



# On the nature of $X(3960)$

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**Abstract** A near-threshold enhancement in the  $D_s^+ D_s^-$  system, dubbed as  $X(3960)$ , is observed by the LHCb collaboration recently. A combined analysis on  $\chi_{c0}(3930) (\rightarrow D^+ D^-)$ ,  $X(3960) (\rightarrow D_s^+ D_s^-)$ , and  $X(3915) (\rightarrow J/\psi \omega)$  is performed using both a  $K$ -matrix approach of  $D_{(s)} \bar{D}_{(s)}$  four-point contact interactions and a model of Flatté-like parameterizations. The use of the pole counting rule and spectral density function sum rule indicate, under current statistics, that this  $D_s^+ D_s^-$  near-threshold state has probably the mixed nature of a  $c\bar{c}$  confining state and  $D_s^+ D_s^-$  continuum.

## 1 Introduction

Most recently, the LHCb experiment [1,2] announced a new hadron,  $X(3960)$ , observed in the  $B^+ \rightarrow D_s^+ D_s^- K^+$  process, where the properties of this state are measured to be

$$M = 3956 \pm 5 \pm 10 \text{ MeV},$$

$$\Gamma = 43 \pm 13 \pm 8 \text{ MeV},$$

$$J^{PC} = 0^{++}.$$

If taking the  $X(3960)$  state and the  $\chi_{c0}(3930)$  in the  $D^+ D^-$  mass distribution [3] as the same particle, the partial-width ratio [2] is calculated to be

$$\frac{\Gamma(X \rightarrow D^+ D^-)}{\Gamma(X \rightarrow D_s^+ D_s^-)} = 0.29 \pm 0.09 \pm 0.10 \pm 0.08,$$

which probably implies the exotic nature of this hadron, since it is harder to excite an  $s\bar{s}$  pair than  $u\bar{u}$  or  $d\bar{d}$  pairs from the

vacuum if this state is a pure charmonium [2]. Several recent theoretical models [4–8] believe that the  $X(3960)$  state is a molecular  $D_s^+ D_s^-$  structure, while others take it as a scalar  $[c\bar{s}][\bar{c}s]$  tetraquark [9–11].

Besides the  $X(3960)$  and  $\chi_{c0}(3930)$  states, the  $X(3915)$  found in the  $J/\psi \omega$  mass spectrum [12] is also near the  $D_s^+ D_s^-$  threshold. As the three states have compatible masses and widths, as well as the preferred  $J^{PC} = 0^{++}$  assignment [12], we assume that they are the same hadron, noted  $X$ , in this work. A combined analysis on the nature for this  $X$  state is performed using both a model of  $D\bar{D}$  and  $D_s^+ D_s^-$  four-point contact interactions and an energy-dependent Flatté-like parameterization. The pole counting rule (PCR) [13], which has been generally applied to the studies of “XYZ” physics in Refs. [14–19], and spectral density function sum rule (SDFSR) [18–23] (only available for a Flatté(-like) model) are employed to distinguish whether this  $X$  state is more inclined to be confining state bound by color force, or composite hadronic molecule loosely bound by deuteron-like meson-exchange force.

In this paper, Sect. 2 presents a couple-channel  $K$ -matrix approach without the explicitly-introduced  $X$  state<sup>1</sup> to model the  $D_{(s)} \bar{D}_{(s)}$  rescattering, Sect. 3 exploits a parameterization with explicitly-introduced  $X$  state, using an energy-dependent Flatté-like formula, to describe the couplings of  $X \rightarrow D^+ D^-$ ,  $D_s^+ D_s^-$ , and  $J/\psi \omega$ , and a brief summary and discussion are carried out in the last section.

## 2 $K$ -matrix approach

For a comprehensive study of this  $X$  state’s nature, four decays, *i.e.*  $B^+ \rightarrow D^+ D^- K^+$ ,  $B^+ \rightarrow D_s^+ D_s^- K^+$ ,  $B^+ \rightarrow J/\psi \omega K^+$ , and  $\gamma^* \gamma^* \rightarrow J/\psi \omega$ , are utilized in this article.

<sup>1</sup> Explicit  $X$  state means that a field of  $X$  is introduced in the Lagrangians.

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In this section, a couple-channel  $K$ -matrix approach with the implicitly-introduced  $X$  state is employed to get the unitarized amplitudes. Then, poles of unitarized amplitudes will be searched for in the complex  $s$  planes, and PCR [13] is implemented to investigate the nature of this  $X$  structure near the  $D_s^+ D_s^-$  threshold.

### 2.1 Amplitudes of $D_{(s)} \bar{D}_{(s)}$ rescattering effect

The effective Lagrangians for the above four channels under  $S$ -wave couplings can be constructed as follows:

$$\begin{aligned}
 \mathcal{L}_{B^+ D \bar{D} K^+} &= g_{B1} B^+ D \bar{D} K^+, \\
 \mathcal{L}_{B^+ D_s^+ D_s^- K^+} &= g_{B2} B^+ D_s^+ D_s^- K^+, \\
 \mathcal{L}_{\gamma^* \gamma^* D \bar{D}} &= g_{\gamma 1} F^{\mu\nu} F_{\mu\nu} D \bar{D}, \\
 \mathcal{L}_{\gamma^* \gamma^* D_s^+ D_s^-} &= g_{\gamma 2} F^{\mu\nu} F_{\mu\nu} D_s^+ D_s^-, \\
 \mathcal{L}_{D \bar{D} D \bar{D}} &= g_{11} D \bar{D} D \bar{D}, \\
 \mathcal{L}_{D \bar{D} D_s^+ D_s^-} &= g_{12} D \bar{D} D_s^+ D_s^-, \\
 \mathcal{L}_{D \bar{D} J/\psi \omega} &= g_{13} D \bar{D} \psi_\mu \omega^\mu, \\
 \mathcal{L}_{D_s^+ D_s^- D_s^+ D_s^-} &= g_{22} D_s^+ D_s^- D_s^+ D_s^-, \\
 \mathcal{L}_{D_s^+ D_s^- J/\psi \omega} &= g_{23} D_s^+ D_s^- \psi_\mu \omega^\mu,
 \end{aligned} \tag{1}$$

where  $D = (D^0, D^+)^T$ ,  $\bar{D} = (\bar{D}^0, D^-)$ ,  $g$  with subscripts stand for coupling constants, and the subscripts 1, 2 and 3 refer to the  $D \bar{D}$ ,  $D_s^+ D_s^-$  and  $J/\psi \omega$  channels, respectively. For example,  $g_{B1}$  means the coupling constant of  $B^+ \rightarrow D \bar{D} K^+$  four-point vertex.

