Eur. Phys. J. C (2023) 83:1072
https://doi.org/10.1140/epjc/s10052-023-12246-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Thermodynamics and tachyon condensation of the dressed-dynamical unstable Dp-branes

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Received: 28 June 2023 / Accepted: 11 November 2023 / Published online: 23 November 2023

Abstract Using the boundary state formalism and thermo field dynamics approach, we study a Dp-brane at finite temperature in which the background Kalb–Ramond field, a $U(1)$ gauge potential, and tachyon field are turned on together with a general tangential dynamics. The thermal entropy of the brane will be studied. In addition, the behavior of the entropy after the tachyon condensation process will be investigated and some thermodynamic interpretations will be extracted.

1 Introduction

Dp-branes are absolutely necessary to comprehend string theory and its relationship to field theories and gravity [1]. Since the introduction of Dp-branes, some of the most fundamental physical results in string theory have been discovered by examining their properties [2,3]. In addition, they have significantly contributed to our grasp of dualities [1,3]. Furthermore, by adding dynamics, various backgrounds, and internal fields to the brane, several intriguing features of the Dp-branes can be revealed through so-called boundary state formalism [4–40].

Among various configurations, thermalizing Dp-branes have been a considerable focus of research. On the one hand, the relationship between Dp-branes and field theory at finite temperature is an intriguing subject in and of itself that might aid in our comprehension of the physical features of Dp-branes. In the low energy limit of string theory, where Dp-branes are solitonic solutions to supergravity, some investigation has been made in this area. In this limit, the thermodynamics of Dp-branes have been stated within the context of path-integral field theory formulation at finite temperature [41–48]. On the other hand, they have been utilized to understand the statistical features of different systems, such as the Hawking temperature, energy-entropy relation and Hagedorn transition of extreme, near-extreme, and Schwarzschild black holes [49–62]. Despite the relative understanding of strings at finite temperature and the promising discoveries from Dp-brane ensembles, little is known about the statistical features of Dp-branes.

To implement temperature to the structure of Dp-branes, one must be able to modify, at the Fock space level, an adaption from zero temperature to finite temperature. Since in the boundary state language, the Dp-brane is expressed in terms of string operators acting on the vacuum, a convenient way is to employ the thermo field dynamics (TFD) formalism [63,64]. Uniting such an approach with the D-branes boundary state formalism and generalizing it to include more extending configurations has been accomplished in remarkable Refs. [65–83]. Previously, TFD was used to examine the renormalization of open bosonic strings at finite temperature. Also, the compatibility of the renormalization with the thermal Veneziano amplitude has been demonstrated [84–88]. In Ref. [89], the global phase structure of the thermal bosonic string ensemble and its connection to the thermal string amplitude are explained.

In addition to thermalizing Dp-branes, the existence of an open string tachyon on it inherently renders it unstable, which is another fascinating topic in string theory. Sen conjectured that the open string tachyon condensation represents the decay of unstable Dp-branes into the closed string vacuum [90–95]. In other words, Through the process of tachyon condensation, An unstable Dp-brane subsequently either decays to a stable D$(p-1)$-brane or collapses to the closed string vacuum. This indicates that Dp-branes may be established as a source of closed string [96–100]. Using the boundary state and tachyon condensation, it is possible to determine the time evolution of the source for each mode of a closed string [101,102]. Also, it has been claimed that the boundary state description of the rolling tachyon is valid for

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a finite duration dictated by string coupling, after which we may encounter energy dissipation into the bulk [103]. Such physical phenomena, which are involved with the decay of an object, resulted in the development of the background-independent string theory formulation.

The thermal feature of the Dp-brane, conjointly with the turned-on tachyonic field on it, motivated and stimulated us to study the entropy of the brane. However, to construct the most general thermal boundary state, some other extensions have been also implemented. Precisely, in this paper, we calculate the entropy of the bosonic Dp-brane dressed with a $U(1)$ gauge potential and a tachyonic profile in the presence of the background B-field which is thermalized in the context of the TFD approach. We also considered general tangential dynamics to the brane. In addition, the entropy of the brane is also computed after tachyon condensation. As will be shown, since the tachyon condensation leads to a change in value of the entropy, the second law of thermodynamics will also be examined for our system.

