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Spectroscopic behavior of fully heavy tetraquarks

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Abstract Stimulated by the observation of the X(6900) from LHCb in 2020 and the recent results from CMS and ATLAS in the di- J/ψ invariant mass spectrum, in this work we systematically study all possible configurations for the ground states of fully heavy tetraquark in the constituent quark model. By our calculation, we present their spectroscopic behaviors including binding energy, lowest meson-meson thresholds, specific wave function, magnetic moment, transition magnetic moment, radiative decay width, rearrangement strong width ratio, internal mass contributions, relative lengths between (anti)quarks, and the spatial distribution of four valence (anti)quarks. We cannot find a stable S-wave state for the fully heavy tetraquark system. We hope that our results will be valuable to further experimental exploration of fully heavy tetraquark states.

1 Introduction

With the birth of the quark model [1–3], exotic states beyond conventional hadrons were proposed. The search for exotic hadronic states is full of challenges and opportunities. Since the X(3872) was first reported by the Belle Collaboration in 2003 [4–6], a series of charmonium-like or bottomoniumlike exotic states [7–21] and P_c states [22–24] have been observed experimentally, which stimulates extensive discussions of their properties by introducing the assignments of conventional hadron, compact multiquark states, molecular state, hybrid, glueball, and kinematic effects [7–21].

In 2003, the BaBar Collaboration observed a narrow heavy-light state D_{s0}^* (2317) in the $D_s^+ \pi^0$ invariant mass spectrum [25]. However, since the mass of the observed $D_{s0}^{*}(2317)$ is about 100 MeV below the quark model predictions in Ref. [26], it is difficult to understand the $D_{s0}^*(2317)$ in a conventional quark model directly, which is referred to as the "low-mass puzzle" of $D_{s0}^*(2317)$. In order to solve the low-mass puzzle, the tetraquark explanation with the $Qq\bar{q}\bar{q}$ configuration was proposed in Refs. [27–29]. Later, the CLEO Collaboration [30] confirmed the $D_{s0}^{*}(2317)$ and announced another narrow resonance $D_{s1}(2460)$ in the $D_{s}^{*+}\pi^{0}$ final states. The low-mass puzzle also happens to the $D_{s1}(2460)$ [26]. A discussion of the $D_{s1}(2460)$ as a tetraquark state can be found in Refs. [31-38]. In particular, the LHCb Collaboration reported the discovery of two new exotic structures $X_0(2900)$ and $X_1(2900)$ [39,40], which inspired the study of exotic charmed tetraquarks [41-50]. In addition, theorists began to study the doubly charmed tetraquark states in the earlier works [51–55]. In 2017, the LHCb Collaboration observed a doubly charmed baryon $\Xi_{cc}^{++}(3620)$ in the $\Lambda_c^+ K^- \pi^+ \pi^+$ decay mode [56]. Using the Ξ_{cc}^{++} (3620) as the scaling point, the theorists further explored the possible stable doubly charmed tetraquark states with the $QQ\bar{q}\bar{q}$ configuration [57–66]. Surprisingly, as a candidate of the doubly charmed tetraquark, the T_{cc}^+ was detected by LHCb in the $D^0 D^0 \pi^+$ invariant mass spectrum, which has a minimal quark configuration of $cc\bar{u}\bar{d}$ [67]. In addition to these singly and doubly charmed tetraquarks, there should be a triply charmed tetraquark. To our knowledge, the triply charmed tetraquark states with the $QQQ\bar{q}$ configuration have also been studied by various approaches [68–72].

Briefly reviewing the status of heavy flavor tetraquark states, we must mention the fully heavy tetraquark with the $QQ\bar{Q}\bar{Q}$ configuration, which has attracted the attention of both theorists and experimentalists. Chao et al. suggested

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that the peculiar resonance-like structures of $R(e^+e^- \rightarrow$ hadrons) for $\sqrt{s} = 6-7$ GeV may be due to the production of the predicted P-wave (cc)- $(c\bar{c})$ states in the energy range of 6.4-6.8 GeV, which could dominantly decay into charmed mesons [73]. The calculation of the fully heavy tetraquark was then carried out using the potential model [74,75] and the MIT bag model with the Born-Oppenheimer approximation [76]. This system has also been studied in a nonrelativistic potential model, where no $QQ\bar{Q}\bar{Q}$ bound state can be found [77]. However, Lloyd et al. adopted a parameterized nonrelativistic Hamiltonian to study such system [78], where they found several closely lying bound states with a large oscillator basis. Later, Karliner et al. estimated the masses of the fully heavy tetraquark states by a simple quark model, and obtained $M(X_{cc\bar{c}\bar{c}}) = 6192 \pm 25$ MeV and $M(X_{bb\bar{b}\bar{b}}) = 18826 \pm 25$ MeV for the fully charmed and fully bottom tetraquarks with the $J^{PC} = 0^{++}$ quantum number, respectively [79]. Anwar et al. calculated the ground-state energy of the *bbbb* bound state in a nonrelativistic effective field theory with one-gluon-exchange (OGE) color Coulomb interaction, and the ground-state $bb\bar{b}\bar{b}$ tetraquark mass was predicted to be (18.72 ± 0.02) GeV [80]. In Ref. [81], Bai et al. presented a calculation of the bbbb tetraquark ground-state energy using a diffusion Monte Carlo method to solve the nonrelativistic many-body system. Debastiani et al. extended the updated Cornell model to study the fully charmed tetraquark in a diquark-antidiquark configuration [82]. Chen et al. used a moment quantum chromodynamics (QCD) sum rule method to give the existence of the exotic states $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ in the compact diquark-antidiquark configuration, where they suggested searching for them in the $J/\psi J/\psi$ and $\eta_c(1S)\eta_{1S}$ channels [83].

With the accumulation of experimental data, many collaborations have tried to search for it. The CMS Collaboration reported the first observation of the $\Upsilon(1S)$ pair production in pp collisions, where there is evidence that a structure around 18.4 GeV with a global significance of 3.6 σ exists in the fourlepton channel, which is probably a fully bottom tetraquark state [84]. However, this structure was not confirmed by the later CMS analysis [85]. Subsequently, the LHCb Collaboration studied the $\Upsilon(1S)_{\mu^+\mu^-}$ invariant mass distribution to search for a possible $bb\bar{b}\bar{b}$ exotic meson, but they did not see any significant excess in the range 17.5–20.0 GeV [86]. By 2020, the LHCb Collaboration declared a narrow resonance X(6900) in the di- J/ψ mass spectrum with a significance of more than 5σ [87]. In addition, a broad structure ranging from 6.2 to 6.8 GeV and an underlying peak near 7.3 GeV were reported at the same time [87]. The ATLAS and CMS collaborations recently published their measurements on the $di-J/\psi$ invariant mass spectrum. Here, they not only confirmed the existence of the X(6900), but also found some new peaks [88–90]. There have been extensive discussions

about the observed X(6900) from different approaches and with different assignments [91–108].

The problem of the stability of the fully heavy tetraquark state has long been debated. Debastiani et al. found that the lowest S-wave $cc\bar{c}\bar{c}$ tetraquarks may be below the dicharmonium thresholds in their updated Cornell model [109]. The 1^+ $bb\bar{b}\bar{c}$ state is thought to be a narrow state in the extended chromomagnetic model [110]. However, many other studies have suggested that the ground state of fully heavy tetraquarks is above the di-meson threshold. Wang et al. also calculated the fully heavy tetraquark state in two nonrelativistic quark models with different OGE Coulomb, linear confinement, and hyperfine potentials [111]. Based on the numerical calculations, they suggested that the ground states should be located about 300-450 MeV above the lowest scattering states, indicating that there is no bound tetraquark state. The lattice nonrelativistic QCD method was applied to study the lowest energy eigenstate of the bbbb system, and no state was found below the lowest bottomonium-pair threshold [112]. In another work, Richard et al. claimed that the fully heavy configuration $QQ\bar{Q}\bar{Q}$ is not stable if one adopts a standard quark model and treats the four-body problem appropriately [113]. Jin et al. studied full-charm and full-bottom tetraquarks using the quark delocalization color screening model and the chiral quark model, respectively, and the results within the quantum numbers $J^P = 0^+, 1^+,$ and 2^+ show that the bound state exists in both models [114]. Frankly, theorists have not come to an agreement on the stability of the fully heavy tetraquark state.

Facing the present status of the fully heavy tetraquark, in this work we adopt the variational method to systematically study the fully heavy tetraquark states, where the mass spectrum of the fully heavy tetraquark is given in the framework of the nonrelativistic quark model associated with a potential containing Coulomb, linear, and hyperfine terms. The constructed total wave functions involved in these systems satisfy the requirement of the Pauli principle. We should emphasize that we can also reproduce the masses of these conventional hadrons with the same parameters, which is a test of our adopted framework. With this preparation, we calculate the binding energies, the lowest meson-meson thresholds, and the rearrangement strong width ratio, and study the stability of the fully heavy tetraquark states against the decay into two meson states. Furthermore, we discuss whether these tetraquarks have a compact configuration based on the eigenvalue of the hyperfine potential matrix. According to specific wave functions, we obtain the magnetic moments, transition magnetic moments, and radiative decay widths, which may reflect their electromagnetic properties and internal structures. We also give the size of the tetraquarks, the relative distances between (anti)quarks, and the spatial distribution of the four valence (anti)quarks for each state. Through the

present systematic work, we can test whether compact bound fully heavy tetraquarks exist within the given Hamiltonian.

This paper is organized as follows. After the introduction, we present the Hamiltonian of the constituent quark model and list the corresponding parameters in Sect. 2. Then, in Sect. 3, we give the spatial function with a simple Gaussian form and construct the flavor, color, and spin wave functions of the fully heavy tetraquark states. In Sect. 4, we show the numerical results obtained by the variational method and further calculate their magnetic moment, transition magnetic moment, radiative decay width, rearrangement strong width ratio, internal mass contributions, and relative lengths between (anti) quarks. In addition, we compare our results with those of other theoretical groups in Sect. 5. Finally, we end the paper with a short summary in Sect. 6.

2 Hamiltonian

We choose a nonrelativistic Hamiltonian for the fully heavy tetraquark system, which is written as

$$H = \sum_{i=1}^{4} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i(1)$$

Here, m_i is the (anti)quark mass, λ_i^c is the SU(3) color operator for the *i*-th quark, and for the antiquark, λ_i^c is replaced by $-\lambda_i^{c*}$. The internal quark potentials V_{ij}^{Con} and V_{ij}^{SS} have the following forms:

$$V_{ij}^{\text{Con}} = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{SS} = \frac{\kappa'}{m_i m_j} \frac{1}{r_{0ij} r_{ij}} e^{-r_{ij}^2 / r_{0ij}^2} \vec{\sigma}_i \cdot \vec{\sigma}_j,$$
(2)

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between the *i*-th (anti)quark and the *j*-th (anti)quark, and σ_i is the SU(2) spin operator for the *i*-th quark. As for the r_{0ij} and κ' , we have

$$r_{0ij} = 1/\left(\alpha + \beta \frac{m_i m_j}{m_i + m_j}\right),$$

$$\kappa' = \kappa_0 \left(1 + \gamma \frac{m_i m_j}{m_i + m_j}\right).$$
(3)

The corresponding parameters appearing in Eqs. (2–3) are shown in Table 1. Here, κ and κ' are the couplings of the Coulomb and hyperfine potentials, respectively, and they are proportional to the running coupling constant $\alpha_s(r)$ of QCD. The Coulomb and hyperfine interactions can be deduced from the one-gluon-exchange model. $1/a_0^2$ represents the strength of the linear potential. r_{0ij} is the Gaussian-smearing parameter. Furthermore, we introduce κ_0 and γ in κ' to better describe the interaction between different quark pairs [115].

Table 1 Parameters of the Hamiltonian

Parameter	К	<i>a</i> ₀	D
Value	120.0 MeV fm	$0.0318119 (MeV^{-1}fm)^{1/2}$	983 MeV
Parameter	α	β	m_c
Value	1.0499 fm^{-1}	0.0008314 (MeV fm) ⁻¹	1918 MeV
Parameter	ко	γ	m_b
Value	194.144 MeV	$0.00088 \; {\rm MeV^{-1}}$	5343 MeV

 Table 2
 All possible flavor combinations for the fully heavy tetraquark system

System	Flavor combinations									
QQQQ										
	$cc\bar{b}\bar{b}~(bb\bar{c}\bar{c})$	$cc\bar{c}\bar{b}~(bc\bar{c}\bar{c})$	$bb\bar{b}\bar{c}$ ($cb\bar{b}$							

3 Wave functions

Here, we focus on the ground states of fully heavy tetraquark. We present the flavor, spatial, and color-spin parts of the total wave function for fully heavy tetraquark system. In order to consider the constraint by the Pauli principle, we use a diquark–antidiquark picture to analyze this tetraquark system.

3.1 Flavor part

First we discuss the flavor part. Here, we list all the possible flavor combinations for the fully heavy tetraquark system in Table 2.

In Table 2, the three flavor combinations in the first row are purely neutral particles, and the *C*-parity is a "good" quantum number. For the other six states in the second row, each state has a charge conjugation anti-partner, and their masses, internal mass contributions, and relative distances between (anti)quarks are absolutely identical, so we only need to discuss one of the pair.

Furthermore, the $cc\bar{c}c$, $bb\bar{b}b$, and $cc\bar{b}b$ states have the two pairs of (anti)quarks which are identical, but only the first two quarks in the $cc\bar{c}b$ and $bb\bar{b}c$ states are identical.

3.2 Spatial part

In this part, we construct the wave function for the spatial part in a simple Gaussian form. We denote the fully heavy tetraquark state as the $Q(1)Q(2)\overline{Q}(3)\overline{Q}(4)$ configuration, and choose the Jacobian coordinate system as follows:

$$\mathbf{x}_{1} = \sqrt{1/2}(\mathbf{r}_{1} - \mathbf{r}_{2}),$$

$$\mathbf{x}_{2} = \sqrt{1/2}(\mathbf{r}_{3} - \mathbf{r}_{4}),$$

$$\mathbf{x}_{3} = \left[\left(\frac{m_{1}\mathbf{r}_{1} + m_{2}\mathbf{r}_{2}}{m_{1} + m_{2}} \right) - \left(\frac{m_{3}\mathbf{r}_{3} + m_{4}\mathbf{r}_{4}}{m_{3} + m_{4}} \right) \right].$$
(4)

Here, we set the Jacobi coordinates with the following conditions:

$$\begin{split} m_1 &= m_2 = m_3 = m_4 = m_c, & \text{for } cc\bar{c}\bar{c}, \\ m_1 &= m_2 = m_3 = m_4 = m_b, & \text{for } bb\bar{b}\bar{b}, \\ m_1 &= m_2 = m_c, m_3 = m_4 = m_b, & \text{for } cc\bar{b}\bar{b}, \\ m_1 &= m_2 = m_3 = m_c, m_4 = m_b, & \text{for } cc\bar{c}\bar{b}, \\ m_1 &= m_2 = m_3 = m_b, m_4 = m_c, & \text{for } bb\bar{b}\bar{c}, \\ m_1 &= m_c, m_2 = m_b, m_3 = m_c, m_4 = m_b, & \text{for } cb\bar{c}\bar{b}. \end{split}$$

Based on this, we construct the spatial wave functions of the $QQ\bar{Q}\bar{Q}$ states in a single Gaussian form. The spatial wave function can satisfy the required symmetry property:

$$R^{s} = \exp[-C_{11}\mathbf{x}_{1}^{2} - C_{22}\mathbf{x}_{2}^{2} - C_{33}\mathbf{x}_{3}^{2}],$$
(5)

where C_{11} , C_{22} , and C_{33} are the variational parameters.

It is also useful to introduce the center of mass frame so that the kinetic term in the Hamiltonian of Eq. (1) can be appropriately reduced for our calculations. The kinetic term, denoted by T_c , is as follows:

$$T_{c} = \sum_{i=1}^{4} \frac{\mathbf{p}_{i}^{2}}{2m_{i}} - \frac{\mathbf{p}_{rC}^{2}}{2M} = \frac{\mathbf{p}_{x_{1}}^{2}}{2m_{1}'} + \frac{\mathbf{p}_{x_{2}}^{2}}{2m_{2}'} + \frac{\mathbf{p}_{x_{3}}^{2}}{2m_{3}'},$$
(6)

where different states have different reduced masses m'_i , which are listed in Table 3.

3.3 Color-spin part

In the color space, the color wave functions can be analyzed using the SU(3) group theory, where the direct product of the diquark and antidiquark components reads

$$(3_c \otimes 3_c) \otimes (\bar{3}_c \otimes \bar{3}_c) = (6_c \oplus \bar{3}_c) \otimes (\bar{6}_c \oplus 3_c).$$
(7)

Based on this, we get two types of color-singlet states:

$$\phi_1 = |(Q_1 Q_2)^{\bar{3}} (\bar{Q}_3 \bar{Q}_4)^3 \rangle, \phi_2 = |(Q_1 Q_2)^6 (\bar{Q}_3 \bar{Q}_4)^{\bar{6}} \rangle.$$
(8)

In the spin space, the allowed wave functions are in the diquark–antidiquark picture:

$$\begin{aligned} \chi_1 &= |(Q_1Q_2)_1(Q_3Q_4)_1\rangle_2, \ \chi_2 &= |(Q_1Q_2)_1(Q_3Q_4)_1\rangle_1, \\ \chi_3 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_0\rangle_1, \ \chi_4 &= |(Q_1Q_2)_0(\bar{Q}_3\bar{Q}_4)_1\rangle_1, \\ \chi_5 &= |(Q_1Q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_0, \ \chi_6 &= |(Q_1Q_2)_0(\bar{Q}_3\bar{Q}_4)_0\rangle_0. \end{aligned}$$

$$(9)$$

In the notation $|(Q_1Q_2)_{\text{spin}1}(Q_3Q_4)_{\text{spin}2}\rangle_{\text{spin}3}$, the spin1, spin2, and spin3 represent the spin of the diquark, the spin

of the antidiquark, and the total spin of the tetraquark state, respectively.

Since the flavor part and spatial parts are chosen to be fully symmetric for the (anti)diquark, the color-spin part of the total wave function should be fully antisymmetric. Combining the flavor part, we show all possible color-spin parts satisfying the Pauli principle with J^{PC} in Table 4.

In addition, it is convenient to consider the strong decay properties, and we again use the meson–meson configuration to represent color-singlet and spin wave functions. The color wave functions in the meson–meson configuration can be derived from the following direct product:

$$(\mathbf{3}_c \otimes \bar{\mathbf{3}}_c) \otimes (\mathbf{3}_c \otimes \bar{\mathbf{3}}_c) = (\mathbf{1}_c \oplus \mathbf{8}_c) \otimes (\mathbf{1}_c \oplus \mathbf{8}_c).$$
(10)

Based on Eq. (10), they can be expressed as

$$\psi_1 = |(Q_1\bar{Q}_3)^1 (Q_2\bar{Q}_4)^1\rangle, \psi_2 = |(Q_1\bar{Q}_3)^8 (Q_2\bar{Q}_4)^8\rangle.$$
(11)

Similarly, the spin wave functions in the meson-meson configuration read as

$$\begin{aligned} \xi_{1} &= |(Q_{1}\bar{Q}_{3})_{1}(Q_{2}\bar{Q}_{4})_{1}\rangle_{2}, \\ \xi_{2} &= |(Q_{1}\bar{Q}_{3})_{0}(Q_{2}\bar{Q}_{4})_{1}\rangle_{1}, \\ \xi_{3} &= |(Q_{1}\bar{Q}_{3})_{1}(Q_{2}\bar{Q}_{4})_{0}\rangle_{1}, \\ \xi_{4} &= |(Q_{1}\bar{Q}_{3})_{1}(Q_{2}\bar{Q}_{4})_{1}\rangle_{1}, \\ \xi_{5} &= |(Q_{1}\bar{Q}_{3})_{1}(Q_{2}\bar{Q}_{4})_{1}\rangle_{0}, \\ \xi_{6} &= |(Q_{1}\bar{Q}_{3})_{0}(Q_{2}\bar{Q}_{4})_{0}\rangle_{0}. \end{aligned}$$

$$(12)$$

4 Numerical analysis

4.1 Mass spectrum, internal contribution, and spatial size

In this subsection, we check the consistency between the experimental masses and the masses of traditional hadrons obtained using the variational method based on the Hamiltonian of Eq. (1) and the parameters in Table 1. We show the results in Table 5 and note that our values are relatively reliable since the deviations for most states are less than 20 MeV.

In addition, in the previous section we systematically constructed the total wave function satisfied by the Pauli principle. The corresponding total wave function can be expanded as follows:

$$|\Psi_{\alpha}\rangle = \sum_{ij} C_{ij}^{\alpha} |F\rangle |R^{s}\rangle |[\phi_{i}\chi_{j}]\rangle.$$
(13)

To study the mass of the fully heavy tetraquarks with the variational method, we calculate the Schrödinger equation $H|\Psi_{\alpha}\rangle = E_{\alpha}|\Psi_{\alpha}\rangle$, diagonalize the corresponding matrix, and then determine the ground state masses for the fully heavy tetraquarks. According to the corresponding variational parameters, we also give the internal mass contributions, including the quark mass part, the kinetic energy part, the confinement potential part, and the hyperfine potential

		1					
States	m'_1	m'_2	m'_3	States	m'_1	m'_2	m'_3
ccēē	m_c	m_c	m_c	$cc\bar{c}\bar{b}$	m_c	$\frac{2m_cm_b}{m_c+m_b}$	$\frac{(m_c+m_b)m_c}{2(3m_c+m_b)}$
$bb\bar{b}\bar{b}$	m_b	m_b	m_b	$bbar{b}ar{c}$	m_b	$\frac{2m_cm_b}{m_c+m_b}$	$\frac{(m_c+m_b)m_b}{2(3m_b+m_c)}$
$cc\bar{b}\bar{b}$	m_c	m_b	$\frac{2m_cm_b}{m_c+m_b}$	$cbar{c}ar{b}$	$\frac{2m_cm_b}{m_c+m_b}$	$\frac{2m_cm_b}{m_c+m_b}$	$\frac{m_c+m_b}{2}$

Table 3 The reduced mass m'_i in different states

 Table 4
 The allowed color-spin parts for each flavor configuration

Туре	$J^{P(C)}$	Color-spin part								
ccēc bbbb ccbb	2+(+)	$\phi_1 \chi_1$								
	$1^{+(-)}$	$\phi_1 \chi_2$								
	$0^{+(+)}$	$\phi_1 \chi_5$	$\phi_2\chi_6$							
$cc\bar{c}\bar{b}\ bb\bar{b}\bar{c}$	2^{+}	$\phi_1 \chi_1$								
	1+	$\phi_1 \chi_2$	$\phi_1 \chi_3$	$\phi_2 \chi_4$						
	0^{+}	$\phi_1 \chi_5$	$\phi_2\chi_6$							
$cb\bar{c}\bar{b}$	2++	$\phi_1 \chi_1$	$\phi_2 \chi_1$							
	1+-	$\phi_1 \chi_2$	$\phi_2 \chi_2$	$\frac{1}{\sqrt{2}}(\phi_1\chi_3+\phi_1\chi_4)$						
		$\frac{1}{\sqrt{2}}(\phi_2\chi_3+\phi_3)$	2 χ4)							
	1++	$\frac{1}{\sqrt{2}}(\phi_1\chi_3-\phi$	1 χ4)	$\frac{1}{\sqrt{2}}(\phi_2\chi_3-\phi_2\chi_4)$						
	0++	$\phi_1 \chi_5$	$\phi_2\chi_5$	$\phi_1 \chi_6 \qquad \phi$	2χ6					

part. For comparison, we also show the lowest meson–meson thresholds for the tetraquarks with different quantum numbers and their internal contributions. This is how we define the binding energy:

$$B_T = M_{\text{tetraquark}} - M_{\text{meson1}} - M_{\text{meson2}}, \qquad (14)$$

where $M_{\text{tetraquark}}$, M_{meson1} , and M_{meson2} are the masses of the tetraquark and the two mesons at the lowest threshold allowed in the rearrangement decay of the tetraquark, respectively. To facilitate the discussion in the next subsection, we also define the V^C , which is the sum of the Coulomb potential and the linear potential.

Here, it is also useful to investigate the spatial size of the tetraquarks, which is strongly related to the magnitude of the various kinetic energies and the potential energies between the quarks. It is also important to understand the relative lengths between the quarks in the tetraquarks and their lowest thresholds, and the relative distance between the heavier quarks is generally shorter than that between the lighter quarks [61]. This tendency is also maintained in each tetraquark state according to the corresponding tables.

4.1.1 Magnetic moments, transition magnetic moments, and radiative decay widths

The magnetic moment of hadrons is a physical quantity that reflects their internal structures [121]. The total magnetic

moment $\vec{\mu}_{total}$ of a compound system contains the spin magnetic moment $\vec{\mu}_{spin}$ and the orbital magnetic moment $\vec{\mu}_{orbital}$ from all of its constituent quarks. For ground hadron states, their contribution of the orbital magnetic moment $\vec{\mu}_{orbital}$ is zero, and so we only concentrate on the spin magnetic moment $\vec{\mu}_{spin}$. The explicit expression for the spin magnetic moment $\vec{\mu}_{spin}$ is written as

$$\vec{\mu}_{\rm spin} = \sum_{i} \mu_i \vec{\sigma}_i = \sum_{i} \frac{\mathcal{Q}_i^{\rm eff}}{2M_i^{\rm eff}} \vec{\sigma}_i, \tag{15}$$

where Q_i^{eff} and M_i^{eff} are the effective charge and effective mass of the *i*-th constituent quark, respectively. The $\vec{\sigma}_i$ denotes the Pauli spin matrix of the *i*-th constituent quark. According to Ref. [122], the effective charge of the quark is affected by other quarks in the inner hadron. We now assume that the effective charge is linearly dependent on the charge of the shielding quarks. Therefore, the effective charge Q_i^{eff} is defined as

$$Q_i^{\text{eff}} = Q_i + \sum_{i \neq j} \alpha_{ij} Q_j, \qquad (16)$$

where Q_i is the bare charge of the *i*-th constituent quark, and α_{ij} is a corrected parameter that reflects the extent to which the charge of other quarks affects the charge of the *i*th quark. To simplify the calculation, we also set α_{ij} always equal to 0.033 according to Ref. [122]. The effective quark masses M_i^{eff} contain the contributions from both the bare quark mass terms and the interaction terms in the chromomagnetic model, and their values are taken from Ref. [123].

To obtain the magnetic moment of the discussed hadron, we calculate the *z*-component of the magnetic moment operator $\hat{\mu}^z$ sandwiched by the corresponding total wave function Ψ_{α} (Eq. (9)). Now, only the spin part of the total wave function is involved. The total spin wave functions of the discussed hadrons are written as

$$|\chi_{\text{total}}\rangle = C_1\chi_1 + C_2\chi_2 + \cdots \tag{17}$$

Based on this, we can quantitatively obtain the magnetic moment of the discussed hadron

$$\mu = \langle \Psi_{\alpha} | \hat{\mu}^{z} | \Psi_{\alpha} \rangle = \langle \chi_{\alpha} | \hat{\mu}^{z} | \chi_{\alpha} \rangle$$

= $C_{1}^{2} \mu(\chi_{1}) + C_{2}^{2} \mu(\chi_{2}) + \dots + 2C_{1}C_{2}\mu^{\text{tr}}(\chi_{1},\chi_{2}) + \dots$
(18)

where μ^{tr} is the cross-term representing the transition moment, and C_1 , C_2 are the eigenvectors of the given mixing state [124]. Similarly, the transition magnetic moments between the hadrons can be obtained as $\mu_{H' \to H\gamma} = \langle \Psi_{H_f} | \hat{\mu}^z | \Psi_{H_i} \rangle$.

According to Eq. (18), the numerical values for the magnetic moments of the traditional hadrons have been listed in Table 5. Here, $\mu_N = e/2m_N$ is the nuclear magnetic moment with $m_N = 938$ MeV as the nuclear mass, which is the unit of the magnetic moment. For comparison, we also show the experimental values and other theoretical results from Refs. [121,122,124–127]. Because of the $\mu_Q = -\mu_{\bar{Q}}$, the magnetic moment of all the $J^P = 0^+$ ground mesons and tetraquarks and the ground states with certain *C*-parity is 0.

The decay property is another important aspect to investigate the nature of the exotic hadron. According to the transition magnetic moments in the above subsection, we can further obtain the radiative decay widths around fully heavy tetraquarks [128–137].

$$\Gamma = \frac{|\mathbf{k}|^2}{\pi} \frac{2}{2J_i + 1} \frac{M_f}{M_i} \sum_{M_{J_f}, M_{J_i}} |\mathcal{M}_{M_{J_f}, M_{J_i}}|^2,$$
(19)

where J_i and J_f are the total angular momentum of the initial and final hadrons, respectively. The M_i and M_f in Eq. (19) represent initial and final hadron masses, respectively.

4.1.2 Relative decay widths of tetraquarks

In addition to radiative decay, we also consider the rearrangement strong decay properties for fully heavy tetraquarks. Based on Eqs. (10–12), the color wave function also falls into two categories: the color-singlet $\psi_1 = |(Q_1\bar{Q}_3)^1(Q_2\bar{Q}_4)^1\rangle$, which can easily decay into two S-wave mesons, and the color-octet $\psi_2 = |(Q_1\bar{Q}_3)^8(Q_2\bar{Q}_4)^8\rangle$, which can only fall apart by gluon exchange. Thus we transform the total wave functions Ψ_{α} into the new configuration,

$$\Psi_{\alpha}\rangle = \sum_{ij} C_{ij}^{\prime\alpha} |F\rangle |R^{s}\rangle |[\psi_{i}\zeta_{j}]\rangle.$$
⁽²⁰⁾

Among the decay behaviors of the tetraquarks, one decay mode is that the quarks simply fall apart into the final decay channels without quark pair creation or annihilation, which is denoted as "Okubo–Zweig–Iizuka (OZI)-superallowed" decays. In this part, we will only focus on this type of decay channel. For two-body decay by *L*-wave, the partial decay width is expressed as [72, 110, 138–140]:

$$\Gamma_i = \gamma_i \alpha \frac{k^{2L+1}}{m^{2L}} |c_i|^2, \qquad (21)$$

where α is an effective coupling constant, c_i is the overlap corresponding exactly to $C_{ii}^{\prime\alpha}$ of Eq. (20), *m* is the mass of the initial state, and k is the momentum of the final state in the rest frame of the initial state. For the decays of the S-wave tetraquarks, $(k/m)^{-2}$ is of order $\mathcal{O}(10^{-2})$ or even smaller, so all higher-wave decays are suppressed, and thus we only need to consider the S-wave decays. As for γ_i , it is determined by the spatial wave functions of the initial and final states, which are different for each decay process. In the quark model in the heavy quark limit, the spatial wave functions of the ground S-wave pseudoscalar and the vector meson are the same. The relations of γ_i for fully heavy tetraquarks are given in Table 6. Based on this, the branching fraction is proportional to the square of the coefficient of the corresponding component in the eigenvectors, and the strong decay phase space, i.e., $k \cdot |c_i|^2$, for each decay mode. From the value of $k \cdot |c_i|^2$, one can roughly estimate the ratios of the relative decay widths between different decay processes of different initial tetraquarks.

In the following subsections, we concretely discuss all possible configurations for fully heavy tetraquarks.

4.2 $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states

First we investigate the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems. There are two $J^{PC} = 0^{++}$ states, one $J^{PC} = 1^{+-}$ state, and one $J^{PC} = 2^{++}$ state according to Table 4. We show the masses of the ground states, the variational parameters, the internal mass contributions, the relative lengths between the quarks, their lowest meson–meson thresholds, the specific wave function, the magnetic moments, the transition magnetic moments, the radiative decay widths, and the rearrangement strong width ratios in Tables 7, 8, and 9, respectively.

Here, we take the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state as an example, and the others have similar discussions, according to Tables 7, 8, and 9. We now analyze the numerical results obtained from the variational method. For the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state, its mass is 19240.0 MeV and the corresponding binding energy B_T is +461.9 MeV. Its variational

Table 5	Masses an	nd magnetic	moments	of some	ground h	adrons
obtained	from the	theoretical	calculation	s. M_{result} ,	μ_{results} ,	$\mu_{\mathrm{bag}},$
$\mu_{\text{the}(1)}$, ar	nd $\mu_{the(2)}$	are theoretic	cal masses a	and magne	tic mome	nts for
Eq. (1), E	q. (<mark>15</mark>), and	l Refs. [121,	122,124], r	respectively	. M _{exp} an	d μ_{exp}

are the observed values of masses and magnetic moments. The masses and errors are in units of MeV. The magnetic moment is in units of the nuclear magnetic moment μ_N . The variational parameter is in units of fm⁻²

Hadron	Σ^+	Σ^0	Σ^{-}	Ξ^0	Ξ^-	Σ_c^{++}	Σ_c^+	Σ_c^0	Σ_c^{*++}	Σ_c^{*+}	Σ_c^{*0}	Σ_b^+	Σ_b^0	Σ_b^-	Σ_b^{*+}	Σ_b^{*0}	Σ_b^{*-}
M_{result}		1187.	7	129	95.4		2445.2			2518.3		Ся	832.1			5860.	8
Parameters		2.1		3	.3		2.1			2.0			2.1			2.0	
i arameters		3.1		2	.9	3.7			3.4			4.0			3.4		
M_{exp}		1189.	4	131	4.9	2454.0 2851.4 5811.3					5832.1						
Error		-1.7		-19	9.5		-8.8			-0.1			20.8			28.7	
μ_{result}	2.53	0.75	-1.04	-1.31	-0.52	2.34	0.55	-1.24	4.09	1.38	-1.32	2.37	0.58	-1.21	3.48	0.78	-1.92
$\mu_{bag} \ [124]$	2.72	0.86	-1.01	-1.58	-0.64	2.13	0.41	-1.31	4.07	1.39	-1.29	2.23	0.58	-1.07	3.29	0.76	-1.77
$\mu_{the(1)}[121]$	2.74	0.84	-1.06	-1.47	-0.52	2.36	0.50	-1.37	4.09	1.30	-1.49						
$\mu_{the(2)}~[122]$	2.46	0.47	-1.10	-1.61	-0.65	3.57	1.96	0.04									
μ_{exp}	2.46		-1.16	-1.25	-0.65								6.14	2.70			-2.02
Hadron	D^{*0}	D^{*+}	D_s^{*+}	B^{*0}	B^{*+}	B_{s}^{*0}	J/ψ	η_c	Υ	η_b	B_c	B_c^*	Δ^{++}	Δ^+	Δ^0	Δ^0	Ω^{-}
M_{result}	199	6.9	2093.3	536	33.6	5434.7	3092.2	2998.5	9468.9	9389.0	6287.9	6350.5		124	5.6		1675.8
Parameters	3.	8	6.2	4	.2	7.5	12.5	15.0	49.7	57.4	22.9	20.2		1.	8		3.3
M_{exp}	201	0.3	2112.2	532	24.7	5415.4	3096.9	2983.9	9460.3	9399.0	6274.9	(6332)		123	2.0		1672.5
Error	-13	8.4	-18.9	38	3.8	19.3	-4.7	14.6	8.6	10.0	13.0	(17.5)		13	.6		3.1
μ_{result}	-1.37	1.24	1.00	-0.78	1.83	0.51	0	-	0	-	-	0.44	5.57	-2.78	0	-2.78	-1.86
$\mu_{bag}~[124]$	-0.98	1.21	1.08	-0.53	1.21	1.01	0	-	0	-	-	0.52	5.70	2.85	0	-2.85	-2.20
$\mu_{the(1)}$ [121]	-1.49	1.30	1.07				0	_					5.58	2.79	0	-2.79	-1.88

Table 6 The approximate relation for γ_i for the $QQ\bar{Q}\bar{Q}$ system

States	γ_i
ccēē	$\gamma_{J/\psi J/\psi} = \gamma_{\eta_c J/\psi} = \gamma_{\eta_c \eta_c}$
$bb\bar{b}\bar{b}$	$\gamma_{\Upsilon\Upsilon}=\gamma_{\eta_b\Upsilon}=\gamma_{\eta_b\eta_b}$
$cc\bar{b}\bar{b}$	$\gamma_{B_c^*B_c^*} = \gamma_{B_cB_c^*} = \gamma_{B_cB_c}$
$cc\bar{c}\bar{b}$	$\gamma_{J/\psi} B_c^* = \gamma_{J/\psi} B_c = \gamma_{\eta_c} B_c^* = \gamma_{\eta_c} B_c$
bbbc	$\gamma_{\Upsilon \bar{B}_c^*} = \gamma_{\Upsilon \bar{B}_c} = \gamma_{\eta_b \bar{B}_c^*} = \gamma_{\eta_b \bar{B}_c}$
$cb\bar{c}\bar{b}$	$\gamma_{J/\psi} \Upsilon = \gamma_{J/\psi} \eta_b = \gamma_{\eta_c} \Upsilon = \gamma_{\eta_c} \eta_b$
	$\gamma_{B_c^*\bar{B}_c^*} = \gamma_{B_c^*\bar{B}_c} = \gamma_{B_c\bar{B}_c^*} = \gamma_{B_c\bar{B}_c}$

parameters are given as $C_{11} = 7.7 \text{ fm}^{-2}$, $C_{22} = 7.7 \text{ fm}^{-2}$, and $C_{33} = 11.4 \text{ fm}^{-2}$, giving roughly the inverse ratios of the size for the diquark, the antidiquark, and between the center of the diquark and the antidiquark, respectively. We naturally find that C_{11} is equal to C_{22} , so the distance of (b-b) would be equal to that of $(\bar{b} - \bar{b})$, and the reason is that the $bb\bar{b}\bar{b}$ system is a neutral system.

The total wave function in the diquark–antidiquark configuration is given by

$$|\Psi_{\text{tot}}\rangle = -0.936|F\rangle|R^{s}\rangle|[\phi_{2}\chi_{6}]\rangle + 0.352|F\rangle|R^{s}\rangle|[\phi_{1}\chi_{5}]\rangle.$$
(22)

The meson–meson configuration is connected to the diquark– antidiquark configuration by a linear transformation. We then obtain the total wave function in the meson–meson configuration:

$$\begin{aligned} |\Psi_{\text{tot}}\rangle &= 0.558 |F\rangle |R^s\rangle |[\psi_1\zeta_5]\rangle + 0.560 |F\rangle |R^s\rangle |[\psi_1\zeta_6]\rangle \\ &+ 0.021 |F\rangle |R^s\rangle |[\psi_2\zeta_5]\rangle + 0.612 |F\rangle |R^s\rangle |[\psi_2\zeta_6]\rangle. \end{aligned}$$

$$(23)$$

According to Eq. (23), we are sure that the overlaps c_i of $\eta_b \eta_b$ and $\Upsilon \Upsilon$ are 0.560 and 0.558, respectively. Then, based on Eq. (21), the rearrangement strong width ratios are

$$\frac{\Gamma_{T_{b^2\bar{b}^2}(19240.0,0^{++})\to\Upsilon\Upsilon}}{\Gamma_{T_{b^2\bar{b}^2}(19240.0,0^{++})\to\eta_b\eta_b}} = 1:1.2,$$
(24)

i.e., both the $\Upsilon \Upsilon$ and $\eta_b \eta_b$ are dominant decay channels for the $T_{b^2 \bar{b}^2}(19240.0, \Upsilon \Upsilon)$ state.

As for the magnetic moments of the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ ground states, their values are all 0, because the same quark and antiquark have exactly opposite magnetic moments,

which cancel each other out. We also discuss the transition magnetic moment of the $T_{b^2\bar{b}^2}(19303.9, 1^{+-}) \rightarrow T_{b^2\bar{b}^2}(19240.0, 0^{++})\gamma$ process. We construct their flavor \otimes spin wave functions as

$$\begin{split} |\Psi\rangle_{T_{b^{2}\bar{b}^{2}}(1^{+-})}^{S=1;S_{s}=0} &= |R^{s}\rangle|\psi\rangle|bb\bar{b}\bar{b}\rangle|\frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow\downarrow-\downarrow\downarrow\uparrow\uparrow\rangle)\\ |\Psi\rangle_{T_{b^{2}\bar{b}^{2}}(0^{++})}^{S=0;S_{s}=0} &= |R^{s}\rangle|\psi\rangle|bb\bar{b}\bar{b}\rangle\\ &= |0.352\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow\downarrow+\downarrow\downarrow\uparrow\uparrow)+\cdots\rangle. \end{split}$$
(25)

And then the transition magnetic momentum of the $T_{b^2\bar{b}^2}(19303.9, 1^{+-}) \rightarrow T_{b^2\bar{b}^2}(19240.0, 0^{++})\gamma$ process can be given by the *z*-component of the magnetic moment operator $\hat{\mu}^z$ sandwiched by the flavor-spin wave functions of the $T_{b^2\bar{b}^2}(19303.9, 1^{+-})$ and $T_{b^2\bar{b}^2}(19240.0, 0^{++})$. Therefore, the corresponding transition magnetic momentum is

$$\mu_{T_{b^2\bar{b}^2}(1^{+-})\to T_{b^2\bar{b}^2}(0^{++})} = \langle \Psi_{\text{tot}}^{1^{+-}} | \hat{\mu^z} | \Psi_{\text{tot}}^{0^{++}} \rangle$$

= 0.352 × $\frac{1}{\sqrt{6}} (4\mu_b - 4\mu_{\bar{b}}) = -0.072 \ \mu_N.$ (26)

As for the transition magnetic moment of the $T_{b^2\bar{b}^2}(19327.9, 2^{++}) \rightarrow T_{b^2\bar{b}^2}(19240.0, 0^{++})\gamma$ process, its value is 0 due to the *C*-parity conservation restriction.

Furthermore, according to Eqs. (19) and (26), we also obtain the corresponding radiative decay widths

$$\Gamma_{T_{b^{2}\bar{b}^{2}}(19303.9,1^{+-}) \to T_{c^{2}\bar{b}^{2}}(19240.0,0^{++})\gamma} = 2.8 \,\text{keV}, \qquad (27)$$

$$\Gamma_{T_{b^2\bar{b}^2}(19327.9,2^{++})\to T_{c^2\bar{b}^2}(19240.0,0^{++})\gamma} = 0 \text{ keV}.$$
 (28)

4.2.1 Relative distances and symmetry

Here, we concentrate on the relative distances between the (anti)quarks in tetraquarks. Looking at the relative distances in Table 9, we find that the relative distances of (1,2) and (3,4) pairs are the same, and other relative distances are the same in all the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states. This is due to the permutation symmetry for the ground state wave function in each tetraquark [65]. For the $c_1c_2\bar{c}_3\bar{c}_4$ and $b_1b_2\bar{b}_3\bar{b}_4$ states, they need to satisfy the Pauli principle for identical particles as follows:

$$A_{12}|\Psi_{\text{tot}}\rangle = A_{34}|\Psi_{\text{tot}}\rangle = -|\Psi_{\text{tot}}\rangle,\tag{29}$$

where the operator A_{ij} means exchanging the coordinates of Q_i (\bar{Q}_i) and Q_j (\bar{Q}_j).

Meanwhile, they are pure neutral particles with definite C-parity, so the permutation symmetries for total wave functions are as follows:

$$A_{12-34}|\Psi_{\text{tot}}\rangle = \pm |\Psi_{\text{tot}}\rangle,\tag{30}$$

where A_{12-34} means that the coordinates of the diquark and the antidiquark are exchanged.

Based on this, the relationship of the relative distances for all the $c_1c_2\bar{c}_3\bar{c}_4$ and $b_1b_2\bar{b}_3\bar{b}_4$ states can be obtained as follows:

and

Obviously, our theoretical derivations are in perfect agreement with the calculated results in Table 9.

We can also prove that three Jacobi coordinates, $\mathbf{R}_{1,2} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{R}_{3,4} = \mathbf{r}_3 - \mathbf{r}_4$, and $\mathbf{R}' = 1/2(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4)$, are orthogonal to each other for all the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states:

$$\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}_{3,4}) | \Psi_{\text{tot}} \rangle$$

$$= \langle \Psi_{\text{tot}} | (34)^{-1} (34) | (\mathbf{R}_{1,2} \cdot \mathbf{R}_{3,4}) | (34)^{-1} (34) | \Psi_{\text{tot}} \rangle$$

$$= -\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}_{3,4}) | \Psi_{\text{tot}} \rangle = 0,$$

$$\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle$$

$$(33)$$

$$= \langle \Psi_{\text{tot}} | (12)^{-1} (12) | (\mathbf{R}_{1,2} \cdot \mathbf{R}') | (12)^{-1} (12) | \Psi_{\text{tot}} \rangle$$

$$= -\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle = 0, \qquad (34)$$

and

$$\begin{aligned} \langle \Psi_{\text{tot}} | (\mathbf{R}_{3,4} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle \\ &= \langle \Psi_{\text{tot}} | (34)^{-1} (34) | (\mathbf{R}_{1,2} \cdot \mathbf{R}') | (34)^{-1} (34) | \Psi_{\text{tot}} \rangle \\ &= - \langle \Psi_{\text{tot}} | (\mathbf{R}_{3,4} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle = 0. \end{aligned}$$
(35)

According to the relative distances in Table 9 and the relationship of Eqs. (29–35), the relative positions of the four valence quarks can be well described for all the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states. Meanwhile, using the relative distances between (anti)quarks and the orthogonal relation, we can also determine the relative distance of (12)–(34), which is consistent with our results in Table 9. We can also give the relative position of R_c and the spherical radius of the tetraquarks. Here, we define R_c as the geometric center of the four quarks (the center of the sphere). Based on these results, we show the spatial distribution of the four valence quarks for the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state in Fig. 1.