In this work,  $D \bar{D}$  will be written in isospin eigenstate, i.e.  $|D \bar{D}\rangle^{I=0} = \frac{1}{\sqrt{2}}(|D^0 \bar{D}^0\rangle + |D^+ D^-\rangle)$  and  $|D \bar{D}\rangle^{I=1} = \frac{1}{\sqrt{2}}(|D^0 \bar{D}^0\rangle - |D^+ D^-\rangle)$ .<sup>2</sup> Two channels,  $D \bar{D}$  and  $D_s^+ D_s^-$ , are considered to construct the  $K$ -matrix. From the Lagrangians in Eq. (1), the  $K$ -matrix can be written as

$$\mathbf{K}^{I=0} = \begin{pmatrix} 6g_{11} & -\sqrt{2}g_{12} \\ -\sqrt{2}g_{12} & 4g_{22} \end{pmatrix}, \tag{2}$$

in which  $\mathbf{K}_{11}$  stands for the process  $D \bar{D} \rightarrow D \bar{D}$ , and  $\mathbf{K}_{12}$  for  $D \bar{D} \rightarrow D_s^+ D_s^-$ , etc. The unitarized amplitudes can be obtained by

$$\mathbf{T} = \mathbf{K} \cdot [1 - \mathbf{GK}]^{-1}, \tag{3}$$

where  $\mathbf{G}$  is the diagonal matrix of two-point loop integrals of these two channels, i.e.  $\mathbf{G} = \text{diag}[B_0(m_D, m_{\bar{D}}), B_0(m_{D_s^+}, m_{D_s^-})]$ . The definition of two-point loop integral is,

$$\begin{aligned}
 B_0(p^2, m_1, m_2) &= \frac{\mu^\epsilon}{i} \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_1^2 + i0^+} \frac{i}{(p-k)^2 - m_2^2 + i0^+}
 \end{aligned}$$

<sup>2</sup> Note that the isospin state of  $|D^+\rangle$  is  $-|I, I_3\rangle = -|\frac{1}{2}, \frac{1}{2}\rangle$ ,  $|D^-\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$ ,  $|D^0\rangle = |\frac{1}{2}, \frac{-1}{2}\rangle$ , and  $|\bar{D}^0\rangle = |\frac{1}{2}, \frac{1}{2}\rangle$ , where these conventions can guarantee  $G$  and  $C$  parities are conserved [24].

$$\begin{aligned}
 &\sim \frac{1}{16\pi^2} \left[ -R + 1 - \ln \frac{m_1^2}{\mu^2} + \frac{m_1^2 - m_2^2 - p^2}{2p^2} \ln \frac{m_2^2}{m_1^2} \right. \\
 &\quad \left. + \frac{p^2 - (m_1 - m_2)^2}{p^2} \alpha(p^2) \ln \frac{\alpha(p^2) - 1}{\alpha(p^2) + 1} \right], \tag{4}
 \end{aligned}$$

where a  $\overline{MS}$  renormalization is understood to be taken,  $\alpha(p^2) \equiv \sqrt{\frac{p^2 - (m_1 + m_2)^2}{p^2 - (m_1 - m_2)^2}}$ ,  $\mu$  is renormalization scale set to be 1 GeV,  $a(\mu) = -R + 1$  is the subtraction constant as a parameter which is 8.05 with large uncertainties obtained by the fit.<sup>3</sup>

Since  $X(3960)$  has isospin  $I = 0$ , only the iso-singlet states,  $|D \bar{D}\rangle^{I=0}$  and  $|D_s^+ D_s^-\rangle$ , are included in the unitarized amplitudes  $\mathbf{T}$ . But the experimental results have been reported only in the  $D^+ D^-$  final states to date, so the approximations

$$\begin{aligned}
 T_{D^0 \bar{D}^0 \rightarrow D^+ D^-} &= \frac{1}{2}(T^{I=0} - T^{I=1}) \sim \frac{1}{2}T^{I=0}, \\
 T_{D^+ D^- \rightarrow D^+ D^-} &= \frac{1}{2}(T^{I=0} + T^{I=1}) \sim \frac{1}{2}T^{I=0},
 \end{aligned} \tag{5}$$

are used. With the help of  $\mathbf{T}$ , the amplitude of  $B^+ \rightarrow D^+ D^- K^+$  can be written as

$$\mathcal{A}_{B^+ \rightarrow D^+ D^- K^+} = g_{B1} \mathbf{T}_{11} - \frac{1}{\sqrt{2}} g_{B2} \mathbf{T}_{21}, \tag{6}$$

which satisfies the constraint on final-state interactions [26].<sup>4</sup>

Other amplitudes of  $B^+ \rightarrow D_s^+ D_s^- K^+$ ,  $B^+ \rightarrow J/\psi \omega K^+$ ,  $\gamma^* \gamma^* \rightarrow J/\psi \omega$ , in which the interactions of intermediate  $D \bar{D}$  and  $D_s^+ D_s^-$  states are included, can be expressed as

$$\mathcal{A}_{B^+ \rightarrow D_s^+ D_s^- K^+} = -\sqrt{2}g_{B1} \mathbf{T}_{12} + g_{B2} \mathbf{T}_{22}, \tag{7}$$