This paper is organized as follows. In Sect. 2, the boundary state corresponding to a dressed-dynamical unstable Dp-branes at zero temperature is reviewed. In Sect. 3, we shall construct the general thermal boundary state when all fields in our configuration are turned on together with general tangential dynamics. We calculate the entropy of the Dp-brane. In Sect. 4, at first, the tachyon condensation is briefly introduced, and, after that, the effect of the tachyon condensation on the entropy is studied. Besides, some thermodynamical interpretations will be provided. Section 5 is devoted to the conclusions.

2 Review of the boundary state

In this section, the boundary state corresponds to an unstable-dressed Dp-brane with tangential dynamics in the zero temperature, $T=0$, is introduced. Consequently, we start with the following sigma-model action for closed string

$$S=-\frac{1}{4\pi\alpha'}\int_{\Sigma}\mathrm{d}^2\sigma\left(\sqrt{-\mathcal{G}}\mathcal{G}^{ab}G_{\mu\nu}+\varepsilon^{ab}B_{\mu\nu}\right)\partial_aX^\mu\partial_bX^\nu +\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}\mathrm{d}\sigma\left(A_\alpha\partial_\sigma X^\alpha+\omega_{\alpha\beta}J_\tau^{\alpha\beta}+T(X^\alpha)\right),\quad(1)$$

where Σ is the closed string worldsheet, while $\partial\Sigma$ indicates its boundary. To avoid dealing with ghosts, we choose to work in the light-cone gauge $X^0\pm X^{d-1}$. Hence, $\mu,\nu\in\{1,\ldots,d-2\}$, $\alpha,\beta\in\{1,\ldots,p\}$ and $i,j\in\{p+1,\ldots,d-2\}$ are spacetime indices, worldvolume directions of the Dp-brane, and its perpendicular directions in light-cone coordinates, respectively. Additionally, the metrics of the target spacetime are denoted by $\eta_{\mu\nu}=\mathrm{diag}(-1,+1,\ldots,+1)$, whereas $\mathcal{G}_{ab}$ with $a,b\in\{\tau,\sigma\}$ signifies the metrics of the string worldsheet. The background fields are the constant Kalb–Ramond field $B_{\mu\nu}$, the $U(1)$ internal gauge field A_α and the open string tachyon field $T(X^\alpha)$. In order to preserve the quadratic structure of the action to become path integrally solvable, we employ the Landau gauge $A_\alpha=-\frac{1}{2}F_{\alpha\beta}X^\beta$ with the constant field strength $F_{\alpha\beta}$, and the tachyon profile $T=\frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$ where $U_{\alpha\beta}$ is a constant symmetric matrix. The constant antisymmetric tensor $\omega_{\alpha\beta}$ with the explicit form $\omega_{\alpha\beta}J_\tau^{\alpha\beta}=2\omega_{\alpha\beta}X^\alpha\partial_\tau X^\beta$ designates the angular velocity which expresses the tangential dynamics of the brane. Due to the presence of background fields on the worldvolume of the brane, the Lorentz symmetry has been manifestly broken. Thus, the tangential dynamics of the brane in the directions of its worldvolume is sensible.

By setting the variation of the action to zero we receive the equation of motion and the following boundary state equations

$$\left[(\eta_{\alpha\beta}+4\omega_{\alpha\beta})\partial_\tau X^\beta+\mathcal{F}_{\alpha\beta}\partial_\sigma X^\beta+U_{\alpha\beta}X^\beta\right]_{\tau=0}|\mathcal{B}\rangle=0,$$
$$\delta X^i|_{\tau=0}|\mathcal{B}\rangle=0,\quad(2)$$

where the total field strength is $\mathcal{F}_{\alpha\beta}=F_{\alpha\beta}-B_{\alpha\beta}$.

The well-known closed string mode expansion simply allows us to rewrite Eq. (2), in terms of the zero modes and oscillators. The coherent state method enables us to obtain the solution of the oscillatory portion of the boundary state equations

$$|\mathcal{B}\rangle^{(\mathrm{osc})}=\prod_{n=1}^{\infty}[\det Q_{(n)}]^{-1}\times\exp\left[-\sum_{m=1}^{\infty}A_m^{\dagger\mu}\Omega_{(m)\mu\nu}B_m^{\dagger\nu}\right]|0\rangle_\alpha\otimes|0\rangle_\beta,\quad(3)$$

where we have used α_n^μ and β_n^μ as left- and right-moving oscillators, respectively. Furthermore, the subsequent notations were used

