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Two-dimensional regular string black hole via complete α' corrections

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Abstract In string theory, an important challenge is to show if the singularity of black holes can be smoothed out by the complete α' corrections. The simplest case is to consider a 2D string black hole or 3D black string. This problem was discussed in a gauged Wess–Zumino–Witten (WZW) model and the results are supposed to be correct to all orders in α' corrections. Based on the recent remarkable progress on classifying all the α' corrections, in this work, we re-study this problem with the low energy effective spacetime action, and provide classes of exact non-perturbative and non-singular solutions of the 2D black hole/3D black string via complete α' corrections.

How to resolve a singularity of a black hole is a long-standing problem in general relativity. It is well-known that the black hole possesses two kinds of infinities: one is the coordinate singularity which can be removed by coordinates transformations; another is the curvature singularity which cannot be smoothed out and is unavoidable in Einstein's gravity [1,2]. The regular black hole solutions must satisfy two requirements: (I) The curvature invariants (such as Kretschmann scalar) are regular everywhere; (II) The geodesics are complete. However, there is yet no systematic methods to obtain regular solutions from solving Einstein's equation exactly without any ad hoc settings. People therefore expect a theory of quantum gravity will provide inspirations to solve this problem completely. String theory as one candidate of quantum gravitational theories should be able to answer this question.

The simplest black hole solution in string theory, namely 1+1 dimensional string black hole, was obtained by solving a 2D closed string's low energy effective action [3,4]. This solution is only valid when the string length scale $\sqrt{\alpha'}$ is small compared to the radius of spacetime curvature. One may wonder how to obtain the exact string black hole solutions of the full action, and whether the spacetime singularity disappears in these solutions. However, it looks impossible since the low energy effective action with complete α' corrections is unknown. As an alternative, Witten obtained the exact metric which described the region outside the event horizon of 2D string black hole through the $SL(2,R)/U(1)$ gauged WZW model, this result is conformally invariant to all orders in $1/k$ when $k\to\infty$ ($k\sim 1/\alpha'$ is the Kac–Moody level) [5]. This metric still possesses a curvature singularity in the maximally extended spacetime. Then, in Ref. [6], Dijkgraaf, Verlinde and Verlinde discovered the exact 2D string black hole for general k, which was supposed to be correct to all orders in α'. Based on this work, Perry and Teo [7], and Yi [8] studied its maximally extended spacetime. The Weyl invariance of this solution is then verified by Tseytlin up to 3-loops (α'^2) in the bosonic sigma model [10]. The supersymmetric 4-loops (α'^3) was also checked in Ref. [11].

In recent works [12–14], Hohm and Zwiebach reconsidered how to classify all orders α' corrections of the low energy effective action. This progress makes it possible to re-study the exact 2D string black hole systematically. Hohm and Zwiebach's motivation was based on two reasons: (I) the tree-level string effective action and its first order α' correction can be put into an explicit $O(d,d)$ covariant form by suitable field redefinitions [15–19]; (II) Sen proved that for configurations independent of m coordinates, all orders in α' expansion possess the $O(m,m)$ symmetry [17,18]. Hohm and Zwiebach therefore assumed that the standard $O(d,d)$ matrix always keep its form unchanged to all orders in α', meanwhile the $O(d,d)$-breaking terms could be absorbed into the standard matrix by the field redefinitions. With this assumption, Hohm and Zwiebach showed that all orders in α' are classified by even powers of Hubble parameter in FLRW cosmological background. The dilaton appears trivially and only first order time derivatives need to be included. Since the equations of motion (EOM) only include first two deriva-

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tives of spacetime metric, the theory is ghost free and exactly solvable. This significant progress makes it possible to study non-perturbative stringy effects. Based on this work, we have found that the big-bang singularity could be smoothed out by the complete α' corrections [20,21]. It is also proved that the Hohm and Zwiebach's derivation can also apply to the domain wall background [22], so that the naked singularities can be resolved [23]. Based on these attempts, it is reasonable to believe that the α' corrections will resolve the singularity of black hole [9]. However, the process is tricky for the general spherically symmetric black holes, because they break the $O(d, d)$ symmetry and Hohm-Zwiebach action cannot be derived from this background. As an alternative, we consider the 2D black hole/3D black string at first.