$$\begin{aligned}
 \mathcal{A}_{B^+ \rightarrow J/\psi \omega K^+} &= \epsilon^*(p_\psi) \cdot \epsilon^*(p_\omega) \times \left( 2g_{B1} \mathbf{T}_{11} g_{13} \right. \\
 &\quad - \sqrt{2}g_{B1} \mathbf{T}_{12} g_{23} \\
 &\quad \left. - \sqrt{2}g_{B2} \mathbf{T}_{21} g_{13} + g_{B2} \mathbf{T}_{22} g_{23} \right), \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A}_{\gamma^* \gamma^* \rightarrow J/\psi \omega} &= \left( p_{\gamma 1}^\mu p_{\gamma 2}^\nu - p_{\gamma 1} \cdot p_{\gamma 2} g^{\mu\nu} \right) \epsilon_\mu(p_{\gamma 1}) \epsilon_\nu(p_{\gamma 2}) \\
 &\quad \times \left( 8g_{\gamma 1} \mathbf{T}_{11} g_{13} - 4\sqrt{2}g_{\gamma 1} \mathbf{T}_{12} g_{23} \right. \\
 &\quad \left. - 4\sqrt{2}g_{\gamma 2} \mathbf{T}_{21} g_{13} + 4g_{\gamma 2} \mathbf{T}_{22} g_{23} \right) \\
 &\quad \times \epsilon^*(p_\psi) \cdot \epsilon^*(p_\omega). \tag{9}
 \end{aligned}$$

Note that in Eqs. (8) and (9), the final processes  $D_{(s)} \bar{D}_{(s)} \rightarrow J/\psi \omega$  are included by multiplying corresponding tree-level diagrams, as it is suppressed by the Okubo–Zweig–Iizuka (OZI) rule.

<sup>3</sup> In Ref. [25],  $a(\mu)$  is taken to be  $\simeq 2$ . There is no physics here, however. If we take  $a(\mu) = 2$ , the fit gives  $\chi^2/d.o.f. = 56.29/51 = 1.10$ , with the pole position almost unchanged, i.e.  $\sqrt{s} = (3.9192 - 0.0130i)$  GeV.

<sup>4</sup> Our parameterization is actually equivalent to another widely used form, e.g., in Ref. [27], up to a mild background contribution.

### 2.2 Numerical results and pole analysis

In the previous subsection, the amplitude involving this  $X$  state in each decay is obtained. Now a simultaneous analysis for the aforementioned four processes is performed to fit the experimental data [1–3, 28, 29]. The background (BKG) shapes are parameterized to be similar as those in the experiments. For the  $D^+D^-$  chain, the incoherent background contains two charmonia,  $\psi(3770)$  and  $\chi_{c2}(3930)$ , modelled by Breit–Wigner functions, and the mass reflection of the  $X_1(2900)$  resonance described by a 1st-order polynomial times a Gaussian  $\mathcal{G}(\mu, \sigma)$  where the parameters are extracted by fitting to the  $X_1(2900)$  component in the LHCb data [3]; and the three-body phase space of  $B^+ \rightarrow D^+D^-K^+$  is applied to describe other potential coherent backgrounds with unconsidered intermediate states. For the  $D_s^+D_s^-$  mode, the backgrounds below 4.25 GeV in the invariant  $D_s^+D_s^-$  mass are the  $X_0(4140)$  state and the non-resonant three-body phase space of  $B^+ \rightarrow D_s^+D_s^-K^+$ , which are coherent with the  $X$  state on grounds of the LHCb analysis [2]. For the two  $J/\psi\omega$  decays, only incoherent backgrounds are taken into account, which is in agreement with the experiments [28, 29].

Finally, the number of events for each decay can be expressed by [30]

$$\begin{aligned}
 N_{B^+ \rightarrow D^+D^-K^+}(s) &= \frac{\mathcal{R}_{D^+D^-} \mathcal{N}_{X \rightarrow D^+D^-}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D^+D^-) \Gamma_B (2\pi)^3 32m_B} \frac{1}{\times 2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_D, m_D)} \\
 &\times |\mathcal{A}_{B^+ \rightarrow D^+D^-K^+} + a_1 e^{i\phi_1}|^2 \\
 &+ \left| \frac{g_{\psi(3770)}}{s - m_{\psi(3770)}^2 + im_{\psi(3770)} \Gamma_{\psi(3770)}} \right|^2 \\
 &+ \left| \frac{g_{\chi_{c2}(3930)}}{s - m_{\chi_{c2}(3930)}^2 + im_{\chi_{c2}(3930)} \Gamma_{\chi_{c2}(3930)}} \right|^2 \\
 &+ a\sqrt{s} e^{-\frac{(\sqrt{s}-\mu)^2}{2\sigma^2}}, \tag{10}
 \end{aligned}$$

$$\begin{aligned}
 N_{B^+ \rightarrow D_s^+D_s^-K^+}(s) &= \frac{\mathcal{R}_{D_s^+D_s^-} \mathcal{N}_{X \rightarrow D_s^+D_s^-}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D_s^+D_s^-) \Gamma_B (2\pi)^3 32m_B} \frac{1}{\times 2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_{D_s}, m_{D_s})} \\
 &\times \left| \mathcal{A}_{B^+ \rightarrow D_s^+D_s^-K^+} + a_2 e^{i\phi_{21}} \right. \\
 &\left. + \frac{g_{X_0(4140)} e^{i\phi_{22}}}{s - m_{X_0(4140)}^2 + im_{X_0(4140)} \Gamma_{X_0(4140)}} \right|^2, \tag{11}
 \end{aligned}$$

$$\begin{aligned}
 N_{B^+ \rightarrow J/\psi\omega K^+}(s) &= \frac{\mathcal{R}_{J/\psi\omega} \mathcal{N}_{X \rightarrow J/\psi\omega}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow J/\psi\omega) \Gamma_B (2\pi)^3 32m_B}
 \end{aligned}$$

$$\begin{aligned}
 &\times 2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_{J/\psi}, m_\omega) \\
 &\times |\mathcal{A}_{B^+ \rightarrow J/\psi\omega K^+}|^2 \\
 &+ a_{31} \rho(\sqrt{s}, m_{J/\psi}, m_\omega) e^{a_{32}(\sqrt{s} - m_{J/\psi} - m_\omega)}, \tag{12} \\
 N_{\gamma^*\gamma^* \rightarrow J/\psi\omega}(s) &= n_4 \rho(\sqrt{s}, m_{J/\psi}, m_\omega) |\mathcal{A}_{\gamma^*\gamma^* \rightarrow J/\psi\omega}|^2 \\
 &+ (a_{41}\sqrt{s} + a_{42}), \tag{13}
 \end{aligned}$$