$$A_n^\mu=\frac{\alpha_n^\mu}{\sqrt{n}},\quad A_n^{\dagger\mu}=\frac{\alpha_{-n}^\mu}{\sqrt{n}},$$
$$B_n^\mu=\frac{\beta_n^\mu}{\sqrt{n}},\quad B_n^{\dagger\mu}=\frac{\beta_{-n}^\mu}{\sqrt{n}},\quad(4)$$

for $n>0$ with the algebras

$$[A_n^\mu,A_m^{\dagger\nu}]=[B_n^\mu,B_m^{\dagger\nu}]=\delta_{n,m}\eta^{\mu\nu},$$
$$[B_n^\mu,A_m^{\dagger\nu}]=[A_n^\mu,B_m^{\dagger\nu}]=\text{all other commutators}=0.\quad(5)$$

The normalization factor $\prod_{n=1}^{\infty}[\det Q_{(n)}]^{-1}$ in Eq. (3) comes from the disk partition function [35,36]. The mode-dependent matrices possess the following definitions

$$Q_{(m)\alpha\beta}=\eta_{\alpha\beta}+4\omega_{\alpha\beta}-\mathcal{F}_{\alpha\beta}+\frac{i}{2m}U_{\alpha\beta},\quad(6)$$
$$\Omega_{(m)\mu\nu}=(\Delta_{(m)\alpha\beta}\ ,\ -\delta_{ij}),\quad(7)$$
$$\Delta_{(m)\alpha\beta}=(Q_{(m)}^{-1}N_{(m)})_{\alpha\beta},\quad(8)$$

$$N_{(m)\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta} + \mathcal{F}_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta}. \tag{9}$$

For the zero-mode part of the boundary state, by using higher dimensional Gaussian integral, we receive the following expression

$$|\mathcal{B}\rangle^{(0)} = \sqrt{\frac{(2\pi)^p}{\det \mathcal{R}}} \prod_{\alpha=1}^{p} |p^\alpha\rangle \prod_i \delta(x^i - y^i)|p^i\rangle, \tag{10}$$

where $\mathcal{R}_{\alpha\beta}$ possesses the definition

$$\mathcal{R}_{\alpha\beta} = -2i\alpha'\left[\mathcal{U} + U^{-1}(\eta+4\omega) + (\eta+4\omega)^T U^{-1}\right]_{\alpha\beta},$$

$$\mathcal{U} = \begin{pmatrix} \left(U^{-1}(\eta+4\omega)\right)_{11} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left(U^{-1}(\eta+4\omega)\right)_{pp} \end{pmatrix}.$$

In the light-cone gauge, one can construct the total boundary state as

$$|\mathcal{B}\rangle^{(\text{tot})} = \frac{\mathcal{T}_p}{2} |\mathcal{B}\rangle^{(\text{osc})} \otimes |\mathcal{B}\rangle^{(0)}, \tag{11}$$

where $\mathcal{T}_p$ is the Dp-brane tension.

3 The entropy

In this section, at first, the boundary states corresponding to a dressed-dynamical unstable Dp-branes at finite temperatures are constructed. Given that the Eqs. (2)–(11) are represented in terms of bosonic string operators and states, these entities must be merely mapped at $T \neq 0$. An intriguing way to do that is by utilizing TFD approach. To ensure that the content of this paper may be understood without being side-tracked by excessive computations, all fields of the brane are independent of temperature.

According to the TFD approach, the thermodynamics of the system is given in an extended Fock space consisting of the original Fock space and an identical copy of it (this is indeed true for the path-integral approach). The total thermic system is comprised of the original string and its duplicate indicated as 'tilde' strings. These two copies are independent and the total Fock space is $\mathcal{H}_{\text{total}} = \mathcal{H} \otimes \tilde{\mathcal{H}}$. To execute this construction in the context of the bosonic string theory, we use $\tilde{A}_n^\mu$, $\tilde{B}_n^\mu$ and... as identical operators belonging to the copy of the system with independent algebras.

