Eur. Phys. J. C (2023) 83:733
https://doi.org/10.1140/epjc/s10052-023-11903-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Quantum interaction between two entangled non point-like objects in the presence of boundaries

Dan Wen[1,a], **Yongshun Hu**[1,2,3,b]

[1] College of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[2] School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
[3] School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

Received: 27 March 2023 / Accepted: 4 August 2023 / Published online: 18 August 2023

Abstract We explore, in the framework of linearized quantum gravity, the quantum gravitational quadrupole-quadrupole interaction between two entangled non point-like objects in the presence of both Dirichlet and Neumann boundaries. The results show that, compared to the case without boundaries, the interaction can be either enhanced or weakened depending on the geometrical arrangement of the objects with respect to the boundaries. In the limit when the two-object system is placed very close to the Dirichlet boundary, the near-regime interaction potential is larger than that of the pure vacuum case when the two objects are placed perpendicular to the boundary but smaller when parallel to it, while, in the far regime, such strong and weak relations between potentials are just opposite to that in the near regime. And, there exists a new r^{-2} far-regime behavior of the interaction potential under the perpendicular configuration. For the case of Neumann boundary, the strong and weak relations between the interaction potentials under perpendicular or parallel configurations and the case without boundary are opposite to the Dirichlet circumstance both in the near and far regimes. Besides, the novel r^{-2} far-regime behavior occurs for the parallel rather than perpendicular configuration in the presence of Neumann boundary.

1 Introduction

Quantum vacuum fluctuation, which is an inevitable consequence of quantum theory, may induce some observable effects. The electromagnetic Casimir–Polder (CP) interaction is one of the well-known examples, which arises from the dipole–dipole interaction between two neutral atoms or molecules induced by electromagnetic vacuum fluctuations [1]. Such interatomic or intermolecular CP effects have been widely studied in different circumstances and found to be influenced significantly by the atomic or molecular states [2–16], the external electromagnetic fields [17–27] and boundaries [28,29]. For instance, the CP interaction potential behaves as r^{-6} and r^{-7} in the near and far regimes respectively when the atoms are in their ground states [1], while it decreases as r^{-3} and r^{-1} in the near and far regimes respectively when the atoms are prepared in an entangled state [15] or placed in external electromagnetic fields [17,19].

Likewise, in the gravitational case, there may also exist a CP-like quantum gravitational quadrupole–quadrupole interactions between non point-like objects if one accepts that basic quantum principles are applicable to gravity. Unfortunately, a full theory of quantum gravity is elusive at present. Nonetheless, one can still study low energy quantum gravitational effects in the framework of effective field theory or linearized quantum gravity. Similar to the electromagnetic case, the CP-like quantum gravitational quadrupole–quadrupole interaction potentials also have been found to be significantly different when the non point-like objects are prepared in different states or placed in certain environments. For example, the gravitational CP-like quadrupole–quadrupole potential between two ground-state objects behaves as r^{-10} and r^{-11} in the near and far regimes respectively [30–33], while it decreases as r^{-5} and r^{-1} in the near and far regimes respectively when the objects are in an entangled state or in external gravitational radiations [34–36]. Moreover, in the presence of gravitational boundaries, the quantum gravitational quadrupole–quadrupole interaction between two ground-state objects is found to be always strengthened as compared with the case without boundaries [37].

The CP-like quantum gravitational interactions discussed above are obtained respectively under a particular condition.

[a] e-mail: wendan@cqupt.edu.cn (corresponding author)
[b] e-mail: huyongshun@ucas.ac.cn

Since the entanglement, the external gravitational radiations and gravitational boundaries can all enhance the quantum gravitational quardrupole–quardrupole interaction in vacuum, a natural question arises as to whether such quantum gravitational effects can be more significant under certain composite circumstances. Physically, the gravitational CP-like interaction for two entangled objects in vacuum is a second-order effect while it is a fourth-order effect for objects in their ground states or in an external gravitational field, due to the fact that the former is related to the single-graviton processes while the latter is relevant to the two-graviton processes. Hence, the second-order quantum effect correlated to the single-graviton processes is expected preferentially and the composite case of two entangled objects together with gravitational boundaries is thus taken into account first. Note here that, although the reflection of gravitational waves by ordinary materials is hardly possible [38], it might be realized by quantum matter such as superconducting films [39,40].

In this paper, we explore the quantum gravitational quardrupole–quardrupole interaction between two entangled non point-like objects in the presence of both Dirichlet and Neumann boundaries based on the method proposed by Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) [41,42]. First, we give a brief description of the formulae for the interaction between the two objects. Then, we obtain the general expression of the interaction energy between the two objects in the presence of Dirichlet and Neumann boundaries respectively, and discuss these results in specific cases. Throughout this paper, the Latin indices run from 1 to 3 and the Einstein summation convention for repeated indices is assumed.