where  $\mathcal{R}_{D^+D^-} = 0.0173$  GeV,  $\mathcal{R}_{D_s^+D_s^-} = 0.02$  GeV, and  $\mathcal{R}_{J/\psi\omega} = 0.01$  GeV, are intervals of invariant mass spectra in the experimental data;  $\mathcal{N}_{X \rightarrow D^+D^-}$ ,  $\mathcal{N}_{X \rightarrow D_s^+D_s^-}$ , and  $\mathcal{N}_{X \rightarrow J/\psi\omega}$ , are the numbers of the expected  $X$  signal events in the  $B^+ \rightarrow D^+D^-K^+$ ,  $D_s^+D_s^-K^+$ , and  $J/\psi\omega K^+$  decays, respectively, with  $\mathcal{N}_{X \rightarrow D^+D^-} = 46.6$  and  $\mathcal{N}_{X \rightarrow D_s^+D_s^-} = 91.4$  obtained by experiments [2, 3], while  $\mathcal{N}_{X \rightarrow J/\psi\omega}$  is an unknown parameter to be fitted; the decay branching fractions  $\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D^+D^-) = 8.1 \times 10^{-6}$  [31],  $\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D_s^+D_s^-) = \mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D^+D^-) \frac{\Gamma(X \rightarrow D_s^+D_s^-)}{\Gamma(X \rightarrow D^+D^-)} = \frac{8.1 \times 10^{-6}}{0.29}$ , and  $\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow J/\psi\omega) = 3.0 \times 10^{-5}$  [28];  $\Gamma_B = 4.018 \times 10^{-10}$  MeV, is the width of the  $B^+$  meson in accordance with the relationship  $\Gamma_B = \hbar/\tau_B$ ;  $n_4$  is a constant to be fitted in the  $\gamma^*\gamma^* \rightarrow J/\psi\omega$  coupling, which absorbs the number of the expected  $X$  events and corresponding branching fractions;  $\rho$  is the phase space factor defined as

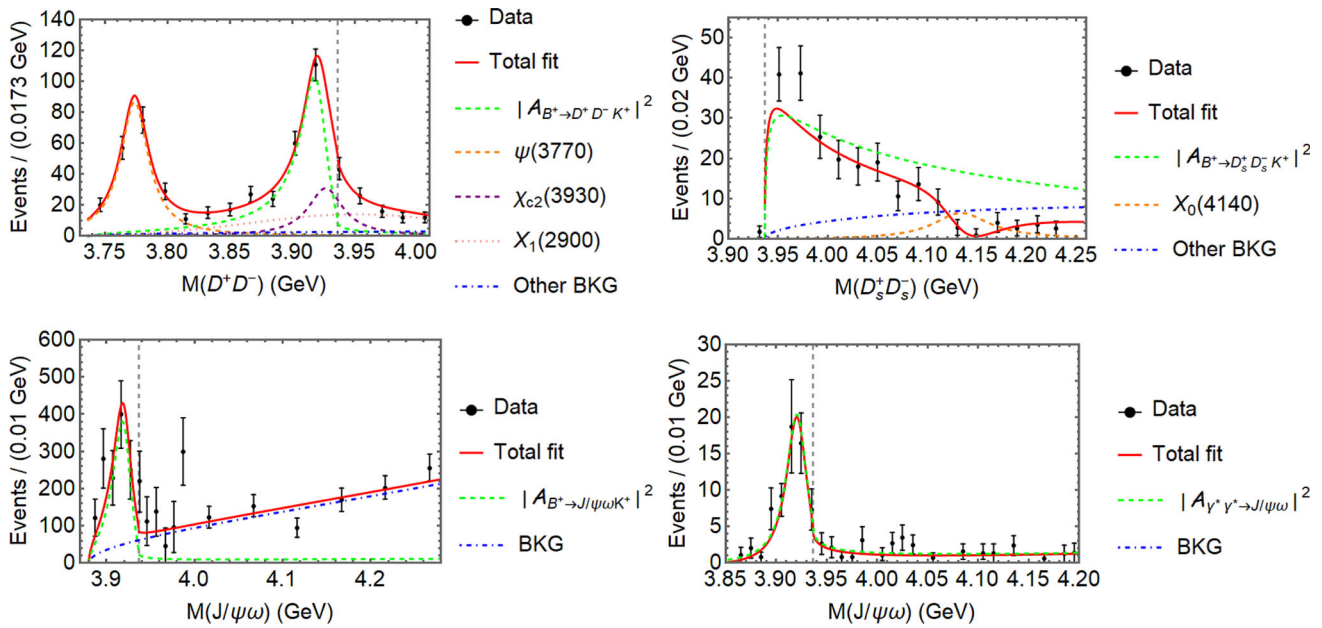
$$\rho(s, m_1, m_2) = \sqrt{\frac{[s - (m_1 - m_2)^2][s - (m_1 + m_2)^2]}{s^2}};$$

$m$  ( $\Gamma$ ) denotes the mean mass (width) of a particle [31];  $a$  (with subscripts) are free parameters.

The fit projections are shown in Fig. 1, where the fit goodness is gained to be  $\chi^2/d.o.f. = 56.22/50 = 1.12$ . Arrayed by signs of phase space factors, a set of Riemann sheets is defined as Table 1. The pole positions in complex  $s$  planes are searched for, and also summarized in Table 1 and sketched in Fig. 2. Only one pole, located on sheet II, is found near the  $D_s^+D_s^-$  threshold. According to PCR [13], this manifests that the  $X$  structure has molecular  $D_s^+D_s^-$  nature. More exactly, the dynamically molecular picture without the explicit  $X$  state is able to describe the current experimental data.

