



Purely virtual extension of quantum field theory for gauge invariant fields: quantum gravity

Damiano Anselmi^{1,2,a}

¹ Dipartimento di Fisica “E.Fermi”, Università di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

² INFN, Sezione di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

Received: 21 June 2023 / Accepted: 6 November 2023 / Published online: 21 November 2023
© The Author(s) 2023

Abstract Quantum gravity is extended to include purely virtual “cloud sectors”, which allow us to define a complete set of point-dependent observables, including a gauge invariant metric and gauge invariant matter fields, and calculate their off-shell correlation functions perturbatively. The ordinary on-shell correlation functions and the S matrix elements are unaffected. Each extra sector is made of a cloud field, its anticommuting partner, a “cloud-fixing” function and a cloud Faddeev-Popov determinant. The additional fields are purely virtual, to ensure that no ghosts propagate. The extension is unitary. In particular, the off-shell, diagrammatic version of the optical theorem holds. The one-loop two-point functions of dressed scalars, vectors and gravitons are calculated. Their absorptive parts are positive, cloud independent and gauge independent, while they are unphysical if non purely virtual clouds are used. We illustrate the differences between our approach to the problem of finding a complete set of observables in quantum gravity and other approaches available in the literature.

1 Introduction

Defining point-dependent observables in general relativity is tricky, because the coordinates are not physical quantities, but just parametrizations of the location. A simple way out is available when the spacetime point is associated with a matter distribution. Consider, for definiteness, four scalar fields $\phi^i(x)$, $i = 1, \dots, 4$, and assume that they depend on the coordinates x^μ in such a way that it is possible to invert x^μ as functions $x^\mu(\phi)$ of ϕ^i . Then, every further field, say a fifth scalar $\varphi(x)$, can be written as a function of the reference fields ϕ^i : $\varphi(x) \rightarrow \varphi(x(\phi)) \equiv \tilde{\varphi}(\phi)$. The function $\tilde{\varphi}(\phi)$ is obviously invariant under general changes of coordinates.

^a e-mail: damiano.anselmi@unipi.it (corresponding author)

The basic idea is to go back to the physical location every time we change coordinates. The ϕ^i do not need to be new, independent fields, but can be functions of the metric itself.

This line of thinking has been pursued in the literature for a long time [1–12, 15, 16]. In Refs. [2–5] Komar and Bergmann use functions of the metric. In Refs. [6–12] the fields ϕ^i describe physical matter. Donnelly and Giddings [15, 16] view them as functions of the metric, for purposes similar to the Coulomb-Dirac dressing of QED [17], the Lavelle–McMullan dressing of non-Abelian gauge theories [18–20], the worldline dressing and the Wilson lines.

Ultimately, the presence of the observer, which is “matter”, is what breaks general covariance, so we may want to view the reference fields as independent matter. In this spirit, we have to take into account that the fundamental theory is changed by the presence of the fields ϕ^i . In Ref. [12] Rovelli questions the need to change the physical world for this purpose and proposes an improvement inspired by the GPS technology, based on a minimal amount of additional matter. Yet, one still needs to provide the physics of the additional matter and add it to the physics of the fundamental theory. Viewing the reference fields as functions of the metric is more appealing from the conceptual point of view, since it does not force us to leave the realm of pure gravity.

These problems are more challenging at the fundamental level, especially in quantum gravity, where they concern our understanding of the fundamental physics of nature.

In this paper we pursue a new strategy, which may shed a different light on the issue. We introduce purely virtual [13, 14] independent fields ζ^μ , which we call cloud fields, and their anticommuting partners H^μ . We extend quantum field theory to include such fields perturbatively in quantum gravity (and general relativity) without affecting the fundamental laws of physics.

The cloud fields play the roles of ϕ^i . Precisely, the fields ϕ^i should be imagined as the differences $x^\mu - \zeta^\mu(x)$. An

arrangement of this type also appears in [15, 16]. In our approach, however, the ζ^μ are neither additional matter, nor functions of the metric, but independent fields, with their own (higher-derivative) propagators, and their own interactions. Moreover, they must be accompanied by anticommuting partners H^μ in a suitable way, and rendered purely virtual, because only in that case the fundamental physics does not change, and no extra degrees of freedom (which would include ghosts) are propagated.

The cloud fields ζ^μ are used to surround the elementary fields of the theory, such as the metric tensor, with appropriate dressings, in order to render them invariant under infinitesimal changes of coordinates. The correlation functions of the dressed fields are new physical quantities, and provide predictions that can in principle be tested experimentally.

The anticommuting partners H^μ are used to endow the extended theory with a certain cloud symmetry, to ensure that the fundamental interactions are unaffected by the presence of the cloud sectors. Specifically: (i) the correlation functions of the undressed fields are unchanged, and (ii) the S matrix amplitudes of the dressed fields coincide with the usual S matrix amplitudes of the undressed fields. Once these goals are achieved, we can concentrate on the new correlation functions. Moreover, we can view the usual (undressed) fields as mere integration and diagrammatic tools, and use dressed fields everywhere else. This way, gauge invariance and gauge independence become manifest in every operation we make.

Because they are independent fields, ζ^μ and H^μ might be viewed as a sort of matter. Then, however, their purely virtual nature makes them “fake matter”. Because they are introduced to be ultimately projected away (that is to say, integrated out), they might be understood as “functions of everything else”, at least in some particular cases (like the classical limit). Nevertheless, they cannot be viewed as functions of the other fields beyond the tree level, since they keep circulating in loops. In view of these remarks, it is better to understand ζ^μ and H^μ as new entities, defined by the very same formalism we develop in the paper.

Note that the dressed metric just propagates the two graviton helicities. In the approaches of Refs. [6–12] it may propagate six degrees of freedom (the additional ones coming from the four reference scalars).

We achieve the goals we have stated in a fully perturbative regime. By construction, the extended theory is local and unitary. Moreover, it is renormalizable (if the underlying gravity theory is renormalizable), up to the cloud sectors, which may be nonrenormalizable due to their arbitrariness. Although the observables that we define are invariant under infinitesimal changes of coordinates, they are not necessarily invariant under global changes of coordinates. From the conceptual and physical points of view, this is what we need: we

break the global symmetry (our observations do that most of the times) without violating unitarity.

In a parallel paper [21], we explore similar issues in gauge theories.

Throughout the paper, “on shell” means on the mass shell, and refers to the S matrix asymptotic states. The correlation functions of the dressed fields differ from the usual correlation functions anytime the external legs are not asymptotic states on the mass shell. The word “virtuality” is used in connection with the concept of pure virtuality, and refers to the removal of all the on-mass-shell contributions to the correlation functions due to a given particle, preserving unitarity [14]. That particle is then called purely virtual.

The notion of pure virtuality relies on a new diagrammatics [14], which allows us to introduce particles that mediate interactions without ever being on shell. The construction is compatible with unitarity, and takes advantage of the possibility of splitting the usual optical theorem [22–27] into independent, spectral optical identities, associated with different (multi)thresholds [14]. When we want to render certain particles χ purely virtual, and calculate diagrams involving them, we need to start from ordinary Feynman diagrams, as if χ were physical particles or ghosts, and remove the contributions of the χ -dependent nontrivial thresholds, as explained in Ref. [14]. Since the spectral optical identities involving those thresholds drop out altogether from the optical theorem, unitarity is manifestly preserved, or enforced (if all the potential ghosts are rendered purely virtual).

The removed degrees of freedom can also be understood as fake particles, or “fakeons”. The main application of this concept is the formulation of a consistent theory of quantum gravity [28], which leads to observationally testable predictions in inflationary cosmology [29]. At the phenomenological level, fakeons evade common constraints that preclude the usage of normal particles [30, 31].

Before plunging into the technical details, we illustrate some applications of the results of this paper. Although the usual S matrix amplitudes do not change, we can define new types of scattering processes. Consider, for example, “short-distance” scattering processes among gravitons (or quarks and gluons), that is to say, processes that occur within distances such that the incoming and outgoing states are not allowed to become free (interactions in a quark gluon plasma, interactions between quarks and gluons at distances that are smaller or much smaller than the proton radius, interactions within the Planck scale, or some slightly larger scales, in strongly interacting quantum gravity, etc.). We cannot advocate the notion of asymptotic state to study such processes. In our language, they do not need to be on the mass shell and, therefore, they may be gauge dependent. The results of this paper show that we can actually define these scattering processes by means of dressed fields. The price is that, although the results we obtain are physical (i.e., they are

gauge invariant and obey the optical theorem), they depend on the clouds, through the cloud parameters, which we denote by $\tilde{\lambda}$. These parameters do not belong to the fundamental theory. Rather, they describe features of the experimental setup (experimental resolutions, finite volume effects, finite temperature effects, dependence on a background, etc.). The $\tilde{\lambda}$ dependence of the results means that it is impossible to eliminate the influence of the observer on the observed phenomenon. Yet, we can get rid of the $\tilde{\lambda}$ dependences at a second stage, by sacrificing a few measurements to calibrate the instrumentation. Once the values of the parameters $\tilde{\lambda}$ are determined, everything else is predicted uniquely, and can be confirmed or falsified. Something similar occurs in the study of infrared divergences of the S matrix amplitudes in gauge and gravity theories [32–35], where it is necessary to specify the resolution of the apparatus. The resolution is also necessary to describe the observation of unstable particles, like the muon [36], which do not admit asymptotic states in a strict sense.

The dependence on the clouds is less surprising if we think that even the gauge invariant correlation functions built by means of Wilson lines depend on the Wilson lines themselves. Different Wilson lines describe situations of different interest, experimentally. Yet, those dependences must be there, since they are a reflection on the choices of the observer.

Another application concerns precisely the infrared behaviors of gauge and gravity theories. The standard way to deal with this problem is to resum another type of dressing, made of soft and collinear photons, gluons and gravitons [32–35]. An alternative way to regularize the infrared divergences is by going a little bit off the mass shell. Working with the usual correlation functions, however, this operation violates gauge invariance. The new correlation functions considered here, made of dressed fields, allow us to go off shell without breaking the local symmetries, and can provide an alternative way to probe the infrared behaviors of gauge and gravity theories.

Before plunging into the topics of the paper, we point out some crucial differences between our methods and purposes, and the ones of other approaches, which may appear to have something in common with ours at first, but are actually very different. We are referring to the Stueckelberg formalism [37] and the compensator-field approach [38]. The former is used to describe massive vectors, which is not our goal here. The latter is used to rephrase the theory in a way that is convenient for several applications, but is not meant to change the physical cohomology.

In particular, after extending the theory, we still define the physical observables as being gauge invariant: they are not required to be invariant under the extra (cloud) transformations. This changes the cohomology of physical observables, and makes the extension nontrivial. A possible source of trouble, however, comes from the fact that the extension

is higher-derivative, and may inject unphysical degrees of freedom in the theory, in the form of ghosts. What saves the day is the last ingredient of our construction, that is to say, the requirement that the whole extension be purely virtual. This makes the construction unitary, and essentially different from the other proposals available in the literature. Moreover, it guarantees that the fundamental spectrum of the theory is unmodified.

On these premises, we manage to build the gauge-invariant, cloud-dependent fields. While the usual correlation functions do not change, we are able to treat new physical correlation functions: those that contain insertions of the gauge-invariant fields, such as point-dependent observables, including a gauge invariant metric tensor, as well as gauge invariant matter fields. We can also consider new scattering processes, like the short distance processes mentioned above. We stress that the ones we study are not just gauge invariant correlation functions: they are (non-asymptotic) correlation functions of gauge invariant fields.

The explicit calculations we make show that if the extra fields are not purely virtual, but quantized by means of the usual Feynman prescription, for example, they cause disasters, by propagating ghosts. This is what marks a crucial distinction between the construction of this paper and its alternatives. In principle, one can form correlation functions of gauge invariant fields in other approaches, including the compensator-field method. Then, however, one must address the problem of ghosts, otherwise the results turn out to be unphysical. This problem also plagues the correlation functions of Wilson lines, as shown in Ref. [21]. It can only be cured, as far as we know today, by resorting to a purely virtual extension. Typically, issues with Lorentz invariance arise in approaches employing Dirac clouds [17], which are of the Coulomb type. These and other problems are overcome, or addressed in much easier ways, in the approach of this paper.

Everything works as long as we keep the usual sector and the cloud sectors to some extent separated. We show it is indeed possible, because renormalization preserves the unmixing. In particular, the functions that define the clouds should be gauge invariant, while the usual gauge-fixing functions should be cloud invariant. In this respect, we recall that restrictions on the gauge-fixing choice are not unusual. A familiar one is adopted in the context of the background field method, where the gauge-fixing must be invariant under the background transformations, otherwise it spoils the virtues of the method. Similarly, the choices we make in our approach highlight a number of virtues that are more difficult to uncover otherwise.

The approach of this paper can offer a better understanding of the physics that lies beyond the realm of scattering processes in quantum field theory, and provides an answer to the problem of finding a complete set of observables in quantum gravity.

Throughout the paper we work with the dimensional regularization [39–42], $\varepsilon = 4 - D$ denoting the difference between the physical dimension and the continued one.

The paper is organized as follows. In Sect. 2 we introduce the fields we need to build the cloud sectors. In Sect. 3 we recall the standard Batalin–Vilkovisky formalism for gravity and the Zinn–Justin master equation. In Sect. 4 we define the cloud sector. In Sect. 5 we show that the ordinary correlation functions are unaffected by the cloud sector, and so are the S matrix amplitudes. In Sect. 6 we build the correlation functions of the dressed fields. In Sect. 7 we prove that the gauge-trivial sector of the theory and the cloud sector are mirrored into one another by a certain duality relation. In Sect. 8 we add several copies of the cloud sector and show that each insertion, in a correlation function, can be dressed with its own, independent cloud. In Sects. 9 and 10 we compute the one-loop two-point functions of the dressed scalars, vectors and gravitons. We do so in Einstein gravity and in quantum gravity with purely virtual particles. In Sect. 9 we work with covariant clouds, and show that the absorptive parts are unphysical. In Sect. 10 we turn to purely virtual clouds, and show that the absorptive parts are then physical. In Sect. 11 we prove that the extended theory is renormalizable. Section 12 contains the conclusions.

