# Semileptonic baryonic B decays 

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#### Abstract

We study the semileptonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ decays with $\mathbf{B} \overline{\mathbf{B}}^{\prime}\left(L \bar{L}^{\prime}\right)$ representing a baryon (lepton) pair. Using the new determination of the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors, we obtain $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)=(5.4 \pm 2.0) \times 10^{-6}$ agreeing with the current data. Besides, $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{v}\right)=$ $(3.5 \pm 1.0) \times 10^{-8}$ is calculated to be 20 times smaller than the previous prediction. In particular, we predict $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow\right.$ $\left.p \bar{\Lambda} e^{-} \bar{v}_{e}, p \bar{\Lambda} \mu^{-} \bar{v}_{\mu}, p \bar{\Lambda} \tau^{-} \bar{v}_{\tau}\right)=(2.1 \pm 0.6,2.1 \pm 0.6,1.7 \pm$ $1.0) \times 10^{-6}$ and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{\nu}\right)=(0.8 \pm 0.2) \times 10^{-8}$, which can be accessible to the LHCb experiment.


## 1 introduction

In the non-leptonic baryonic $B$ decays, the observation of $B \rightarrow p \bar{p}\left(\pi, K^{(*)}, D^{(*)}\right)$ and $B^{-} \rightarrow \Lambda \bar{p}(J / \psi, \gamma)$ suggests the unique existence of the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition [1-3], with which the $C P$ asymmetries of $B^{-} \rightarrow p \bar{p}\left(\pi^{-}, K^{(*)-}\right)$ [4, 5] and the branching fractions of $B^{-} \rightarrow \Lambda \bar{p} D^{(*) 0}, \bar{B}^{0} \rightarrow$ $\Sigma^{0} \bar{\Lambda} D^{0}$ [6] have been predicted, and verified by the later measurements [7].

The semileptonic $B$ decays of $B^{-} \rightarrow p \bar{p} \ell^{-} \bar{\nu}_{\ell}$ and $B^{-} \rightarrow \Lambda \bar{p} v_{\ell} \bar{v}_{\ell}$ with $\ell$ denoting $e, \mu$ or $\tau$ can provide another evidence for the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition [8,9]. Like the studies of the semileptonic $B^{-} \rightarrow \pi^{+} \pi^{-} \ell^{-} \bar{\nu}_{\ell}$ decays [10,11], the full dibaryon invariant mass spectrum can be used to test the possible co-existence of the resonant and non-resonant contributions. Therefore, we have predicted $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}\right)=(1.04 \pm 0.26 \pm 0.12) \times 10^{-4}[8]$ and $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{v}\right)=(7.9 \pm 1.9) \times 10^{-7}$ [9]. We have also predicted $\mathcal{R}_{e / \mu} \equiv \mathcal{B}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}\right) / \mathcal{B}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right) \simeq$ 1 [8]. By contrast, the pole model argument leads to the evaluation of $\mathcal{B}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \ell \bar{v}_{\ell}\right)=10^{-5}-10^{-6}$ [12].

Experimentally, it has been measured that [13-16]

$$
\mathcal{B}_{e x}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}\right)=(5.8 \pm 3.7 \pm 3.6)
$$

[^0]\[

$$
\begin{align*}
& \quad \times 10^{-4}\left(<1.2 \times 10^{-3}\right)[\text { Cleo }] \\
& \mathcal{B}_{e x}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}\right)=\left(8.2_{-3.2}^{+3.7} \pm 0.6\right) \\
& \quad \times 10^{-6}[\text { Belle }] \\
& \mathcal{B}_{e x}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)=\left(3.1_{-2.4}^{+3.1} \pm 0.7\right) \\
& \quad \times 10^{-6}[\text { Belle }] \\
& \mathcal{B}_{e x}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)=\left(5.27_{-0.24}^{+0.23} \pm 0.21 \pm 0.15\right) \\
& \quad \times 10^{-6}[\mathrm{LHCb}] \\
& \mathcal{B}_{e x}\left(B^{-} \rightarrow \Lambda \bar{p} v \bar{v}\right)=(0.4 \pm 1.1 \pm 0.6) \\
& \quad \times 10^{-5}\left(<3.0 \times 10^{-5}\right)[\text { Babar }] . \tag{1}
\end{align*}
$$
\]