2 Basic equations

We consider two entangled non point-like objects (labeled as A and B) coupled with the fluctuating gravitational fields in vacuum in the presence of gravitational plane boundaries. For simplicity, the objects A and B are modeled as two-level systems with the ground and excited states being $|g\rangle$ and $|e\rangle$ respectively, and the corresponding energy spacing is labeled as ω_0. The total Hamiltonian of the system is

$$H = H_S + H_F + H_I, \tag{1}$$

where H_S denotes the Hamiltonian of the two-level systems (A and B), H_F denotes the Hamiltonian of the gravitational fields in vacuum, and H_I denotes the interaction Hamiltonian between the non point-like objects and the gravitational fields. Here H_I takes the form

$$H_I = -\frac{1}{2} Q^A_{ij} E_{ij}(\vec{x}_A) - \frac{1}{2} Q^B_{ij} E_{ij}(\vec{x}_B), \tag{2}$$

where Q^{ξ}_{ij} is the quadrupole moment operator of object ξ ($\xi = A, B$), and E_{ij} is the gravitoelectric tensor of the fluctuating gravitational fields in vacuum defined as $E_{ij} = -c^2 C_{0i0j}$ by an analogy between the linearized Einstein field equations and the Maxwell equations [43–49], where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor and c the speed of light. Under the weak-field approximation, the metric tensor for the fluctuating gravitational fields can be expanded as a sum of the flat spacetime metric and a linearized perturbation $h_{\mu\nu}$. Then, the gravitoelectric tensor E_{ij} can be expressed as

$$E_{ij} = \frac{1}{2}\ddot{h}_{ij}. \tag{3}$$

where a dot denotes the derivative with respect to time t. For the pure vacuum case, the quantized metric perturbations can be expressed as (in the transverse traceless gauge) [50]

$$h_{ij} = \sum_{\vec{p},\lambda} h^{\lambda}_{ij,\vec{p}} = \sum_{\vec{p},\lambda} \sqrt{\frac{\hbar G}{\omega c^2 \pi^2}} [a_\lambda(\omega) e^{(\lambda)}_{ij}(\vec{p}) e^{i(\vec{p}\cdot\vec{x}-\omega t)} + \text{H.c.}], \tag{4}$$

where $h^{\lambda}_{ij,\vec{p}}$ is the gravitational field mode with wave vector $\vec{p}$, $a_\lambda(\omega)$ represents the annihilation operator of the fluctuating gravitational fields, λ denotes the polarization states, $\omega = c|\vec{p}| = c(p_x^2 + p_y^2 + p_z^2)^{1/2}$, $e^{(\lambda)}_{ij}(\vec{p})$ are polarization tensors, and H.c. denotes the Hermitian conjugate.

We assume that the two objects are in their maximally entangled states, i.e., the symmetric or antisymmetric state,

$$\psi_\pm = \frac{1}{\sqrt{2}}(|g_A\rangle|e_B\rangle \pm |e_A\rangle|g_B\rangle), \tag{5}$$

and denote the vacuum state of the gravitational field as $|0\rangle$. To investigate the quantum gravitational quadrupole–quadrupole interaction potential between two entangled objects in vacuum, the second-order DDC formalism has ever been exploited and shown to be really convenient, see in Ref. [34]. Hence, in the presence of gravitational boundaries, we employ this formalism again to calculate interaction energy shift between objects A and B, which is [34]

$$\Delta E_{AB} = -\frac{i}{4}\int_{t_0}^{t} dt' \chi^F_{ijkl}(\vec{x}_A(t), \vec{x}_B(t')) C^{AB}_{ijkl}(t,t') + (A \rightleftharpoons B \text{ terms}). \tag{6}$$

Here $C^{AB}_{ijkl}(t,t')$ and $\chi^F_{ijkl}(\vec{x}_A(t), \vec{x}_B(t'))$ are respectively the statistical functions introduced for the gravitational field and the objects, which take the form

$$C^{AB}_{ijkl}(t,t') = \frac{1}{2}\langle\psi_\pm|\{Q^{AF}_{ij}(t), Q^{BF}_{kl}(t')\}|\psi_\pm\rangle, \tag{7}$$

$$\chi^F_{ijkl}(\vec{x}(t), \vec{x}(t')) = \frac{1}{2}\langle 0|[E^F_{ij}(\vec{x}(t)), E^F_{kl}(\vec{x}(t'))]|0\rangle, \tag{8}$$

where the label "F" denotes the free part of the operator, i.e., the part presents even in the absence of interaction.

3 Dirichlet boundary condition

Now let us consider the quantum gravitational quadrupole–quadrupole potential between two entangled non point-like objects when a Dirichlet boundary is present. For simplicity, this plane boundary is assumed to be placed at $z = 0$, so that the metric perturbations satisfies $h^{\lambda}_{ij,\vec{p}}|_{z=0} = 0$ and the gravitational field mode $h^{\lambda}_{ij,\vec{p}}$ can be expressed as

$$h^{\lambda}_{ij,\vec{p}} = \sqrt{\frac{\hbar G}{2c^2\omega\pi^2}}\left\{a_\lambda(\omega)\left[e^{(\lambda)}_{ij}(\vec{p})e^{i(\vec{p}\cdot\vec{x}-\omega t)} - e^{(\lambda)}_{ij}(\vec{p}_-)\times e^{i(\vec{p}_-\cdot\vec{x}-\omega t)}\right] + \text{H.c.}\right\}, \tag{9}$$

where $\vec{p}_- = \{p_x, p_y, -p_z\}$. Then, according to Eqs. (3) and (8), the statistical function of the gravitational field $\chi^F_{ijkl}(\vec{x}_A(t), \vec{x}_B(t'))$ in the presence of a Dirichlet boundary can be expressed as