### 3 Flatté-like parameterization

Flatté(-like) formula [32] is a general model with an explicitly-introduced hadron used to parameterize a resonant structure near a hadron-hadron threshold in particle physics, especially in experimental data analyses. In this subsection, an energy-dependent Flatté-like parameterization for this  $X$  state coupling to  $D^+D^-$ ,  $D_s^+D_s^-$ , and  $J/\psi\omega$  are used to fit the experimental data and seek pole positions in the complex  $s$  planes. PCR [13] and SDFSR [21, 22] are carried out



**Fig. 1** Fit distributions with the  $K$ -matrix approach for  $B^+ \rightarrow D^+ D^- K^+$  (top left),  $B^+ \rightarrow D_s^+ D_s^- K^+$  (top right),  $B^+ \rightarrow J/\psi \omega K^+$  (bottom left), and  $\gamma^* \gamma^* \rightarrow J/\psi \omega$  (bottom right). Here, the data are from Refs. [1–3, 28, 29], and the vertical dashed lines are located at the  $D_s^+ D_s^-$  threshold

to distinguish whether this  $X$  state is a confining state or a molecular  $D_s^+ D_s^-$  hadron.

### 3.1 Parameterization models

The effective Lagrangians of  $X$  coupling to the three channels are given by

$$\begin{aligned} \mathcal{L}_{XD^+D^-} &= g_1 X D^+ D^-, \\ \mathcal{L}_{XD_s^+D_s^-} &= g_2 X D_s^+ D_s^-, \\ \mathcal{L}_{XJ/\psi\omega} &= g_3 X \psi_\mu \omega^\mu. \end{aligned} \tag{14}$$

**Table 1** Definition of Riemann sheets for coupled  $D_{(s)}\bar{D}_{(s)}$  channels and pole positions

Sheet	$\rho_{D\bar{D}}$	$\rho_{D_s^+D_s^-}$	Pole position
I	+	+	...
II	-	+	3.9207 - 0.0129i
III	-	-	...
IV	+	-	...

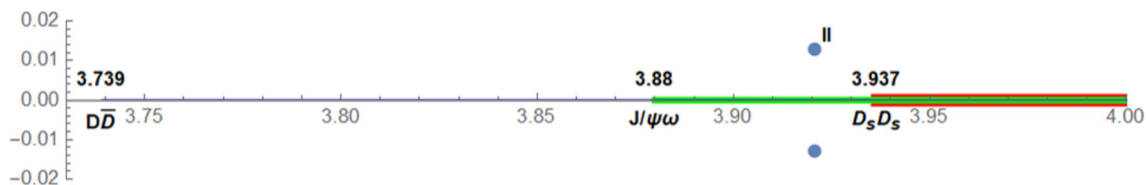
Then, the corresponding squared amplitudes read

$$|\mathcal{M}_{B^+ \rightarrow D^+ D^- K^+}|^2 = \left| ig \frac{i}{s - m_X^2 + im_X \Gamma_X} ig_1 \right|^2, \tag{15}$$

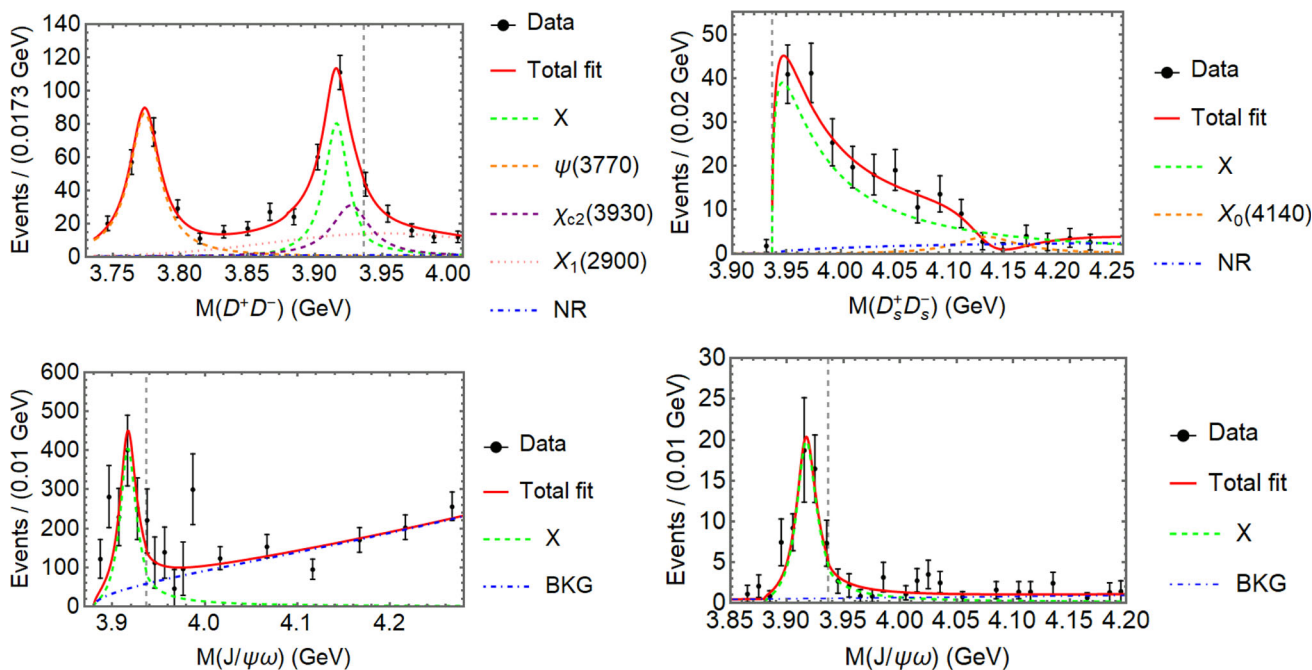
$$|\mathcal{M}_{B^+ \rightarrow D_s^+ D_s^- K^+}|^2 = \left| ig \frac{i}{s - m_X^2 + im_X \Gamma_X} ig_2 \right|^2, \tag{16}$$

$$\begin{aligned} |\mathcal{M}_{B^+ \rightarrow J/\psi \omega K^+}|^2 &= \left| ig \frac{i}{s - m_X^2 + im_X \Gamma_X} ig_3 \right|^2 \\ &\times \sum_{pol} \epsilon_\mu^*(\vec{p}_{J/\psi}) \epsilon_\rho(\vec{p}_{J/\psi}) \sum_{pol} \epsilon_\nu^*(\vec{p}_\omega) \epsilon_\sigma(\vec{p}_\omega) g^{\rho\sigma} g^{\mu\nu}, \end{aligned} \tag{17}$$