The threshold effect commonly observed in $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ is also observed in $B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{e}$ [15], which is drawn as a peak around the threshold area of $m_{\mathbf{B B}^{\prime}} \simeq m_{\mathbf{B}}+m_{\bar{B}^{\prime}}$ in the $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ invariant mass spectrum. There is no sign that the $B$ to $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition receives a resonant contribution. Nonetheless, it is clearly seen that $\mathcal{B}_{e x}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)$ is 20 times smaller than the prediction [8]. This has been pointed out as the theoretical challenge to alleviate the discrepancy [17]. On the other hand, the ratio $\mathcal{R}_{e / \mu} \simeq 1$ as a test of the lepton universality is not conclusive, and the prediction of $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)$ is within the experimental upper bound.

In Ref. [6], the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors ( $F_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ ) are extracted with the data from $B \rightarrow \mathbf{B}^{\prime} M$, which cause the overestimation of $\mathcal{B}(B \rightarrow p \bar{p} \ell \bar{\nu})$. With the same theoretical inputs, $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{v}\right)$ might be overestimated as well [9]. A question is hence raised: whether there exist the universal $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors to explain the nonleptonic and semileptonic baryonic $B$ decays.

In this paper, we propose to perform a new global fit, in order to accommodate the current data of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ with $L \bar{L}^{\prime}$ denoting a lepton pair and $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$. With $F_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ determined from the new global fit, we will re-investigate $B^{-} \rightarrow \Lambda \bar{p} \nu \bar{v}$. Since LHCb has been able to accumulate more events for the $\bar{B}_{s}^{0}$ decays, we will study $\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell^{-} \bar{v}$ and $\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{v}$ decays for future measurements.


(b)


Fig. 1 Feynman diagrams for the $B \rightarrow \mathbf{B}^{\prime} L \bar{L}^{\prime}$ decays, where (a) depicts $B^{-} \rightarrow p \bar{p} \ell^{-} \bar{\nu}_{\ell}$ and $\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell^{-} \bar{\nu}_{\ell}$, while $(b, c) B^{-} \rightarrow \Lambda \bar{p} \nu_{\ell} \bar{\nu}_{\ell}$ and $\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu_{\ell} \bar{\nu}_{\ell}$

## 2 Formalism

The semileptonic baryonic $B$ decays come from the quarklevel $b \rightarrow u \ell \bar{v}_{\ell}$ and $b \rightarrow s \nu_{\ell} \bar{v}_{\ell}$ processes. In Fig. 1a, $b \rightarrow$ $u \ell \bar{v}_{\ell}$ appear as the tree-level $b \rightarrow u W, W \rightarrow \ell \bar{v}_{\ell}$ decays. Due to the loop contributions from the penguin-level $b \rightarrow$ $s Z, Z \rightarrow \nu_{\ell} \bar{\nu}_{\ell}$ and box diagrams in Fig. 1b,c, respectively [18], $b \rightarrow s \nu_{\ell} \bar{\nu}_{\ell}$ can be rarer than $b \rightarrow u \ell \bar{v}_{\ell}$. The effective Hamiltonians for the above semileptonic $b$ decays are given by $[18,19]$

$$
\begin{align*}
& \mathcal{H}\left(b \rightarrow u \ell \bar{v}_{\ell}\right)=\frac{G_{F} V_{u b}}{\sqrt{2}} \bar{u} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\ell} \gamma^{\mu}\left(1-\gamma_{5}\right) v_{\ell} \\
& \mathcal{H}\left(b \rightarrow s v_{\ell} \bar{v}_{\ell}\right) \\
& \quad=\frac{G_{F}}{\sqrt{2}} \frac{\alpha_{e m}}{2 \pi \sin ^{2} \theta_{W}} \lambda_{t} D\left(x_{t}\right) \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{v}_{\ell} \gamma^{\mu}\left(1-\gamma_{5}\right) v_{\ell} \tag{2}
\end{align*}
$$

where $G_{F}$ is the Fermi constant, $V_{u b}$ and $\lambda_{t} \equiv V_{t s}^{*} V_{t b}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $D\left(x_{t}\right)$ with $x_{t} \equiv m_{t}^{2} / m_{W}^{2}$ is the top-quark loop function [18-21]. According to $\mathcal{H}\left(b \rightarrow u \ell \bar{v}_{\ell}, s v_{\ell} \bar{v}_{\ell}\right)$, the amplitudes of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ with $L \bar{L}^{\prime}=\left(\ell \bar{v}_{\ell}, v_{\ell} \bar{v}_{\ell}\right)$ can be derived as [8,9]