2 The cloud field, its anticommuting partner, and the dressed fields

In this section we lay out the basic notions that are needed to build the cloud sectors. Let

$$\delta_\xi g_{\mu\nu} = \xi^\rho \partial_\rho g_{\mu\nu} + g_{\mu\rho} \partial_\nu \xi^\rho + g_{\nu\rho} \partial_\mu \xi^\rho \tag{2.1}$$

denote the transformation of the metric tensor $g_{\mu\nu}$ under infinitesimal changes of coordinates $\delta x^\mu = -\xi^\mu(x)$. The closure relations read

$$[\delta_\xi, \delta_\eta]g_{\mu\nu} = \delta_{[\xi, \eta]}g_{\mu\nu}, \quad [\xi, \eta]^\rho \equiv \eta^\sigma \partial_\sigma \xi^\rho - \xi^\sigma \partial_\sigma \eta^\rho. \tag{2.2}$$

We define the basic cloud field as an independent “vector” $\zeta^\mu(x)$ that transforms according to the rule

$$\delta_\xi \zeta^\mu(x) = \xi^\mu(x - \zeta(x)). \tag{2.3}$$

In the next sections we explain how to include ζ^μ into the action. For the moment, we just study its properties. A relation like (2.3) and similar ones below can be meant as expansions in powers of ζ . From now on, we understand that the argument of a function is x , whenever it is not specified.

It is easy to check that the definition (2.3) is meaningful, since it closes:

$$[\delta_\xi, \delta_\eta]\zeta^\mu = -\xi^\rho(x - \zeta)\eta^\mu_{,\rho}(x - \zeta) + \eta^\rho(x - \zeta)\xi^\mu_{,\rho}(x - \zeta) = \delta_{[\xi, \eta]}\zeta^\mu, \tag{2.4}$$

where $X_{,\mu} \equiv \partial_\mu X$. To avoid a certain confusion that may arise when the argument of a function is $x - \zeta(x)$, we need to pay attention to the notation. An expression like $\partial_\rho X^\mu(x - \zeta)$ is ambiguous, because the total derivative acts on the x dependence inside ζ^ν , while the partial derivative is not supposed to. We have

$$\partial_\rho(X^\mu(x - \zeta)) = X^\mu_{,\rho}(x - \zeta) - \zeta^\sigma_{,\rho} X^\mu_{,\sigma}(x - \zeta). \tag{2.5}$$

The point-dependent dressed fields are then

$$\begin{aligned} \varphi_d(x) &= \varphi(x - \zeta(x)), \\ A_{\mu d}(x) &= A_\mu(x - \zeta) - \zeta^\nu_{,\mu} A_\nu(x - \zeta), \\ g_{\mu\nu d}(x) &= g_{\mu\nu}(x - \zeta) - \zeta^\rho_{,\mu} g_{\nu\rho}(x - \zeta) \\ &\quad - \zeta^\rho_{,\nu} g_{\mu\rho}(x - \zeta) + \zeta^\rho_{,\mu} \zeta^\sigma_{,\nu} g_{\rho\sigma}(x - \zeta), \end{aligned} \tag{2.6}$$

for scalars, vectors and the metric, respectively. Indeed, using the Taylor expansion of $\varphi(x - \zeta)$, it is easy to check that $\delta_\xi \varphi = \xi^\rho \varphi_{,\rho}$ implies

$$\delta_\xi \varphi_d(x) = \xi^\rho(x - \zeta)\varphi_{,\rho}(x - \zeta) - \delta_\xi \zeta^\rho \varphi_{,\rho}(x - \zeta) = 0.$$

Moreover, $\delta_\xi A_\mu = \xi^\rho A_{\mu,\rho} + A_\rho \xi^\rho_{,\mu}$ implies

$$\begin{aligned} \delta_\xi A_{\mu d}(x) &= A_\rho(x - \zeta)\xi^\rho_{,\mu}(x - \zeta) \\ &\quad - \zeta^\nu_{,\mu} A_\rho(x - \zeta)\xi^\rho_{,\nu}(x - \zeta) \\ &\quad - A_\rho(x - \zeta)\partial_\mu(\xi^\rho(x - \zeta)) = 0, \end{aligned}$$

where we have used (2.5) in the last step with $X^\mu = \xi^\mu$. Similarly, (2.1) implies $\delta_\xi g_{\mu\nu d}(x) = 0$.

Generically, if $T_{\mu_1 \dots \mu_n}(x)$ is a tensor, its gauge invariant, dressed version is

$$T_{\mu_1 \dots \mu_n d}(x) = (\delta^{\nu_1}_{\mu_1} - \zeta^{\nu_1}_{,\mu_1}) \dots (\delta^{\nu_n}_{\mu_n} - \zeta^{\nu_n}_{,\mu_n}) T_{\nu_1 \dots \nu_n}(x - \zeta). \tag{2.7}$$

We can also define dual fields, which allow us to raise and lower the indices and invert the definitions of the dressed fields given above. The dual cloud field $\tilde{\zeta}^\mu(x)$ is defined as the solution of the equation

$$\tilde{\zeta}^\mu(x) = -\zeta^\mu(x - \tilde{\zeta}(x)), \tag{2.8}$$

which can be worked out recursively by expanding in powers of ζ^μ :

$$\tilde{\zeta}^\mu(x) = -\zeta^\mu(x + \zeta(x + \zeta(x + \zeta(x + \dots))))).$$

Differentiating (2.8), we find

$$\left[\delta^\mu_\rho - \zeta^\mu_{,\rho}(x - \tilde{\zeta}(x)) \right] \left[\delta^\rho_\nu - \tilde{\zeta}^\rho_{,\nu}(x) \right] = \delta^\mu_\nu. \tag{2.9}$$

Using this identity and (2.3) we derive the infinitesimal transformation of $\tilde{\zeta}^\mu$, which reads

$$\delta_\xi \tilde{\zeta}^\mu(x) = -(\delta^\mu_\nu - \tilde{\zeta}^\mu_{,\nu}(x))\xi^\nu(x). \tag{2.10}$$

It is straightforward to check its closure.

The inverse relations are

$$\begin{aligned} \varphi(x) &= \varphi_d(x - \tilde{\zeta}(x)), \\ A_\mu(x) &= A_{\mu d}(x - \tilde{\zeta}) - \tilde{\zeta}_{,\mu}^v A_{vd}(x - \tilde{\zeta}), \\ g_{\mu\nu}(x) &= g_{\mu\nu d}(x - \tilde{\zeta}) - \tilde{\zeta}_{,\mu}^\rho g_{\nu\rho d}(x - \tilde{\zeta}) \\ &\quad - \tilde{\zeta}_{,\nu}^\rho g_{\mu\rho d}(x - \tilde{\zeta}) + \tilde{\zeta}_{,\mu}^\rho \tilde{\zeta}_{,\nu}^\sigma g_{\rho\sigma d}(x - \tilde{\zeta}), \\ T_{\mu_1 \dots \mu_n}(x) &= (\delta_{\mu_1}^{v_1} - \tilde{\zeta}_{,\mu_1}^{v_1}) \dots (\delta_{\mu_n}^{v_n} - \tilde{\zeta}_{,\mu_n}^{v_n}) T_{v_1 \dots v_n d}(x - \tilde{\zeta}). \end{aligned} \tag{2.11}$$

Observe that (2.8) implies

$$x^\mu - \zeta^\mu(x)|_{x \rightarrow x - \tilde{\zeta}} = x^\mu - \tilde{\zeta}^\mu(x) - \zeta^\mu(x - \tilde{\zeta}(x)) = x^\mu.$$

Defining $y^\mu = x^\mu - \tilde{\zeta}^\mu(x)$ and relabelling $x \leftrightarrow y$, we find the dual identity

$$\zeta^\mu(x) = -\tilde{\zeta}^\mu(x - \zeta(x)). \tag{2.12}$$

Differentiating this relation, we also find

$$\left[\delta_\rho^\mu - \tilde{\zeta}_{,\rho}^\mu(x - \zeta) \right] \left[\delta_\nu^\rho - \zeta_{,\nu}^\rho(x) \right] = \delta_\nu^\mu. \tag{2.13}$$

Vectors and tensors with upper indices are dressed as follows:

$$\begin{aligned} A_d^\mu(x) &= A^\mu(x - \zeta) - \tilde{\zeta}_{,\nu}^\mu(x - \zeta) A^\nu(x - \zeta), \\ g_d^{\mu\nu}(x) &= (\delta_\rho^\mu - \tilde{\zeta}_{,\rho}^\mu(x - \zeta)) (\delta_\sigma^\nu - \zeta_{,\sigma}^\nu(x - \zeta)) g^{\rho\sigma}(x - \zeta), \\ T_d^{\mu_1 \dots \mu_n}(x) &= (\delta_{v_1}^{\mu_1} - \tilde{\zeta}_{,\nu_1}^{\mu_1}(x - \zeta)) \dots \\ &\quad (\delta_{v_n}^{\mu_n} - \tilde{\zeta}_{,\nu_n}^{\mu_n}(x - \zeta)) T^{v_1 \dots v_n}(x - \zeta). \end{aligned} \tag{2.14}$$

Indeed, (2.9) with $x \rightarrow x - \zeta(x)$ ensures that

$$\begin{aligned} A_d^\mu(x) A_{\mu d}(x) &= (\delta_\nu^\mu - \tilde{\zeta}_{,\nu}^\mu(x - \zeta)) A^\nu(x - \zeta) (\delta_\mu^\rho - \zeta_{,\mu}^\rho) A_\rho(x - \zeta) \\ &= A^\mu(x - \zeta) A_\mu(x - \zeta), \end{aligned}$$

as required for a scalar. Similarly, $g_d^{\mu\nu}(x) g_{\nu\rho d}(x) = \delta_\rho^\mu$. Moreover, the behaviors of upper indices under infinitesimal transformations, as in $\delta_\xi A^\mu = \xi^\rho A_{,\rho}^\mu - A^\rho \xi_{,\rho}^\mu$, imply that the fields (2.14) are invariant. For example,

$$\begin{aligned} \delta_\xi A_d^\mu(x) &= -(\delta_\nu^\mu - \tilde{\zeta}_{,\nu}^\mu(x - \zeta)) \xi_{,\rho}^\nu(x - \zeta) A^\rho(x - \zeta) \\ &\quad + (\delta_\nu^\mu - \tilde{\zeta}_{,\nu}^\mu(x - \zeta)) \xi_{,\rho}^\nu(x - \zeta) A^\rho(x - \zeta) = 0. \end{aligned}$$

Here we have used

$$\delta_\xi \tilde{\zeta}_{,\rho}^\mu(x - \zeta) = -(\delta_\nu^\mu - \tilde{\zeta}_{,\nu}^\mu(x - \zeta)) \xi_{,\rho}^\nu(x - \zeta),$$

which follows from (2.10).

It is also crucial to introduce anticommuting partners H^μ of ζ^μ , defined by the transformation law

$$\delta_\xi H^\mu = -H^\nu \xi_{,\nu}^\mu(x - \zeta). \tag{2.15}$$

The consistency of this transformation follows from its closure:

$$[\delta_\xi, \delta_\eta] H^\mu = H^\rho \xi_{,\rho}^\nu(x - \zeta) \eta_{,\nu}^\mu(x - \zeta)$$

$$\begin{aligned} &+ H^\nu \xi^\rho(x - \zeta) \eta_{,\nu\rho}^\mu(x - \zeta) - (\xi \leftrightarrow \eta) \\ &= -H^\rho(x) (\partial_\rho (\eta^\nu \xi_{,\nu}^\mu - \xi^\nu \eta_{,\nu}^\mu))(x - \zeta) \\ &= \delta_{[\xi, \eta]} H^\mu. \end{aligned} \tag{2.16}$$

The anticommuting partner \tilde{H}^μ of $\tilde{\zeta}^\mu$ is a field transforming exactly as H^μ .

We have achieved what we wanted, that is to say, define point-dependent observables in general relativity. However, we have done it at the cost of introducing new fields, the cloud fields (and their anticommuting partners). The next problem is to include the extra fields into the action, and ensure that the extension does not change the fundamental theory, and does not propagate unphysical degrees of freedom. First, we develop a formalism to ensure that the correlation functions of the undressed fields and the S matrix amplitudes are unmodified, despite the presence of new interactions. Then, we render the whole new sectors purely virtual.

We also want the construction to be perturbative (expanding the metric around flat space), diagrammatic and local. We do not require polynomiality, though, since in quantum gravity we have to renounce it anyway.

3 Batalin–Vilkovisky formalism and Zinn–Justin master equation

In this section we recall the standard Batalin–Vilkovisky formalism [43,44] for gravity, which is a convenient tool to study the Ward–Takahashi–Slavnov–Taylor identities [45–48] to all orders in a compact form.

The classical action $S_{cl}(g, A, \varphi)$ can be any action of classical gravity, possibly coupled to matter. For concreteness, we assume that the matter sector is made of an Abelian vector A_μ and a neutral scalar field φ . The specific form of S_{cl} is not important for the theoretical setup we are going to develop. However, particular forms of S_{cl} will be used in the computations.