$$\begin{aligned} |\mathcal{M}_{\gamma^* \gamma^* \rightarrow J/\psi \omega}|^2 &= \left| 4ig'(-p_1 \cdot p_2 g^{\mu\nu} + p_1^\nu p_2^\mu) \frac{i}{s - m_X^2 + im_X \Gamma_X} ig_3 g^{\rho\sigma} \right|^2 \\ &\times \frac{1}{9} \sum_{pol} [\epsilon_\mu(\vec{p}_1) \epsilon_{\mu'}^*(\vec{p}_1)] \sum_{pol} [\epsilon_\nu(\vec{p}_2) \epsilon_{\nu'}^*(\vec{p}_2)] \\ &\times \sum_{pol} [\epsilon_\rho^*(\vec{p}_{J/\psi}) \epsilon_{\rho'}(\vec{p}_{J/\psi})] \sum_{pol} [\epsilon_\sigma^*(\vec{p}_\omega) \epsilon_{\sigma'}(\vec{p}_\omega)], \end{aligned} \tag{18}$$



**Fig. 2** Schematic diagram of pole positions for the  $K$ -matrix approach



**Fig. 3** Fit distributions with Flatté-like parameterizations for  $B^+ \rightarrow D^+D^-K^+$  (top left),  $B^+ \rightarrow D_s^+D_s^-K^+$  (top right),  $B^+ \rightarrow J/\psi\omega K^+$  (bottom left), and  $\gamma^*\gamma^* \rightarrow J/\psi\omega$  (bottom right). Here, data are from

Refs. [1–3,28,29], and the vertical dashed lines are located at the  $D_s^+D_s^-$  threshold

where  $g$  and  $g'$  stand for coupling constants of  $B^+ \rightarrow XK^+$  and  $\gamma^*\gamma^* \rightarrow X$ , respectively,  $\sum_{pol}$  denotes summation of polarization, and

$$\Gamma_X = \frac{1}{16\pi m_X} \times \left[ g_1^2 \rho_1 + g_2^2 \rho_2 + g_3^2 \rho_3 \left( 2 + \frac{(s - m_\psi^2 - m_\omega^2)^2}{4m_\psi^2 m_\omega^2} \right) \right]. \tag{19}$$

### 3.2 Numerical results and pole analysis

A simultaneous fit to the experimental data [1–3,28,29] is imposed for the mentioned-above decays in this subsection. The background shapes are parameterized similarly as the  $K$ -matrix approach. Then each distribution of the number of events can be expressed as

$$N_{B^+ \rightarrow D^+D^-K^+}(s) = \frac{\mathcal{R}_{D^+D^-} \mathcal{N}_{X \rightarrow D^+D^-}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D^+D^-) \Gamma_B (2\pi)^3 32m_B} \frac{1}{2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_D, m_D)} \times \left| \frac{gg_1}{D_X(s)} + a_1 e^{i\phi_1} \right|^2 + \left| \frac{g_{\psi(3770)}}{s - m_{\psi(3770)}^2 + im_{\psi(3770)} \Gamma_{\psi(3770)}} \right|^2$$

$$+ \left| \frac{g_{\chi_{c2}(3930)}}{s - m_{\chi_{c2}(3930)}^2 + im_{\chi_{c2}(3930)} \Gamma_{\chi_{c2}(3930)}} \right|^2 + a\sqrt{s} e^{-\frac{(\sqrt{s}-\mu)^2}{2\sigma^2}}, \tag{20}$$

$$N_{B^+ \rightarrow D_s^+D_s^-K^+}(s) = \frac{\mathcal{R}_{D_s^+D_s^-} \mathcal{N}_{X \rightarrow D_s^+D_s^-}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow D_s^+D_s^-) \Gamma_B (2\pi)^3 32m_B} \frac{1}{2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_{D_s}, m_{D_s})} \times \left| \frac{gg_2}{D_X(s)} + a_2 e^{i\phi_{21}} \right|^2 + \left| \frac{g_{X_0(4140)} e^{i\phi_{22}}}{s - m_{X_0(4140)}^2 + im_{X_0(4140)} \Gamma_{X_0(4140)}} \right|^2, \tag{21}$$

$$N_{B^+ \rightarrow J/\psi\omega K^+}(s) = \frac{\mathcal{R}_{J/\psi\omega} \mathcal{N}_{X \rightarrow J/\psi\omega}}{\mathcal{B}(B^+ \rightarrow XK^+) \mathcal{B}(X \rightarrow J/\psi\omega) \Gamma_B (2\pi)^3 32m_B} \frac{1}{2\sqrt{s} \rho(m_B, m_K, \sqrt{s}) \rho(\sqrt{s}, m_{J/\psi}, m_\omega)} \times \left| \frac{gg_3}{D_X(s)} \right|^2 \left[ 2 + \frac{(s - m_\psi^2 - m_\omega^2)^2}{4m_\psi^2 m_\omega^2} \right] + a_{31} \rho(\sqrt{s}, m_{J/\psi}, m_\omega) e^{a_{32}(\sqrt{s} - m_{J/\psi} - m_\omega)}, \tag{22}$$

$$N_{\gamma^*\gamma^* \rightarrow J/\psi\omega}(s) = n_4 \rho(\sqrt{s}, m_{J/\psi}, m_\omega) \left| \frac{4g'g_3}{D_X(s)} \right|^2$$