$$\begin{aligned}\chi^F_{ijkl}(\vec{x}(t), \vec{x}(t')) &= \frac{1}{8}\langle 0|[\ddot{h}_{ij}(\vec{x}(t)), \ddot{h}_{kl}(\vec{x}(t'))]|0\rangle \\ &= \frac{\hbar c G}{16\pi^2}\int d^3\vec{p}\; p^3 \sum_\lambda \Big[e^{(\lambda)}_{ij}(\vec{p})e^{(\lambda)}_{kl}(\vec{p})e^{i\vec{p}\cdot\vec{r}} \\ &\quad + e^{(\lambda)}_{ij}(\vec{p}_-)e^{(\lambda)}_{kl}(\vec{p}_-)e^{i\vec{p}_-\cdot\vec{r}_-} \\ &\quad - e^{(\lambda)}_{ij}(\vec{p})e^{(\lambda)}_{kl}(\vec{p}_-)e^{i\vec{p}\cdot\vec{r}_-} \\ &\quad - e^{(\lambda)}_{ij}(\vec{p}_-)e^{(\lambda)}_{kl}(\vec{p})e^{i\vec{p}_-\cdot\vec{r}}\Big]\left(e^{-i\omega\Delta t'} - e^{i\omega\Delta t'}\right),\end{aligned} \tag{10}$$

where $\Delta t' = t - t'$, $\vec{r} = \{x - x', y - y', z - z'\}$ and $\vec{r}_- = \{x - x', y - y', z + z'\}$. Note here that $r = |\vec{r}|$ is the distance between the two objects and $r_- = |\vec{r}_-|$ is the distance between an object and the image of another one. Here the summation of polarization tensors in the TT gauge gives [50]

$$\begin{aligned}\sum_\lambda e^{(\lambda)}_{ij}(\vec{p})e^{(\lambda)}_{kl}(\vec{p}') &= \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} \\ &\quad + \hat{p}_i\hat{p}_j\hat{p}'_k\hat{p}'_l + \hat{p}_i\hat{p}_j\delta_{kl} + \hat{p}'_k\hat{p}'_l\delta_{ij} \\ &\quad - \hat{p}_i\hat{p}'_k\delta_{jl} - \hat{p}_i\hat{p}'_l\delta_{jk} - \hat{p}_j\hat{p}'_k\delta_{il} - \hat{p}_j\hat{p}'_l\delta_{ik},\end{aligned} \tag{11}$$

where $\hat{p}_i$ is the i-th component of the unit vector $\vec{p}_i/|\vec{p}|$. From this summation of polarization tensors, we can obtain

$$\begin{aligned}\sum_\lambda e^{(\lambda)}_{ij}(\vec{p})e^{(\lambda)}_{kl}(\vec{p})e^{i\vec{p}\cdot\vec{r}} &= \frac{1}{p^4}[(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl})\nabla^4 + (\partial_i\partial_j\delta_{kl} \\ &\quad + \partial_k\partial_l\delta_{ij} - \partial_i\partial_k\delta_{jl} \\ &\quad - \partial_i\partial_l\delta_{jk} - \partial_j\partial_k\delta_{il} - \partial_j\partial_l\delta_{ik})\nabla^2 + \partial_i\partial_j\partial_k\partial_l]e^{i\vec{p}\cdot\vec{r}} \\ &= \frac{1}{p^4}H^r_{ijkl}e^{i\vec{p}\cdot\vec{r}},\end{aligned} \tag{12}$$

and

$$\begin{aligned}\sum_\lambda e^{(\lambda)}_{ij}(\vec{p})e^{(\lambda)}_{kl}(\vec{p}_-)e^{i\vec{p}\cdot\vec{r}_-} &= \frac{1}{p^4}\sigma_{km}\sigma_{ln}[(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm} - \delta_{ij}\delta_{mn})\nabla^4 \\ &\quad + (\partial_i\partial_j\delta_{mn} + \partial_m\partial_n\delta_{ij} \\ &\quad - \partial_i\partial_m\delta_{jn} - \partial_i\partial_n\delta_{jm} - \partial_j\partial_m\delta_{in} - \partial_j\partial_n\delta_{im})\nabla^2 \\ &\quad + \partial_i\partial_j\partial_m\partial_n]e^{i\vec{p}\cdot\vec{r}_-} \\ &= \frac{1}{p^4}\sigma_{km}\sigma_{ln}H^{r_-}_{ijmn}e^{i\vec{p}\cdot\vec{r}_-},\end{aligned} \tag{13}$$

where H^{ρ}_{ijkl} denotes the differential operator to ρ, $\nabla^2 = \partial_i\partial^i$, and $\sigma_{ij} = \delta_{ij} - 2\delta_{i3}\delta_{j3}$. Substituting Eqs. (12) and (13) into Eq. (10) and performing the integration in the spherical coordinate, we obtain

$$\begin{aligned}\chi^F_{ijkl}(\vec{x}(t), \vec{x}(t')) &= \frac{\hbar c G}{2\pi}\int_0^\infty dp \\ &\quad \times\left[H^r_{ijkl}\frac{\sin pr}{r} - \sigma_{km}\sigma_{ln}H^{r_-}_{ijmn}\frac{\sin pr_-}{r_-}\right] \\ &\quad \times(e^{-i\omega\Delta t'} - e^{i\omega\Delta t'}) \\ &= -\frac{i\hbar c G}{2}\Big[H^r_{ijkl}\frac{1}{r}[\delta(r - c\Delta t') - \delta(r + c\Delta t')] \\ &\quad - \sigma_{km}\sigma_{ln}H^{r_-}_{ijmn}\frac{1}{r_-}[\delta(r_- - c\Delta t') - \delta(r_- + c\Delta t')]\Big].\end{aligned} \tag{14}$$