From now on we use the usual notation in the literature to compute the entropy. According to the Refs. [63–80], $|\,\rangle\rangle$ signifies a state from $\mathcal{H}_{\text{total}}$. The vacuum state is given by $|0\rangle\rangle^{(\text{osc})} \equiv |0\rangle\rangle_\alpha^{(\text{osc})} \otimes |0\rangle\rangle_\beta^{(\text{osc})}$. Owing to the fact that two Fock spaces are independent, we may write vacuums as $|0\rangle\rangle_{(\alpha,\beta)}^{(\text{osc})} = |0\rangle_{(\alpha,\beta)}^{(\text{osc})} \otimes \widetilde{|0\rangle}_{(\alpha,\beta)}^{(\text{osc})}$. One should note that to have a total vacuum state, the enlarging procedure must be also applied to the zero-modes.

Thermal features of the system in $\mathcal{H}$ space are accomplished by introducing a set of Bogoliubov unitary operators, i.e,

$$|0(\beta_T)\rangle\rangle_{(\alpha,\beta)} = \prod_{n>0} e^{-i\mathcal{G}_n^{(\alpha,\beta)}} |0\rangle\rangle_{(\alpha,\beta)}, \tag{12}$$

in which

$$\mathcal{G}_n^\alpha = -i\theta(\beta_T)\left(A_n.\tilde{A}_n - A_n^\dagger.\tilde{A}_n^\dagger\right), \tag{13}$$

$$\mathcal{G}_n^\beta = -i\theta(\beta_T)\left(B_n.\tilde{B}_n - B_n^\dagger.\tilde{B}_n^\dagger\right), \tag{14}$$

acting on the states and on the operators of the enlarged space. In Eqs. (13) and (14), $\beta_T = (k_B T)^{-1}$ where k_B the Boltzmann's constant and $\theta(\beta_T)$ is a temperature parameter whose value depends on the mode statistics. Since we merely deal with the bosonic string theory, the value of $\theta(\beta_T)$ is

$$\cosh\theta_n(\beta_T) = u_n(\beta_T) = \frac{1}{\sqrt{1 - e^{-\beta_T w_n}}},$$

$$\sinh\theta_n(\beta_T) = v_n(\beta_T) = \sqrt{\frac{e^{-\beta_T w_n}}{1 - e^{-\beta_T w_n}}}. \tag{15}$$

Given the fact that the Bogoliubov operators do not combine the left- and right-moving states, it also is possible to produce a direct product of the states as $|0(\beta_T)\rangle\rangle = |0(\beta_T)\rangle\rangle_\alpha \otimes |0(\beta_T)\rangle\rangle_\beta$. The action of Bogoliubov transformations on oscillator operators translates them to new temperature-dependent operators through the Heisenberg picture,

$$\begin{aligned} &\{A_n^\mu(\beta_T), \tilde{A}_n^\mu(\beta_T), B_n^\mu(\beta_T), \tilde{B}_n^\mu(\beta_T)\} \\ &\quad = e^{-i\mathcal{G}_n^\alpha}\{A_n^\mu, \tilde{A}_n^\mu, B_n^\mu, \tilde{B}_n^\mu\} e^{i\mathcal{G}_n^\alpha}, \end{aligned} \tag{16}$$

in which the following results can be received

$$\begin{aligned} A_n^\mu(\beta_T) &= u_n(\beta_T)A_n^\mu - v_n(\beta_T)\tilde{A}_n^{\dagger\mu}, \\ \tilde{A}_n^\mu(\beta_T) &= u_n(\beta_T)\tilde{A}_n^\mu - v_n(\beta_T)A_n^{\dagger\mu}, \end{aligned} \tag{17}$$

$$\begin{aligned} B_n^\mu(\beta_T) &= u_n(\beta_T)B_n^\mu - v_n(\beta_T)\tilde{B}_n^{\dagger\mu}, \\ \tilde{B}_n^\mu(\beta_T) &= u_n(\beta_T)\tilde{B}_n^\mu - v_n(\beta_T)B_n^{\dagger\mu}. \end{aligned} \tag{18}$$

At finite temperature, there are three possible formulations for Dp-brane. One of them, which follows in this paper, is to map all associated operators and states to their thermal counterparts (for other possibilities and relations between them, see Refs. [69–76]). Therefore, the boundary state of dressed dynamical unstable Dp-branes at $T \neq 0$ is