We introduce the set of fields $\Phi^\alpha = (g_{\mu\nu}, C^\mu, \bar{C}^\mu, B^\mu, A_\mu, \varphi)$, where C^μ are the Faddeev–Popov ghosts [49], \bar{C}^μ are the antighosts and B^μ are the Nakanishi–Lautrup Lagrange multipliers [50,51]. The superscript α collects all the indices. We couple sources $K^\alpha = (K_g^{\mu\nu}, K_\mu^C, K_\mu^{\bar{C}}, K_\mu^B, K_A^\mu, K_\varphi)$ to the field transformations by means of the functional

$$\begin{aligned} S_K(\Phi, K) &= - \int (C^\rho \partial_\rho g_{\mu\nu} + g_{\mu\rho} \partial_\nu C^\rho + g_{\nu\rho} \partial_\mu C^\rho) K_g^{\mu\nu} \\ &\quad - \int (C^\rho \partial_\rho A_\mu + A_\rho \partial_\mu C^\rho) K_A^\mu \\ &\quad - \int (C^\rho \partial_\rho \varphi) K_\varphi - \int C^\rho (\partial_\rho C^\mu) K_\mu^C \\ &\quad - \int B^\mu K_\mu^{\bar{C}}. \end{aligned} \tag{3.1}$$

This way, the transformations of the fields can be written as

$$\delta_\xi \Phi^\alpha = \theta(S_K, \Phi^\alpha) = -\theta \frac{\delta_r S_K}{\delta K^\alpha}, \tag{3.2}$$

where $\xi^\mu = \theta C^\mu$, θ is a constant anticommuting (Grassmann) variable and

$$(X, Y) = \int \left(\frac{\delta_r X}{\delta \Phi^\alpha} \frac{\delta_l Y}{\delta K^\alpha} - \frac{\delta_r X}{\delta K^\alpha} \frac{\delta_l Y}{\delta \Phi^\alpha} \right) \tag{3.3}$$

are the Batalin–Vilkovisky antiparentheses [43,44], the subscripts r and l denoting the right and left derivatives, respectively.

The closure of the algebra of the transformations is encoded in the identity

$$(S_K, S_K) = 0. \tag{3.4}$$

The Jacobi identity satisfied by the antiparentheses implies the nilpotence relation $(S_K, (S_K, X)) = 0$ for every X .

The gauge-fixed action reads

$$S_{\text{gf}}(\Phi) = S_{\text{cl}}(g, A, \varphi) + (S_K, \Psi(\Phi)), \tag{3.5}$$

where $\Psi(\Phi)$ is the ‘‘gauge fermion’’, that is to say, a local functional that is introduced to fix the gauge. For example, in a generic covariant gauge we may choose

$$\begin{aligned} \Psi(\Phi) &= \int \sqrt{-g} \bar{C}^\mu (G_\mu(g) - \lambda g_{\mu\nu} B^\nu), \\ G_\mu(g) &= g^{\nu\rho} \partial_\rho g_{\mu\nu} - \frac{\lambda'}{2} g^{\nu\rho} \partial_\mu g_{\nu\rho}, \end{aligned} \tag{3.6}$$

where λ, λ' are gauge-fixing parameters and $G_\nu(g)$ is the gauge-fixing function. We have

$$\begin{aligned} (S_K, \Psi) &= \int \sqrt{-g} B^\mu (G_\mu(g) - \lambda g_{\mu\nu} B^\nu) \\ &\quad + S_{\text{ghost}} \rightarrow \frac{1}{4\lambda} \int \sqrt{-g} G_\mu g^{\mu\nu} G_\nu + S_{\text{ghost}}, \\ S_{\text{ghost}} &= - \int \bar{C}^\mu (S_K, \sqrt{-g} G_\mu - \lambda \sqrt{-g} g_{\mu\nu} B^\nu), \end{aligned}$$

where the arrow denotes the integration over B^μ and S_{ghost} is the ghost action. Other gauge choices will be considered in the paper.

The total action is

$$S(\Phi, K) = S_{\text{gf}}(\Phi) + S_K(\Phi, K) \tag{3.7}$$

and satisfies the Zinn–Justin equation [52]

$$(S, S) = 0, \tag{3.8}$$

also known as master equation. This identity collects the gauge invariance of the classical action, the trivality of the gauge-fixing sector, as well as the closure of the algebra. We have the nilpotence relation $(S, (S, X)) = 0$ for every X .

4 Cloud sector

In this section we build the cloud sector. To trivialize its effects on the usual correlation functions and the S matrix elements, we mimick the key aspects of the gauge-fixing procedure. In particular, we need:

1. the cloud field ζ^μ ;
2. its anticommuting partner H^μ ;
3. a new symmetry (which we call cloud symmetry), which shifts ζ^μ by (minus) H^μ ; and
4. anticommuting H^μ -partners \bar{H}^μ , as well as Lagrange multipliers E^μ .

The reason why ζ^μ and H^μ must have opposite statistics is precisely that the latter is the shift of the former by the new symmetry. The reason why \bar{H}^μ and E^μ must be included is that they allow us to ‘‘fix the could’’, in the same way as we normally fix the gauge. Indeed, the fields H^μ can be seen as the ‘‘Faddeev–Popov ghosts’’ of the cloud symmetry, while \bar{H}^μ are the ‘‘cloud antighosts’’, and E^μ are the Lagrange multipliers for the ‘‘cloud-fixing’’. Finally, the reason why ζ^μ alone is not sufficient, but H^μ, \bar{H}^μ and E^μ are needed as well, is that the multiplet $\tilde{\Phi}^\alpha = (\zeta^\mu, H^\mu, \bar{H}^\mu, E^\mu)$ provides the easiest way to ensure that the contributions of the extra fields mutually compensate in all the usual, on-shell correlation functions and the S matrix amplitudes. This way, the fundamental physics does not change, and the cloud sector makes a difference only in the new correlations functions, which are those built with the gauge invariant fields.

Then, we further extend the construction to a whole Batalin–Vilkovisky formalism for the cloud sector. First, we include a new set of sources $\tilde{K}^\alpha = (\tilde{K}_\mu^\zeta, \tilde{K}_\mu^H, \tilde{K}_\mu^{\bar{H}}, \tilde{K}_\mu^E)$, coupled to the $\tilde{\Phi}^\alpha$ transformations. Second, we extend the definition (3.3) of antiparentheses to the new sector:

$$(X, Y) = \int \left(\frac{\delta_r X}{\delta \Phi^\alpha} \frac{\delta_l Y}{\delta K^\alpha} - \frac{\delta_r X}{\delta K^\alpha} \frac{\delta_l Y}{\delta \Phi^\alpha} + \frac{\delta_r X}{\delta \tilde{\Phi}^\alpha} \frac{\delta_l Y}{\delta \tilde{K}^\alpha} - \frac{\delta_r X}{\delta \tilde{K}^\alpha} \frac{\delta_l Y}{\delta \tilde{\Phi}^\alpha} \right).$$

Third, we collect the gauge transformations (2.3) and (2.15) of the new fields ζ^μ and H^μ , and the cloud transformations, into the functionals

$$\begin{aligned} S_K^{\text{gauge}} &= S_K - \int C^\mu (x - \zeta) \tilde{K}_\mu^\zeta - \int H^\nu C_{,\nu}^\mu (x - \zeta) \tilde{K}_\mu^H, \\ S_K^{\text{cloud}} &= \int H^\mu \tilde{K}_\mu^\zeta - \int E^\mu \tilde{K}_\mu^{\bar{H}}, \quad S_K^{\text{tot}} = S_K^{\text{gauge}} + S_K^{\text{cloud}}. \end{aligned}$$

The cloud transformations, which are encoded into the second functional, are just the most general shifts of ζ^μ and \bar{H} . For example, the total (gauge plus cloud) transformation of ζ^μ reads

$$\begin{aligned} \delta_{\xi, \mathcal{H}} \zeta^\mu &= \theta (S_K^{\text{tot}}, \zeta^\mu) = -\theta H^\mu(x) + \theta C^\mu(x - \zeta(x)) \\ &= -\mathcal{H}^\mu(x) + \xi^\mu(x - \zeta(x)), \end{aligned}$$

where $\mathcal{H} = \theta H$.

It is easy to check the identities

$$(S_K^{\text{gauge}}, S_K^{\text{gauge}}) = (S_K^{\text{cloud}}, S_K^{\text{cloud}}) = 0, \tag{4.1}$$

which express the closures of both types of transformations. The first identity follows from (2.2), (2.16) and (3.4).

We also have:

$$S_K^{\text{gauge}} - S_K = - \left(S_K^{\text{cloud}}, \int C^\mu(x - \zeta) \tilde{K}_\mu^H \right),$$

$$S_K^{\text{cloud}} = - \left(S_K^{\text{cloud}}, \int \zeta^\mu \tilde{K}_\mu^\zeta + \int \tilde{H}^\mu \tilde{K}_\mu^{\tilde{H}} \right). \tag{4.2}$$

These formulas show that the functionals $S_K^{\text{gauge}} - S_K$ and S_K^{cloud} are cohomologically exact under the cloud symmetry, i.e., they have the form $(S_K^{\text{cloud}}, \text{local functional})$. Together with $(S_K, S_K^{\text{cloud}}) = 0$ (which is trivial), they imply the further identity

$$(S_K^{\text{gauge}}, S_K^{\text{cloud}}) = 0, \tag{4.3}$$

which gives, together with (4.1),

$$(S_K^{\text{tot}}, S_K^{\text{tot}}) = 0. \tag{4.4}$$

4.1 The cloud and the total action

We fix the cloud by adding $(S_K^{\text{tot}}, \tilde{\Psi})$ to the action, where $\tilde{\Psi}(\Phi, \tilde{\Phi})$ is the ‘‘cloud fermion’’. A typical form of it is

$$\tilde{\Psi}(\Phi, \tilde{\Phi}) = \int \sqrt{-g_d} \tilde{H}^\mu \left(V_\mu - \tilde{\lambda} g_{\mu\nu d} E^\nu \right), \tag{4.5}$$

where g_d is the determinant of $g_{\mu\nu d}$ and V_μ denotes the ‘‘cloud function’’, i.e., the function that specifies the cloud. We assume that V_μ is gauge invariant:

$$(S_K^{\text{gauge}}, V_\mu) = 0, \quad (S_K^{\text{gauge}}, \tilde{\Psi}) = 0. \tag{4.6}$$

Basically, we can view V_μ as a function of the dressed metric $g_{\mu\nu d}$.

We find

$$(S_K^{\text{tot}}, \tilde{\Psi}) = (S_K^{\text{cloud}}, \tilde{\Psi}) = \int \sqrt{-g_d} E^\mu \left(V_\mu - \tilde{\lambda} g_{\mu\nu d} E^\nu \right)$$

$$+ \int \tilde{H}^\mu \frac{\delta}{\delta \zeta^\rho} \left[\sqrt{-g_d} \left(V_\mu - \tilde{\lambda} g_{\mu\nu d} E^\nu \right) \right] H^\rho. \tag{4.7}$$

The last term gives a ‘‘Faddeev–Popov determinant’’ for the cloud, which is crucial for the diagrammatic properties that we derive in the next sections.

The total action of the extended theory is then

$$S_{\text{tot}}(\Phi, K, \tilde{\Phi}, \tilde{K}) = S_{\text{cl}} + (S_K^{\text{tot}}, \Psi + \tilde{\Psi}) + S_K^{\text{tot}} \tag{4.8}$$

and satisfies its own master equation

$$(S_{\text{tot}}, S_{\text{tot}}) = 0. \tag{4.9}$$

Note that S_{tot} is gauge invariant, since Eqs. (4.1), (4.4) and (4.6) imply

$$(S_K^{\text{gauge}}, S_{\text{tot}}) = (S_K^{\text{cloud}}, S_{\text{tot}}) = 0. \tag{4.10}$$

We can also write

$$S_{\text{tot}}(\Phi, K, \tilde{\Phi}, \tilde{K}) = S(\Phi, K) + (S_K^{\text{cloud}}, \Theta),$$

$$\Theta = \tilde{\Psi} - \int C^\mu(x - \zeta) \tilde{K}_\mu^H - \int \zeta^\mu \tilde{K}_\mu^\zeta - \int \tilde{H}^\mu \tilde{K}_\mu^{\tilde{H}},$$

which shows that the difference between the dressed action and the ordinary action is cohomologically exact with respect to the cloud symmetry.

4.2 Covariant cloud

To make explicit calculations, we need to choose the cloud function V_μ in (1). A convenient starting point is the covariant cloud function

$$V_\mu(g, \zeta) = g_d^{\nu\rho} \partial_\rho g_{\mu\nu d} - \frac{\tilde{\lambda}'}{2} g_d^{\nu\rho} \partial_\mu g_{\nu\rho d}, \tag{4.11}$$

where $\tilde{\lambda}'$ is a further cloud parameter. This choice mimicks the gauge-fixing function of formula (3.6), and makes the cloud sector as similar as possible to the gauge-fixing one.

Other choices are considered in the paper (see formula (10.1) and comments right below it). It should be kept in mind that the freedom we have for V_μ is enormous, given that it describes the influence of the surrounding, classical environment on the observation of a quantum phenomenon. When we are interested in a renormalizable theory of quantum gravity, like the one of [28] (see Sect. 10), it is convenient to restrict to gauge-fixing functions and cloud functions that are manifestly renormalizable by power counting. In this paper, we focus on those.

Note that we are allowed to use second metrics, inside the clouds (as well as inside the gauge-fixing function). For example, we can use the flat-space metric $\eta_{\mu\nu}$ to raise the indices of ∂_μ . Sometimes, however, it may be convenient to use a unique metric everywhere, for a better control on the renormalization properties of the dressed theory.