As for the statistical function $C^{AB}_{ijkl}(t, t')$ of the objects, its form can be obtained as

$$C^{AB}_{ijkl}(t, t') = \pm\frac{1}{2}\hat{Q}^A_{ij}\hat{Q}^B_{kl}(e^{-i\omega_0\Delta t'} + e^{i\omega_0\Delta t'}), \tag{15}$$

where the sign $\pm$ correspond to the symmetric and anti-symmetric states respectively, $\hat{Q}^{\xi}_{ij} = e^{i\omega_0 t}\langle g_\xi|Q^{\xi F}_{ij}|e_\xi\rangle$, $\hat{Q}^{\xi *}_{ij} = e^{-i\omega_0 t}\langle e_\xi|Q^{\xi F}_{ij}|g_\xi\rangle$, and $\hat{Q}^{\xi}_{ij} = \hat{Q}^{\xi *}_{ij}$ is assumed.

Substituting Eqs. (10) and (15) into Eq. (6), the interobject quantum gravitational interaction potential in the presence of a Dirichlet boundary can then be obtained as

$$\begin{aligned}\Delta E_{AB} = \mp\frac{G}{4}\hat{Q}^A_{ij}\hat{Q}^B_{kl}\Big[&\theta(c\Delta t - r)H^r_{ijkl}\frac{\cos p_0 r}{r} \\ &- \theta(c\Delta t - r_-)H^{r_-}_{ijkl}\frac{\cos p_0 r_-}{r_-}\Big],\end{aligned} \tag{16}$$

where we have introduced $p_0 = \omega_0/c$ and $\Delta t = t - t_0$. $\theta(x)$ is a step function, which equals to 1 when $x > 0$ but 0 when $x < 0$. Let us note here that

$$\begin{aligned}H^r_{ijkl}\frac{\cos p_0 r}{r} &= \frac{1}{r^5}[(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} \\ &\quad + \hat{r}_i\hat{r}_j\delta_{kl} + \hat{r}_k\hat{r}_l\delta_{ij} - \hat{r}_i\hat{r}_k\delta_{jl} \\ &\quad - \hat{r}_i\hat{r}_l\delta_{jk} - \hat{r}_j\hat{r}_k\delta_{il} - \hat{r}_j\hat{r}_l\delta_{ik} \\ &\quad + \hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_l)r^4 p_0^4\cos p_0 r\end{aligned}$$

$$+2(-\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}+\delta_{ij}\delta_{kl}$$
$$-\hat{r}_i\hat{r}_j\delta_{kl}-\hat{r}_k\hat{r}_l\delta_{ij}+2\hat{r}_j\hat{r}_k\delta_{il}$$
$$+2\hat{r}_j\hat{r}_l\delta_{ik}+2\hat{r}_i\hat{r}_k\delta_{jl}+2\hat{r}_i\hat{r}_l\delta_{jk}$$
$$-5\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_l)r^3p_0^3\sin p_0r$$
$$+(-3\delta_{ik}\delta_{jl}-3\delta_{il}\delta_{jk}+\delta_{ij}\delta_{kl}+3\hat{r}_i\hat{r}_j\delta_{kl}$$
$$+3\hat{r}_k\hat{r}_l\delta_{ij}+9\hat{r}_j\hat{r}_k\delta_{il}$$
$$+9\hat{r}_j\hat{r}_l\delta_{ik}+9\hat{r}_i\hat{r}_k\delta_{jl}+9\hat{r}_i\hat{r}_l\delta_{jk}$$
$$-45\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_l)r^2p_0^2\cos p_0r$$
$$+3(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}+\delta_{ij}\delta_{kl}$$
$$-5\hat{r}_i\hat{r}_j\delta_{kl}-5\hat{r}_k\hat{r}_l\delta_{ij}-5\hat{r}_j\hat{r}_k\delta_{il}$$
$$-5\hat{r}_j\hat{r}_l\delta_{ik}-5\hat{r}_i\hat{r}_k\delta_{jl}$$
$$-5\hat{r}_i\hat{r}_l\delta_{jk}+35\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_l)rp_0\sin p_0r$$
$$+3(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}+\delta_{ij}\delta_{kl}-5\hat{r}_i\hat{r}_j\delta_{kl}$$
$$-5\hat{r}_k\hat{r}_l\delta_{ij}-5\hat{r}_j\hat{r}_k\delta_{il}$$
$$-5\hat{r}_j\hat{r}_l\delta_{ik}-5\hat{r}_i\hat{r}_k\delta_{jl}-5\hat{r}_i\hat{r}_l\delta_{jk}$$
$$+35\hat{r}_i\hat{r}_j\hat{r}_k\hat{r}_l)\cos p_0r], \quad (17)$$

where $\hat{r}_i$ is the i-th component of the unit vector $\vec{r}_i/r$. The first term in Eq. (16) corresponds to the interaction between two entangled objects in vacuum [34], while the second term is the additional contribution due to the Dirichlet boundary which can be regarded as the interaction between an object and the image of the other one. It shows that, both the two terms exist simultaneously only when the distance r and r_- are smaller than the distance characterized by $c\Delta t$, which means that such interaction potential appears only when one object (as well as its image) is in the light cone of the other.