$$
\begin{align*}
\mathcal{M} & \left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \ell^{-} \bar{v}_{\ell}\right) \\
= & \frac{G_{F} V_{u b}}{\sqrt{2}}\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{u} \gamma_{\mu}\left(1-\gamma_{5}\right) b|B\rangle \bar{\ell}^{\mu} \gamma^{\mu}\left(1-\gamma_{5}\right) v_{\ell}, \\
\mathcal{M} & \left(B \rightarrow \mathbf{B}^{\prime} \nu_{\ell} \bar{v}_{\ell}\right) \\
= & \frac{G_{F}}{\sqrt{2}} \frac{\alpha_{\mathrm{em}}}{2 \pi \sin ^{2} \theta_{W}} \lambda_{t} D\left(x_{t}\right)\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b|B\rangle \\
& \times \bar{v}_{\ell} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu_{\ell} \tag{3}
\end{align*}
$$

with $\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right|(\bar{q} b)|B\rangle$ representing the matrix elements of the $B$ meson to $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition. In Fig. 1a-c, $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ occur as $B^{-} \rightarrow p \bar{p} \ell \bar{v}_{\ell}, \Lambda \bar{p} v_{\ell} \bar{v}_{\ell}$ and $\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell \bar{v}_{\ell}, \Lambda \bar{\Lambda} \nu_{\ell} \bar{v}_{\ell}$ for our study.

The amplitudes of the non-leptonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decays have two forms [1,22,23]: $\mathcal{M}_{C} \propto\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right|\left(\bar{q} q^{\prime}\right)|0\rangle \times\langle M|(\bar{q} b)$ $|B\rangle$ and $\mathcal{M}_{T} \propto\langle M|\left(\bar{q} q^{\prime}\right)|0\rangle\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right|(\bar{q} b)|B\rangle$, where $\mathcal{M}_{C}$ denotes the current amplitude with $\mathbf{B} \bar{B}^{\prime}$ produced from the quark current [22-26], and $\mathcal{M}_{T}$ the transition amplitude with $\mathbf{B} \bar{B}^{\prime}$ from the $B$ meson transition [1-3]. Clearly, $\mathcal{M}_{T}(B \rightarrow$ $\left.\mathbf{B} \overline{\mathbf{B}}^{\prime} M\right)$ and $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ can be related by $\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right|(\bar{q} b)|B\rangle$ [8,9]. As seen in Fig. 2, $B \rightarrow p \bar{p} M$ with $M=(\pi, K)$, $B \rightarrow p \bar{p} V$ with $V=\left(\rho, K^{*}\right)$, and $\bar{B}^{0} \rightarrow p \bar{p} D^{0(*)}$ involve the transition amplitudes, given by [1,27-29]

$$
\begin{align*}
& \mathcal{M}(B \rightarrow p \bar{p} M)=\frac{G_{F}}{\sqrt{2}}\left(\hat{\mathcal{M}}_{1}+\hat{\mathcal{M}}_{6}\right) \\
& \hat{\mathcal{M}}_{1}=\alpha_{1}^{q q^{\prime}}\langle M| \bar{q}^{\prime} \gamma_{\mu}\left(1-\gamma_{5}\right) u|0\rangle\langle p \bar{p}| \bar{q} \gamma^{\mu}\left(1-\gamma_{5}\right) b|B\rangle \\
& \hat{\mathcal{M}}_{6}=\alpha_{6}^{q q^{\prime}}\langle M| \bar{q}^{\prime}\left(1+\gamma_{5}\right) u|0\rangle\langle p \bar{p}| \bar{q}\left(1-\gamma_{5}\right) b|B\rangle \\
& \mathcal{M}(B \rightarrow p \bar{p} V)=\frac{G_{F}}{\sqrt{2}} \alpha_{1}^{q q^{\prime}}\langle V| \bar{q}^{\prime} \gamma_{\mu}\left(1-\gamma_{5}\right) u|0\rangle \\
& \quad \times\langle p \bar{p}| \bar{q} \gamma^{\mu}\left(1-\gamma_{5}\right) b|B\rangle \\
& \mathcal{M}\left(\bar{B}^{0} \rightarrow p \bar{p} D^{0(*)}\right)=\frac{G_{F}}{\sqrt{2}} V_{c b} V_{u d}^{*} a_{2}\left\langle D^{0(*)}\right| \bar{c} \gamma_{\mu} \\
& \quad \times\left(1-\gamma_{5}\right) u|0\rangle\langle p \bar{p}| \bar{d} \gamma^{\mu}\left(1-\gamma_{5}\right) b\left|\bar{B}^{0}\right\rangle \tag{4}
\end{align*}
$$