In the quark model, a compact tetraquark state has no color-singlet substructure, while a hadronic molecule is a loosely bound state which contains several color-singlet

Table 7 The masses, binding energies, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width ratios, and the relative lengths between the quarks for the $J^{PC} = 0^{++}$, $1^{+-} cc\bar{c}\bar{c}$ states and their lowest meson–meson thresholds. Here, (i, j) denotes the contribution of the *i*-th and *j*-th quarks. The number is

given as i = 1 and 2 for the quarks, and 3 and 4 for the antiquark. The masses and corresponding contributions are given in units of MeV, and the relative lengths (variational parameters) are in units of fm (fm⁻²). Meanwhile, we present a comparison with the other two CMI models [110, 123] to further secure the effective quark mass

$cc\overline{c}\overline{c}$	The c	ontributi	on from	each term	Rela	tive Le	engths (fm)	Orrana 11	Present Work		CMI	Model
$J^{PC} = 0^{++}$		Value	$\eta_c \eta_c$	Difference	(i,j)	Value	$\eta_c \eta_c$	Overan	Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	6384.4	5997.0	387.4	(1,2)	0.406			$2m_c$	3836.0		
Variational	C_{11}	7.7	15.0		(1.3)	0.371	$0.290(\eta_c)$		$\frac{\mathbf{p}_{x_1}^2}{2m'}$	233.9		
Parameters (fm ⁻²)	C_{22} C_{22}	11.4	15.9 _		(2.3)	0.371			$\underline{m_{\overline{c}}}$ $\underline{\mathbf{p}_{x_3}^2}$	123.1		
Ouark	Mass	7672.0	7672.0	0.0	(1.4)	0.371		<i>c</i> -quark:	$\frac{m_c + m_{\bar{c}} \ 2m'_3}{V^C(12)}$	-6.8	$-\frac{1}{2}m_{-1}$	
Confinemen	t Potential	-2083.8	-2440.4	356.6	(1,1) (2.4)	0.371	0.290(n)	m_c^{eff}	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$		-792.9	
Kinetic	Energy	814.0	915.1	-101.1	(2, 1)	0.011	$0.250(\eta_c)$	č	$_{2}[V^{C}(23) + V^{C}(24)]$	-52.1	-102.0 5m -	9m
CS Inter	raction	22.7	-150.0	172.7	(0, 4)	(3.4).	0.235 fm		-D	-083.0	3835.6	2///c 3//0.6
	(1.2)	-6.8	100.0	112.1	(1,2)- B	dius.	0.235 fm		Subtotal	3151.1	3042.7	3449.6
	(2,2)	-0.0		ļ	10	iarus.	0.200 III		2m	3836.0	0012.1	011010
V^C	(1, 4)	-20.1			(1.9)	96.1	997.9()		$\mathbf{p}_{x_2}^2$	0000.0		
	(1,4)	-20.1			(1,3)	-20.1	$-231.2(\eta_c)$		$\frac{\overline{2m_2^7}}{\overline{p_2^2}}$	255.9		
	Subtotal	-117.8	-474.4	356.6	(3,4)	-6.8		<i>ē</i> -quark:	$\frac{m_{\bar{c}}}{m_c + m_{\bar{c}}} \frac{P_{X_3}}{2m'_3}$	123.1		
Total Con	tribution	718.9	291.0	427.9	(2,4)	-26.1	$-237.2(\eta_c)$	off	$V^{C}(34)$	-6.8	$-\frac{1}{4}m_{cc}$	
Total Wave	function:							$m_{\overline{c}}^{c,r,r}$	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	-52.1	-792.4	
$\Psi_{tot} = 0.$	$535 F\rangle R^s\rangle$	$ [\phi_1\chi_5]\rangle$	-0.845 1	$ F\rangle R^s\rangle [\phi_2\chi]$	$\langle 6 \rangle = 0$	0.612 F	$ R^s\rangle R^s\rangle [\psi_1\zeta_5]\rangle$		$+V^{C}(23) + V^{C}(24)]$		$\frac{5}{4}m_{c\overline{c}}$	$2m_c$
+0	$.443 F\rangle R^s$	$\langle [\psi_1\zeta_6]\rangle$	= 0.612	$F\rangle R^s\rangle [\psi_1]$	$\langle \zeta_5] \rangle + 0$	0.443 F	$ R^s\rangle R^s\rangle [\psi_1\zeta_6]\rangle$		- <i>D</i>	-983.0	3835.6	3449.6
The rearran	gement str	ong widt	h ratios:						Subtotal	3151.1	3042.7	3449.6
$\Gamma_{T_{c^2\bar{c}^2}(6384.3}$	$(,0^{++}) \rightarrow J/\psi J$	$\Gamma_{/\psi}:\Gamma_{T_{c^2}}$	\bar{c}^2 (6384.3,	$(0^{++}) \rightarrow \eta_c \eta_c$	= 1 : 2.	8			$\frac{3}{2}V^{SS}(12)$	11.4	$4v_{cc}$	$4C_{cc}$
The radiativ	ve decay wi	dths: Γ_T	c2 c2 c2 (6482.	$7,2^{++}) \rightarrow T_{c^2}$	$\frac{1}{c^2}$ (6384.3	3,0 ⁺⁺)Υ	h = 0 keV	CC	4, (12)	1111	14.2	21.2
		Γ_{T_c}	$2\bar{c}^2$ (6451.5	$(1^{+-}) \rightarrow T_{c^2 \overline{c}^2}$	(6384.3,	0 ⁺⁺)Y	= 238.1 keV	Interaction	$\frac{3}{2}V^{SS}(12)$	11.4	$4v_{\overline{c}\overline{c}}$	$4C_{\overline{c}\overline{c}}$
The magnet	ic moments	5:	$\mu_{T_{c^2}}$	$\overline{c}^{2}(6384.3,0^{+})$	$\Psi \rangle = \langle \Psi \rangle$	$\mathcal{P}_{tot}^{0^{++}} \hat{\mu}$	$z \Psi_{tot}^{0^{++}}\rangle = 0$		$\frac{1}{4}v$ (12)	11.4	14.2	21.2
The transiti	on magnet	ic momer	nts:						Subtotal	22.7	28.4	42.4
$\mu_{T_{c^2\bar{\sigma}^2}(6451.5)}$	$(1^{+-}) \rightarrow T_{c^2 \overline{c^2}}$	$(6384.3,0^{+})$	$_{++)\gamma} = \langle \Psi$	$\Psi_{tot}^{1^{+-}} \hat{\mu^z} \Psi_t^0 $	$\langle t_{ot}^{++} \rangle =$	0.671	μ_N	Matrix nondiagonal element -40			-60.9	159.2
$\mu_{T_{c^2\bar{c}^2}(6482.7)}$	$(,2^{++}) \rightarrow T_{c^2 \bar{c}^2}$	(6384.3,04	$(+)_{\gamma} = \langle \Psi$	$\Psi_{tot}^{2^{++}} \hat{\mu^{z}} \Psi_{t}^{0}$	$\langle t_{ot}^{++} \rangle =$	0		Total contri	bution	6384.4	6044.9	7016.0
$J^{PC} = 1^{+-}$		Value	$J/\psi\eta_c$	Difference	Rela	tive Le	engths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	6451.5	6090.7	360.8	(i,j) `	Value	$J/\psi \eta_c$		$2m_c$	3836.0		
Variational	C_{11}	9.1	15.0		(1.2)	0.373			$\underline{\mathbf{p}_{x_1}^2}$	077.0		
Parameters (fm ⁻²)	L Caa		1.1.1		()				2m'	277.2		
(1111)	C22 C22	9.1 7.3	12.5		(1.3)	0.395	$0.290(n_c)$	<i>c</i> -quark:	$\frac{2m'_1}{m_{\tilde{c}}} \frac{\mathbf{p}_{x_3}^2}{\mathbf{p}_{x_3}^2}$	111.2	$\frac{1}{2}m_{cc}$	
Quark	C ₃₃	9.1 7.3 7672.0	7672.0	0.0	(1,3) (2,3)	0.395 0.395	$0.290(\eta_c)$	<i>c</i> -quark:	$\frac{\frac{2m_1'}{m_{\bar{c}}} \mathbf{p}_{x_3}^2}{\frac{m_{\bar{c}}}{m_c + m_{\bar{c}}} \frac{\mathbf{p}_{x_3}^2}{2m_3'}}{V^C(12)}$	277.2 111.2 -19.4	$\frac{1}{2}m_{cc}$	
Quark	C_{33} Mass t Potential	9.1 7.3 7672.0 -1998.8		0.0	(1,3) (2,3) (1,4)	0.395 0.395 0.395	$0.290(\eta_c)$	c-quark: m_c^{eff}	$\frac{\frac{2m_1'}{m_c + m_{\tilde{c}}} \mathbf{p}_{x_3}^2}{V^C(12)}$ $\frac{\frac{1}{2} [V^C(13) + V^C(14)]}{V^C(14) + V^C(14)}$	277.2 111.2 -19.4	$\frac{1}{2}m_{cc}$ 1585.8 $\frac{1}{2}m_{cc}$	$2m_{-}$
Quark Confinemen Kinetic	$\begin{array}{c} & C_{22} \\ C_{33} \\ \\ Mass \\ t \text{ Potential} \\ \\ Energy \end{array}$	9.1 7.3 7672.0 -1998.8 767.2	-2367.4 839.0	0.0 368.6	(1,3) (2,3) (1,4) (2,4)	0.395 0.395 0.395 0.395	$0.290(\eta_c)$ $0.318(J/\psi)$	c -quark: m_c^{eff}	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{p_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ - D \end{array}$	277.2 111.2 -19.4 3.0 -983.0	$\frac{1}{2}m_{cc}$ 1585.8 $\frac{1}{2}m_{c\bar{c}}$ 1534.3	$2m_c$ 3449 6
Quark Confinemen Kinetic CS Inter	$\begin{array}{c} C_{32} \\ C_{33} \\ \\ \text{Mass} \\ \text{t Potential} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	9.1 7.3 7672.0 -1998.8 767.2 1.5	-2367.4 -53.9	0.0 368.6 -71.8 54.4	(1.3) (2.3) (1.4) (2.4) (3.4)	0.395 0.395 0.395 0.395 0.395 0.373	$0.290(\eta_c)$ $0.318(J/\psi)$	c -quark: m_c^{eff}	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{23}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2}[V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \end{array}$	277.2 111.2 -19.4 3.0 -983.0 3225.0	$\frac{\frac{1}{2}m_{cc}}{1585.8}$ $\frac{\frac{1}{2}m_{c\bar{c}}}{1534.3}$ 3120.0	$2m_c$ 3449.6 3449.6
Quark Confinemen Kinetic CS Inter	$\begin{array}{c} C_{32} \\ C_{33} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4	-2367.4 839.0 -53.9	0.0 368.6 -71.8 54.4	(1,3) (2,3) (1,4) (2,4) (3,4) (1,2)-	$\begin{array}{c} 0.395 \\ 0.395 \\ 0.395 \\ 0.395 \\ 0.373 \\ \hline (3.4) \end{array}$	$0.290(\eta_c)$ $0.318(J/\psi)$	c-quark: m_c^{eff}	$\frac{2m_1'}{m_{\tilde{\pi}}} \frac{p_{x_3}^2}{m_{\tilde{\pi}}^2 2m_3'} \frac{p_{x_3}^2}{2m_3'} V^C(12)$ $\frac{1}{2} [V^C(13) + V^C(14) + V^C(23) + V^C(24)] - D$ Subtotal $\frac{2m_{\tilde{\pi}}}{2m_{\tilde{\pi}}}$	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0	$\frac{\frac{1}{2}m_{cc}}{1585.8}$ $\frac{\frac{1}{2}m_{c\bar{c}}}{1534.3}$ 3120.0	$2m_c$ 3449.6 3449.6
Quark Confinemen Kinetic CS Inter	$\begin{array}{c} C_{33} \\ C_{33} \\ Mass \\ t \text{ Potential} \\ Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ \end{array}$	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4	12.5 - 7672.0 -2367.4 839.0 -53.9	0.0 368.6 -71.8 54.4	(1.3) (2.3) (1.4) (2.4) (3.4) (1.2) - B = B = B = B = B = B = B = B = B = B	0.395 0.395 0.395 0.395 0.373 (3,4):	$0.290(\eta_c)$ $0.318(J/\psi)$ 0.294 fm 0.235 fm	c -quark: m_c^{eff}	$\frac{2m_1'}{m_c m_z} \frac{\mathbf{p}_{x_3}^2}{\mathbf{p}_c^{+m_z} \frac{2m_3}{2m_3}} V^C(12)$ $\frac{1}{2}[V^C(13) + V^C(14) + V^C(23) + V^C(24)] - D$ Subtotal $\frac{2m_c}{\mathbf{p}_{x_2}^2}$	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2	$\frac{\frac{1}{2}m_{cc}}{1585.8}$ $\frac{\frac{1}{2}m_{c\bar{c}}}{1534.3}$ 3120.0	$2m_c$ 3449.6 3449.6
Quark Confinemen Kinetic CS Inter V ^C	$\begin{array}{c} C_{33} \\ \text{Mass} \\ \text{t Potential} \\ \hline \text{Energy} \\ \hline \text{raction} \\ \hline (1,2) \\ (2,3) \\ (1,4) \end{array}$	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4 1.5	12.5 - 7672.0 -2367.4 839.0 -53.9	0.0 368.6 -71.8 54.4	$\begin{array}{c} (1.2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	0.395 0.395 0.395 0.395 0.373 (3,4): adius:	$0.290(\eta_c)$ $0.318(J/\psi)$ 0.294 fm 0.235 fm	c-quark: m_c^{eff}	$\begin{array}{c} 2m_{1}^{2m_{1}^{\prime}} \\ \frac{2m_{c}^{2m_{c}}}{m_{c}+m_{c}} \frac{\mathbf{P}_{x_{3}}^{2}}{2m_{3}^{\prime}} \\ V^{C}(12) \\ \frac{1}{2}[V^{C}(13) + V^{C}(14) \\ + V^{C}(23) + V^{C}(24)] \\ -D \\ \hline \\$	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2	$\frac{1}{2}m_{cc}$ 1585.8 $\frac{1}{2}m_{c\bar{c}}$ 1534.3 3120.0	$2m_c$ 3449.6 3449.6
$\begin{array}{c} \mbox{Quark}\\ \mbox{Confinemen}\\ \mbox{Kinetic}\\ \mbox{CS Inter}\\ \mbox{V}^{C} \end{array}$	$\begin{array}{c} C_{33} \\ \hline C_{33} \\ \hline Mass \\ t \text{ Potential} \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline \end{array}$	$9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\$	12.5 - 7672.0 -2367.4 839.0 -53.9	0.0 368.6 -71.8 54.4	$\begin{array}{c} (1.2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ Ra \\ (1,3) \\ \end{array}$	0.395 0.395 0.395 0.395 0.373 (3,4): adius: 1.5	$0.290(\eta_c)$ $0.318(J/\psi)$ 0.294 fm 0.235 fm $-237.2(\eta_c)$	c-quark: m_c^{eff} \bar{c} -quark:	$\frac{2m_1'}{m_c + m_c} \frac{p_{x_3}^2}{2m_3'} \frac{p_{x_3}^2}{V^C(12)} \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \frac{2m_c}{m_c + m_c} \frac{p_{x_3}^2}{2m_3'} \\ \frac{m_c}{V_c^C(12)} \frac{p_{x_3}^2}{2m_3'} \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 10.4	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ \frac{1}{2}m_{cc$	$2m_c$ 3449.6 3449.6
Quark Confinemen Kinetic CS Inter V ^C	$\begin{array}{c} C_{33} \\ \text{Mass} \\ \text{t Potential} \\ \text{Energy} \\ \text{raction} \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline \text{Subtotal} \\ \end{array}$	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4 1.5 1.5 -32.8	12.5 7672.0 -2367.4 839.0 -53.9 -401.4	0.0 368.6 -71.8 54.4 368.6	(1.2) $(1,3)$ $(2,3)$ $(1,4)$ $(2,4)$ $(3,4)$ $(1,2)-$ Ra $(1,3)$ $(3,4)$ $(3,4)$	0.395 0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4	$0.290(\eta_c)$ $0.318(J/\psi)$ 0.294 fm 0.235 fm $-237.2(\eta_c)$	c-quark: m_c^{eff} \bar{c} -quark:	$\begin{array}{c} \frac{2m_1'}{m_c} \frac{p_{x_3}^2}{m_c + m_c} \frac{p_{x_3}^2}{2m_3} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ 1 \\ 1585.8 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ $	$2m_c$ 3449.6 3449.6
Quark Confinemen Kinetic CS Inter V ^C	$\begin{array}{c} C_{33} \\ \text{Mass} \\ \text{t Potential} \\ \hline \text{Energy} \\ \text{raction} \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline \text{Subtotal} \\ \text{tribution} \end{array}$	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4 1.5 1.5 -32.8 718.9	12.5 7672.0 -2367.4 839.0 -53.9 -401.4 291.0	0.0 368.6 -71.8 54.4 368.6 427.9	(1.2) $(1,3)$ $(2,3)$ $(1,4)$ $(2,4)$ $(3,4)$ $(1,2)-$ Ra $(1,3)$ $(3,4)$ $(2,4)$	0.395 0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4 -1.5	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \ \mathrm{fm} \\ 0.235 \ \mathrm{fm} \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} $	$2m_c$ 3449.6 3449.6 $2m_c$
Quark Confinemen Kinetic CS Inter V^C Total Con Total Wave	$\begin{array}{c} C_{33} \\ \hline \\ C_{33} \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	9.1 7.3 7672.0 -1998.8 767.2 1.5 -19.4 1.5 1.5 -32.8 718.9 Ψ	$ \begin{array}{c} 12.5 \\ 7672.0 \\ -2367.4 \\ 839.0 \\ -53.9 \\ \hline -401.4 \\ 291.0 \\ tot = F\rangle \end{array} $	$\begin{array}{c} 0.0\\ 368.6\\ -71.8\\ 54.4\\ \end{array}$ $\begin{array}{c} 368.6\\ 427.9\\ R^s\rangle [\phi_1\chi_2]\end{array}$	(1,3) (2,3) (1,4) (2,4) (3,4) (1,2) - Ra (1,3) (3,4) (2,4)	$\begin{array}{c} 0.395\\ 0.395\\ 0.395\\ 0.395\\ 0.373\\ (3,4):\\ adius:\\ 1.5\\ -19.4\\ -1.5\\ 0.408 F\end{array}$	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \text{ fm} \\ 0.235 \text{ fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ R^s\rangle [\psi_1\zeta_2]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ \frac{1}{3}449.6 \\ \\ \frac{1}{2}449.6 \\ \\ \frac{1}{2}m_{c\bar{c}} \\ $	$2m_c$ 3449.6 3449.6 $2m_c$
Quark Confinemen Kinetic CS Inter V ^C Total Con Total Wave -0	C_{33} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: 0.408 F \rangle R ^s	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \Psi \\ \rangle [\psi_1 \zeta_3]\rangle \end{array}$	$ \begin{array}{c} 12.5 \\ 7672.0 \\ -2367.4 \\ 839.0 \\ -53.9 \\ \hline -401.4 \\ 291.0 \\ \hline tot = F\rangle \\ + 0.577 . \end{array} $	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ \hline \\ 427.9\\ \hline \\ R^s\rangle [\phi_1\chi_2]\\ F\rangle R^s\rangle [\psi_2,\psi_3] \\ \hline \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2)- \\ \hline Ra \\ (1,3) \\ (3,4) \\ (2,4) \\ \hline (2,4) \\ \hline) \rangle = -C \\ \zeta_5 \rangle \rangle + C \end{array}$	0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4 -1.5 0.408 F 0.577 F	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \ \mathrm{fm} \\ 0.235 \ \mathrm{fm} \\ \hline \\ -237.2(\eta_c) \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{1}{2}m_{c\bar{c}} \\ \frac{1}{2}m_{c\bar{c}$	$2m_c$ 3449.6 3449.6 $2m_c$ 3449.6
Quark Confinemen Kinetic CS Inter V ^C Total Con Total Wave -C The rearran	C_{33} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: 0.408 F \rangle R ^s gement str	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \hline \\ \psi \\ \gamma [\psi_1 \zeta_3] \rangle \\ \psi \\ \psi \\ \eta \ deca \end{array}$	$ \begin{array}{c} 12.5 \\ \hline 7672.0 \\ -2367.4 \\ \hline 839.0 \\ -53.9 \\ \hline -401.4 \\ \hline 291.0 \\ \hline tot = F\rangle \\ + 0.577 . \\ y \ channel \end{array} $	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ \hline \\ 427.9\\ \hline \\ R^s\rangle [\phi_1\chi_2]\\ F\rangle R^s\rangle [\psi_{2^i}\\ \psi_{2^i}\\ \vdots\\ J/\psi\eta_c \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ R_{1} \\ (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ (2,4) \\ (2,4) \\ (2,5] \rangle + 0 \end{array}$	0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4 -1.5 0.408 F 0.577 F	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \text{ fm} \\ 0.235 \text{ fm} \\ \hline \\ -237.2(\eta_c) \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D	$\begin{array}{c} \frac{2m_1'}{m_c} \frac{p_{x_3}^2}{m_c + m_c} \frac{p_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{8}{3}v_{cc} \\ \\ \end{array}$	$2m_c$ 3449.6 3449.6 $2m_c$ 3449.6 $\frac{8}{3}C_{cc}$
Quark Confinemen Kinetic CS Inter V ^C Total Con Total Wave -C The rearran The radiativ	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \hline \\ \psi_1 \zeta_3] \rangle \\ \phi_{\text{ong deca}} \\ \phi_{\text{ths:}} \end{array}$	$ \begin{array}{c} 12.5 \\ 7672.0 \\ -2367.4 \\ \overline{839.0} \\ -53.9 \\ \hline -401.4 \\ 291.0 \\ \hline tot = F\rangle \\ + 0.577 \\ \hline y \ channe \\ \end{array} $	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ \hline \\ 427.9\\ \hline \\ R^s\rangle [\phi_1\chi_2]\\ F\rangle R^s\rangle [\psi_2, \psi_3]\\ \hline \\ E_1 & J/\psi\eta_c \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2)- \\ R_{4} \\ \hline (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ \hline (2,4) \\ \hline (2,5] \rangle + C \end{array}$	$\begin{array}{c} 0.395\\ 0.395\\ 0.395\\ 0.395\\ 0.373\\ (3,4):\\ adius:\\ \hline 1.5\\ -19.4\\ -1.5\\ 0.408 F\\ 0.577 F\\ \hline \end{array}$	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \text{ fm} \\ 0.235 \text{ fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: m_c^{eff} -D	$\begin{array}{c} \frac{2m_1'}{m_c} \frac{P_{x_3}^2}{m_c + m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7	$\frac{1}{2}m_{cc}$ 1585.8 $\frac{1}{2}m_{c\bar{c}}$ 1534.3 3120.0 $\frac{1}{2}m_{cc}$ 1585.8 $\frac{1}{2}m_{c\bar{c}}$ 3449.6 3120.0 $\frac{8}{3}v_{cc}$ 9.5	$2m_c$ 3449.6 3449.6 $2m_c$ 3449.6 $\frac{8}{3}C_{cc}$ 14.1
$\begin{tabular}{ c c c c }\hline Quark \\ Confinemen \\ Kinetic \\ CS Inter \\ \hline \\ V^C \\ \hline \\ $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ \hline 32.8 \\ 718.9 \\ \hline \Psi \\ \rangle [\psi_1 \zeta_3] \rangle \\ \text{ong deca} \\ \text{dths:} \\ (6451.5, 1^+ \end{array}$	$\begin{array}{c} 12.5 \\ \hline 7672.0 \\ -2367.4 \\ \hline 839.0 \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -91.0 \\ \hline \\ -91.0 \\ \hline \\ + 0.577 \\ \hline \\ y \ channel \\ \hline \\ y \ channel \\ \hline \\)\Upsilon = 70 \end{array}$	$\begin{array}{c} 0.0\\ 368.6\\ -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ 427.9\\ R^s\rangle [\phi_1\chi_2]\\ F\rangle R^s\rangle [\psi_2\eta_2]\\ cl: J/\psi\eta_c\\ 0.4 \text{ keV} \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ \\ \hline \\ (1,3) \\ (2,4) \\ (2,4) \\ \hline \\ (2,4) \\ \hline \\ (2,4) \\ \hline \\ (2,5] \rangle + 0 \end{array}$	0.395 0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4 -1.5 0.408 F 0.577 F	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \text{ fm} \\ 0.235 \text{ fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1 \zeta_2]\rangle \\ \hline \\ \gamma R^s\rangle [\psi_2 \zeta_6]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D CS	$\begin{array}{c} \frac{2m_1'}{m_c} \frac{p_{x_3}^2}{m_c + m_c} \frac{p_{x_3}^2}{2m_3} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7 8.7	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{8}{3}v_{cc} \\ 9.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ \end{array}$	$\frac{2m_{c}}{3449.6}$ $\frac{3449.6}{3449.6}$ $\frac{2m_{c}}{3449.6}$ $\frac{8}{3}C_{cc}$ 14.1 $\frac{8}{3}C_{c\overline{c}}$
$\begin{tabular}{ c c c c }\hline Quark \\ Confinemen \\ Kinetic \\ CS Inter \\ \hline \\ V^C \\ \hline \\ $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ 1.5 \\ \hline 32.8 \\ 718.9 \\ \Psi \\ \rangle [\Psi_1 \zeta_3] \rangle \\ \text{ong deca} \\ \text{dths:} \\ (6451.5, 1^4 \\ (6384.3, 0^{-1}) \\ (6454.5, 1^{-1}) \\ (6384.3, 0^{-1}) \\ (6384.3, $	$\begin{array}{c} 12.5 \\ \hline 7672.0 \\ -2367.4 \\ \hline 839.0 \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ -91.0 \\ \hline \\ +0.577 \\ -9.0 \\ $	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ \hline 54.4\\ \hline \\ \hline$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ \\ Ra \\ (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ \rangle = -C \\ \zeta_5 \rangle + C \\ \zeta_5 $	0.395 0.395 0.395 0.395 0.373 (3,4): adius: 1.5 -19.4 -1.5 0.408 F 0.577 F	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \text{ fm} \\ 0.235 \text{ fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \hline \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D CS Interaction	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7 8.7	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{8}{3}v_{cc} \\ 9.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 9.5 \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 9.5 \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 9.5 \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 1.5 \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 1.5 \\ \frac{8}{3}v_{\bar{c}\bar{c}} \\ 1.5 \\ \frac{8}{3}v_{\bar{c}} \\ 1.5$	$\frac{2m_c}{3449.6}$ $\frac{3449.6}{3449.6}$ $\frac{2m_c}{3449.6}$ $\frac{8}{3}C_{cc}$ 14.1 $\frac{8}{3}C_{c\bar{c}}$ 14.1
$\begin{tabular}{ c c c c }\hline & \end{tabular} \\ \hline & \end{tabular} $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \Psi \\ \rangle [(\mu_1 \zeta_3] \rangle \\ 0 \text{ng deca} \\ d \text{ths:} \\ (6451.5, 1^+ \\ (6384.3, 0^- \\ s: \ \mu_{T_{c2}} \\ (6384.3, 0^- \\ s: \ \mu_{T_{c2}} \\ \end{array}$	$\begin{array}{c} 12.5 \\ \hline \\ 7672.0 \\ -2367.4 \\ \hline \\ 839.0 \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ 291.0 \\ \hline \\ 291.0 \\ \hline \\ 291.0 \\ \hline \\ +0.577 \\ + 0.577 \\ \hline \\ y \text{ channel} \\ \hline \\ \cdot \\ -)\gamma = 70 \\ \cdot \\ \cdot \\ \gamma \text{ channel} \\ \hline \\ \cdot \\ \gamma \text{ channel} \\ \cdot \\ \gamma \text{ channel} \\ \hline \\ \cdot \\ \gamma \text{ channel} \\ \cdot \\ \gamma \text{ channel} \\ \hline \\ \cdot \\ \gamma \text{ channel} \\ \hline \\ \cdot \\ \gamma \text{ channel} \\ \hline \\ \cdot \\ \gamma \text{ channel} \\ \cdot \\ \gamma channe$	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ \hline 427.9\\ \hline \\ R^{s}\rangle [\phi_{1}\chi_{2}]\\ \hline \\ F\rangle R^{s}\rangle [\psi_{2t}\\ \hline \\ R^{s}\rangle [\psi_{2t}\\ \hline \\ U_{1}+\gamma] = \langle \Psi_{t}^{1}\\ \hline \\ \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ Ra \\ (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ (2,4) \\ (2,5) \\ \rangle = -C \\ \zeta_5 \\ \rangle + C \\ z_5 \\ z_5 \\ \rangle$	$\begin{array}{c} 0.395\\ 0.395\\ 0.395\\ 0.395\\ 0.373\\ (3,4):\\ adius:\\ 1.5\\ -19.4\\ -1.5\\ 0.408 F\\ 0.577 F\\ \hline\\ \Psi_{tot}^{1+-}\rangle \end{array}$	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \ \text{fm} \\ 0.235 \ \text{fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ N R^s\rangle [\psi_1\zeta_2]\rangle \\ \hline \\ N R^s\rangle [\psi_2\zeta_6]\rangle \\ \hline \\ \hline \\ = 0 \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D CS Interaction	$\frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \frac{V^C(12)}{V^C(12)} \frac{\frac{1}{2}[V^C(13) + V^C(14) + V^C(24)] - D}{-D}$ Subtotal $\frac{2m_c}{\frac{P_{x_3}^2}{2m_2'} \frac{P_{x_3}^2}{2m_3'} \frac{V^C(12)}{V^C(12)} \frac{\frac{1}{2}[V^C(13) + V^C(14) + V^C(23) + V^C(24)] - 983.0}{Subtotal} \frac{\frac{1}{2}V^{SS}(12)}{\frac{1}{2}V^{SS}(34)} - \frac{1}{4}(V^{SS}(13) + V^{SS}(14) + V^{SS}(13) + V^{SS}(14)$	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7 8.7 -15.8	$\begin{array}{c} \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \\ \frac{8}{3}v_{cc} \\ 9.5 \\ \\ \frac{8}{3}v_{c\bar{c}} \\ 9.5 \\ -\frac{16}{3}v_{c\bar{c}} \\ 9.5 \\ -\frac{16}{3}v_{c\bar{c}} \end{array}$	$\frac{2m_c}{3449.6}$ $\frac{3449.6}{3449.6}$ $\frac{2m_c}{3449.6}$ $\frac{8}{3}C_{cc}$ 14.1 $\frac{8}{3}C_{\bar{c}\bar{c}}$ 14.1 $-\frac{16.1}{3}C_{c\bar{c}}$
$\begin{tabular}{ c c c c }\hline Quark \\ Confinemen \\ Kinetic \\ CS Inter \\ \hline CS Inter \\ \hline V^C \\ \hline \hline \\ \hline $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \Psi \\ \rangle [\psi_1 \zeta_3] \rangle \\ \text{ong deca } \\ \text{dths:} \\ \varphi (6384.3.0^{-3} \\ \varphi (6384.3^{-3} \\ \varphi (6384$	$\begin{array}{c} 12.5 \\ \hline \\ 7672.0 \\ -2367.4 \\ \hline \\ 839.0 \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ 401.4 \\ \hline \\ 291.0 \\ \hline \\ tot = F\rangle \\ + 0.577 , \\ y \text{ channel} \\ \hline \\ y \text{ channel} \\ \hline \\)\Upsilon = 70 \\ \hline \\ y \text{ channel} \\ \hline \\ r^2(6451.5, \\ \text{its:} \\ \end{array}$	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline \\ 368.6\\ \hline \\ 427.9\\ \hline \\ R^s\rangle [\phi_1\chi_2]\\ \hline \\ F\rangle R^s\rangle [\psi_{2i}\\ \hline \\ S [k^s\rangle [\psi_{2i}\\ \hline \\ \psi_{\eta_c}\\ \hline \\ \hline \\ 0.4 \text{ keV}\\ \hline \\ 38.1 \text{ keV}\\ \hline \\ 1+-) = \langle \Psi_t^1 \\ \hline \\ \end{array}$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ \hline \\ (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ \rangle = -C \\ \zeta_5 \rangle + C \\ \vdots \end{array}$	$\begin{array}{c} 0.395\\ 0.395\\ 0.395\\ 0.395\\ 0.373\\ (3,4):\\ adius:\\ \hline 1.5\\ -19.4\\ -1.5\\ 0.408 F\\ 0.577 F\\ \hline \\ \Psi_{tot}^{1+-} \rangle \end{array}$	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \ \text{fm} \\ 0.235 \ \text{fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \hline \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \\ \hline \\ = 0 \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D CS Interaction	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7 8.7 -15.8	$\frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 1534.3 \\ 3120.0 \\ \\ \frac{1}{2}m_{cc} \\ 1585.8 \\ \frac{1}{2}m_{c\bar{c}} \\ 3449.6 \\ 3120.0 \\ \\ \frac{8}{3}v_{cc} \\ 9.5 \\ \frac{8}{3}v_{c\bar{c}} \\ 9.5 \\ -\frac{16}{3}v_{c\bar{c}} \\ -28.4 \\ \\ \end{array}$	$\frac{2m_c}{3449.6}$ $\frac{3449.6}{3449.6}$ $\frac{2m_c}{3449.6}$ $\frac{8}{3}C_{cc}$ 14.1 $\frac{8}{3}C_{c\bar{c}}$ 14.1 $-\frac{16}{3}C_{c\bar{c}}$ -28.2
$\begin{tabular}{ c c c c }\hline Quark \\ Confinemen \\ Kinetic \\ CS Inter \\ \hline \\ V^C \\ \hline \\ $	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 9.1 \\ 7.3 \\ 7672.0 \\ -1998.8 \\ 767.2 \\ 1.5 \\ -19.4 \\ 1.5 \\ 1.5 \\ -32.8 \\ 718.9 \\ \hline \\ \psi \\ \psi \\ [\psi_1 \zeta_3]\rangle \\ \text{ong deca} \\ \text{dths:} \\ \psi \\ $	$\begin{array}{c} 12.5 \\ \hline 7672.0 \\ -2367.4 \\ \hline 839.0 \\ -53.9 \\ \hline \\ -53.9 \\ \hline \\ +0.571] \\ \psi channel \\ +0.577] \\ \psi channel \\)\gamma = 70 \\ \hline \\ \gamma channel \\)\gamma = 70 \\ \hline \\ \gamma channel \\)\gamma = 70 \\ \hline \\ \gamma channel \\)\gamma = \sqrt{10} \\ \hline \\ \gamma channel \\)\gamma = \sqrt{10} \\ \hline \\ \gamma channel \\\gamma \gamma = 70 \\ \hline \\ \gamma channel \\\gamma \gamma = \sqrt{10} \\ \hline \\ \gamma channel $	$\begin{array}{c} 0.0\\ \hline 368.6\\ \hline -71.8\\ 54.4\\ \hline \\ \hline$	$\begin{array}{c} (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2)- \\ \hline \\ (1,3) \\ (2,4) \\ (2,4) \\ (2,4) \\ \rangle = -C \\ \zeta_{5} \rangle + C \\ \zeta_{5} \rangle + C \\ \vdots \\$	$\begin{array}{c} 0.395\\ 0.395\\ 0.395\\ 0.395\\ 0.373\\ (3,4):\\ adius:\\ \hline 1.5\\ -19.4\\ -1.5\\ 0.408 F\\ 0.577 F\\ \hline \\ \Psi_{tot}^{1+-}\rangle\\ 0.750\mu\end{array}$	$\begin{array}{c} 0.290(\eta_c) \\ \\ 0.318(J/\psi) \\ \hline \\ 0.294 \ \text{fm} \\ 0.235 \ \text{fm} \\ \hline \\ -237.2(\eta_c) \\ \hline \\ -164.2(J/\psi) \\ \hline \\ \gamma R^s\rangle [\psi_1\zeta_2]\rangle \\ \hline \\ \gamma R^s\rangle [\psi_2\zeta_6]\rangle \\ \hline \\ \hline \\ = 0 \\ \hline \\ \mu_N \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ -D CS Interaction	$\begin{array}{c} \frac{2m_1'}{m_c+m_c} \frac{P_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \hline \\ $	277.2 111.2 -19.4 3.0 -983.0 3225.0 3836.0 277.2 111.2 -19.4 3.0 1534.3 3225.0 8.7 8.7 -15.8 1.5	$\frac{1}{2}m_{cc}$ $\frac{1}{2}m_{c\bar{c}}$ $\frac{1}{2}m_{c\bar{c}}$ $\frac{1}{2}m_{c\bar{c}}$ $\frac{1}{2}m_{cc}$ $\frac{1}{2}m_{cc}$ $\frac{1}{2}m_{c\bar{c}}$ $\frac{1}{2}m_{c\bar{c}}$ $\frac{3449.6}{3120.0}$ $\frac{8}{3}v_{cc}$ 9.5 $\frac{8}{3}v_{\bar{c}\bar{c}}$ 9.5 $-\frac{16}{3}v_{c\bar{c}}$ -28.4 -9.5	$\begin{array}{c} 2m_{c} \\ 3449.6 \\ \hline 3449.6 \\ \hline \\ 2m_{c} \\ \hline \\ 3449.6 \\ \hline \\ \frac{8}{3}C_{cc} \\ 14.1 \\ \frac{8}{3}C_{c\bar{c}} \\ 14.1 \\ -\frac{16}{3}C_{c\bar{c}} \\ -28.2 \\ \hline \\ 0.0 \\ \hline \end{array}$

Table 8 The masses, binding energies, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width

ratios, and the relative lengths between quarks for the $J^{PC} = 2^{++} cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states and their lowest meson–meson thresholds. The notation is the same as that in Table 7