3.1 Two special cases

Now Let us discuss the interaction potential in some special circumstances. The first case is that the two objects are placed perpendicular to the Dirichlet boundary. We take $\vec{r}=\{0,0,r\}$ and, for convenience, assume $r<c\Delta t$. When the two-object system is very close to the boundary, i.e., $r\sim r_-$, the interaction potential Eq. (16) becomes

$$\Delta E_{AB}=\mp\frac{\hbar G\omega_0}{r^5}\alpha(8p_0^3r^3\sin p_0r+24p_0^2r^2\cos p_0r$$
$$-48p_0r\sin p_0r-48\cos p_0r), \quad (18)$$

where the isotropic gravitational polarizability $\alpha=\alpha_{ijkl}=\hat{Q}_{ij}\hat{Q}_{kl}/\hbar\omega_0$ defined in Ref. [34] has been employed again. In the near regime, i.e., $r\ll c/\omega_0$, the leading term of the potential Eq. (18) takes the form

$$\Delta E_{AB}\simeq\pm\frac{48\hbar G\omega_0}{r^5}\alpha\cos p_0r, \quad (19)$$

while in the far regime, i.e., $r\gg c/\omega_0$, it becomes

$$\Delta E_{AB}\simeq\mp\frac{8\hbar G\omega_0^4}{r^2c^3}\alpha\sin(p_0r+\phi_1), \quad (20)$$

where $\phi_1=\arcsin\frac{3}{\sqrt{9+p_0^2r^2}}$. This shows that, when the two entangled objects are placed perpendicular to the Dirichlet boundary, the interobject quantum gravitational quadrupole–quadrupole interaction behaves as r^{-5} in the near regime and r^{-2} in the far regime. Compare to the case without gravitational boundary [34], the Dirichlet boundary increases the near-regime potential about 2.3 times in the leading order since the coefficient in the pure vacuum case is 21, and modifies the r^{-1} far-regime behavior of the interaction in the absence of boundary.

The second case is that the two objects are placed parallel to the Dirichlet boundary. In the limit of $r\sim r_-$, the interaction potential Eq. (16) becomes

$$\Delta E_{AB}=\mp\frac{\hbar G\omega_0}{r^5}\alpha(2p_0^4r^4\cos p_0r$$
$$+6p_0^2r^2\cos p_0r-18p_0r\sin p_0r-18\cos p_0r), \quad (21)$$

where $\vec{r}=\{r,0,0\}$ has been applied. In the near regime, i.e., $r\ll c/\omega_0$, the leading term of Eq. (21) becomes

$$\Delta E_{AB}\simeq\pm\frac{18\hbar G\omega_0}{r^5}\alpha\cos p_0r, \quad (22)$$

while in the far regime, i.e., $r\gg c/\omega_0$, it reduces to

$$\Delta E_{AB}\simeq\mp\frac{2\hbar G\omega_0^5}{rc^4}\alpha\cos p_0r. \quad (23)$$

In this case, the interobject quantum gravitational quadrupole–quadrupole interaction behaves as r^{-5} and r^{-1} in the near and far regimes, respectively. Compare to the case without gravitational boundary [34], the Dirichlet boundary weakens the near-regime interaction potential, but increases the far-regime potential 2 times in the leading order since the coefficient in the pure vacuum case is 1.

4 Neumann boundary condition

For the Neumann boundary condition, the field mode satisfies $\partial_z h^\lambda_{ij,\vec{p}}|_{z=0}=0$ and thus can be written as

$$h^\lambda_{ij,\vec{p}}=\sqrt{\frac{\hbar G}{2c^2\omega\pi^2}}\left\{a_\lambda(\omega)\left[e^{(\lambda)}_{ij}(\vec{p})e^{i(\vec{p}\cdot\vec{x}-\omega t)}\right.\right.$$
$$\left.\left.+e^{(\lambda)}_{ij}(\vec{p}_-)e^{i(\vec{p}_-\cdot\vec{x}-\omega t)}\right]+\text{H.c.}\right\}. \quad (24)$$

From the above equation, one can show that the statistical function of the gravitational field $\chi^F_{ijkl}(\vec{x}_A(t),\vec{x}_B(t'))$ becomes

$$\chi^F_{ijkl}(\vec{x}(t),\vec{x}(t'))=\frac{\hbar cG}{2\pi}\int_0^\infty dp$$
$$\times\left[H^r_{ijkl}\frac{\sin pr}{r}+\sigma_{km}\sigma_{ln}H^{r_-}_{ijmn}\frac{\sin pr_-}{r_-}\right]$$

$$\times(e^{-i\omega\Delta t'} - e^{i\omega\Delta t'})$$
$$= -\frac{i\hbar cG}{2}\Big[H^{r}_{ijkl}\frac{1}{r}[\delta(r - c\Delta t') - \delta(r + c\Delta t')]$$
$$+\sigma_{km}\sigma_{ln}H^{r_-}_{ijmn}\frac{1}{r_-}[\delta(r_- - c\Delta t') - \delta(r_- + c\Delta t')]\Big], \tag{25}$$

and the interaction potential Eq. (6) then reads

$$\Delta E_{AB} = \mp\frac{G}{4}\hat{Q}^A_{ij}\hat{Q}^B_{kl}\Big[\theta(c\Delta t - r)H^{r}_{ijkl}\frac{\cos p_0 r}{r}$$
$$+\theta(c\Delta t - r_-)H^{r_-}_{ijkl}\frac{\cos p_0 r_-}{r_-}\Big]. \tag{26}$$

Obviously, the boundary-dependent interaction energy exists also only when an object as well as its image are in the light cone of the other one, similar to the Dirichlet case.