In this paper, our aim is to find regular solutions which exactly solve the EOM of completely α' corrected closed string theory. In the perturbative region $\alpha' \to 0$, it reduces to traditional 2D black hole/3D black string. At first, let us start by the 2D low energy effective action of closed string:

$$S = \int d^2x\sqrt{-g}e^{-2\phi}\left(R + 4(\nabla\phi)^2 + \lambda^2\right), \tag{1}$$

where $g_{\mu\nu}$ is the string metric, ϕ is the physical dilaton, $\lambda^2 = -\frac{2(D-26)}{3\alpha'}$ and we set Kalb-Ramond field $b_{\mu\nu} = 0$ for simplicity. This two dimensional gravity has dynamics due to a pre-factor $e^{-2\phi}$. The black hole solution of this action is given by [3]:

$$\begin{aligned} ds^2 &= -\left(1-\frac{M}{r}\right)dt^2 + \left(1-\frac{M}{r}\right)^{-1}\frac{1}{\lambda^2 r^2}dr^2, \\ \phi &= -\frac{1}{2}\ln\left(\frac{2}{M}r\right). \end{aligned} \tag{2}$$

where we set an integral constant $\phi_0 = 0$ in ϕ for simplicity. If we add a direction $d\varphi^2$ in this metric, it is called a black string solution and firstly discovered by Horne and Horowitz [24]. The difference between 2D and 3D theories is only λ due to its definition. When we consider the extra dimension φ, the 3D black string solution is a simple product of $d\varphi^2$ and the two dimensional metric, which does not affect the action [24]. Therefore, although we only study the 2D black hole in this paper, our result also applies to the 3D black string. Back to the metric (2), the event horizon is located at $r = M$ and its curvature singularity is $r = 0$ due to a scalar curvature $R = \frac{\lambda^2 M}{r}$. Keep in mind that there are two kinds of coordinate transformations which cover different regions of maximally extended spacetime. The first one is

$$\frac{r}{M} = \cosh^2\left(\frac{\lambda}{2}x\right), \tag{3}$$

where $r \geq M$ and $x \geq 0$. Utilizing this coordinate transformation, the metric (2) becomes

$$\begin{aligned} ds^2 &= -\tanh^2\left(\frac{\lambda}{2}x\right)dt^2 + dx^2, \\ \Phi &= -\ln\left(\sinh(\lambda x)\right), \end{aligned} \tag{4}$$

where $O(d, d)$ invariant dilaton is defined by

$$\Phi = 2\phi - \ln\sqrt{\det g_{ij}}. \tag{5}$$

This metric is well-known as Witten's 2D black hole solution, which was obtained by the $SL(2, R)/U(1)$ gauged WZW model [5]. This metric could be applied in Hohm-Zwiebach action directly. However, it does not possess a curvature singularity since it only describes the region outside the event horizon ($x = 0$), and the scalar curvature $R_0 = \lambda^2\cosh^{-2}\left(\frac{\lambda x}{2}\right)$ is regular in this region. To discover the curvature singularity of the metric (2), we need to adopt the second kind of coordinate transformations:

$$\frac{r}{M} = \cos^2\left(\frac{\lambda}{2}x\right), \tag{6}$$

where $0 \leq r \leq M$ and we only consider one period, namely $0 \leq x \leq \frac{\pi}{\lambda}$. Based on this transformation, the metric (2) becomes

$$\begin{aligned} ds^2 &= -dx^2 + \tan^2\left(\frac{\lambda}{2}x\right)dt^2, \\ \Phi &= -\ln\left(\sin(\lambda x)\right), \end{aligned} \tag{7}$$

which describes the inner metric of black hole, and x here plays a role as the time-like direction.This metric (7) topologically corresponds to an annulus. The event horizon is located at $x = 0$ and the curvature singularity is the boundary $x = \frac{\pi}{\lambda}$ due to the scalar curvature $R_0 = \lambda^2\cos^{-2}\left(\frac{\lambda x}{2}\right)$. Our aim is to remove the curvature singularity of (7) by the complete α' corrections.