$$\begin{aligned} |\mathcal{B}(\beta_T)\rangle\rangle^{(\text{tot})} &= \frac{\mathcal{T}_p^2}{4}\frac{(2\pi)^p}{\det\mathcal{R}}\left\{\prod_{n=1}^{\infty}[\det Q_{(n)}]^{-2}\right\} \\ &\times \prod_{\alpha=1}^{p} |p^\alpha\rangle\widetilde{|p^\alpha\rangle} \prod_{i=p+1}^{d-2} \delta(x^i - y^i)\widetilde{\delta(x^i - y^i)}|p^i\rangle\widetilde{|p^i\rangle} \\ &\times\exp\left[-\sum_{m=1}^{\infty} A_m^\dagger(\beta_T).\Omega_{(m)}.B_m^\dagger(\beta_T)\right] \end{aligned}$$

$$\times \exp\left[-\sum_{k=1}^{\infty}\tilde{A}_k^{\dagger}(\beta_T).\Omega_{(k)}.\tilde{B}_k^{\dagger}(\beta_T)\right]|0(\beta_T)\rangle. \tag{19}$$

Now we study thermal properties of this system. This can be done by introducing the entropy operator in the context of TFD and sandwiching it between the established boundary states (19), i.e,

$$\mathcal{S} = k_B\ {}^{(\mathrm{tot})}\langle\langle\mathcal{B}(\beta_T)|\mathcal{K}(A, B; \theta(\beta_T)|\mathcal{B}(\beta_T)\rangle\rangle^{(\mathrm{tot})}. \tag{20}$$

Since we are dealing with a closed string, the operator $\mathcal{K}$ takes the feature

$$\mathcal{K}(A, B; \theta(\beta_T)) = \sum_{n=1}^{\infty}\Big[(A_n^{\dagger}.A_n + B_n^{\dagger}.B_n)\ln\sinh^2\theta_n(\beta_T) -(A_n.A_n^{\dagger} + B_n.B_n^{\dagger})\ln\cosh^2\theta_n(\beta_T)\Big]. \tag{21}$$

Equations (20) and (21) can be rewritten to include tilde strings. However, given the postulation of Refs. [63,64], to achieve the physical properties of the system, the computation of $\tilde{\mathcal{S}} = k_B\ {}^{(\mathrm{tot})}\langle\langle\mathcal{B}(\beta_T)|\tilde{\mathcal{K}}(A, B; \theta(\beta_T)|\mathcal{B}(\beta_T)\rangle\rangle^{(\mathrm{tot})}$ must be dropped out.

By plugging Eqs. (19) and (21) into Eq. (20), The entropy of the dressed-dynamical unstable Dp-brane is

$$\mathcal{S} = \mathcal{T}_p^2 k_B\frac{(d-2)\mathcal{V}_{d-2}\tilde{\mathcal{V}}_{d-2}}{(2\pi)^{2(d-2p-2)}}\det{}^{-1}_{p\times p}(\mathcal{R}^{\dagger}\mathcal{R}) \times\left[\prod_{s=1}^{\infty}[\det{}_{p\times p}Q_{(s)}]^{-4}\right] \times\sum_{n=1}^{\infty}\xi_n(\theta_n(\beta_T))\prod_{m=1}^{\infty}\left[\det\left(\mathbb{I}-\Omega_{(m)}^{\dagger}\Omega_{(m)}\right)\right]^{-2}, \tag{22}$$

where $\mathcal{V}_{d-2}$ and $\tilde{\mathcal{V}}_{d-2}$ are the volume of the $\mathcal{H}$-space and $\tilde{\mathcal{H}}$-space in light-cone gauge, respectively, and

$$\xi_n(\theta_n(\beta_T)) \equiv \ln\sinh^2\theta_n(\beta_T) +(1+3\sinh^2\theta_n(\beta_T))\ln\tanh^2\theta_n(\beta_T), \tag{23}$$

is the thermal function associated with the entropy. In the $T\to 0$ limit, the contribution of the oscillators diverges. In contrast, in $T\to\infty$ limit, the oscillator's contribution is proportional to $\ln(-1)$. This may indicate that the concept of temperature breaks down at arbitrarily high temperatures owing to similar phenomena that happen at Hagedorn temperature in string theory. Some plausible explanations are provided in Refs. [65–76] in order to comprehend these values of entropy.

Another interesting result is that if we simply turn on the tachyon profile and off all background and internal fields, together with the tangential dynamics of the Dp-brane, the resulted entropy will be zero. This may demonstrate the difference between the tachyonic instability and the thermal instability of the D-brane.