4.1 Two special cases

For the special case of two entangled objects placed perpendicular to the Neumann boundary, we take $\vec{r} = \{0, 0, r\}$ and consider the case when the two-object system is very close to the boundary, i.e., $r \sim r_-$. The interaction potential Eq. (26) becomes

$$\Delta E_{AB} = \mp\frac{\hbar G\omega_0}{r^5}\alpha(2p_0^4r^4\cos p_0r - 4p_0^3r^3\sin p_0r$$
$$-6p_0^2r^2\cos p_0r + 6p_0r\sin p_0r + 6\cos p_0r), \tag{27}$$

where $r < c\Delta t$ is assumed. In the near regime, i.e., $r \ll c/\omega_0$, the leading term of the potential Eq. (27) is

$$\Delta E_{AB} \simeq \mp\frac{6\hbar G\omega_0}{r^5}\alpha\cos p_0r, \tag{28}$$

while in the far regime, i.e., $r \gg c/\omega_0$, it takes the form

$$\Delta E_{AB} \simeq \mp\frac{2\hbar G\omega_0^5}{rc^4}\alpha\cos(p_0r + \phi_2), \tag{29}$$

where $\phi_2 = \arcsin\frac{2}{\sqrt{4+p_0^2r^2}}$. It shows, when the two objects are placed perpendicular to the Neumann boundary, the interobject quantum gravitational quadrupole–quadrupole interaction behaves as r^{-5} and r^{-1} in the near and far regimes, respectively. Compare to the case without boundary [34], the Neumann boundary weakens the near-regime interaction potential, and changes its sign which means the attractive and repulsive properties of the interaction are modified. Also, the far-regime interaction is enhanced about 2 times compared to the pure vacuum case due to the boundary.

For the case when two entangled objects are placed parallel and close to the Neumann boundary, the interaction potential Eq. (26) becomes

$$\Delta E_{AB} = \mp\frac{\hbar G\omega_0}{r^5}\alpha(4p_0^3r^3\sin p_0r + 12p_0^2r^2\cos p_0r$$
$$-24p_0r\sin p_0r - 24\cos p_0r), \tag{30}$$

where $\vec{r} = \{r, 0, 0\}$ has been taken. In the near regime, i.e., $r \ll c/\omega_0$, the leading term of Eq. (30) takes the form

$$\Delta E_{AB} \simeq \pm\frac{24\hbar G\omega_0}{r^5}\alpha\cos p_0r, \tag{31}$$

while in the far regime, i.e., $r \gg c/\omega_0$, it becomes

$$\Delta E_{AB} \simeq \mp\frac{4\hbar G\omega_0^4}{r^2c^3}\alpha\sin(p_0r + \phi_1). \tag{32}$$

It shows, the interobject quantum gravitational quadrupole–quadrupole interaction behaves as r^{-5} and r^{-2} in the near and far regimes, respectively. Compare to the case without boundary [34], the Neumann boundary enhances the near-regime interaction potential about 1.1 times in the leading order and modifies the behavior of the interaction in the far regime. Also, one can find that the interaction potentials in these two special cases under Neumann boundary condition are quite different from that of the Dirichlet circumstance. Moreover, it is worth mentioning here that, thought we do not know how to realize the gravitational Neumann boundary condition through specific physical setup, a theoretical exploration may deserve.

5 Discussion

In this paper, we explore the quantum gravitational quadrupole–quadrupole interaction between two entangled non point-like objects in the presence of plane gravitational boundaries, based on the second-order DDC formalism. Two kinds of boundary conditions, i.e., Neumann and Dirichlet, are considered and our result shows that the interaction can be either enhanced or weakened depending on the geometrical arrangement of the two-object system with respect to the boundaries. In the limit when the two-object system is placed very close to the Dirichlet boundary, the near-regime interaction potential is larger than that of the pure vacuum case when the two objects are placed perpendicular to the boundary but smaller when parallel to it, while, in the far regime, such strong and weak relations between potentials are just opposite to that in the near regime. And, there exists a new r^{-2} far-regime behavior of the interaction potential under the perpendicular configuration. For the case of Neumann boundary, the strong and weak relations between the interaction potentials under perpendicular or parallel configurations and the case without boundary are opposite to the Dirichlet circumstance both in the near and far regimes. Besides, the novel r^{-2} far-regime behavior occurs for the parallel rather than perpendicular configuration in the presence of Neumann boundary.

Acknowledgements This work was supported in part by the NSFC under Grants no. 12205032 and no. 12275037, the CQ RLSBJ under Grant no. cx2021044, the Talent Introduction Program of Chongqing University of Posts and Telecommunications under Grant no. E012A2021209, and China Postdoctoral Science Foundation under Grant no. 2022M713104.

Data availability statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This paper is a theoretical work.]