$cc\overline{c}\overline{c}$	The	contribut	tion from	each term	Rela	tive L	engths (fm)	Overall	Present Work	ζ.	CMI N	Model
$J^{PC} = 2^{++}$		Value	$J/\psi J/\psi$	Difference	(i,j) \mathbb{N}	Vaule	$J/\psi J/\psi$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	B_T	6482.7	6184.5	298.2	(1,2) 0).377			$2m_c$	3836.0		
Variational	C_{11}	8.9	12.5		(1.3) 0).403	$0.318(J/\psi)$		$\frac{\mathbf{p}_{x_1}^2}{\mathbf{p}_{x_1}}$	270.4		
Parameters (fm^{-2})	C_{22}	8.9	12.5		(2.3) 0	1 403	(-/ / / /	e quark:	$m_{\overline{c}}^{2m_{1}} p_{x_{3}}^{2}$	105.6	1	
(m -)	C ₃₃	0.9	-		(2,3) 0	0.405		<i>c</i> -quark:	$\overline{\frac{m_c+m_{\overline{c}}}{V^C(12)}}$	-14.6	$\overline{2}^{m_{cc}}$	
Quark	Mass	1072.0	7672.0	0.0	(1,4) 0	0.403		m_c^{eff}	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	1110	1585.8	
Confinemen	t Potential	-1973.6	-2294.4	320.8	(2,4) 0).403	$0.318(J/\psi)$		$+V^{C}(23) + V^{C}(24)]$	10.8	$\frac{1}{2}m_{c\bar{c}}$	$2m_c$
Kinetic	Energy	752.0	769.9	-10.9	(3,4) 0).377			- <i>D</i>	-983.0	1534.3	3449.6
CS Inter	raction	32.3	43.9	-11.6	(1,2)-((3,4):	0.302 fm		Subtotal	3225.2	3120.0	3449.6
	(1,2)	-14.6			Ra	dius:	0.241 fm		$2m_c$	3836.0		
V^C	(2,3)	5.4							$\frac{\mathbf{P}_{x_2}^{*}}{2m'_2}$	270.4		
	(1,4)	5.5			(1,3)	5.4	$-164.2(J/\psi)$		$\frac{m_{\tilde{c}}}{m_{+}+m_{-}} \frac{\mathbf{p}_{x_{3}}^{2}}{2m'_{-}}$	105.6	$\frac{1}{2}m_{cc}$	
	Subtotal	-7.6	-328.4	320.8	(3.4) -	-14.6		<i>c</i> -quark:	$V^{C}(12)$	-14.6	1585.8	
Total Con	tribution	776.7	478.5	298.2	(2.4)	5.4	$-164.2(J/\psi)$	$m_{\overline{c}}^{eff}$	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	10.9	$\frac{1}{2}m_{o\bar{o}}$	$2m_{\circ}$
Total Wave	function:		110.0		(=, 1)	0.1	101.2(0) \$	Ŭ	$+V^{\circ}(23)+V^{\circ}(24)$	-983.0	2 ^{,1000}	3449.6
$\Psi_{L,L} = F\rangle I$	$\mathbb{R}^{s} \setminus [[d_{1}, \chi_{2}]]$	- 0 577	$F \setminus [R^s \setminus [a])$	$(c_1) = 0.81$	$ 6 F \setminus B^s $	5 \ [[a/1_a /	-1)	Subtotal	3225.2	3120.0	3449.6	0110.0
$\frac{\Psi_{tot} - \Gamma / \Gamma }{T_{be}}$	$\frac{\iota}{ [\psi_1\chi_1] }$	- 0.377	$\frac{r}{ n } \frac{ \psi }{ \psi }$	$\frac{1\zeta_{1}}{1\zeta_{2}} = 0.81$	10 17 / 11	$/ [\psi_2 \varsigma] $	1]/	Subtotai	0220.2	5120.0	8	80
The rearran	gement stro	ong decay	y channel	$J/\psi J/\psi$					$\frac{1}{2}V^{SS}(12)$	8.5	$\frac{1}{3}v_{cc}$	$\frac{1}{3}C_{cc}$
The radiativ	ze decay wi	dths:									9.5	14.1
$\Gamma_{T_{c^2\bar{c}^2}(6482.7)}$	$,2^{++}) \rightarrow T_{c^2 \overline{c}^2}$	(6384.3,0+	$(+)\gamma = 0$	κeV				\mathbf{CS}	$\frac{1}{2}V^{SS}(34)$	8.5	$\frac{6}{3}v_{\overline{c}\overline{c}}$	$\frac{6}{3}C_{\overline{c}\overline{c}}$
$\Gamma_{T_{c^2\bar{c}^2}(6482.7)}$	$,2^{++}) \rightarrow T_{c^2 \overline{c}^2}$	(6451.5,1+	$(-)\gamma = 70$.4 keV		. 1		Interaction	2		9.5	14.1
The magnet	ic moments	$\mu_{T_{c^2}}$	$\overline{c}^{2}(6451.5,1)$	$_{+-)} = \langle \Psi_{tot}^{1} \rangle$	$ \hat{\mu^z} \Psi^1_t$	$\left t \right _{tot} \rangle =$: 0.		$\frac{1}{4}(V^{SS}(13) + V^{SS}(14))$	15.3	$\frac{16}{3}v_{c\bar{c}}$	$\frac{16}{3}C_{c\bar{c}}$
The transiti	on magneti	c momen	nts:						$+V^{55}(23)+V^{55}(24)$		28.4	28.2
$\mu_{T_{c^2\bar{c}^2}(6482.7}$	$,2^{++}) \rightarrow T_{c^2 \bar{c}^2}$	(6384.3,0+	$_{+)\gamma} = \langle \Psi \rangle$	$L_{tot}^{2^{++}} \hat{\mu^z} \Psi_{tot}^{0^{+}}$	$\left t^{+} \right\rangle = 0$				Subtotal	32.3	47.4	56.5
$\mu_{T_{c^2\bar{c}^2}(6482,7,2^{++})\to T_{c^2\bar{c}^2}(6384.3,0^{++})\gamma} = \langle \Psi_{tot} \mid \mu^* \mid \Psi_{tot} \rangle = 0$					$\langle \bar{r} \rangle = \mu_c - \mu_{\bar{c}} = 0.750 \mu_N$ Total c			Total contrib	ution	6482.7	6287.3	6056.0
$\mu T_{c^2 \overline{c}^2}(6482.7)$	$,2$ $(1) \rightarrow 1_{c^2 \overline{c}^2}$	(6451.4,1+)7 1-	tot $ \mu = to$	$t / - \mu_c$	$e^{-\mu_c}$	$= 0.150 \mu_N$	rotar contrib	ution	0402.1	0201.0	0950.0
$\frac{\mu_{T_{c^2\bar{c}^2}(6482.7)}}{bb\bar{b}\bar{b}} J^{PC}$	$= 2^{++}$	Value	ΥΥ ΥΥ	Difference	$t / - \mu_c$ Rela	tive L	engths (fm)	Contribution	Value	Ref. [110]	Ref. [123]	0950.0
$\frac{\mu T_{c^2 \overline{c}^2}(6482.7)}{bb \overline{b} \overline{b}} J^{PC}}{Mass}$	$= 2^{++}$ $/B_T$	Value 19327.9	ΥΥ 18938.8	$\frac{1}{2}$ Difference 390.1	$t / - \mu_c$ Rela $(i, j) V$	tive L \sqrt{alue}	engths (fm) $\Upsilon\Upsilon$	Contribution	Value 2m _b	Ref. [110] 10686.0	Ref. [123]	0930.0
$\frac{\mu T_{c^2 \overline{c}^2}(6482.7)}{bb \overline{b} \overline{b}} J^{PC}}{Mass}$	$= 2^{++} \\ B_T \\ C_{11}$	Value 19327.9 30.0	γ γ ΥΥ 18938.8 49.4	$\frac{1}{2}$ Difference 390.1	$\begin{array}{c} r & - \mu_c \\ \hline \text{Rela} \\ \hline (i,j) & V \\ \hline (1.2) & 0 \end{array}$, μ _c , tive L Value).205	$\frac{1}{2} \frac{0.130 \mu_N}{\text{(fm)}}$	Contribution	Value $2m_b$ $\frac{\mathbf{p}_{x_1}^2}{2m_2}$	Ref. [110] 10686.0 328.0	Ref. [123]	0930.0
$\frac{\mu T_{c^2 \bar{c}^2} (6482.7)}{b b \bar{b} \bar{b}} J^{PC}$ Mass, Variational Parameters (f_{m}^{-2})	$= 2^{++} \\ B_T \\ C_{22} \\ C_{11} \\ C_{22} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_$	Value 19327.9 30.0 30.0 22.0	$\frac{\Upsilon \Upsilon}{18938.8}$ 49.4 49.4	$\frac{1}{2}$ $\frac{1}$	$f_{t} = \mu_{c}$ Relat (i, j) = V (1.2) = 0 (1.3) = 0	tive L value 0.205	$\frac{-0.150\mu_N}{\Upsilon}$	Contribution	Value $2m_b$ $\frac{p_{x_1}^2}{2m_1^2}$ $m_b^2 = p_{x_3}^2$	Ref. [110] 10686.0 328.0	¹ mu	0930.0
$\frac{\mu_{T_{c^2\bar{c}^2}}(6482.7)}{bb\bar{b}\bar{b}} J^{PC}$ Mass, Variational Parameters (fm ⁻²)	$\begin{array}{c} = 2^{++} \\ B_T \\ \hline \\ C_{22} \\ C_{33} \\ \hline \\ M_{2-7} \end{array}$	Value 19327.9 30.0 30.0 23.0	$\gamma \gamma$ (1 $\gamma \gamma$ 18938.8 49.4 49.4 -	Difference 390.1	$\begin{array}{c} i & j - \mu_c \\ \hline \text{Rela} \\ (i,j) & V \\ (1.2) & 0 \\ (1,3) & 0 \\ (2,2) & 0 \end{array}$	itive Lo Value).205).220	$\frac{\text{engths (fm)}}{\Upsilon\Upsilon}$ 0.160(\Upsilon)	Contribution	Value $2m_b$ $\frac{P_{x_1}^2}{2m_1'}$ $\frac{m_b}{m_b+m_b}\frac{P_{x_3}^2}{2m_b'}$ $V^C(12)$	Ref. [110] 10686.0 328.0 126.0 -269.1	$\frac{1}{2}m_{bb}$	0930.0
$\frac{\mu_{T_{c^2 \overline{c}^2}}(6482.7)}{bb\overline{bb}} J^{PC}$ Mass Variational Parameters (fm ⁻²) Quark	$=2^{++}$ $/B_T$ C_{11} C_{22} C_{33} Mass t Detection	Value 19327.9 30.0 30.0 23.0 21372.0	γ (1 ΥΥ 18938.8 49.4 49.4 21372.0 2550.5	$\frac{1}{0.0}$	$\begin{array}{c} r & -\mu_c \\ \hline Rela \\ (i,j) & V \\ (1.2) & 0 \\ (1,3) & 0 \\ (2,3) & 0 \\ (1,4) & 0 \end{array}$	itive L value).205).220).220	$\frac{10.150 \mu_N}{\Upsilon }$ $0.160(\Upsilon)$	Contribution b-quark: m_b^{eff}	$\begin{array}{c} \mbox{Value} \\ & 2m_b \\ & \frac{\mathbf{P}_{s_1}^2}{2m_1'} \\ & \frac{m_b}{2m_1'} \mathbf{P}_{s_3}^2 \\ & \frac{m_b}{2m_b'} \mathbf{P}_{s_3}^{(2)} \\ & \frac{1}{2}[V^C(12) + V^C(12) \\ \end{array}$	Ref. [110] 10686.0 328.0 126.0 -269.1	Ref. [123] $\frac{1}{2}m_{bb}$ 4764.8	0930.0
$\frac{\mu_{I_{c}2\pi^{2}}(6482.7, bbb\overline{b} J^{PC})}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen	$2^{++} \rightarrow 2_{c^2 \overline{c}^2}$ $= 2^{++}$ $/B_T$ C_{11} C_{22} C_{33} Mass t Potential	Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3	γ (1 ΥΥ 18938.8 49.4 49.4 21372.0 - -3559.5 1087.2	0.0 582.2	$\begin{array}{c} r & -\mu_c \\ \hline Rela \\ \hline (i,j) & V \\ \hline (1.2) & 0 \\ (1,3) & 0 \\ (2,3) & 0 \\ (1,4) & 0 \\ (2,4) & 0 \end{array}$	value 0.205 0.220 0.220 0.220	$\frac{10.150\mu_N}{\Upsilon\Upsilon}$ $0.160(\Upsilon)$	Contribution b-quark: m_b^{eff}	$\begin{array}{c} \hline \text{Value} \\ \hline & 2m_b \\ & \frac{\mathbf{p}_{s_1}^2}{2m_1'} \\ & \frac{m_{\tilde{b}}}{m_b+m_{\tilde{b}}} \frac{\mathbf{p}_{s_3}^2}{2m_3'} \\ & V^C(12) \\ & \frac{1}{2}[V^C(13) + V^C(14) \\ & + V^C(23) + V^C(24)] \end{array}$	0432.1 Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 002.0	Ref. [123] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$	2mb
$\frac{PT_{c2\pi2}(6482.7, BPC)}{bb\overline{b}} \frac{J^{PC}}{J^{PC}}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic	$\frac{(2+1) \rightarrow l_{c^2 S^2}}{(2+1) \rightarrow l_{c^2 S^2}}$ $= 2^{++}$ $/B_T$ C_{11} C_{22} C_{33} Mass t Potential Energy	Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1	γ (1) $\Upsilon\Upsilon$ 18938.8 49.4 49.4 21372.0 -3559.5 1087.3 20.0	0.0 582.2 -179.2	$\begin{array}{c} r = \mu_c \\ \hline r = \mu_c \\ \hline r = 0 \\ \hline r = 0$	value 0.205 0.220 0.220 0.220 0.220		Contribution b-quark: m_b^{eff}	$\begin{array}{c c} \hline & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \hline & & \\ & & \\ & & \\ & & \\ \hline & & \\ & &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0	Ref. [123] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5	$2m_b$ 10105.8
$\frac{PT_{c_{2}\overline{c}\overline{c}}(648.7, PC)}{bb\overline{b} J^{PC}}$ Mass Variational Parameters (fm^{-2}) Quark Confinemen Kinetic CS Inter	$\frac{(2+1) \rightarrow l_{c}^{2} z^{2}}{2} = 2^{++}$ $\frac{B_{T}}{B_{T}}$ $\frac{C_{11}}{C_{22}}$ C_{33} Mass t Potential Energy raction $\frac{(2+2)}{C_{33}}$	Value 19327.9 30.0 23.0 21372.0 -2977.3 908.1 25.1	$\begin{array}{c} \gamma & (1 \\ \Upsilon \\ 18938.8 \\ 49.4 \\ 49.4 \\ - \\ 21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ \end{array}$	0.0 582.2 -179.2 -12.9	$\begin{array}{c} r & \mu_c \\ \hline Rela \\ (i, j) & V \\ (1.2) & 0 \\ (1.3) & 0 \\ (2.3) & 0 \\ (1.4) & 0 \\ (2.4) & 0 \\ (3.4) & 0 \end{array}$	yalue √alue 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.225		Contribution b-quark: m_b^{eff}	$\begin{array}{c c} \hline & & \\ & & &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3	Ref. [123] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$ 2m_b 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8 $
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} \frac{J^{PC}}{J^{PC}}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter	$\begin{array}{c} (2^{+}) \rightarrow c_{c}^{2}z^{2}\\ =2^{++}\\ \\ B_{T}\\ \hline \\ C_{11}\\ C_{22}\\ C_{33}\\ \\ Mass\\ t \text{ Potential}\\ \\ Energy\\ \\ exaction\\ \hline \\ (1,2) \end{array}$	(6431.4,1) Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1	γ γ ΥΥ 18938.8 49.4 49.4 - 21372.0 -3559.5 1087.3 38.0 38.0	0.0 582.2 -179.2 -12.9	$\begin{array}{c} \text{Rela}\\\hline(i,j) & \text{V}\\(1.2) & 0\\(1.3) & 0\\(2.3) & 0\\(1.4) & 0\\(2.4) & 0\\(3.4) & 0\\(1.2)\text{-}(1.2$	yalue Value 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.220 (3,4):	$ \begin{array}{c} \text{engths (fm)} \\ \underline{\Upsilon\Upsilon} \\ 0.160(\Upsilon) \\ 0.165 \text{ fm} \end{array} $	Contribution b-quark: m_b^{eff}	$ \begin{array}{c} \hline & Value \\ & 2m_b \\ & \frac{\mathbf{p}_{x_1}^2}{2m_1} \\ & \frac{\mathbf{m}_{b}}{m_b + \mathbf{m}_b^2} \frac{\mathbf{p}_{x_3}}{2m_3} \\ & \frac{\mathbf{m}_{b}}{m_b + \mathbf{m}_b^2} \frac{\mathbf{p}_{x_3}}{2m_3} \\ & \frac{1}{2} [V^C(12) + V^C(12) \\ & + V^C(23) + V^C(24)] \\ & -D \\ \hline & Subtotal \\ \hline & 2m_b \\ & 2 \end{array} $	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \end{array}$	$2m_b$ 10105.8 10105.8
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} J^{PC}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter V^{C}	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $/B_{T}$ C_{11} C_{22} C_{33} Mass t Potential Energy caction $(1,2)$ $(2,3)$	(644)1.4,11 Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3	<u>уу</u> (1 <u>YY</u> 18938.8 49.4 49.4 21372.0 -3559.5 1087.3 38.0	0.0 582.2 -179.2 -12.9	$\begin{array}{c} r & r = \mu_c \\ \hline r & r = \mu_c \\ \hline r & r = r \\ r & r \\ r &$	yaling (2007)	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \hline \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \end{array}$	Contribution b-quark: m_b^{eff}	$\begin{array}{c c} \hline & & \\ & & &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \end{array}$	$2m_b$ 10105.8 10105.8
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} \frac{J^{PC}}{J^{PC}}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{B_{T}}$ $\frac{C_{11}}{C_{22}}$ C_{33} Mass t Potential Energy caction $(1,2)$ $(2,3)$ $(1,4)$	(444)14,17 Value 19327.9 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3	γγ (1 ΥΥ 18938.8 49.4 49.4 21372.0 -3559.5 1087.3 38.0	0.0 582.2 -179.2 -12.9	$\begin{array}{c} \text{Rela} \\ \hline (i,j) & V \\ \hline (1.2) & 0 \\ (1.3) & 0 \\ (2.3) & 0 \\ (2.3) & 0 \\ (1.4) & 0 \\ (2.4) & 0 \\ (3.4) & 0 \\ (1.2) \text{-}(\\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ $	yalue Value 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.205 (3,4): udius: 118.3	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \hline \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ \hline \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \end{array}$	Contribution b-quark: m_b^{eff}	$\begin{array}{c c} \hline & & \\ &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0	$\frac{1}{2}m_{bb}$ $\frac{1}{2}m_{b\bar{b}}$ $\frac{1}{2}m_{b\bar{b}}$ $\frac{1}{2}m_{b\bar{b}}$ $\frac{1}{2}m_{b\bar{b}}$ $\frac{1}{2}m_{b\bar{b}}$	$2m_b$ 10105.8 10105.8
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} \frac{J^{PC}}{J^{PC}}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{11}}$ $\frac{C_{11}}{C_{22}}$ C_{33} Mass t Potential Energy caction $(1,2)$ $(2,3)$ $(1,4)$ Subtotal	(8431.4,1) Value 19327.9 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -1011.3	γγ (12) ΥΥ 18938.8 49.4 - 21372.0 - -3559.5 1087.3 38.0 - -1593.5 -	tot µ ¥ tot Difference 390.1 0.0 582.2 -179.2 -12.9 582.2 582.2	$\begin{array}{c} \mathbf{Rela} \\ (i,j) & \mathbf{V} \\ (1.2) & 0 \\ (1.3) & 0 \\ (2,3) & 0 \\ (1,4) & 0 \\ (2,4) & 0 \\ (2,4) & 0 \\ (3,4) & 0 \\ (1,2)-(\mathbf{Ra} \\ (1,3) & -1 \\ (3,4) & -2 \end{array}$	j j	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \end{array}$	Contribution b-quark: m_b^{eff} \bar{b} -quark:	$\begin{array}{c c} \hline & & \\ &$	0432.1 Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{bb} \\ 4764.8 \end{array}$	$2m_b$ 10105.8 10105.8
$\begin{array}{c} \frac{PT_{e2\pi2}(6482.7)}{PC}\\ \hline bb\overline{b} & J^{PC}\\ \hline Mass,\\ Variational\\ Parameters\\ (fm^{-2})\\ \hline Quark\\ Confinemen\\ \hline Kinetic\\ \hline CS Inter\\ \hline V^{C}\\ \hline \hline Total Con\\ \end{array}$	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{11}}$ $\frac{C_{11}}{C_{22}}$ C_{23} Mass t Potential Energy caction $(1,2)$ $(2,3)$ $(1,4)$ Subtotal tribution	(6431.4,1) Value 19327.9 30.0 330.0 23.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -1101.3 -78.1	γγ (1-2) ΥΥ 18938.8 49.4 - 21372.0 - -3559.5 1087.3 38.0 - -1593.5 - -468.2 -	tot µ ¥ tot Difference 390.1 0.0 582.2 -179.2 -12.9 582.2 390.1	$\begin{array}{c} \mathbf{Rela} \\ (i,j) & \mathbf{V} \\ (1.2) & 0 \\ (1.3) & 0 \\ (2,3) & 0 \\ (2,3) & 0 \\ (1,4) & 0 \\ (2,4) & 0 \\ (2,4) & 0 \\ (3,4) & 0 \\ (1,2)-(\mathbf{Ra} \\ (1,3) & -1 \\ (3,4) & -2 \\ (2,4) & -1 \end{array}$	μc tive L √alue 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.205 (3,4): udius: 118.3 269.1 118.3	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \end{array}$	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$	$\begin{array}{c c} \hline & Value \\ \hline & 2m_b \\ & \frac{p_{s_1}^2}{2m_1} \\ \frac{m_5}{m_b+m_5} \frac{p_{s_3}}{2m_3} \\ \hline & \frac{m_5}{m_b+m_5} \frac{p_{s_3}}{2m_3} \\ \hline & \frac{1}{2} [V^C(12) + V^C(14) \\ + V^C(23) + V^C(24)] \\ & -D \\ \hline & Subtotal \\ \hline & 2m_b \\ & \frac{p_{s_2}^2}{2m_5} \frac{2m_5}{2m_5} \frac{p_{s_3}}{2m_5} \\ \hline & \frac{p_{s_3}}{2m_5} \\ \frac{m_5}{2m_5} \frac{p_{s_3}^2}{2m_5} \\ \hline & \frac{p_{s_3}}{2m_5} \\ \hline & V^C(12) \\ \hline & \frac{1}{2} [V^C(13) + V^C(14) \\ V^C(20) + V^C(10) \\ \hline \end{array}$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \end{array}$	$2m_b$ 10105.8 10105.8 $2m_b$
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} J^{PC}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter V^C Total Con Total Wave	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{22}}$ C_{23} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function:	(44114,1) Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -118.3 -118.3 -78.1	γγ (12) ΥΥ 18938.8 49.4 - 21372.0 - -3559.5 1087.3 38.0 - -1593.5 - -468.2 -	tot µ ¥ tot Difference 390.1 0.0 582.2 -179.2 -12.9 582.2 390.1	$\begin{array}{c} \text{Rela} \\ \hline (i,j) & V \\ (1.2) & 0 \\ (1.3) & 0 \\ (2.3) & 0 \\ (2.4) & 0 \\ (3.4) & 0 \\ (1.2)-(\\ \hline \\ \text{Ra} \\ (1.3) & -1 \\ (3.4) & -2 \\ (2.4) & -1 \end{array}$	μ ₂ tive L √alue 0.205 0.220 0.220 0.220 0.220 0.220 0.205 (3,4): udius: 118.3 269.1 118.3	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \end{array}$	Contribution b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c c} \hline & & \\ &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \end{array}$	$2m_b$ 10105.8 10105.8 $2m_b$ 10105.8
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} J^{PC}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter V^C Total Con Total Wave $\Psi_{u,v} = F l$	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{22}}$ C_{23} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: $2^{*} [(\phi, y_{0}])$	(44114,1) Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -118.3 -78.1 = 0.577[]	γγ (1) ΥΥ 18938.8 49.4 - 21372.0 - -3559.5 1087.3 38.0 - -1593.5 - -468.2 -	$tot \mu + tot$ Difference 390.1 0.0 582.2 -179.2 -12.9 582.2 390.1 (c,1) = 0.81	$\begin{array}{c} \mathbf{Rela} \\ \mathbf{Rela} \\ (i,j) \mathbf{V} \\ (1.2) 0 \\ (1.3) 0 \\ (2.3) 0 \\ (2.4) 0 \\ (2.4) 0 \\ (3.4) 0 \\ (1.2) \mathbf{-} \\ (3.4) 0 \\ (1.2) \mathbf{-} \\ (1.3) \mathbf{-} \\ (2.4) \mathbf{-} \\ (2.4$	j. µc view Like Like Value 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.205 (3,4): 118.3 269.1 118.3 *Mikbac *Mikbac	$\begin{array}{c} 0.130 \mu_{N} \\ \hline \\ engths (fm) \\ \hline \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ \hline \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ \end{array}$	Contribution b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c c} \hline & Value \\ & 2m_b \\ & \frac{\mathbf{p}_{s_1}^2}{2m_1^2} \frac{2m_b}{2m_3^2} \\ & \frac{m_b}{m_b} \frac{\mathbf{p}_{s_2}^2}{2m_3} \frac{m_b}{2m_3} \\ & \frac{\mathbf{p}_{s_1}^{C}(12)}{2} \frac{1}{2} [V^C(13) + V^C(14) \\ & + V^C(23) + V^C(24)] \\ & -D \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline \\$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3	$\begin{array}{c} \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$ $\begin{array}{c} \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\overline{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$	$2m_b$ 10105.8 10105.8 10105.8 10105.8 10105.8
$\frac{PT_{c_2\pi^2}(648.7, T_{c_2\pi^2}(648.7, T_{c_2\pi^2}(648.7, T_{c_2}))}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter V^C Total Con Total Wave $\Psi_{tot} = F\rangle I$	$\frac{(2+1) \rightarrow c_{0}^{2}z^{2}}{z^{2}z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{22}}$ C_{23} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: $R^{*} \rangle [[\phi_{1}\chi_{2}] \rangle$	$\begin{array}{l} (443)(4,1)\\ Value\\ 19327.9\\ 30.0\\ 30.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 23.0\\ 25.1\\ -269.1\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -11011.3\\ -78.1\\ = 0.577[],\\ ung decay$	$\gamma \sim (-1)^{\gamma} \sim (-1)^$	$tot \mu + tot$ Difference 390.1 0.0 582.2 -179.2 -12.9 582.2 390.1 $(\zeta_1] > -0.81$	$\begin{array}{c} \mathbf{Rela} \\ \mathbf{Rela} \\ (i,j) \mathbf{V} \\ (1.2) 0 \\ (1.3) 0 \\ (2.3) 0 \\ (2.4) 0 \\ (2.4) 0 \\ (3.4) 0 \\ (1.2) \mathbf{-} \\ (3.4) 0 \\ (1.2) \mathbf{-} \\ (1.3) \mathbf{-} \\ (2.4) \mathbf{-} \\ (2.4$	tive L \sqrt{alue} 0.220 0.205 (3.4): 118.3 269.1 118.3	$\begin{array}{c} 0.130 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ \hline \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ 1] \rangle \end{array}$	Contribution b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c} \mbox{Value} \\ & \mbox{$\frac{2m_b}{p_{s_1}^2}$} \\ & \mbox{$\frac{p_{s_2}^2}{m_b+m_b p_{s_3}^2}$} \\ \hline & \mbox{$\frac{m_b}{p_s} p_{s_3}^2$} \\ \hline & \mbox{$\frac{m_b}{p_s} p_{s_3}^2$} \\ \hline & \mbox{$\frac{V^C(12)}{(13) + V^C(14)}$} \\ & \mbox{$-D$} \\ \hline & \mbox{$Subtotal$} \\ \hline & \mbox{$\frac{p_{s_2}^2}{2m_b^2} p_{s_3}^2$} \\ \hline & \mbox{$\frac{p_{s_2}^2}{2m_b^2} p_{s_3}^2$} \\ \hline & \mbox{$\frac{V^C(12)}{(12) + p_{s_3}^C(12)}$} \\ \hline & \mbox{$\frac{1}{2}[V^C(13) + V^C(14)]$} \\ & \mbox{$+V^C(23) + V^C(24)]$} \\ \hline & \mbox{$-D$} \\ \hline & \mbox{$Subtotal$} \\ \hline \end{array}$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$ $\begin{array}{c} \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$	$2m_b$ 10105.8 10105.8 10105.8 10105.8 10105.8 10105.8
$\frac{PT_{c_2\pi^2}(648.7, PC)}{bb\bar{b}} J^{PC}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inter V^C Total Con Total Wave $\Psi_{tot} = F\rangle I$ The rearran	$\frac{(2+1) \rightarrow c_{c}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $\frac{B_{T}}{C_{22}}$ $\frac{C_{23}}{C_{33}}$ Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: $R^{*} \rangle [\phi_{1}\chi_{2}]\rangle$ gement strue gement strue	(44114,1) Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -118.3 -118.3 -78.1 = 0.577[]. ong decay	γ (- $\gamma\gamma$ 18938.8 49.4 - 21372.0 -3559.5 1087.3 38.0 -1593.5 -468.2 $F\rangle R^s\rangle [\psi$ y channel	$\begin{array}{c} & 0.0 \\ & 582.2 \\ & -179.2 \\ & -12.9 \\ \hline \\ & 582.2 \\ & 390.1 \\ \hline \\ & 582.2 \\ & 390.1 \\ \hline \\ & 582.2 \\ & 390.1 \\ \hline \\ & 582.2 \\ \hline \\ \\ \\ & 582.2 \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} \text{Rela}\\ \hline (i,j) & \text{V}\\ \hline (1.2) & 0\\ (1.3) & 0\\ (2.3) & 0\\ (2.4) & 0\\ (2.4) & 0\\ (3.4) & 0\\ \hline (1.2)-(\\ \hline \\ \text{Ra}\\ (1.3) & -1\\ \hline \\ (2.4) & -1\\ \hline \\ (2.4$	tive L tive L \sqrt{alue} 0.205 0.220 0.205 (3.4): 118.3 269.1 118.3	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ \hline \\ 0.165 \text{ fm} \\ \hline \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ 1] \rangle \end{array}$	Contribution b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c c} & & \\ & &$	Best 110 Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -269.1 -269.1 -269.1 9651.3 6.6	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5 \\ 1 \\ \hline \end{array}$	$ \begin{array}{c} 2m_b \\ 10105.8 \\ 10105.8 \\ \hline 2m_b \\ 10105.8 \\ \hline 10105.8 \\ \hline \frac{3}{5}C_{bb} \\ \hline 7.7 \\ \hline 7.7 \\ \hline $
$\frac{PT_{c_{2}\overline{c}\overline{c}}(648.7, PC)}{bb\overline{b} J^{PC}}$ Mass, Variational Parameters (fm ⁻²) Quark Confinemen Kinetic : CS Inter V^{C} Total Con Total Wave $\Psi_{tot} = F\rangle I$ The rearran The rearran	$\begin{array}{c} (2+1) \rightarrow c_{0}^{2}z^{2}\\ =2^{++}\\ \\ /B_{T}\\ \hline C_{11}\\ C_{22}\\ C_{33}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	(44114,1) Value 19327.9 30.0 30.0 23.0 21372.0 -2977.3 908.1 25.1 -269.1 -118.3 -118.3 -118.3 -1011.3 -78.1 = 0.577[]. ong decay dths:	$\gamma ~ (- \gamma \gamma)$ 18938.8 49.4 - 21372.0 - 3559.5 1087.3 38.01593.5 -468.2 $F \rangle R^s\rangle [\psi$ y channel	$\begin{array}{c} tot \mu \ \ \ tot \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} \text{Rela}\\ \hline (i,j) & \text{V}\\ \hline (1.2) & 0\\ (1.3) & 0\\ (2.3) & 0\\ (2.4) & 0\\ (2.4) & 0\\ (3.4) & 0\\ \hline (1.2)-(\\ \hline \\ \text{Ra}\\ (1.3) & -1\\ \hline \\ (2.4) & -1\\ \hline \\ (2.4$	$\frac{1}{\sqrt{alue}} \frac{1}{\sqrt{alue}}$ ().205 ().220 ().220 ().220 ().220 ().220 ().220 ().205 (().4): ().205 (().4): ().205 (().4): ().205 (().2)().200 (().2)()().200 (().2)()().200 (().2)()().2	$\begin{array}{c} 0.130 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ 1] \rangle \end{array}$	Contribution b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c c} & & \\ & &$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 6.6	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ 8 \end{array}$	$ \begin{array}{c} 2m_b \\ 10105.8 \\ 10105.8 \\ \hline 10105.8 \\ \hline 10105.8 \\ \hline 10105.8 \\ \hline \frac{8}{3}C_{bb} \\ 7.7 \\ 8 \\ \hline C \\ 8 \\ \hline \end{array} $
$\begin{array}{c} \frac{PT_{c_{2}\overline{c}^{2}}(642,7)}{bb\overline{b}} J^{PC} \\ \hline \\ bb\overline{b} J^{PC} \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \hline \\ Quark \\ Confinemen \\ \hline \\ Kinetic \\ CS Inter \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Vave \\ \hline \\ \Psi_{tot} = F\rangle J \\ \hline \\ The rearran \\ \hline \\ The rearran \\ \hline \\ The rearian \\ \hline \\ The readiative \\ \Gamma_{T_{b^{2}\overline{c}^{2}}(19327, \mathbb{T}) \\ \hline \end{array}$	$\begin{array}{l} (2+1) \rightarrow t_{c^2 z^2} \\ = 2^{++} \\ \hline B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline C_{33} \\ \hline Mass \\ t \ Potential \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline Subtotal \\ \hline tribution \\ function: \\ R^* \rangle [\phi_1 \chi_2] \rangle \\ gement \ strue \\ ve \ decay \ wide \\ g_{9,2^{++}} \rightarrow T_{b^2 \overline{b}} \\ \end{array}$	$\begin{array}{l} (4431.4,1)\\ Value\\ 19327.9\\ 30.0\\ 30.0\\ 23.0\\ 21372.0\\ -2977.3\\ 908.1\\ 25.1\\ -269.1\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -1011.3\\ -78.1\\ = 0.577],\\ \text{ong decay}\\ \text{dths:}\\ _2(19240.0,0)\end{array}$	$\begin{array}{c} \gamma\gamma & (-\gamma\gamma) \\ \gamma\gamma \\ 18938.8 \\ 49.4 \\ -21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ \end{array}$ $\begin{array}{c} -1593.5 \\ -468.2 \\ F\rangle R^s\rangle [\psi \\ \psi \\ channel \\ 0^{++}\rangle \gamma = 0 \end{array}$	$\begin{array}{c} tot \mu \ \ \ tot \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} r = -\mu_c \\ \hline Rela \\ (i, j) V \\ (1.2) 0 \\ (1.3) 0 \\ (2.3) 0 \\ (2.4) 0 \\ (2.4) 0 \\ (3.4) 0 \\ (1.2)-(\\ \hline Ra \\ (1.3) -1 \\ (2.4) -1 \\ \hline (3.4) -2 \\ (2.4) -1 \\ \hline (6 F) R^s \end{array}$	$\frac{1}{\sqrt{alue}} \frac{1}{\sqrt{alue}}$ ().205 ().220 ().220 ().220 ().220 ().220 ().220 ().205 (().4): ().205 (().4): ().205 (().2)().205 (().2	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \ fm \\ 0.132 \ fm \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ 1] \rangle \end{array}$	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$	$\begin{array}{c c} & & \\ & &$	0432.1 Ref. [110] 10686.0 328.0 126.0 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 6.6 6.6	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \hline \\ 1.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 $	$2m_b \\ 10105.8 \\ 10105.8 \\ 10105.8 \\ 10105.8 \\ \hline 10105.8 \\ \hline \frac{8}{3}C_{bb} \\ 7.7 \\ \hline \frac{8}{3}C_{b\overline{b}} \\ \hline 7.7 \\ \hline 8.7 \\ \hline 7.7 \\ \hline 8.7 \\ \hline 7.7 \\ \hline 8.7 \\ \hline 7.7 \\ \hline 7.7$
$\begin{array}{c} \frac{PT_{c_{2}\overline{c}^{2}}(642.7)}{bb\overline{b}} J^{PC} \\ \hline \\ bb\overline{b} J^{PC} \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \hline \\ Quark \\ Confinemen \\ \hline \\ Kinetic \\ CS Inter \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Vave \\ \hline \\ \Psi_{tot} = F\rangle J \\ \hline \\ The rearran \\ \hline \\ The rearran \\ \hline \\ The rearian \\ \hline \\ The readiative \\ \hline \\ \Gamma_{L^{2}\overline{L^{2}}(19327)} \\ \hline \\ \Gamma_{L^{2}\overline{L^{2}}(19327)} \\ \hline \end{array}$	$\begin{array}{l} (2^{+}) \rightarrow t_{c^2 z^2} \\ = 2^{++} \\ \hline B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline Mass \\ t \ Potential \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline Subtotal \\ tribution \\ function: \\ R^* \rangle [\phi_1 \chi_2] \rangle \\ gement \ strue \\ re \ decay \ wide \\ g_{2^{+}} \rightarrow T_{b^2 \overline{b}} \\ \end{array}$	$\begin{array}{l} (4431.4,1)\\ Value\\ 19327.9\\ 30.0\\ 30.0\\ 21372.0\\ -2977.3\\ 908.1\\ 25.1\\ -269.1\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -1011.3\\ -78.1\\ = 0.577],\\ \text{ong decay}\\ \text{dths:}\\ {}_{2}(19240.0,4\\ {}_{2}(19303.9,-)\\ (19303.9,-)\\ \end{array}$	$\begin{array}{c} \gamma\gamma & (1-\gamma)\gamma \\ \gamma\gamma \\ 18938.8 \\ 49.4 \\ -21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ -1593.5 \\ -468.2 \\ F\rangle R^s\rangle [\psi \\ \psi \\ channel \\ 0^{++})\gamma = 0 \\ 0^{++})\gamma = 1 \end{array}$	$\begin{array}{c} tot \mu \ \ \ tot \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} \mathbf{Rela}\\ \mathbf{Rela}\\ (i,j) \mathbf{V}\\ (1.2) 0\\ (1.3) 0\\ (2.3) 0\\ (2.4) 0\\ (2.4) 0\\ (3.4) 0\\ (1.2)-(\mathbf{Ra}\\ (1.3) \mathbf{-1}\\ (2.4) \mathbf{-1}\\ (2.4) \mathbf{-1}\\ (3.4) \mathbf{-2}\\ (2.4) \mathbf{-1}\\ (4.5) \mathbf{Rs}\\ (4.5) \mathbf{Rs}\\$	$ \psi_c = \psi_c $ tive L \sqrt{alue} 0.205 0.220 0.220 0.220 0.220 0.220 0.220 0.220 0.205 (3.4): 118.3 269.1 269.1 26.1 2	$\begin{array}{c} 0.150 \mu_{N} \\ \hline \\ engths (fm) \\ \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \ fm \\ 0.132 \ fm \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ 1] \rangle \end{array}$	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$ CS Interaction	$\begin{array}{c c} & & \\ & &$	0432.1 Ref. [110] 10686.0 328.0 126.0 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 6.6 6.6	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{\bar{b}\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{\bar{b}\bar$	$2m_b \\ 10105.8 \\ 10105.8 \\ 10105.8 \\ 10105.8 \\ \hline 10105.8 \\ \hline \frac{8}{3}C_{bb} \\ 7.7 \\ \hline \frac{8}{3}C_{\overline{b}\overline{b}} \\ 7.7 \\ \hline 7.7 \\ \hline 7.7 \\ 7$
$\begin{array}{c} \frac{PT_{c_{2}\overline{z}\overline{z}}(642.7,}{bb\overline{b}\overline{b} J^{PC}}\\ \hline \\ bb\overline{b} J^{PC}\\ \hline \\ Mass, \\ Variational \\ Parameters\\ (fm^{-2}) \\ \hline \\ Quark \\ Confinemen \\ \hline \\ Kinetic \\ CS Inter \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ Total Vave \\ \Psi_{tot} = F\rangle J \\ \hline \\ The rearran \\ \hline \\ The magnet \\ \hline \end{array}$	$\begin{array}{c} (2+1) \rightarrow t_{o}^{2} g^{2} \\ = 2^{++} \\ \hline B_{T} \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline Mass \\ t \ Potential \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline Subtotal \\ tribution \\ function: \\ R^{*}\rangle [[\phi_{1}\chi_{2}]\rangle \\ gement \ stree \\ qe \ decay \ wide \\ g,2^{++}) \rightarrow T_{b^{2}\overline{b}} \\ g,2^{++}) \rightarrow T_{b^{2}\overline{b}} \\ ic \ moments \end{array}$	$\begin{array}{l} (4431.4,1)\\ \hline\\ Value\\ 19327.9\\ 30.0\\ 30.0\\ 21372.0\\ -2977.3\\ 908.1\\ 25.1\\ -269.1\\ -118.3\\ -1011.3\\ -118.3\\ -1011.3\\ -78.1\\ \hline\\ = 0.577],\\ \\ \text{ong decay}\\ \text{fths:}\\ _{2} (19240.0,4\\ _{2} (19303.9,\\ \\ \vdots \mu_{T_{b2}},\\ \end{array}$	$\begin{array}{c} \gamma\gamma & (1-\gamma) \\ \gamma\gamma & \gamma\gamma \\ 18938.8 \\ 49.4 \\ 49.4 \\ - \\ 21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ \end{array}$ $\begin{array}{c} -1593.5 \\ -468.2 \\ F\rangle R^s\rangle [\psi \\ \psi \\ channel \\ 0^{++}\gamma = 0 \\ 0^{++}\gamma = 1 \\ \frac{1}{5^2} (19303.9), \end{array}$	$\begin{array}{c} tot \mu \ \ \ tot \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} r = -\mu_c \\ \hline Rela \\ (i, j) & V \\ (1.2) & 0 \\ (1.3) & 0 \\ (2.3) & 0 \\ (2.4) & 0 \\ (2.4) & 0 \\ (3.4) & 0 \\ (1.2)-(\\ \hline Ra \\ (1.3) & -1 \\ (2.4) & -1 \\ (2.4) & -1 \\ (2.4) & -1 \\ (3.4) & -2 \\ (2.4) & -1 \\ (3.4) & -2 \\ (2.4) & -1 \\ (4.5) $	$\frac{1}{2} \frac{\mu_c}{bc}$ tive L tive L \sqrt{alue} 0.205 0.220 0.220 0.220 0.220 0.220 0.205 (3,4): dius: 118.3 269.1 118.3 269.1 118.3 269.1 118.3	$ \begin{array}{c} 0.130 \mu_{N} \\ \hline \\ engths (fm) \\ \hline \Upsilon \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.160(\Upsilon) \\ \hline \\ 0.165 \text{ fm} \\ 0.132 \text{ fm} \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ -796.7(\Upsilon) \\ \hline \\ 1 \\ \end{array} \right) $	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$ CS Interaction	$\begin{array}{c c} & \text{Value} \\ & & \frac{2m_b}{\frac{\mathbf{P}_{s_1}^2}{m_b + m_b^2 2m_1^2}} \\ & & \frac{m_b}{m_b + m_b^2 2m_3^2} \\ & & \frac{m_b}{V^C(12)} \\ \frac{1}{2}[V^C(13) + V^C(14) \\ & + V^C(23) + V^C(24)] \\ & & -D \\ \hline \\ & & \text{Subtotal} \\ \hline \\ & & \frac{\mathbf{P}_{s_2}^2}{2m_2^2} \\ & & \frac{m_b}{2m_b} \\ & & \frac{\mathbf{P}_{s_3}^2}{2m_2^2} \\ & & \frac{m_b}{V^C(12)} \\ & & \frac{1}{2}[V^C(13) + V^C(14) \\ & & + V^C(23) + V^C(14) \\ & & + V^C(23) + V^C(24)] \\ & & -D \\ \hline \\ & & \text{Subtotal} \\ \hline \\ & & \frac{1}{2}V^{SS}(12) \\ & & \frac{1}{4}(V^{SS}(13) + V^{SS}(14) \\ & & \frac{1}{4}(V^{SS}(13) + V^{SS}(14)) \\ \hline \end{array}$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 6.6 6.6 6.6 11.9	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{16}{3}v_{b\bar{b}} \\ 5.1 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \hline \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \\ 0$	$\begin{array}{c} 2m_b \\ 10105.8 \\ \hline 3C_{b\bar{b}} \\ 7.7 \\ \hline \frac{8}{3}C_{b\bar{b}} \\ 7.7 \\ \hline \frac{16}{3}C_{b\bar{b}} \\ \hline \end{array}$
$\begin{array}{c} \frac{PT_{c_{2}\overline{c}\overline{c}}(642.7,}{bb\overline{b}\overline{b}} J^{PC} \\ \hline \\ bb\overline{b}\overline{b} J^{PC} \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \hline \\ Quark \\ Confinemen \\ \hline \\ Kinetic \\ CS Inter \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Vave \\ \Psi_{tot} = F\rangle J \\ \hline \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ \\ \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ \\ \\ \\ The rearran \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\frac{(2+1) \rightarrow t_{o}^{2}z^{2}}{z^{2}}$ $= 2^{++}$ $/B_{T}$ C_{11} C_{22} C_{33} Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: $R^{*}\rangle [[\phi_{1}\chi_{2}]\rangle$ gement strue re decay with gement strue re decay wi	$\begin{array}{l} (4431.4,1)\\ \hline Value\\ 19327.9\\ 30.0\\ 30.0\\ 21372.0\\ -2977.3\\ 908.1\\ 25.1\\ -269.1\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -1011.3\\ -78.1\\ \hline \\ = 0.577],\\ \mbox{ong decay}\\ thts:\\ _2(19240.0,4\\ _2(19303.9,\\ :: \ \mu_{T_{h2}},\\ \mbox{c moment}\\ c \ moment\\ c \ moment\\ \end{array}$	$\begin{array}{c} \gamma\gamma & (1-\gamma) \\ \gamma\gamma \\ 18938.8 \\ 49.4 \\ 49.4 \\ - \\ 21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ \end{array}$ $\begin{array}{c} -1593.5 \\ -468.2 \\ F\rangle R^s\rangle [\psi \\ \psi \\ channel \\ 0^{++})\gamma = 0 \\ 0^{++})\gamma = 1 \\ \frac{52}{12}(19303.9, \\ \tau ts: \end{array}$	$\begin{array}{c} tot \mu \ \ \ tot \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	$\begin{array}{c} \mathbf{k} & \mathbf{j} = -\hat{\mu}c\\ \mathbf{Rela}\\ (i,j) & \mathbf{V}\\ (1.2) & 0\\ (1.3) & 0\\ (2.3) & 0\\ (2.4) & 0\\ (2.4) & 0\\ (3.4) & 0\\ (1.2)-(\\ \mathbf{Ra}\\ (1.3) & -1\\ (2.4) & -1\\ (2.4) & -1\\ (2.4) & -1\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2\\ (2.4) & -1\\ (3.4) & -2$	$\frac{1}{t_{tot}} \frac{\mu_c}{2}$ titve L (value) 0.205 0.220 0.205 0.220 0.220 0.205	$\frac{-0.130 \mu_N}{\Upsilon \Upsilon}$ engths (fm) $\Upsilon \Upsilon$ 0.160(\Upsilon) 0.160(\Upsilon) 0.165 fm 0.132 fm -796.7(\Upsilon) -796.7(\Upsilon) 1]> = 0	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$ CS Interaction	$\begin{array}{c} \mbox{Value} \\ & \mbox{$\frac{2m_b$}{\frac{P_{s_1}^2}{m_b + m_b^2 2m_b^2}$}} \\ & \mbox{$\frac{p_{s_2}^2}{m_b + m_b^2 2m_b^2}$} \\ \hline & \mbox{$\frac{VC(12)$}{\frac{1}{2}[V^C(13) + V^C(14)$} \\ + V^C(23) + V^C(24)]$} \\ & \mbox{$-D$} \\ \hline & \mbox{$Subtotal$} \\ \hline & \mbox{$\frac{P_{s_2}^2}{2m_2^2}$} \\ & \mbox{$\frac{m_b}{\frac{P_{s_2}^2}}$} \\ & \mbox{$\frac{T_{s_2}^2}{2m_2^2}$} \\ & \\mbox{$\frac{T_{s_2}^2}{2m_2^2}$} \\ & \\mbox{$\frac{T_{s_2}^2}{2m_2^$	Best 110 Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -269.1 -236.6 -983.0 9651.3 6.6 -983.0 9651.3	$\begin{array}{c} \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\overline{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{3}{8}v_{b\overline{b}} \\ 5.1 \\ \hline \\ \frac{8}{3}v_{\overline{b}\overline{b}} \\ 5.1 \\ \hline \\ \frac{8}{3}v_{\overline{b}\overline{b}} \\ 5.1 \\ \hline \\ \frac{16}{3}v_{b\overline{b}} \\ 15.3 \\ \hline \end{array}$	$\begin{array}{c} 2m_b \\ 10105.8 \\ \hline 3C_{b\bar{b}} \\ 7.7 \\ \hline \frac{8}{3}C_{b\bar{b}} \\ 7.7 \\ \hline \frac{6}{3}C_{b\bar{b}} \\ \hline 15.5 \\ \hline \end{array}$
$\begin{array}{c} \frac{\rho T_{c_{2}\overline{c}^{2}}(642,7)}{bb\overline{b}} J^{PC} \\ \hline \\ bb\overline{b} J^{PC} \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \hline \\ Quark \\ Confinemen \\ \hline \\ Kinetic \\ CS Inter \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ Total Con \\ \hline \\ V^{C} \\ \hline \\ \hline \\ \hline \\ Total Con \\ \hline \\ Total Vave \\ \hline \\ \Psi_{tot} = F\rangle J \\ \hline \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ The rearran \\ \hline \\ The rearran \\ \hline \\ \\ The rearran \\ \hline \\ The rearran \\ $	$\begin{array}{c} (2^{+}) \rightarrow J_{c^2 z^2} \\ = 2^{++} \\ \hline B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline Mass \\ t \ Potential \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline Subtotal \\ tribution \\ \hline function: \\ R^* \rangle [\phi_1 \chi_2] \rangle \\ gement \ stree \\ e \ decay \ wide \\ 9,2^{++}) \rightarrow T_{b^2 \overline{b}} \\ 9,2^{++}) \rightarrow T_{b^2 \overline{b}} \\ ic \ moments \\ on \ magneti \\ 9,2^{++}) \rightarrow T_{b^2 \overline{b}} \\ generation \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline (2,3) \\ (1,4) \\ \hline (2,3) \\ (1,4) \\ \hline (2,3) \\ (2,3) \\ (2,3) \\ (2,3) \\ (2,3) \\ (2,3) \\ (2,3) \\ (3,$	$\begin{array}{l} (4401,4,1)\\ \hline \\ Value\\ 19327.9\\ 30.0\\ 30.0\\ 21372.0\\ -2977.3\\ 908.1\\ 25.1\\ -269.1\\ -118.3\\ -118.3\\ -118.3\\ -118.3\\ -1011.3\\ -78.1\\ \hline \\ = 0.577],\\ \text{ong decay}\\ \text{fths:}\\ _{2}(19240.0,4\\ _{2}(19303.9,\\ \vdots \mu_{T_{h2}}\\ c \text{ momer}\\ _{2}(19240.0,4) \\ (19240.0,4) \\$	$\begin{array}{c} \gamma\gamma (1-\gamma) \\ \gamma\gamma \\ \gamma\gamma \\ 18938.8 \\ 49.4 \\ 49.4 \\ -1 \\ 21372.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ -3559.5 \\ 1087.3 \\ 38.0 \\ -3559.5 \\ -468.2 \\ F\rangle R^s\rangle [\psi \\ \psi \\ channel \\ 0^{++})\gamma = 0 \\ 0^{++})\gamma = 0 \\ 0^{++})\gamma = 1 \\ \frac{1}{52} (19303.9, \\ 18.5 \\ 0^{++})\gamma = \langle 0 \\ 18.5 \\ 0^{++})\gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 18.5 \\ 0^{++} \rangle \gamma = \langle 0 \\ 0^{++} \rangle \gamma = \langle 0$	$\begin{array}{c} & 0.0 \\ \hline 0.0 \\ \hline 390.1 \\ \hline \\ 0.0 \\ \hline 582.2 \\ \hline -179.2 \\ \hline -179.2 \\ \hline -12.9 \\ \hline \\ \hline \\ 582.2 \\ \hline 390.1 \\ \hline \\ \hline \\ \hline \\ 582.2 \\ \hline \\ 390.1 \\ \hline \\ \hline \\ \hline \\ \hline \\ 8^{*}B^{*}_{c} \\ \hline \\ keV \\ \hline \\ 0 \ keV \\ \hline \\ \hline \\ +- \right) = \langle \Psi^{1}_{tot} \hat{\mu^{z}} \Psi^{j}_{tot} \Psi^{j}_{tot}$	$\begin{array}{c} \mathbf{Rela}\\ (i,j) & \mathbf{V}\\ (1.2) & 0\\ (1.3) & 0\\ (2.3) & 0\\ (2.4) & 0\\ (2.4) & 0\\ (2.4) & 0\\ (3.4) & 0\\ (1.2)-(\\ \mathbf{Ra}\\ (1.3) & -1\\ (2.4) $	$\frac{1}{t_{tot}} \sum_{i=1}^{t_{tot}} \frac{1}{t_{iot}} \sum_{j=1}^{t_{tot}} \frac{1}{t_{iot}} \sum_{j=1}^{t_{to$	$= 0.130 \mu_N$ engths (fm) $\Upsilon \Upsilon$ $0.160(\Upsilon)$ 0.165 fm 0.132 fm $-796.7(\Upsilon)$ $-796.7(\Upsilon)$ $1]\rangle$	Contribution b-quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$ Interaction	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Ref. [110] 10686.0 328.0 126.0 -269.1 -236.6 -983.0 9651.3 10686.0 328.0 126.0 -269.1 -269.1 -269.1 -269.1 -269.1 -269.1 -236.6 -983.0 9651.3 6.6 6.6 11.9 25.1 25.1	$\begin{array}{c} \frac{1}{2}m_{bb} \\ \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \hline \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{16}{3}v_{b\bar{b}} \\ 15.3 \\ 25.5 \\ \end{array}$	$\begin{array}{c} 2m_b \\ 10105.8 \\ \hline 3C_{b\bar{b}} \\ 7.7 \\ \hline \frac{8}{3}C_{b\bar{b}} \\ 7.7 \\ \hline \frac{6}{3}C_{b\bar{b}} \\ \hline 15.5 \\ \hline 30.9 \\ \hline \end{array}$

Table 9 The masses, binding energies, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width

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ratios, and the relative lengths between quarks for the $J^{PC} = 0^{++}$, $1^{+-}bb\bar{b}\bar{b}$ states and their lowest meson–meson thresholds. The notation is the same as that in Table 7