Due to the presence of the tachyon field and the general tangential dynamics of the branes, our results are more general than those presented in Refs. [65–76]. However, by quenching the tachyonic field, the matrix $\Delta_{(n)}$ becomes mode-independent, and the results in mentioned reference are obtained when $\omega_{\alpha\beta}\to 0$ is also applied. With the attainment of entropy, it becomes feasible to deduce various other thermodynamic properties. Given the renormalization problem of the string tension from a finite-temperature renormalization-group approach, however, calculating the free energy, which helps us better comprehend phase transition and thermodynamic stability of the system, is quite difficult in the context of TFD [84–88,104,105]. This may indicate future research that must be conducted within the context of Dp-branes.

4 The effects of the tachyon condensation on the entropy

When Dp-branes are studied in the presence of open string tachyonic fields, instability of it, whose investigation is crucial to our understanding of the vacuum of string theory, will take place. Through so-called tachyon condensation, a phase transition follows. During this process, the Dp-brane collapses drastically, and we are left with a collection of closed strings.

From a mathematical standpoint, at least one element of the tachyon matrix $U_{\alpha\beta}$ must be infinite to form the tachyon condensation. For simplicity, we impose the condensation of the tachyon field just in the x^p-direction, that is, $U_{pp}\to\infty$. There are three matrices involving the tachyonic field in Eq. (22); $\mathcal{R}$, $Q_{(n)}$ and $\Delta_{(n)}$, in which the limit must be considered. By implementing $U_{pp}\to\infty$, we have $\lim_{U_{pp}\to\infty}(U^{-1})_{p\alpha} = \lim_{U_{pp}\to\infty}(U^{-1})_{\alpha p} = 0$. Consequently, the matrix $\mathcal{R}$ loses its final row and column. Let us denote it as $\bar{\mathcal{R}}$. Similarly, the effect of tachyon condensation on the component $\prod_{n=1}^{\infty}[\det_{p\times p}Q_{(n)}]^{-4}$, with the use of zeta function regularization, becomes

$$\lim_{U_{pp}\to\infty}\prod_{n=1}^{\infty}[\det{}_{p\times p}Q_{(n)}]^{-4} = \pi^2 U_{pp}^2\prod_{n=1}^{\infty}\left[\det{}_{(p-1)\times(p-1)}Q_{(n)}^{[p-1]}\right]^{-4}, \tag{24}$$

where the matrix $Q_{(n)}^{[p-1]}$ can be obtained by removing the final row and column of matrix $Q_{(n)}$, resulting in a $(p-1)\times(p-1)$ matrix. The limit of the matrix $\Delta_{(n)}$, since it is not the product of the limits of $Q_{(m)}^{-1}$ and $N_{(m)}$, must be determined after the executing the product $Q_{(m)}^{-1}N_{(m)}$. It gives rise to

$$\lim_{U_{pp}\to\infty}\Delta_{(m)} = \begin{pmatrix}\left(\mathbf{\Delta}_{(m)}\right)_{(p-1)\times(p-1)} & \mathbf{0}_{(p-1)\times 1}\\ \mathbf{0}_{1\times(p-1)} & -1\end{pmatrix}, \tag{25}$$

where

$$\left(\mathbf{\Delta}_{(m)}\right)_{(p-1)\times(p-1)} = \left(Q^{[p-1]}_{(m)}\right)^{-1} N^{[p-1]}_{(m)}. \quad (26)$$

Now it is evident that a Neumann direction (x^p-direction) has been altered into a Dirichlet direction. Hence, the tachyon condensation phenomenon deforms an unstable Dp-brane into a stable D$(p-1)$-brane.

Adding all these together, the effect of the tachyon condensation on the entropy, Eq. (22), takes the feature

$$\mathcal{S}' = (d-2)k_B \frac{(U_{pp}\mathcal{T}_p)^2 \mathcal{V}_{d-2}\tilde{\mathcal{V}}_{d-2}}{4(2\pi)^{2(d-2p-3)}} \det{}^{-1}_{(p-1)\times(p-1)}(\bar{\mathcal{R}}^\dagger \bar{\mathcal{R}}) \times \left[\prod_{s=1}^{\infty} [\det{}_{p\times p} Q^{[p-1]}_{(s)}]^{-4}\right] \times \sum_{n=1}^{\infty} \xi_n(\theta_n(\beta_T)) \prod_{m=1}^{\infty} \left[\det\left(\mathbb{I} - \mathbf{\Omega}^\dagger_{(m)}\mathbf{\Omega}_{(m)}\right)\right]^{-2}, \quad (27)$$