5 Cloud independence of the non-cloud sector

In this section we prove that the ordinary correlation functions of elementary and composite fields are unmodified. This also ensures that the vertices and diagrams of the cloud sector do not affect the renormalization of the non-cloud sector of the theory. Moreover, we show that the S matrix amplitudes of the dressed fields are cloud independent and coincide with the usual S matrix amplitudes of the undressed fields.

The generating functional of the correlation functions is

$$Z(J, K, \tilde{J}, \tilde{K}) = \int [d\Phi d\tilde{\Phi}] \exp \left(i S_{\text{tot}}(\Phi, K, \tilde{\Phi}, \tilde{K}) + i \int \Phi^\alpha J_\alpha + i \int \tilde{\Phi}^\alpha \tilde{J}_\alpha \right) \tag{5.1}$$

and $W(J, K, \tilde{J}, \tilde{K}) = -i \ln Z(J, K, \tilde{J}, \tilde{K})$ is the generating functional of the connected ones. The ordinary correlation functions are the functional derivatives with respect to the sources J^α , calculated at $\tilde{J} = \tilde{K} = 0$. They are collected in

$$\begin{aligned} Z(J, K, 0, 0) &= \int [d\Phi d\tilde{\Phi}] \exp \left(i S(\Phi, K) + i (S_K^{\text{tot}}, \tilde{\Psi}) + i \int \Phi^\alpha J_\alpha \right) \\ &= \int [d\Phi] \exp \left(i S(\Phi, K) + i \int \Phi^\alpha J_\alpha \right) \\ &\quad \times \int [d\tilde{\Phi}] e^{i(S_K^{\text{cloud}}, \tilde{\Psi})}. \end{aligned} \tag{5.2}$$

We want to prove that this expression coincides with the ordinary generating functional, thanks to the identity

$$\int [d\tilde{\Phi}] e^{i(S_K^{\text{cloud}}, \tilde{\Psi})} = 1. \tag{5.3}$$

Since $\tilde{\Psi}$ depends on both Φ and $\tilde{\Phi}$, the left-hand side of (5.3) is in principle a functional of Φ . To show that it is actually a constant, we consider arbitrary infinitesimal deformations of the fields Φ . Let $\delta\tilde{\Psi}$ denote the variation of $\tilde{\Psi}$ due to them. The variation of the integral is then

$$\delta \int [d\tilde{\Phi}] e^{i(S_K^{\text{cloud}}, \tilde{\Psi})} = i \int [d\tilde{\Phi}] (S_K^{\text{cloud}}, \delta\tilde{\Psi}) e^{i(S_K^{\text{cloud}}, \tilde{\Psi})}, \tag{5.4}$$

Performing the change of field variables $\tilde{\Phi}^\alpha \rightarrow \tilde{\Phi}^\alpha + \theta(S_K^{\text{cloud}}, \tilde{\Phi}^\alpha)$ in the integral

$$\begin{aligned} &\int [d\tilde{\Phi}] \delta\tilde{\Psi} e^{i(S_K^{\text{cloud}}, \tilde{\Psi})}, \\ &\text{we obtain} \\ &\int [d\tilde{\Phi}] \delta\tilde{\Psi} e^{i(S_K^{\text{cloud}}, \tilde{\Psi})} \\ &= \int [d\tilde{\Phi}] \left[\delta\tilde{\Psi} + \theta(S_K^{\text{cloud}}, \delta\tilde{\Psi}) \right] e^{i(S_K^{\text{cloud}}, \tilde{\Psi})}. \end{aligned} \tag{5.5}$$

We have used the fact that $(S_K^{\text{cloud}}, \tilde{\Psi})$ is independent of the sources, so $(S_K^{\text{cloud}}, \tilde{\Psi}) \rightarrow (S_K^{\text{cloud}}, \tilde{\Psi}) + \theta(S_K^{\text{cloud}}, (S_K^{\text{cloud}}, \tilde{\Psi})) = (S_K^{\text{cloud}}, \tilde{\Psi})$. The equality (5.5) shows that the right-hand side of (5.4) vanishes, as we wished to prove.

5.1 Cloud independence of the S matrix amplitudes

Now we prove that the scattering amplitudes of the dressed fields coincide with the usual scattering amplitudes (of undressed fields). Specifically, the clouds have no effect on the mass shell, when the polarizations are attached to the amputated external legs.

First, we recall a general result about the invariance of the S matrix amplitudes under a perturbative change of field variables (see, for example, [21] for the proof). Consider a generic theory of scalar fields φ , described by some classical action $S(\varphi)$. If $\mathcal{O}(\varphi)$ denotes a composite field that is at least quadratic in φ , define new fields $\varphi' \equiv \varphi + \mathcal{O}(\varphi)$.

Then, the following results hold. The locations $p^2 = m_{\text{ph}}^2$ of the poles of the two-point functions $\langle \varphi | \varphi \rangle$ and $\langle \varphi' | \varphi' \rangle$ coincide perturbatively, where m_{ph} denotes the physical mass (possibly equipped with an imaginary part, if the particle is unstable). We have

$$\langle \varphi | \varphi \rangle \simeq \frac{iZ}{p^2 - m_{\text{ph}}^2 + i\epsilon}, \quad \langle \varphi' | \varphi' \rangle \simeq \frac{iZ'}{p^2 - m_{\text{ph}}^2 + i\epsilon},$$

for $p^2 \simeq m_{\text{ph}}^2$, for suitable factors Z and Z' .

Moreover, the correlation functions that contain more than two φ' insertions satisfy

$$\begin{aligned} &\left\langle \prod_{a=1}^j \frac{p_a^2 - m_{\text{ph}}^2}{\sqrt{Z'}} \varphi'(p_a) \right\rangle_{\text{on-shell}} \\ &= \left\langle \prod_{a=1}^j \frac{p_a^2 - m_{\text{ph}}^2}{\sqrt{Z}} \varphi(p_a) \right\rangle_{\text{on-shell}}, \end{aligned} \tag{5.6}$$

which proves that the S matrix amplitudes are invariant under the change of field variables $\varphi \rightarrow \varphi'$.

Applying this theorem to the cloud extension of the gravity theory, we have, in momentum space,

$$\begin{aligned} &\left\langle \prod_{i=1}^n k_i^2 \varepsilon_i^{\mu_i \nu_i}(k_i) g_{\mu_i \nu_i \text{d}}(k_i) \right. \\ &\quad \times \left. \prod_{a=1}^j p_a^2 \varepsilon_a^{\rho_a}(p_a) A_{\rho_a \text{d}}(p_a) \prod_{b=1}^l \frac{q_b^2 - m_{\text{ph}}^2}{\sqrt{Z'_\varphi}} \varphi_{\text{d}}(q_b) \right\rangle_{\text{on-shell}} \\ &= \left\langle \prod_{i=1}^n k_i^2 \varepsilon_i^{\mu_i \nu_i}(k_i) g_{\mu_i \nu_i}(k_i) \right. \\ &\quad \times \left. \prod_{a=1}^j p_a^2 \varepsilon_a^{\rho_a}(p_a) A_{\rho_a}(p_a) \prod_{b=1}^l \frac{q_b^2 - m_{\text{ph}}^2}{\sqrt{Z_\varphi}} \varphi(q_b) \right\rangle_{\text{on-shell}}. \end{aligned} \tag{5.7}$$

The polarizations $\varepsilon^{\mu\nu}(k)$ and $\varepsilon^\mu(p)$ of the gravitons and the vector fields, respectively, satisfy $k_\mu \varepsilon^{\mu\nu}(k) = p_\mu \varepsilon^\mu(p) = 0$, and include the normalization factors $1/\sqrt{Z}$. The ‘‘dressed’’ polarizations $\varepsilon_{\text{d}}^{\mu\nu}(k)$ and $\varepsilon_{\text{d}}^\mu(p)$ are the same, apart from having normalization factors $1/\sqrt{Z'}$. With an abuse of notation, we use the same symbols for the fields and their Fourier transforms, since the meaning is clear from the context. By the theorem proved in the first part of this section, the right-hand side of (5.7) is cloud independent and coincides with the usual S matrix amplitude.

Note that formulas (2.6) show that the expansion of $g_{\mu\nu\text{d}}$ in powers of ζ^ρ , combined with the expansion of $g_{\mu\nu}$ around the flat-space metric $\eta_{\mu\nu}$, contains a linear contribu-

tion $-\zeta_{\mu,\nu} - \zeta_{\nu,\mu}$, besides $g_{\mu\nu}$ itself, plus nonlinear terms (which can be regarded as composite fields $\mathcal{O}(\varphi)$), where $\zeta_\mu = \eta_{\mu\nu}\zeta^\nu$. Thus, $g_{\mu\nu d}$ is not of the form $\varphi' = \varphi + \mathcal{O}(\varphi)$. Nevertheless, the linear terms $-\zeta_{\mu,\nu} - \zeta_{\nu,\mu}$ are killed by the polarization $\varepsilon^{\mu\nu}(k)$, after the Fourier transform. This means that $\varepsilon_d^{\mu\nu}(k)g_{\mu\nu d}(k)$ is of the required form, apart from an unimportant normalization factor.

Also note that we are not comparing correlation functions of the same theory, as in (5.6). We are jumping from one theory (the extended one, to which the left-hand side of (5.7) refers) to another theory (the non extended one, to which the right-hand side of (5.7) refers), thanks to the result proved previously in this section.

In the end, the product $k^2\varepsilon_d^{\mu\nu}(k)g_{\mu\nu d}(k)$ is gauge invariant (and, therefore, gauge independent, for the arguments given below) and its dressing is trivial:

$$\lim_{k^2 \rightarrow 0} k^2\varepsilon_d^{\mu\nu}(k)\langle g_{\mu\nu d}(k) \dots \rangle = \lim_{k^2 \rightarrow 0} k^2\varepsilon^{\mu\nu}(k)\langle g_{\mu\nu}(k) \dots \rangle.$$

The same result holds for the insertions of vectors and scalars (and fermions, if present).

The identity (5.7) proves that the ordinary theory of scattering can be understood as a theory of scattering of dressed fields. We can even forget about the undressed fields altogether, and always work with the dressed fields. So doing, gauge invariance and gauge independence become manifest. In particular, the S -matrix amplitudes are automatically ensured to be gauge independent.

Clearly, the proof of this section relies heavily on the notion of asymptotic state, which is crucial to build the S matrix elements. For this reason, it does not generalize to short-distance scattering processes, where the incoming and outgoing states are not allowed to become free. Those processes can be studied from the correlation functions of gauge-invariant fields (which are not constrained to be on the mass shell), once they are equipped with the notion of pure virtuality.

6 Dressed correlation functions

In this section we study the correlation functions that contain insertions of dressed fields. It is possible to study them systematically by coupling new sources to them and extending the generating functionals again. We replace the action S_{tot} inside (5.1) by

$$S_{\text{tot}}^{\text{ext}} = S_{\text{tot}} + \int (J_d^{\mu\nu} g_{\mu\nu d} + J_d^\mu A_{\mu d} + J_d \varphi_d), \tag{6.1}$$

and denote the extended functionals by $Z_{\text{tot}}^{\text{ext}}(J, K, \tilde{J}, \tilde{K}, J_d) = \exp(iW_{\text{tot}}^{\text{ext}}(J, K, \tilde{J}, \tilde{K}, J_d))$. The insertions of dressed fields can be studied by taking the functional derivatives with respect to the new sources $J_d^{\mu\nu}, J_d^\mu, J_d$. The extended action

is gauge invariant, since (4.10) implies

$$(S_K^{\text{gauge}}, S_{\text{tot}}^{\text{ext}}) = 0. \tag{6.2}$$

Clearly, $S_{\text{tot}}^{\text{ext}}$ is not cloud invariant.

It is straightforward to prove that the correlation functions of the dressed fields, collected in the functional $Z_{\text{tot}}^{\text{ext}}(J_d) = \exp(iW_{\text{tot}}^{\text{ext}}(J_d)) \equiv Z_{\text{tot}}^{\text{ext}}(0, 0, 0, 0, J_d) = \exp(iW_{\text{tot}}^{\text{ext}}(0, 0, 0, 0, J_d))$, are gauge independent. The argument is identical to the one of subsection 7.1 of [21], so we do not repeat it here. Gauge independence will be verified explicitly in the computations.

7 Gauge/cloud duality

In this section we show that the gauge-trivial sector and the cloud sector are dual to each other. We call this property *gauge/cloud duality*. Sometimes, it can be used to simplify the computations.