**Table 2** Key Riemann sheets for three coupled channels and pole positions

Sheet	$\rho_{D\bar{D}}$	$\rho_{J/\psi\omega}$	$\rho_{D_s^+ D_s^-}$	Pole position
II	–	+	+	...
III	–	–	+	3.9163 – 0.0107i
VII	–	–	–	3.8986 – 0.0108i

$$\begin{aligned} & \times \left[ 2 + \frac{(s - m_\psi^2 - m_\omega^2)^2}{4m_\psi^2 m_\omega^2} \right] \\ & \times \frac{1}{9} \left[ 4q_1^2 q_2^2 - \frac{(s + q_1^2 + q_2^2)^2}{2} + \frac{(s + q_1^2 + q_2^2)^4}{16q_1^2 q_2^2} \right] \\ & + (a_{41}\sqrt{s} + a_{42}), \end{aligned} \tag{23}$$

where

$$D_X(s) = s - m_X^2 + im_X\Gamma_X. \tag{24}$$

As shown in Fig. 3, the fit gives  $\chi^2/d.o.f. = 52.42/55 = 0.95$ , which is slightly better than the previous  $K$ -matrix approach. According to signs of phase space factors, eight Riemann sheets can be generated for the three coupled channels, among which only three sheets have the largest impact on observables [12], as listed in Table 2 and sketched in Fig. 4. The pole positions in complex  $s$  planes are searched for and also summarized in Table 2. According to PCR [13], the phenomenon that two poles are found near the  $D_s^+ D_s^-$  threshold indicates that the  $X$  structure gets inclined to attribute with a confining state. Thus, it can be seen that both the implicit and explicit  $X$  interpretations can meet the experimental data well, but the latter is a little better.

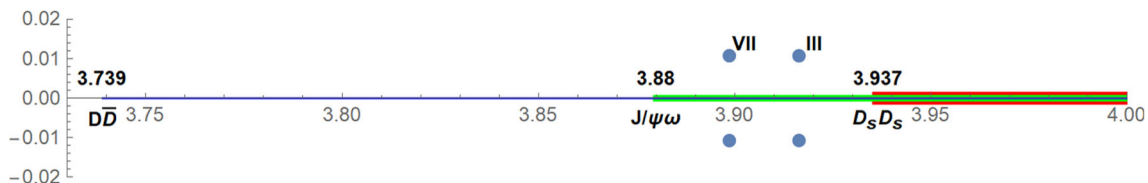
To further gain an insight on the nature of this near-threshold state, SDFSR is carried out, which is utilized in an  $S$ -wave Flatté-like parameterization. From Refs. [19–23], a renormalization constant  $\mathcal{Z}$  can be calculated by integrating a spectrum density function with respect to energy, which refers to probability of finding a confining particle in the continuous spectrum: the greater the tendency of  $\mathcal{Z}$  to 1, the more the resonant structure is likely to be a confining state; conversely, the closer the value of  $\mathcal{Z}$  is to 0, the more the hadron tends to be a hadronic molecule. The fit gives  $\mathcal{Z} = 0.458$  when the integral interval belongs to  $[E_f - \Gamma_X, E_f + \Gamma_X]$ ;  $\mathcal{Z} = 0.670$  when  $[E_f - 2\Gamma_X, E_f + 2\Gamma_X]$ , where  $E_f$  is energy difference between  $M_X$  and the  $D_s^+ D_s^-$  threshold ( $m_{th}$ ), i.e.  $E_f = m_X - m_{th}$ . The result that the  $\mathcal{Z}$  value is slightly less

than 0.5 in  $[E_f - \Gamma_X, E_f + \Gamma_X]$  but mildly greater than 0.5 in  $[E_f - 2\Gamma_X, E_f + 2\Gamma_X]$ , implies that this  $X$  state may neither be a pure confining state nor a pure molecule. Together with the previous pole analyses, this  $X$  resonant structure is more probably a mixture of a confining state and a  $D_s^+ D_s^-$  hadronic molecule.

### 4 Summary and discussion

Based on the assumption that  $\chi_{c0}(3930) (\rightarrow D^+ D^-)$ ,  $X(3960) (\rightarrow D_s^+ D_s^-)$ , and  $X(3915) (\rightarrow J/\psi\omega)$  are the same hadron, a combined analysis is performed using both the  $K$ -matrix approach of  $D_{(s)}\bar{D}_{(s)}$  four-point contact interactions and the model of energy-dependent Flatté-like parameterizations. It is found that both the implicit and explicit  $X$  interpretations can meet the experimental data well. The use of PCR and SDFSR demonstrate that this  $X$  hadron is not like a pure  $D_s^+ D_s^-$  molecule, but might be the mixed nature of a  $c\bar{c}$  confining state and  $D_s^+ D_s^-$  continuum. One possible scenario is that the  $X$  hadron has a  $c\bar{c}$  core strongly renormalized by the  $D_s^+ D_s^-$  coupling, like the  $\chi_{c1}(3872)$  as a  $c\bar{c}$  resonance with a contribution of the  $D^* \bar{D}$  couple-channel effect [14, 33].

To further analyze the nature of this  $X$  state, a number of theoretical predictions for the  $^3P$  charmonia are summarized in Table 3. If this  $X$  hadron is indeed a charmonium, it is most likely to be the  $\chi_{c0}(2P)$  candidate, which is favored by the relativistic Godfrey–Isgur model (GIM) [34], the couple-channel potential model (CPM) [35], and Literature [39]. However, it is not in agreement with the other theoretical expectations, whose masses are predicted in the range of 3842–3868 MeV [34, 36–38]. Another phenomenon is that two candidates can be treated as the  $\chi_{c0}(2P)$  charmonium:  $\chi_{c0}(3860)$  [40] discovered in the  $D\bar{D}$  decays via  $e^+e^- \rightarrow J/\psi D\bar{D}$  and the  $X$  state discussed in this work. Yet Ref. [41] argued that the  $\chi_{c0}(3860)$  peak is due to a bound state around 3695 MeV. It needs to be confirmed in the forthcoming experimental measurements. Whatever, more accurate studies based on potential models, as well as other methods, are needed to shed light on the nature of the  $X$  state. For example, Ref. [42] estimated the branching fraction of  $B^+ \rightarrow X(3960)K^+$  to be  $(2.9 - 13.3) \times 10^{-4}$  if assuming  $X(3960)$  as a  $D_s^+ D_s^-$  bound state, which can be helpful in the future experiments to test if the  $X(3960)$  hadron is a bound state.