Based on (7), we first assume the ansatz,

$$ds^2 = -dx^2 + a(x)^2 dt^2. \tag{8}$$

The closed string fields which depend on this metric possess $O(1, 1)$ symmetry. It is worth noting that we can also use the ansatz (2), which shares the same result with (7) by the coordinate transformation (6). Based on this ansatz, Hohm and Zwiebach showed that the following low energy effective action with complete α' corrections could be rewritten as

$$\begin{aligned} I_{HZ} &= \int d^2x\sqrt{-g}e^{-2\phi}\Big(R + 4(\partial\phi)^2 \\ &\quad + \frac{1}{4}\alpha'\left(R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + \ldots\right) + \alpha'^2(\ldots) + \ldots\Big), \\ &= \int dx e^{-\Phi}\left(-\dot{\Phi}^2 - \sum_{k=1}^{\infty}(-\alpha')^{k-1}2^{2k+1}c_k H^{2k}\right), \end{aligned} \tag{9}$$

where $\dot{f}(x) \equiv \partial_x f(x)$, $H(x) \equiv \frac{\dot{a}(x)}{a(x)}$, $c_1 = -\frac{1}{8}$, $c_2 = \frac{1}{64}$, $c_3 = -\frac{1}{3.2^7}$, $c_4 = \frac{1}{2^{15}} - \frac{1}{2^{12}}\zeta(3)$ and $c_{k>4}$'s are unknown coefficients for a bosonic case [25]. In addition, the $O(d,d)$ symmetry of the action (9) is manifested by the following transformations:

$$\Phi \to \Phi, \qquad a \to a^{-1}, \qquad H \to -H. \tag{10}$$

To match the model (1), we need to add an $O(d,d)$ invariant constant into the action (9):

$$I_m = \int d^2x e^{-\Phi}\lambda^2. \tag{11}$$

which is also a scalar under the general coordinate transformations. The EOM then is

$$\begin{aligned}
&\ddot{\Phi} + \frac{1}{2}Hf(H) = 0,\\
&\frac{d}{dx}\left(e^{-\Phi}f(H)\right) = 0,\\
&\dot{\Phi}^2 + g(H) + \lambda^2 = 0,
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
f(H) &= \sum_{\mathrm{k}=1}^{\infty}\left(-\alpha'\right)^{k-1}2^{2(k+1)}kc_kH^{2k-1}\\
&= -2H - \alpha' 2H^3 + \cdots,\\
g(H) &= \sum_{\mathrm{k}=1}^{\infty}\left(-\alpha'\right)^{k-1}2^{2k+1}(2k-1)c_kH^{2k}\\
&= -H^2 - \alpha'\frac{3}{2}H^4 + \cdots,
\end{aligned} \tag{13}$$

and there is an extra constraint $\dot{g}(H) = H\dot{f}(H)$. It is easy to check that the solution (4) and (7) satisfy the EOM (12) at zeroth order in α'. Furthermore, we wish to stress that α' is any positive real number in the EOM (12) and the action (9). Therefore, Eq. (13) are not the simple expansions in the limit $\alpha' \to 0$, but the non-perturbative series in general α'.

To remove the curvature singularity of (7) to obtain a non-perturbative and non-singular solution of EOM (12) with complete α' corrections, there are two requirements:

- The solutions are assumed to be regular everywhere for general α'.
- In the perturbative regime, namely, as $\alpha' \to 0$, the solutions must reduce to the perturbative solutions of EOM (12). Although, we know that $\alpha'\lambda^2 = -\frac{2(D-26)}{3\lambda^2} = 16$ when $D = 2$ which satisfies the condition $\alpha'\lambda^2 > 0$, we treat $\alpha'\lambda^2$ as a general value here as in [6].

In the body, we present one class of general non-perturbative solution which covers all coefficients c_k. In the Appendix A, we will give other possible general solutions. Now, let us start by calculating the perturbative solutions of EOM (12).