bbbb	The	contribut	ion from	each term	Relat	ive Lei	ngths (fm)	Orronall	Present Work		CMI	Model
$J^{PC} = 0^{++}$		Value	$\eta_b \eta_b$	Difference	(i, j)	Vaule	$\eta_b \eta_b$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	19240.0	18778.1	461.9	(1,2)	0.227			$2m_b$	10686.0		
Variational	C_{11}	24.6	57.4		(1.3)	0.204	$0.148(\eta_b)$		$\frac{\mathbf{p}_{x_1}^2}{2}$	269.2		
Parameters (fm^{-2})	C_{22}	24.6	57.4		(2.3)	0.204	(1-)		$m_{\bar{c}} p_{x_3}^2$	216.0		
(IIII) Ouenh	Magg	01270.0	91979.0	0.0	(2,0)	0.204		<i>b</i> -quark:	$m_b + m_{\overline{b}} 2m'_3$ $V^C(12)$	111.8	1	
Confinamon	Mass t Potontial	21072.0	21372.0	0.0 602 0	(1,4)	0.204	0.149()	m^{eff}	V (12) $1_{[UC(12)]} + U^{C}(14)$	111.0	$-\frac{1}{4}m_{bb}$	
Vinctio	Enoneri	-3101.0	1955.0	023.2	(2,4)	0.204	$0.148(\eta_b)$	mb	$\frac{1}{2}[V(13) + V(14)]$	-697.4	-2382.4	0
COLL (Energy	970.4	1255.9	-280.0	(3,4)	0.227	0.100.0		$+V^{\circ}(23)+V^{\circ}(24)$]		$\frac{3}{4}m_{b\overline{b}}$	$2m_b$
CS Inter	raction	17.0	-125.5	142.5	(1,2)-	-(3,4):	0.126 fm			-983.0	11806.2	10105.8
	(1,2)	-111.8			R	adius:	0.130 fm		Subtotal	9602.6	9423.8	10105.8
V^C	(2,3)	-339.7							$\frac{2m_b}{p^2}$	10686.0		
	(1,4)	-339.7			(1,3) ·	-339.7	$-879.1(\eta_b)$		$\frac{P_{x_2}}{2m'_2}$	216.0		
	Subtotal	-1135.0	-1758.2	623.2	(3,4) ·	-111.8		The quarks	$\frac{m_{\bar{c}}}{m_b + m_{\bar{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_2'}$	216.0		
Total Con	tribution	-147.6	-627.9	480.3	(2,4) ·	-339.7	$-879.1(\eta_b)$	o-quark.	$V^{C}(34)$	111.8	$-\frac{1}{4}m_{bb}$	
Total Wave	function:							$m_{\overline{b}}^{eff}$	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	C07 4	-2382.4	
$\Psi_{tot} = 0.3$	$352 F\rangle R^s\rangle $	$\left[\phi_1\chi_5\right]$ –	0.936 F	$\langle R^s \rangle [\phi_2 \chi_6]$	$]\rangle = 0.5$	$558 F\rangle $	$ R^s\rangle [\psi_1\zeta_5]\rangle$		$+V^{C}(23)+V^{C}(24)$]	-697.4	$\frac{5}{4}m_{b\overline{b}}$	$2m_b$
+0.	$560 F\rangle R^s\rangle$	$ [\psi_1\zeta_6]\rangle$ -	+ 0.021 F	$ R^s\rangle R^s\rangle [\psi_2\zeta_5]$	$ \rangle + 0.6$	$512 F\rangle $	$ R^s\rangle [\psi_2\zeta_6]\rangle$		-D	-983.0	11806.2	10105.8
The rearran	gement str	ong widt	h ratios:	71 7167 - 3-		. , ,	1 7117 - 3-37		Subtotal	9602.6	9423.8	10105.8
$\Gamma_{T_{0,0}(19240)}$	0 0++) - m	$: \Gamma_T \circ \circ ($	1924 0 0++	$\rightarrow n, n, = 1$: 1.2				9 6 6 / · · · ·		$4v_{bb}$	$4C_{bb}$
The radiativ	ve decav wi	idths:	1024.0,0	/ //6//6					$\frac{3}{4}V^{33}(12)$	8.5	7.7	11.6
Гл. (10207	0.0++).7	(100.40.0		0 keV				CS			$4v_{\overline{1}\overline{1}}$	$4C_{\overline{1}\overline{1}}$
$\Gamma_{b^2\bar{b}^2}(19327)$	$9,2++) \rightarrow I_{b^2}$	52 (19240.0,	$\gamma = \gamma$	2.8 keV				Interaction	$\frac{3}{4}V^{SS}(34)$	8.5	77	11.6
The magnet	$i_{b2} \xrightarrow{g_{1}} \xrightarrow{g_{1}} \xrightarrow{g_{1}} \xrightarrow{g_{1}} \xrightarrow{g_{2}}$	$\frac{5}{52}(19240.0)$	(100.10.0	$= \sqrt{\Psi_{i}^{0}}$	$p^{++} _{\hat{\mu^{z}} }$	$\Psi_{i}^{0^{++}}$	= 0		Subtotal	17.0	15.4	23.2
The transiti	on magnet	ic momen	<u>52(19240.0</u>	,011) (11	or P	- 101 /		Matrix non	diagonal element	17.8	-27.0	-40.2
The transition magnetic moments:							Total contri	10040.0	10026.0	10.2		
$\mu_{T_{b^2\bar{b}^2}(19327.9,2^{++})\to T_{b^2\bar{b}^2}(19240.0,0^{++})\gamma} = \langle \Psi_{tot}^{2^{++}} \hat{\mu^z} \Psi_{tot}^{0^{++}} \rangle = 0$					= 0		TOTAL COULT	DUUION	19240.0	1 10000.0	20275.0	
$\mu_{T_{b^2\bar{b}^2}(19327.}$	$9,2^{++}) \rightarrow T_{b^2}$	52 (19240.0,	$_{0^{++})\gamma} =$	$\langle \Psi_{tot}^{2++} \mu^{z} \Psi \rangle$ $\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi \rangle$	$\langle v_{tot}^{0^{++}} \rangle = \langle v_{tot}^{0^{++}} \rangle =$	= 0 = 0.352	$\times \frac{4}{-}(\mu_b -$	$(\mu_{\bar{x}}) = -0.072$		19240.0	18830.0	20275.0
$ \mu_{T_{b^2\bar{b}^2}(19327.} \\ \mu_{T_{b^2\bar{b}^2}(19393.} \\ I^{PC} - 1^{+-} $	$9,2^{++}) \rightarrow T_{b^2}$ $9,1^{+-}) \rightarrow T_{b^2}$	52 (19240.0, 52 (19240.0, Value	$0^{(++)\gamma} = 0^{(++)\gamma} = 0^{(++)\gamma}$	$\langle \Psi_{tot}^{2} \mu^{z} \Psi$ $\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi$ Difference	$\left< \frac{P_{tot}^{0++}}{P_{tot}^{0++}} \right> =$	= 0 = 0.352 :ive Lei	$\times \frac{4}{\sqrt{6}}(\mu_b - \mu_b)$	$\mu_{\overline{b}}) = -0.072$	$2\mu_N$	Value	18830.0	20275.0
$\begin{array}{c} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.}\\ J^{PC}=1^{+-}\\ \end{array}$ Mass	$9,2^{++}) \rightarrow T_{b^{2}}$ $9,1^{+-}) \rightarrow T_{b^{2}}$	$\begin{array}{c} & \begin{array}{c} & \\ & \\ \hline \\ \hline$	$\frac{1}{18857.9}$		$ \frac{P_{tot}^{0++}}{P_{tot}^{0++}} = \frac{P_{tot}^{0++}}{\text{Relat}} $	= 0 = 0.352 tive Lei Value	$\frac{4}{\sqrt{6}}(\mu_b - \mu_b)$ ngths (fm)	$\mu_{\overline{b}}) = -0.072$	$\frac{2\mu_N}{\text{Contribution}}$	Value	Ref. [110]	20275.0 Ref. [123]
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.}\\ J^{PC}=1^{+-}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$9,2^{++}) \rightarrow T_{b2}$ $9,1^{+-}) \rightarrow T_{b2}$ $/B_T$ C_{11}	$\begin{array}{c} & \begin{array}{c} & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & $	$ \begin{array}{r} {}_{0^{++})\gamma} = \\ {}_{0^{++})\gamma} = \\ \hline {} \underline{\Upsilon \eta_b} \\ \hline 18857.9 \\ \hline 57.4 \end{array} $		$\frac{\mathcal{P}_{tot}^{0++}}{\left(i,j\right)} = \frac{\mathcal{P}_{tot}^{0++}}{\left(i,j\right)}$ Relat	= 0 = 0.352 :ive Lei Value	$ heta imes rac{4}{\sqrt{6}}(\mu_b - rac{1}{2} rac{1}{2} rac{4}{\sqrt{6}}(\mu_b - rac{1}{2} r{1}{2} ra$	$\mu_{\overline{b}}) = -0.072$	$\frac{2\mu_N}{Contribution}$ $\frac{2m_b}{\mathbf{p}_{x^*}^2}$	Value 10686.0	Ref. [110]	20275.0 Ref. [123]
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ J^{PC}=1^{+-}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$9,2^{++}) \to T_{b21}$ $9,1^{+-}) \to T_{b22}$ $/B_T$ C_{11} C_{22}	$\begin{array}{c} & \begin{array}{c} & \\ & \\ & \\ \hline & \\ & \\ \hline & \\ & \\ \hline & \\ & \\$	$ \begin{array}{l} _{0^{++})\gamma} = \\ _{0^{++})\gamma} = \\ \hline & \underline{\gamma \eta_b} \\ \hline 18857.9 \\ \hline & 57.4 \\ \hline & 49.4 \end{array} $	$\frac{\langle \Psi_{tot}^{1+-} \mu^{z} \Psi}{\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi}$ $\frac{\text{Difference}}{446.0}$	$\frac{\mathcal{P}_{tot}^{0++}}{\left(i,j\right)} = \frac{\mathcal{P}_{tot}^{0++}}{\left(i,j\right)}$ (1.2)	= 0 = 0.352 :ive Lei Value 0.203	$\frac{4}{\sqrt{6}}(\mu_b - \frac{4}{\sqrt{6}})$ ngths (fm) $\Upsilon \eta_b$	$\mu_{\overline{b}}) = -0.072$	$\frac{2m_b}{\frac{\mathbf{p}_{x_1}^2}{2m_1}}$	Value 10686.0 335.6	Ref. [110]	20275.0 Ref. [123]
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.}\\ J^{PC}=1^{+-}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} 9,2^{++}) \to T_{b^2} \\ 9,1^{+-}) \to T_{b^2} \\ \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \end{array}$	$\begin{array}{c c} & (19240.0, \\ & & \\ \hline \\ \\ & & \\ \hline \\ \\ \\ & & \\ \hline \\ \\ \\ & & \\ \hline \\ \\ \hline \\ \\ \\ \hline \\ \\ \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \hline \\ \\$	$ \begin{array}{r} _{0^{++})\gamma} = \\ _{0^{++})\gamma} = \\ \hline \\ \hline \\ \hline \\ 18857.9 \\ \hline \\ 57.4 \\ 49.4 \\ \hline \\ - \end{array} $	$\frac{\langle \Psi_{tot}^{2+} \mu^{z} \Psi}{\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi}$ $\frac{\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi}{1}$ $\frac{\langle \Psi_{tot}^{1+-} \hat{\mu^{z}} \Psi}{446.0}$	$ \begin{array}{c} \mathcal{I}_{tot}^{0^{++}} \rangle = \\ \mathcal{I}_{tot}^{0^{++}} \rangle = \\ \hline \\ \hline (i,j) \\ \hline (1.2) \\ (1.3) \end{array} $	= 0 = 0.352 ive Lei Value 0.203 0.217	$\frac{4}{\sqrt{6}} \left(\mu_b - \frac{1}{\sqrt{6}} \left(\mu_b - \frac{1}{\sqrt{6}} \right) \right)$ $\frac{\gamma \eta_b}{0.148(\eta_b)}$	$\mu_{\overline{b}}) = -0.072$	$\frac{2m_b}{2m_b}$ $\frac{2m_b}{\frac{p_{x_1}^2}{2m_1^2}}$ $\frac{m_b}{m_b + m_b} \frac{p_{x_3}^2}{2m_3}$	Value 10686.0 335.6 131.4	Ref. [110] $\frac{1}{2}m_{bb}$	Ref. [123]
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.}\\ J^{PC}=1^{+-}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} 9,2^{++}) \to T_{b^{2}1} \\ 9,1^{+-}) \to T_{b^{2}1} \\ \hline \\ /B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \\ \\ Mass \end{array}$	2 (19240.0, 52 (19240.0, Value 19303.9 30.7 30.7 24.0 21372.0	$ \begin{array}{r} {}_{0^{++})\gamma} = \\ {}_{0^{++})\gamma} = \\ \hline {} \underline{\gamma \eta_b} \\ \hline 18857.9 \\ \hline 57.4 \\ 49.4 \\ - \\ \hline 21372.0 \end{array} $	$\frac{\langle \Psi_{tot}^{1+} \mu^z \Psi_{tot}^{1+} \mu^z \Psi}{\langle \Psi_{tot}^{1+-} \mu^z \Psi}$ Difference $\frac{446.0}{0.0}$	$ \begin{array}{c} I_{tot}^{0++} \rangle = \\ P_{tot}^{0++} \rangle = \\ \hline Relat \\ \hline (i,j) \\ \hline (1.2) \\ (1.3) \\ (2,3) \end{array} $	= 0 = 0.352 Sive Ler Value 0.203 0.217 0.217	$\frac{4}{\sqrt{6}}(\mu_b - \frac{1}{\sqrt{6}}(\mu_b) - \frac{1}{2}\eta_b}{\gamma_{\eta_b}}$ $0.148(\eta_b)$	$(\mu_{\overline{b}}) = -0.072$	$\frac{2m_b}{2m_b}$ $\frac{2m_b}{\frac{P_{x_1}^2}{\frac{2m_1^2}{2m_1^2}}}$ $\frac{m_b}{V^C(12)} P_{x_3}^2$ $\frac{1}{2[V^C(12) + V^C(14)]}$	Value 10686.0 335.6 131.4 -274.5	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8	Ref. [123]
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.}\\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.}\\ J^{PC}=1^{+-}\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b21} \\ 9,1^{+-}) \rightarrow T_{b22} \\ \hline \\ /B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ t \mbox{ Potential} \end{array}$	$\begin{array}{c c} & (19240.0, \\ & & \\ \hline \\ & & \\ & \\$	$\begin{array}{c} _{0^{++})\gamma} = \\ _{0^{++})\gamma} = \\ \hline & \Upsilon \eta_b \\ \hline 18857.9 \\ \hline 57.4 \\ 49.4 \\ - \\ \hline 21372.0 \\ - 3641.9 \end{array}$	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{z} \Psi_{}^{z} \Psi_$	$ \begin{array}{c} 0^{+} \\ i_{tot} \rangle = \\ \hline \\ 0^{0^{++}} \\ i_{tot} \rangle = \\ \hline \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \end{array} $	= 0 = 0.352 :ive Ler Value 0.203 0.217 0.217 0.217	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{\gamma_{\eta_b}}$ $0.148 (\eta_b)$	$(\mu_{\overline{b}}) = -0.072$	$\frac{2m_b}{2m_b} \\ \hline \frac{2m_b}{\frac{p_{x_1}^2}{2m_1^2} \frac{p_{x_2}^2}{m_b + m_b \frac{p_{x_3}^2}{2m_3^2}}} \\ \frac{m_b}{V^C(12)} \\ \frac{1}{2} [V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ \end{bmatrix}$	Value 10686.0 335.6 131.4 -274.5 -244.2	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$	20275.0 Ref. [123] 2m _b
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19393.} \\ J^{PC} = 1^{+-} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2}};\\ 9,1^{+-}) \rightarrow T_{b^{2}};\\ \hline\\ \\ B_{T}\\ \hline\\ C_{11}\\ C_{22}\\ C_{33}\\ \hline\\ Mass\\ t \ Potential\\ \hline\\ Energy\\ \end{array}$	22 (19240.0, 22 (19240.0, Value 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0	$\begin{array}{c} _{0^{++})\gamma} = \\ _{0^{++})\gamma} = \\ \hline & \Upsilon \eta_b \\ 18857.9 \\ 57.4 \\ 49.4 \\ - \\ 21372.0 \\ -3641.9 \\ 1171.6 \end{array}$	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{z} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{z} \Psi \\ Difference \\ \hline \\ 446.0 \\ \hline \\ \hline \\ 0.0 \\ \hline \\ 638.4 \\ \hline \\ -237.6 \end{array}$	$ \begin{array}{c} \langle U_{tot}^{0+} \rangle = \\ \langle U_{tot}^{0++} \rangle = \\ \hline \langle U_{tot}^{0++} \rangle = \\ \hline \langle (i,j) \\ (1.2) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \end{array} $	= 0 = 0.352 :ive Lei 0.203 0.217 0.217 0.217 0.217	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.160(\Upsilon)$	(b-quark:) = -0.072	$\begin{array}{c} \begin{array}{c} \hline \\ \hline $	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5	Ref. [123] $2m_b$ 10105.8
$\begin{array}{l} \mu_{T_{b^2 \tilde{b}^2}(19327.} \\ \mu_{T_{b^2 \tilde{b}^2}(19393.} \\ J^{PC} = 1^{+-} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b21} \\ 9,1^{+-}) \rightarrow T_{b22} \\ \hline \\ /B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ Mass \\ t \ Potential \\ \hline \\ Energy \\ raction \\ \end{array}$	32 (19240.0, 19240.0, Value 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3	$_{0^{++})\gamma} =$ $_{0^{++})\gamma} =$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8	$\begin{array}{c} \langle \Psi_{tot}^{z} \mid \mu^{z} \Psi \\ \langle \Psi_{tot}^{1+-} \mid \mu^{z} \Psi \\ \hline \text{Difference} \\ \hline \\ $	$ \begin{array}{c} \langle 0_{tot}^{0} \rangle = \\ \langle i_{tot} \rangle \rangle = \\ \hline \rho_{tot}^{0++} \rangle = \\ \hline \langle i,j \rangle \\ (1.2) \\ (1.3) \\ (2.3) \\ (1.4) \\ (2.4) \\ (3.4) \end{array} $	= 0 = 0.352 :ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.203	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{\gamma_{\eta_b}}$ $0.148 (\eta_b)$ $0.160 (\Upsilon)$	(b-quark:) = -0.072	$ \frac{2m_b}{\frac{2m_b}{\frac{p_{x_1}^2}{2m_1}}} \\ \frac{\frac{m_b}{2m_1} p_{x_2}^2}{\frac{m_b}{2m_3} p_{x_3}^2} \\ V^C(12) \\ \frac{1}{2}[V^C(13) + V^C(14) \\ + V^C(23) + V^C(24)] \\ -D \\ \\ \hline \\ Subtotal \\ \end{array} $	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8 \end{array}$
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \hline \\ \text{Mass}_{j} \\ \hline \\ \text{Variational} \\ \text{Parameters} \\ (\text{fm}^{-2}) \\ \\ \\ \text{Quark} \\ \text{Confinement} \\ \\ \\ \text{Kinetic} \\ \\ \\ \\ \\ \\ \text{CS Inter} \\ \end{array}$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2}1} \\ g,1^{+-}) \rightarrow T_{b^{2}1} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \\ Mass \\ t \text{ Potential} \\ \hline \\ Energy \\ raction \\ \hline \\ (1,2) \end{array}$	30.7 30.7 30.7 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5	$_{0^{++})\gamma} = \frac{\gamma \eta_{b}}{\gamma}$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{z} \Psi_{tot}^{z} \mu^{z} \Psi_{tot}^{+} \mu^{z} \Psi_{tot}^{-} \Psi_{tot}^{-} \mu^{z} $	$\begin{array}{l} \left \begin{array}{c} 0_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} 0_{tot}^{0++} \right\rangle = \\ \left \begin{array}{c} 0_{$	= 0 = 0.352 ive Lee 0.203 0.217 0.217 0.217 0.217 0.203 -(3,4):	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7}} \frac{1}{\sqrt{7}}$	$\mu_{\overline{b}}) = -0.072$	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ \text{Ref. [123]}\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \end{array}$
	$\begin{array}{c} 9.2^{++}) \rightarrow T_{b^{2}1} \\ 9.1^{+-}) \rightarrow T_{b^{2}1} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \\ C_{33} \\ \\ \\ Mass \\ t \ Potential \\ \\ Energy \\ raction \\ \hline \\ (1,2) \\ (2,3) \\ \end{array}$	52 (19240.0, 192 (19240.0, 19303.9 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5 -122.1 -122.1	$_{0^{++})\gamma} = \frac{\gamma \eta_{b}}{\gamma} = \frac{\gamma \eta_{b}}{18857.9}$ $\frac{57.4}{49.4}$ $\frac{1}{2}$ 21372.0 -3641.9 1171.6 -43.8	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{z} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{z} \Psi \\ \text{Difference} \\ \hline \\ 446.0 \\ \hline \\ 0.0 \\ \hline \\ 638.4 \\ \hline \\ -237.6 \\ \hline \\ 45.1 \\ \end{array}$	$ \begin{array}{c} \left \begin{array}{c} 0 \\ i \\ i \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ t \\ i \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ t \\ i \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ i \\ t \\ i \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ i \\ t \\ i \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ i \\ t \\ t \\ t \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ t \\ t \\ t \\ t \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ t \\ t \\ t \\ t \\ t \\ t \end{array} \right\rangle = \\ \left \begin{array}{c} 0 \\ i \\ t \\ t$	= 0 = 0.352 :ive Lei Value 0.203 0.217 0.217 0.217 0.203 -(3,4): adius:	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.160 (\Upsilon)$ 0.162 fm 0.130 fm	$\mu_{\overline{b}}) = -0.072$	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ \text{Ref. [123]}\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \end{array}$
$\begin{split} & \mu_{T_{b^2 \overline{b}^2}(19327.} \\ & \mu_{T_{b^2 \overline{b}^2}(19393.} \\ & J^{PC} = 1^{+-} \\ & \text{Mass}_{j} \\ & \text{Variational} \\ & \text{Parameters} \\ & \text{(fm}^{-2)} \\ & \text{Quark} \\ & \text{Confinement} \\ & \text{Kinetic} \\ & \text{CS Inter} \\ \\ & V^C \end{split}$	$\begin{array}{c} 9.2^{++}) \rightarrow T_{b21} \\ 9.1^{+-}) \rightarrow T_{b22} \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ t \ Potential \\ Energy \\ raction \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \end{array}$	52 (19240.0, 192 (19240.0, Value 19303.9 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5 -122.1 -122.1 -122.1	$_{0^{++})\gamma} = \frac{\gamma \eta_{b}}{\gamma}$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{z} \Psi_{}^{z} \Psi_$	$\begin{array}{c} \left \begin{array}{c} v_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} v_{tot}^{0} \right\rangle = \\ \hline v_{tot}^{0} \right\rangle = \\ \hline v_{tot}^{0} \left\rangle = \\ \hline v_{tot}^{0} \right\rangle = \\ \hline v_{tot}^{0} \left\langle 1, 2 \right\rangle \\ \hline v_{$	= 0 = 0.352 ive Lee Value 0.203 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.148 (\eta_b)$ $0.160 (\Upsilon)$ 0.162 fm 0.130 fm $-879.1 (\eta_b)$	$\mu_{\overline{b}}) = -0.072$	$\begin{array}{c} \frac{2m_b}{\frac{P_{x_1}^2}{2m_1^2}} \\ \hline \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ \text{Ref. [123]}\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \end{array}$
$\begin{split} & \mu_{T_{b^2 \overline{b}^2}(19327.} \\ & \mu_{T_{b^2 \overline{b}^2}(19393.} \\ & J^{PC} = 1^{+-} \\ & \text{Mass}_{j} \\ & \text{Variational} \\ & \text{Parameters} \\ & \text{(fm}^{-2)} \\ & \text{Quark} \\ & \text{Confinement} \\ & \text{Kinetic} \\ & \text{CS Inter} \\ \\ & V^C \end{split}$	$\begin{array}{c c} 9,2^{++}) \to T_{b^{2}1} \\ \hline 9,1^{+-}) \to T_{b^{2}1} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ t \ Potential \\ Energy \\ raction \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ \end{array}$	52 (19240.0, 192(19240.0, Value 19303.9 30.7 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5 -122.1 -122.1 -122.7 -1037.4	$_{0^{++})\gamma} =$ $_{0^{++})\gamma} =$ $\gamma \eta_b$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{++-} \mu^{2} \Psi \\ \hline \text{Difference} \\ \hline \\ 446.0 \\ \hline \\ 0.0 \\ \hline \\ 638.4 \\ -237.6 \\ \hline \\ 45.1 \\ \hline \\ \hline \\ 638.5 \\ \hline \end{array}$	$\begin{array}{c} p_{tot}^{0} \rangle = \\ p_{tot}^{0} \rangle = \\ \hline \\ Relat \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline \\ (1,2) \\ \hline \\ R \\ \hline \\ \hline \\ (1,3) \\ - \\ \hline \\ (3,4) \\ \hline \end{array}$	= 0 = 0.352 ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.148 (\eta_b)$ $0.160 (\Upsilon)$ 0.162 fm 0.130 fm $-879.1 (\eta_b)$	b-quark: m_b^{eff} \overline{b} -quark:	$\begin{array}{r} \begin{array}{c} \begin{array}{c} \\ \hline \\ $	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{bb}$ 4764.8	$\begin{array}{c} 20275.0\\ \hline \\ \text{Ref. [123]}\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \end{array}$
$\begin{array}{c} \mu_{T_{b^2 \overline{b}^2}(19327.} \\ \mu_{T_{b^2 \overline{b}^2}(19393.} \\ J^{PC} = 1^{+-} \\ \\ \\ \hline \\ \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2};} \\ 9,1^{+-}) \rightarrow T_{b^{2};} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{22} \\ C_{33} \\ \hline \\ C_{23} \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ tribution \end{array}$	52 (19240.0, 52 (19240.0, Value 19303.9 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5 -122.1 -122.1 -1037.4	$_{-1675.9}^{0++)\gamma} =$ γ_{η_b} γ	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ Difference \\ \hline 446.0 \\ \hline \\ 0.0 \\ 638.4 \\ -237.6 \\ 45.1 \\ \hline \\ 638.5 \\ \hline \\ 446.0 \\ \end{array}$	$\begin{array}{c} p_{tot}^{0} \rangle = \\ p_{tot}^{0} \rangle = \\ \hline \text{Relat} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2) \\ \hline \\ (1,3) \\ (2,4) \\ (3,4) \\ \hline \\ (2,4) \\ (2,4) \\ \end{array}$	= 0 = 0.352 ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.160 (\Upsilon)$ 0.162 fm 0.130 fm $-879.1 (\eta_b)$	b-quark: m_b^{eff} \overline{b} -quark: m_b^{eff}	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5	18830.0 Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 2m_t \end{array}$
$\mu_{T_{b^2 \bar{b}^2}(19327.}$ $\mu_{T_{b^2 \bar{b}^2}(19327.}$ $J^{PC} = 1^{+-}$ Mass, Variational Parameters (fm ⁻²) Quark Confinement Kinetic I CS Inter V^C Total Com	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2}1} \\ g,1^{+-}) \rightarrow T_{b^{2}1} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ t \ Potential \\ \hline \\ Energy \\ raction \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ tribution \\ function; \end{array}$	⁵² (19240.0, ⁵² (19240.0, ⁷² (19240.0, ^{19303.9} 19303.9 30.7 30.7 24.0 21372.0 -3003.4 934.0 1.3 -274.5 -122.1 -122.1 -1037.4 -102.1 Ψ.	$_{-1675.9}^{0++)\gamma} =$ $_{0^{++)\gamma}}^{\gamma} =$ $_{18857.9}^{\gamma}$ $_{57.4}^{\gamma}$ $_{-21372.0}^{\gamma}$ $_{-3641.9}^{\gamma}$ $_{1171.6}^{\gamma}$ $_{-43.8}^{\gamma}$	$\frac{\langle \Psi_{tot}^{+} \mu^{2} \Psi}{\langle \Psi_{tot}^{+-} \mu^{2} \Psi}$ Difference 446.0 0.0 638.4 -237.6 45.1 638.5 446.0 $R^{s \setminus [(\phi_{1}, y_{0}])}$	$\begin{array}{c} p_{tot}^{0} \rangle = \\ p_{tot}^{0} \rangle = \\ \hline p_{tot}^{0} \rangle = \\ \hline relat \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2.3) \\ (1.4) \\ (2.4) \\ (3.4) \\ \hline (1.2) \\ \hline (1.3) \\ (2.4) \\ \hline (1.3) \\ \hline (2.4) \\ \hline (2.4)$	= 0 = 0.352 ive Lee Value 0.203 0.217 0.217 0.217 0.217 0.217 0.217 0.217 0.213 -(3,4): adius: -122.1 -274.5 -122.1 108[<i>F</i>])	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b}}{0.148 (\eta_b)}$ $0.148 (\eta_b)$ $0.160 (\Upsilon)$ 0.162 fm 0.130 fm $-879.1 (\eta_b)$ $-796.7 (\Upsilon)$ $ R^8 \rangle [R_{2} \rangle \langle c_2 \rangle\rangle$	b-quark: m_b^{eff} \overline{b} -quark: m_b^{eff}	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \\ \\ \\ \hline \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0	18830.0 Ref. [110] $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4772.5	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 2m_b\\ 10105.8\\ \hline \end{array}$
$\begin{array}{c} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ Quark \\ Confinement \\ \\ Confinement \\ \\ CS Inter \\ \\ \\ \\ CS Inter \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$9,2^{++}) \rightarrow T_{b2};$ $9,1^{+-}) \rightarrow T_{b2};$ $/B_T$ C_{11} C_{22} C_{33} Mass t Potential Energy raction $(1,2)$ $(2,3)$ $(1,4)$ Subtotal tribution function: $408 E \backslash B^{*}\rangle$	$\begin{array}{c c} & & & \\ & & \\ & & \\ & & \\ \hline & & \\$	$_{0^{++})\gamma} =$ $\gamma \eta_{b}$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8 -43.8 -1675.9 -548.1 $\sigma = F\rangle _{b^{+}}$	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ \Psi_{tot}^{+-} \mu^{2} \Psi \\ \hline \\ 0.0 \\ \hline \\ \hline \\ 0.0 \\ \hline \\ \hline \\ 0.0 \\ \hline \\ 638.4 \\ \hline \\ -237.6 \\ \hline \\ 45.1 \\ \hline \\ $	$\begin{array}{c} \left \begin{array}{c} \left $	= 0 = 0.352 :ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -122.1 408 F) 577 F)	$\frac{1}{2} \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7}\eta_b} \\ 0.148(\eta_b) \\ 0.148(\eta_b) \\ 0.160(\Upsilon) \\ 0.162 \text{ fm} \\ 0.130 \text{ fm} \\ -879.1(\eta_b) \\ -796.7(\Upsilon) \\ R^s\rangle [\psi_1\zeta_2]\rangle \\ R^s\rangle [\psi_1\zeta_2]\rangle \\ R^s\rangle [\psi_1\zeta_2]\rangle $	b-quark: m_b^{eff} \overline{b} -quark: m_b^{eff}	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3	13830.0 Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \end{array}$
$\begin{array}{l} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ Quark \\ Confinement \\ \\ Kinetic \\ \\ CS Inter \\ \\ \\ V^{C} \\ \\ \hline \\ \hline \\ Total Control \\ \\ Total Control \\ \\ \hline \\ \\ Total Wave \\ -0. \\ \\ \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} 9.2^{++}) \rightarrow T_{b21} \\ 9.1^{+-}) \rightarrow T_{b22} \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ C_{33} \\ \hline \\ C_{23} \\ C_{23} \\ C_{23} \\ C_{33} \\ \hline \\ C_{23} \\ C_{$	$\begin{array}{c c} & & & \\ &$	$_{0^{++})\gamma} = \frac{\gamma \eta_b}{\gamma}$ 18857.9 57.4 49.4 -2 21372.0 -3641.9 1171.6 -43.8 -1675.9 -548.1 $\sigma_t = F\rangle .+$ 0.577 F w channel	$\frac{\langle \Psi_{tot}^{+} \mu^{2} \Psi}{V_{tot}^{+-} \mu^{2} \Psi} \frac{ \Psi_{tot}^{+} \mu^{2} \Psi}{V_{tot}^{+-} \mu^{2} \Psi} \frac{ \Psi_{tot}^{+} \mu^{2} \Psi}{V_{tot}^{+} \mu^{2} \Psi} \frac{ \Phi_{tot}^{-} \mu^{2} \Psi}{ \Phi_{tot}^{-} \mu^{2} \Psi} \frac{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi}{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi} \frac{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi}{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi} \frac{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi}{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi} \frac{ \Phi_{tot}^{-} \Psi}{ \Phi_{tot}^{-} \Phi_{tot}^{-} \Psi} \frac{ \Phi_{tot}^{-} \Psi}{ \Phi} \frac{ \Phi}{ \Phi} \frac{ \Phi}$	$\begin{array}{l} \left \begin{array}{c} 0_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} 0_{tot$	$= 0$ $= 0.352$ ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -274.5 -122.1 408 $ F\rangle $ 577 $ F\rangle $	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b} \\ 0.148(\eta_b) \\ 0.148(\eta_b) \\ 0.160(\Upsilon) \\ 0.162 \text{ fm} \\ 0.130 \text{ fm} \\ -879.1(\eta_b) \\ -796.7(\Upsilon) \\ R^s\rangle [\psi_1\zeta_2]\rangle \\ R^s\rangle [\psi_2\zeta_6]\rangle$	b-quark: m_b^{eff} \bar{b} -quark: m_b^{eff}	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3	Ref. [110] $\frac{1}{2}m_{bb}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3	$\begin{array}{c} 20275.0\\ \hline \\ \text{Ref. [123]}\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 8C_{*} \end{array}$
$\begin{array}{c} \mu_{T_{b^2 \overline{b}^2}(19327.} \\ \mu_{T_{b^2 \overline{b}^2}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ Quark \\ Confinement \\ \\ Kinetic \\ \\ CS Inter \\ \\ \\ CS Inter \\ \\ \\ \\ V^C \\ \\ \hline \\ \hline \\ Total Com \\ \\ \hline \\ \hline \\ Total Com \\ \\ \hline \\ Total Com \\ \\ \hline $	$\begin{array}{c} 9.2^{++}) \rightarrow T_{b21} \\ 9.1^{+-}) \rightarrow T_{b22} \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ c_{33} \\ \hline \\ C_{23} \\ c_{33} \\ \hline \\ C_{23} \\ c_{33} \\ c_{33} \\ \hline \\ C_{23} \\ c_{33} \\ c_{33} \\ c_{33} \\ \hline \\ C_{23} \\ c_{33} \\ $	$\begin{array}{c c} & & & \\ &$	$_{0^{++})\gamma} = \frac{\gamma \eta_b}{\gamma \theta_b}$ 18857.9 57.4 49.4 -2 21372.0 -3641.9 1171.6 -43.8 -1675.9 -548.1 $\rho_t = F\rangle .$ + 0.577 F y channe	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ \\ \Psi_{tot}^{+-} \mu^{2} \Psi \\ \\ Difference \\ \hline \\ $	$\begin{array}{c} \left \begin{array}{c} 0_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} \mathrm{Relat} \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1.4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \\ (3,4) \\ (1,2) \\ \\ (3,4) \\ \\ (2,4) \\ \\ (2,4) \\ \\ (2,4) \\ \\ (3,4) \\ \\ \end{array} \right \\ \left \begin{array}{c} 0_{i} \\ 0_{i$	= 0 = 0.352 ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -274.5 -122.1 $408 F\rangle $ 577 $ F\rangle $	$\frac{1}{\chi} \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\chi} \eta_b - \frac{1}{\chi} \eta$	b-quark: m_b^{eff} \overline{b} -quark: m_b^{eff}	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	19240.0 Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -274.5 -274.5 -274.5 -274.5 -274.5 -274.5 -244.2 -983.0 9653.3 6.7	$\frac{\frac{1}{2}m_{bb}}{4764.8}$ $\frac{\frac{1}{2}m_{b\bar{b}}}{4722.5}$ 9487.3 $\frac{\frac{1}{2}m_{b\bar{b}}}{4764.8}$ $\frac{\frac{1}{2}m_{b\bar{b}}}{4764.8}$ $\frac{\frac{1}{2}m_{b\bar{b}}}{4722.5}$ 9487.3 $\frac{\frac{8}{3}v_{b\bar{b}}}{5.1}$	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \frac{8}{3}C_{bb}\\ \hline \\ 7.7 \end{array}$
$\mu_{T_{b^2\bar{b}^2}(19327.}$ $\mu_{T_{b^2\bar{b}^2}(19327.}$ $J^{PC} = 1^{+-}$ Mass, Variational Parameters (fm ⁻²) Quark Confinement Kinetic I CS Inter V^C Total Com Total Wave -0. The rearran The radiative	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2}};\\ 9,1^{+-}) \rightarrow T_{b^{2}};\\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ c_{33} \\ \hline \\ C_{23};\\ c_{33} \\ c_{33} \\ \hline \\ C_{23};\\ c_{33} \\ c_{33}$	$\begin{array}{c c} & & & \\ & & \\ & & \\ & & \\ \hline & & \\$	$_{0^{++}\gamma} =$ $_{0^{++}\gamma} =$ $\Upsilon \eta_b$ 18857.9 57.4 49.4 - 21372.0 -3641.9 1171.6 -43.8 - -1675.9 -548.1 $_{0^{+}} = F\rangle .$ $\pm 0.577 F $ y channe	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ Difference \\ \hline \\ 446.0 \\ \hline \\ 0.0 \\ 638.4 \\ -237.6 \\ 45.1 \\ \hline \\ 638.5 \\ \hline \\ 446.0 \\ \hline \\ R^{s} \rangle [\phi_{1} \chi_{2}] \rangle \\ \hline \\ \Gamma \rangle R^{s} \rangle [\psi_{2} \zeta_{5} \\ 1: J/\psi \eta_{c} \\ \hline \\ 1.0 \ c M \end{array}$	$\begin{array}{c} \left \begin{array}{c} 0_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} \mathrm{Relat} \\ \overline{(t,j)} \\ (1,2) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ (1,2) \\ (3,4) \\ (2,4) $	$= 0$ $= 0.352$ ive Ler Value 0.203 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -274.5 -122.1 408 $ F\rangle $ 577 $ F\rangle $	$\frac{1}{\sqrt{6}} \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}}) + \frac{1}{\sqrt{7}\eta_b} = \frac{1}{\sqrt{7}\eta_b} = \frac{1}{\sqrt{7}\eta_b} = \frac{1}{\sqrt{7}} = \frac$	b-quark: m_b^{eff} \bar{b} -quark: m_b^{eff}	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 -244.2 -983.0 9653.3 6.7	$\frac{1}{2}m_{bb}$ $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{8}{3}v_{bb}$ 5.1 $\frac{8}{3}v_{b\bar{b}}$	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \hline \\ \frac{8}{3}C_{bb}\\ \hline \\ 7.7\\ \hline \\ \frac{8}{5}C_{}\end{array}$
$\begin{array}{c} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2};} \\ 9,1^{+-}) \rightarrow T_{b^{2};} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{22} \\ C_{33} \\ \hline \\ C_{23} \\$	$\begin{array}{c c} & & & \\ & & & \\ & & & \\ \hline & & & \\ & &$	$_{0^{++})\gamma} =$ $_{0^{++})\gamma} =$ $\Upsilon \eta_b$ 18857.9 57.4 49.4 -21372.0 -3641.9 1171.6 -43.8 -1675.9 -548.1 $_{0^{+}} = F\rangle _{+}$ + 0.577 F y channe $_{1^{+-})\gamma} =$	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ \hline \\ \text{Difference} \\ \hline \\ $	$\begin{array}{c} \left \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \end{array} \right\rangle = \\ \hline \\ Relat \\ \hline \\ (i,j) \\ \hline \\ (1,2) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline \\ (1,3) \\ \hline \\ (2,4) \\ \hline \\ (2$	= 0 = 0.352 ive Lee Value 0.203 0.217 0.217 0.217 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -274.5 -122.1 $408 F\rangle $ 577 $ F\rangle $	$\frac{1}{\sqrt{6}} \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}}) + \frac{1}{\sqrt{7}\eta_b} +$	\overline{b} -quark: m_b^{eff} \overline{b} -quark: m_b^{eff} \overline{cs}	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 9653.3 9653.3 6.7 6.7	$\frac{1}{2}m_{bb}$ $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{8}{3}v_{b\bar{b}}$ 5.1 $\frac{8}{3}v_{b\bar{b}}$ 5.1	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \frac{8}{3}C_{bb}\\ \hline \\ 7.7\\ \hline \\ \frac{8}{3}C_{b\bar{b}}\\ \hline \\ 7.7\end{array}$
$\begin{split} & \mu_{T_{b^2 \bar{b}^2}(19327.} \\ & \mu_{T_{b^2 \bar{b}^2}(19327.} \\ & J^{PC} = 1^{+-} \\ & \text{Mass}_{\bar{b}} \\ & \text{Variational} \\ & \text{Parameters} \\ & (\text{fm}^{-2}) \\ & \text{Quark} \\ & \text{Confinement} \\ & \text{Kinetic} \\ & \text{CS Inter} \\ \\ & V^C \\ & \text{Total Com} \\ & \text{Total Com}$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b^{2}};\\ 9,1^{+-}) \rightarrow T_{b^{2}};\\ \hline \\ B_{T}\\ \hline \\ C_{11}\\ C_{22}\\ C_{33}\\ \hline \\ Mass\\ t \ Potential\\ Energy\\ raction\\ \hline \\ (1,2)\\ (2,3)\\ (1,4)\\ \hline \\ (1,2)\\ (2,3)\\ (1,4)\\ \hline \\ Subtotal\\ tribution\\ function:\\ 408 F\rangle R^{s}\rangle\\ gement \ str\\ ve \ decay \ wi\\ 9,2^{++}) \rightarrow T_{b^{2}};\\ 9,1^{+-}) \rightarrow T_{b^{2}};\\ \end{array}$	$\begin{array}{c c} & & & \\ & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ &$	$\begin{array}{l} _{0^{++})\gamma} = \\ {2} \gamma_{\eta_b} \\ 18857.9 \\ 57.4 \\ 49.4 \\ - \\ 21372.0 \\ -3641.9 \\ 1171.6 \\ -43.8 \\ \end{array}$ $\begin{array}{l} - \\ -1675.9 \\ -548.1 \\ -548.1 \\ -548.1 \\ -577 F \\ y \ channe \\ 1^{+-})\gamma = \\ 0^{++})\gamma = \\ 0^{++})\gamma = \end{array}$	$\frac{\langle \Psi_{tot}^{+} \mu^{2} \Psi}{\langle \Psi_{tot}^{+-} \mu^{2} \Psi}$ Difference 446.0 0.0 638.4 -237.6 45.1 638.5 446.0 $\frac{638.5}{446.0}$ $\frac{638.5}{2\langle \psi_{1}\chi_{2} \rangle}$ $\frac{\langle \psi_{1}\chi_{2} \rangle}{\langle \psi_{2}\zeta_{5} \psi_{2}\zeta_{5} \psi_{2}\zeta_{5} \psi_{2}\zeta_{5} \psi_{1}\chi_{2} \rangle}$ 1.0 keV 2.8 keV	$\begin{array}{l} \left \begin{array}{c} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ \end{array} \right\rangle = \\ \hline \\$	$= 0$ $= 0.352$ ive Lee Value 0.203 0.217 0.217 0.217 0.217 0.217 0.203 -(3,4): adius: -122.1 -274.5 -122.1 $408 F\rangle $ 577 F \rangle	$\frac{1}{\sqrt{6}} \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}}) + \frac{1}{\sqrt{7}\eta_b} +$	$(b-\text{quark:} m_b^{eff}) = -0.072$	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 6.7 6.7	$\begin{array}{c} \frac{1}{2}m_{bb} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$ $\begin{array}{c} \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4764.8 \\ \frac{1}{2}m_{b\bar{b}} \\ 4722.5 \\ 9487.3 \\ \end{array}$ $\begin{array}{c} \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ \frac{8}{3}v_{b\bar{b}} \\ 5.1 \\ 16 \end{array}$	$\begin{array}{c} 20275.0\\ \hline \\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \hline \\ 10105.8\\ \hline \\ \hline$
$\begin{array}{c} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ Quark \\ Confinement \\ \\ Confinement \\ \\ Confinement \\ \\ CS Inter \\ \\ CS Inter \\ \\ V^{C} \\ \\ \hline \\ Total Cont \\ \\ CS Inter \\ \\ V^{C} \\ \\ \hline \\ Total Cont \\ \\ Total Cont \\ \\ Total Cont \\ \\ Total Cont \\ \\ \\ \\ Total Cont \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b21} \\ 9,1^{+-}) \rightarrow T_{b22} \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ t \ Potential \\ Energy \\ raction \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ tribution \\ function: \\ 408 F\rangle R^s\rangle \\ gement \ str \\ ve \ decay \ wi \\ 9,2^{++}) \rightarrow T_{b2} \\ gement \ str \\ ve \ decay \ wi \\ 9,1^{+-}) \rightarrow T_{b2} \\ ic \ moment \end{array}$	$\begin{array}{c c} & & & \\ &$	$\begin{array}{l} _{0^{++})\gamma} = \\ _{0^{++})\gamma} = \\ \hline & \Upsilon \eta_b \\ 18857.9 \\ 57.4 \\ 49.4 \\ - \\ 21372.0 \\ -3641.9 \\ 1171.6 \\ -43.8 \\ \hline \\ -1675.9 \\ -548.1 \\ -43.8 \\ \hline \\ -1675.9 \\ -548.1 \\ -10^{-} \\ -43.8 \\ \hline \\ -1675.9 \\ -548.1 \\ -10^{-} \\ -1675.9 \\ -548.1 \\ -10^{-} \\ -10^{-$	$\begin{array}{c} \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ \\ \hline \\ Difference \\ \hline \\ $	$\begin{array}{c} \left \begin{array}{c} 0_{tot}^{0} \right\rangle = \\ \left \begin{array}{c} \mathrm{Relat} \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1.4) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline \\ (1,2) \\ \hline \\ (1,2) \\ \hline \\ (1,3) \\ - \\ \hline \\ (2,4) \\ - \\ \hline \\ (2,4) \\ - \\ \hline \\ (1,3) \\ - \\ \hline \\ (2,4) \\ - \\ \hline \\ (1,2) \\ - \\ \\ (1,2) \\ - \\ \hline \\ (1,2) \\ - \\ \hline \\ (1,2) \\ - \\ \hline \\ (1,$	$ \begin{split} &= 0 \\ &= 0.352 \\ &: ive Lee \\ &Value \\ &0.203 \\ &0.217 \\ &0.$	$\frac{4}{\sqrt{6}}(\mu_{b} - \frac{1}{\sqrt{6}}(\mu_{b} - \frac{1}{\sqrt{6}})$ $\frac{1}{2} \sqrt{10} + \frac{1}{2} $	$\mu_{\overline{b}} = -0.072$ b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$ CS Interaction	$\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \\ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 6.7 6.7 -12.2	$\frac{1}{2}m_{bb}$ $\frac{1}{2}m_{bb}$ $\frac{1}{2}m_{b\bar{b}}$	$\begin{array}{c} 20275.0\\ \hline \\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \hline \\ \hline \\ 10105.8\\ \hline \\ \hline$
$\begin{array}{c} \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ J^{PC} = 1^{+-} \\ \\ \hline \\ Mass, \\ Variational \\ Parameters \\ (fm^{-2}) \\ \\ Quark \\ Confinement \\ \\ \\ \\ Confinement \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} 9,2^{++}) \rightarrow T_{b2};\\ 9,1^{+-}) \rightarrow T_{b2};\\ \hline\\ B_{T} \\ \hline\\ C_{11} \\ C_{22} \\ C_{33} \\ \hline\\ C_{33} \\ $	$\begin{array}{c c} & & & \\ & & & \\ & & & \\ & & & \\ \hline & & & \\ & &$	$\begin{array}{l} & & & \\ & &$	$\frac{\langle \Psi_{tot}^{+} \mu^{2} \Psi}{\langle \Psi_{tot}^{+-} \mu^{2} \Psi} \frac{ \Psi}{\langle \Psi_{tot}^{++} \mu^{2} \Psi} \frac{ \Psi}{\langle \Psi} \frac{ \Psi}$	$\begin{array}{c} \left \begin{array}{c} \left $	$ \begin{split} &= 0 \\ &= 0.352 \\ &: ive \ Ler \\ &Value \\ &0.203 \\ &0.217 \\ &$	$\frac{4 \times \frac{4}{\sqrt{6}} (\mu_b - \frac{1}{\sqrt{6}} (\mu_b) - \frac{1}{\sqrt{7} \eta_b} \\ 0.148 (\eta_b) \\ 0.148 (\eta_b) \\ 0.160 (\Upsilon) \\ 0.162 \text{ fm} \\ 0.130 \text{ fm} \\ -879.1 (\eta_b) \\ -796.7 (\Upsilon) \\ R^s\rangle [\psi_1 \zeta_2]\rangle \\ R^s\rangle [\psi_2 \zeta_6]\rangle \\ = 0 \\ 25 \dots$	b-quark: m_b^{eff} \bar{b} -quark: m_b^{eff} \bar{b} -quark: $m_{\bar{b}}^{eff}$ Interaction	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 6.7 6.7 -12.2 13	18830.0 Ref. [110] $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4762.5 9487.3 $\frac{8}{3}v_{b\bar{b}}$ 5.1 $\frac{-16}{3}v_{b\bar{b}}$ 5.1 $-\frac{16}{3}v_{b\bar{b}}$ -15.3	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \hline \\ 3C_{bb}\\ \hline \\ 7.7\\ -\frac{16}{3}C_{b\bar{b}}\\ \hline \\ 7.7\\ -\frac{16}{3}C_{b\bar{b}}\\ -15.5\\ \hline \\ 0.0 \end{array}$
$\begin{split} & \mu_{T_{b^{2}\bar{b}^{2}}(19327.} \\ & \mu_{T_{b^{2}\bar{b}^{2}}(19393.} \\ & J^{PC} = 1^{+-} \\ & \text{Mass}_{j} \\ & \text{Variational} \\ & \text{Parameters} \\ & (\text{fm}^{-2}) \\ & \text{Quark} \\ & \text{Confinement} \\ & \text{Kinetic} \\ & \text{CS Inter} \\ \\ & V^{C} \\ \\ & \text{Total Com} \\ & Tot$	$\begin{array}{c c} 9,2^{++}) \rightarrow T_{b21} \\ \hline 9,1^{+-}) \rightarrow T_{b22} \\ \hline \\ 9,1^{+-}) \rightarrow T_{b22} \\ \hline \\ Parameter \\ \hline \\ Parameter \\ Parameter \\ \hline \\ Parameter \\ Parameter \\ \hline \hline \hline \hline \hline \\ Parameter \\ \hline \hline \hline \hline \hline P$	$\begin{array}{c c} & & & \\ &$	$\begin{array}{l} _{0^{++})\gamma} = \\ \frac{\gamma \eta_{b}}{\gamma} = \\ \hline \gamma \eta_{b} \\ 18857.9 \\ 57.4 \\ 49.4 \\ - \\ 21372.0 \\ -3641.9 \\ 1171.6 \\ -43.8 \\ \hline \\ -43.8 \\ \hline \\ -548.1 \\ \alpha_{t} = F\rangle . \\ + 0.577 F \\ y \text{ channe} \\ 1^{+-})\gamma = \\ 0^{++})\gamma = \\ 1^{+-})\gamma = \\ 1^{+-}\gamma = \\ 1^{++}\gamma =$	$\begin{split} & \langle \Psi_{tot}^{+} \mu^{2} \Psi \\ \langle \Psi_{tot}^{+-} \mu^{2} \Psi \\ & \text{Difference} \\ \hline \\ & 446.0 \\ \hline \\ & 0.0 \\ \hline \\ & 638.4 \\ \hline \\ & -237.6 \\ \hline \\ & 45.1 \\ \hline \\ \hline \\ & 638.5 \\ \hline \\ & 446.0 \\ \hline \\ & R^{s} \rangle [\phi_{1} \chi_{2}] \rangle \\ \hline \\ & R^{s} \rangle [\phi_{1} \chi_{2}] \rangle \\ \hline \\ & R^{s} \rangle [\phi_{1} \chi_{2}] \rangle \\ \hline \\ & 1.0 \text{ keV} \\ \hline \\ & 2.8 \text{ keV} \\ \hline \\ & , 1+- \rangle = \langle \Psi_{tot}^{1} \mu^{2} \Psi \\ \langle \Psi_{tot}^{1+-} \mu^{2} \Psi \\ \langle \Psi_{tot}^{1+-} \mu^{2} \Psi \\ \end{split}$	$\begin{array}{c} \left \begin{array}{c} \left $	$ \begin{split} &= 0 \\ &= 0.352 \\ &:ive \ Ler \\ &Value \\ &0.203 \\ &0.217 \\ &0$	$\frac{4}{\sqrt{6}} \left(\mu_{b} - \frac{1}{\sqrt{6}} \right)\right)\right)$ $0.148(\eta_{b})$ $0.160(\Upsilon)$ 0.162 fm 0.130 fm $-879.1(\eta_{b})$ $-796.7(\Upsilon)$ $ R^{s}\rangle [\psi_{1}\zeta_{2}]\rangle$ $ R^{s}\rangle [\psi_{2}\zeta_{6}]\rangle$ $= 0$ $25\mu_{N}$ $72\mu_{N}$	$\mu_{\overline{b}} = -0.072$ b-quark: m_{b}^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$ CS Interaction	$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	Value 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 10686.0 335.6 131.4 -274.5 -244.2 -983.0 9653.3 6.7 9653.3 6.7 -12.2 1.3	12830.0 Ref. [110] $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4722.5 9487.3 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 4764.8 $\frac{1}{2}m_{b\bar{b}}$ 5.1 $\frac{8}{3}v_{b\bar{b}}$ 5.1 $-\frac{16}{3}v_{b\bar{b}}$ -15.3 -5.1 18606.4	$\begin{array}{c} 20275.0\\ \hline \\ Ref. [123]\\ \hline \\ 2m_b\\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ 10105.8\\ \hline \\ \hline \\ 3C_{b\bar{b}}\\ \hline \\ 7.7\\ \hline \\ \frac{8}{3}C_{b\bar{b}}\\ \hline \\ 7.7\\ \hline \\ -\frac{16}{3}C_{b\bar{b}}\\ -15.5\\ \hline \\ 0.0\\ \hline \\ 20211.6\\ \hline \end{array}$

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Fig. 1 Relative positions for four valence quarks and R_c in the $J^{PC} = 0^{++} b b \bar{b} \bar{b}$ ground state. Meanwhile, we label the relative distances of $R_{b,b}$, $R_{b,\bar{b}}$, $R_{\bar{b},\bar{b}}$, R', and the radius (units: fm)

hadrons. According to Table 9, we easily find that the relative distances of (1,2), (1,3), (1,4), (2,3), (2,4), and (3,4) quark pairs are all 0.227 or 0.204 fm. Meanwhile, the radius of the state is only 0.130 fm. Thus, in this state, all the distances between the quark pairs are roughly the same order of magnitude. If it is a molecular configuration, the distances between two quarks and two antiquarks should be much greater than the distances in the compact multiquark scheme, and the radius of molecular configuration can reach several femtometers. Therefore, our calculations are consistent with the compact tetraquark expectations.

4.2.2 The internal contribution

Let us now turn our discussion to the internal mass contribution for the $J^{PC} = 0^{++} b b \bar{b} \bar{b}$ ground state.

First, for the kinetic energy, this $bb\bar{b}\bar{b}$ state has 814.0 MeV, which can be understood as the sum of three internal kinetic energies: kinetic energies of two pairs of the $b - \bar{b}$, and the $(b\bar{b}) - (b\bar{b})$ pair. Accordingly, the sum of the internal kinetic energies of the $\eta_b \eta_b$ state only comes from the two pairs of the $b - \bar{b}$. Therefore, this *bbbb* state has an additional kinetic energy needed to bring the $\eta_b \eta_b$ into a compact configuration. The actual kinetic energies of two pairs of the $b - \bar{b}$ in the $J^{PC} = 0^{++} b b \bar{b} \bar{b}$ ground state are smaller than those in the $\eta_b \eta_b$ state. This is because, as can be seen in Table 9, the distance of $b - \bar{b}$ is larger in the tetraquark state than in the meson: the distance of $b - \bar{b}$ is 0.204 fm in this $bb\bar{b}\bar{b}$ state, while it is 0.148 fm in η_b . Meanwhile, we find that even if we consider the additional kinetic energy between the $(b\bar{b}) - (b\bar{b})$ pair, the total kinetic energy in this $b\bar{b}b\bar{b}$ state is still smaller than that in the $\eta_b \eta_b$ state. However, this does not lead the ground $J^{PC} = 0^{++} b b \bar{b} \bar{b}$ state to a stable state because of the confinement potential part.

As for the confinement potential part, the contributions from V^C for the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state in Table 9 are all attractive. Thus, this state has a large positive binding energy. However, it is still above the meson–meson threshold because the $V^C(b\bar{b})$ in η_b is very attractive. As for the other internal contributions, the quark contents of this state are the same as the corresponding rearrangement decay threshold. Moreover, the mass contribution from the hyperfine potential term is negligible compared to the contributions from other terms.

4.2.3 Comparison with two models of chromomagnetic interaction

Now, we compare the numerical values for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems between the constituent quark model and two CMI models [110,123] in Tables 7, 8, and 9. The comparisons of the values for $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states, which are in the last three columns of Tables 7, 8, and 9, can be summarized in the following important conclusions.

First, we find that there is no stable state below the lowest heavy quarkonium pair thresholds in any of the models. In all three models, we consider two possible color configurations, the color-sextet $|(QQ)^{3_c}(\bar{Q}\bar{Q})^{\bar{3}_c}\rangle$ and the color-triplet $|(QQ)^{6_c}(\bar{Q}\bar{Q})^{\bar{6}_c}\rangle$. According to the extended chromomagnetic model [110], the ground state is always dominated by the color-sextet configuration. This view is consistent with the specific wave function of the ground state in Eq. (22) given by the constituent quark model.

In contrast, the masses obtained from the constituent quark model are systematically larger than those from the extended CMI model [110] according to Tables 7, 8, and 9. Meanwhile, the masses obtained from the CMI model [123] are obviously larger than those of the constituent quark model. Their mass differences are mainly due to the effective quark masses as given in the last three columns of Tables 7, 8, and 9. The effective quark masses are the sum of the quark mass, the relevant kinetic term, and all the relevant interaction terms in the constituent quark model, which indeed seems to approximately reproduce the effective quark mass from two CMI models [110, 123]. We compare the subtotal values of the c and \bar{c} quark part in Table 8. The c effective quark mass in the constituent quark model is 3225 MeV, which is about 100 MeV larger than that of the extended CMI model in the $J^{PC} = 0^{++} cc\bar{c}\bar{c}$ state. Correspondingly, we also find that the c effective quark mass in the CMI model [123] is 3450 MeV, and about 200 MeV larger than that of the constituent quark model. The effective quark masses in the extended CMI model depend on the parameters of the traditional hadron. However, the effective quark masses should be different depending on whether they are inside a meson, a baryon, or a tetraquark. The effective quark masses trend to be large when they are inside configurations with larger constituents in the extended CMI model, as can be seen from the comparisons of the last three columns in Tables 7, 8, and 9. Moreover, we note that a similar situation occurs in the $ud\bar{c}\bar{c}$ state in Table X of Ref. [61].

4.3 $cc\bar{b}\bar{b}$ state

Here, we will concentrate on the $cc\bar{b}\bar{b}$ system. Similar to the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems, the $cc\bar{b}\bar{b}$ system is satisfied with fully antisymmetry for diquarks and antiquarks. There are two $J^P = 0^+$ states, one $J^P = 1^+$ state, and one $J^P = 2^+$ state in the $cc\bar{b}\bar{b}$ system. We show the masses of the ground states, the variational parameters, the internal mass contributions, the relative lengths between the quarks, their lowest meson–meson thresholds, the specific wave function, the magnetic moments, the transition magnetic moments, the radiative decay widths, and the rearrangement strong width ratios in Tables 11 and 12.

First, we take the $J^P = 0^+ cc\bar{b}\bar{b}$ ground state as an example to discuss its properties with the variational method. A similar situation occurs in the other two quantum numbers according to Tables 11 and 12. The mass of the lowest $J^P = 0^+ cc\bar{b}\bar{b}$ state is 12,920.0 MeV, and the corresponding binding energy B_T is +344.2 MeV according to Table 11. Thus, the state is obviously higher than the corresponding rearrangement meson-meson thresholds. The wave function is given by

$$|\Psi_{\text{tot}}\rangle = -0.966|F\rangle|R^{s}\rangle|[\phi_{1}\chi_{5}]\rangle + 0.259|F\rangle|R^{s}\rangle|[\phi_{2}\chi_{6}]\rangle.$$
(36)

Here, we see that the mass contribution of the ground state comes mainly from the $|(Q_1Q_2)_1^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_1^3\rangle_0$ component, and the $|(Q_1Q_2)_0^6(\bar{Q}_3\bar{Q}_4)_0^{\bar{6}}\rangle_0$ component is negligible. Its variational parameters are given as $C_{11} = 23.9$ fm⁻², $C_{22} = 10.5$ fm⁻², and $C_{33} = 12.3$ fm⁻².

The meson-meson configuration is connected to the diquark-antidiquark configuration by a linear transformation. Then, we obtain the total wave function in the mesonmeson configuration:

$$\begin{aligned} |\Psi_{\text{tot}}\rangle &= -0.589|F\rangle|R^{s}\rangle|[\psi_{1}\zeta_{5}]\rangle + 0.095|F\rangle|R^{s}\rangle|[\psi_{1}\zeta_{6}]\rangle \\ &+ 0.608|F\rangle|R^{s}\rangle|[\psi_{2}\zeta_{5}]\rangle + 0.524|F\rangle|R^{s}\rangle|[\psi_{2}\zeta_{6}]\rangle. \end{aligned}$$

$$(37)$$

According to Eq. (37), we are sure that the overlaps c_i of $B_c B_c$ and $B_c^* B_c^*$ are 0.095 and 0.589, respectively. Then, based on Eq. (21), the rearrangement strong width ratios is

$$\frac{\Gamma_{T_c2\bar{b}^2}(12920.0,0^+) \to B_c^* B_c^*}{\Gamma_{T_c2\bar{b}^2}(12920.0,0^+) \to B_c B_c} = 1:50,$$
(38)

i.e., $B_c B_c$ is the dominant rearrangement decay channel for the $T_{c^2\bar{h}^2}(12920.0, 0^+)$ state. As for the magnetic moment of the $J^P = 0^+ cc\bar{b}\bar{b}$ ground state, its value is 0, while the magnetic moment of all $J^P = 0^+$ tetraquark states is 0. As for the $J^P = 1^+ cc\bar{b}\bar{b}$ state, we construct its flavor \otimes spin wave functions as

$$\begin{split} |\Psi\rangle_{T_{c^{2}\bar{b}^{2}}(12939.9,1^{+})}^{S=1;S_{s}=1} \\ &= |R^{s}\rangle|\psi\rangle|cc\bar{b}\bar{b}\rangle|\frac{1}{2}(\uparrow\uparrow\uparrow\downarrow+\uparrow\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow\downarrow-\downarrow\uparrow\downarrow\uparrow). \end{split}$$

$$(39)$$

So the corresponding transition magnetic momentum is

$$\mu_{T_{c^2\bar{b}^2}(12939.9,1^+)} = \langle \Psi_{\text{tot}}^{1^+} | \hat{\mu^z} | \Psi_{\text{tot}}^{1^+} \rangle = \mu_c + \mu_{\bar{b}} = 0.490 \mu_N.$$
(40)

We also discuss the transition magnetic moment of the $T_{c^2\bar{b}^2}(12939.9, 1^+) \rightarrow T_{c^2\bar{b}^2}(12920.0, 0^+)\gamma$ process. We still construct their flavor \otimes spin wave functions as

$$\begin{split} |\Psi\rangle_{T_{c^{2}\bar{b}^{2}}(1^{+})}^{S=1;S_{s}=0} &= |R^{s}\rangle|\psi\rangle|cc\bar{b}\bar{b}\rangle|\frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow\downarrow-\downarrow\downarrow\uparrow\uparrow\rangle),\\ |\Psi\rangle_{T_{c^{2}\bar{b}^{2}}(0^{+})}^{S=0;S_{s}=0} &= |R^{s}\rangle|\psi\rangle|cc\bar{b}\bar{b}\rangle\\ &= |0.966\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow\downarrow+\downarrow\downarrow\uparrow\uparrow)+\cdots\rangle. \end{split}$$
(41)

And then the transition magnetic momentum of the $T_{c^2\bar{b}^2}(12939.9, 1^+) \rightarrow T_{c^2\bar{b}^2}(12920.0, 0^+)\gamma$ process can be described by the *z*-component of the magnetic moment operator $\hat{\mu}^z$ sandwiched by the flavor-spin wave functions of the $T_{c^2\bar{b}^2}(12939.9, 1^+)$ and $T_{c^2\bar{b}^2}(12920.0, 0^+)$. Thus the corresponding transition magnetic momentum is

$$\mu_{T_{c^2\bar{b}^2}(12939.9,1^+) \to T_{c^2\bar{b}^2}(12920.0,0^+)\gamma} = \langle \Psi_{\text{tot}}^{1^+} | \hat{\mu}^{\hat{z}} | \Psi_{\text{tot}}^{0^+} \rangle$$
$$= 0.966 \times \frac{1}{\sqrt{6}} (4\mu_c - 4\mu_{\bar{b}}) = 0.534\mu_N.$$
(42)

Further, according to Eqs. (19) and (42), we also obtain the radiative decay widths

$$\Gamma_{T_{c^2\bar{b}^2}(12960.9,2^+)\to T_{c^2\bar{b}^2}(12939.9,1^+)\gamma} = 3.6 \,\text{keV}.$$
(43)

Finally, we turn to the internal contribution for the $cc\bar{b}\bar{b}$ ground state. For the kinetic energy part, the $J^P = 0^+$ $cc\bar{b}\bar{b}$ state receives 835.9 MeV, which is smaller than that of the meson-meson threshold B_cB_c . The potential part of this state is much smaller than that of the lowest meson-meson threshold. Furthermore, we find that all the V^C for this state are attractive. However, compared to the V^C of B_cB_c , these attractive values seem trivial. This is because the length between $c - \bar{b}$ in tetraquarks is longer than that in B_c according to Tables 11 and 12. In summary, we tend to think that these $cc\bar{b}\bar{b}$ states are unstable compact states.

4.4 $cc\bar{c}\bar{b}$ and $bb\bar{b}\bar{c}$ states

Here, we discuss the $cc\bar{c}b$ and $bb\bar{b}c$ systems. For these two systems, they only need to satisfy the antisymmetry for the diquark. Thus, compared to the above three systems, the $cc\bar{c}b$ and $bb\bar{b}c$ systems have more allowed states. There are two $J^P = 0^+$ states, three $J^P = 1^+$ states, and one $J^P = 2^+$ state in the $cc\bar{c}b$ and $bb\bar{b}c$ systems. We calculate the masses of the ground states, the corresponding variational parameters, the various internal contributions, the relative lengths between the quarks, their lowest meson-meson thresholds, specific wave functions, magnetic moments, transition magnetic moments, radiative decay widths, and rearrangement strong width ratios in Tables 13, 14, and 15, respectively.

We now analyze the numerical results of the $J^P = 1^+$ ground $bb\bar{b}\bar{c}$ state obtained from the variational method according to Table 14. Other states would have similar discussions from Tables 13, 14, and 15. The mass of the lowest $J^P = 1^+ bb\bar{b}\bar{c}$ state is 16043.2 MeV, and the corresponding binding energy B_T is +303.7 MeV. Thus, the state is obviously above the lowest rearrangement meson-meson decay channel $B_c^* \eta_b$, and it is an unstable tetraquark state. Its variational parameters are given as $C_{11} = 12.4$ fm⁻², $C_{22} = 21.0$ fm⁻², and $C_{33} = 28.9$ fm⁻². The corresponding wave function is given by

$$\begin{split} |\Psi_{\text{tot}}\rangle &= 0.984 |F\rangle |R^s\rangle |[\phi_2\chi_4]\rangle + 0.171 |F\rangle |R^s\rangle |[\phi_1\chi_3]\rangle \\ &- 0.044 |F\rangle |R^s\rangle |[\phi_1\chi_2]\rangle. \end{split}$$
(44)

Here, we note that the mass contribution of the ground state comes mainly from the $|(Q_1Q_2)_0^6(\bar{Q}_3\bar{Q}_4)_1^{\bar{6}}\rangle_1$ component, and the other two components are negligible. Then we transform Eq. (44) into the meson–meson configuration via a linear transformation, and the corresponding wave function is given as

$$\begin{split} |\Psi_{\text{tot}}\rangle &= 0.494 |F\rangle |R^s\rangle |[\psi_1\zeta_2]\rangle - 0.396 |F\rangle |R^s\rangle |[\psi_1\zeta_3]\rangle \\ &- 0.487 |F\rangle |R^s\rangle |[\psi_1\zeta_4]\rangle + 0.111 |F\rangle |R^s\rangle |[\psi_2\zeta_2]\rangle \\ &- 0.246 |F\rangle |R^s\rangle |[\psi_2\zeta_3]\rangle - 0.537 |F\rangle |R^s\rangle |[\psi_2\zeta_4]\rangle. \end{split}$$

$$(45)$$

Furthermore, we can be sure that its rearrangement strong width ratios are

$$\Gamma_{T_{b}2\bar{b}\bar{c}}^{(16043,2,1^{+})\to B_{c}^{*}\Upsilon}:\Gamma_{T_{b}2\bar{b}\bar{c}}^{(16043,2,1^{+})\to B_{c}^{*}\Upsilon}:$$

$$\Gamma_{T_{b}2\bar{b}\bar{c}}^{(16043,2,1^{+})\to B_{c}^{*}\eta_{b}} = 1:1.3:1.5.$$

$$(46)$$

And its radiative decay widths are

$$\Gamma_{T_{b^{2}\bar{b}\bar{c}}(16149.2,2^{+})\to T_{c^{2}\bar{c}\bar{b}}(16043.2,1^{+})\gamma} = 435.0 \text{ keV},$$

$$\Gamma_{T_{c^{2}\bar{c}\bar{b}}(16043.9,0^{+})\to T_{c^{2}\bar{c}\bar{b}}(16043.2,1^{+})\gamma} = 10^{-6} \text{ keV}.$$

Let us now focus on the internal contributions for this state and the relative lengths between the quarks. For the kinetic energy part, the state obtains 876.1 MeV, which is obviously smaller than that of the lowest meson–meson threshold $B_c \eta_b$. The actual kinetic energy of the $b - \bar{b} (b - \bar{c})$ in the $J^P = 1^+ bb\bar{b}\bar{c}$ state is smaller than that in the $\eta_b (B_c^*)$ meson. The reason for this can be seen in Table 14. The size of this pair is larger in the $J^P = 1^+ bb\bar{b}\bar{c}$ state than in the meson: the distance (3,4) is 0.245 fm in this tetraquark, while it is 0.148 fm in η_b .

Here, let us turn our discussion to the potential parts. The potential part of this state is much smaller than that of its lowest meson–meson threshold. Although the V^C between quark and antiquark are attractive, the V^C in the diquark and antiquark are repulsive. However, relative to the η_b and B_c mesons, the V^C in the tetraquark are less attractive. Therefore, they still have relatively large positive binding energy in this state.