We begin by noting that the transformation law (2.3) and the definitions (2.6) imply that the cloud transformation of the dressed metric is just an infinitesimal diffeomorphism (2.1) with parameters H_d^μ such that

$$H^\mu = (\delta_v^\mu - \zeta_{,\nu}^\mu)H_d^\nu. \tag{7.1}$$

Precisely,

$$(S_K^{\text{tot}}, g_{\mu\nu d}) = (S_K^{\text{cloud}}, g_{\mu\nu d}) = H_d^\rho \partial_\rho g_{\mu\nu d} + g_{\mu\rho d} \partial_\nu H_d^\rho + g_{\nu\rho d} \partial_\mu H_d^\rho = \delta_{H_d}^{\text{diff}} g_{\mu\nu d}. \tag{7.2}$$

Similarly, for scalars and vectors we have

$$\begin{aligned} (S_K^{\text{cloud}}, \varphi_d) &= H_d^\rho \partial_\rho \varphi_d, \\ (S_K^{\text{cloud}}, A_{\mu d}) &= H_d^\rho \partial_\rho A_{\mu d} + A_{\rho d} \partial_\mu H_d^\rho. \end{aligned} \tag{7.3}$$

It is easy to check that H_d^μ is gauge invariant $(S_K^{\text{gauge}}, H_d^\mu) = 0$. Moreover, the cloud transformation of H_d^μ mimics the gauge transformation of the ghosts C^μ :

$$(S_K^{\text{cloud}}, H_d^\mu) = H_d^\rho \partial_\rho H_d^\mu. \tag{7.4}$$

We define the dressed cloud field ζ_d^μ as the dual field of formula (2.8):

$$\zeta_d^\mu = \tilde{\zeta}^\mu. \tag{7.5}$$

Using (7.1), it is easy to derive the cloud transformation of ζ_d^μ , which reads

$$(S_K^{\text{cloud}}, \zeta_d^\mu) = H_d^\mu(x - \zeta_d(x)). \tag{7.6}$$

Note that ζ_d^μ is not gauge invariant. Using (2.10), its gauge transformation can be used to define the dressed Faddeev-Popov ghosts

$$C_d^\mu \equiv (\delta_\nu^\mu - \zeta_{d,\nu}^\mu)C^\nu = -(S_K^{\text{gauge}}, \zeta_d^\mu), \tag{7.7}$$

which, instead, are gauge invariant by construction. Their cloud transformations read

$$\begin{aligned}
 (S_K^{\text{cloud}}, C_d^\mu) &= (S_K^{\text{gauge}}, (S_K^{\text{cloud}}, \zeta_d^\mu)) \\
 &= (S_K^{\text{gauge}}, H_d^\mu(x - \zeta_d(x))) = C_d^\rho H_{,\rho d}^\mu(x - \zeta_d),
 \end{aligned}
 \tag{7.8}$$

having used (4.3) and $(S_K^{\text{gauge}}, H_d^\mu) = 0$.

Now, collecting (2.6), (7.1), (7.5) and (7.7), we define the change of field variables

$$\Phi, \tilde{\Phi} \rightarrow \Phi_d, \tilde{\Phi}_d \tag{7.9}$$

from undressed fields to dressed fields, leaving all the other fields unchanged: $\tilde{C}_d^\mu = \tilde{C}^\mu$, $B_d^\mu = B^\mu$, $\tilde{H}_d^\mu = \tilde{H}^\mu$ and $E_d = E$. The transformations are perturbatively local, which means that when we use them as changes of field variables in the functional integral, the Jacobian determinant is equal to one, using the dimensional regularization.

To ensure that the antiparentheses are preserved, so that all the properties derived till now continue to hold, we embed (7.9) into a canonical transformation

$$\Phi, \tilde{\Phi}, K, \tilde{K} \rightarrow \Phi_d, \tilde{\Phi}_d, K_d, \tilde{K}_d, \tag{7.10}$$

of the Batalin–Vilkovisky type. Its generating functional is

$$F(\Phi, \tilde{\Phi}, K_d, \tilde{K}_d) = \int \Phi_d(\Phi, \tilde{\Phi}) K_d + \int \tilde{\Phi}_d(\Phi, \tilde{\Phi}) \tilde{K}_d.$$

At the practical level, the whole operation amounts to work out the transformations of the dressed fields, which we have already done, and couple them to the dressed sources. Collecting the gauge transformations (7.7) and the cloud transformations (7.2), (7.3), (7.4), (7.6) and (7.8), we find

$$\begin{aligned}
 S_K^{\text{gauge}} &= \int C_d^\mu \tilde{K}_{\mu d}^\zeta - \int B_d^\mu K_{\mu d}^{\tilde{C}}, \\
 S_K^{\text{cloud}} &= - \int (H_d^\rho \partial_\rho g_{\mu\nu d} + g_{\mu\rho d} \partial_\nu H_d^\rho + g_{\nu\rho d} \partial_\mu H_d^\rho) K_{gd}^{\mu\nu} \\
 &\quad - \int H_d^\rho (\partial_\rho H_d^\mu) K_{\mu d}^H - \int H_d^\rho (\partial_\rho \varphi_d) K_{\varphi d} \\
 &\quad - \int (H_d^\rho \partial_\rho A_{d\mu} + A_{\rho d} \partial_\mu H_d^\rho) K_{\mu d}^A \\
 &\quad - \int E_d^\mu \tilde{K}_{\mu d}^{\tilde{H}} - \int H_d^\mu(x - \zeta_d) \tilde{K}_{\mu d}^\zeta \\
 &\quad - \int C_d^\nu H_{,\nu d}^\mu(x - \zeta_d) \tilde{K}_{\mu d}^{\tilde{C}}.
 \end{aligned}$$

We see that the canonical transformation (7.10) switches the gauge transformations and the cloud transformations. Similarly, it exchanges the roles of the gauge-fixing function G_μ and the cloud function $V_\mu: G_\mu(g(\zeta, g_d)) \leftrightarrow V_\mu(g_d)$. We may also say that it exchanges the quantization prescription of the gauge-trivial sector with the one of the cloud sector (see below).

The correlation functions of the dressed fields coincide with the ones of the undressed fields in a specific gauge. For example, choosing the covariant gauge (3.6) and the covariant cloud (1), (4.11), we have

$$\begin{aligned}
 &\langle g_{\mu_1\nu_1 d}(x_1) \cdots g_{\mu_n\nu_n d}(x_n) \varphi_d(y_1) \cdots \varphi_d(y_j) \\
 &\quad \times A_{\rho_1 d}(z_1) \cdots A_{\rho_k d}(z_k) \rangle \\
 &= \langle g_{\mu_1\nu_1}(x_1) \cdots g_{\mu_n\nu_n}(x_n) \varphi(y_1) \cdots \varphi(y_j) \\
 &\quad \times A_{\rho_1}(z_1) \cdots A_{\rho_k}(z_k) \rangle_{\lambda \rightarrow \tilde{\lambda}, \lambda' \rightarrow \tilde{\lambda}'}.
 \end{aligned}
 \tag{7.11}$$

Combined with the cloud independence of the right-hand side, proved in Sect. 5, this property ensures that the dressed correlation function can be calculated by replacing λ with $\tilde{\lambda}$ and λ' with $\tilde{\lambda}'$ in a usual correlation function. The left-hand side of (7.11) normally includes a huge number of diagrams. However, (7.11) implies that most contributions cancel out in the end.

8 Multiclouds

In this section we show how to equip each insertion with its own, independent dressing. To do so, we extend the formalism of the previous sections by adding several copies of the cloud sector.

We introduce many cloud fields $\zeta^{\mu i}$, where i labels the copies. Then we add copies of their anticommuting partners $H^{\mu i}$ (the cloud ghosts), the antighosts $\tilde{H}^{\mu i}$ and the Lagrange multipliers $E^{\mu i}$. We collect them in $\tilde{\Phi}^{\alpha i} = (\zeta^{\mu i}, H^{\mu i}, \tilde{H}^{\mu i}, E^{\mu i})$. We also couple sources $\tilde{K}^{\alpha i}$ to their transformations. Next, we extend the definition (3.3) of antiparentheses to include all the copies:

$$\begin{aligned}
 (X, Y) &= \int \left[\frac{\delta_r X}{\delta \Phi^\alpha} \frac{\delta_l Y}{\delta K^\alpha} - \frac{\delta_r X}{\delta K^\alpha} \frac{\delta_l Y}{\delta \Phi^\alpha} \right. \\
 &\quad \left. + \sum_i \left(\frac{\delta_r X}{\delta \tilde{\Phi}^{\alpha i}} \frac{\delta_l Y}{\delta \tilde{K}^{\alpha i}} - \frac{\delta_r X}{\delta \tilde{K}^{\alpha i}} \frac{\delta_l Y}{\delta \tilde{\Phi}^{\alpha i}} \right) \right].
 \end{aligned}
 \tag{8.1}$$

Finally, we extend the gauge transformations and introduce cloud transformations for each copy:

$$\begin{aligned}
 S_K^{\text{gauge}} &= S_K - \sum_i \int C^\mu(x - \zeta^i) \tilde{K}_\mu^{\zeta i} \\
 &\quad - \sum_i \int H^{\nu i} C_{,\nu}^\mu(x - \zeta^i) \tilde{K}_\mu^{H i}, \\
 S_K^{\text{cloud} i} &= \int (H^{\mu i} \tilde{K}_\mu^{\zeta i} - E^{\mu i} \tilde{K}_\mu^{\tilde{H} i}), \\
 S_K^{\text{cloud}} &= \sum_i S_K^{\text{cloud} i}, \quad S_K^{\text{tot}} = S_K^{\text{gauge}} + S_K^{\text{cloud}}.
 \end{aligned}
 \tag{8.2}$$

It is easy to check that the identities (4.1) and (4.4) continue to hold.

The simplest cloud fermion is just the sum of the cloud fermions of each copy:

$$\tilde{\Psi}(\Phi, \tilde{\Phi}) = \sum_i \int \sqrt{-g_d^i} \tilde{H}^{\mu i} \left(V_\mu^i + \frac{\tilde{\lambda}_i}{2} g_{\mu\nu d}^i E^{vi} \right), \quad (8.3)$$

where $g_{\mu\nu d}^i$ is the dressed metric tensor built with the i th cloud field $\zeta^{\mu i}$, and V_μ^i is the i th cloud function, assumed to be gauge invariant, $(S_K^{\text{gauge}}, V_\mu^i) = 0$. For simplicity, we also assume that each V_μ^i depends only on the i th cloud field $\zeta^{\mu i}$ (besides $g_{\mu\nu}$), i.e., different cloud sectors are not mixed. We can just take each V_μ^i to be a function of $g_{\mu\nu d}^i$.

The total action of the extended theory is still (4.8), and satisfies the master equations (4.9) and (4.10). Moreover,

$$(S_K^{\text{cloud}i}, S_{\text{tot}}) = 0 \quad (8.4)$$

for every i .

It is always possible to build gauge invariant functions with two cloud fields. For example, the functions

$$\zeta_{1i}^\mu(x) \equiv \zeta^{\mu 1}(x) + \tilde{\zeta}^{\mu i}(x - \zeta^1(x)) \quad (8.5)$$

are gauge invariant, since (2.3) and (2.10) imply

$$\delta_\xi \zeta_{1i}^\mu(x) = \xi^\mu(x - \zeta^1) - \tilde{\zeta}_{,v}^{\mu i}(x - \zeta^1) \xi^\mu(x - \zeta^1) - (\delta_v^\mu - \tilde{\zeta}_{,v}^{\mu i}(x - \zeta^1)) \xi^v(x - \zeta^1) = 0.$$

We do not have control on such functions, when they are turned on. For this reason, it may be important to prove, when possible, that the operations we make preserve the unmixing stated above.

The insertions of dressed fields can be studied by means of the extended action

$$S_{\text{tot}}^{\text{ext}} = S_{\text{tot}} + \sum_i \int \left(J_d^{\mu\nu i} g_{\mu\nu d}^i + J_d^{\mu i} A_{\mu d}^i + J_d^i \varphi_d^i \right). \quad (8.6)$$

The correlation functions that do not contain insertions belonging to some cloud sector are independent of that cloud sector. Indeed, the proof of (5.3) can be repeated for every cloud sector separately.

The gauge/cloud duality is less powerful in the presence of many clouds. It can be used to eliminate one cloud, or a combination of clouds, but not all of them. For example, a correlation function

$$\langle g_{\mu_1\nu_1 d}^{(1)}(x_1) \cdots g_{\mu_n\nu_n d}^{(n)}(x_n) A_{\rho_1 d}^{(n+1)}(y_1) \cdots A_{\rho_j d}^{(n+j)}(y_j) \varphi_d^{(n+j+1)}(z_1) \cdots \varphi_d^{(n+j+k)}(z_k) \rangle, \quad (8.7)$$

with different clouds for every field, can be simplified to

$$\langle g_{\mu_1\nu_1 d}^{(1)'}(x_1) g_{\mu_2\nu_2 d}^{(2)'}(x_2) \cdots g_{\mu_n\nu_n d}^{(n)'}(x_n) A_{\rho_1 d}^{(n+1)'}(y_1) \cdots A_{\rho_j d}^{(n+j)'}(y_j) \varphi_d^{(n+j+1)'}(z_1) \cdots \varphi_d^{(n+j+k)'}(z_k) \rangle, \quad (8.8)$$

by means of a field redefinition that exchanges the first dressed field with its undressed version. The primes mean

that the clouds of the other fields must be redefined as a consequence.

These operations preserve the unmixing, after further redefinitions of the cloud fields. For example, the transformation (7.10) leads to

$$\varphi_d^{(i)}(x) = \varphi_d^{(1)}(x - \zeta^{i'}(x)), \quad i > n + j,$$

where $\varphi_d^{(1)}$ is the scalar field dressed with the first cloud (which does not even appear in (8.7), but this does not matter for what we are saying) and $\zeta^{i'}(z_i)$ is the solution of

$$\zeta^{\mu i'}(x) = \zeta^{\mu i}(x) - \zeta^{\mu 1}(x - \zeta^{i'}(x)).$$

To restore the unmixing, it is sufficient to define the new i th cloud field as $\zeta^{\mu i'}$, $i > 1$, after relabelling $\varphi_d^{(1)}$ as φ . The same can be done for the other insertions of (8.7).

Repeating the arguments of Sect. 5, it is possible to extend the results of that section to the multicloud case, i.e., prove that the usual correlation functions are cloud independent, and that the S matrix amplitudes coincide with the usual ones, formula (5.7), even when each insertion is dressed with its own, independent cloud.

In the explicit calculations of this paper we work with a unique cloud, for simplicity.

9 One-loop two-point functions

In this section we calculate the one-loop two-point functions of the basic dressed fields in the covariant gauge, with a covariant cloud. We show that the absorptive parts are in general unphysical. In the next section we turn to purely virtual clouds, and show that the absorptive parts then become physical.