For convenience, we introduce a new variable Ω as

$$\Omega \equiv e^{-\Phi}, \tag{14}$$

where $\dot{\Omega} = -\dot{\Phi}\Omega$ and $\ddot{\Omega} = \left(-\ddot{\Phi} + \dot{\Phi}^2\right)\Omega$. And the EOM (12) become

$$\begin{aligned}
&\ddot{\Omega} - \left(h(H) - \lambda^2\right)\Omega = 0,\\
&\frac{d}{dt}\left(\Omega f(H)\right) = 0,\\
&\dot{\Omega}^2 + \left(g(H) + \lambda^2\right)\Omega^2 = 0,
\end{aligned} \tag{15}$$

where we define a new function

$$h(H) \equiv \frac{1}{2}Hf(H) - g(H) = \alpha'\frac{1}{2}H^4 + \ldots, \tag{16}$$

It is easy to see that $h(H) = 0$ at the zeroth order in α'. Then, we assume the perturbative solutions of the EOM (15) take the following forms when $\alpha' \to 0$:

$$\begin{aligned}
\Omega(x) &= \Omega_0(x) + \alpha'\Omega_1(x) + \alpha'^2\Omega_2(x) + \ldots,\\
H(x) &= H_0(x) + \alpha' H_1(x) + \alpha'^2 H_2(x) + \ldots,
\end{aligned} \tag{17}$$

where we denote Ω_i and H_i as the i-th order of the perturbative solutions. Therefore, the perturbative solution can be calculated order by order

$$\begin{aligned}
H(x) &= \lambda\csc(\lambda x) - \frac{\lambda^3}{4}\frac{(\cos(2\lambda x) + 4)}{\sin^3(\lambda x)}\alpha' + \cdots,\\
\Omega(x) &= \sin(\lambda x) + \frac{\lambda^2}{4}\frac{\cos(2\lambda x)}{\sin(\lambda x)}\alpha' + \cdots.
\end{aligned} \tag{18}$$

Due to the Eq. (14), we also have

$$\Phi(x) = -\log(\sin(\lambda x)) - \frac{1}{4}\lambda^2\left(\cot^2(\lambda x) - 1\right)\alpha' + \cdots. \tag{19}$$

Based on the perturbative solutions (18) and (19) up to any higher order, we can figure out the general non-perturbative and non-singular solution which exactly solves the EOM (12) and covers all coefficients c_k:

$$\Phi(x) = \frac{1}{2}\log\left(\frac{\sum_{\mathrm{k}=1}^{N}\left(\alpha'\lambda^2\right)^{k-1}}{\sum_{\mathrm{k}=1}^{N}\sigma_k(\lambda x, c_k)\left(\alpha'\lambda^2\right)^{k-1}}\right), \tag{20}$$

where σ_k's are functions of λx and c_k. After obtained regular $\Phi(x)$, the regularity of $H(x)$, $f(x)$ and $g(x)$ is guaranteed due to the EOM (12). Moreover, in the perturbative regime $\alpha' \to 0$, the general solution $\Phi(x)$ is expanded as,

$$\begin{aligned}
\Phi(x) = &-\frac{1}{2}\log(\sigma_1) + \frac{(\sigma_1 - \sigma_2)}{2\sigma_1}\alpha'\lambda^2\\
&+ \frac{\left(\sigma_1^2 - 2\sigma_3\sigma_1 + \sigma_2^2\right)}{4\sigma_1^2}\left(\alpha'\lambda^2\right)^2 + \cdots.
\end{aligned} \tag{21}$$

To analyze the general solution (20), we choose the special case whose perturbative expansion only covers $c_1 = -\frac{1}{8}$ and $c_2 = \frac{1}{64}$. It is not difficult to check that the general solution (20) which covers the coefficients $c_{N>2}$ does not affect our following argument due to the Ref. [21]. Therefore, to match the expansion (21) with the perturbative solution (19), we can fix the functions $\sigma_1 = \sin^2(\lambda x)$, $\sigma_2 = \frac{1}{2}$ and $N = 2$ such that