4.5 $cb\bar{c}\bar{b}$ state

Finally, we investigate the $cb\bar{c}\bar{b}$ system. Similar to the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems, the $cb\bar{c}\bar{b}$ system is a pure neutral system and has a certain *C*-parity. Thus the corresponding magnetic moment is $0\mu_N$ for all the ground $cb\bar{c}\bar{b}$ states. Moreover, the Pauli principle does not impose any constraints on the wave functions of the $cb\bar{c}\bar{b}$ system. Thus, compared to other tetraquark systems discussed above, the $cb\bar{c}\bar{b}$ system has more allowed states. There are four $J^{PC} = 0^{++}$ states, four $J^{PC} = 1^{+-}$ states, two $J^{PC} = 1^{++}$ states, and two $J^{PC} = 2^{++}$ states in the $cb\bar{c}\bar{b}$ system.

We now analyze the numerical results for the $cb\bar{c}\bar{b}$ system obtained from the variational method. Here, we take the $J^{PC} = 0^{++} cb\bar{c}\bar{b}$ ground state as an example for discussion, and others would have similar discussions. The mass of the lowest $J^{PC} = 0^{++} cb\bar{c}\bar{b}$ state is 12,759.3 MeV, and the corresponding binding energy B_T is +371.8 MeV. Thus, the state obviously has a larger mass than the lowest rearrangement meson-meson decay channel $\eta_b\eta_c$, and it should be an unstable compact tetraquark state. Its variational parameters are given as $C_{11} = 11.9 \text{ fm}^{-2}$, $C_{22} = 11.9 \text{ fm}^{-2}$, and $C_{33} = 22.9 \text{ fm}^{-2}$. Since this state is a pure neutral state, we naturally find that the value of C_{11} is equal to C_{22} , which means that the distance of (b - b) is equal to $(\bar{b} - \bar{b})$. Our results also reflect these properties according to Table 16. The corresponding wave function is given as

$$\Psi_{\text{tot}} = 0.961|F\rangle|R^{s}\rangle|[\phi_{2}\chi_{5}]\rangle + 0.114|F\rangle|R^{s}\rangle|[\phi_{2}\chi_{6}]\rangle - 0.069|F\rangle|R^{s}\rangle|[\phi_{1}\chi_{5}]\rangle - 0.241|F\rangle|R^{s}\rangle|[\phi_{1}\chi_{6}]\rangle.$$

$$(47)$$

Based on Eq. (47), we find that its mass contribution to the ground state comes mainly from the $6 \otimes \overline{6}$ component, the corresponding $3 \otimes \overline{3}$ component being negligible. Then we transform Eq. (47) into $c\overline{c} - b\overline{b}$ and $c\overline{b} - b\overline{c}$ configurations via a linear transformation, and the corresponding two wave functions are given as

Further, we can ascertain its rearrangement strong width ratios. For the $c\bar{b} - b\bar{c}$ decay mode

$$\Gamma_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to B_{c}^{*}\bar{B}_{c}^{*}} : \Gamma_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to B_{c}^{*}\bar{B}_{c}} : \\ \Gamma_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to B_{c}\bar{B}_{c}^{*}} = 1:3.9:3.9,$$

$$(49)$$

where both the $B_c^* \bar{B}_c$ and $B_c \bar{B}_c^*$ channels are the dominant decay modes for the $T_{cb\bar{c}\bar{b}}(12796.9, 1^{+-})$ tetraquark state.

$$\Gamma_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to \eta_b J/\psi} : \Gamma_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to \Upsilon\eta_c}$$

$$= 1:18.4.$$
(50)

The dominant decay channel is the $\Upsilon \eta_c$ final states in the $c\bar{c} - b\bar{b}$ decay mode.

We also calculate the transition magnetic moments for this state:

$$\begin{split} &\mu_{T_{cb\bar{c}\bar{b}}(12882.4,2^{++}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})\gamma} = 0, \\ &\mu_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})\gamma} = 0, \\ &\mu_{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})\gamma} = 0.081, \end{split}$$
(51)

which are in units of μ_N . Furthermore, we can obtain its radiative decay widths as

$$\begin{split} & \Gamma_{T_{cb\bar{c}\bar{b}}(12882.4,2^{++}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})_{\gamma}} = 0, \\ & \Gamma_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})_{\gamma}} = 0, \\ & \Gamma_{T_{cb\bar{c}\bar{b}}(12797.3,1^{+-}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})_{\gamma}} = 33.1, \end{split}$$

which are in units of keV.

Let us now turn our discussion to the internal contribution for the $J^{PC} = 1^{+-} cb\bar{c}\bar{b}$ ground state. For the kinetic energy part, the state obtains 858.5 MeV, which is smaller than the 1001.2 MeV of the lowest meson-meson threshold $B_c\eta_b$ according to Table 16. As for the potential part, although the V^C between quark and antiquark are attractive, the V^C in the diquark and antiquark are repulsive. However, relative to the lowest meson-meson threshold $B_c\eta_b$, the total V^C is less attractive than the $B_c\eta_b$, which leads to this state having a relatively larger mass.

We also note that the $V^{C}(1, 3)$, $V^{C}(2, 3)$, $V^{C}(1, 4)$, and $V^{C}(2, 4)$ are absolutely the same, and meanwhile the distances of (1,3), (1,4), (2,3), and (2,4) are also the same. These actually reflect $\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}_{3,4}) | \Psi_{\text{tot}} \rangle = \langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle = \langle \Psi_{\text{tot}} | (\mathbf{R}_{3,4} \cdot \mathbf{R}') | \Psi_{\text{tot}} \rangle = 0$. Obviously, it is unreasonable that the distance of $c\bar{c}$ is exactly the same as that of the $c\bar{b}$ and $b\bar{b}$. According to Sec IV of Ref. [65], it is not sufficient to consider only the single Gaussian form where the spatial part of the wave function is $l_1 = l_2 = l_3 = 0$ in the spatial part of the total wave function is not sufficient. These lead to the $cb\bar{c}\bar{b}$ state, which is far away from the real structures in nature. We have reason enough to believe that the $\langle \Psi_{\text{tot}} | (\mathbf{R}_{1,2} \cdot \mathbf{R}_{3,4}) | \Psi_{\text{tot}} \rangle$ should not be zero. Meanwhile, considering other spatial basis would reduce the corresponding binding energy B_T [65]. But these corrections would be powerless against the higher binding energy B_T of the ground $J^{PC} = 1^{+-} cb\bar{c}\bar{b}$. In conclusion, we tend to think that the $J^{PC} = 1^{+-} cb\bar{c}\bar{b}$ ground state should be an unstable compact state.

5 Comparison with other work

Mass spectra have been studied with different approaches such as different nonrelativistic constituent quark models, different chromomagnetic models, relativistic quark models, nonrelativistic chiral quark model, diquark models, the diffusion Monte Carlo calculation, and the QCD sum rule. In addition, these fully heavy tetraquark systems have been discussed with different color structures such as the $8_{Q\bar{Q}} \otimes 8_{Q\bar{Q}}$ configuration, the diquark–antiquark configuration ($3 \otimes \bar{3}$ and the $6 \otimes \bar{6}$), and the couplings between the above color configurations. For comparison, we briefly list our results and other theoretical results in Table 10.

Compared to other systems, the most extensive discussion is found for the $cc\bar{c}\bar{c}$ system. Thus, we will concentrate on the $cc\bar{c}\bar{c}$ system, but other systems can be discussed in a similar way. After comparing our results with those of other studies, we can see that most theoretical masses of $cc\bar{c}\bar{c}$ in ground states lie in a wide range of 6.0-6.8 GeV in Table 10. Our results are 6.38, 6.45, and 6.48 GeV for the 0^{++} , 1^{+-} , and 2^{++} cccc ground states, respectively. These three ground states are expected to be broad because they can all decay to charmonium pairs $\eta_c \eta_c$, $\eta_c J/\psi$, or $J/\psi J/\psi$ through the quark (antiquark) rearrangements. Therefore, these types of decays are favored both dynamically and kinematically. According to Table 10, we can conclude that the obtained masses of the ground states are obviously smaller than the X(6900) observed by the LHCb Collaboration. The observed X(6900) is less likely to be the ground compact tetraquark state and could be a first or second radial excited $cc\bar{c}\bar{c}$ state.

Although we all use a similar Hamiltonian expression as in the nonrelativistic constituent quark model [111, 141–143], the spatial wave function is mostly expanded in the Gaussian basis according to Ref. [144], while we treat the spatial function as a Gaussian function, which is convenient for use in further variational methods to handle calculations in the fourbody problem. Our results for the $cc\bar{c}\bar{c}$ system are roughly compatible with other nonrelativistic constituent quark models, although different papers have chosen different potential forms.

Table 10 Comparison of the results of different methods for the $QQ\bar{Q}\bar{Q}$ tetraquark states

<u> </u>								1.1	77				77	
Systems $T^{P(C)}$			0	c	$c\overline{c}\overline{c}$ 1+-	o++	of	bt ⊦+	1+-	o++		+	:bb 1+	o+
<u> </u>	Our res	ault	6384	6512	6452	6483	19352	19240	19304	4	12920	13008	12940	12961
	-041 105	un	6377	6425	6425	6432	19215	19247	19247	19249	12847	12866	12864	12868
The second stimistic	Ref. [11	11]	6371	6483	6450	6479	19243	19305	19311	19325	12886	12946	12924	12940
constituent	Ref. [14	42]	6487	6518	6500	6524	19332	19338	19329	19341	12947	13039	12960	12972
quark models	Ref. [14	43]	6500	6411	6453	6475	19200	19235	19216	19225	12880	12981	12890	12902
	Ref. [78	31	6477	6695	6528	6573	-	-	_	-	-	-	-	-
		,	64	91	6580	6607	193	357	19413	19429	129	963	13024	13041
Multiquark color flux-tube model	Ref. [14	41]	64	07	6463	6486	193	329	19373	19387	129	906	12946	12960
			60	35	6139	6194	188	834	18890	18921	125	597	12660	12695
	Ref. [79	9]	61	92	-	-	188	826	-	-		-	-	-
The chromo-	D.f. [10	201	6899	7016	6899	6956	20155	20275	20212	20243	13496	13634	13560	13595
magnetic model	Ref. [12	23]	6035	6253	6137	6194	18834	18954	18890	18921	12597	12734	12660	12695
	Ref. [11	10]	6045	6271	6231	6287	18836	18981	18969	19000	12596	12712	12672	12703
	Ref. [10	07]	6034	6254	6137	6194	18834	18953	18890	18921	-	-	-	-
The Bethe- Salpeter equations	Ref.[150	D]	64	19	6456	6516	192	205	19221	19253	-	-	-	-
The relativistic	Ref. [10	[00	61	90	6271	6367	193	314	19320	19330	128	846	12859	12883
quark model	Ref. [94	4]	6435	6542	6515	6543	19201	19255	19251	19262	-	-	-	-
Monte Carlo metho	d Ref. [14	46]	63	51	6441	6471	193	199	19276	19289	128	865	12908	12926
The diquark mode	Ref. [14	19]	59	66	6051	6223	187	754	18808	18916	-	-	-	-
	Ref. [14	47,151]	63	22	6354	6385	196	366	19673	19680	124	401	12409	12427
The QCD sum	Ref. [14	45]	6360	6540	6470	6520	18130	18150	18140	18150	-	-	-	-
rule method	Ref. [15	52, 153]	59	90	6050	6090	188	840	18840	18850	-	-	-	-
The nonrelativistic chiral quark model	Ref. [14	48,154]	65	10	6600	6708		-	-	-	126	584	12737	12791
An effective potential model	Ref. [91	1]	6346	6476	6441	6475	19154	19226	19214	19232	-	-	-	-
Systems			$cb\overline{c}\overline{b}$					cc	:cb			bt	bē	
$J^{P(C)}$	0	++	1^{+}		1^{++}	2^{++}	0+	1	+	2^{+}	0+	1	+	2^{+}
O	12760	12851	12797	12856	12857	128824	9621	96246	9706	9731	16044	16043	16125	16149
Our result	12989	13008	12999	13056	12960	12971	9766	9729			16163	16144		
Dof [149]	12783	12850	12802	12835	12851	12852	9665	9676	9699	9713	16061	16046	16079	16089
Itel. [145]	12966	13035	12949	12964	12938	12964	9732	9718			16100	16089		
Rof [142]	12835	12864	12852	12864	12870	12864	9740	9746	9749	9768	16158	16157	16164	16176
Itel. [142]	12864	13050	13047	13052	13056	13070	9763	9757			16158	16167		
	128	394		12955		13000	9735	97	66	9839	16175	16	179	16274
Ref. [141]	128	329		12881		12925	9670	96	83	9732	16126	16	130	16182
	123	354		12436		12548	9705	97	05	9732	15713	15'	729	15806
Ref. [110]	12363	12509	12425	12477	12524	12537	9318	9335	9384	9526	15712	15719	15851	15882
	12682	12747	12720	12744	12703	12755	9506	9499			15862	15854		
	13396	13634	13478	13592	13510	13590	10144	10174	10231	10273	16832	16840	16884	16917
	13483	13553	13520	13555	13599	13599	10322	10282			16952	16915		
Ref. [123]	12354	12592	12436	12550	12468	12548	9313	9343	9400	9442	15713	15729	15773	15806
	12441	12511	12478	12513	12557	12557	9491	9451			15841	15804		
	12578	12620	12496	12583	12611	12690								
	12656	12693	12653	12735	12700	12700								
Ref. [100]	12813	12824	12826	12831	12831	12849	9572	9602	9619	9647	16109	16117	16117	16132
Ref. [146]	125	534	125	510	12569	12582	9615	96	10	9719	16040	160	013	16129
Ref. [149]	12359	12471	12424	12488	12485	12566	-	-	-	-	-	-	-	-
Ref. [154]		-	-	-	-	-	95	79	9590	9613	160	J60	16062	16068

It is also interesting to note that relatively larger results are also given by the QCD sum rules [145], the Monte Carlo method [146], the diquark model [147], and the chiral quark model [148]. However, the results given by the QCD sum rules [145] are about 1 GeV below those of the constituent quark models for the $bb\bar{b}\bar{b}$ system. In contrast, our results are obviously larger than the chromomagnetic models [79,107,110,123] and the diquark models [100,149], where these models usually neglect the kinematic term and explicitly include confining potential contributions or adopt a diquark picture.

6 Summary

The discovery of exotic structures in the di- J/ψ invariant mass spectrum from the LHCb, CMS, and ATLAS collaborations gives us strong confidence to investigate the fully heavy tetraquark system. Thus, we use the variational method to systematically calculate the masses of all possible configurations for fully heavy tetraquarks within the framework of the constituent quark model. Meanwhile, we also give the corresponding internal mass contributions, the relative lengths between (anti)quarks, their lowest meson–meson thresholds, the specific wave function, magnetic moments, transition magnetic moments, the radiative decay widths, rearrangement strong width ratios, and comparisons with the two different CMI models.

To obtain the above results, we need to construct the total wave functions of the tetraquark states, including the flavor part, color part, spin part, and spatial part, which is chosen to be a simple Gaussian form. Here, we first estimate the theoretical values of traditional hadrons, which are used to compare the experimental values to prove the reliability of this model. Before discussing the numerical analysis, we analyze the stability condition using only the color-spin interaction. Then, we obtain the specific numerical values and show them in corresponding tables and the spatial distribution of valence quarks for the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state in Fig. 1.

For the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems, there are two pure neutral systems with definite *C*-parity. There are only two $J^{PC} = 0^{++}$ states, one $J^{PC} = 1^{+-}$ state, and one $J^{PC} = 2^{++}$ state, due to the Pauli principle. We also find that these states with different quantum numbers are all above the lowest thresholds, and have larger masses. Since these states are pure neutral particles, the corresponding magnetic moments are all 0 for the ground $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states. Meanwhile, of course, the variational parameters C_{11} and C_{22} are the same, so the distances of the diquark and antidiquark are also the same. Moreover, the distances between quark and antiquark are all the same according to the symmetry analysis of Eqs. (31–32). Furthermore, three Jacobi coordinates are orthogonal to each other according to Eqs. (33–35). Based on this, we take the

 $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state as an example to show the spatial distribution of four valence quarks. As for the internal contribution, although the kinetic energy part is smaller than that of the $\eta_b\eta_b$ state, the V^C in η_b is much more attractive relative to the $J^{PC} = 0^{++} bb\bar{b}\bar{b}$ ground state, which is the main reason that this state has a larger mass than the meson-meson threshold. Similar situations occur in other systems.

Similar to the $cc\bar{c}\bar{c}$ and *bbbb* systems, the *ccbb* system has the same number of allowed ground states. According to the specific function, their mass contribution comes mainly from the $\bar{3} \otimes 3$ component within the diquark–antiquark configuration. Furthermore, we obtain the relevant values of the magnetic moments, the transition magnetic moments, and the radiative decay widths. We also obtain the rearrangement strong width ratios within the meson–meson configuration.

As for the $cc\bar{c}b$ and $bb\bar{b}c$ systems, there are more allowed states due to fewer symmetry restrictions. Considering only the hyperfine potential, we can expect to have a compact stable state for the $J^P = 1^+ bb\bar{b}c$ configuration. However, since the V^C of the tetraquark are less attractive than the corresponding mesons, this state still has a mass larger than the meson-meson threshold.

In the $cb\bar{c}b$ system, these states are also pure neutral particles, and we naturally obtain that their variational parameters C_{11} and C_{22} are the same. There is no constraint from the Pauli principle, so there are four $J^{PC} = 0^{++}$ states, four $J^{PC} = 1^{+-}$ states, two $J^{PC} = 1^{++}$ states, and two $J^{PC} = 2^{++}$ states. All of the $cb\bar{c}\bar{b}$ states have larger masses relative to the lowest thresholds. Moreover, they all have two different rearrangement strong decay modes: $c\bar{c} - b\bar{b}$ and $c\bar{b} - b\bar{c}$.

Then we compare our results with other theoretical work. Our results are roughly compatible with other nonrelativistic constituent quark models, although different papers have chosen different potential forms. Meanwhile, it is also interesting to find that similar mass ranges are given by the QCD sum rules, the Monte Carlo method, and the chiral quark model. This shows that our results are quite reasonable.

In summary, our theoretical calculations show that the masses of the $cc\bar{c}\bar{c}$ ground states are around 6.45 GeV, which is obviously lower than 6.9 GeV. Thus, the experimentally observed X (6900) state does not seem to be a ground $cc\bar{c}\bar{c}$ tetraquark state, but could be a radially or orbitally excited state. We also find that these lowest states all have a large positive binding energy B_T . In other words, all these states are found to have masses greater than the corresponding two meson decay thresholds via the quark rearrangement. Hence, we conclude that there is no compact bound fully heavy tetraquark ground state which is stable against the strong decay into two mesons within the constituent quark model. Finally, we hope that more relevant experimental analyses will be able to focus on this system in the near future.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data.]

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Appendix

In this appendix, we show the masses, binding energies, variational parameters, internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width ratios, and the relative lengths between the quarks for the $cc\bar{b}\bar{b}$, $cc\bar{c}\bar{b}$, $bb\bar{b}\bar{c}$, and $cb\bar{c}\bar{b}$ states with different $J^{P(C)}$ quantum numbers and their lowest meson-meson thresholds (Tables 11, 12, 13, 14, 15, 16, 17).
 Table 11
 The masses, binding energies, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong

width ratios, and the relative lengths between quarks for the $J^P = 0^+$, $1^+ cc\bar{b}\bar{b}$ states and their lowest meson–meson thresholds. The notation is the same as that of Table 7

$cc\overline{b}\overline{b}$	The o	ontribut	ion from	each term	Relative Le	engths (fm)	Overall	Present Work		CMI	Model
$J^{P} = 0^{+}$		Value	$B_c B_c$	Difference	(i, j) Value	$B_c B_c$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	12920.0	12575.8	344.2	(1,2) 0.348			$2m_c$	3836.0		
Variational	C_{11}	23.9	22.9		(1.3) 0.308	$0.235(B_c)$		$\frac{\mathbf{p}_{x_1}^2}{2m'}$	319.0		
Parameters (fm^{-2})	C_{22} C_{22}	10.5 12.3	22.9		(2.3) 0.308			$m_{\overline{b}}^{2m_1} \mathbf{p}_{x_3}^2$	188.0		
	Maga	14599.0	14522.0	0.0	(2,3) 0.300		<i>c</i> -quark:	$m_c + m_{\overline{b}} 2m'_3$ $V^C(12)$	-46.9	1	
Confinamon	Mass t Potontial	2420 1	2705.5	275.4	(1,4) 0.308	0.995(D)	m^{eff}	V (12) 1[VC(12) + VC(14)	-40.5	2 ¹¹¹ cc	
Kinotio	Enongra	-2420.1 825.0	-2195.5	111.4	(2,4) 0.508	$0.235(D_c)$	m _c	$\frac{1}{2}[V(13) + V(14)]$	-95.0	1080.8	0
COL	Energy	035.9	947.5	-111.4	(3,4) 0.230			$+V^{-}(23) + V^{-}(24)$		$\frac{1}{m_{\overline{b}}+m_c}m_{c\overline{b}}$	$2m_c$
CS Inte	raction	-7.0	-98.0	91.0	(1,2)- $(3,4)$:	0.226 fm		-D	-983.0	1578.7	3449.6
	(1,2)	-46.9			Radius:	0.151 fm		Subtotal	3218.1	3164.5	3449.6
V^C	(2,3)	-47.5						$2m_b$	10686.0		
	(1,4)	-47.5			(1,3) -47.5	$-414.8(B_c)$		$\frac{P_{x_2}}{2m'_2}$	216.4		
	Subtotal	-454.0	-829.5	375.3	(3,4) -217.1		T. en enler	$\frac{m_c}{m_c + m_{\overline{b}}} \frac{\mathbf{P}_{x_3}^2}{2m_2'}$	67.5		
Total Con	tribution	-374.9	19.8	355.1	(2,4) -47.5	$-414.8(B_c)$	<i>o</i> -quark:	$V^{C}(34)$	-217.1	$\frac{1}{2}m_{bb}$	
Total Wave	function:						$m_{\overline{b}}^{eff}$	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	05.0	4764.8	
$\Psi_{tot} = 0.25$	$9 F\rangle R^s\rangle [\phi$	$\langle p_2 \chi_6] \rangle = 0$	$0.966 F\rangle I$	$ \langle R^s \rangle [\phi_1 \chi_5] \rangle$	$ = -0.589 F\rangle $	$ R^s\rangle [\psi_1\zeta_5]\rangle$		$+V^{C}(23)+V^{C}(24)$]	-95.0	$\frac{m_{\overline{b}}}{m_{\overline{b}}+m_{c\overline{b}}}m_{c\overline{b}}$	$2m_b$
+0.	$.095 F\rangle R^s\rangle$	$ [\psi_1 \zeta_6]\rangle$ -	+ 0.608 F	$ R^s\rangle [\psi_2\zeta]$	$(5) + 0.524 F\rangle$	$ R^s\rangle [\psi_2\zeta_6]\rangle$		-D	-983.0	4743.6	10105.8
The rearran	gement str	ong widt	h ratios :	71 - 710 - 43	-1/ 1 /	1 - 7117 2 3017		Subtotal	9674.8	9508.4	10105.8
Γπ (10000	0	 	(10000 0 0	0++) . D .	$= 1 \cdot 50$			$\frac{1}{2}V^{SS}(12)$	9.6	$\frac{8}{2}v_{aa} + \frac{8}{2}v_{bb}$	$\frac{8}{2}C_{aa} + \frac{8}{2}C_{bb}$
The radiativ	$(0,0++) \rightarrow B_c^2 I$ we decay wi	$\frac{S_c^2 + I_c^2 \overline{b}}{dths}$	2(12920.0.0	$(,0^{++}) \rightarrow B_c B$	c 1100			$\frac{1}{2}V^{SS}(34)$	5.5	95+51	141+77
Γ	ie deedy in	cromb .	-0k	v ·			CS	$\frac{2}{1}$ (USS(12) + USS(14)	0.0	$-\frac{32}{2}u =$	$-\frac{32}{C}$
$\Gamma T_{c^2 \overline{b}^2}(12960.$	$9,2^+) \rightarrow T_{c^2 \overline{b}^2}$	(12920.0,0-	$\gamma = 0 M$	leaV			Interaction	$V^{SS}(23) + V^{SS}(24)$	-22.1	3 ⁰ cb	3 0 cb
$\frac{1}{T_{c^2 \overline{b}^2}(12939)}$	$9,1^+) \rightarrow T_{c^2\overline{b}^2}$	(12920.0,0	$F_{\gamma} = 5.8$	/II($(1+1)^{+}$	0		Subtotal	-7.0	-31.5	12.2
The magnet	ic moments	μ_{T_c}	$2\overline{b}2(12920.0)$	$(0,0^+) = \{\Psi_t\}$	$ \Psi^* \Psi_{tot} =$	0	Matrix non	diagonal element	-1.0	-10.5	-13.3
ine transiti	on magnet.	ic momer	us. /Ju	2^{+} 1.3 10^{+}			Total contr	ibution	-10.7	-09.7	40.1
$\mu_{T_{c^2\overline{b}^2}(12960.}$	$9,2^+) \rightarrow T_c^2 \overline{b}^2$	(19240.0,0-	$F_{\gamma} = \langle \Psi_t \rangle$	$ \mu^{\sim} \Psi_{tot} $	a = 0			Ibution	12920.0	12596.3	13496.0
$\mu_{T_{c^2 \overline{b}^2}(12939.}$	$9,1^+) \rightarrow T_{c^2 \overline{b}^2}$	(12920.0,0	$\psi_{\gamma} \equiv \langle \Psi_{i} \rangle$	$ \mu^* \Psi_{tot} $	$\rangle = 0.966 \times \frac{1}{\sqrt{2}}$	$\overline{\overline{6}}(\mu_c - \mu_{\overline{b}}) =$	$0.534 \mu_N$				
$J^{r} = 1^{+}$		Value	$B_c^*B_c$	Difference	Relative Le	ngths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
Mass	B_T	12939.9	12638.4	301.5	(i, j) Value	$B_c^*B_c$		$2m_c$	3836.0		
Parameters	C_{11} C_{22}	24.8 10.3	20.2 22.9		(1,2) 0.351			2	$\frac{P_{x_1}}{2m'_1}$	312.7	
$({\rm fm}^{-2})$	C_{33}	11.1	-		(1,3) 0.317	$0.250(B_c^*)$	<i>c</i> -quark:	$\frac{m_{\overline{b}}}{m_c + m_{\overline{b}}} \frac{\mathbf{P}_{x_3}^z}{2m'_3}$	169.1	$\frac{1}{2}m_{cc}$	
Quark	Mass	14522.0	14522.0	0.0	(2,3) 0.317		meff	$V^{C}(12)$	-43.5	1585.8	
Confinemen	t Potential	-2400.7	-2741.1	340.4	(1,4) 0.317		m_c	$\frac{1}{2}[V^{C}(13) + V^{C}(14) + V^{C}(23) + V^{C}(24)]$	-83.2	$\frac{m_c}{m_{\overline{r}}+m_c}m_{c\overline{b}}$	$2m_c$
Kinetic	Energy	814.0	891.5	-77.5	(2,4) 0.317	$0.235(B_c)$		-D	-983.0	1587.7	3449.6
CS Inter	raction	4.6	-34.0	38.6	(3,4) 0.226			Subtotal	3208.1	3164.5	3449.6
	(1,2)	-43.5			(1,2)- $(3,4)$:	0.238 fm		$2m_b$	10686.0		
	(2,3)	-41.6			Radius:	0.157 fm		$\frac{\mathbf{p}_{x_2}^2}{\mathbf{p}_{x_2}}$	271.5		
$V^{\mathbb{C}}$	(1.4)	41.6			(1.2) 41.6	260 4(D *)		$m_{c}^{2m'_{2}} \mathbf{p}_{x_{3}}^{2}$	60.7	1	
	(1,4) Subtotal	-41.0	000 F	975.9	(1,3) = 41.0	$-300.4(D_c)$	\overline{b} -quark:	$\frac{\overline{m_c + m_{\bar{b}}}}{V^C(34)}$	-225.5	2711bb	
	Subtotal	-454.0	-829.5	375.3	(3,4) -225.5	414 Q(D)	meff	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$		4704.8 m .	2
Total Con	tribution	383.9	82.4	301.5	(2,4) -41.6	$-414.8(B_c)$	$m_{\overline{b}}$	$+V^{C}(23) + V^{C}(24)]$	-83.2	$\frac{b}{m_{\overline{b}}+m_c}m_{c\overline{b}}$	$2m_b$
Total Wave	function:	$\Psi_{tot} =$	$ F\rangle R^{*}\rangle [$	$\phi_1 \chi_2] \rangle =$				- <i>D</i>	-983.0	4743.6	10105.8
-0.	$419 F\rangle R^s\rangle$	$ [\psi_1\zeta_2]\rangle$ -	- 0.393 F	$\langle R^s\rangle [\psi_1\zeta]$	$ \rangle = 0.066 F\rangle$	$ R^s\rangle [\psi_1\zeta_4]\rangle$		Subtotal	9726.5	9508.4	10105.8
+0.	$.587 F\rangle R^s\rangle$	$ [\psi_2\zeta_2]\rangle$ -	+ 0.557 F	$ \langle R^s \rangle \psi_2 \zeta$	$ \rangle + 0.105 F\rangle$	$ R^s\rangle [\psi_2\zeta_4]\rangle$		$\frac{1}{2}V^{33}(12)$	5.7	$\frac{8}{3}v_{cc} + \frac{8}{3}v_{\overline{b}\overline{b}}$	$\frac{8}{3}C_{cc} + \frac{8}{3}C_{\overline{b}\overline{b}}$
The rearran	gement str	ong deca	y channel	$: B^*B_c$			CS	$\frac{1}{2}V^{55}(34)$	4.6	9.5 + 5.1	14.1 + 7.7
The radiativ	ve decay wi	dths: $\Gamma_{T_{q}}$	$2\overline{b}^{2}$ (12960.	$9,2^+) \rightarrow T_{c^2 \overline{b}}$	$_{2}(12939.9.9,1^{+})\gamma$	$= 3.6 \mathrm{keV}$	Interaction	$-\frac{1}{4}(V^{SS}(13) + V^{SS}(14))$	-10.6	$-\frac{16}{3}v_{\overline{c}\overline{b}}$	$-\frac{16}{3}C_{\overline{c}\overline{b}}$
			$\Gamma_{T_{c^2 \overline{b}^2}(129)}$	$(339.9,1^+) \rightarrow T$	c2b2(12920.0,0+)	$\gamma = 3.8 \mathrm{keV}$		$+V^{55}(23)+V^{55}(24)$		-15.7	-17.6
The magnet	ic moments	s:						Subtotal	4.6	-1.2	4.3
$\mu_{T_{c^2\bar{b}^2}(12939.9,1^+)} = \langle \Psi_{tot}^{1^+} \hat{\mu^z} \Psi_{tot}^{1^+} \rangle = \mu_c + \mu_{\bar{b}} = 0.490 \mu_N $							Total contribution 12939.9 12671.6 13560.0				
The transiti	on magnet	ic momer	nts: $\mu_{T_b^2 \overline{b}}$	$_{2}(12960.9, 2^{+})$	$) \rightarrow T_{b^2 \overline{b}^2} (12939.5)$	$_{9,1^+)\gamma} = \langle \Psi_{ta}^2 \rangle$	$\hat{\mu}_{t}^{z} \Psi_{tot}^{1^{+}}\rangle =$	$\mu_c - \mu_{\overline{b}} = 0.342 \mu_N$			
			$\mu_{T_{o2\overline{b}2}}$	(12939.9,1+)	$\rightarrow T_{a2\overline{b}2}$ (12920.0,	$_{(0^+)\gamma} = \langle \Psi_{tot}^{1^+} \rangle$	$ \hat{\mu^{z}} \Psi_{tot}^{0^{+}}\rangle = 0$	$0.966 \times \frac{4}{\sqrt{6}} (\mu_c - \mu_{\bar{b}}) = 0.$	$534\mu_N$		

Table 12The masses, binding energies, variational parameters, theinternal contribution, total wave functions, magnetic moments, transi-tion magnetic moments, radiative decay widths, rearrangement strong

width ratios, and the relative lengths between quarks for the $J^P = 2^+ cc\bar{b}\bar{b}$ state and its lowest meson–meson threshold. The notation is the same as that in Table 7

$cc\overline{b}\overline{b}$	The o	contribut	ion from	each term	Rela	ative L	engths (fm)	Overall	Present Work		CMI N	Model
$J^P = 2^+$		Value	$B_c^* B_c^*$	Difference	(i,j)	Value	$B_c^*B_c^*$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	12960.9	12700.9	260.0	(1,2)	0.355			$2m_c$	3836.0		
Variational	C_{11}	24.5	20.2		(1.3)	0.322	$0.250(B_c^*)$		$\frac{\mathbf{p}_{x_1}^2}{2m_1'}$	306.2		
(fm^{-2})	$C_{22} \\ C_{33}$	10.1	20.2 -		(2,3)	0.322		<i>c</i> -quark:	$\frac{m_{\overline{b}}}{m_{a}+m_{\overline{a}}} \frac{\mathbf{p}_{x_{3}}^{2}}{2m_{a}^{\prime}}$	163.1	$\frac{1}{2}m_{cc}$	
Quark	Mass	14522.0	14522.0	0.0	(1,4)	0.322		meff	$V^C(12)$	-38.9	1585.8	
Confinemen	t Potential	-2382.1	-2686.8	304.7	(2,4)	0.322	$0.250(B_c^*)$	m_c	$\frac{1}{2}[V^{C}(13) + V^{C}(14) + V^{C}(23) + V^{C}(24)]$	-77.4	$\left \frac{m_c}{m_{\overline{b}}+m_c}m_{c\overline{b}}\right $	$2m_c$
Kinetic	Energy	795.6	835.7	-40.1	(3,4)	0.227			-D	-983.0	1587.7	3449.6
CS Inter	raction	25.3	30.0	-4.7	(1,2)	-(3,4):	$0.243~\mathrm{fm}$		Subtotal	3206.0	3164.5	3449.6
	(1,2)	-38.9			F	adius:	$0.160~{\rm fm}$		$2m_b$	10686.0		
V^C	(2,3)	-38.7							$\frac{\mathbf{p}_{x_2}^2}{2m_2'}$	268.8		
	(1,4)	-38.7			(1,3)	-38.7	$-360.4(B_c^*)$	b quark	$\frac{m_c}{m_c+m_{\overline{b}}}\frac{\mathbf{p}_{x_3}^2}{2m_3'}$	58.5	$\frac{1}{2}m_{bb}$	
	Subtotal	-416.0	-720.8	304.8	(3,4)	-222.4		<i>o</i> -quark.	$V^{C}(34)$	-222.4	4764.8	
Total Con	tribution	404.9	144.9	260.0	(2,4)	-38.7	$-360.4(B_c^*)$	$m_{\overline{b}}^{eff}$	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$ + $V^{C}(23) + V^{C}(24)]$	-77.4	$\left \frac{m_{\overline{b}}}{m_{\overline{b}}+m_c}m_{c\overline{b}}\right $	$2m_b$
Total Wave	function:								-D	-983.0	4743.6	10105.8
$\Psi_{tot} = F\rangle I$	$ \langle R^s \rangle [\phi_1 \chi_2] \rangle$	= 0.577	$F\rangle R^s\rangle [e$	$\langle \psi_1 \zeta_1] \rangle - 0.8$	$316 F\rangle$	$ R^s\rangle [\psi$	$_{2}\zeta_{1}]\rangle$		Subtotal	9730.5	9508.4	10105.8
The rearran	gement str	ong deca	y channe	el: $B_c^* B_c^*$					$\frac{1}{2}V^{SS}(12)$	03	$\frac{8}{3}v_{cc}$	$\frac{8}{3}C_{cc}$
The radiativ	ve decay wi	dths:	$\Gamma_{T_{c^2\bar{b}^2}(1}$	$2960.9,2^+) \rightarrow 7$	$C_{c^2\bar{b}^2}(12)$	2920.0,0+	$_{)\gamma} = 0 \mathrm{keV}$	đđ	$\frac{1}{2}$ v (12)	5.5	9.5	14.1
			$\Gamma_{T_{c^2\overline{b}^2}(12)}$	$960.9,2^+) \to T_c$	$2\bar{b}^2(129)$	39.9.9,1	$_{+)\gamma} = 3.6 \mathrm{keV}$	US Interaction	$\frac{1}{2}V^{SS}(34)$	4.6	$\frac{8}{3}v_{\overline{b}\overline{b}}$	$\frac{8}{3}C_{\overline{b}\overline{b}}$
The magnet	ic moment	s:							$\frac{1}{2}V$ (34)	4.0	5.1	7.7
$\mu_{T_{c^2\bar{b}^2}(12969.}$	$_{9,2^{+})}=\langle \Psi_{i}^{2}$	$\hat{\mu}_{ot}^{z} \hat{\mu}^{z} \Psi_{to}^{z} $	$_{ot}^{+}\rangle = 2\mu_{c}$	$+ 2\mu_{\overline{b}} = 0.$	$982\mu_N$	r			$\frac{1}{4}[V_{aa}^{SS}(13) + V_{aa}^{SS}(14)]$	10.3	$\frac{16}{3}v_{\overline{c}\overline{b}}$	$\frac{16}{3}C_{\overline{c}\overline{b}}$
The transiti	on magnet	ic momer	nts :						$+V^{SS}(23)+V^{SS}(24)]$	10.0	15.7	17.6
$\mu_{T_{c^2\bar{b}^2}(12960.}$	$9,2^+) \rightarrow T_{c^2 \bar{b}^2}$	(19240.0,0	$_{+)\gamma} = \langle \Psi$	$\hat{\mu}_{tot}^{2^+} \hat{\mu}^z \Psi_{tot}^{0^+} \rangle$	0 = 0;				Subtotal	25.3	30.3	39.5
$\mu_{T_{b^2\bar{b}^2}(12960.}$	$9,2^+) \rightarrow T_{b^2\bar{b}^2}$	(12939.9, 1)	$_{+)\gamma} = \langle \Psi$	$\frac{2^+}{tot} \hat{\mu^z} \Psi^{1^+}_{tot}\rangle$	$= \mu_c$	$-\mu_{\overline{b}} =$	$0.342\mu_N$	Total contri	ibution	12960.9	12703.1	13595.0

 Table 13
 The masses, binding energy, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width
 ratios, and the relative lengths between quarks for the $J^P = 0^+$, $1^+ cc\bar{c}\bar{b}$ states and their lowest meson–meson thresholds. The notation is the same as that in Table 7