**Fig. 4** Schematic diagram of pole positions for the Flatté-like parameterization

**Table 3** Summary of  $^3P$  charmonia’s properties between experimental measurements [12]. Here the experimental measurements refer to the mean values from the 2022 Particle Data Group (PDG) [12], and the theoretical predictions in different potential models from GIM [34], CPM [35], the non-relativistic model (nRM) [34], the relativistic perturbative

model (RPM) [36], the relativistic non-perturbative model (RnPM) [36], the screened potential model (SPM) [37], and the framework of chiral quark model by solving the Schrödinger equation with the Gaussian expansion method (GEM) [38]. The 2022 PDG [12] takes  $X(3915)$  and  $\chi_{c0}(3930)$  as the same particle, denoted as  $\chi_{c0}(3915)$ . (In units of MeV)

State		EXP	GIM	CPM	nRM	RPM	RnPM	SPM	GEM
$1^3P_0$	$\chi_{c0}(1P)$	$3414.71 \pm 0.30$	3445	3441	3424	3415.7	3415.2	3433	3430
$1^3P_1$	$\chi_{c1}(1P)$	$3510.67 \pm 0.05$	3510	3520	3505	3508.2	3510.6	3510	3491
$1^3P_2$	$\chi_{c2}(2P)$	$3556.17 \pm 0.07$	3550	3565	3556	3557.7	3556.2	3554	3523
$2^3P_0$	$\chi_{c0}(3860)$	$3862^{+48}_{-35}$	3916	3915	3852	3843.7	3864.3	3842	3868
	$\chi_{c0}(3915)$	$3921.7 \pm 1.8$							
$2^3P_1$	$\chi_{c1}(3872)$	$3871.65 \pm 0.06$	3953	3875	3925	3939.7	3950.0	3901	3911
$2^3P_2$	$\chi_{c2}(3930)$	$3922.5 \pm 1.0$	3979	3966	3972	3993.7	3992.3	3937	3935
$3^3P_0$			4292		4202			4131	4172
$3^3P_1$			4317		4271			4178	4204
$3^3P_2$			4337		4317			4208	4222

**Table 4** Pole positions without the  $J/\psi\omega$  channel

Case	Sheet	$\rho_{D\bar{D}}$	$\rho_{D_s^+D_s^-}$	Pole position
$K$ -matrix	II	–	+	$3.9279 - 0.0079i$
Flatté-like	II	–	+	$3.9303 - 0.0041i$
	III	–	–	$3.8702 - 0.0109i$

**Table 5** Pole positions without the  $D_s^+D_s^-$  channel

Case	Sheet	$\rho_{D\bar{D}}$	$\rho_{J/\psi\omega}$	Pole position
$K$ -matrix	II	–	+	$3.9189 - 0.0115i$
Flatté-like	II	–	+	$3.9164 - 0.0046i$
	III	–	–	$3.9157 - 0.0101i$

Due to limited data statistics, however, we cannot draw a solid conclusion in this work. More experimental data are expected to further clarify the nature of  $\chi_{c0}(3930)/X(3960)/X(3915)$ , for instance, the  $\gamma\gamma \rightarrow D_{(s)}\bar{D}_{(s)}$  reactions, the  $e^+e^- \rightarrow \psi D_{(s)}\bar{D}_{(s)}$  productions, the amplitude analysis for the  $X(3915) \rightarrow J/\psi\omega$  chain, and the ratio of  $\Gamma(X \rightarrow D_{(s)}\bar{D}_{(s)})/\Gamma(X \rightarrow J/\psi\omega)$ . Without doubt, other decay modes are also valuable to elucidate the nature of the  $D_s^+D_s^-$  near-threshold structure, such as  $X \rightarrow \eta^{(\prime)}\eta_c, \pi\pi\chi_{c0,2}, \gamma J/\psi, \gamma\psi(3686), \gamma\psi(3770), \gamma D^{(*)}\bar{D}, \gamma D_s^+D_s^-,$  etc.

Nevertheless, it is noteworthy that the  $0^{++}$  assignment for the  $X(3915)(\rightarrow J/\psi\omega)$  state is not completely determined by experiments. Several works take the  $X(3915)(\rightarrow J/\psi\omega)$  as the  $2^{++}$  charmonium  $\chi_{c2}(3930)$  [5,43], but the  $\chi_{c0}(3930)(\rightarrow D^+D^-)$  and  $X(3960)(\rightarrow D_s^+D_s^-)$  are the same  $0^{++}$  molecular hadron [5,44,45]. In view of this assumption, fits without the  $J/\psi\omega$  channel are also tested, where the numerical results are summarized in Table 4. These

pole positions are roughly consistent with the nominal results though the elementariness of  $\chi_{c0}(3930)/X(3960)$  is less favored here. In addition, Refs. [5, 11] regards the  $X(3960)$  as a different state from  $\chi_{c0}(3930)(\rightarrow D^+D^-)/X(3915)(\rightarrow J/\psi\omega)$ , so that fits without the  $D_s^+D_s^-$  decay are used to check. As listed in Table 5, the numerical values are compatible with the nominal ones, which shed light on the mixed nature of  $X(3915)/\chi_{c0}(3930)$ . That is, it does not shake the conclusion of this article in case that the three decays are not from the same hadron.

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### Appendix A: Relevant fitted parameters

The parameters associated with pole positions are summarized in Tables 6 and 7.

**Table 6** Parameters associated with pole positions in the  $K$ -matrix method, which are  $g_{11}$ ,  $g_{12}$ ,  $g_{22}$  in Eq. 2 and subtraction constant  $a(\mu)$  of  $B_0$  function in Eq. 4, where  $g_{11} = g(1 + \delta_1)$ ,  $g_{22} = g(1 + \delta_2)$ , and  $g_{12} = 2g$

Parameter	Value
$\delta_1$	$-0.0450 \pm 0.0653$
$\delta_2$	$2.4954 \pm 1.4267$
$g$	$1.6543 \pm 0.6575$
$a(\mu)$	$8.0516 \pm 2.4454$

**Table 7** Parameters associated with pole positions in Flatté-like parameterizations

Parameter	Value
$m_X$ (GeV)	$3.9108 \pm 0.0104$
$g_1$ (GeV)	$1.2287 \pm 1.9256$
$g_2$ (GeV)	$5.2372 \pm 4.0788$
$g_3$ (GeV)	$3.6111 \pm 0.6263$

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