omega}{a} \sinh(at) - \frac{ik_x}{a} (\cosh(at) - 1) \right) \right|^2 \\
& = I_a^{(0)} + I_a^{(2)} + \mathcal{O}(a^4), \\
I_a^{(0)} = & \, 2\pi\sigma^2 \exp\left(-(\omega + \Omega)^2 \sigma^2 \right), \\
I_a^{(2)} = & \, \frac{1}{3}\pi a^2 \sigma^6 e^{-(\omega+\Omega)^2\sigma^2} \left(6k_x^2 \Omega^2 \sigma^2 + 12k_x^2 \Omega \sigma^2 \omega \right. \\
& + 6k_x^2 \sigma^2 \omega^2 - 3k_x^2 + 2\Omega^3 \sigma^2 \omega \\
& \left. + 6\Omega^2 \sigma^2 \omega^2 + 6\Omega \sigma^2 \omega^3 - 6\Omega\omega + 2\sigma^2 \omega^4 - 6\omega^2 \right).
\end{aligned}
\tag{A28}
$$

Using

$$
\int d^d k f(|k|) k_i^2 = \int d^d k f(|k|) \frac{k^2}{d}, \tag{A29}
$$

we obtain

$$
\begin{aligned}
& \frac{1}{16\pi^3} \int d^d k \frac{1}{\omega} I_a^{(2)} = \frac{1}{4\pi^2} \int_0^{\infty} dk \frac{k^2}{\omega} \frac{1}{3} \pi a^2 \sigma^6 e^{-(\omega+\Omega)^2\sigma^2} \\
& \times \left(2k^2 \Omega^2 \sigma^2 + 4k^2 \Omega \sigma^2 \omega + 2k^2 \sigma^2 \omega^2 - k^2 \right. \\
& \left. + 2\Omega^3 \sigma^2 \omega + 6\Omega^2 \sigma^2 \omega^2 + 6\Omega \sigma^2 \omega^3 - 6\Omega\omega + 2\sigma^2 \omega^4 - 6\omega^2 \right).
\end{aligned}
\tag{A30}
$$

It is possible to obtain analytical results when $m \to 0$, that is,

$$
\begin{aligned}
P_{\pm}^{D=3+1} = & \frac{1}{2\pi\sigma^2} \left(P_a^{(0)} + P_a^{(2)} \right) + \mathcal{O}(a^4) \\
P_a^{(2)} = & \frac{a^2 \sigma^6}{12\pi} \int_0^{\infty} dk k^2 e^{-(k+\Omega)^2\sigma^2} \\
& \times \left(8k\Omega^2 \sigma^2 + 10k^2 \Omega \sigma^2 + 4k^3 \sigma^2 - 7k \right. \\
& \left. + 2\Omega^3 \sigma^2 - 6\Omega \right) + \mathcal{O}(m) = \frac{a^2 \sigma^2}{24\pi} e^{-\Omega^2 \sigma^2} + \mathcal{O}(m).
\end{aligned}
\tag{A31}
$$

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