$ccc\overline{b}$ The contribution from each term Relativ						ative Le	engths (fm)	Owenell	Present Work		CMI M	odel
$J^{P} = 0^{+}$		Value	$B_c \eta_c$	Difference	(i, j)	Value	$B_c \eta_c$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	9620.5	9286.4	332.1	(1,2)	0.418			$2m_c$	3836.0	$-\frac{1}{4}m_{cc}$	
Variational	C_{11}	11.4	22.9		(1.3)	0.325	$0.290(\eta_{c})$	<i>c</i> -quark:	$\frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_c + m_{\overline{b}}}{3m_c + m_{\overline{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_2'}$	451.6	-792.9	
(fm^{-2})	C_{22} C_{33}	15.2	15.0 -		(2,3)	0.325		eff	$V^{C}(12)$	-12.6	$\frac{4(m_c+m_{\bar{c}})}{1917.8}m_{c\bar{c}}$	
Quark	Mass	11097.0	11097.0	0.0	(1,4)	0.336		$m_c^{e_f f}$	$\frac{1}{2}[V^{C}(13) + V^{C}(23)]$ $\frac{1}{2}[V^{C}(14) + V^{C}(24)]$	-91.8 -74.8	$\frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{b}}$	$2m_c$
Confinemen	t Potential	-2280.0	-2618.0	338.0	(2,4)	0.336	$0.235(B_c)$		$\frac{1}{2} \begin{bmatrix} v & (14) + v & (24) \end{bmatrix} \\ -D$	-983.0	1973.7	3449.6
Kinetic	Energy	810.3	931.2	-120.9	(3,4)	0.333			Subtotal	3125.4	3098.6	3449.6
CS Inte	raction	18.2	-123.8	142.0	(1,2)	-(3,4):	$0.204~{\rm fm}$		2 m_c	1918.0	$\frac{-m_{\overline{c}}}{4(m_{\overline{b}}+m_{\overline{c}})}m_{\overline{c}\overline{b}}$	
	(1,2)	-12.6			(1,3)	-91.9	$-237.2(\eta_c)$		$\frac{m_{\bar{b}}}{m_{\bar{c}}+m_{\bar{b}}} \frac{\mathbf{p}_{x_2}^{*}}{2m_2'} + \frac{m_c}{3m_c+m_{\bar{b}}} \frac{\mathbf{p}_{x_3}^{*}}{2m_3'}$	235.0	-400.5	
V^C	(1,4)	-74.8			(2,3)	-91.9		<i>ē</i> -quark:	$\frac{1}{2}V^{C}(34)$	-91.9	$\frac{5m_c}{4(m_c+m_c)}m_{c\bar{b}}$	$m_{\overline{c}}$
	Subtotal	-314.0	-652.0	338.0	(2,4)	-74.8	$-414.8(B_c)$	m^{eff}_{-}	$\frac{1}{2}[V^{\circ}(13) + V^{\circ}(23)] - \frac{1}{2}D$	-491.5	$\frac{4(m_c+m_b)}{1917.8}$	1724.8
Total Con	tribution	514.4	155.4	359.0	(3,4)	32.0	(-)	_c	Subtotal	1585.6	1517.3	1724.8
Total Wave	function:								m _b	5343.0	$\frac{-m_{\overline{b}}}{4(m_{\overline{c}}+m_{\overline{c}})}m_{\overline{c}\overline{b}}$	
$\Psi_{tot} = 0.402$	$1 F\rangle R^s\rangle [\phi$	$_1\chi_5]\rangle - 0$	$0.916 F\rangle I$	$ \langle R^s \rangle [\phi_2 \chi_6] \rangle$	= -(0.574 F	$ R^s\rangle [\psi_1\zeta_5]\rangle$	\overline{b} -quark:	$\frac{m_{\bar{c}}}{m_{\bar{c}}+m_{\bar{b}}}\frac{\mathbf{p}_{x_2}^2}{2m_2'}+\frac{m_c}{3m_c+m_{\bar{b}}}\frac{\mathbf{p}_{x_3}^2}{2m_3'}$	123.6	-1203.5	
-0.	$532 F\rangle R^s\rangle$	$ [\psi_1\zeta_6]\rangle -$	+ 0.019 F	$ R^s\rangle [\psi_2\zeta]$	$ \rangle = 0$	0.622 F	$ R^s\rangle [\psi_2\zeta_6]\rangle$	meff	$\frac{1}{2}V^{C}(34)$	$16.0 \\ 74.8$	$\frac{5m_{\overline{b}}}{1}m_{\overline{b}}$	m _T
The rearran	gement str	ong deca	v channe).	5]/ ·		/ -* / [+250]/	$m_{\overline{b}}$	$\frac{1}{2}[V^{C}(14) + V^{C}(24)] - \frac{1}{2}D$	-491.5	$4(m_c+m_{\overline{b}}) = 200$	5052.9
Г	.gement str	онд ucca • г	y chamic	. — 1	·19				$-\frac{1}{2}D$ Subtotal	4916.3	4726.0	5052.9
$\frac{T_{c^2\bar{b}^2}(9620.5)}{\text{The radiativ}}$	$(0^+) \rightarrow B_c^* J/\psi$	$\frac{1}{dthe} T_{c^2 \overline{b}^2}$	(9620.5,0+)	$\rightarrow B_c \eta_c = 1$	1.2		- OkoV		2 66.	1010.0	4v	4Cm
The faulation	ve decay wi	uuns. 1	$T_{c^2 \bar{c} \bar{b}}(9730$	$(5,2^+) \rightarrow T_{c^2}$	<u>5</u> (9620	$.5,0^{+})\gamma$	= 0.007 lmV	CS	$\frac{3}{4}V^{SS}(12)$	10.8	14.3	21.2
	•		$T_{c^2 \overline{c} \overline{b}}(962)$	$24.6,1^+) \rightarrow T_c$	$\frac{2}{cb}(962)$	$\frac{20.5,0^+)\gamma}{1}$	= 0.007 keV	Interaction	$\frac{3}{4}V^{SS}(34)$	7.3	$4v_{\overline{c}\overline{b}}$	$4C_{\overline{c}\overline{b}}$
The magnet	ic moment	s: $\mu_{T_{c^2 \overline{c} \overline{b}}}$	(9620.5,0 ⁺)	$=\langle \Psi_{tot} \mu$	$\sim \Psi_{tot} $	$ _{t}\rangle = 0$			Subtotal	10.0	0.9	24.4
The transiti	on magnet	ic momei	nts:	↓ ^+ ↓ • • 0 ⁺ ↓	0			M.t.t.:	Subtotal	10.2	22.1	34.4
$\mu_{T_{c^2\overline{c}\overline{b}}(9730.5}$	$,2^+) \rightarrow T_{c^2 \overline{c} \overline{b}}(9)$	9620.5,0+)	$\gamma = \langle \Psi_{tot}^z \rangle$	$ \mu^z \Psi_{tot}^0\rangle =$	= 0			Matrix non	diagonal element	-25.0	-46.2	-117.7
$\mu_{T_{0}} = -(9624.6)$	$(1+) \ \sqrt{T} = (0)$	0620 5 0±).	$ = \langle \Psi_{tot}^{\dagger} \rangle $	$ \mu^z \Psi_{\pm\pm\pm}^0\rangle =$	= -0.0	$196\mu_{M}$		Total contri	bution	19620.5	9317.5	1 10144.0
1 1 c2 cb (002 110	$(1^{+}) \rightarrow 1_{c^2 \overline{c} \overline{b}}$	5020.3,0 ·)	γ (-ιοι	P 1 - 101/	- 0.0	000µN					001110	
$J^P = 1^+$	$(1^{+}) \rightarrow 1_{c^2 \overline{c} \overline{b}}(1^{+})$	Value	$B_c^* \eta_c$	Difference	Rela	ative Le	engths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
$\frac{J^P = 1^+}{Mass}$	$/B_T$	Value 9624.6	$B_c^* \eta_c$	Difference 275.6	Rela (i, j)	ative Le Value	engths (fm) $B_c^* \eta_c$		Contribution $2m_c$	Value 3836.0	Ref. [110] $-\frac{1}{4}m_{cc}$	Ref. [123]
$\frac{J^{P} = 1^{+}}{J^{P} = 1^{+}}$ Mass Variational Parameters	B_T C_{11} C_{2zb}	Value 9624.6 11.1 6.9	$\frac{B_c^* \eta_c}{9349.0}$ 20.2 15.0	$\frac{1775.6}{275.6}$	Rela (i, j) (1.2)	ative Le Value 0.429	engths (fm) $B_c^* \eta_c$	<i>c</i> -quark:	$\frac{2m_c}{\frac{\mathbf{P}_{x_1}^2}{2m_1^2} + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{\mathbf{P}_{x_3}^2}{2m_3^2}}$	Value 3836.0 442.8	$\begin{array}{c} \text{Ref. [110]} \\ \hline -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline 5m_c \\ \hline m_c \\ \hline m_c \\ \hline \end{array}$	Ref. [123]
$\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²)	$ \frac{B_T}{C_{22}} \\ C_{22} \\ C_{33} $	Value 9624.6 11.1 6.9 15.3	$ \frac{B_c^* \eta_c}{9349.0} 20.2 15.0 - $	1000000000000000000000000000000000000	Rela (i, j) (1.2) (1.3)	ative Le Value 0.429 0.328	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$	<i>c</i> -quark:	Contribution $\frac{2m_c}{\frac{\mathbf{p}_{x_1}^2}{2m_1^{\prime}} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3^{\prime}}}{V^C(12)}$ $UC(22) = UC(22)$	Value 3836.0 442.8 -17.4	$\begin{array}{c} \text{Ref. [110]} \\ \hline \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline \\ \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{c}} \\ 1917.8 \end{array}$	Ref. [123]
$ \frac{J^P = 1^+}{J^P = 1^+} $	B_T C_{11} C_{22} C_{33} Mass	Value 9624.6 11.1 6.9 15.3 11097.0	$ \begin{array}{r} & & & \\ & & & \\ \hline B_c^* \eta_c \\ \hline 9349.0 \\ \hline 20.2 \\ 15.0 \\ \hline - \\ 11097.0 \end{array} $	0.0	Rela (i, j) (1.2) (1.3) (2,3)	ative Le Value 0.429 0.328 0.328	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$	c-quark: m_c^{eff}	$\begin{array}{c} \text{Contribution} \\ \hline 2m_c \\ \frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{4} [V^C(14) + V^C(24)] \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \frac{5m_c}{4(m_c+m_{\bar{c}})}m_{c\bar{c}} \\ \hline & 1917.8 \\ \hline & \frac{5m_c}{4(m_c+m_{\bar{b}})}m_{c\bar{b}} \end{array}$	Ref. [123] 2m _c
$\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen	B_T C_{11} C_{22} C_{33} Mass t Potential	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2	$\begin{array}{c} B_c^* \eta_c \\ \hline \\ 9349.0 \\ \hline \\ 20.2 \\ 15.0 \\ \hline \\ 11097.0 \\ -2563.6 \end{array}$	0.0 297.4	Rela (i, j) (1.2) (1.3) (2,3) (1,4)	ative Le Value 0.429 0.328 0.328 0.340	$\frac{\text{engths (fm)}}{B_c^*\eta_c}$ $0.290(\eta_c)$	c-quark: m_c^{eff}	$\begin{array}{c} \hline \\ & 2m_c \\ \hline \\ \frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_c + m_b}{3m_c + m_b} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0	$\begin{array}{c} \hline & \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{c}} \\ 1917.8 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{b}} \\ \hline & 1973.7 \end{array}$	Ref. [123] $2m_c$ 3449.6
$ \frac{J^{P} = 1^{+}}{J^{P} = 1^{+}} $ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic	B_T C_{11} C_{22} C_{33} Mass t Potential Energy	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2	$\begin{array}{c} B_c^* \eta_c \\ \hline \\ 9349.0 \\ \hline \\ 20.2 \\ 15.0 \\ \hline \\ -11097.0 \\ -2563.6 \\ \hline \\ 875.4 \end{array}$	0.0 297.4 -80.2		ative Le Value 0.429 0.328 0.328 0.340 0.340	rangths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$	c-quark: m_c^{eff}	$\begin{array}{c} \hline \\ & 2m_c \\ \hline & 2m_c \\ \frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_c + m_5}{2m_a} \frac{\mathbf{p}_{x_3}^2}{2m_1^2} \\ & V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ & \text{Subtotal} \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \frac{5m_c}{4(m_c+m_{\bar{c}})}m_{c\bar{c}} \\ \hline & 1917.8 \\ \hline & \frac{5m_c}{4(m_c+m_{\bar{c}})}m_{c\bar{b}} \\ \hline & 1973.7 \\ \hline & 3098.6 \\ \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6
$\frac{J^{P} = 1^{+}}{J^{P} = 1^{+}}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte	B_{T} C_{11} C_{22} C_{33} Mass t Potential Energy raction	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0	$\begin{array}{c} B_c^* \eta_c \\ \hline \\ 9349.0 \\ \hline \\ 20.2 \\ 15.0 \\ \hline \\ - \\ 11097.0 \\ -2563.6 \\ \hline \\ 875.4 \\ -59.9 \end{array}$	0.0 297.4 -80.2 67.9	$\begin{array}{c} \text{Rela} \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \end{array}$	Soop ₁ /v ative Le Value 0.429 0.328 0.328 0.340 0.340	rangths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$	c-quark: m_c^{eff}		Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & 5m_c \\ \hline & 4(m_c+m_{\bar{c}})m_{c\bar{c}} \\ \hline & 1917.8 \\ \hline & 5m_c \\ \hline & 4(m_c+m_{\bar{c}})m_{c\bar{b}} \\ \hline & 1973.7 \\ \hline & 3098.6 \\ \hline & \hline & -\frac{m_c}{4(m_{\bar{c}}+m_{\bar{c}})}m_{\bar{c}\bar{b}} \\ \hline & \hline & \\ \hline & \\ \hline & -\frac{m_c}{4(m_{\bar{c}}+m_{\bar{c}})}m_{\bar{c}\bar{b}} \\ \hline \end{array}$	Ref. [123] 2m_c 3449.6 3449.6 3449.6
$\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte	B_T C_{11} C_{22} C_{33} Mass t Potential Energy raction (1,2)	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4	$\begin{array}{c} B_c^* \eta_c \\ \hline \\ 9349.0 \\ \hline \\ 20.2 \\ 15.0 \\ \hline \\ 11097.0 \\ -2563.6 \\ \hline \\ 875.4 \\ -59.9 \end{array}$	0.0 297.4 -80.2 67.9		ative Le Value 0.429 0.328 0.328 0.340 0.340 0.338 (3,4):	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm	c-quark: m_c^{eff}	$\begin{array}{c} \hline \\ \hline \\ & 2m_c \\ \hline \\ \frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ & V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ \hline \\ \hline \\ \hline \\ \frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{m_{\tilde{c}}^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -983.0 3122.5 1918.0 230.3	$\begin{array}{c} \hline & \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{c}} \\ 1917.8 \\ \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{b}} \\ 1973.7 \\ \hline & 3098.6 \\ \hline & \frac{-m_{\overline{c}}}{4(m_{\overline{c}}+m_{\overline{c}})}m_{\overline{c}\overline{b}} \\ -400.5 \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6
$\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte	B_T C_{11} C_{22} C_{33} Mass t Potential Energy raction $(1,2)$ $(2,3)$	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4 -87.0	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ \hline 20.2 \\ 15.0 \\ \hline \\ 11097.0 \\ -2563.6 \\ \hline 875.4 \\ -59.9 \end{array}$	0.0 297.4 -80.2 67.9	$\begin{array}{c} \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2) \end{array}$	Solp Jy ative Le Value 0.429 0.328 0.328 0.328 0.328 0.340 0.338 I-(3,4):	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm	c-quark: m_c^{eff} \bar{c} -quark:	$\begin{array}{c} \hline & \\ & 2m_c \\ \hline & 2m_c \\ \hline & \frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ & V^C(12) \\ \hline & \frac{1}{2} [V^C(13) + V^C(23)] \\ \hline & \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline & \\ \hline & \\ \hline & \\ \hline & \frac{m_5}{m_c + m_5} \frac{\mathbf{p}_{x_2}^2}{2m_2'} + \frac{m_c}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ & \frac{1}{2} V^C(34) \\ & VVC(24) - UC(26) \\ \hline & \\ \hline \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{c}} \\ 1917.8 \\ \hline & \\ \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{b}} \\ 1973.7 \\ \hline & \\ 3098.6 \\ \hline & \\ \frac{-m_{\overline{c}}}{4(m_{\overline{b}}+m_{\overline{c}})}m_{\overline{c}\overline{b}} \\ -400.5 \\ \hline & \\ \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{b}} \\ \hline \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6 $m_{\bar{c}}$
$ \frac{J^{P} = 1^{+}}{J^{P} = 1^{+}} $ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C}	$/B_T$ C_{11} C_{22} C_{33} Mass t Potential Energy raction $(1,2)$ $(2,3)$ $(1,4)$	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4 -87.0 -68.9	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ 20.2 \\ 15.0 \\ - \\ 11097.0 \\ -2563.6 \\ 875.4 \\ -59.9 \end{array}$	0.0 297.4 -80.2 67.9	$ \begin{array}{c} \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2) \\ \end{array} $	-68.9	rangths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$	c-quark: m_c^{eff} \bar{c} -quark: m_c^{eff}	$\begin{array}{c} \hline \\ Contribution \\ \hline 2m_c \\ p_{x_1}^2 + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \\ \hline \\ \hline \\ \hline \\ \frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(34) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ -\frac{-1}{2} D \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{c}} \\ \hline & \frac{1917.8}{4(m_c+m_{\overline{b}})}m_{c\overline{b}} \\ \hline & \frac{1973.7}{4(m_{\overline{b}}+m_{\overline{c}})}m_{\overline{c}\overline{b}} \\ \hline & -400.5 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{b}} \\ \hline & 1917.8 \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6 $m_{\tilde{c}}$ 1724.8
$\frac{J^{P}}{J^{P}} = 1^{+}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte	$(B_T) \rightarrow c_{2256} (C_{225}) (C_{225$	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4 -87.0 -68.9 -300.2	-597.6	0.0 297.4 -80.2 67.9 297.4	$ \begin{array}{c} \text{Rela} \\ \hline \text{(}i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2) \\ \end{array} $	ative Le Value 0.429 0.328 0.328 0.340 0.340 0.340 0.338 (3,4): -68.9 29.0	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$	$\begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$\frac{p_{x_1}^2}{2m_1^2} + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{p_{x_3}^2}{2m_3^2}$} \\ \mbox{$V^C(12)$} \\ \mbox{$\frac{V^C(13) + V^C(23)]}{\frac{1}{2}[V^C(14) + V^C(24)]$} \\ -D \\ \mbox{$Subtotal$} \\ \mbox{$\frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{p_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{p_{x_3}^2}{2m_3^2}$} \\ \mbox{$\frac{1}{2}V^C(34)$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{-\frac{1}{2}D$}$} \\ \mbox{$Subtotal$} \\ \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{c}} \\ \hline & 1917.8 \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{c}})}m_{c\overline{b}} \\ \hline & 1973.7 \\ \hline & 3098.6 \\ \hline & \frac{-m_{\overline{c}}}{4(m_{\overline{c}}+m_{\overline{c}})}m_{c\overline{b}} \\ \hline & \frac{-m_{\overline{c}}}{4(m_{\overline{c}}+m_{\overline{c}})}m_{c\overline{b}} \\ \hline & \frac{5m_c}{4(m_c+m_{\overline{b}})}m_{c\overline{b}} \\ \hline & 1917.8 \\ \hline & 1517.3 \\ \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6 $m_{\bar{c}}$ 1724.8 1724.8
$\frac{J^{P} = 1^{+}}{J^{P} = 1^{+}}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con	$(B_T) \rightarrow c_2 c_3 c_3 c_2 c_3 c_3 c_3 c_3 c_3 c_3 c_3 c_3 c_3 c_3$	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4 -87.0 -68.9 -300.2 503.0	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ 20.2 \\ 15.0 \\ - \\ 11097.0 \\ -2563.6 \\ 875.4 \\ -59.9 \\ -597.6 \\ 218.0 \end{array}$	0.0 297.4 -80.2 67.9 297.4 297.4 297.4	$ \begin{array}{c} \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \end{array} $	-68.9 29.0 -87.0	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(n_c)$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$	$\begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$\frac{\mathbf{p}_{x_1}^2 + \frac{m_c + m_{\tilde{b}}}{2m_1} \mathbf{p}_{x_3}^2$} \\ V^C(12) \\ \mbox{$V^C(12)$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$-D$} \\ \mbox{$Subtotal$} \\ \hline \mbox{$\frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{-1}{2}D$} \\ \mbox{$Subtotal$} \\ \hline \mbox{$m_{\tilde{b}}$} \\ \hline \end{array} $	Value 3836.0 442.8 -17.4 -87.0 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0		Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 1724.8 1724.8
$\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte $\frac{V^{C}}{Total Con}$ Total Con	$(B_T) \rightarrow c_2 c_5 c_6 c_6 c_7 c_7 c_7 c_7 c_7 c_7 c_7 c_7 c_7 c_7$	Value 9624.6 11.1 6.9 15.3 11097.0 -2266.2 795.2 8.0 -17.4 -87.0 -68.9 -300.2 503.0	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ 20.2 \\ 15.0 \\ - \\ 11097.0 \\ -2563.6 \\ 875.4 \\ -59.9 \\ \hline \\ -597.6 \\ 218.0 \end{array}$	0.0 297.4 -80.2 67.9 297.4 297.4 297.4 297.4	$\begin{array}{c} \text{Relat} \\ \hline \text{Relat} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \hline \\ (2,4) \\ (3,4) \\ (1,3) \\ \hline \end{array}$	-68.9 29.0 -87.0	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark:	$ \begin{array}{c} \hline \text{Contribution} \\ \hline 2m_c \\ p_{x_1}^2 + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_1^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \\ \hline \\ \hline \\ \hline \\ \frac{m_5}{m_c + m_5} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ -\frac{1}{2} D \\ \hline \\ \hline \\ \hline \\ \hline \\ \frac{m_5}{m_c + m_5} \frac{\mathbf{p}_{x_2}^2}{2m_5^2} + \frac{m_c}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_6^2} \\ \hline \end{array} $	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 $m_{\tilde{c}}$ 1724.8 1724.8 -1203.5
$J^{P} = 1^{+}$ $Mass$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con Total Wave $\Psi_{tot} = 0.2$	$ B_T = C_{22} C_{33}$ $ B_T = C_{11} C_{22} C_{33}$ Mass t Potential Energy raction $(1,2)$ $(2,3)$ $(1,4)$ Subtotal tribution function: $20 F\rangle B^s\rangle B^s\rangle $	$\begin{array}{c} \text{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \end{array}$	γ (2-10) $B_c^* \eta_c$ (2) 9349.0 20.2 15.0 -2563.6 875.4 -59.9 -597.6 218.0 -0.968[F]	0.0 297.4 -80.2 67.9 297.4 297.4 285.0	$\begin{array}{c} \text{Relation} \\ \text{Relation} \\ (i,j) \\ (1,2) \\ (1,2) \\ (2,3) \\ (1,4) \\ (2,4) \\ (1,2) \\ (2,4) \\ (3,4) \\ (1,3) \\ (1,$	2000 µN ative Le Value 0.429 0.328 0.328 0.340 0.340 0.340 0.338 (3,4): 68.9 29.0 -87.0 → 118 E [†]	engths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark: $m_{\bar{c}}^{eff}$	$\begin{array}{c} \begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$2m_1^2$} + \frac{m_c + m_5}{3m_c + m_5} \frac{p_{23}^2}{2m_3} \\ V^C(12) \\ \mbox{$1p$} \\ \mb$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 63.0	Ref. [110] $-\frac{1}{4}m_{cc}$ -792.9 $\frac{5m_c}{4(m_c+m_{\bar{c}})}m_{c\bar{c}}$ 1917.8 $\frac{5m_c}{4(m_c+m_{\bar{c}})}m_{c\bar{b}}$ 1973.7 3098.6 $\frac{-m_{\bar{c}}}{4(m_c+m_{\bar{c}})}m_{c\bar{b}}$ $\frac{4(m_c+m_{\bar{b}})}{4(m_c+m_{\bar{b}})}m_{c\bar{b}}$ 1917.8 1517.3 $\frac{-m_{\bar{b}}}{4(m_{\bar{b}}+m_{\bar{c}})}m_{c\bar{b}}$ $\frac{-5m_{\bar{b}}}{4(m_{\bar{b}}+m_{\bar{c}})}m_{\bar{c}\bar{b}}$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 1724.8 1724.8 -1203.5 $m_{\overline{c}}$
$\frac{J^{P} = 1^{+}}{Mass}$ $\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte: $\frac{V^{C}}{Total \ Con}$ Total Con Total Wave $\Psi_{tot} = 0.2$	$ B_T = C_{11} C_{22} C_{33}$ Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: 20 F > R^* >	$\begin{array}{c} \text{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \end{array}$	y (=10 B _c [*] η _c] 9349.0 20.2 15.0 -2563.6 875.4 -59.9 -597.6 218.0 -0.3968[F] -0.3968[F]	$\frac{1}{297.4}$ $\frac{297.4}{285.0}$ $\frac{297.4}{285.0}$	$\begin{array}{c} \text{Relation} \\ \hline \text{Relation} \\ \hline (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ (1,2) \\ (2,4) \\ (1,2) \\ (2,4) \\ (1,3) \\$	ative Le Value 0.429 0.328 0.328 0.328 0.340 0.340 0.338 (3,4): -68.9 29.0 -87.0	engths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\phi_1\chi_2]\rangle$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark: $m_{\bar{b}}^{eff}$	$\begin{array}{c} \begin{array}{c} \text{Contribution} \\ \hline & 2m_c \\ & 2m_c \\ \frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ & V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ \hline \\ \hline \\ \frac{m_5}{m_c + m_5} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(34) + V^C(23)] \\ & -\frac{1}{2} D \\ \hline \\ \hline \\ \frac{m_c}{m_c + m_5} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \hline \\ \frac{m_c}{12} V^C(34) \\ \frac{1}{2} [V^C(34) + V^C(24)] \\ & \frac{1}{2} [V^C(14) + V^C(24)] \\ \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 5343.0 122.1 14.5 -68.9 -491.5	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline \\$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 1724.8 1724.8 -1203.5 $m_{\tilde{b}}$ 5052.9
$\frac{J^{P} = 1^{+}}{Mass}$ $\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte $\frac{V^{C}}{V^{C}}$ Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0.$	$ B_{T} = C_{11} C_{22} C_{33}$ Mass t Potential Energy raction (1,2) (2,3) (1,4) Subtotal tribution function: $20 F\rangle R^{s}\rangle $ $494 F\rangle R^{s}\rangle $	$\begin{array}{c} \text{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ \left[\phi_1\chi_3\right]\rangle + \\ \left[\left[\psi_1\zeta_2\right]\right\rangle - \\ \hline \\ \left[\psi_1\zeta_2\right]\rangle - \\ \hline \\ \\ \left[\psi_1\zeta_2\right]\rangle - \\ \hline \\ \\ \\ \left[\psi_1\zeta_2\right]\rangle - \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	(-100) $B_c^* \eta_c$ 9349.0 20.2 15.0 -2563.6 875.4 -59.9 -597.6 218.0 $-0.396 F\rangle$ $-0.396 F\rangle$ $0.246 F\rangle$	$ \frac{1}{297.4} $ 297.4 -80.2 67.9 297.4 -80.2 67.9 297.4 285.0 $ R^{s}\rangle [\phi_{2}\chi_{4}]$	$\begin{array}{c} \text{Relative for a strength of } \\ \hline \text{Relative for a strength of } \\ \hline \text{(1,2)} \\ \hline \text{(1,2)} \\ \hline \text{(1,3)} \\ \hline \text{(2,3)} \\ \hline \text{(1,3)} \\ \hline \text{(2,4)} \\ \hline \text{(1,2)} \\ \hline \text{(2,4)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \text{(1,3)} \\ \hline \text{(3,4)} \\ \hline \ \text{(3,4)} \\ \hline \text{(3,4)} \\ \hline \ $	obsorp/in ative Le Value 0.429 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.328 0.338 -(3,4): -68.9 29.0 -87.0 0.1118 F) 0.2487 F	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$	$c-quark:$ m_{c}^{eff} $\bar{c}-quark:$ $m_{\bar{c}}^{eff}$ $\bar{b}-quark:$ $m_{\bar{b}}^{eff}$	$\begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$2m_1$} + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \mbox{$V^C(12)$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$-D$} \\ \mbox{$Subtotal$} \\ \mbox{$\frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{-1}{2}D$} \\ \mbox{$Subtotal$} \\ \mbox{$\frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_3^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$\frac{-1}{2}D$} \\ \mbox{$Subtotal$} \\ \end{tabular}$	Value 3836.0 442.8 -17.4 -87.0 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 1724.8 1724.8 1724.8 1724.8 5052.9 5052.9
$J^{P} = 1^{+}$ $Mass$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0.$ $+0.$	$\begin{array}{c} & B_T \\ \hline B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline Mass \\ t \text{ Potential} \\ \hline Energy \\ raction \\ \hline (1,2) \\ (2,3) \\ (1,4) \\ \hline Subtotal \\ tribution \\ function: \\ 20 F\rangle R^s\rangle \\ 494 F\rangle R^s\rangle \\ 494 F\rangle R^s\rangle \\ math tribular \\$	$\begin{array}{c} \text{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \end{array}$	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ 20.2 \\ 15.0 \\ -1 \\ 11097.0 \\ -2563.6 \\ 875.4 \\ -59.9 \\ \hline \\ -597.6 \\ 218.0 \\ -0.968 F \\ -0.396 F \\ -0.246 F \\ -0.24$	$\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 $	$\begin{array}{c} \text{Relative for a constraint of } \\ \text{Relative for a constraint of } \\ \text{(1, j)} \\ (1, 2) \\ (1, 3) \\ (2, 3) \\ (1, 3) \\ (2, 4) \\ (2, 4) \\ (3, 4) \\ (1, 2) \\ (2, 4) \\ (3, 4) \\ (1, 3) \\ (1, $	obsorption ative Le Value 0.429 0.328 0.340 0.338 -(68.9 29.0 -87.0 0.118 F \rangle 0.487 F 0.537 F	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $\langle R^*\rangle [\psi_1\zeta_4]\rangle$ $\langle R^*\rangle [\psi_2\zeta_4]\rangle$	c-quark: m_c^{eff} \overline{c} -quark: $m_{\overline{c}}^{eff}$ \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$2m_{x_1}$} + \frac{m_c + m_{\tilde{b}}}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3} \\ \mbox{$V^C(12)$} \\ \mbox{$V^C(12)$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$-D$} \\ \mbox{$Subtotal$} \\ \mbox{$\frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^3} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{1}{2}[V^C(13) + V^C(23)]$} \\ \mbox{$\frac{-\frac{1}{2}D$} \\ \\ \mbox{$Subtotal$} \\ \\ \mbox{$\frac{m_{\tilde{c}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^3} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$\frac{1}{2}[V^C(14) + V^C(24)]$} \\ \mbox{$\frac{-\frac{1}{2}D$} \\ \\ \mbox{$Subtotal$} \\ \\ \mbox{$Subtotal$} \\ \end{tabular}} \end{array}$	Value 3836.0 442.8 -17.4 -87.0 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 1724.8 1724.8 1724.8 1724.8 5052.9 5052.9 5052.9
$\frac{1}{J^{P}} = 1^{+}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0.$ $+0.$ The rearran	$\begin{array}{c} B_{T}\rangle \rightarrow c_{2} c_{5} c_{6} \\ B_{T}\rangle \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} \text{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ [\phi_1\chi_3]\rangle + \\ [\psi_1\zeta_2]\rangle - \\ [\psi_2\zeta_2]\rangle - \\ \text{ong deca} \\ \downarrow \end{array}$	$\begin{array}{c} B_c^* \eta_c \\ 9349.0 \\ 20.2 \\ 15.0 \\ -2563.6 \\ 875.4 \\ -59.9 \\ \end{array}$ $\begin{array}{c} -597.6 \\ 218.0 \\ -0.968 F\rangle \\ -0.396 F \\ -0.246 F \\ y \text{ channe} \end{array}$	$\begin{array}{c} 0.0 \\ 297.4 \\ \hline 297.4 \\ \hline 297.4 \\ \hline 285.0 \\ \hline \\ R^{s}\rangle [\phi_{2}\chi_{c}] \\ R^{s}\rangle [\psi_{1}\zeta_{c}] \\ R^{s}\rangle [\psi_{2}\zeta_{c}] \\ \hline \\ R^{s}\rangle [\psi_{2}\zeta_{c}] \\ \hline \\ \hline \\ \end{array}$	$\begin{array}{c} Relative for a strength of the streng$	0.000μ ative Le Value 0.429 0.328 0.328 0.328 0.340 0.338 -(3,4): -68.9 29.0 -87.0 0.118 F⟩ 0.487 F 0.537 F 0.537 F	$\begin{array}{c} \text{engths (fm)} \\ \hline B_c^* \eta_c \\ \hline 0.290(\eta_c) \\ \hline 0.250(B_c^*) \\ \hline 0.204 \text{ fm} \\ \hline -360.4(B_c^*) \\ \hline -237.2(\eta_c) \\ \hline R^*\rangle [\phi_1 \chi_2]\rangle \\ \hline \rangle R^*\rangle [\psi_1 \zeta_4]\rangle \\ \hline \rangle R^*\rangle [\psi_2 \zeta_4]\rangle \\ \hline z_{J}/\psi : \end{array}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark: $m_{\bar{b}}^{eff}$ CS	$\begin{array}{r} \begin{array}{c} \text{Contribution} \\ & 2m_c \\ & 2m_c \\ \frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_c + m_{\tilde{b}}}{2m_1} \frac{\mathbf{p}_{x_3}^2}{\mathbf{p}_{x_3}} \\ & V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ \hline \\ \hline \\ \frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ & -\frac{1}{2} D \\ \hline \\ \hline \\ \frac{m_{\tilde{b}}}{m_{\tilde{c}} + m_{\tilde{b}}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_c}{3m_c + m_{\tilde{b}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -\frac{1}{2} D \\ \hline \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -\frac{1}{2} D \\ \hline \\$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline & \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \\ \hline \hline$	$\begin{array}{c} 2m_c \\ 3449.6 \\ 3449.6 \\ 3449.6 \\ \hline \\ m_{\overline{c}} \\ 1724.8 \\ 1724.8 \\ 1724.8 \\ 1724.8 \\ \hline \\ 5052.9 \\ 5052.9 \\ \hline \\ 5052.9 \\ \hline \\ -\frac{4}{3}C_{\overline{c}\overline{b}} \\ -4.4 \\ \end{array}$
$\frac{1}{J^{P}} = 1^{+}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con Total Wave $\Psi_{tot} = 0.2$ = 0. $\pm 0.$ The rearran $\Gamma_{T_{c}cxb}$ (9624.6	$\begin{array}{c} /B_T \\ \hline \\ /B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ \hline \\ \hline \\ \hline \\ C_{23} \\ \hline \\ $	$\begin{array}{c} \mathrm{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ [\psi_1 \chi_3] \rangle + \\ [[\psi_1 \zeta_2] \rangle - \\ [[\psi_2 \zeta_2] \rangle - \\ \mathrm{ong \ deca} \\ \vdots \ \Gamma_{T_{c2\pi b}} \\ \end{array}$	$\begin{array}{c} 8^{\circ}_{c} \gamma_{c} \\ 9349.0 \\ 20.2 \\ 15.0 \\ - \\ 11097.0 \\ - \\ 2563.6 \\ 875.4 \\ - \\ 59.9 \\ \hline \\ - \\ 597.6 \\ 218.0 \\ - \\ 0.968 F\rangle \\ - \\ 0.968 F\rangle \\ - \\ 0.396 F \\ - \\ 0.246 F \\ y \ channe \\ 9624.6,1^{+}) \end{array}$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ 297.4 \\ \hline -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ \hline 285.0 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ 285.0 \\ \rangle R^s\rangle [\psi_1\zeta_5 \\ \rangle R^s\rangle [\psi_2\zeta_5 \\ \hline \\ R^s\rangle [\psi_2\zeta_5 \\ \hline \\ R^s\rangle [\psi_2\zeta_5 \\ \hline \\ R^s\rangle [\psi_2\zeta_5 \\ R^s\rangle [\psi_2$	$\begin{array}{c} Relative for a strength of the streng$	$\begin{array}{l} \text{bis} 600 \mu_N \\ \text{ative Le} \\ \hline \text{Value} \\ 0.429 \\ 0.328 \\ 0.328 \\ 0.328 \\ 0.340 \\ 0.338 \\ 0.340 \\ 0.338 \\ 0.340 \\ 0.338 \\ 0.340 \\ 0.328 \\ 0.3$	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $\frac{B_c^* \eta_c}{0.290(\eta_c)}$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$	$c-quark:$ m_c^{eff} $\bar{c}-quark:$ $m_{\bar{c}}^{eff}$ $\bar{b}-quark:$ $m_{\bar{b}}^{eff}$ CS Interaction	$\begin{array}{r} \begin{array}{c} \text{Contribution} \\ & 2m_c \\ & 2m_c \\ \hline & 2m_1^2 + \frac{m_c + m_5}{2m_1} \mathbf{P}_{x_3}^2 \\ & V^C(12) \\ \hline & V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline & \\ \hline \\ \hline$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3 10.4	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline \\ \hline & \\ \hline \hline \\ \hline \\$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 $m_{\bar{c}}$ 1724.8 1724.8 -1203.5 $m_{\bar{b}}$ 5052.9 5052.9 $-4\frac{4}{3}C_{\bar{c}\bar{b}}$ -4.4 $4C_{cc}$ 22
$\frac{1}{J^{P}} = 1^{+}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Intee V^{C} Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0,$ $+0.$ The rearran $\frac{\Gamma_{T_{c^{2}c\bar{b}}}(9624.6)}{\Gamma}$ The radiative	$\begin{array}{c} B_{T}\rangle \rightarrow L_{c} 2\pi h \\ \hline B_{T} \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{23} \\ \hline \\ \hline \\ \hline \\ C_{23} \\ \hline \\ \hline \\ C_{23} \\ \hline \\ $	$\begin{array}{c} \mathrm{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ [\psi_1 \zeta_2] \rangle - \\ [[\psi_2 \zeta_2] \rangle - \\ \mathrm{ong} \ \mathrm{deca} \\ : \Gamma_{T_c^2 x \overline{b}} (\mathrm{dths} : \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$\begin{array}{c} & & & & \\ & & &$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ 297.4 \\ \hline -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ \hline 285.0 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ 285.0 \\ \hline \\ R^s\rangle [\psi_2\zeta_5 \\ K^s\rangle [\psi_2\zeta_5 \\ \hline \\ R^s\rangle [\psi_2\zeta_5 \\ $	$\begin{array}{c} \text{Relat} \\ \hline \text{Relat} \\ \hline (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \hline \\ (2,4) \\ (1,2) \\ \hline \\ (3,4) \\ (1,3) \\ \hline \\ (3,4) \\ (3,4) \\ (1,3) \\ \hline \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4)$	$\begin{array}{l} & \text{bis}(\rho_{IN}) \\ & \text{ative Le} \\ & \text{Value} \\ & 0.429 \\ & 0.328 \\ & 0.328 \\ & 0.328 \\ & 0.340 \\ & 0.338 \\ & 0.340 \\ & 0.338 \\ & 0.328 \\ &$	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $\frac{B_c^* \eta_c}{0.290(\eta_c)}$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $(R^*)[[\phi_1\chi_2]]\rangle$ $ R^*\rangle [[\phi_1\chi_2]\rangle\rangle$ $ R^*\rangle [[\psi_1\zeta_4]\rangle\rangle$ $ R^*\rangle [[\psi_1\zeta_4]\rangle\rangle$ $ R^*\rangle [[\psi_1\zeta_4]\rangle\rangle$ $ R^*\rangle [[\psi_1\zeta_4]\rangle\rangle$ $ R^*\rangle [[\psi_1\zeta_4]\rangle$	$c-quark:$ m_{c}^{eff} $\bar{c}-quark:$ $m_{\bar{c}}^{eff}$ $\bar{b}-quark:$ $m_{\bar{b}}^{eff}$ Interaction	$\begin{array}{c} \mbox{Contribution} \\ \mbox{$2m_c$} \\ \mbox{$2m_c$} \\ \mbox{$2m_1$} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{P}_{x_3}^2}{2m_3} \\ V^C(12) \\ \mbox{$V^C(12)$} \\ \mbox{$1p$} \\ \$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3 10.4	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 7724.8 1724.8 1724.8 -1203.5 $m_{\tilde{b}}$ 5052.9 5052.9 $-\frac{4}{3}C_{c\bar{b}}$ -4.4 $4C_{cc}$ 21.2
$\frac{1}{J^{P}} = 1^{+}$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte V^{C} Total Con Total Vave $\Psi_{tot} = 0.2$ $= 0.$ $+0.$ The rearran $\Gamma_{T,2255}(9624.6)$ The radiative	$ B_{T} \rightarrow L_{c} 2\pi h C$ $ B_{T} $ C_{11} C_{22} C_{33} Mass t Potential Energy raction $(1,2)$ $(2,3)$ $(1,4)$ Subtotal tribution function: $20 F\rangle R^{s}\rangle $ $111 F\rangle R^{s}\rangle$ gement str $,1^{+} \rightarrow B_{c} J/\psi$ we decay wi	$\begin{array}{c} \mathrm{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ (\phi_1\chi_3]\rangle + \\ [\psi_1\zeta_2]\rangle - \\ ([\psi_1\zeta_2]\rangle - \\ 0 \mathrm{ong} \ \mathrm{deca} \\ : \Gamma_{T_c^2 c \overline{c}}(d \mathrm{ths}; d \overline{c}) \\ \end{array}$	$\begin{array}{c} 9349.0\\ \hline 9349.0\\ \hline 20.2\\ 15.0\\ \hline -11097.0\\ -2563.6\\ \hline 875.4\\ -59.9\\ \hline \\ -59.9\\ \hline \\ -597.6\\ \hline 218.0\\ \hline \\ -0.396 F\\ -0.396 F\\ -0.396 F\\ -0.396 F\\ -0.396 F\\ y \ channe\\ 9624.6,1+)\\ \Gamma_{T_c2z5}(973)\\ \Gamma_{T_c2z5}(973)\\ \hline \\ \Gamma_{T_c2z5}(973)\\ \hline \end{array}$	$\begin{array}{c} 0.0 \\ 275.6 \\ \hline \\ 0.0 \\ 297.4 \\ -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ -80.2 \\ 67.9 \\ \hline \\ 100.5,9 \\ 0.5,9 \\ 0.5,2^{+}) \\ 100.5,2^{+}) \\ 0.5$	$\begin{array}{c} Relative for a state of the state o$	$\begin{array}{c} 1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0$	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_2\zeta_4]\rangle$ $z_{J/\psi}:$ $= 145.0 \text{keV}$ $= 0.007 \text{keV}$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark: $m_{\bar{b}}^{eff}$ Interaction	$\begin{tabular}{ c c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3 10.4 8.0 0.2	$\begin{array}{c} \overline{\text{Ref. [110]}} \\ \overline{\text{Ref. [110]}} \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \overline{4(m_c + m_{\bar{c}})}m_{c\bar{c}} \\ \overline{1917.8} \\ \overline{4(m_c + m_{\bar{c}})}m_{c\bar{b}} \\ \overline{1973.7} \\ \overline{3098.6} \\ -\frac{m_{\bar{c}}}{4(m_c + m_{\bar{c}})}m_{c\bar{b}} \\ -400.5 \\ \overline{4(m_c + m_{\bar{c}})}m_{c\bar{b}} \\ \overline{1917.8} \\ \overline{1917.8} \\ \overline{1917.8} \\ \overline{1517.3} \\ -\frac{m_{\bar{b}}}{4(m_c + m_{\bar{c}})}m_{c\bar{b}} \\ \overline{5929.5} \\ \overline{4726.0} \\ -\frac{4}{3}v_{c\bar{b}} \\ -2.6 \\ 4v_{cc} \\ 14.2 \\ \overline{11.6} \\ \overline{11.6} \\ \overline{11.6} \\ \overline{11.6} \\ \overline{11.6} \\ \end{array}$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 3449.6 $m_{\overline{c}}$ 1724.8 1724.8 -1203.5 $m_{\overline{b}}$ 5052.9 5052.9 $-\frac{4}{3}C_{\overline{c}\overline{b}}$ -4.4 $4C_{cc}$ 21.2 16.8
$\frac{1}{J^P} = 1^+$ Mass Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte: V^C Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0.$ +0. The rearran $\Gamma_{L^2x5}(9624.6$ The radiative The magnet	$\begin{array}{c} B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{22} \\ C_{33} \\ \hline \\ C_{23} \\ \hline \\ \hline \\ \hline \\ \hline \\ C_{23} \\ \hline \\ $	$\begin{array}{c} \mathrm{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ -300.2 \\ 503.0 \\ \hline \\ (\phi_1\chi_3]\rangle + \\ [\psi_1\zeta_2]\rangle - \\ 0 ng \ \mathrm{deca} \\ \vdots \ \Gamma_{T_c2x5}(\\ \mathrm{dths}; \\ \vdots \\ \mu_{T_c2} \\ \end{array}$	$\begin{array}{c} & & & & \\ & & &$	$\begin{array}{c} 0.0 \\ \hline 275.6 \\ \hline 0.0 \\ 297.4 \\ \hline -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ \hline 285.0 \\ \hline \\ 297.4 \\ \hline \\ -80.2 \\ 67.9 \\ \hline \\ \hline \\ 297.4 \\ \hline \\ 285.0 \\ \hline \\ R^s \rangle [\phi_2 \chi_4 \\ 285.0 \\ \hline \\ R^s \rangle [\psi_1 \zeta_1 \\ \zeta_2 \\ \hline \\ R^s \rangle [\psi_2 \zeta_1 \\ \hline \\ R^s \rangle [\psi_2 \zeta_1 \\ \hline \\ R^s \rangle [\psi_1 \zeta_1 \\ \zeta_2 \\ \hline \\ R^s \rangle [\psi_1 \zeta_1 \\ \zeta_1 \\ \hline \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\ \zeta_1 \\ \hline \\ \\ R^s \rangle [\psi_1 \\]]]] $	$\begin{array}{c} \text{Relation} \\ \text{Relation} \\ \text{Relation} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,3) \\ (1,2) \\ (2,4) \\ (3,4) \\ (1,2) \\ (3,4) \\ (3,4) \\ (1,2) \\ (3,4)$	$\begin{array}{l} & \text{(5.6)} \\ & \text{(5.6)} $	$\frac{\text{engths (fm)}}{B_c^* \eta_c}$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_2\zeta_4]\rangle$ $= 145.0\text{keV}$ $= 0.007\text{keV}$ $-0.233\mu_N$	c-quark: m_c^{eff} \bar{c} -quark: $m_{\bar{c}}^{eff}$ \bar{b} -quark: $m_{\bar{b}}^{eff}$ Interaction Matrix non	$\begin{tabular}{ c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3 10.4 8.0 -9.3	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 3449.6 3449.6 1724.8 1724.8 1724.8 1724.8 -1203.5 $m_{\overline{b}}$ 5052.9 5052.9 $-\frac{4}{3}C_{\overline{cb}}$ -4.4 $4C_{cc}$ 21.2 16.8 -100.0
$\frac{J^{P} = 1^{+}}{Mass}$ $\frac{J^{P} = 1^{+}}{Mass}$ Variational Parameters (fm ⁻²) Quark Confinemen Kinetic CS Inte: $\frac{V^{C}}{Total Con}$ Total Con Total Wave $\Psi_{tot} = 0.2$ $= 0.$ $\pm 0.$ The rearran $\Gamma_{L_{2\pi\bar{b}}(6624.6}$ The radiative The magneter The transition	$\begin{array}{c} B_T \\ \hline B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline B_{33} \\ \hline C_{33} \\ \hline C_{11} \\ C_{22} \\ C_{33} \\ \hline C_{23} \hline \hline C_{23} \\ \hline C_{23} \hline \hline C_{23} \\ \hline C_{23} \hline \hline C_{23} $	$\begin{array}{c} \mathrm{Value} \\ 9624.6 \\ 11.1 \\ 6.9 \\ 15.3 \\ 11097.0 \\ -2266.2 \\ 795.2 \\ 8.0 \\ -17.4 \\ -87.0 \\ -68.9 \\ -300.2 \\ 503.0 \\ \hline \\ (\phi_1\chi_3]\rangle + \\ [\psi_1\zeta_2]\rangle - \\ 00g \ \mathrm{deca} \\ \vdots \ \Gamma_{T_c2\pi5} \\ \mathrm{dths} \\ \vdots \\ \mathbf{s} : \ \mu_{T_c2} \\ \mathrm{ic} \ \mathrm{moment} \end{array}$	$\begin{array}{c} & & B_c^* \eta_c \\ & & B_c^* \eta_c \\ & & 9349.0 \\ & & 20.2 \\ & & 15.0 \\ & & -2563.6 \\ & & 875.4 \\ & & -2563.6 \\ & & 875.4 \\ & & -59.9 \\ \end{array}$	$\begin{array}{c} 0.0 \\ 275.6 \\ \hline \\ 0.0 \\ 297.4 \\ -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ -80.2 \\ 67.9 \\ \hline \\ 297.4 \\ 285.0 \\ \hline \\ 297.4 \\ \hline \\ 285.0 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ 285.0 \\ \hline \\ R^s\rangle [\psi_1\zeta_2 \\ \psi_1\zeta_2 \\ $	$\begin{array}{c} \text{Relation} \\ \text{Relation} \\ \text{Relation} \\ \text{Relation} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ (3,4) \\ (3,4) \\ (1,2) \\ (3,4) \\$	$\begin{array}{l} & (5.6 \mu)_{V} \\ \hline (5.6 $	engths (fm) $B_c^* \eta_c$ $0.290(\eta_c)$ $0.250(B_c^*)$ 0.204 fm $-360.4(B_c^*)$ $-237.2(\eta_c)$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $\rangle R^*\rangle [\psi_1\zeta_4]\rangle$ $\rangle R^*\rangle [\psi_2\zeta_4]\rangle$ $_{*}J/\psi$: = 145.0keV = 0.007 keV $-0.233 \mu_N$	$c-quark:$ m_{c}^{eff} $\bar{c}-quark:$ $m_{\bar{c}}^{eff}$ $\bar{b}-quark:$ $m_{\bar{b}}^{eff}$ CS Interaction Matrix non- Total contri	$\begin{tabular}{ c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 3836.0 442.8 -17.4 -87.0 -68.9 -983.0 3122.5 1918.0 230.3 14.5 -87.0 -491.5 1584.3 5343.0 122.1 14.5 -68.9 -491.5 4919.2 -2.3 10.4 8.0 -9.3 9624.6	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & \\ -\frac{1}{4}m_{cc} \\ -792.9 \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline & \\ \hline \\ \hline$	Ref. [123] $2m_c$ 3449.6 3449.6 3449.6 3449.6 3449.6 1724.8 1724.8 1724.8 1724.8 -1203.5 $m_{\tilde{b}}$ 5052.9 5052.9 5052.9 $-\frac{4}{3}C_{c\bar{c}}$ -4.4 $4C_{cc}$ 21.2 16.8 -100.0 10144

Table 14 The masses, binding energy, variational parameters, the inter-nal contribution, total wave functions, magnetic moments, transitionmagnetic moments, radiative decay widths, rearrangement strong width

ratios, and the relative lengths between quarks for the $J^P = 0^+$, $1^+ bb\bar{b}\bar{c}$ states and their lowest meson–meson thresholds. The notation is the same as that in Table 7