Funded by SCOAP^3. SCOAP^3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

1. H.B.G. Casimir, D. Polder, The influence of retardation on the London-van der Waals forces. Phys. Rev. **73**, 360 (1948). https://doi.org/10.1103/PhysRev.73.360
2. R.R. McLone, E.A. Power, The long range van der Waals forces between non-identical systems. Proc. R. Soc. A **286**, 573 (1965). https://doi.org/10.1098/rspa.1965.0165
3. L. Gomberoff, R.R. McLone, E.A. Power, Long-range retarded potentials between molecules. J. Chem. Phys. **44**, 4148 (1966). https://doi.org/10.1063/1.1726597
4. E.A. Power, T. Thirunamachandran, Quantum electrodynamics with nonrelativistic sources. V. Electromagnetic field correlations and intermolecular interactions between molecules in either ground or excited states. Phys. Rev. A **47**, 2539 (1993). https://doi.org/10.1103/PhysRevA.47.2539
5. E.A. Power, T. Thirunamachandran, A new insight into the mechanism of intermolecular forces. Chem. Phys. **171**, 1 (1993). https://doi.org/10.1016/0301-0104(93)85127-T
6. E.A. Power, T. Thirunamachandran, Dispersion forces between molecules with one or both molecules excited. Phys. Rev. A **51**, 3660 (1995). https://doi.org/10.1103/PhysRevA.51.3660
7. L. Rizzuto, R. Passante, F. Persico, Dynamical Casimir-Polder energy between an excited- and a ground-state atom. Phys. Rev. A **70**, 012107 (2004). https://doi.org/10.1103/PhysRevA.70.012107
8. Y. Sherkunov, Casimir-Polder interaction between an excited atom and a gas dielectric medium. Phys. Rev. A **75**, 012705 (2007). https://doi.org/10.1103/PhysRevA.75.012705
9. J. Preto, M. Pettini, Resonant long-range interactions between polar macromolecules. Phys. Lett. A **377**, 587 (2013). https://doi.org/10.1016/j.physleta.2012.12.034
10. M. Donaire, R. Guérout, A. Lambrecht, Quasiresonant van der Waals Interaction between Nonidentical Atoms. Phys. Rev. Lett. **115**, 033201 (2015). https://doi.org/10.1103/PhysRevLett.115.033201
11. P.W. Milonni, S.M.H. Rafsanjani, Distance dependence of two-atom dipole interactions with one atom in an excited state. Phys. Rev. A **92**, 062711 (2015). https://doi.org/10.1103/PhysRevA.92.062711
12. P.R. Berman, Interaction energy of nonidentical atoms. Phys. Rev. A **91**, 042127 (2015). https://doi.org/10.1103/PhysRevA.91.042127
13. U.D. Jentschura, V. Debierre, Long-range tails in van der Waals interactions of excited-state and ground-state atoms. Phys. Rev. A **95**, 042506 (2017). https://doi.org/10.1103/PhysRevA.95.042506
14. J. Sánchez-Cánovas, M. Donaire, Two-atom van der Waals forces with one atom excited: the identical-atom limit. Phys. Rev. A **104**, 052814 (2021). https://doi.org/10.1103/PhysRevA.104.052814
15. D.P. Craig, T. Thirunamachandran, *Molecular Quantum Electrodynamics* (Dover, Mineola, 1998)
16. A. Salam, *Molecular Quantum Electrodynamics* (Wiley, Hoboken, 2010)
17. T. Thirunamachandran, Intermolecular interactions in the presence of an intense radiation field. Mol. Phys. **40**, 393 (1980). https://doi.org/10.1080/00268978000101561
18. P.W. Milonni, M.-L. Shih, Source theory of the Casimir force. Phys. Rev. A **45**, 4241 (1992). https://doi.org/10.1103/PhysRevA.45.4241
19. P.W. Milonni, A. Smith, van der Waals dispersion forces in electromagnetic fields. Phys. Rev. A **53**, 3484 (1996). https://doi.org/10.1103/PhysRevA.53.3484
20. D.S. Bradshaw, D.L. Andrews, Optically induced forces and torques: interactions between nanoparticles in a laser beam. Phys. Rev. A **72**, 033816 (2005). https://doi.org/10.1103/PhysRevA.72.033816
21. D.L. Andrews, R.G. Crisp, D.S. Bradshaw, Optically induced interparticle forces: from the bonding of dimers to optical electrostriction in molecular solids. J. Phys. B **39**, S637 (2006). https://doi.org/10.1088/0953-4075/39/15/S11
22. A. Salam, Intermolecular interactions in a radiation field via the method of induced moments. Phys. Rev. A **73**, 013406 (2006). https://doi.org/10.1103/PhysRevA.73.013406
23. A. Salam, Two alternative derivations of the static contribution to the radiation-induced intermolecular energy shift. Phys. Rev. A **76**, 063402 (2007). https://doi.org/10.1103/PhysRevA.76.063402
24. Y. Sherkunov, Casimir-Polder interaction between two atoms in electromagnetic fields. J. Phys. Conf. Ser. **161**, 012041 (2009). https://doi.org/10.1088/1742-6596/161/1/012041
25. W.C. Mackrodt, The coupling of intermolecular interactions to a static electric field. Mol. Phys. **27**, 933 (1974). https://doi.org/10.1080/00268977400100861
26. G. Fiscelli, L. Rizzuto, R. Passante, Dispersion interaction between two hydrogen atoms in a static electric field. Phys. Rev. Lett. **124**, 013604 (2020). https://doi.org/10.1103/PhysRevLett.124.013604
27. Y. Hu, J. Hu, H. Yu, Quantum corrections to the classical electrostatic interaction between induced dipoles. Phys. Rev. A **103**, 042803 (2021). https://doi.org/10.1103/PhysRevA.103.042803
28. E.A. Power, T. Thirunamachandran, Quantum electrodynamics in a cavity. Phys. Rev. A **25**, 2473 (1982). https://doi.org/10.1103/PhysRevA.25.2473
29. S. Spagnolo, R. Passante, L. Rizzuto, Field fluctuations near a conducting plate and Casimir–Polder forces in the presence of boundary conditions. Phys. Rev. A **73**, 062117 (2006). https://doi.org/10.1103/PhysRevA.73.062117
30. L.H. Ford, M.P. Hertzberg, J. Karouby, Quantum gravitational force between polarizable objects. Phys. Rev. Lett. **116**, 151301 (2016). https://doi.org/10.1103/PhysRevLett.116.151301
31. P. Wu, J. Hu, H. Yu, Quantum correction to classical gravitational interaction between two polarizable objects. Phys. Lett. B **763**, 40 (2016). https://doi.org/10.1016/j.physletb.2016.10.025
32. P. Wu, J. Hu, H. Yu, Interaction between two gravitationally polarizable objects induced by thermal bath of gravitons. Phys. Rev. D **95**, 104057 (2017). https://doi.org/10.1103/PhysRevD.95.104057