We define the expansion around flat space by writing $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$ and $g_{\mu\nu d} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu d}$, where $\kappa = \sqrt{8\pi G}$ and G is Newton's constant. It is convenient to make the replacements

$$\begin{aligned} C^\mu &\rightarrow \kappa C^\mu, & B^\mu &\rightarrow \kappa B^\mu, \\ \bar{C}^\mu &\rightarrow \kappa \bar{C}^\mu, & \Psi &\rightarrow \kappa^{-2} \Psi, \\ \zeta^\mu &\rightarrow \kappa \zeta^\mu, & H^\mu &\rightarrow \kappa H^\mu, \\ E^\mu &\rightarrow \kappa E^\mu, & \bar{H}^\mu &\rightarrow \kappa \bar{H}^\mu, & \tilde{\Psi} &\rightarrow \kappa^{-2} \tilde{\Psi}, \end{aligned}$$

so that the loop expansion coincides with the expansion in powers of κ .

The two-point functions can be calculated by expanding the dressed fields (2.6) to the first order in κ , where we find

$$\begin{aligned} \varphi_d &= \varphi - \kappa \zeta^\mu \varphi_{,\mu}, & A_{\mu d} &= A_\mu - \kappa \zeta^\rho A_{\mu,\rho} - \kappa \zeta_{,\mu}^\rho A_\rho, \\ h_{\mu\nu d} &= h_{\mu\nu} - \frac{1}{2} (\zeta_{\mu,\nu} + \zeta_{\nu,\mu}) - \kappa \zeta^\rho h_{\mu\nu,\rho} \\ &\quad - \kappa \zeta_{,\mu}^\rho h_{\nu\rho} - \kappa \zeta_{,\nu}^\rho h_{\mu\rho} + \frac{\kappa}{2} \zeta_{,\mu}^\rho \eta_{\rho\sigma} \zeta_{,\nu}^\sigma, \end{aligned}$$

where $\zeta_\mu = \eta_{\mu\nu}\zeta^\nu$. The higher-order corrections can be neglected in our calculations, since they give only tadpoles.

We start from Einstein gravity minimally coupled to a massless scalar field φ and a vector field A_μ . The action is

$$-\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} (\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma}.$$

The two-point function of the dressed scalar field reads

$$\langle \varphi_d | \varphi_d \rangle = \langle \varphi | \varphi \rangle - \kappa \langle \zeta^\mu \varphi_{,\mu} | \varphi \rangle - \kappa \langle \varphi | \zeta^\mu \varphi_{,\mu} \rangle + \kappa^2 \langle \zeta^\mu \varphi_{,\mu} | \zeta^\nu \varphi_{,\nu} \rangle + \mathcal{O}(\kappa^3),$$

to the quadratic order in κ . A vertical bar separates the (elementary or composite) field of momentum p (to the left) from the one of momentum $-p$ (to the right). The diagrams are shown in Fig. 1.

The covariant gauge defined by (3.6) gives the ordinary scalar self-energy

$$\langle \varphi | \varphi \rangle = \frac{i}{p^2 + i\epsilon} + \frac{3i\kappa^2(\lambda\lambda' - 3\lambda - \lambda' + 5)(\lambda' - 1)}{16\pi^2\epsilon(\lambda' - 2)^2} \times (-p^2 - i\epsilon)^{-\epsilon/2} + \mathcal{O}(\kappa^3).$$

Using the covariant cloud (4.11), the remaining diagrams of Fig. 1 give the total

$$\langle \varphi_d | \varphi_d \rangle = \frac{i}{p^2 + i\epsilon} + \frac{3i\kappa^2(\tilde{\lambda}\tilde{\lambda}' - 3\tilde{\lambda} - \tilde{\lambda}' + 5)(\tilde{\lambda}' - 1)}{16\pi^2\epsilon(\tilde{\lambda}' - 2)^2} \times (-p^2 - i\epsilon)^{-\epsilon/2} + \mathcal{O}(\kappa^3). \tag{9.1}$$

The dependence on the gauge-fixing parameters λ and λ' has disappeared, as expected. The result depends on the choice of the cloud, through the parameters $\tilde{\lambda}$ and $\tilde{\lambda}'$, and satisfies formula (7.11), due to the gauge/cloud duality.

The off-shell absorptive part of the two-point function is defined by amputating the external legs and taking the real part, multiplied by minus 2 (see [21] for details). We find

$$\text{Abso}[\langle \varphi_d | \varphi_d \rangle] = -2\text{Re}[(ip^2)\langle \varphi_d | \varphi_d \rangle(ip^2)] = -\frac{3\kappa^2(\tilde{\lambda}\tilde{\lambda}' - 3\tilde{\lambda} - \tilde{\lambda}' + 5)(\tilde{\lambda}' - 1)}{16\pi(\tilde{\lambda}' - 2)^2} \times (p^2)^2\theta(p^2) + \mathcal{O}(\kappa^3).$$

The sign of the lowest-order contribution is positive or negative, depending on the cloud parameters $\tilde{\lambda}$ and $\tilde{\lambda}'$, so $\text{Abso}[\langle \varphi_d | \varphi_d \rangle]$ is not physical.

In the case of the vector field, the undressed two-point function reads

$$\langle A_\mu | A_\nu \rangle = \langle A_\mu | A_\nu \rangle_0 + \frac{i\kappa^2(3\lambda - (1 - 2\lambda')^2)}{24\pi^2\epsilon(\lambda' - 2)^2} \times \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) (-p^2 - i\epsilon)^{-\epsilon/2},$$

where $\langle A_\mu | A_\nu \rangle_0$ is the free propagator. After the gravitational dressing, we find

$$\langle A_{\mu d} | A_{\nu d} \rangle = \langle A_\mu | A_\nu \rangle_0 + \frac{i\kappa^2(3\tilde{\lambda} - (1 - 2\tilde{\lambda}')^2)}{24\pi^2\epsilon(\tilde{\lambda}' - 2)^2} \times \left[\eta_{\mu\nu} + f(\lambda, \lambda', \tilde{\lambda}, \tilde{\lambda}', \lambda_A) \frac{p_\mu p_\nu}{p^2} \right] (-p^2 - i\epsilon)^{-\epsilon/2},$$

where f is a function that we do not report here, while λ_A is the gauge-fixing parameter of the A_μ propagator. To get rid of λ_A , we must include a gauge dressing for A_μ (besides the gravitational dressings we have already included). This operation is straightforward, since it is sufficient to consider the two-point function $\langle F_{\mu\nu d} | F_{\rho\sigma d} \rangle$ of the field strength. We obtain $\langle F_{\mu\nu d} | F_{\rho\sigma d} \rangle = \langle F_{\mu\nu} | F_{\rho\sigma} \rangle_{\lambda \rightarrow \tilde{\lambda}, \lambda' \rightarrow \tilde{\lambda}'}$, in agreement with (7.11). The absorptive part reads

$$\begin{aligned} \text{Abso}[\langle F_{\mu\nu d} | F_{\rho\sigma d} \rangle] &= -2\text{Re}[(ip^2)\langle F_{\mu\nu d} | F_{\rho\sigma d} \rangle(ip^2)] \\ &= -\frac{\kappa^2(3\tilde{\lambda} - (1 - 2\tilde{\lambda}')^2)}{24\pi(\tilde{\lambda}' - 2)^2} (p^2)^2\theta(p^2) \\ &\quad \times (\eta_{\mu\rho} p_\nu p_\sigma - \eta_{\mu\sigma} p_\nu p_\rho \\ &\quad - \eta_{\nu\rho} p_\mu p_\sigma + \eta_{\nu\sigma} p_\mu p_\rho). \end{aligned}$$

Again, it is not physical.

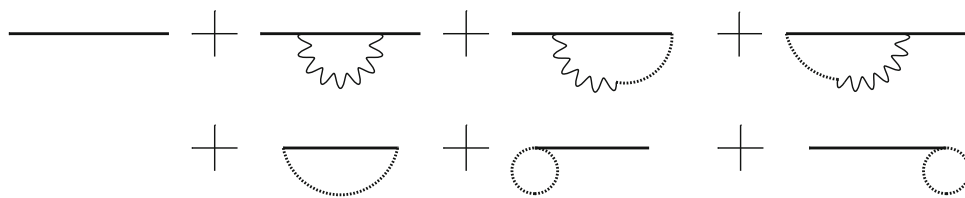
10 Purely virtual clouds and physical absorptive parts

To find physical absorptive parts, we must turn to purely virtual clouds. This is achieved as follows. The free propagators mix the graviton field $h_{\mu\nu}$ and the cloud field ζ^μ . Inside the propagators, we can distinguish three types of poles in p^2 : the physical poles, the gauge-trivial poles and the cloud poles. The cloud poles are those introduced by the cloud, and appear in $\langle h_{\mu\nu} | \zeta^\rho \rangle_0$ and $\langle \zeta^\rho | \zeta^\sigma \rangle_0$. The gauge-trivial poles are those involving the unphysical components of $h_{\mu\nu}$, which are h_{00} , h_{0i} , the longitudinal components $p^j h_{ij}(p)$ and the trace h_{ii} (in some reference frame), where i, j are space indices. The physical poles are the remaining ones.

The three classes of poles can be clearly distinguished in the special gauge of Ref. [54], which can be extended to the could sector straightforwardly. The gauge fermion (3.6) is replaced by

$$\begin{aligned} \Psi(\Phi) &= \int \bar{C}^0 \left(G_0(g) - \frac{\lambda + 3}{4} B_0 \right) \\ &\quad + \int \bar{C}^i \left(G_i(g) - \lambda \frac{\lambda + 3}{4} B_i \right), \\ G_0(g) &= \lambda \partial_0 h_{00} + \partial_0 h_{ii} - 2\partial_i h_{0i}, \\ G_i(g) &= 2\lambda \partial_0 h_{0i} - \lambda \partial_i h_{00} - \frac{\lambda + 3}{2} \partial_j h_{ij} + \frac{\lambda + 1}{2} \partial_i h_{jj}. \end{aligned} \tag{10.1}$$

Fig. 1 Two-point function of the dressed scalar field to order κ^2



The cloud fermion (1) is replaced by an analogous formula, with $\lambda \rightarrow \tilde{\lambda}$, $\tilde{C}^\mu \rightarrow \tilde{H}^\mu$, $B^\mu \rightarrow E^\mu$, $G_\mu \rightarrow V_\mu$, $h_{\mu\nu} \rightarrow h_{\mu\nu d} = h_{\mu\nu} - (\partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu)/2 + \mathcal{O}(\kappa)$.

We do not report the free propagators explicitly, because they are quite lengthy and not strictly necessary for our calculations (see [21] for their expressions in gauge theories). We just report that they contain only single poles (which is what makes the special gauge “special”), and that the gauge-trivial poles are located at $\lambda E^2 - \mathbf{p}^2 = 0$ and $4\lambda E^2 - \mathbf{p}^2(3 + \lambda) = 0$, the cloud poles are located at $\tilde{\lambda} E^2 - \mathbf{p}^2 = 0$ and $4\tilde{\lambda} E^2 - \mathbf{p}^2(3 + \tilde{\lambda}) = 0$, and the physical poles are obviously located at $E^2 - \mathbf{p}^2 = 0$, where $p^\mu = (E, \mathbf{p})$ is the propagator momentum. What is important is that a unique gauge-fixing parameter, λ , and a unique cloud parameter, $\tilde{\lambda}$, are sufficient to distinguish the three classes of poles in a manifest way.

The cloud poles must be quantized as purely virtual [14]. This means that, after performing the threshold decomposition of a diagram as explained in Ref. [14], the (multi) thresholds receiving contributions from those poles must be removed. The gauge-trivial poles can be quantized in the way we want (because the correlation functions of the dressed fields are gauge independent). The physical poles must be quantized by means of the Feynman $i\epsilon$ prescription.

It is convenient to quantize the gauge-trivial poles as purely virtual as well, like the cloud poles. Since the purely virtual poles do not contribute to the absorptive parts of the two-point functions at one loop, we can just ignore all of them.

At the end, each calculation amounts to just one diagram, the usual self-energy diagram (second drawing of Fig. 1), with a caveat: we must replace the internal graviton and vector propagators with their physical parts, which are

$$\begin{aligned} \langle A^{\mu a} | A^{\nu b} \rangle_{0 \text{ phys}} &= i \frac{\delta^{ab} \delta_i^\mu \delta_j^\nu \Pi^{ij}}{p^2 + i\epsilon}, \\ \langle h_{\mu\nu} | h_{\rho\sigma} \rangle_{0 \text{ phys}} &= \frac{i}{2} \frac{\delta_i^\mu \delta_j^\nu \delta_k^\rho \delta_l^\sigma}{p^2 + i\epsilon} (\Pi^{ik} \Pi^{jl} \\ &\quad + \Pi^{il} \Pi^{jk} - \Pi^{ij} \Pi^{kl}), \end{aligned} \tag{10.2}$$

where $\Pi^{ij} = \delta^{ij} - (p^i p^j / \mathbf{p}^2)$.

In the scalar case, the absorptive part of $\langle \varphi_d | \varphi_d \rangle$ turns out to be zero. We can partially understand this result by noting that the optical theorem relates it to the cross section of a process (graviton emission by a scalar field), which cannot occur on shell. Nevertheless, the correlation function we are

studying is not on shell. Yet, the result is still zero, due to the graviton polarizations, which are implicit in (10.2).

In the case of the vectors, we find, at rest,

$$\begin{aligned} \text{Abso}[\langle A_{id} | A_{jd} \rangle] &= -2\text{Re}[(ip^2) \langle A_{id} | A_{jd} \rangle (ip^2)] \\ &= \frac{\kappa^2 \delta_{ij}}{12\pi} (p^2)^2 \theta(p^2) + \mathcal{O}(\kappa^3), \end{aligned} \tag{10.3}$$

which is positive, as expected.