with $\left(q, q^{\prime}\right)=(u, d)$ for $B^{-} \rightarrow p \bar{p} \pi^{-}$and $B^{-} \rightarrow p \bar{p} \rho^{-}$, $\left(q, q^{\prime}\right)=(u, s)$ for $B^{-} \rightarrow p \bar{p} K^{-}$and $B^{-} \rightarrow p \bar{p} K^{*-}$, and $\left(q, q^{\prime}\right)=(d, s)$ for $\bar{B}^{0} \rightarrow p \bar{p} \bar{K}^{0}$ and $\bar{B}^{0} \rightarrow p \bar{p} \bar{K}^{* 0}$. The


Fig. 2 Feynman diagrams for the three-body baryonic $B$ decays, where $(a, b)$ and $(c)$ depict $B \rightarrow p \bar{p} M(V)$ and $\bar{B}^{0} \rightarrow p \bar{p} D^{0(*)}$, respectively
parameters in Eq. (4) result from the factorization approach [30], written as

$$
\begin{align*}
& \alpha_{1}^{u q^{\prime}}=V_{u b} V_{u q^{\prime}}^{*} a_{1}-V_{t b} V_{t q^{\prime}}^{*} a_{4} \\
& \alpha_{1}^{d s}=-V_{t b} V_{t s}^{*} a_{4} \\
& \alpha_{6}^{u q^{\prime}}=\alpha_{6}^{d q^{\prime}}=V_{t b} V_{t q^{\prime}}^{*} 2 a_{6} \tag{5}
\end{align*}
$$

with $a_{i}=c_{i}^{\text {eff }}+c_{i \pm 1}^{\text {eff }} / N_{c}$ for $i=\operatorname{odd}$ (even), where $c_{i}^{\text {eff }}$ are the effective Wilson coefficients, and $N_{c}$ the color number [30].

The matrix elements of the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition in Eqs. (3) and (4) can be presented as $[1,3,6,17,27-29]$

$$
\begin{align*}
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| V_{\mu}^{b}|B\rangle= & i \bar{u}\left[g_{1} \gamma_{\mu}+g_{2} i \sigma_{\mu \nu} p^{\nu}+g_{3} p_{\mu}\right. \\
& \left.+g_{4}\left(p_{\overline{\mathbf{B}}^{\prime}}+p_{\mathbf{B}}\right)_{\mu}+g_{5}\left(p_{\overline{\mathbf{B}}^{\prime}}-p_{\mathbf{B}}\right)_{\mu}\right] \gamma_{5} v \\
\left\langle\overline{\mathbf{B}}^{\prime}\right| A_{\mu}^{b}|B\rangle= & i \bar{u}\left[f_{1} \gamma_{\mu}+f_{2} i \sigma_{\mu \nu} p^{v}+f_{3} p_{\mu}\right. \\
& \left.+f_{4}\left(p_{\overline{\mathbf{B}}^{\prime}}+p_{\mathbf{B}}\right)_{\mu}+f_{5}\left(p_{\overline{\mathbf{B}}^{\prime}}-p_{\mathbf{B}}\right)_{\mu}\right] v \\
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| S^{b}|B\rangle= & i \bar{u}\left[\bar{g}_{1 \mid p}+\bar{g}_{2}\left(E_{\overline{\mathbf{B}}^{\prime}}+E_{\mathbf{B}}\right)+\bar{g}_{3}\left(E_{\overline{\mathbf{B}}^{\prime}}-E_{\mathbf{B}}\right)\right] \gamma_{5} v \\
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| P^{b}|B\rangle= & i \bar{u}\left[\bar{f}_{1 \not p}+\bar{f}_{2}\left(E_{\overline{\mathbf{B}}^{\prime}}+E_{\mathbf{B}}\right)+\bar{f}_{3}\left(E_{\overline{\mathbf{B}}^{\prime}}-E_{\mathbf{B}}\right)\right] v \tag{6}
\end{align*}
$$