$$\begin{aligned}
\Phi(x) &= \log\sqrt{\frac{1+\alpha'\lambda^2}{\sin^2(\lambda x)+\frac{1}{2}\alpha'\lambda^2}},\\
H(x) &= -\frac{\sqrt{2}\lambda\left(\left(\alpha'\lambda^2+1\right)\cos(2\lambda x)-1\right)}{\left(\alpha'\lambda^2+1\right)^{1/2}\left(\alpha'\lambda^2+1-\cos(2\lambda x)\right)^{3/2}},\\
f(x) &= -2\sqrt{2}\lambda\left(\frac{\alpha'\lambda^2+1}{\alpha'\lambda^2+1-\cos(2\lambda x)}\right)^{1/2},\\
g(x) &= \frac{\lambda^2}{\left(\alpha'\lambda^2+1-\cos(2\lambda x)\right)^2}\Big(-\alpha'\lambda^2\left(\alpha'\lambda^2+2\right)\\
&\quad +2\left(\alpha'\lambda^2+1\right)\cos(2\lambda x)-2\Big).
\end{aligned} \tag{22}$$

Based on the ansatz (8), Kretschmann scalar is given by $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{1}{2}R_{\mu\nu}R^{\mu\nu} = R^2 = 4\left(\dot{H}+H^2\right)^2$. Therefore, the regular solution exists when $\alpha'\lambda^2 > 0$ in the region $0 \le x \le \frac{\pi}{\lambda}$. From the solution (22), we will get

$$\begin{aligned}
a(x) = C\exp\sqrt{2}\Bigg[&\sqrt{\frac{\alpha'\lambda^2+1}{\alpha'\lambda^2}}\,\mathbb{F}\left(x\lambda\middle|-\frac{2}{\alpha'\lambda^2}\right)\\
&-\sqrt{\frac{\alpha'\lambda^2}{\alpha'\lambda^2+1}}\,\mathbb{E}\left(x\lambda\middle|-\frac{2}{\alpha'\lambda^2}\right)\\
&-\frac{\sin(2\lambda x)}{\sqrt{\left(\alpha'\lambda^2+1\right)\left(\alpha'\lambda^2+1-\cos(2\lambda x)\right)}}\Bigg],
\end{aligned} \tag{23}$$

and physical dilaton,

$$\phi(x) = \frac{1}{2}\Phi(x) + \frac{1}{2}\ln a(x), \tag{24}$$

where $\mathbb{F}(\phi|m)$ and $\mathbb{E}(\phi|m)$ are elliptic integrals of the first and second kinds, and C is an integral constant. Now, we set $\alpha'\lambda^2 = 16$, $\lambda = 1$, $C = 1$ and plot $R(x)$, $a(x)$ and $\phi(x)$ in the region $0 \le x \le \frac{\pi}{\lambda}$ as an example, see Fig. 1.

In Fig. 1, it is easy to see that $R(x)$, $a(x)$ and $\phi(x)$ are regular in the region $0 \le x \le \frac{\pi}{\lambda}$. To see how α' corrections affects the property of the curvature singularity, we can see the following figure.

In Fig. 2, it presents the behavior of Ricci scalar $R(x)$ of the solution (22). When α' goes to zero, $R(x)$ reduces to $R_0(x) = \lambda^2\cos^{-2}\left(\frac{\lambda x}{2}\right)$ (Red solid line), which possesses the curvature singularity at $x = \frac{\pi}{\lambda}$. On the other hand, when α' grows up, the curvature singularity disappears and $R(x)$ becomes regular everywhere.