$bb\overline{b}\overline{c}$	The c	ontributio	on from e	each term	Rela	tive Le	engths (fm)	Overall	Present Work		CMI M	lodel
$J^P = 0^+$		Value	$B_c\eta_b$]	Difference	(i, j)	Vaule	$B_c \eta_b$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass/2	B_T	16043.9 1	5676.9	367.0	(1,2)	0.242			$2m_b$	10686.0	$-\frac{1}{4}m_{bb}$	
Variational	C_{11}	12.5	22.9		(1.3)	0.239	$0.148(\eta_b)$	<i>b</i> -quark:	$\frac{\mathbf{p}_{x_1}^2}{2m'} + \frac{m_c + m_{\bar{b}}}{2m + m_{\bar{c}}} \frac{\mathbf{p}_{x_3}^2}{2m'}$	393.8	$-2\hat{3}82.4$	
Parameters (fm ⁻²)	C_{22} C_{22}	21.7	58.8		(2.3)	0.239		o quanto	$V^{C}(12)$	97.7	$\frac{\overline{4(m_b+m_{\overline{b}})}}{5002}m_{b\overline{b}}$	
Ouark N	Vlass	17947.0 1	7947.0	0.0	(1.4)	0.249		m_b^{eff}	$\frac{1}{2}[V^{C}(13) + V^{C}(23)]$	-251.3	$\frac{5m_b}{4(m_b-1)}m_{b\overline{c}}$	$2m_{\rm b}$
Confinement	Potential	-2786.7 -	3259.9	473.2	(2, 4)	0.249	$0.235(B_{\star})$		$\frac{1}{2}[V^{C}(14) + V^{C}(24)]$	-228.3 -983.0	5929.5	10105.8
$\text{Communication for the second of the second se$							0.200(Dc)		Subtotal	9714.9	9450.2	10105.8
CS Intera	action	15.5	-111.8	127.3	(0, 1)	(3.4).	0.148 fm		m _b	5949.0	$-m_{\overline{b}}$ $m_{-\overline{c}}$	10100.0
	(1.2)	07.7	111.0	121.0	(1,2)	-(0,4). 051.2	870 1(m)		$\frac{m_{\overline{c}}}{m_{\overline{c}}} \frac{\mathbf{p}_{x_2}^2}{\mathbf{p}_{x_2}^2} + \frac{m_b}{\mathbf{p}_{x_3}} \frac{\mathbf{p}_{x_3}^2}{\mathbf{p}_{x_3}^2}$	183.8	$4(m_{\bar{c}}+m_{\bar{b}})^{m_{cb}}$	
VC	(1,2)	91.1			(1,3)	-201.0	$-879.1(\eta_b)$	\overline{b} -quark:	$\frac{m_{\bar{b}} + m_{\bar{c}}}{\frac{1}{2}V^{C}(34)}$	20.4	-1203.3 5mb	
V	(1,4)	-225.8	1000.0	479.0	(2,3)	-251.3	414 O(D)	1	$\frac{1}{2}[V^{C}(13) + V^{C}(23)]$	-251.3 -491.5	$\overline{4(m_b+m_{\overline{c}})}m_{b\overline{b}}$	m _b
	Subtotal	-820.7 -	-1293.9	473.2	(2,4)	-228.3	$-414.8(B_c)$	$m_{\overline{b}}^{e_{ff}}$	$-\frac{1}{2}D$	-101.0	5903.1	5052.9
Total Contr	ribution	78.1	-304.1	382.2	(3,4)	40.8			Subtotal	4804.4	4099.0 -m=	5052.9
Total Wave f	unction:							a quark:	$m_{\overline{b}} = \mathbf{p}_{x_2}^2 + m_b = \mathbf{p}_{x_3}^2$	1918.0 205.7	$\frac{m_c}{4(m_{\overline{c}}+m_{\overline{b}})}m_{\overline{b}\overline{c}}$	
$\Psi_{tot} = 0.308$	$ F\rangle R^s\rangle [\phi$	$_1\chi_5]\rangle - 0.$	$951 F\rangle F$	$R^s \rangle [\phi_2 \chi_6] \rangle$	y = -0	$.543 F\rangle$	$ R^s\rangle [\psi_1\zeta_5]\rangle$	c-quark.	$\frac{\overline{m_{\bar{b}} + m_{\bar{c}}}}{\frac{1}{2}W^{C}(3A)} + \frac{1}{3m_{b} + m_{\bar{c}}} \frac{1}{2m'_{3}}$	20.4	_	-400.5
-0.58	$ F\rangle R^s\rangle $	$ \psi_1\zeta_6]\rangle -$	$0.056 F\rangle$	$ R^s\rangle [\psi_2\zeta_5$	$ \rangle - 0.$	$602 F\rangle$	$ R^s\rangle [\psi_2\zeta_6]\rangle.$	$m_{\overline{c}}^{eff}$	$\frac{1}{2}[V^{C}(14) + V^{C}(24)]$	-228.3	$\frac{5m_{\overline{c}}}{4(m_b+m_{\overline{c}})}m_{b\overline{c}}$	$m_{\overline{c}}$
The rearrang	gement stro	ong decay	channel	:					$-\frac{1}{2}D$	-491.5	1973.8.8	1724.8
$\underline{\Gamma_{T_{b^2\overline{b}\overline{c}}(16043.9,}}$	$,0^+) \rightarrow B_c^* \Upsilon$	$: \Gamma_{T_{b^2 \overline{b} \overline{c}}(16)}$	043.9,0+)	$_{\rightarrow B_c \eta_b} = 1$: 1.4.				Subtotal	1524.3	1573.3	1724.8
The radiative	e decay wi	dths: Γ	$T_{b^2 \overline{b} \overline{c}}(161$	$(49.2,2^+) \to T_{l}$	$b^{2}\overline{b}\overline{c}(160)$	043.9,0+)	$\gamma = 0 \mathrm{keV}$	CC	$\frac{3}{4}V^{SS}(12)$	7.9	$4v_{bb}$	$4C_{bb}$
		Γ_T	$2 \overline{cb} (16043)$	$(.9,0^+) \rightarrow T_{c^2}$	$\bar{c}\bar{b}(16043)$	$3.2, 1^+)\gamma$	$= 10^{-6}{\rm keV}$	Interaction	299		$4v_{\overline{ab}}$	$4C_{\overline{a}\overline{b}}$
The magnetic	c moments	s: $\mu_{T_{b^2\overline{b}}}$	$\bar{c}(16043.9,$	$_{0^+)} = \langle \Psi_{to}^0 \rangle$	$\hat{\mu}_{pt}^{+} \hat{\mu^{z}} \Psi$	$\langle P_{tot}^{0^+} \rangle =$	0		$\frac{3}{4}V^{SS}(34)$	7.7	7.9	13.2
The transitio	on magneti	ic moment	ts:						Subtotal	15.5	15.5	24.8
$\mu_{T_{h^2\bar{h}\bar{\sigma}}(16149.2,}$	$(2^+) \rightarrow T_{h2\bar{h}\bar{a}}$	(16043.9,0+)	$_{\gamma} = \langle \Psi_t^2 \rangle$	$_{ot}^{+} \hat{\mu^{z}} \Psi_{tot}^{0^{+}}\rangle$	$\rangle = 0$			Matrix non	diagonal element	-15.2	-26.7	43.7
	0-00		$-/\pi^{0}$	$^{+} \hat{uz} \Psi^{1+} \rangle$		06		Total contr	ibution	16043.9	15711.9	16932.0
$\mu_{T_{b2\overline{b}\overline{a}}}(16043.9,$	$,0^+) \rightarrow T_{h2\bar{h}\bar{a}}$	(16043.2, 1+)	$\gamma = \langle \Psi_t \rangle$	ot µ *tot/	i = 0.0	$90\mu_N$						
$\frac{\mu_{T_{b^2\bar{b}\bar{c}}}(16043.9,}{J^P = 1^+}$	$(0^+) \rightarrow T_{b^2 \overline{b} \overline{c}}$	(16043.2,1+) Value	$\frac{\partial \gamma}{\partial r} = \langle \Psi_t \rangle$ $B_c^* \eta_b 1$	Difference	Rela	tive Le	engths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
$\frac{\frac{\mu_{T_b 2\bar{b}\bar{c}}(16043.9,}{J^P = 1^+}}{Mass/J}$	B_T	Value 16043.2 1	$\frac{B_c^* \eta_b}{15739.5}$	Difference 303.7	i = 0.0 Rela (i, j)	tive Le Value	engths (fm) $B_c^* \eta_b$		Contribution $2m_b$	Value 10686.0	Ref. [110] $-\frac{1}{4}m_{bb}$	Ref. [123]
$\frac{\mu_{T_b 2\bar{b}\bar{c}}(16043.9,}{J^P = 1^+}$ $\frac{Mass/J}{Variational}$	$\frac{B_T}{C_{11}}$	(16043.2,1+) Value 16043.2 1 12.4	$\frac{B_c^* \eta_b}{B_c^* \eta_b}$ 1 15739.5 20.2	Difference 303.7		utive Le Value 0.245	engths (fm) $B_c^* \eta_b$	h quark:	$\frac{2m_b}{\frac{\mathbf{p}_{x_1}^2}{2m_b} + \frac{m_b + m_z}{2m_b}}$	Value 10686.0 387.9	Ref. [110] $-\frac{1}{4}m_{bb}$ -2382.4	Ref. [123]
$\frac{\mu_{T_b 2\bar{b}\bar{c}}(16043.9,}{J^P = 1^+}$ $\frac{Mass/J}{Variational}$ Parameters (fm^{-2})	$B_T \\ C_{11} \\ C_{22} \\ C \\ $	(16043.2,1+) Value 16043.2 1 12.4 21.0 28 0	$\frac{B_c^* \eta_b}{15739.5}$ 20.2 57.4	$\int \int $	Rela (i, j) (1.2)	utive Le Value 0.245 0 240	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$	<i>b</i> -quark:	Contribution $\frac{2m_b}{\frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_b + m_c}{m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3'}}{V^C(12)}$	Value 10686.0 387.9 94.5	Ref. [110] $-\frac{1}{4}m_{bb}$ -2382.4 $\frac{5m_b}{4(m_b+m_{\overline{b}})}m_{b\overline{b}}$	Ref. [123]
$\frac{\mu_{T_b 2\bar{b}\bar{c}}(16043.9,}{J^P = 1^+}$ $\frac{Mass/J}{Variational}$ Parameters $\frac{(fm^{-2})}{Ouark}$	$B_T = \begin{bmatrix} C_{11} \\ C_{22} \\ C_{33} \end{bmatrix}$	(16043.2,1+) Value 16043.2 1 12.4 21.0 28.9	$\frac{B_c^* \eta_b}{20.2} = \frac{1}{20.2}$	$\frac{1}{0} \frac{1}{100} \frac{1}{1$	Rela (i, j) (1.2) (1.3) (2.3)	 utive Le Value 0.245 0.240 	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$	<i>b</i> -quark: m_b^{eff}	$\frac{Contribution}{2m_b} \\ \frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \end{bmatrix}$	Value 10686.0 387.9 94.5 -248.2	Ref. [110] $-\frac{1}{4}m_{bb}$ -2382.4 $\frac{5m_b}{4(m_b+m_{\overline{b}})}m_{b\overline{b}}$ 5903.1 $\frac{5m_b}{5m_b}m_{b\overline{c}}$	Ref. [123]
$\frac{\mu_{T_{b^{2}\bar{b}\pi}}(16043.9,}{J^{P}=1^{+}}$ $\frac{J^{P}=1^{+}}{Mass/J}$ Variational Parameters (fm ⁻²) Quark M Confinement	B_T C_{11} C_{22} C_{33} Mass Potential	(16043.2,1+) Value 16043.2 1 12.4 21.0 28.9 17947.0 1 -2779.8 -	$\frac{B_c^* \eta_b}{15739.5}$ 20.2 57.4 - 17947.0 3205 5	$\frac{1}{303.7}$	$ \begin{array}{c} \text{Rela} \\ (i, j) \\ (1.2) \\ (1.3) \\ (2.3) \\ (1.4) \end{array} $	Value 0.245 0.240 0.240 0.240	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$	<i>b</i> -quark: m_b^{eff}	$\frac{2m_b}{\frac{\mathbf{p}_{x_1}^2}{2m_1^2} + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2}}{V^C(12)} \\ \frac{\frac{1}{2}[V^C(13) + V^C(23)]}{\frac{1}{2}[V^C(14) + V^C(24)]}$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0	$\begin{array}{c} \hline & \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline & 5m_b \\ 4(m_b+m_{\overline{b}})m_{b\overline{b}} \\ 5903.1 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ 5929.5 \end{array}$	Ref. [123] $2m_b$ 10105 8
$\frac{\mu_{T_{b^2\bar{b}\bar{a}}}(16043.9,}{J^P = 1^+}$ $\frac{J^P = 1^+}{Mass/J}$ Variational Parameters $\frac{(fm^{-2})}{Quark M}$ Confinement Kinetic E	$B_T = \frac{C_{11}}{C_{22}} C_{33}$ Mass Potential	(16043.2,1+) Value 16043.2 1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876 1	$\frac{B_c^* \eta_b}{2} \frac{1}{2} \frac{1}$	0.0 425.7	$\begin{array}{c} \text{Rela} \\ \hline \\ (i,j) \\ (1.2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \end{array}$	value 0.245 0.240 0.240 0.250 0.250	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$	<i>b</i> -quark: m_b^{eff}	$\frac{2m_b}{\frac{\mathbf{p}_{x_1}^2}{2m_1} + \frac{m_b + m_z}{3m_b + m_z} \frac{\mathbf{p}_{x_3}^2}{2m_3^2}}{V^C(12)} \\ \frac{\frac{1}{2}[V^C(13) + V^C(23)]}{\frac{1}{2}[V^C(14) + V^C(24)]} \\ -D \\ Subtotal$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4	Ref. [110] $-\frac{1}{4}m_{bb}$ -2382.4 $\frac{5m_b}{4(m_b+m_5)}m_{b\bar{b}}$ 5903.1 $\frac{5m_b}{4(m_b+m_{\bar{c}})}m_{b\bar{c}}$ 5929.5 9450.2	Ref. [123] $2m_b$ 10105.8 10105.8
$\frac{\mu_{T_{b^2\bar{b}\bar{a}}}(16043.9,}{J^P = 1^+}$ $\frac{J^P = 1^+}{Mass/J}$ Variational Parameters (fm ⁻²) Quark M Confinement Kinetic E CS Integra	B_{T} C_{11} C_{22} C_{33} Mass Potential Chergy extion	(16043.2,1+) Value 16043.2,1 12.4 21.0 28.9 17947.0,1 -2779.8 - 876.1 4,8	$\frac{B_c^* \eta_b}{15739.5}$ 20.2 57.4 - 17947.0 3205.5 961.5 47.8	0.0 425.7 -123.3 52.6	$\begin{array}{c} \hline \text{Rela} \\ \hline (i,j) \\ \hline (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ \end{array}$	value 0.245 0.240 0.240 0.250 0.250 0.250	$\frac{\text{ngths (fm)}}{B_c^* \eta_b} \\ 0.148(\eta_b) \\ 0.250(B_c^*)$	b-quark: m_b^{eff}	$ \begin{array}{c} \hline \\ Contribution \\ \hline \\ 2m_b \\ \hline \\ \frac{\mathbf{P}_{x_1}^2}{2m_1^2} + \frac{m_{\tilde{b}} + m_{\tilde{c}}}{3m_b + m_{\tilde{c}}} \frac{\mathbf{P}_{x_3}^2}{2m_3^2} \\ V^C(12) \\ \hline \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \hline \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \\ \hline \\ Subtotal \\ \hline \\ m_{\tilde{c}} \end{array} $	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4	$\begin{array}{c} \text{Ref. [110]} \\ \hline -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline 5m_{b} \\ \hline 4(m_{b}+m_{\overline{b}})m_{b\overline{b}} \\ \hline 5903.1 \\ \hline 5903.1 \\ \hline 5929.5 \\ \hline 9450.2 \\ \hline -m_{\overline{b}} \\ m^{-1} \\ \hline m^{-1} \\ m^$	Ref. [123] $2m_b$ 10105.8 10105.8
$\frac{\mu_{T_{b^2\bar{b}\bar{a}}}(16043.9,}{J^P = 1^+}$ $\frac{J^P = 1^+}{Mass/J}$ Variational Parameters $\frac{(fm^{-2})}{Quark M}$ Confinement Kinetic E CS Intera	B_{T} C_{11} C_{22} C_{33} Mass Potential Cnergy action $(1, 2)$	(16043.2,1+) Value 16043.2,1 12.4 21.0 28.9 17947.0 1 -2779.8 876.1 4.8	$\begin{array}{c} B_c^* \eta_b \ 1\\ \hline B_c^* \eta_b \ 1\\ \hline 5739.5\\ \hline 20.2\\ 57.4\\ \hline \\ \hline \\ 7947.0\\ 3205.5\\ \hline 961.5\\ -47.8 \end{array}$	0.0 425.7 -123.3 52.6	$\begin{array}{c} \text{Rela} \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \end{array}$	value 0.245 0.240 0.240 0.250 0.250 0.250 0.320	rangths (fm) $B_c^* \eta_b$ $0.148(\eta_b)$ $0.250(B_c^*)$	b-quark: m_b^{eff}	$\begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline & \\ \frac{\mathbf{P}_{x_1}^2}{2m_1^2} + \frac{m_{\overline{b}} + m_{\overline{c}}}{3m_b + m_{\overline{c}}} \frac{\mathbf{P}_{x_3}^2}{2m_3^2} \\ & \\ & \\ & \\ & \\ V^C(12) \\ \hline & \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \hline & \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ & \\ & \\ \hline & \\ & \\ & \\ \hline & \\ & \\ &$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8		Ref. [123] 2mb 10105.8 10105.8 10105.8
$\begin{array}{c} \mu_{T_{b^2 \overline{b} \overline{c}}}(16043.9,\\ J^P = 1^+\\ \hline \\ Mass/J\\ \hline \\ Variational\\ Parameters\\ (fm^{-2})\\ \hline \\ \\ Quark M\\ \hline \\ Confinement\\ \hline \\ \hline \\ Kinetic E\\ \hline \\ \\ CS Intera\\ \hline \end{array}$	B_T C_{11} C_{22} C_{33} Mass Potential Chergy action $(1,2)$ $(2,2)$	(16043.2,1+) Value 16043.2,1 12.4 21.0 28.9 17947.0,1 -2779.8 - 876.1 4.8 94.5	$\begin{array}{c} & & & & \\ & & &$	0.0 425.7 -123.3 52.6	$\begin{array}{c} \text{Rela} \\ \hline \\ (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline \\ (1,2) \end{array}$	$\begin{array}{c} \text{iso}\mu_{N} \\ \text{value} \\ \text{Value} \\ 0.245 \\ 0.240 \\ 0.240 \\ 0.250 \\ 0.250 \\ 0.320 \\ \text{-}(3,4): \end{array}$	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm	b-quark: m_b^{eff} \overline{b} -quark:	$ \begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline & \\ \hline & \\ \frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_{\tilde{b}} + m_{\tilde{c}}}{3m_b + m_{\tilde{c}}} \frac{\mathbf{p}_{x_3}^2}{2m_3}}{W^C(12)} \\ & \\ & \\ \hline & \\ \frac{1}{2}[V^C(13) + V^C(23)] \\ \hline & \\ \frac{1}{2}[V^C(14) + V^C(24)] \\ & -D \\ \hline \\ \hline & \\ \frac{m_{\tilde{c}}}{m_{\tilde{b}} + m_{\tilde{c}}} \frac{\mathbf{p}_{x_2}^2}{2m_2'} + \frac{m_b}{3m_b + m_{\tilde{c}}} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ \hline & \\ \hline \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline & \\ \hline & \\ \hline \hline & \\ \hline \hline & \\ \hline \hline \\ \hline \hline & \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9		Ref. [123] 2mb 10105.8 10105.8 10105.8
$\begin{array}{c c} \mu_{T_{b}2\overline{b}\overline{c}}(16043.9,\\ \hline J^P=1^+\\ \hline Mass/J\\ \hline Variational\\ Parameters\\ (fm^{-2})\\ \hline Quark M\\ \hline Confinement\\ \hline Kinetic E\\ \hline CS Intera\\ \hline V^C \end{array}$	B_T C_{11} C_{22} C_{33} Mass Potential Energy action $(1,2)$ $(2,3)$ $(4,1)$	(16043.2,1+ Value 16043.2 1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876.1 4.8 94.5 -248.2 202.6	$\begin{array}{c} B_c^* \eta_b \\ B_c^* \eta_b \\ \hline 5739.5 \\ \hline 20.2 \\ 57.4 \\ \hline - \\ 7947.0 \\ \hline 3205.5 \\ 961.5 \\ -47.8 \end{array}$	0.0 425.7 -123.3 52.6	$\begin{array}{c} \text{Rela} \\ \hline \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline \\ (1,2) \end{array}$	value Value 0.245 0.240 0.240 0.250 0.250 0.250 0.320 -(3,4):	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm	b-quark: m_b^{eff} \overline{b} -quark:	$\begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline & \\ & \\ \hline & \\ & \\$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -401.5		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\bar{b}}$
$\frac{\mu_{T_b 2\bar{u}\bar{c}}(16043.9,}{J^P = 1^+}$ $\frac{J^P = 1^+}{Mass/J}$ Variational Parameters (fm ⁻²) Quark M Confinement Kinetic E CS Intera V^C	B_T C_{11} C_{22} C_{33} Mass Potential Caregy action $(1,2)$ $(2,3)$ $(1,4)$	(16043.2,1+ Value 16043.2 1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876.1 4.8 94.5 -248.2 -225.8	$\begin{array}{c} & & & \\ & & & \\ & & & \\$	0.0 425.7 -123.3 52.6	$\begin{array}{c} \hline \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \end{array}$	Hold No. 2017 No.	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$	b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c} \hline & \\ & \\ & \\ & \\ \hline & \\ & \\ & \\ & \\ & \\$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9
$\frac{\mu_{T_{b^2\bar{b}\pi}}(16043.9,}{J^P = 1^+}$ $\frac{J^P = 1^+}{Mass/J}$ Variational Parameters (fm ⁻²) Quark M Confinement Kinetic E CS Intera V^C	B_{T} C_{11} C_{22} C_{33} Mass Potential Chergy action (1,2) (2,3) (1,4) Subtotal U	(16043.2,1+ Value 16043.2,1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876.1 4.8 94.5 -248.2 -225.8 -813.7 -	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b 1$ 5739.5 20.2 57.4 - 7947.0 3205.5 961.5 -47.8 - -1239.5	0.0 425.7 -123.3 52.6	$\begin{array}{c} \hline \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2.3) \\ (1.4) \\ (2.4) \\ (3.4) \\ \hline (1.2) \\ (2.4) \\ (3.4) \\ \hline (3.4) \\ \hline (3.4) \\ \hline \end{array}$	 1990µN 1990µN	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$	b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c} \text{Contribution} \\ & 2m_b \\ & \frac{2m_b}{2m_1} + \frac{m_b + m_\pi}{2m_3} \frac{p_{x_3}^2}{2m_1^2} + \frac{m_b + m_\pi}{2m_3^2} \frac{2m_3^2}{2m_3^2} \\ & V^C(12) \\ & \frac{1}{2} [V^C(13) + V^C(23)] \\ & \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ & \frac{m_\pi}{2m_b + m_\pi} \frac{p_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_\pi} \frac{p_{x_3}^2}{2m_3^2} \\ & \frac{1}{2} [V^C(34) \\ & \frac{1}{2} [V^C(13) + V^C(23)] \\ & -\frac{1}{2} D \\ \hline \\ & \text{Subtotal} \\ \end{array}$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9
$\begin{array}{c c} \mu_{T_{b^2 \overline{b} \overline{c}}}(16043.9,\\ \hline J^P = 1^+\\ \hline \\ \hline \\ Mass/J\\ \hline \\ Variational\\ Parameters\\ (fm^{-2})\\ \hline \\ \\ Quark M\\ \hline \\ Confinement\\ \hline \\ Kinetic E\\ \hline \\ \\ CS Intera\\ \hline \\ \\ V^C\\ \hline \\ \hline \\ \hline \\ \hline \\ Total Contral \\ \hline \end{array}$	$\begin{array}{c} {}_{,0^+) \rightarrow T_b 2 \overline{b} \overline{c}^{,1}} \\ \\ \hline \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	(16043.2,1+ Value 16043.2,1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876.1 4.8 94.5 -248.2 -225.8 -813.7 - 67.1	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b$ 1 15739.5 20.2 57.4 - 7947.0 3205.5 961.5 -47.8 - -1239.5 -241.5	0.0 425.7 -123.3 52.6 425.7 -308.6	$\begin{array}{c} \hline \text{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \end{array}$	 100µN 100µN 1100 <	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$	b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$	$\begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline & \\ & \\ \hline & \\ & \\$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 204.4		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9
$\begin{array}{c} \mu_{T_{b^2 \overline{b} \overline{c}}}(16043.9,\\ \hline J^P = 1^+\\ \hline Mass/J\\ \hline Variational\\ Parameters\\ (fm^{-2})\\ \hline Quark M\\ \hline Confinement\\ \hline Kinetic E\\ \hline CS Intera\\ \hline V^C\\ \hline \hline Total Contral\\ \hline Total Contral\\ \hline \end{array}$	$\begin{array}{c} B_{T} \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ Mass \\ Potential \\ Chergy \\ action \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ \hline \\ ribution \\ \hline \\ \hline \\ unction: \\ \hline \end{array}$	(16043.2,1+ Value 16043.2 1 12.4 21.0 28.9 17947.0 1 -2779.8 - 876.1 4.8 94.5 -248.2 -225.8 -813.7 - 67.1	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b 1$ 5739.5 20.2 57.4 - 7947.0 3205.5 961.5 -47.8 -47.8	0.0 425.7 -123.3 52.6 425.7 308.6	$\begin{array}{c} \hline \mathbf{Rela} \\ \hline \mathbf{Rela} \\ \hline (i,j) \\ \hline (1.2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ \hline (1,2) \\ (2,4) \\ (3,4) \\ \hline (3,4) \\ (1,3) \\ \end{array}$	 100µN 100µN 1100 <	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$	b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$ \overline{c} -quark:	$ \begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline \hline \\ \hline & \\ \hline & \\ \hline \hline \\ \hline & \\ \hline \\ \hline$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9	$\begin{array}{c} \text{Ref. [110]} \\ \hline -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline \frac{5m_b}{4(m_b+m_{\overline{b}})}m_{b\overline{b}} \\ 5903.1 \\ \hline \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ 5929.5 \\ \hline 9450.2 \\ \hline \frac{-m_{\overline{b}}}{4(m_{\overline{c}}+m_{\overline{b}})}m_{\overline{b}\overline{c}} \\ -1203.5 \\ \hline \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline 5903.1 \\ \hline 4699.6 \\ \hline \frac{-m_{\overline{c}}}{4(m_{\overline{c}}+m_{\overline{b}})}m_{\overline{b}\overline{c}} \\ -400.5 \\ \end{array}$	Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9
$\begin{array}{c} \mu_{T_{b}2\overline{b}\overline{c}}(16043.9,\\ J^{P}=1^{+}\\ \hline\\ Mass/J\\ \hline\\ Variational\\ Parameters\\ (fm^{-2})\\ \hline\\ Quark M\\ Confinement\\ \hline\\ Kinetic E\\ CS Intera\\ \hline\\ V^{C}\\ \hline\\ \hline\\ Total Contr\\ \hline\\ Total Contr\\ \hline\\ Total Wave fr\\ \Psi_{tot}=0.17\\ \hline\end{array}$	$\begin{array}{c} B_{T} \\ \hline \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\$	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3]\rangle + \end{array}$	$\begin{array}{c} & & & \\ & & & \\ & & & \\$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_{c}] \\ \end{array}$	$\begin{array}{c} \hline \mathbf{Rela} \\ \hline \mathbf{Rela} \\ \hline (i,j) \\ (1.2) \\ (1.3) \\ (2.3) \\ (1.4) \\ (2.4) \\ (2.4) \\ (1.2) \\ (2.4) \\ (3.4) \\ (1.3) \\ \hline (1.3) \\ (1.3) \\ \end{array}$	$\begin{array}{l} & \text{(i)} \\ & $	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\phi_1\chi_2]\rangle$	$b-\text{quark:}$ m_b^{eff} $\overline{b}-\text{quark:}$ $m_{\overline{b}}^{eff}$ $\overline{c}-\text{quark:}$ $m_{\overline{c}}^{eff}$	$\begin{array}{c} \hline & \\ & \\ \hline & \\ & \\ \hline \\ \hline$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\bar{b}}$ 5052.9 5052.9 $m_{\bar{c}}$
$\begin{array}{c} \mu_{T_{b}2\overline{b}\overline{c}}(16043.9,\\\\ J^P = 1^+\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\\\Kinetic E\\\\CS Intera\\\\V^C\\\\\hline\\Total Contr\\\\Total Contr\\\\Total Wave fr\\\\\Psi_{tot} = 0.17\\\\= 0.43\end{array}$	$\begin{array}{c} B_T\\ \hline \\ C_{11}\\ C_{22}\\ C_{33}\\ \hline \\ Potential\\ \hline \\ Potential\\ \hline \\ chergy\\ action\\ \hline \\ (1,2)\\ (2,3)\\ (1,4)\\ \hline \\ Subtotal\\ \hline \\ ribution\\ \hline \\ ribution\\ \hline \\ Table (R^s) \\ Subtotal\\ \hline \\ Subtotal\\ \hline \\ ribution\\ \hline \\ Subtotal\\ \hline \\ ribution\\ \hline \\ Subtotal\\ \hline \\ Subtotal\\ \hline \\ Final (R^s) \\ Subtotal\\ \hline \\ \\ \\ Subtotal\\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ [\phi_1\chi_3]) + +\\ [[\psi_1\zeta_2]) - \end{array}$	$\begin{array}{c} B_c^* \eta_b \\ B_c^* \eta_b \\ 1\\ 5739.5 \\ 20.2 \\ 57.4 \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ $	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \phi_3 [\psi_1 \zeta_5]] \\ \hline \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \hline \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \hline \\ \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \\ \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \\ \\ \\ \\ R^s\rangle [\psi_1 \zeta_5]] \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \end{array}$ $\begin{array}{c} (2,4) \\ (3,4) \\ (1,3) \\ (1,3) \\ \end{array}$	$\begin{array}{l} & \text{(i)} \\ & $	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$	$b-\text{quark:}$ m_b^{eff} $\overline{b}-\text{quark:}$ $m_{\overline{b}}^{eff}$ $\overline{c}-\text{quark:}$ $m_{\overline{c}}^{eff}$	$\begin{array}{r} \begin{array}{c} \text{Contribution} \\ \hline & 2m_b \\ \hline & 2m_b \\ \hline & \frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ & V^C(12) \\ \hline & \frac{1}{2}[V^C(13) + V^C(23)] \\ \hline & \frac{1}{2}[V^C(14) + V^C(24)] \\ & -D \\ \hline \\ \hline & \frac{m_c}{m_b - \frac{\mathbf{p}_{x_2}^2}{2m_2'}} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ \hline & \frac{1}{2}[V^C(13) + V^C(23)] \\ \hline & \frac{1}{2}[V^C(13) + V^C(23)] \\ & -\frac{1}{2}D \\ \hline \\ \hline & \\ \hline \\ & \frac{m_b}{m_b - \frac{\mathbf{p}_{x_2}^2}{2m_2'}} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3'} \\ \hline \\ & \frac{m_b}{12}[V^C(14) + V^C(24)] \\ \hline \\ & \frac{1}{2}[V^C(14) + V^C(24)] \\ \hline \\ & \frac{1}{2}[V^C(14) + V^C(24)] \\ & -\frac{1}{2}D \end{array}$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5	$\begin{array}{c} \hline & \\ \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline & \\ 5903.1 \\ \hline & \\ \hline \\ \hline$	Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8
$\begin{array}{c} \mu_{T_{b}2\overline{b}\overline{c}}(16043.9,\\\\ J^P = 1^+\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\\\Kinetic E\\\\CS Intera\\\\V^C\\\\\hline\\Total Contr\\\\Total Contr\\\\Total Wave fr\\\\\Psi_{tot} = 0.17\\\\= 0.4;\\+0.2e\\\end{array}$	$\begin{array}{c} B_T \\ \hline \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C$	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) \\ (\phi_1\chi_3)\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3])$	$\begin{array}{c} & P_{\gamma} - \langle \psi_t \\ B_c^* \eta_b & 1 \\ \hline B_c^* \eta_b & 1 \\ \hline S_c^* 739.5 \\ \hline 20.2 \\ 57.4 \\ \hline \\ -1239.5 \\ \hline \\ -1239.5 \\ \hline \\ -47.8 \\ \hline \\ -1239.5 \\ \hline \\ -241.5 \\ \hline \\ 0.984 F \rangle \\ 0.467 F \\ 0.188 F \\ \end{array}$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \phi_3 [\psi_1\zeta_3] \\ R^s\rangle [\psi_2\zeta_3] \\ R^s\rangle [\psi_2\zeta_3] \\ \hline \end{array}$	$\begin{array}{c} \hline \mathbf{Rela} \\ \hline \mathbf{Rela} \\ \hline (i,j) \\ \hline (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ \hline (1,2) \\ \hline (2,4) \\ (1,2) \\ \hline (2,4) \\ (1,2) \\ \hline (3,4) \\ \hline (1,3) \\ \hline (1,3)$	$\begin{array}{c} \text{iso} \mu_N \\ \text{trive Le} \\ \hline \text{Value} \\ 0.245 \\ 0.240 \\ 0.250 \\ 0.250 \\ 0.250 \\ 0.320 \\ -(3,4): \\ -225.8 \\ 39.7 \\ -248.2 \\ 0.044 F\rangle \\ 0.044 F\rangle \\ 0.500 F\rangle \\ 0.502 F\rangle \end{array}$	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$	$b-\text{quark:}$ m_b^{eff} $\overline{b}-\text{quark:}$ $m_{\overline{b}}^{eff}$ $\overline{c}-\text{quark:}$ $m_{\overline{c}}^{eff}$	$ \begin{array}{c} \hline \text{Contribution} \\ \hline 2m_b \\ p_{x_1}^2 + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_1^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \\ \hline \\ \hline \\ \hline \\ \frac{m_c}{m_b + m_c} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ -\frac{1}{2} D \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \frac{m_b}{m_b - m_c} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{m_b}{2} V^C(34) \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -\frac{1}{2} D \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \\ \hline \\ \hline \\ \\ \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \\ \hline \hline$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0	$\begin{array}{c} \hline & \ \ \ \ \ \ \ \ \ \ \ \ \$	Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8 1724.8
$\frac{\mu_{T_b 2\bar{b} \bar{c}}(16043.9,}{J^P = 1^+}$ Mass/J Variational Parameters (fm ⁻²) Quark M Confinement Kinetic E CS Intera V^C Total Contri Total Wave fr $\Psi_{tot} = 0.17$ = 0.43 ± 0.22	$\begin{array}{c} B_{T} \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} $	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3]\rangle + (\phi_1\chi_3)\rangle +$	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b 1$ 5739.5 20.2 57.4 - 3205.5 961.5 -47.8 - 1239.5 -241.5 $0.984 F\rangle$ 0.467 F 0.188 F channel	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \langle R^s\rangle [\psi_1\zeta_2 \\ \langle R^s\rangle [\psi_1\zeta_2 \\ \langle R^s\rangle [\psi_2\zeta_3 \\ \vdots \\ \Gamma_{T_b^2 \overline{h_s} c} [\psi_1\zeta_3 \\ \langle R^s\rangle [\psi$	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \end{array}$ $\begin{array}{c} (2,4) \\ (3,4) \\ (1,3) \\ (3,4) \\ (1,3) \\ (3,4) \\ $	$\begin{array}{l} & \text{(i)} & ($	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_2\zeta_4]\rangle$ $\gamma :$	b-quark: m_b^{eff} \overline{b} -quark: $m_{\overline{b}}^{eff}$ \overline{c} -quark: $m_{\overline{c}}^{eff}$	$ \begin{array}{c} \hline \text{Contribution} \\ \hline 2m_b \\ p_{x_1}^2 + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_1^2} \\ V^C(12) \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(14) + V^C(24)] \\ -D \\ \hline \\ \hline \\ \hline \\ \frac{m_c}{m_b - \mathbf{p}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ \frac{1}{2} [V^C(13) + V^C(23)] \\ -\frac{1}{2} D \\ \hline \\ \hline \\ \\ \hline \\ \frac{m_b}{m_b - \mathbf{p}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \frac{m_b}{2m_b - \mathbf{p}} \frac{\mathbf{p}_{x_2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x_3}^2}{2m_3^2} \\ \hline \\ \hline \\ \\ \frac{m_b}{2} [V^C(14) + V^C(24)] \\ -\frac{1}{2} D \\ \hline \\ \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \hline \\ \\ \hline \\ \hline \\ \hline \\ \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \\ \hline \hline$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -2.6		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8 1724.8 $-\frac{4}{3}C_{\overline{b}\overline{c}}$
$\begin{array}{c} \mu_{T_{b}2\overline{b}\overline{c}}(16043.9,\\\\ J^P=1^+\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\Kinetic E\\\\CS Intera\\\\\hline\\V^C\\\\\hline\\Total Contri\\\\Total Contri\\\\Total Wave fr\\\Psi_{tot}=0.17\\\\=0.43\\\\+0.24\\\\\hline\\The rearrang\\\\\Gamma_{t_{b}2\overline{b}\overline{c}}(16043.2,\\\\\end{array}$	$\begin{array}{c} B_{T} \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{$	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0, 1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ [\phi_1\chi_3]\rangle + ([\psi_1\zeta_2]\rangle - \\ [[\psi_1\zeta_2]\rangle - \\ [[\psi_2\zeta_2]\rangle - \\ 0 ng \ decay\\ \vdots \ \Gamma_{L^2\bar{L}^n}(16) \end{array}$	$\begin{array}{c} & \gamma = \langle \psi_t \\ B_c^* \eta_b & 1 \\ \hline B_c^* \eta_b & 1 \\ \hline 15739.5 \\ \hline 20.2 \\ 57.4 \\ \hline \\ -1239.5 \\ \hline 3205.5 \\ 961.5 \\ -47.8 \\ \hline \\ 961.5 \\ -47.8 \\ \hline \\ -47.8 \\ \hline \\ 0.188 F \\ 0.188 F \\ channel \\ 043.2,1^+) - \\ \end{array}$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ 425.7 \\ \hline -123.3 \\ 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ 425.7 \\ \hline \\ 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \langle R^s\rangle [\psi_1\zeta_5] \\ \langle R^s\rangle [\psi_2\zeta_5] \\ \hline \\ \Gamma_{T_b2bc}(\\ \rightarrow B_c^*\eta_b = 1 \\ \end{array}$	$\begin{array}{c} \hline \mathbf{Rela} \\ \hline \mathbf{Rela} \\ \hline (i,j) \\ \hline (1.2) \\ (1.3) \\ (2,3) \\ (1,4) \\ (2,4) \\ \hline (1,2) \\ \hline (1,2) \\ \hline (1,2) \\ \hline (1,2) \\ \hline (1,3) \\ \hline ($	$\begin{array}{l} & (1,2) \\$	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\psi_1 \chi_2]\rangle$ $ R^*\rangle [\psi_1 \zeta_4]\rangle$ $ R^*\rangle [\psi_2 \zeta_4]\rangle$ $r_{\Upsilon}:$	$b-quark:$ m_b^{eff} $\overline{b}-quark:$ $m_{\overline{b}}^{eff}$ $\overline{c}-quark:$ $m_{\overline{c}}^{eff}$ CS Interaction	$\begin{array}{r} \begin{array}{c} & \text{Contribution} \\ & & 2m_b \\ & \frac{\mathbf{p}_{x1}^2}{2m_1} + \frac{m_b + m_c}{3m_b + m_c} \frac{\mathbf{p}_{x3}^2}{2m_3} \\ & V^C(12) \\ & \frac{1}{2} [V^C(13) + V^C(23)] \\ & \frac{1}{2} [V^C(14) + V^C(24)] \\ & -D \\ \hline \\ & & \text{Subtotal} \\ \end{array} \\ \hline \\ & \frac{m_c}{m_b - m_c} \frac{\mathbf{p}_{x2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x3}^2}{2m_3^2} \\ & \frac{1}{2} [V^C(34) + V^C(23)] \\ & \frac{1}{2} [V^C(31) + V^C(23)] \\ & -\frac{1}{2} D \\ \hline \\ & \text{Subtotal} \\ \hline \\ & \frac{m_c}{m_b + m_c} \frac{\mathbf{p}_{x2}^2}{2m_2^2} + \frac{m_b}{3m_b + m_c} \frac{\mathbf{p}_{x3}^2}{2m_3^2} \\ & \frac{1}{2} [V^C(14) + V^C(24)] \\ & \frac{1}{2} [V^C(14) + V^C(24)] \\ & -\frac{1}{2} D \\ \hline \\ & \text{Subtotal} \\ \hline \\ & -\frac{1}{4} V^{SS}(12) \\ \hline \end{array} $	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -226		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8 1724.8 $-\frac{4}{3}C_{\overline{b}\overline{c}}$ $-\frac{4}{4}C_{\overline{b}\overline{c}}$
$\begin{array}{c} \mu_{T_{b^2 \overline{b} \overline{c}}}(16043.9,\\\\ J^P = 1^+\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\Kinetic E\\\\CS Intera\\\\\hline\\V^C\\\\\hline\\Total Contr\\\\Total Contr\\\\Total Wave fr\\\\\Psi_{tot} = 0.17\\\\= 0.42\\\\+0.24\\\\The rearrang\\\\\hline\\\Gamma_{b^2 \overline{b} \overline{c}}(16043.2,\\The radiativ\\\\\end{array}$	$\begin{array}{c} {}_{,0^+) \rightarrow T_b 2 \overline{b} \overline{c}'} \\ \\ B_T \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0, 1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3] \rangle + ([\psi_1\zeta_2]) - \\ [[\psi_1\zeta_2] \rangle - \\ [[\psi_2\zeta_2] \rangle - \\ 0ng \ decay\\ \vdots \ \Gamma_{T_k2E_k}(16\\ idths: \ \Gamma_{T_i} \\ \end{array}$	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b 1$ $B_c^* \eta_b 1$ 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.2 57.4 - 20.5 - 20.5 - 20.5 - 20.5 - - 20.5 - - - - - - - -	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ 425.7 \\ \hline -123.3 \\ 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ 425.7 \\ \hline \\ 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \langle R^s\rangle [\psi_1\zeta_5 \\ \langle R^s\rangle [\psi_1\zeta_5 \\ \langle R^s\rangle [\psi_2\zeta_5 \\ \hline \\ \Gamma_{b_2b_5} \\ \hline \\ \rightarrow B^*_c\eta_b = 1 \\ \hline \\ 0.2,2^+) \rightarrow T_{-2} \end{array}$	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ (1,2)^2 \\ (1,2)^2 \\ (1,2)^2 \\ (1,3)^2 \\ ($	$\begin{array}{l} & \text{iso} \mu_N \\ & \text{trive Le} \\ & \text{Value} \\ & 0.245 \\ & 0.240 \\ & 0.240 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.320 \\ & -(3,4): \\ & -225.8 \\ & 39.7 \\ & -225.8 \\ & -25.8 \\ & -25.8 \\ & -25.8 \\ & -25.8 \\ & -25.8 \\ & -25$	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_2\zeta_4]\rangle$ $r :$ $= 435.0 \text{keV}$	$b-quark:$ m_b^{eff} $\overline{b}-quark:$ $m_{\overline{b}}^{eff}$ $\overline{c}-quark:$ $m_{\overline{c}}^{eff}$ CS Interaction	$ \begin{array}{c} \hline & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -225.0 -22.6 7.5	$\begin{array}{c} \hline & \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{b}})}m_{b\overline{b}} \\ \hline & 5903.1 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & 5929.5 \\ \hline & 9450.2 \\ \hline & \frac{-m_{\overline{b}}}{4(m_c+m_{\overline{c}})}m_{\overline{b}\overline{c}} \\ \hline & -1203.5 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & 5903.1 \\ \hline & 4699.6 \\ \hline & \frac{-m_{\overline{c}}}{4(m_c+m_{\overline{b}})}m_{\overline{b}\overline{c}} \\ \hline & -400.5 \\ \hline & \frac{5m_r}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & 1973.8 \\ \hline & 1573.3 \\ \hline & -\frac{4}{3}v_{\overline{b}\overline{c}} \\ -2.6 \\ \hline & 4v_{bb} \\ \hline & 7.7 \\ \hline \end{array}$	$\begin{array}{c c} & \text{Ref. [123]} \\ & 2m_b \\ 10105.8 \\ \hline \\ 10105.8 \\ \hline \\ 10105.8 \\ \hline \\ 5052.9 \\ \hline \\ 5052.9 \\ \hline \\ 5052.9 \\ \hline \\ \\ 5052.9 \\ \hline \\ \hline \\ 1724.8 \\ \hline \\ 11.6 \\ \hline \end{array}$
$\begin{array}{c} \mu_{T_{b^2 \overline{b} \overline{c}}}(16043.9,\\ \hline J^P = 1^+\\ \hline Mass/J\\ \hline Variational\\ Parameters\\ (fm^{-2})\\ \hline Quark M\\ \hline Confinement\\ \hline Kinetic E\\ \hline CS Intera\\ \hline V^C\\ \hline \hline Total Contr\\ \hline Total Contr\\ \hline Total Wave fr\\ \Psi_{tot} = 0.17\\ = 0.43\\ \pm 0.24\\ \hline The rearrang\\ \hline \Gamma_{T_{b^2 \overline{b} \overline{c}}}(16043.2,\\ \hline The radiativ\\ \hline \end{array}$	$\begin{array}{c} B_{T} \\ \hline \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ Mass \\ Potential \\ \hline \\ Concept \\ action \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ \hline \\ ribution \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ \hline \\ ribution \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ Subtotal \\ \hline \\ ribution \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ (2,3) \\ (1,4) \\ \hline \\ (2,3) \\ (1,4) \\ \hline \\ (2,3) \\ (1,2) \\ (2,3) \\ (1,4) \\ \hline \\ (1,2) \\ (2,3) \\ (1,4) \\ (2,3) \\ (1,4) \\ (2,3) \\ (1,4) \\ (2,3) \\ (1,4) \\ (2,3) \\ (1,4) \\ (2,3) \\$	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -876.1\\ -248.2\\ -225.8\\ -10000\\ -2000\\ -2000\\$	$\begin{array}{c} & \gamma = \langle \psi_t \\ B_c^* \eta_b \ 1 \\ \hline B_c^* \eta_b \ 1 \\ \hline S739.5 \\ \hline 20.2 \\ 57.4 \\ \hline - \\ 7947.0 \\ 3205.5 \\ 961.5 \\ \hline - \\ 961.5 \\ \hline - \\ -47.8 \\ \hline 961.5 \\ \hline -47.8 \\ \hline 0.984 F \rangle \\ 0.984 F \rangle \\ 0.984 F \rangle \\ 0.0984 F \rangle \\ 0.0984 F \rangle \\ 0.188 F \\ \hline 0.18$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline \\ 308.6 \\ \hline \\ \hline \\ 8^s \rangle [\phi_2 \chi_4 \\ \gamma \chi_5 \\ \rangle R^s \rangle [\psi_2 \zeta_5 \\ \hline \\ \gamma R^s \rangle $	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ \hline (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ (1,2) \\ (2,4) \\ (1,2) \\ (3,4) \\ (1,3) \\ \end{array}$	$\begin{array}{l} & \text{(1)} \\ & $	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $\langle R^*\rangle [\psi_1\zeta_4]\rangle$ $\langle R^*\rangle [\psi_2\zeta_4]\rangle$ $\langle T^*\rangle =435.0\text{keV}$ $\zeta_1=10^{-6}\text{keV}$	$b-quark:$ m_b^{eff} $\overline{b}-quark:$ $m_{\overline{b}}^{eff}$ $\overline{c}-quark:$ $m_{\overline{c}}^{eff}$ CS Interaction	$\begin{tabular}{ c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -226 7.5	$\begin{array}{c} \hline & \text{Ref. [110]} \\ \hline & -\frac{1}{4}m_{bb} \\ -2382.4 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{b}})}m_{b\overline{b}} \\ \hline & 5903.1 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & 5929.5 \\ \hline & 9450.2 \\ \hline & \frac{-m_{\overline{b}}}{4(m_c+m_{\overline{b}})}m_{b\overline{c}} \\ \hline & -1203.5 \\ \hline & \frac{5m_b}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & -1203.5 \\ \hline & \frac{5m_b}{4(m_{\overline{c}}+m_{\overline{b}})}m_{b\overline{c}} \\ \hline & -400.5 \\ \hline & \frac{5m_\pi}{4(m_{\overline{c}}+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & -400.5 \\ \hline & \frac{5m_\pi}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & -400.5 \\ \hline & \frac{5m_\pi}{4(m_b+m_{\overline{c}})}m_{b\overline{c}} \\ \hline & -2.6 \\ \hline & 4v_{bb} \\ \hline & 7.7 \\ \hline & 5.0 \\ \hline \end{array}$	$\begin{array}{c c} & \text{Ref. [123]} \\ & 2m_b \\ 10105.8 \\ \hline \\ 10105.8 \\ \hline \\ 5052.9 \\ \hline \\ 5052.9 \\ \hline \\ 5052.9 \\ \hline \\ \\ 5052.9 \\ \hline \\ \\ 1724.8 \\ \hline \\ 100000000000000000000000000000000$
$\begin{array}{c} \mu_{T_{b^{2}\overline{b}\pi}}(16043.9,\\\\ J^{P}=1^{+}\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\\\Kinetic E\\\\CS Intera\\\\V^{C}\\\\\hline\\Total Contr\\\\Total Contr\\\\Total Wave fn\\\\\Psi_{tot}=0.17\\\\=0.4;\\+0.2e\\\\The rearrang\\\\\Gamma_{h^{2}\overline{b}\pi}(16043.2,\\The radiativ\\\\\hline\end{array}$	$\begin{array}{c} \begin{array}{c} B_T\\ \hline\\ C_{11}\\ C_{22}\\ C_{33}\\ \hline\\ C_{33}\\ \hline\\ Mass\\ Potential\\ \hline\\ Corrector \\ C_{33}\\ \hline\\ Mass\\ Potential\\ \hline\\ Corrector \\ C_{33}\\ \hline\\ (1,2)\\ (2,3)\\ (1,4)\\ \hline\\ Subtotal\\ \hline\\ ribution\\ \hline\\ (1,2)\\ (2,3)\\ (1,4)\\ \hline\\ Subtotal\\ \hline\\ ribution\\ \hline\\ Tibution\\ \hline\\ Tibution\\ \hline\\ Tibution\\ \hline\\ (1,2)\\ (2,3)\\ (1,4)\\ \hline\\ Subtotal\\ \hline\\ ribution\\ \hline\\ Tibution\\ $	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3]\rangle + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([\phi_1\chi_3])) + ([\phi_1\chi_3]) + ([$	$\gamma - \langle \psi_t \rangle$ $B_c^* \eta_b 1$ $\overline{B_c^* \eta_b 1}$ $\overline{B_c^* \eta_b 1}$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline \\ -123.3 \\ \hline \\ 52.6 \\ \hline \\ \hline \\ 425.7 \\ \hline \\ -123.3 \\ \hline \\ 52.6 \\ \hline \\ \hline \\ \hline \\ 1000000000000000000000000$	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ (1,3) \\ ($	$\begin{array}{l} & \text{(1)} \\ & $	$\frac{\text{engths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $-879.1(\eta_b)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $\langle R^*\rangle [\psi_1\zeta_4]\rangle$ $\langle R^*\rangle [\psi_2\zeta_4]\rangle$ $\langle qr:$ $= 435.0 \text{keV}$ $-0.346\mu_N$	$b-quark:$ m_b^{eff} $\overline{b}-quark:$ $m_{\overline{b}}^{eff}$ $\overline{c}-quark:$ $m_{\overline{c}}^{eff}$ Interaction Matrix non	$\begin{tabular}{ c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -22.6 7.5 4.8 4.8		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8 1724.8 $-\frac{4}{3}C_{\overline{b}\overline{c}}$ -4.4 $4C_{bb}$ 11.6 7.2 24.3
$\begin{array}{c} \mu_{T_{b^{2}\overline{b}\overline{c}}}(16043.9,\\\\ J^{P}=1^{+}\\\\\hline Mass/J\\\\Variational\\Parameters\\(fm^{-2})\\\\Quark M\\\\Confinement\\\\Kinetic E\\\\CS Intera\\\\\hline\\V^{C}\\\\\hline\\Total Contr\\\\Total Contr\\\\Total Wave fr\\\\\Psi_{tot}=0.17\\\\=0.4;\\+0.2e\\\\The rearrang\\\\\hline{\Gamma_{b^{2}\overline{b}\overline{c}}}(16043.2,\\The radiativ\\\\\hline\\The magnetic\\\\The transitio\\\\\hline\end{array}$	$\begin{array}{c} 0^{+} \rightarrow T_{b} 2_{b} z_{b} \\ \hline \\ B_{T} \\ \hline \\ C_{11} \\ C_{22} \\ C_{33} \\ \hline \\ C_{33} \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\begin{array}{c} (16043.2,1+)\\ \hline \\ Value\\ 16043.2,1\\ 12.4\\ 21.0\\ 28.9\\ 17947.0,1\\ -2779.8\\ -\\ 876.1\\ 4.8\\ 94.5\\ -248.2\\ -225.8\\ -813.7\\ -\\ 67.1\\ \hline \\ (\phi_1\chi_3]\rangle + (\phi_1\chi_3)\rangle +$	$\begin{array}{c} & \gamma = \langle \psi_t \\ B_c^* \eta_b \ 1 \\ \hline B_c^* \eta_b \ 1 \\ \hline S_c^* \eta_b$	$\begin{array}{c} 0.0 \\ \hline 0.0 \\ \hline 425.7 \\ \hline -123.3 \\ \hline 52.6 \\ \hline \\ 425.7 \\ \hline 308.6 \\ \hline \\ R^s\rangle [\phi_2\chi_4 \\ \langle R^s\rangle [\psi_1\zeta_5 \\ \langle R^s\rangle [\psi_1\zeta_5 \\ \langle R^s\rangle [\psi_2\zeta_5 \\ \vdots \\ \Gamma_{T_b2\overline{b}c}(\\ \rightarrow B_c^*\eta_b = 1 \\ \hline \\ \cdot .2,2^+) \rightarrow T_{c^2} \\ \hline \\ 3.9,0^+) \rightarrow T_{c^2} \\ \hline \\ 1^+) = \langle \Psi_{tc}^1 \\ \hline \end{array}$	$\begin{array}{c} \operatorname{Rela} \\ \hline \operatorname{Rela} \\ (i,j) \\ (1,2) \\ (1,3) \\ (2,3) \\ (1,4) \\ (2,4) \\ (3,4) \\ (1,2) \\ \end{array}$ $\begin{array}{c} (2,4) \\ (1,2) \\ (1,2) \\ (1,3) \\ \end{array}$ $\begin{array}{c} (2,4) \\ (1,2) \\ (1,3) \\$	$\begin{array}{l} & \text{iso} \mu_N \\ & \text{trive Le} \\ \hline & \text{Value} \\ & 0.245 \\ & 0.240 \\ & 0.240 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.250 \\ & 0.320 \\ & -248.2 \\ & 0.320 \\ & -248.2 \\ & 0.320 \\ & -248.2 \\ & 0.320 \\ & -248.2 \\ & 0.320 \\$	$\frac{\text{ngths (fm)}}{B_c^* \eta_b}$ $0.148(\eta_b)$ $0.250(B_c^*)$ 0.148 fm $-360.4(B_c^*)$ $ R^*\rangle [\phi_1\chi_2]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_1\zeta_4]\rangle$ $ R^*\rangle [\psi_2\zeta_4]\rangle$ $(T^*) = 435.0 \text{keV}$ $-0.346 \mu_N$	$b-quark:$ m_b^{eff} $\overline{b}-quark:$ $m_{\overline{b}}^{eff}$ $\overline{c}-quark:$ $m_{\overline{c}}^{eff}$ Interaction Matrix non Total contr	$\begin{tabular}{ c c c c c } \hline Contribution & & & & & & & & & & & & & & & & & & &$	Value 10686.0 387.9 94.5 -248.2 -225.8 -983.0 9711.4 5342.0 183.8 19.9 -248.2 -491.5 4807.0 1918.0 304.4 19.9 -225.8 -491.5 1525.0 -2.6 7.5 4.8 -5.0 16048.1		Ref. [123] $2m_b$ 10105.8 10105.8 $m_{\overline{b}}$ 5052.9 5052.9 $m_{\overline{c}}$ 1724.8 1724.8 $-\frac{4}{3}C_{\overline{b}\overline{c}}$ -4.4 $4C_{bb}$ 11.6 7.2 24.3 16915.0

 Table 15
 The masses, binding energy, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong width
 ratios, and the relative lengths between quarks for the $J^P = 2^+ cc\bar{c}\bar{b}$ and $bb\bar{b}\bar{c}$ states and their lowest meson–meson thresholds. The notation is the same as that in Table 7

$cc\overline{c}\overline{b}$	The contribution from each term Relative Lengths (fm)						engths (fm)	Overall	Present Work		CMI 1	Model
$J^P = 2^+$		Value	B_c^*J/ψ]	Difference	(i, j)	Vaule	$B_c J/\psi$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass/ <i>I</i> Variational Parameters	$\begin{array}{c} B_T \\ C_{11} \\ C_{22} \\ C_{22} \end{array}$	9730.5 13.7 8.9	9442.7 20.2 12.5	287.8	(1,2) (1.3)	0.378 0.350	$0.318(J/\psi)$	c-quark	$\frac{2m_c}{\frac{\mathbf{p}_{x_1}^2}{2m_1'} + \frac{m_c + m_5}{3m_c + m_5} \frac{\mathbf{p}_{x_3}^2}{2m_3'}}{V^C(12)}$	3836.0 408.0	$\frac{\frac{1}{2}m_{cc}}{1585.8}\\\frac{m_{c}}{\frac{m_{c}+m_{\bar{c}}}{4(m_{c}+m_{\bar{c}})}}m_{c\bar{c}}$	
(fm ⁻²) Quark M Confinement	L ₃₃ Iass Potential	9.1 11097.0 -2158.4	- 11097.0 -2490.6	0.0 332.2	(2,3) (1,4) (2,4)	0.359 0.359 0.359	$0.250(B_c^*)$	1.1.1.1.1.1	$\frac{\frac{1}{2}[V^{C}(13) + V^{C}(23)]}{\frac{1}{2}[V^{C}(14) + V^{C}(24)]} -D$	-21.9 -17.3 -983.0	$\frac{\frac{767.1}{m_c}}{\frac{m_c}{4(m_c+m_{\bar{b}})}}m_{c\bar{b}}}$ 789.3	$2m_c$ 3449.6
Kinetic E	nergy	763.9	799.3	-35.4	(3,4)	0.304			Subtotal	3207.5	3142.2	3449.6
CS Intera	ction	28.0	36.9	-8.9	(1,2)	-(3,4):	$0.264~{\rm fm}$		$m_{\overline{c}}$	1918.0	$\frac{m_{\overline{c}}}{4(m_{\overline{c}}+m_{\overline{c}})}m_{c\overline{c}}$	
V^C	(1,2) (1,4)	-14.3 -17.3	594.6	220.0	(1,3) (2,3)	-21.9 -21.9	$-164.2(J/\psi)$	<i>ē</i> -quark	$\frac{\frac{m_b}{m_c + m_{\bar{b}}} \frac{1}{2m_z'} + \frac{m_c}{3m_c + m_{\bar{b}}} \frac{1}{2m_3'}}{\frac{1}{2}V^C(34)} \\ \frac{1}{2}[V^C(13) + V^C(23)]$	244.7 -49.8 -21.9 -491.5	767.1 $\frac{m_{\bar{c}}}{4(m_{\bar{c}}+m_{\bar{b}})}m_{\bar{c}\bar{b}}$	$m_{\overline{c}}$
Total Contr	ibution	-192.4	-024.0	007.0	(2,4)	-17.5	$-300.4(D_c)$		$-\frac{1}{2}D$	1500.5	1568.2	1724.8
Total Wave fi	inction:	099.0	311.7	201.0	(3,4)	-99.0			m _b	1000.0 E949.0	$\frac{m_{\overline{b}}}{m_{\overline{b}}} m_{\overline{c}}$	1124.0
$\frac{\Psi_{tot} = F\rangle R^s}{\text{The rearrange}}$	$\langle \phi_1 \chi_1 \rangle$ ement streed decay wi	$= 0.577 \vec{h}$ ong decay dths:	$F\rangle R^s\rangle [u]$	$ \psi_1\zeta_1] angle - 0$ l: $B_c^*J/\psi.$	816 <i>F</i>	$\langle R^s \rangle [a]$	$\psi_2\zeta_1]\rangle$	b -quark	$\frac{\frac{m_{\tilde{\tau}}}{m_{\tilde{\tau}}+m_{\tilde{\tau}}}\frac{\mathbf{p}_{x_{2}}^{2}}{p_{x_{2}}^{2}} + \frac{m_{c}}{3m_{c}+m_{\tilde{\tau}}}\frac{\mathbf{p}_{x_{3}}^{2}}{p_{x_{3}}^{2}}}{\frac{\frac{1}{2}V^{C}(34)}{\frac{1}{2}[V^{C}(14) + V^{C}(24)]}} \\ -\frac{1}{2}D$	5343.0 111.2 -49.8 -17.4 -491.5	$\frac{4(m_{\overline{b}}+m_{\overline{c}})}{4(m_{c}+m_{\overline{b}})}m_{c\overline{b}}$ $\frac{2371.8}{4779.8}$	2407.0 $m_{\overline{b}}$ 5052.9
$\Gamma_{c^2 \bar{c} \bar{b}}^{(9730.5,2)}$	$^+) \rightarrow T_{c^2 \overline{c} \overline{b}} (9)$	9620.5,0 ⁺)γ	145.0	lX					Subtotal	4090.0	4110.0	5052.9
$\frac{\Gamma_{T_c^2 \bar{c} \bar{b}}(9730.5,2)}{\text{The magnetic}}$ $\frac{\mu_{T_c^2 \bar{c} \bar{b}}(9730.5,2)}{\text{The transition}}$	$ \begin{array}{l} \overset{+) \to T_{c^2 \overline{c} \overline{b}}(\underline{s}) \\ \approx \text{moments} \\ \overset{+) = \langle \Psi_{to}^{2^+} \\ \approx \text{magnetian} \end{array} $	$\frac{1}{2} \frac{1}{2} \frac{1}$	$\phi = 2\mu_c + \frac{1}{15}$	$+ \mu_{\overline{c}} + \mu_{\overline{b}} =$	= 0.46	$4\mu_N$		CS Interaction	$\frac{\frac{1}{2}V^{SS}(12)}{\frac{1}{2}V^{SS}(34)}$ $\frac{1}{4}(V^{SS}(13) + V^{SS}(23))$ $\frac{1}{4}(V^{SS}(14) + V^{SS}(24))$	$5.6 \\ 8.5 \\ 9.5 \\ 4.3$	$\frac{\frac{8}{3}v_{cc} + \frac{8}{3}v_{\bar{c}\bar{b}}}{9.5+5.2} \\ \frac{\frac{8}{3}v_{c\bar{c}} + \frac{8}{3}v_{c\bar{b}}}{14.2+7.9}$	$\frac{\frac{8}{3}C_{cc} + \frac{8}{3}C_{\bar{c}\bar{b}}}{14.1 + 8.8}$ $\frac{\frac{8}{3}C_{c\bar{c}} + \frac{8}{3}C_{c\bar{b}}}{14.1 + 8.8}$
			$-/\Psi_{+}^{2^{+}}$	$ \hat{u^z} \Psi^{0^+}\rangle$	- 0				Subtotal	28.0	36.8	45.9
$\mu_{T_{c^2\bar{c}\bar{b}}}^{(9730.5,2)}$	$^+) \rightarrow T_{c^2 \overline{c} \overline{b}} (9)$	$9620.5,0^{+})\gamma$	$= \langle \Psi_{tot}^{2^+} \rangle$	$ \hat{\mu}^{z} \Psi_{tot}^{1^{+}}\rangle$	= -0.5	294µ _N		Total contr	ibution	9730.5	9526.0	10273.2
$\frac{I I_{c^2 \overline{cb}}(9730.3,2)}{b b \overline{b} \overline{c} J^P = 2^+}$	$() \rightarrow 1_{c^2 \overline{c} \overline{b}}$	Value	B*Υ]	Difference	Rela	tive Le	engths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
Mass/1	B_T	16149.2	15819.4	329.8	(i, j)	Value	$B_c^*\Upsilon$		$2m_c$	10686.0	$\frac{1}{2}m_{bb}$	
Variational Parameters (fm ⁻²) Quark M Confinement	$\begin{bmatrix} C_{11} \\ C_{22} \\ C_{33} \end{bmatrix}$ fass Potential	14.4 28.6 16.9 17947.0	20.2 49.7 - 17497.0 -3123.1	0.0 463.4	(1.2) (1,3) (2,3) (1,4)	0.210 0.256 0.256 0.266	0.160(Ύ)	<i>b</i> -quark	$\frac{\frac{\mathbf{p}_{x_1}^2}{2m_1^{\prime}} + \frac{m_b + m_{\tilde{c}}}{3m_b + m_{\tilde{c}}} \frac{\mathbf{p}_{x_3}^2}{2m_3^{\prime}}}{V^C(12)} \\ \frac{1}{2}[V^C(13) + V^C(23)] \\ \frac{1}{2}[V^C(14) + V^C(24)] \\ -D$	404.4 -257.5 -85.5 -77.6 -983.0	$\frac{\frac{2}{4764.8}}{\frac{m_b}{4(m_b+m_{\overline{b}})}}m_{b\overline{b}}$ $\frac{2361.2}{\frac{m_b}{4(m_b+m_{\overline{c}})}}m_{b\overline{c}}$ $\frac{2371.8}$	$2m_b$ 10105.8
Kinetic E	nergy	838.2	961.5	-123.3	(2,4)	0.266	$0.250(B_c^*)$		Subtotal	9527.1	9497.8	10105.8
CS Intera	ction (1,2)	23.8 -257.5	34.0	-10.2	(3,4) (1,2)	0.296	0.194 fm		$\frac{m_{\overline{b}}}{\frac{m_{\overline{c}}}{m_{\overline{c}}+m_{\overline{b}}}\frac{\mathbf{p}_{x_2}^2}{2m_2'} + \frac{m_b}{3m_b+m_{\overline{c}}}\frac{\mathbf{p}_{x_3}^2}{2m_3'}}$	5343.0 146.6	$\frac{\frac{m_{\overline{b}}}{4(m_{\overline{b}}+m_{\overline{b}})}m_{b\overline{b}}}{2382.4}$	
V^C	(2,3) (1,4) Subtotal	-85.5 -77.6 -693.7	-1157.1	463.4	(2,4) (3,4)	-77.6 -110.1	$-360.4(B_c^*)$	<i>b</i> -quark	$\frac{\frac{1}{2}V^{C}(34)}{\frac{1}{2}[V^{C}(13) + V^{C}(23)]} - \frac{1}{2}D}$ Subtotal	-35.1 -85.5 -491.5 4857.5	$\frac{\frac{m_{\bar{b}}}{4(m_{\bar{b}}+m_{\bar{c}})}m_{\bar{b}\bar{c}}}{2360.6}$ 4743.0	$m_{\overline{b}}$ 5052.9 5052.9
Total Contr	ibution	168.2	-161.6	329.8	(1,3)	-85.5	$-796.7(\Upsilon)$		$m_{\bar{c}}$	1918.0	$\frac{m_{\overline{c}}}{4(m_{\overline{b}}+m_{\overline{c}})}m_{\overline{c}\overline{b}}$	
Total Wave fu	inction:							ē-quark	$\frac{m_{\bar{b}}}{m_{\bar{c}}+m_{\bar{b}}} \frac{p_{x_2}}{2m_2'} + \frac{m_b}{3m_b+m_{\bar{c}}} \frac{p_{x_3}}{2m_3'}$	287.2		801.3
$\Psi_{tot} = F\rangle R^s$	$ \phi_1\chi_1\rangle$	= 0.577 I	$F\rangle R^s\rangle [y]$	$\langle \psi_1 \zeta_1] \rangle - 0.5$	816 F	$\langle R^s \rangle [q$	$\langle \psi_2 \zeta_1] \rangle$	C-quark	$\frac{1}{2}V^{C}(34)$ $\frac{1}{2}[V^{C}(14) + V^{C}(24)]$	-77.6	$\frac{m_{\overline{c}}}{4(m_b+m_{\overline{c}})}m_{b\overline{c}}$	$m_{\bar{c}}$
The rearrange	ement str	ong decay	channe	l: $B_c^*\Upsilon$.					$-\frac{1}{2}D$	-491.5	789.3	1724.8
The radiative	decay wi	dths: I	$T_{b^2\overline{b}\overline{c}}(161$	$49.2,2^+) \to T$	$b^{2}\bar{b}\bar{c}(16$	043.9,0+	$_{)\gamma} = 0 \mathrm{keV}$		Subtotal	1581.2	1590.6	1724.8
The magnetic $\mu_{T_{c^2\bar{c}\bar{b}}(16149.2,3)}$ The transition	moments $_{2^+)} = \langle \Psi_t^2$	$\Gamma_{T_{b^2}}$ s: $\int_{ot}^{+} \hat{\mu^z} \Psi_{tot}^{2+}$ ic momen	$2b\bar{c}^{(16149.3)}$ $b\bar{c}^{(16149.3)}$ $b\bar{c}^{(16149.3)}$ $b\bar{c}^{(16149.3)}$	$+ \mu_{\overline{b}} + \mu_{\overline{c}}$	(16043) = -0	$(472\mu_N)^{-2,1^+)\gamma}$	$= 435.0 \mathrm{keV}$	CS Interaction	$\frac{\frac{1}{2}V^{SS}(12)}{\frac{1}{2}V^{SS}(34)}$ $\frac{1}{4}(V^{SS}(13) + V^{SS}(23))$ $\frac{1}{4}(V^{SS}(14) + V^{SS}(24))$	$5.9 \\ 6.4 \\ 4.6 \\ 6.9$	$\frac{\frac{8}{3}v_{bb} + \frac{8}{3}v_{\bar{c}\bar{b}}}{5.1 + 5.2}$ $\frac{\frac{8}{3}v_{b\bar{b}} + \frac{8}{3}v_{b\bar{c}}}{7.7 + 7.9}$	$\frac{\frac{8}{3}C_{bb} + \frac{8}{3}C_{\bar{c}\bar{b}}}{7.7 + 8.8}$ $\frac{\frac{8}{3}C_{b\bar{b}} + \frac{8}{3}C_{b\bar{c}}}{7.7 + 8.8}$
$\mu_{T_{1},27}$ (16149.2.4	$(2^+) \rightarrow T \rightarrow T$	(16043 2 1+	$_{1\gamma} = \langle \Psi_t^2 \rangle$	$\hat{F}_{ot}^{+} \hat{\mu^{z}} \Psi_{tot}^{1^{+}} $	0 = 0.3	$329\mu_N$			Subtotal	23.8	25.9	33.1
$\mu_{T_{b^2 \bar{b} \bar{c}}(16149.2,2)}$	$(2^+) \rightarrow T_{b^2 \overline{b} \overline{c}}$	(16043.9,0+	$_{)\gamma} = \langle \Psi_t^2 \rangle$	$\hat{\mu}_{ot}^{+} \hat{\mu^{z}} \Psi_{tot}^{0^{+}} $	$\rangle = 0$	/		Total contr	ibution	16149.2	15882.3	16917.0