with $V_{\mu}^{b}\left(A_{\mu}^{b}\right) \equiv \bar{q} \gamma_{\mu}\left(\gamma_{5}\right) b, S^{b}\left(P^{b}\right) \equiv \bar{q} b$, and $p_{\mu} \equiv\left(p_{B}-\right.$ $\left.p_{\mathbf{B}}-p_{\overline{\mathbf{B}}^{\prime}}\right)_{\mu} . F_{\mathbf{B} \overline{\mathbf{B}}^{\prime}} \equiv\left(g_{i}, f_{i}, \bar{g}_{j}, \bar{f}_{j}\right)$ with $i=1,2, \ldots, 5$ and $j=1,2,3$ are the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors.
$F_{\mathbf{B B}^{\prime}}$ are momentum dependent. In terms of perturbative $\mathrm{QCD}(\mathrm{pQCD})$ counting rules $[1,3,6,28,31-33]$, one param-
eterizes that

$$
\begin{align*}
& f_{i}=\frac{D_{f_{i}}}{t^{3}} g_{i}=\frac{D_{g_{i}}}{t^{3}} \\
& \bar{f}_{j}=\frac{D_{\bar{f}_{j}}}{t^{3}} \bar{g}_{j}=\frac{D_{\bar{g}_{j}}}{t^{3}} \tag{7}
\end{align*}
$$

with $t \equiv\left(p_{\mathbf{B}}+p_{\overline{\mathbf{B}}^{\prime}}\right)^{2}$. For $F_{\mathbf{B \overline { B }}^{\prime}} \propto 1 / t^{n}, n=3$ corresponds to the three gluon propagators, which are drawn in Figs. 1a-c and 2a-c. Since $V_{\mu}^{b}$ and $A_{\mu}^{b}$ can be combined as the right-handed chiral current $R_{\mu}=\left(V_{\mu}^{b}+A_{\mu}^{b}\right) / 2$, and the baryon decomposed of the right and left-handed states, that is, $\left|\mathbf{B}_{R+L}\right\rangle=\left|\mathbf{B}_{R}\right\rangle+\left|\mathbf{B}_{L}\right\rangle$, it leads to $[6,17]$

$$
\begin{align*}
& \left\langle\mathbf{B}_{R+L} \overline{\mathbf{B}}_{R+L}^{\prime}\right| R_{\mu}|B\rangle \\
& =i m_{b} \bar{u} \gamma_{\mu}\left[\frac{1+\gamma_{5}}{2} G_{R}+\frac{1-\gamma_{5}}{2} G_{L}\right] v \\
& \quad+i \bar{u} \gamma_{\mu} / p_{b}\left[\frac{1+\gamma_{5}}{2} G_{R}^{k}+\frac{1-\gamma_{5}}{2} G_{L}^{k}\right] v \tag{8}
\end{align*}
$$

where $\left|B_{q}\right\rangle \sim \bar{b} \gamma_{5} q|0\rangle$ has been used. As the chiral charge, $Q \equiv R_{\mu=0}$ annihilates the $b$ quark, and creates a valence quark in $\mathbf{B}$, while the spectator quark in the $B$ meson is transformed as a valence quark $\left(\bar{q}_{i}\right)$ in $\overline{\mathbf{B}}^{\prime}$. We hence obtain $G_{R, L}^{(k)}$ as the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors in the chiral representation. When the chirality states of a spinor $(R, L)$ are taken as the helicity states $(\uparrow, \downarrow)$, one can see $\bar{q}_{i}$ with the helicity to be (anti-)parallel $[\|(\overline{\|})]$ to the helicity of $\overline{\mathbf{B}}^{\prime}$, such that the chiral charge acting on $\bar{q}_{i}$ can be more explicitly defined as
$Q_{\|(\overline{\|})}(i)(i=1,2,3)$. We thus derive that
$G_{R(L)} \propto e_{\|}^{R(L)} G_{\|}+e_{\bar{\Pi}}^{R(L)} G_{\Pi}$
$G_{R(L)}^{k} \propto \bar{e}_{\|}^{R(L)} G_{\|}^{k}+\bar{e}_{\overline{\|}}^{R(L)} G_{\|}^{k}$
where $e_{\|}^{R(L)}$ and $e_{\overline{\|}}^{R(L)}$ sum over the weight factors of $\mathbf{B}_{R(L)} \overline{\mathbf{B}}^{\prime}{ }_{R(L)}$, and $\bar{e}_{\|}^{R(L)}$ and $\bar{e}_{\overline{\|}}^{R(L)}$ those of $\mathbf{B}_{L(R)} \overline{\mathbf{B}}^{\prime}{ }_{R(L)}$. By defining $G_{\|(\bar{\Pi})}^{(k)} \equiv D_{\|(\bar{\Pi})}^{(k)} / t^{3}(k=2,3, \ldots, 5)$, we relate the two sorts of the form factors as [1,3,28]