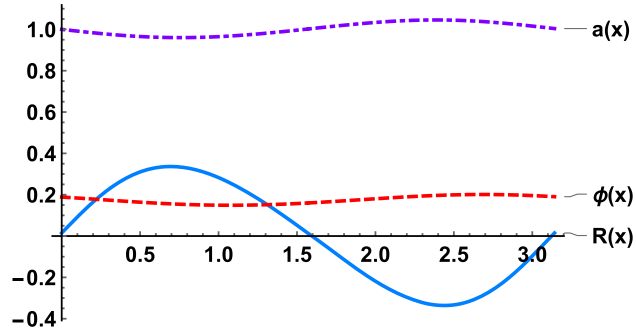

Fig. 1 The figure of $R(x)$, $a(x)$ and $\phi(x)$ at $\alpha'\lambda^2 = 16$

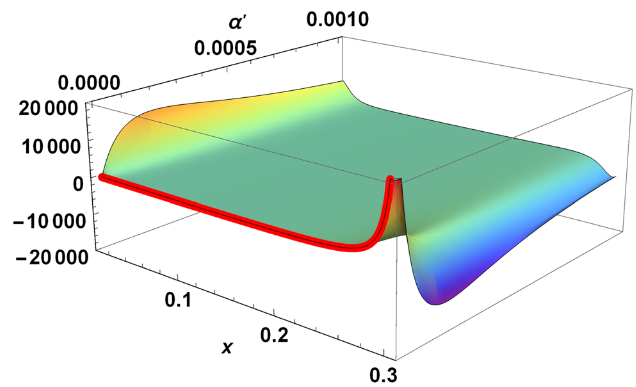

Fig. 2 The figure of $R(x)$, where $\lambda = 10$, $0 \le \alpha' \le \frac{1}{1000}$ and $0 \le x \le \frac{\pi}{\lambda} \simeq 0.3$. The curvature singularity locates at $x = \frac{\pi}{\lambda} \simeq 0.3$ when $\alpha' = 0$

Finally, we need to stress the relations between the Dijkgraaf et al.'s exact string black hole [6] and our result. As expected, the perturbative solutions (18) and (19) of Hohm-Zwiebach action matches with α' expansion of Dijkgraaf et al.'s solution up to two-orders. The verification requires to know the higher-loop β-function and Hohm-Zwiebach action simultaneously, and it is not straightforward since there exits a series of field redefinitions from the higher-loop β-function to the EOM of Hohm-Zwiebach action. Each field redefinition modifies the EOM and their corresponding perturbative solutions. In Appendix B, we present the simple example of field redefinitions up to the first-order α' correction. Beyond second-order α' correction, the β-functions are unknown. It is therefore impossible to verify the correctness of the Dijkgraaf et al.'s solution to all orders in α' through the Hohm-Zwiebach action.

In short, we used the complete α' corrections of closed string theory to remove the singularity of 2D black hole or 3D black string. We looked for the regular black hole solution (22) which exactly solves the EOM of Hohm-Zwiebach action. In the perturbative limit $\alpha' \to 0$, Witten's 2D black hole solution is recovered. The T-dual solutions can be achieved by simply replacing $a \to a^{-1}$ in Eq. (23). Moreover, it is worthwhile to study how to obtain other exact solution (which possesses two spacetime sigularities) of coset model [26] from the Hohm-Zwiebach action.

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Appendix A

Supposing the coefficients $c_{k\leq n}$ are known, to figure out the non-perturbative solutions, we can choose a different kind of ansatz. Here, we present two simple examples. The first possible ansatz is

$$\Phi(x) = -\frac{1}{2}\log\left[\sum_{k=1}^{N}\frac{\rho_k(\lambda x, c_k)}{1+(\alpha'\lambda^2)^k}\right], \tag{A1}$$

where ρ_k's are functions of λx and coefficients c_k. Singularities appear if and only if

$$\sum_{k=1}^{N}\frac{\rho_k(\lambda x, c_k)}{1+(\alpha'\lambda^2)^k} = 0. \tag{A2}$$

If we wish to cover the first two terms of the perturbative solution, we set $N = 2$. In the perturbative regime $\alpha' \to 0$, the ansatz $\Phi(x)$ in (A1) is expanded as

$$\Phi(x) = -\frac{1}{2}\log(\rho_1+\rho_2) + \frac{\rho_1}{2(\rho_1+\rho_2)}\alpha'\lambda^2 - \frac{(\rho_1^2 - 2\rho_2^2)}{4(\rho_1+\rho_2)^2}\left(\alpha'\lambda^2\right)^2 + \mathcal{O}\left(\alpha'^3\right). \tag{A3}$$

To match the perturbative solution, ρ_k's can be fixed as

$$\rho_1 = -\frac{1}{2}\cos(2\lambda x)\csc(\lambda x), \quad \rho_2 = \frac{1}{2}\csc(\lambda x), \quad \ldots \tag{A4}$$