33. B.R. Holstein, Analytical on-shell calculation of low energy higher order scattering. J. Phys. G **44**, 01LT01 (2017). https://doi.org/10.1088/0954-3899/44/1/01LT01
34. Y. Hu, J. Hu, H. Yu, P. Wu, Resonance interaction between two entangled gravitational polarizable objects. Eur. Phys. J. C **80**, 792 (2020). https://doi.org/10.1140/epjc/s10052-020-8375-y
35. Y. Hu, J. Hu, H. Yu, Quantum gravitational interaction between two objects induced by external gravitational radiation fields. Phys. Rev. D **101**, 066015 (2020). https://doi.org/10.1103/PhysRevD.101.066015
36. Y. Hu, J. Hu, H. Yu, Induced quantum gravitational interaction between two objects with permanent quadrupoles in external gravitational fields. Phys. Rev. D **104**, 086011 (2021). https://doi.org/10.1103/PhysRevD.104.086011
37. H. Yu, Z. Yang, P. Wu, Quantum interaction between two gravitationally polarizable objects in the presence of boundaries. Phys. Rev. D **97**, 026008 (2018). https://doi.org/10.1103/PhysRevD.97.026008
38. L. Smolin, On the intrinsic entropy of the gravitational field. Gen. Relativ. Gravit. **17**, 417 (1985). https://doi.org/10.1007/BF00761902
39. S.J. Minter, K. Wegter-McNelly, R.Y. Chiao, Do mirrors for gravitational waves exist? Physica **42E**, 234 (2010). https://doi.org/10.1016/j.physe.2009.06.056
40. R.Y. Chiao, J.S. Sharping, L.A. Martinez, B.S. Kang, A. Castelli, N. Inan, J.J. Thompson, Dynamical Casimir effect and the possibility of laser-like generation of gravitational radiation. arXiv:1712.08680
41. J. Dalibard, J. Dupont-Roc, C. Cohen-Tannoudji, Vacuum fluctuations and radiation reaction: identification of their respective contributions. J. Phys. **43**, 1617 (1982). https://doi.org/10.1051/jphys:0198200430110161700
42. J. Dalibard, J. Dupont-Roc, C. Cohen-Tannoudji, Dynamics of a small system coupled to a reservoir: reservoir fluctuations and self-reaction. J. Phys. **45**, 637 (1984). https://doi.org/10.1051/jphys:01984004504063700
43. W.B. Campbell, T.A. Morgan, Maxwell form of the linear theory of gravitation. Am. J. Phys. **44**, 356 (1976). https://doi.org/10.1119/1.10195
44. A. Matte, Sur de nouvelles solutions oscillatoires des equations de la gravitation. Can. J. Math. **5**, 1 (1953). https://doi.org/10.4153/CJM-1953-001-3
45. W.B. Campbell, T. Morgan, Debye potentials for the gravitational field. Physica (Utrecht) **53**, 264 (1971). https://doi.org/10.1016/0031-8914(71)90074-7
46. P. Szekeres, Linearized gravitation theory in macroscopic media. Ann. Phys. (N. Y.) **64**, 599 (1971). https://doi.org/10.1016/0003-4916(71)90117-5
47. R. Maartens, B.A. Bassett, Gravito-electromagnetism. Class. Quantum Gravity **15**, 705 (1998). https://doi.org/10.1088/0264-9381/15/3/018
48. M.L. Ruggiero, A. Tartaglia, Gravitomagnetic effects. Nuovo Cimento Soc. Ital. Fis. **117B**, 743 (2002)
49. J. Ramos, M. de Montigny, F. Khanna, On a Lagrangian formulation of gravitoelectromagnetism. Gen. Relativ. Gravit. **42**, 2403 (2010). https://doi.org/10.1007/s10714-010-0990-8
50. H. Yu, L.H. Ford, Light-cone fluctuations in flat spacetimes with nontrivial topology. Phys. Rev. D **60**, 084023 (1999). https://doi.org/10.1103/PhysRevD.60.084023