where $\mathbf{\Omega}_{(m)\mu\nu} \equiv \left\{ \left[\left(Q^{[p-1]}_{(m)}\right)^{-1} N^{[p-1]}_{(m)} \right]_{\alpha'\beta'}, -\delta_{i'j'} \right\}$ in which $\alpha', \beta' \in \{1, \ldots, p-1\}$ and $i', j' \in \{p, \ldots d-2\}$. From Eq. (27), one can deduce that portions that consist of the thermal factor remain the same. However, due to the tachyon condensation, the terms corresponding to the fields and dynamics are drastically changed which results in a new value for entropy. Nevertheless, Eq. (27) is not just the entropy of a stable brane. It comprises both the entropy of D$(p-1)$-brane and the entropy associated with the closed strings that were produced during the collapse of the original brane. As can be seen, entropy (27) is a complicated function in terms of temperature, fields and dynamics of the configuration, making their separation rather impossible.

4.1 The second law of the thermodynamics

In this part, we explored some thermodynamic aspects of the process of tachyon condensation. Since the system evolves from its initial state, i.e, the original Dp-brane, to its final state, which is the D$(p-1)$-brane and the released closed strings, studying the second law of thermodynamics must be interesting. Hence, we must examine the validity of the inequality $\Delta\mathcal{S} \equiv \mathcal{S}' - \mathcal{S} > 0$. Given that in Eqs. (27) and (22), the determinant factors are always positive, in the limit of $U_{pp} \to \infty$, the inequality is valid if and only if $\xi(\theta_n(\beta_T))$ is positive which signals the positivity of the entropy. This implies that

$$\ln(e^{\beta_T\omega_n} - 1) + \beta_T\omega_n \frac{e^{\beta_T} + 2}{e^{\beta_T} - 1} < 0. \quad (28)$$

In the other words, the second law of thermodynamics holds true when the frequency value of the nth oscillator, denoted as ω_n, are within the range of $nk_BT \ln X$ where $X \approx (1, \sqrt{\pi/3})$. This condition significantly limits the frequency levels of it.

It should be noted that, in the context of TFD approach, the attainment of a physically admissible configuration that adheres to the principles of the second law of thermodynamics necessitates a positive entropy. Consequently, the instability of the brane, due to the presence of an open string tachyonic field on it, should not cause a negative value in entropy.

5 Conclusions

We presented a boundary state, corresponding to a thermal-dynamical Dp-brane in the presence of an internal $U(1)$ gauge potential, an open string tachyon field and a non-zero constant Kalb–Ramond field. For implementing the thermal aspect to the setup, the temperature has been encoded in the operators using the TFD approach. This enabled us to map all operators in $T = 0$ to $T \neq 0$. Then, the thermal entropy of such branes was computed. By assessing the resulting entropy at $T \to 0$, we observed that the entropy diverges. Besides, in the limit $T \to \infty$, the oscillator contribution to the entropy is proportional to $\ln(-1)$. Owing to the similar phenomena in the string theory at the Hagedorn temperature, this may imply that the concept of temperature breaks down at arbitrary high temperatures.

Note that in simpler setups similar conclusions have been also provided in the Refs. [65–80]. This clarifies that, in the context of the TFD approach, the resultant entropy in the limits $T \to 0$ and $T \to \infty$ is independent of any field and dynamics.

We examined the effect of the tachyon condensation in our setup and in the thermal entropy. Since the thermal factors, associated with the entropy are unaffected by the tachyon condensation, the study of the entropy in the limits of low- and high-temperatures yields the same results as the setup before the tachyon condensation. Despite this, as it can be seen in Eq. (27), the effect of this phenomenon profoundly manifests in the fields and dynamic contributors of entropy.

Finally, we examined the existence of the second law of thermodynamics for our setup during the tachyon condensation process. We observed that if Dp-brane is in a thermal non-equilibrium state, the second law of thermodynamics will be violated after tachyon condensation. Therefore, to maintain a physical system, as long as one considers tachyon condensation, the Dp-brane must not possess negentropy.

Acknowledgements The author would like to thank D. Kamani for the useful discussions and for reading the manuscript.

Data availability This manuscript has no associated data or the data will not be deposited. [Authors' comment: The published paper is a theoretical work. No experimental data were used.]

Funded by SCOAP^3. SCOAP^3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

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