The second ansatz is given by

$$\Phi(x) = -\frac{1}{2}\log\left[\sum_{k=1}^{N}\left(\alpha'\lambda^2\right)^{k-1}\omega_k(\lambda x, c_k)\right], \tag{A5}$$

where ω_k's are functions of λx and coefficients c_k. Singularities appear if and only if

$$\sum_{k=1}^{N}\left(\alpha'\lambda^2\right)^{k-1}\omega_k(\lambda x, c_k) = 0. \tag{A6}$$

In the perturbative regime $\alpha' \to 0$, the ansatz $\Phi(x)$ in (A5) is expanded as

$$\Phi(x) = -\frac{1}{2}\log(\omega_1) - \frac{\omega_2}{2\omega_1}\alpha'\lambda^2 - \frac{(2\omega_1\omega_3 - \omega_2^2)}{4\omega_1^2}\left(\alpha'\lambda^2\right)^2 + \mathcal{O}\left(\alpha'^3\right). \tag{A7}$$

To match the perturbative solution, ω_k's can be fixed as

$$\omega_1 = \sin^2(\lambda x), \qquad \omega_2 = \frac{1}{2}\cos(2\lambda x), \qquad \cdots . \tag{A8}$$

Appendix B

Considering the FLRW ansatz

$$ds^2 = -n(x)^2 dx^2 + a(x)^2 dt^2, \tag{B1}$$

the ordinary low energy effective action with the first-order α' correction becomes

$$\begin{aligned} I_o &= \int d^2x\sqrt{-g}e^{-2\phi} \times \left(R + 4(\partial\phi)^2 + \lambda^2 + \frac{1}{4}\alpha' R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}\right) \\ &= \int dx e^{-\Phi}\left[\frac{1}{n}\left(-\dot{\Phi}^2 + H^2\right) + n\lambda^2 + \alpha'\frac{1}{n^5}\left(H\dot{n} - n\left(\dot{H} + H^2\right)\right)^2\right]. \end{aligned} \tag{B2}$$

The corresponding perturbative solution is

$$\begin{aligned} \Phi_0 &= -\log(\sin(\lambda x)), \\ H_0 &= \lambda\csc(\lambda x), \\ H_1 &= -2\lambda^3\sin^4\left(\frac{\lambda x}{2}\right)\csc^3(\lambda x), \end{aligned} \tag{B3}$$

which is consistent with Dijkgraaf et al.'s result inside the event horizon up to the first-order α' correction. Using the field redefinitions,

$$n = n + \alpha'\delta\mathbf{n},$$

$$H = H + \alpha'\delta\mathbf{H},$$
$$\Phi = \Phi + \alpha'\delta\Phi, \tag{B4}$$

where

$$\begin{aligned}\delta\mathbf{n} &= \frac{1}{32}\lambda^2\csc^4(\lambda x)\left(-64\cos(\lambda x) + 16\cos(2\lambda x)\right.\\ &\quad\left.+36\sin(\lambda x) - 3\sin(3\lambda x) + 5\sin(5\lambda x)\right),\\ \delta\mathbf{H} &= -\frac{1}{4}\lambda^3\left(4\cos(\lambda x) + 1\right)\csc^3(\lambda x),\\ \delta\Phi &= -\frac{1}{4}\lambda^2\left(\cot^2(\lambda x) - 1\right),\end{aligned} \tag{B5}$$

the action becomes Hohm-Zwiebach action with the first-order α' correction after setting $n = 1$,

$$I_{HZ} = \int dx e^{-\Phi}\left(-\dot{\Phi}^2 + H^2 + \frac{1}{2}\alpha' H^4 + \lambda^2\right). \tag{B6}$$

And the corresponding perturbative solution:

$$\begin{aligned}\Phi_0 &= -\log(\sin(\lambda x)),\\ H_0 &= \lambda\csc(\lambda x),\\ \Phi_1 &= -\frac{1}{4}\lambda^2\left(\cot^2(\lambda x) - 1\right),\\ H_1 &= -\frac{\lambda^3}{4}\frac{(\cos(2\lambda x) + 4)}{\sin^3(\lambda x)},\end{aligned} \tag{B7}$$

which is consistent with (18) and (19).

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