Finally, the absorptive part of the dressed graviton two-point function (in pure gravity, at rest) is

$$\begin{aligned} \text{Abso}[\langle h_{ijd} | h_{kld} \rangle] &= -2\text{Re}[(ip^2) \langle h_{ijd} | h_{kld} \rangle (ip^2)] \\ &= \frac{\kappa^2 (3\delta_{ik} \delta_{jl} + 3\delta_{il} \delta_{jk} - 2\delta_{ij} \delta_{kl})}{80\pi} \\ &\quad \times (p^2)^2 \theta(p^2) + \mathcal{O}(\kappa^3), \end{aligned} \tag{10.4}$$

which is again positive definite.

Now we switch to the theory of quantum gravity with purely virtual particles [28]. It is convenient to formulate it in the variables of Ref. [53], to gain an explicit distinction among the graviton, the inflaton ϕ and the massive purely virtual spin-2 particle $\chi_{\mu\nu}$. We can actually ignore $\chi_{\mu\nu}$, because it does not contribute to the absorptive parts that we want to compute. Neglecting the cosmological constant, the relevant terms of the action are

$$\begin{aligned} &-\frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \frac{1}{2} \int \sqrt{-g} \left[g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) \right. \\ &\quad \left. - \frac{3m_\phi^2}{2\kappa^2} (1 - e^{\kappa\phi\sqrt{2/3}})^2 \right] \\ &+ \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} e^{\kappa\phi\sqrt{2/3}} (\partial_\mu \varphi) (\partial_\nu \varphi) \\ &- \frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma}. \end{aligned}$$

The absorptive part (10.3) of the vector two-point function does not change. The one of the scalar two-point function is no longer zero, because it receives a contribution from the inflaton. In the high-energy limit (where we can neglect the mass m_ϕ), we find

$$\text{Abso}[\langle \varphi_d | \varphi_d \rangle] = \frac{\kappa^2}{48\pi} (p^2)^2 \theta(p^2) + \mathcal{O}(\kappa^3).$$

Switching off the matter sector, the absorptive part (10.4) of the dressed graviton two-point function also receives a correction from the inflaton ϕ , and the final result is (10.4) multiplied by 19/18.

11 Renormalization

In this section we study the renormalization of the extended theory. We assume that the starting theory of quantum gravity is renormalizable by power counting, like the theory based on purely virtual particles of Ref. [28]. To have better power-counting behaviors, it may be convenient to use a higher-derivative gauge-fixing, as in [55], and higher-derivative clouds as well. The cloud fields $\zeta^{\mu i}$ have dimension minus one in units of mass, so nonpolynomial functions of them are turned on by renormalization.

The theory of [28] is unitary. If the clouds are purely virtual, the complete dressed theory is unitary as well. However, the arguments of this section do not rely on unitarity, so the results we obtain also apply to nonunitary clouds, and even nonunitary theories, such as the Stelle theory [56], where the Feynman prescription is used for the quantization of the every field (and so $\chi_{\mu\nu}$ is a ghost).

When the arguments work for gravity exactly as they do for gauge theories, we skip the details of the proofs. The reader should refer to [21] for the missing derivations.

First, the master equation (4.9) satisfied by S_{tot} implies an analogous master equation

$$(\Gamma_{\text{tot}}, \Gamma_{\text{tot}}) = 0 \tag{11.1}$$

for the generating functional $\Gamma_{\text{tot}} = W_{\text{tot}}(J, \tilde{J}, K, \tilde{K}) - \int \Phi^\alpha J_\alpha - \sum_i \int \tilde{\Phi}^{\alpha i} \tilde{J}_\alpha^i$ of the one-particle irreducible (1PI) Green functions, where $\Phi^\alpha = \delta_r W_{\text{tot}} / \delta J^\alpha$, $\tilde{\Phi}^{\alpha i} = \delta_r W_{\text{tot}} / \delta \tilde{J}^{\alpha i}$. Second, the i th cloud invariance (4.10) of the total action S_{tot} , which is the identity $(S_K^{\text{cloud } i}, S_{\text{tot}}) = 0$, implies the i th cloud invariance

$$(S_K^{\text{cloud } i}, \Gamma_{\text{tot}}) = 0 \tag{11.2}$$

of the Γ functional.

Proceeding inductively, we can show that the total renormalized action $S_{R\text{tot}}$ satisfies the renormalized master equations

$$(S_{R\text{tot}}, S_{R\text{tot}}) = 0, \quad (S_K^{\text{cloud } i}, S_{R\text{tot}}) = 0. \tag{11.3}$$

Since the cloud symmetry is the most general shift of the cloud fields, the second equation ensures that the total renormalized action is the sum of a cloud-independent renormalized action S_R and some cloud-exact rest. Separating S_K^{cloud} itself, which is nonrenormalized, we can write

$$S_{R\text{tot}} = S_R + (S_K^{\text{cloud}}, \Upsilon_R) + S_K^{\text{cloud}}$$

for some local functional Υ_R . It is possible to extend the arguments of Sect. 5 to $S_{R\text{tot}}$ and show that S_R is cloud independent and coincides with the usual renormalized action, while the scattering amplitudes are gauge independent.

Moreover, the dependence on the gauge-fixing parameters and the dependences on the cloud parameters go through renormalization as canonical transformations. In particular,

the beta functions of the physical parameters are gauge independent and cloud independent.

Some simplification comes from the introduction of “cloud numbers”, besides the usual ghost number. The usual ghost number is defined to be equal to 1 for C^μ , minus 1 for \tilde{C}^μ , K_μ^B , $K_g^{\mu\nu}$, K_A^μ , K_φ , $\tilde{K}_\mu^{\zeta^i}$ and $\tilde{K}_\mu^{H^i}$, minus 2 for K_μ^C , and 0 for every other field and source. The i th cloud number is defined to be equal to one for $H^{\mu i}$, minus one for $\tilde{H}^{\mu i}$, $\tilde{K}_\mu^{H^i}$ and $\tilde{K}_\mu^{E^i}$, and zero in all the other cases.

Every term of the action S_{tot} is neutral with respect to the ghost and cloud numbers just defined, with the exception of the source terms $\int H^{\mu i} \tilde{K}_\mu^{\zeta^i}$. Since, however, such terms cannot be used in nontrivial 1PI diagrams, all the counterterms are neutral. This ensures that each cloud number is separately conserved in 1PI diagrams beyond the tree level.

Power counting is not very helpful in the cloud sectors, since the cloud fields $\zeta^{\mu i}$ have negative dimensions. A simplification can be achieved by combining the background-field method with the Batalin–Vilkovisky formalism, as shown in Ref. [57]. So doing, the gauge and cloud transformations are not renormalized. Yet, the cloud sectors are nonrenormalizable, strictly speaking, since infinitely many counterterms are allowed by power counting and the symmetry constraints. For example, we can always build gauge invariant candidate counterterms that depend nontrivially on the fields of each cloud sector and are exact under every cloud symmetry. Examples are

$$(S_K^{\text{cloud } 1}, (S_K^{\text{cloud } 2}, \dots (S_K^{\text{cloud } N}, \tilde{\Upsilon}))),$$

where N denotes the number of clouds, and $\tilde{\Upsilon}$ is a gauge invariant local functional, built with gauge invariant combinations of cloud fields, such as (8.5). We cannot exclude that different cloud sectors mix under renormalization. Nevertheless, we can prove that the counterterms that do not contain fields and sources of some cloud sector are the same as if that sector were absent.

The renormalization in every non-background-field approach can be reached by means of a (renormalized) canonical transformation. Details on this can be found in Ref. [57].

Equipped with the renormalized action and the renormalized gauge transformations, we can build dressed fields that are gauge-invariant with respect to the latter. The correlation functions of the renormalized dressed fields are gauge independent.

Finally, the arguments that lead to the identity (5.7) continue to hold after renormalization. We have an identity analogous to (5.7), where the dressed and undressed fields are replaced by their renormalized versions. In particular, the S matrix amplitudes of the renormalized dressed fields are cloud independent and coincide with the usual S matrix amplitudes of the renormalized undressed fields. Since the

former are gauge independent by construction, the latter are gauge independent as well.

12 Conclusions

We have extended quantum field theory to include purely virtual cloud sectors, to study point-dependent physical observables in general relativity and quantum gravity, with particular emphasis on the gauge invariant versions of the metric and the matter fields. The cloud diagrammatics and its Feynman rules are derived from a local action, which is built by means of cloud fields $\zeta^{\mu i}$ and their anticommuting partners $H^{\mu i}$. It incorporates the choices of clouds, the cloud Faddeev–Popov determinants and the cloud symmetries. The usual gauge-fixing must be cloud invariant, while the cloud-fixings must be gauge invariant.

The formalism allows us to define physical, off-mass-shell correlation functions of point-dependent observables, and calculate them within the realm of perturbative quantum field theory. Every field insertion can be equipped with its own, independent cloud. We may eventually replace the elementary fields with the dressed ones everywhere, to work in a manifestly gauge independent environment.

The extension does not change the fundamental physics, in the sense that the ordinary correlation functions and the S matrix amplitudes are unmodified. It allows us to compute new correlation functions, those containing insertions of dressed fields. If the clouds are quantized as purely virtual, the extended theory is unitary. In particular, the correlation functions of the dressed fields obey the off-shell, diagrammatic version of the optical theorem. No unwanted degrees of freedom propagate.

A Batalin–Vilkovisky formalism and its Zinn–Justin master equations allow us to study renormalizability and the WTST identities to all orders in the perturbative expansion. A gauge/cloud duality shows that the usual gauge-fixing is nothing but a particular cloud, provided it is rendered purely virtual. A purely virtual gauge-fixing is a natural upgrade of the so-called physical gauges [58].

We have illustrated the key properties of our approach by computing the one-loop two-point functions of dressed scalars, vectors and gravitons, and comparing purely virtual clouds to non-purely virtual clouds, in Einstein gravity as well as in quantum gravity with purely virtual particles. If purely virtual clouds are used, the absorptive parts are positive, cloud independent and gauge independent. This suggests that they are properties of the fundamental theory. The absorptive parts are not positive, in general, if non-purely virtual clouds are used.

Pure virtuality can be a natural environment to extract physical information from off-shell correlation functions. Among the other things, it allows us to break global invari-

ances without breaking the local ones, avoiding undesirable consequences on unitarity. We can also define short-distance scattering processes, where the results depend on the clouds. It emerges that in such processes the observer necessarily disturbs the observed phenomenon. The net effect is that the amplitudes are physical, but depend on the cloud parameters. Yet, after sacrificing a few measurements for the calibration of our instrumentation, we are able to make testable, and possibly falsifiable, predictions.

We did not introduce true matter to define the metric as a physical observable. Instead, we used purely virtual dressings. In this sense, our approach provides the identification of a complete set of observables in quantum gravity. Yet, it raises new issues. It would be interesting to clarify the relation between the Komar–Bergmann classical approach [2–5] and the one formulated here, as well as investigate the nonlocal nature of the algebra of commutators (check [15, 16] for this aspect in the Donnelly–Giddings approach). It is important to recall that pure virtuality at the operatorial level still has to be understood (the formulation we have today being mainly diagrammatic [14]), so we may not be ready to use the results of this paper for a canonical analysis of the observables and a Hamiltonian quantization.

The formulation developed here is perturbative, around flat space. To overcome these limitations, we need to face old and new challenges. An obstacle is that the prescription for purely virtual particles is understood only diagrammatically (hence, perturbatively) at present. Even standard procedures, like the resummation of self-energies into effective propagators, hide unexpected features, as discussed in Ref. [36]. Another challenge is to generalize or adapt the known nonperturbative methods. The numeric (lattice) approaches are not immediately helpful, since they are mostly suited for Euclidean theories, where the crucial aspects of purely virtual particles disappear. Results about the perturbative expansion around a non flat background have been obtained in the context of primordial cosmology [29]. Still, a systematics of purely virtual particles in curved spacetime is awaiting to be developed.

Acknowledgements This work was supported in part by the European Regional Development Fund through the CoE program grant TK133 and the Estonian Research Council grant PRG803. We thank the CERN theory group for hospitality during the final stage of the project.