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$cb\overline{c}\overline{b}$	The co	ontributi	on from	each term	Relati	ive Le	engths (fm)	Orronall	Present Work		CMI	Model
$J^{PC} = 0^{++}$		Value	$\eta_b \eta_c$	Difference	(i,j) V	alue	$\eta_b \eta_c$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass	B_T	12759.6	12387.5	372.1	(1,2) 0.	.315			m_c	1918.0	$\frac{-m_c}{4(m_c+m_b)}m_{cb}$	
Variational	C ₁₁	12.7	15.0		(1.3) 0.	.277	$0.290(\eta_c)$	a guarlu	$\frac{m_b}{m_c+m_b} \frac{\mathbf{p}_{x_1}^2}{2m_1'}$	193.7	-400.5	
Parameters $(f_{m}-2)$	C_{22}	12.7	57.4		(2.3) 0.	277	(1~)	c-quark.	$\frac{m_b}{2m+2m} \frac{\mathbf{p}_{x_3}^2}{2m/2}$	139.6	$\frac{6m_c}{8(m_c+m_{\overline{b}})}m_{c\overline{b}}$	
 Ouark	Mass	14522.0	14522.0	0.0	(1,4) 0	277		m_c^{eff}	$\frac{1}{2}V^{C}(12)$	21.3	$\frac{5m_c}{2}$ $m_c\overline{c}$	m .
Confinemen	t Potential	-2571.3	-3082.3	511.0	(1, 1) 0.	977	0.148(m)		$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	-172.6	$8(m_c + m_{\bar{c}})^{-11}$	1724.8
Kinotia	Fnorm	005.8	1085.5	170.7	(2,4) 0.	915	0.140(1/6)		-1/2D Subtotal	1608 5	1545.1	1724.0
CC Into	na ation	01.1	197.6	-113.1	(3,4) 0.	.510	0.101.0		m	1000.0 E242.0	$-m_b - m_c$	1724.0
CS Inte	raction	-81.1	-137.0	0.06	(1,2)-(3	3,4):	0.164 fm		$m_c \mathbf{p}_{x_1}^2$	5343.0	$\frac{1}{4(m_c+m_b)}m_{cb}$ -1203.8	
C	(1,2)	42.6			(1,3) -1	72.6	$-237.2(\eta_c)$	h quark:	$m_c+m_b \frac{2m_1'}{p_{r_0}^2}$	69.5 50.1	$\frac{5m_b}{8(m_c+m_{\overline{i}})}m_{c\overline{b}}$	
VC	(1,4)	-172.6			(2,3) -1	72.6		o-quark.	$\frac{m_c}{2m_c+2m_b}\frac{m_3}{2m_3'}$	21.3	2965.5	
	Subtotal	-605.3	-1116.3	511.0	(2,4) -1	72.6	$-879.1(\eta_b)$	m_b^{eff}	$\frac{1}{2}V^{C}(12)$ $\frac{1}{2}[V^{C}(23) + V^{C}(24)]$	-172.6	$\frac{3m_b}{8(m_b+m_{\overline{b}})}m_{b\overline{b}}$	m_b
Total Con	itribution	219.3	-168.5	387.8	(3,4) 4	2.6			-1/2D	-491.5	2951.6	5052.9
Total Wave	function:								Subtotal	4819.8	4713.3	5052.9
$\Psi_{tot} = 0.9$	$961 F\rangle R^s\rangle $	$\phi_2 \chi_5] \rangle +$	+0.114 F	$ R^s\rangle R^s\rangle [\phi_2\chi]$	$_{6}]\rangle - 0.0$	069 F	$\langle R^s \rangle [\phi_1 \chi_5] \rangle$		$-\frac{1}{4}(V^{SS}(12) + V^{SS}(34))$	-5.3	$-\frac{8}{3}v_{cb} - \frac{20}{3}v_{\overline{c}\overline{c}}$	$-\frac{8}{3}C_{cb} - \frac{20}{3}C_{\overline{c}\overline{c}}$
-0.2	$241 F\rangle R^s\rangle $	$\left[\phi_1\chi_6\right]$ =	= 0.211 I	$\langle F \rangle R^s \rangle [\psi_1 \phi_1 \phi_2 \phi_2 \phi_3 \phi_3 \phi_1 \phi_1 \phi_2 \phi_2 \phi_3 \phi_3 \phi_1 \phi_1 \phi_2 \phi_2 \phi_3 \phi_3 \phi_1 \phi_2 \phi_3 \phi_3 \phi_3 \phi_1 \phi_2 \phi_1 \phi_2 \phi_2 \phi_1 \phi_2 \phi_2 \phi_1 \phi_2 \phi_2 \phi_2 \phi_1 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2 \phi_2$	$(5_{6}] \rangle - 0.8$	830 F	$\langle R^s \rangle R^s \rangle [\psi_1 \zeta_5] \rangle$		$-\frac{5}{4}V^{SS}(13)$	-33.3	-5.1 - 35.5	-8.8-35.3
-0.5	$367 F\rangle R^s\rangle$	$[\psi_2 \zeta_5]\rangle$ -	+ 0.363 F	$F\rangle R^s\rangle [\psi_2\zeta]$	$[_6]\rangle = 0.3$	333 F	$ \langle R^s \rangle [\psi_1' \zeta_6'] \rangle$	CS Interaction	$-\frac{5}{4}V^{SS}(24)$	-10.1	$-\frac{20}{3}v_{b\overline{b}} - \frac{40}{3}v_{b\overline{c}}$	$-\frac{20}{3}C_{b\overline{b}} - \frac{40}{3}C_{b\overline{c}}$
-0.	$668 F\rangle R^s\rangle$	$ [\psi_1'\zeta_5']\rangle$	-0.398 I	$\langle F \rangle R^s \rangle [\psi_2']$	$(5)^{\prime} = (5)^{\prime} + 0.5$	533 F	$\langle R^s\rangle [\psi_2'\zeta_6']\rangle$	meetaction	$-\frac{5}{4}(V^{SS}(14) + V^{SS}(23))$	-32.5	-19.2-39.3	-19.3 - 44.3
The rearran	ngement str	ong deca	v channe	el:			// ////		Subtotal	-81.1	-99.2	-107.5
Γ_{T} =(19750	6.0++) \n n	$: \Gamma_T$	× (19750.6.0∃	$(++)$, $\tau(+\infty)$	= 21:1			Matrix non	diagonal element	-15.8	-80.7	-51.9
Г _{сbēb} (12739.	$(0,0) \rightarrow \eta_c \eta_b$	$\Gamma_{cb\bar{c}b}$: Γ_{T}	(12739.0,0	$(+) \rightarrow J/\psi I$	= 7.6:	1		Total contr	ibution	12759.6	12336.1	13396.0
The radiati	$(0,0++) \rightarrow B_c B$	<u>c </u>	<u>5(12/59.6,</u>	$(++) \rightarrow B_c^- B_c^-$	The ma	- orneti	c moments: 7	Г. . (12759.6	$(3,0^{++}) - \langle \Psi_{i}^{0^{++}} \hat{\mu_{z}} \Psi_{i}^{0^{++}} \rangle$	≥ -0		
Г	ve decay wi	cr0113.	–	$33.1 \mathrm{keV}$	The tre	neitic	n magnotic n	omonte:	$\frac{1}{10000000000000000000000000000000000$	>= 0	$- < \Psi^{2^{++}} _{\hat{u}z}$	$ \Psi^{0^{++}} > = 0$
¹ T _{cbcb} (12797. Г	$(3,1^{+-}) \rightarrow T_{cb\overline{c}}$	$\overline{b}(12759.6,$	$_{0^{++}\gamma} - $	0.1 Ke V	ine tra	uisitic	n magnetic n	noments.	$\mu_{T_{cb\bar{c}\bar{b}}(12882.4,2^{++}) \to T_{cb\bar{c}\bar{b}}(126676) \to T_{cbc$	2759.6,0++	$\psi_{tot} = \psi_{tot} \mu^{2}$	$ \Psi_{tot}\rangle >= 0$
$T_{cb\bar{c}\bar{b}}(12856.$	$6,1^{++}) \rightarrow T_{cb\overline{c}}$	$\frac{1}{5}(12759.6,$	$_{0^{++}\gamma} = 0$	0	$\mu_{T_{cb\bar{c}\bar{b}}(1)}$	2856.6	$,1^{++}) \rightarrow T_{cb\overline{c}\overline{b}}(12)$	$759.6,0^{++})\gamma =$	$= \langle \Psi_{tot} \mu^{z} \Psi_{tot} \rangle = 0$		$\sqrt{1}$	0.01
$\Gamma_{T_{cb\overline{c}\overline{b}}(12882.}$	$4,2^{++}) \rightarrow T_{cb\bar{c}}$	$\overline{b}(12759.6,$	$_{0^{++})\gamma} =$	0	$\mu_{T_{cb\bar{c}\bar{b}}(1}$	2796.9	$,1^{+-}) \rightarrow T_{cb\overline{c}\overline{b}}(12)$	$759.6,0^{++})\gamma$ =	$= \langle \Psi_{tot}^{z} \mu^{z} \Psi_{tot}^{o} \rangle \geq 0.2$	26×2	$\frac{1}{6}(\mu_c - \mu_b) = 0.$	$.081 \mu_N$
$J^{PC} = 2^{++}$		Value	$\Upsilon J/\psi$	Difference	Relati	ive Le	engths (fm)		Contribution	Value	Ref. [110]	Ref. [123]
Mass	$/B_T$	12882.4	12561.1	321.3	(i,j) V	alue	$\Upsilon J/\psi$		m_c	1918.0	$\frac{-m_c}{4(m_c+m_b)}m_{cb}$	
Variational	C_{11}	11.0	12.5		(1.3) 0.	.340		<i>c</i> -quark:	$\frac{m_b}{m_c + m_b} \frac{1}{2m'_1}$	167.1	-400.5 $-5m_c$ m $-$	
(fm^{-2})	C ₂₂ C ₃₃	21.0			(1,3) 0.	.296	$0.318(J/\psi)$		$\frac{m_b}{2m_c+2m_b} \frac{\mathbf{p}_{x_3}}{2m'_3}$	124.2	$\frac{8(m_c+m_{\overline{b}})}{986.7}$	
Quark	Mass	14522.0	14522.0	0.0	(2,3) 0.	.296		$m_c^{e_f f}$	$\frac{1}{2}V^{C}(12)$	-137.7	$\frac{5m_c}{8(m_c+m_{\overline{c}})}m_{c\overline{c}}$	m_c
Confinemen	t Potential	-2460.7	-2926.9	466.2	(1,4) 0.	.296			$\frac{1}{2}[V^{\circ}(13) + V^{\circ}(14)] - 1/2D$	-491.5	958.9	1724.8
Kinetic	Energy	791.7	925.1	-133.4	(2,4) 0.	.296	$0.160(\Upsilon)$		Subtotal	1594.1	1545.1	1724.8
CS Inte	raction	29.5	41.0	-11.5	(3.4) 0.	.340			m _b	5343.0	$\frac{-m_b}{4(m_c+m_t)}m_{cb}$	
	(1,2)	28.0			(1.2)-(;	3.4):	0.174 fm		$\frac{m_c}{m_c+m_b}\frac{\mathbf{p}_{x_1}^2}{2m_b^2}$	60.0	-1203.8	
	(2,3)	-137 7				- / /		<i>b</i> -quark:	$\frac{m_c}{2}$ $\frac{\mathbf{p}_{x_3}^2}{2}$	44.6	$\frac{6m_b}{8(m_c+m_{\overline{b}})}m_{c\overline{b}}$	
V^C	(1,4)	-137.7			(2.4) = 1	37 7	$-796.7(\Upsilon)$	eff	$\frac{1}{2}V^{C}(12)$	14.0	$\frac{2900.0}{5m_b}$ m_{17}	<i>m</i> 1
	Subtotal	404.7	060.0	466 9	$(2, 4)^{-1}$	01.1	150.1(1)	m_b ,	$\frac{1}{2}[V^{C}(23) + V^{C}(24)]$	-137.7	$8(m_b + m_{\overline{b}})^{1100}$	5052.0
Total Con	tribution	226 5	-300.3	201.4	(3,4) 2 (1,2) 1	97.7	1649(1/4)		-1/2D Subtotal	4822.4	4712.2	5052.9
	IIIIDUIIOII	320.5	5.1	521.4	(1,3) -1	ər.r ·	$-104.2(J/\psi)$		$\frac{1}{1}\left(\frac{VSS}{10} + \frac{VSS}{24}\right)$	4032.4	8 10	8 C + 10 C
I OTAL MARK	Course of the second								$-\frac{1}{4}(v (12) + v (34))$	-4.7	$-\frac{1}{3}v_{cb}+\frac{1}{3}v_{c\bar{c}}$	$-\frac{1}{3}C_{cb}+\frac{1}{3}C_{cc}$
Total wave	function:	1)		D811/ 1			1 1 1 1 1 1 1 1 1 1 1 1 1		57755(10)	15.0	F 1 + 177 77	001177
$\Psi_{tot} = 0.999$	function: $9 F\rangle R^s\rangle [\phi$	$_{2}\chi_{1}] angle - 0$	$0.027 F\rangle $	$R^s \rangle [\phi_1 \chi_1]$	$\rangle = -0.8$	801 F	$\langle R^s \rangle [\psi_1 \zeta_1] \rangle$	CS	$\frac{5}{8}V^{SS}(13)$	15.2	-5.1+17.7	-8.8+17.7
$\Psi_{tot} = 0.999$ -0.59	function: $9 F\rangle R^s\rangle [\phi]$ $9 F\rangle R^s\rangle [y]$	$ \chi_1\rangle = 0$ $ \chi_2\zeta_1\rangle = 0$	$0.027 F\rangle -0.832 F\rangle $	$R^{s}\rangle [\phi_{1}\chi_{1}]$ $\langle F\rangle R^{s} angle [\psi_{1}'\phi_{1}']$	$\rangle = -0.8$ $\zeta_{1}^{\prime}]\rangle + 0.8$	801 F 555 F	$\langle R^s \rangle [\psi_1 \zeta_1] \rangle$ $\langle R^s \rangle [\psi_2' \zeta_1'] \rangle$	CS Interaction	$\frac{5}{8}V^{SS}(13)$ $\frac{5}{8}V^{SS}(24)$	$15.2 \\ 4.5$	-5.1+17.7 $\frac{10}{3}v_{b\bar{b}} + \frac{20}{3}v_{b\bar{c}}$	-8.8+17.7 $\frac{10}{3}C_{b\overline{b}}+\frac{20}{3}C_{b\overline{c}}$
$\Psi_{tot} = 0.999$ -0.59 The rearran	function: $9 F\rangle R^s\rangle [\phi$ $99 F\rangle R^s\rangle [u$ ngement str	$ 2\chi_1\rangle = 0$ $\langle \psi_2 \zeta_1 \rangle = 0$ ong deca	$0.027 F\rangle $ $-0.832 H\rangle$ ay channe	$R^{s}\rangle [\phi_{1}\chi_{1}]$ $\overline{r}\rangle R^{s}\rangle [\psi_{1}'\phi_{2}]$ el: $\Upsilon J/\psi$ a	$\rangle = -0.8$ $f_1' \rangle + 0.8$ and $B_c^* B$	$801 F$ $555 F$ B_c^*	$ \langle R^s \rangle [\psi_1 \zeta_1]\rangle$ $ \langle R^s \rangle [\psi_2' \zeta_1']\rangle$	CS Interaction	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{\frac{5}{8}(V^{SS}(14) + V^{SS}(23))}{\frac{5}{8}(V^{SS}(14) + V^{SS}(23))}$	15.2 4.5 14.7	$-5.1+17.7$ $\frac{10}{3}v_{b\overline{b}} + \frac{20}{3}v_{b\overline{c}}$ $9.6+19.7$	$-8.8+17.7$ $\frac{10}{3}C_{b\bar{b}} + \frac{20}{3}C_{b\bar{c}}$ $9.6+22.0$
$\Psi_{tot} = 0.999$ -0.59 The rearran	function: $9 F\rangle R^s\rangle [\phi]$ $99 F\rangle R^s\rangle [u]$ ngement str tic moment	$ 2\chi_1]\rangle = 0$ $\langle \psi_2 \zeta_1 \rangle = 0$ ong deca s: $T_{cb\bar{c}}$	$0.027 F\rangle -0.832 F\rangle$ ay channe $\overline{b}(12882.5)$	$R^{s} \rangle [\phi_{1}\chi_{1}]$ $\vec{r} \rangle R^{s} \rangle [\psi_{1}' \phi_{1} \phi_{1}]$ $\vec{r} \rangle R^{s} \rangle [\psi_{1}' \phi_{1} \phi_{2}]$ $\vec{r} \rangle R^{s} \rangle [\psi_{1}' \phi_{2}]$ $\vec{r} \rangle R^{s} \rangle E^{s} \rangle E^{s$	$\rangle = -0.8$ and $B_c^* B$ $\langle \Psi_{tot}^{2^{++}} \hat{\mu} \rangle$	801 F 555 F B_{c}^{*} $E^{2} \Psi_{tot}^{2^{+}} $	$\begin{split} \langle R^s \rangle [\psi_1 \zeta_1] \rangle \\ \langle R^s \rangle [\psi_2 \zeta_1] \rangle \\ \\ \tau_t^{+} \rangle = 0 \end{split}$	CS Interaction	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{\frac{5}{8}(V^{SS}(14) + V^{SS}(23))}{\text{Subtotal}}$	15.2 4.5 14.7 29.5	$-5.1+17.7$ $\frac{10}{3}v_{b\overline{b}} + \frac{20}{3}v_{b\overline{c}}$ $9.6+19.7$ 41.7	$-8.8+17.7$ $\frac{10}{3}C_{b\overline{b}} + \frac{20}{3}C_{b\overline{c}}$ 9.6+22.0 40.5
$\Psi_{tot} = 0.999$ -0.59 The rearran The magnet	function: $9 F\rangle R^s\rangle [\phi]$ $99 F\rangle R^s\rangle [u]$ ngement str tic moment ve decay wi	$ 2\chi_1]\rangle - 0$ $ 2\chi_1]\rangle = 0$ $ 2\chi_$	$0.027 F\rangle $ -0.832 <i>I</i> ay channe $\overline{b}(12882)$	$R^{s} \rangle [\phi_{1}\chi_{1}]$ $\vec{r} \rangle R^{s} \rangle [\psi_{1}' \phi_{1} \phi_{2}]$ el: $\Upsilon J/\psi \in$ $4, 2^{++}) =$	$\rangle = -0.8$ and $B_c^* B$ $\langle \Psi_{tot}^{2^{++}} \hat{\mu} \rangle$	801 F 555 F B_{c}^{*} $E^{2} \Psi_{to}^{2^{+}} $	$\begin{array}{l} \langle R^s\rangle [\psi_1\zeta_1]\rangle\\ \langle R^s\rangle [\psi_2'\zeta_1']\rangle\\ \\ t\\ t\\ t \\ t \\ \end{array} = 0 \end{array}$	CS Interaction Matrix non	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{\frac{5}{8}(V^{SS}(14) + V^{SS}(23))}{\text{Subtotal}}$ diagonal element	15.2 4.5 14.7 29.5 0.1	$-5.1+17.7$ $\frac{10}{3}v_{b\overline{b}} + \frac{20}{3}v_{b\overline{c}}$ 9.6+19.7 41.7 -29.1	$-8.8+17.7$ $\frac{10}{3}C_{b\bar{b}} + \frac{20}{3}C_{b\bar{c}}$ $9.6+22.0$ 40.5 3.2
$\begin{split} \Psi_{tot} &= 0.999 \\ & -0.59 \\ \hline & The rearran \\ The magnet \\ \Gamma_{T_{cb\bar{c}\bar{b}}(12882.)} \end{split}$	function: $9 F\rangle R^s\rangle [\phi]$ $99 F\rangle R^s\rangle [\psi]$ ngement str tic moment tic moment we decay wi $4,2^{++}) \rightarrow T_{cb\bar{c}}$	$ 2\chi_1]\rangle - 0$ $ 2\chi_1]\rangle =$ $ 2\chi_1 \rangle =$ 	$0.027 F\rangle $ $-0.832 I\rangle$ ay channel $\overline{b}(12882)$ $1^{+-})\gamma = -$	$R^{s} \rangle [\phi_{1}\chi_{1}]$ $T^{s} \rangle [\psi_{1}^{s}\psi_{1}] [\psi_{1$	$\rangle = -0.8$ $\sum_{i=1}^{2} \rangle + 0.8$ $\max_{i=1}^{2} B_{c}^{2++} \hat{\mu} $	$801 F \\ 555 F \\ \frac{3c}{c} \\ \frac{3c}{c^{2}} \Psi_{tot}^{2^{+}} $	$\begin{split} &\langle R^s\rangle [\psi_1\zeta_1]\rangle\\ &\langle R^s\rangle [\psi_2'\zeta_1']\rangle\\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &$	CS Interaction Matrix non Total contr	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{\frac{5}{8}(V^{SS}(14) + V^{SS}(23))}{\text{Subtotal}}$ diagonal element ibution	15.2 4.5 14.7 29.5 0.1 12882.4	$\begin{array}{r} -5.1 + 17.7 \\ \frac{10}{3} v_{b\overline{b}} + \frac{20}{3} v_{b\overline{c}} \\ 9.6 + 19.7 \\ 41.7 \\ -29.1 \\ 12529.4 \end{array}$	$ \begin{array}{r} -8.8+17.7 \\ \frac{10}{3}C_{b\bar{b}} + \frac{20}{3}C_{b\bar{c}} \\ 9.6+22.0 \\ \hline 40.5 \\ \hline 3.2 \\ \hline 13599.0 \\ \end{array} $
$\begin{split} \Psi_{tot} = 0.99 \\ & -0.59 \\ \hline \text{The rearran} \\ \hline \text{The radiatir} \\ \Gamma_{T_{cb\bar{c}\bar{b}}(12882.} \\ \Gamma_{T_{cb\bar{c}\bar{b}}(12882.} \end{split}$	function: $9 F\rangle R^s\rangle [\phi P^s\rangle [q P^s\rangle [q $	$2\chi_1]\rangle = 0$ $\psi_2\zeta_1]\rangle = 0$ ong deca $S: T_{cb\bar{c}}$ $dths:$ $\overline{b}(12797.3, 0)$ $\overline{b}(12759.6, 0)$	$\begin{array}{l} 0.027 F\rangle \\ -0.832 I\\ \hline \text{ay channel}\\ \overline{b}(12882\\ 1^{+-})\gamma = -\\ 0^{++})\gamma = -\end{array}$	$R^{s} \rangle [\phi_{1}\chi_{1}]$ $\vec{r} \rangle R^{s} \rangle [\psi'_{1}d]$ $\vec{r} J/\psi \epsilon$ $4, 2^{++}) =$ 4.9 keV 0	$\begin{split} \rangle &= -0.8 \\ \tilde{\gamma}_1'] \rangle + 0.8 \\ \frac{1}{\sqrt{\Psi_{tot}^{2^{++}} \hat{\mu}}} \\ \overline{\sqrt{\Psi_{tot}^{2^{++}} \hat{\mu}}} \end{split}$	$801 F$ $555 F$ 3_{c}^{*} 2882.4	$\begin{split} & \langle R^*\rangle [\psi_1\zeta_1]\rangle \\ & \langle \rangle R^*\rangle [\psi_2\zeta_1]\rangle \\ & \vdots \\$	CS Interaction Matrix non Total contr 8856.6,1++) γ =	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{5}{8}(V^{SS}(14) + V^{SS}(23))$ Subtotal diagonal element ibution $= 0$	15.2 4.5 14.7 29.5 0.1 12882.4	$\begin{array}{c} -5.1+17.7\\ \frac{10}{3}v_{b\bar{b}}+\frac{20}{3}v_{b\bar{c}}\\ 9.6+19.7\\ \hline 41.7\\ -29.1\\ \hline 12529.4\\ \hline \end{array}$	$ \begin{array}{r} -8.8+17.7 \\ \frac{10}{3}C_{b\overline{b}} + \frac{20}{3}C_{b\overline{c}} \\ 9.6+22.0 \\ \hline 40.5 \\ \hline 3.2 \\ 13599.0 \\ \end{array} $
$\begin{split} \Psi_{tot} = 0.99 \\ -0.59 \\ \hline \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	function: $\begin{split} & 9 F\rangle R^s\rangle [\phi\\ & 99 F\rangle R^s\rangle [q\\ & 99 F\rangle R^s\rangle [q\\ & \text{remain the moment}\\ & \text{tic moment}\\ & \text{ve decay wit}\\ & 4,2^{++})\rightarrow T_{cb\overline{c}}\\ & 4,2^{++})\rightarrow T_{cb\overline{c}} \end{split}$	$ 2\chi_1\rangle - 0$ $ 2\chi_1\rangle = 0$	$\begin{array}{l} 0.027 F\rangle \\ -0.832 I\\ \hline \\ ay \ channel{eq:basic}\\ \hline \\ \hline$	$R^{s} \rangle [\phi_{1}\chi_{1}]$ $\vec{r} \rangle R^{s} \rangle [\psi_{1}' u]$ $\vec{el:} \Upsilon J/\psi =$ $4, 2^{++} =$ 4.9 keV 0 μ_{T}	$\begin{split} \rangle &= -0.8\\ \zeta_1'] \rangle + 0.8\\ \mathrm{ind} \ B_c^* B\\ \overline{\langle \Psi_{tot}^{2++} \hat{\mu} \rangle}\\ \hline \Gamma_{T_{cb\bar{c}\bar{b}}(1)} \end{split}$	$801 F$ $555 F$ 3_{c}^{*} 2882.4 2882.4 $3.4,2^{++}$	$\begin{split} & \langle R^s \rangle [\psi_1 \zeta_1] \rangle \\ & \langle R^s \rangle [\psi_2' \zeta_1'] \rangle \\ & \vdots \\ $	CS Interaction Matrix non Total contr $\frac{856.6,1++)\gamma}{6,1+-)\gamma} = \langle \Psi$	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{5}{8}(V^{SS}(14) + V^{SS}(23))$ Subtotal diagonal element ibution $= 0$ $= 0$ $\frac{1}{V_{tot}^{2t+}} \hat{\mu}^{z} \Psi_{tot}^{1+-}\rangle = 0.774 \times 0.000$	$ \begin{array}{r} 15.2 \\ 4.5 \\ 14.7 \\ \hline 29.5 \\ 0.1 \\ 12882.4 \\ \hline (\mu_c + \mu_b) \end{array} $	$\begin{array}{c} -5.1{+}17.7\\ \frac{10}{3}v_{b5}+\frac{20}{3}v_{b\overline{c}}\\ 9.6{+}19.7\\ \hline 41.7\\ -29.1\\ 12529.4\\ \hline \end{array}$	$-8.8+17.7$ $\frac{10}{3}C_{b\bar{b}} + \frac{20}{3}C_{b\bar{c}}$ 9.6+22.0 40.5 3.2 13599.0 $-\mu_b) = 0.345\mu_N$
$\begin{split} \Psi_{tot} &= 0.99! \\ &- 0.59 \\ \hline &- 0.59 \\ \hline & The rearran \\ The magnet \\ The radiati \\ \Gamma_{T_{cb\bar{c}\bar{b}}}(1282. \\ \Gamma_{cb\bar{c}\bar{b}}(1282. \\ The transiti \\ \mu_{T_{cb\bar{c}\bar{b}}}(1282. \\ \hline \end{pmatrix}$	function: $9 F\rangle R^{s}\rangle [\phi]$ $9 F\rangle R^{s}\rangle [\phi]$ $9 F\rangle R^{s}\rangle [\psi]$ $1000000000000000000000000000000000000$	$2\chi_1]\rangle - 0$ $\frac{2\chi_1]\rangle = 0$ $\frac{2\chi_1]\rangle = 0}{0}$ $\frac{2\chi_1[\rangle = 0}{0}$ $\frac{\chi_1[\chi_1] = 0}{0}$ $\frac{\chi_1[\chi_2] = 0}{0}$ $\frac{\chi_2[\chi_2] = 0}{0}$	$\begin{array}{l} 0.027 F\rangle \\ -0.832 I\\ \text{ay channel}\\ _{5}(12882.)\\ 1^{+-})\gamma = 0\\ 0^{++})\gamma = 0\\ \text{nts:}\\ 1^{++})\gamma = 0 \end{array}$	$\begin{aligned} R^{s} \rangle [\phi_{1}\chi_{1}] \\ \overline{\gamma} \rangle R^{s} \rangle [\psi_{1}' a \\ \overline{\epsilon} [\psi_{1}' a \\ \overline{\epsilon} [\psi_{1}' a \\ \overline{\epsilon} [\psi_{1}' a \\ \overline{\epsilon}] \\ \overline{\epsilon} [\psi_{1}' a \\ \epsilon$	$\begin{split} \rangle &= -0.8 \\ \tilde{\gamma}_{1}^{\prime} \rangle \rangle + 0.8 \\ md \ B_{c}^{*} B \\ \overline{\Psi}_{tot}^{2++} \hat{\mu} \\ \\ \frac{\Gamma_{T_{cb}\overline{c}\overline{b}}(1)}{\psi_{tot}^{2++} \hat{\mu}} \\ \\ \frac{\Gamma_{T_{cb}\overline{c}\overline{b}}(1)}{\psi_{tot}^{2++} \hat{\mu}} \end{split}$	$801 F$ $555 F$ 3_{c}^{*} 2882.4 $3.4,2^{++}$ $= 0$	$\begin{split} & \langle \rangle R^s \rangle [\psi_1 \zeta_1] \rangle \\ & \langle R^s \rangle [\psi_2 \zeta_1'] \rangle \\ & \vdots \\ &$	CS Interaction Matrix non Total contr $(56.6,1^{++})\gamma = \langle \Psi \mu_{T_{cb}\pi}(12882, 0) \rangle$	$\frac{\frac{5}{8}V^{SS}(13)}{\frac{5}{8}V^{SS}(24)}$ $\frac{5}{8}(V^{SS}(14) + V^{SS}(23))$ Subtotal diagonal element ibution $= 0$ $\frac{7^{2++}}{t^{2++}} \hat{\mu}^{z} \Psi_{tot}^{1+-}\rangle = 0.774 \times 4.2^{++}) \rightarrow T_{cb}\pi(12759.3.0^{++})\gamma$	$ \begin{array}{c} 15.2 \\ 4.5 \\ 14.7 \\ \hline 29.5 \\ \hline 0.1 \\ 12882.4 \\ \hline (\mu_c + \mu_b \\ = \langle \Psi_{tot}^{2^{++}} \\ \end{array} $	$\begin{array}{c} -5.1{+}17.7\\ \frac{10}{3}v_{b\bar{b}}+\frac{20}{3}v_{b\bar{c}}\\ 9.6{+}19.7\\ \hline 41.7\\ -29.1\\ \hline 12529.4\\ \end{array}$	$-8.8+17.7$ $\frac{10}{3}C_{b\bar{b}} + \frac{20}{3}C_{b\bar{c}}$ 9.6+22.0 40.5 3.2 13599.0 $-\mu_b) = 0.345\mu_N$

Table 16 The masses, binding energies, variational parameters, the internal contribution, total wave functions, magnetic moments, transition magnetic moments, radiative decay widths, rearrangement strong

width ratios, and the relative lengths between quarks for the $J^{PC} = 0^{++}$, $2^{++} cb\bar{c}\bar{b}$ states and their lowest meson-meson thresholds. The notation is the same as that in Table 7

 $\Gamma_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++})\to T_{cb\bar{c}\bar{b}}(12796.9,1^{+-})\gamma}=0.1~\rm{keV}$

 $J/\psi\Upsilon$

 $\underline{\Gamma_{T_{cb\bar{c}\bar{b}}(12882.4,2^{++}) \to T_{cb\bar{c}\bar{b}}(12856.6,1^{++})\gamma} = 0 \ \mathrm{keV}$

The radiative decay widths:

internal contion magne	ntribution, tic momen	total wants, radi	ave func ative dec	tions, ma cay width	ignetic mom is, rearrange	ents, transi- ment strong	1 ^{+–} , notatio	$1^{++} cb\bar{c}\bar{b}$ states and the on is the same as that in	eir lowe Table	est meson-mes 7	on thresholds.
$cb\overline{c}\overline{b}$	The co	ontributi	on from	each term	Relative L	ength (fm)	0	Present Work		CMI	Model
$J^{PC} = 1^{+-}$		Value	$\Upsilon \eta_c$	Difference	(i, j) Value	$\Upsilon \eta_c$	Overall	Contribution	Value	Ref. [110]	Ref. [123]
Mass Variational Parameters (fm ⁻²)	$\begin{array}{c} B_T \\ \hline C_{11} \\ C_{22} \\ C_{33} \end{array}$	12796.9 11.9 11.9 22.9	12467.4 15.0 49.7	329.5	$\begin{array}{c} (1,2) & 0.331 \\ (1.3) & 0.289 \\ (2,3) & 0.289 \end{array}$	$0.290(\eta_c)$	<i>c</i> -quark:	$\frac{m_c}{\frac{m_b}{m_c + m_b}} \frac{\mathbf{p}_{x_1}^2}{\frac{2m_1'}{2m_c + 2m_b}} \frac{\mathbf{p}_{x_3}}{\frac{2m_3'}{2m_3'}}$	1918.0 180.6 135.3	$\frac{\frac{-m_c}{4(m_c+m_b)}m_{cb}}{-400.5}$ $\frac{\frac{5m_c}{8(m_c+m_{\bar{b}})}m_{c\bar{b}}}{986.7}$	
Quark	Mass	14522.0	14522.0	0.0	(1,4) 0.289		m_c^{eff}	$\frac{1}{2}V^{C}(12)$	-17.8	$\frac{5m_c}{8(m_c+m_{\overline{c}})}m_{c\overline{c}}$	m_c
Confinemen	t Potential	-2527.1	-3000.0	472.9	(2,4) 0.289	$0.160(\Upsilon)$		$\frac{1}{2}[V^{\circ}(13) + V^{\circ}(14)] - 1/2D$	-491.5	958.9	1724.8
Kinetic	Energy	858.5	1001.2	-142.7	(3,4) 0.331			Subtotal	1602.1	1545.1	1724.8
CS Inte	raction	-41.4	-55.8	14.4	(1,2)- $(3,4)$:	$0.168~{ m fm}$		$m_{\overline{b}}$	5343.0	$\frac{-m_b}{4(m_c+m_b)}m_{cb}$	
	(1,2)	35.6			(1,3) -158.1	$-237.2(\eta_c)$		$\frac{m_c}{m_c+m_b}\frac{\mathbf{p}_{x_1}}{2m'_1}$	64.8	-1203.8 $5m_b$ m -	
V^C	(1,4)	-158.1			(2,3) - 158.1		<i>b</i> -quark:	$\frac{m_c}{2m_c+2m_h} \frac{\mathbf{p}_{x_3}^2}{2m'_2}$	48.6	$\frac{\overline{8(m_c+m_{\bar{b}})}}{2965.5}m_{cb}$	
	Subtotal	-561.1	-1034.0	472.9	(2,4) - 158.1	$-796.7(\Upsilon)$	m_i^{eff}	$\frac{1}{2}V^{C}(12)$	-17.8 -158.1	$\frac{5m_b}{8(m_b+m_{\overline{b}})}m_{b\overline{b}}$	m_b
Total Con	tribution	256.0	-88.6	344.6	(3,4) 35.6		0	$\frac{1}{2}[V^{\circ}(23) + V^{\circ}(24)] - 1/2D$	-491.5	2951.6	5052.9
Total Wave	function:							Subtotal	4824.7	4713.3	5052.9
$\Psi_{tot} = 0.8$	$77 F\rangle R^s\rangle [$	$\phi_1 \chi_2] \rangle -$	$0.064 F\rangle$	$ R^s\rangle [\phi_2\chi]$	$ 2\rangle + 0.320 F $	$\langle R^s\rangle [\phi_1\chi_3]\rangle$		$-\frac{1}{4}[V^{SS}(12) + V^{SS}(34)]$	-5.0	$-\frac{8}{3}v_{cb} - \frac{10}{3}v_{cc}$	$-\frac{8}{3}C_{cb} - \frac{10}{3}C_{\overline{c}\overline{c}}$
+0.3	$20 F\rangle R^{\circ}\rangle [$	$\phi_1\chi_4]\rangle +$	0.105 F	$ R^{\circ}\rangle [\phi_{2}\chi]$	$ _{3} \rangle + 0.105 F $	$\langle R^{\circ}\rangle [\phi_2\chi_4]\rangle$	\mathbf{CS}	$-\frac{1}{8}V^{-2}(13)$	-10.0	-5.2-17.7	-8.8-17.7
= 0.1	$211 F\rangle R^{\circ}\rangle $	$ \psi_1\zeta_2 \rangle \dashv$	-0.854 F	$\langle R^{\circ}\rangle [\psi_{1}\zeta$	$ 3 \rangle + 0.223 F $	$\langle R^{\circ}\rangle [\psi_2\zeta_2]\rangle$	Interaction	$= \frac{1}{8}V^{-1}(24)$	-4.0	$-\frac{1}{3}v_{b\bar{b}} - \frac{1}{3}v_{b\bar{c}}$	$-\frac{1}{3}C_{b\overline{b}} - \frac{1}{3}C_{b\overline{c}}$
0		F / #/1\	0.40117		+0.420 F	$\langle R^{\circ}\rangle [\psi_{2}\zeta_{3}]\rangle$		$\frac{-\frac{1}{8}[V^{-1}(14) + V^{-1}(23)]}{C_{11}L_{12}L_{12}L_{12}}$	-10.0	-9.0-19.7	-9.7-22.0
= -0.4	$481 F\rangle R^{\circ}\rangle $	$ \psi_1\zeta_2 \rangle -$	-0.481 F	$\langle \rangle R^{\circ} \rangle [\psi_1 \zeta]$	$ 3 \rangle + 0.283 F $	$ \langle R^{\circ}\rangle [\psi_{1}\zeta_{4}]\rangle$	Mataina	Subtotal	-41.4	-52.2	-58.1
-0.,	$\frac{399 F\rangle K^{-}\rangle }{1}$	$ \psi_2\zeta_2 \rangle -$	- 0.395 F	$\frac{ R^{-}\rangle [\psi_{2}\zeta] }{ \psi_{2}\zeta }$	$ _{3]} + 0.382 F $	$\langle R^{-}\rangle [\psi_{2}\zeta_{4}]\rangle$	Total contri		19706.0	-54.7	-19.6
The rearran	igement str	ong deca	iy channe	21:	1 10 4 1	-	Total contr.		12790.9	12409.9	1.20.20
$\frac{1}{T_{cb\bar{c}\bar{b}}(12796.)}$ The radiativ	$(9,1^{+-}) \rightarrow \eta_b J_p$ ve decav wi	$\frac{\psi + T_{cb\bar{c}}}{dths}$	<u>b</u> (12796.9,	$(1^{+-}) \rightarrow \Upsilon \eta_c$	The magnet	$T_{cb\overline{c}\overline{b}}(12796.9,2)$ ic moments:	$T_{r+=\overline{r}}(1279)$	$\frac{1}{1} \frac{T_{cb\bar{c}\bar{b}}(12796.9,1^{+-}) \to B_c^* \bar{B}_c}{6.9,1^{+-}} = \langle \Psi_{tot}^{1^{+-}} \hat{\mu}^z \Psi_t^1$	$\frac{T_{cb\bar{c}\bar{b}}}{T_{cb\bar{c}\bar{b}}}$	$(12796.9, 1^{+-}) \rightarrow B_c I$	$\frac{3}{c} = 1:3.9:3.9$
Γπ (10707	2 1+-) . T	(10750.6	o++)=:	33.1 keV	The transiti	on magnetic	moments:	$\frac{\mu_{T}}{\mu_{T}} = (10000 \pm 0.000 \pm 0.0000 \pm 0.0000 \pm 0.00000 \pm 0.000000 \pm 0.00000000$	10750 6 14	$= 0.345 \mu_N$	
$\Gamma_{cb\bar{c}\bar{b}}(12797)$	$(1++) \rightarrow T_{cb\bar{c}}$	E(12759.6,	(+-) = ().1 keV	Um (10050.0	······································	0700 0 1+->	$= \langle \Psi_{i+t}^{1++} \hat{\mu}^{z} \Psi_{i+t}^{1+-} \rangle = 0.1^{+}$	$13(\mu_{a} +$	$(\mu_b) = 0.036\mu_N$	
$\Gamma_{cb\bar{c}\bar{b}}(12830.$	$(0,1^{++}) \rightarrow I_{cb\bar{c}}$	(10700.9,	$(+-)^{\gamma} = ($	4.9 keV	П. (12830.0	$(1^{+}) \rightarrow I_{cb\overline{c}\overline{b}}(1)$	2790.9,1 ·).y	$= \langle \Psi_{t,t}^{1+-} \hat{\mu}^{z} \Psi_{t,t}^{0++} \rangle = 0.22$	26×24	$\frac{1}{2}(\mu_a - \mu_b) = 0.0$)81 <i>11</i> M
$\frac{I_{cb\bar{c}\bar{b}}(12882)}{I^{PC}-1^{++}}$	$4,2$ $() \rightarrow T_{cb\overline{c}}$	Volue	$\gamma I = \gamma I = \gamma$	Difference	$P_{cb\overline{c}\overline{b}}(12796.9$	$(1,1) \rightarrow T_{cb\overline{c}\overline{b}}(1)$	2759.6,011	Contribution	Velue	$\frac{1}{6}(\mu e^{-\mu 0}) = 0.0$	Dof [192]
J = 1 Maga	/ D	19856 6	$10^{-1}\psi$	205 5	(i, i) Value	$\sim I/\omega$			1018 0	$-m_c - m_{}$	nei. [125]
Variational Parameters	$\begin{array}{c c} & & & \\ & & C_{11} \\ & & C_{22} \end{array}$	12850.0 11.4 11.4	12501.1 12.5 49.7	295.5	(i, j) value (1.2) 0.333	$1J/\psi$	<i>c</i> -quark:	$\frac{m_b}{m_c+m_b} \frac{\mathbf{p}_{x_1}^2}{2m_1'}$	174.0	$\frac{4(m_c+m_b)}{-400.5}$ $\frac{5m_c}{8(m_c+m_b)}m_{c\bar{b}}$	
(fm^{-2})	C ₃₃	21.5	-		(1,3) 0.291	$0.318(J/\psi)$	m^{eff}	$\frac{m_b}{2m_c+2m_b} \frac{4x_3}{2m'_3}$	127.2 16.0	986.7	
Quark	Mass	14522.0	14522.0	0.0	(2,3) 0.291		110 _C	$\frac{1}{2}[V^{C}(13) + V^{C}(14)]$	-146.5	$\frac{5m_c}{8(m_c+m_{\overline{c}})}m_{c\overline{c}}$	m_c
Confinemen	t Potential	-2488.0	-2926.9	438.9	(1,4) 0.291			-1/2D	-491.5	958.9	1724.8
Kinetic	Energy	818.6	925.1	-106.5	(2,4) 0.291	$0.160(\Upsilon)$		Subtotal	1597.2	1545.1	1724.8
CS Inte	raction	10.0	41.0	-31.0	(3,4) 0.333			$m_{\overline{b}}$ m_{c} $\mathbf{p}_{x_{1}}^{2}$	5343.0	$\frac{1}{4(m_c+m_b)}m_{cb}$	
	(1,2)	32.0			(1,2)-(3,4):	0.172 fm	h quark:	$\overline{m_c + m_b} \frac{2m'_1}{2m'_2}$	62.5 45.7	$\frac{5m_b}{8(m_c+m_{\overline{t}})}m_{c\overline{b}}$	
V^C	(2,3)	-146.5					o-quark.	$\frac{m_c}{2m_c+2m_b}\frac{\omega_3}{2m'_3}$	16.0	2965.5	
	(1,4)	-146.5			(2,4) - 146.5	$-796.7(\Upsilon)$	m_b^{eff}	$\frac{1}{2}[V^{C}(23) + V^{C}(24)]$	-146.5	$\frac{6m_b}{8(m_b+m_{\overline{b}})}m_b\overline{b}$	m_b
	Subtotal	-522.0	-960.9	438.9	(3,4) 32.0			<u>-1/2D</u>	-491.5	2951.6	5052.9
Total Con	itribution	306.6	5.1	301.5	(1,3) -146.5	$-164.2(J/\psi)$		Subtotal	4829.2	4713.3	5052.9
Total Wave	function:							$\frac{1}{4}[V^{55}(12) + V^{55}(34)]$	4.9	$\frac{5}{3}v_{cb} + \frac{10}{3}v_{\overline{c}\overline{c}}$	$\frac{5}{3}C_{cb} + \frac{10}{3}C_{\overline{c}\overline{c}}$
$\Psi_{tot} = 0.6$	$93 F\rangle R^s\rangle [$	$\phi_2 \chi_3] \rangle -$	$0.693 F\rangle$	$ R^s\rangle [\phi_2\chi]$	$_{4}]\rangle + 0.139 F$	$\langle R^s\rangle [\phi_1\chi_3]\rangle$	CS	$\frac{3}{8}V^{SS}(13)$	15.5	5.2+17.7	8.8+17.7
-0.1	$ 39 F\rangle R^s\rangle $	$\phi_1\chi_4]\rangle =$	= 0.686 <i>F</i>	$\langle R^s \rangle [\psi_1 \zeta]$	$ _{4}\rangle + 0.727 F $	$ \langle R^s\rangle [\psi_2\zeta_4]\rangle$	Interaction	$\frac{3}{8}V^{SS}(24)$	4.6	$\frac{10}{3}v_{b\overline{b}} - \frac{20}{3}v_{b\overline{c}}$	$\frac{10}{3}C_{b\overline{b}} - \frac{20}{3}C_{b\overline{c}}$
= -0.4	$480 F\rangle R^s\rangle $	$ \psi_1'\zeta_2'\rangle$ -	-0.480 F	$\langle R^{s} \rangle \psi_{1}' \zeta \rangle$	$ _{3}\rangle - 0.395 F _{3}\rangle$	$ R^{s}\rangle [\psi_{2}'\zeta_{2}']\rangle$		$\frac{-\breve{8}[V^{SS}(14) + V^{SS}(23)]}{2}$	-15.0	9.6-19.7	9.7-22.0
				. 1.1.5	-0.395 F	$ \rangle R^{s}\rangle [\psi_{2}'\zeta_{3}']\rangle$		Subtotal	10.0	12.9	14.1
The magnet	tic moment	s: $T_{cb\overline{c}\overline{b}}($	12856.6,1	$l^{++}) = \langle \Psi$	$ \mu^z \Psi_{tot}^1 $	$\rangle = 0$	Matrix non	diagonal element	-6.3	-6.1	-59.5
The radiati	ve decay wi	dths:					Total contri	Ibution	12856.6	12523.6	13510.0
$\Gamma_{T_{cb\bar{c}\bar{b}}(12856.}$	$(6,1^{++}) \rightarrow T_{cb\overline{c}}$	$\overline{b}(12759.6,$	$_{0^{++})\gamma} = 0$) keV	The transiti	on magnetic :	moments: μ	$\iota_{T_{cb\overline{c}\overline{b}}(12882.4,2^{++})\rightarrow T_{cb\overline{c}\overline{b}}(12882.4,2^{++})}$	2856.6,1+-	$ \psi_{\gamma} = \langle \Psi_{tot}^{2+1} \hat{\mu^{z}} $	$ \Psi_{tot}^{\perp}\rangle = 0$

 $\left| \mu_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++}) \to T_{cb\bar{c}\bar{b}}(12759.6,0^{++})\gamma} = \langle \Psi_{tot}^{1^{++}} | \hat{\mu^{z}} | \Psi_{tot}^{0^{++}} \rangle = 0 \right|$

 $\Gamma_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++})\to B_c^*\bar{B}_c}:\Gamma_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++})\to B_c\bar{B}_c^*}=1:1$

 $\left| \mu_{T_{cb\bar{c}\bar{b}}(12856.6,1^{++}) \to T_{cb\bar{c}\bar{b}}(12796.9,1^{+-})\gamma} \right| \leq \langle \Psi_{tot}^{1++} | \hat{\mu}^z | \Psi_{tot}^{1+-} \rangle = 0.113(\mu_c + \mu_b) = 0.036\mu_N$

Table 17 The masses, binding energies, variational parameters, the in ti

width ratios, and the relative lengths between quarks for the J^{PC} = nresholds. The

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