Data availability This manuscript has no associated data or the data will not be deposited. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article

are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

Funded by SCOAP³. SCOAP³ supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

- J. Géheniau, R. Debever, Les quatorze invariants de courbure de l'espace Riemannien a quatre dimensions. *Helv. Phys. Acta Suppl.* **4**, 101 (1956)
- A.B. Komar, Construction of a complete set of independent observables in the general theory of relativity. *Phys. Rev.* **111**, 1182 (1958). <https://doi.org/10.1103/PhysRev.111.1182>
- P. Bergmann, A. Komar, Poisson brackets between locally defined observables in general relativity. *Phys. Rev. Lett.* **4**, 432 (1960). <https://doi.org/10.1103/PhysRevLett.4.432>
- P.G. Bergmann, Conservation laws in general relativity as the generators of coordinate transformations. *Phys. Rev.* **112**, 287 (1958). <https://doi.org/10.1103/PhysRev.112.287>
- P.G. Bergmann, Observables in general relativity. *Rev. Mod. Phys.* **33**, 510 (1961). <https://doi.org/10.1103/RevModPhys.33.510>
- B. De Witt, in *Gravitation: An Introduction to Current Research*, ed. by L. Witten (Wiley, New York, 1962)
- B. de Witt, Quantum theory of gravity I. The canonical theory. *Phys. Rev.* **160**, 1113 (1967). <https://doi.org/10.1103/PhysRev.160.1113>
- J. Earman, J. Norton, What price spacetime substantivalism? The hole story. *Br. J. Philos. Sci.* **38**, 515 (1987). <https://doi.org/10.1093/bjps/38.4.515>
- J. Earman, *World Enough and Space-time: Absolute Versus Relational Theories of Spacetime* (MIT Press, Cambridge, 1989)
- C. Rovelli, What is observable in classical and quantum gravity? *Class. Quantum Gravity* **8**, 297 (1991). <https://doi.org/10.1088/0264-9381/8/2/011>
- J.D. Brown, D. Marolf, Relativistic material reference systems. *Phys. Rev. D* **53**, 1835 (1996). <https://doi.org/10.1103/physrevd.53.1835>
- C. Rovelli, GPS observables in general relativity. *Phys. Rev. D* **65**, 044017 (2002). <https://doi.org/10.1103/PhysRevD.65.044017>. [arXiv:gr-qc/0110003](https://arxiv.org/abs/gr-qc/0110003)
- D. Anselmi, A new quantization principle from a minimally non time-ordered product. *J. High Energy Phys.* **12**, 088 (2022). [https://doi.org/10.1007/JHEP12\(2022\)088](https://doi.org/10.1007/JHEP12(2022)088). 22A5 Renorm and <https://renormalization.com/22a5/>. [arXiv:2210.14240](https://arxiv.org/abs/2210.14240) [hep-th]
- D. Anselmi, Diagrammar of physical and fake particles and spectral optical theorem. *J. High Energy Phys.* **11**, 030 (2021). <https://renormalization.com/21a5/>. 21A5 Renormalization.com. [https://doi.org/10.1007/JHEP11\(2021\)030](https://doi.org/10.1007/JHEP11(2021)030). [arXiv:2109.06889](https://arxiv.org/abs/2109.06889) [hep-th]
- W. Donnelly, S.B. Giddings, Diffeomorphism-invariant observables and their nonlocal algebra. *Phys. Rev. D* **93**, 024030 (2016). <https://doi.org/10.1103/PhysRevD.93.024030>. [arXiv:1507.07921](https://arxiv.org/abs/1507.07921) [hep-th]
- W. Donnelly, S.B. Giddings, Observables, gravitational dressing, and obstructions to locality and subsystems. *Phys. Rev. D* **94**, 104038 (2016). <https://doi.org/10.1103/PhysRevD.94.104038>. [arXiv:1607.01025](https://arxiv.org/abs/1607.01025) [hep-th]
- P.A.M. Dirac, Gauge invariant formulation of quantum electrodynamics. *Can. J. Phys.* **33**, 650 (1955). <https://doi.org/10.1139/p55-081>
- M. Lavelle, D. McMullan, Observables and gauge fixing in spontaneously broken gauge theories. *Phys. Lett. B* **347**, 89 (1995). [https://doi.org/10.1016/0370-2693\(95\)00046-N](https://doi.org/10.1016/0370-2693(95)00046-N). [arXiv:hep-ph/9412145](https://arxiv.org/abs/hep-ph/9412145)
- M. Lavelle, D. McMullan, The color of quarks. *Phys. Lett. B* **371**, 83 (1996). [https://doi.org/10.1016/0370-2693\(95\)01571-X](https://doi.org/10.1016/0370-2693(95)01571-X). [arXiv:hep-ph/9509343](https://arxiv.org/abs/hep-ph/9509343)
- M. Lavelle, D. McMullan, Constituent quarks from QCD. *Phys. Rep.* **279**, 1 (1997). [https://doi.org/10.1016/S0370-1573\(96\)00019-1](https://doi.org/10.1016/S0370-1573(96)00019-1). [arXiv:hep-ph/9509344](https://arxiv.org/abs/hep-ph/9509344)
- D. Anselmi, Purely virtual extension of quantum field theory for gauge invariant fields: Yang–Mills theory. *Eur. Phys. J. C* **83**, 544 (2023). <https://doi.org/10.1140/epjc/s10052-023-11717-2>. <https://renormalization.com/22a3/>. 22A3 Renorm and [arXiv:2207.11271](https://arxiv.org/abs/2207.11271) [hep-ph]
- R.E. Cutkosky, Singularities and discontinuities of Feynman amplitudes. *J. Math. Phys.* **1**, 429 (1960). <https://doi.org/10.1063/1.1703676>
- M. Veltman, Unitarity and causality in a renormalizable field theory with unstable particles. *Physica* **29**, 186 (1963). [https://doi.org/10.1016/S0031-8914\(63\)80277-3](https://doi.org/10.1016/S0031-8914(63)80277-3)
- G. 't Hooft, Renormalization of massless Yang–Mills fields. *Nucl. Phys. B* **33**, 173 (1971). [https://doi.org/10.1016/0550-3213\(71\)90395-6](https://doi.org/10.1016/0550-3213(71)90395-6)
- G. Hooft, Renormalizable Lagrangians for massive Yang–Mills fields. *Nucl. Phys. B* **35**, 167 (1971). [https://doi.org/10.1016/0550-3213\(71\)90139-8](https://doi.org/10.1016/0550-3213(71)90139-8)
- G. 't Hooft, M. Veltman, *Diagrammar*, CERN report. CERN-73-09. <https://cdsweb.cern.ch/record/186259>
- M. Veltman, *Diagrammatica. The path to Feynman rules* (Cambridge University Press, New York, 1994)
- D. Anselmi, On the quantum field theory of the gravitational interactions. *J. High Energy Phys.* **06**, 086 (2017). [https://doi.org/10.1007/JHEP06\(2017\)086](https://doi.org/10.1007/JHEP06(2017)086). <https://renormalization.com/17a3/> 17A3 Renormalization.com. [arXiv:1704.07728](https://arxiv.org/abs/1704.07728) [hep-th]
- D. Anselmi, E. Bianchi, M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term. *J. High Energy Phys.* **07**, 211 (2020). [https://doi.org/10.1007/JHEP07\(2020\)211](https://doi.org/10.1007/JHEP07(2020)211) <https://renormalization.com/20a2/>. 20A2 Renormalization.com and [arXiv:2005.10293](https://arxiv.org/abs/2005.10293) [hep-th]
- D. Anselmi, K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Müürsepp, M. Piva, M. Raidal, Phenomenology of a fake inert doublet model. *J. High Energy Phys.* **10**, 132 (2021). [https://doi.org/10.1007/JHEP10\(2021\)132](https://doi.org/10.1007/JHEP10(2021)132). <https://renormalization.com/21a3/> 21A3 Renormalization.com and [arXiv:2104.02071](https://arxiv.org/abs/2104.02071) [hep-ph]
- D. Anselmi, K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Müürsepp, M. Piva, M. Raidal, A fake doublet solution to the muon anomalous magnetic moment. *Phys. Rev. D* **104**, 035009 (2021). <https://doi.org/10.1103/PhysRevD.104.035009>. <https://renormalization.com/21a4/>. 21A4 Renormalization.com and [arXiv:2104.03249](https://arxiv.org/abs/2104.03249) [hep-ph]
- F. Bloch, A. Nordsieck, Note on the radiation field of the electron. *Phys. Rev.* **52**, 54 (1937). <https://doi.org/10.1103/PhysRev.52.54>
- T. Kinoshita, Mass singularities of Feynman amplitudes. *J. Math. Phys.* **3**, 650 (1962). <https://doi.org/10.1063/1.1724268>
- T.D. Lee, M. Nauenberg, Degenerate systems and mass singularities. *Phys. Rev.* **133**, B1549 (1964). <https://doi.org/10.1103/PhysRev.133.B1549>
- S. Weinberg, Infrared photons and gravitons. *Phys. Rev.* **140**, B516 (1965). <https://doi.org/10.1103/PhysRev.140.B516>
- D. Anselmi, Dressed propagators, fakeon self-energy and peak uncertainty. *J. High Energy Phys.* **06**, 058 (2022). [https://doi.org/10.1007/JHEP06\(2022\)058](https://doi.org/10.1007/JHEP06(2022)058). <https://renormalization.com/22a1/>. 22A1 Renormalization.com and [arXiv: 2201.00832](https://arxiv.org/abs/2201.00832) [hep-ph]
- E.C.G. Stueckelberg, Die Wechselwirkungs Kraefte in der Elektrodynamik und in der Feldtheorie der Kernkraefte (I), [The inter-

- action forces in electrodynamics and in the field theory of nuclear forces (I)]. *Helv. Phys. Acta* **11**, 225 (1938). <https://doi.org/10.5169/seals-110852>
38. B. de Wit, M.T. Grisaru, Compensating fields and anomalies, in *Quantum Field Theory and Quantum Statistics*, vol. 2, ed. by I.A. Batalin (Adam Hilger, C.J. Isham and G.A. Vilkovisky, 1987)
39. C.G. Bollini, J.J. Giambiagi, The number of dimensions as a regularizing parameter, *Nuovo Cim. B* **12**, 20 (1972). <https://doi.org/10.1007/BF02895558>
40. C.G. Bollini, J.J. Giambiagi, Lowest order divergent graphs in ν -dimensional space. *Phys. Lett. B* **40**, 566 (1972). [https://doi.org/10.1016/0370-2693\(72\)90483-2](https://doi.org/10.1016/0370-2693(72)90483-2)
41. G. Hoof, M. Veltman, Regularization and renormalization of gauge fields. *Nucl. Phys. B* **44**, 189 (1972). [https://doi.org/10.1016/0550-3213\(72\)90279-9](https://doi.org/10.1016/0550-3213(72)90279-9)
42. G.M. Cicuta, E. Montaldi, Analytic renormalization via continuous space dimension. *Lett. Nuovo Cim.* **4**, 329 (1972). <https://doi.org/10.1007/BF02756527>
43. I.A. Batalin, G.A. Vilkovisky, Gauge algebra and quantization. *Phys. Lett. B* **102**, 27 (1981). [https://doi.org/10.1016/0370-2693\(81\)90205-7](https://doi.org/10.1016/0370-2693(81)90205-7)
44. I.A. Batalin, G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators. *Phys. Rev. D* **28**, 2567 (1983). <https://doi.org/10.1103/PhysRevD.28.2567>. Erratum-ibid. **30**, 508 (1984). <https://doi.org/10.1103/PhysRevD.30.508>
45. J.C. Ward, An identity in quantum electrodynamics. *Phys. Rev.* **78**, 182 (1950). <https://doi.org/10.1103/PhysRev.78.182>
46. Y. Takahashi, On the generalized Ward identity. *Nuovo Cimento* **6**, 371 (1957). <https://doi.org/10.1007/BF02832514>
47. A.A. Slavnov, Ward identities in gauge theories. *Theor. Math. Phys.* **10**, 99 (1972). <https://doi.org/10.1007/BF01090719>
48. J.C. Taylor, Ward identities and charge renormalization of Yang–Mills field. *Nucl. Phys. B* **33**, 436 (1971). [https://doi.org/10.1016/0550-3213\(71\)90297-5](https://doi.org/10.1016/0550-3213(71)90297-5)
49. L.D. Faddeev, V. Popov, Feynman diagrams for the Yang–Mills field. *Phys. Lett. B* **25**, 29 (1967). [https://doi.org/10.1016/0370-2693\(67\)90067-6](https://doi.org/10.1016/0370-2693(67)90067-6)
50. N. Nakanishi, Covariant quantization of the electromagnetic field in the Landau gauge. *Prog. Theor. Phys.* **35**, 1111 (1966). <https://doi.org/10.1143/PTP.35.1111>
51. B. Lautrup, Canonical quantum electrodynamics in covariant gauges. *Kgl. Dan. Vid. Se. Mat. Fys. Medd.* **35**(11), 1 (1967)
52. J. Zinn-Justin, Renormalization of gauge theories, Bonn lectures 1974, in *Trends in Elementary Particle Physics*, ed. by H. Rollnik and K. Dietz, Lecture Notes in Physics, vol. 37, p. 1 (Springer Verlag, Berlin, 1975)
53. D. Anselmi, M. Piva, Quantum gravity, fakeons and micro-causality. *J. High Energy Phys.* **11**, 21 (2018). [https://doi.org/10.1007/JHEP11\(2018\)021](https://doi.org/10.1007/JHEP11(2018)021). <https://renormalization.com/18a3/>. 18A3 Renormalization.com and [arXiv:1806.03605](https://arxiv.org/abs/1806.03605) [hep-th]
54. D. Anselmi, Aspects of perturbative unitarity. *Phys. Rev. D* **94**, 025028 (2016). <https://doi.org/10.1103/PhysRevD.94.025028>. <https://renormalization.com/16a1/>. 16A1 Renorm and [arXiv:1606.06348](https://arxiv.org/abs/1606.06348) [hep-th]
55. D. Anselmi, M. Piva, The ultraviolet behavior of quantum gravity. *J. High Energy Phys.* **05**, 27 (2018). [https://doi.org/10.1007/JHEP05\(2018\)027](https://doi.org/10.1007/JHEP05(2018)027). 18A2 Renormalization.com and [arXiv:1803.07777](https://arxiv.org/abs/1803.07777) [hep-th]
56. K.S. Stelle, Renormalization of higher derivative quantum gravity. *Phys. Rev. D* **16**, 953 (1977). <https://doi.org/10.1103/PhysRevD.16.953>
57. D. Anselmi, Background field method and the cohomology of renormalization. *Phys. Rev. D* **93**, 065034 (2016). <https://doi.org/10.1103/PhysRevD.93.065034>. [arXiv:1511.01244](https://arxiv.org/abs/1511.01244) [hep-th]
58. P. Gaigg, W. Kummer, M. Schweda (eds.), *Physical and Nonstandard Gauges*, Lecture Notes in Physics, vol. 361 (Springer, Heidelberg, 1990). <https://doi.org/10.1007/BFb0015131>