$$
\begin{align*}
D_{g_{1}} & =\frac{5}{3} D_{\|}-\frac{1}{3} D_{\Pi} D_{f_{1}}=\frac{5}{3} D_{\|}+\frac{1}{3} D_{\Pi} D_{g_{k}} \\
& =\frac{4}{3} D_{\|}^{k}=-D_{f_{k}} \\
D_{g_{1}} & =\frac{1}{3} D_{\|}-\frac{2}{3} D_{\Pi} D_{f_{1}}=\frac{1}{3} D_{\|}+\frac{2}{3} D_{\Pi} D_{g_{k}} \\
& =\frac{-1}{3} D_{\|}^{k}=-D_{f_{k}} \\
D_{g_{1}} & =D_{f_{1}}=-\sqrt{\frac{3}{2}} D_{\|} D_{g_{k}}=-D_{f_{k}}=-\sqrt{\frac{3}{2}} D_{\|}^{k} \\
D_{g_{1}} & =D_{f_{1}}=\sqrt{\frac{3}{2}} D_{\|} D_{g_{k}}=-D_{f_{k}}=\sqrt{\frac{3}{2}} D_{\|}^{k} \\
D_{g_{1}} & =D_{f_{1}}=D_{\|} D_{g_{k}}=-D_{f_{k}}=-D_{\|}^{k} \tag{10}
\end{align*}
$$

for $\langle p \bar{p}|(\bar{u} b)\left|B^{-}\right\rangle,\langle p \bar{p}|(\bar{d} b)\left|\bar{B}^{0}\right\rangle,\langle\Lambda \bar{p}|(\bar{s} b)\left|B^{-}\right\rangle$, $\langle p \bar{\Lambda}|(\bar{u} b)\left|\bar{B}_{s}^{0}\right\rangle$, and $\langle\Lambda \bar{\Lambda}|(\bar{s} b)\left|\bar{B}_{s}^{0}\right\rangle$, respectively. Likewise, we perform a derivation for $\bar{g}_{j}\left(\bar{f}_{j}\right)$ through the (pseudo)scalar current, which leads to [28,29]

$$
\begin{align*}
D_{\bar{g}_{1}} & =\frac{5}{3} \bar{D}_{\|}-\frac{1}{3} \bar{D}_{\Pi} D_{\bar{f}_{1}}=\frac{5}{3} \bar{D}_{\|}+\frac{1}{3} \bar{D}_{\Pi} D_{\bar{g}_{2,3}} \\
& =\frac{4}{3} \bar{D}_{\|}^{2,3}=-D_{\bar{f}_{2,3}} \\
D_{\bar{g}_{1}} & =\frac{1}{3} \bar{D}_{\|}-\frac{2}{3} \bar{D}_{\Pi} D_{\bar{f}_{1}}=\frac{1}{3} \bar{D}_{\|}+\frac{2}{3} \bar{D}_{\Pi} D_{\bar{g}_{2,3}} \\
& =\frac{-1}{3} \bar{D}_{\|}^{2,3}=-D_{\bar{f}_{2,3}} \tag{11}
\end{align*}
$$

for $\langle p \bar{p}|(\bar{u} b)\left|B^{-}\right\rangle$and $\langle p \bar{p}|(\bar{d} b)\left|\bar{B}^{0}\right\rangle$, respectively. Note that $R(L) \sim \uparrow(\downarrow)$ is based on the approximation with the large energy transfer, which is conveniently presented as $t \rightarrow \infty$. It is also derived that the correction term is of order $m_{q} / \sqrt{t}$ [31-33]. In fact, $\sqrt{t}$ of a few GeV has been large enough to suppress the correction term [33]. Consequently, the relations with the chirality (helicity) symmetry are shown to be able to describe the scattering processes [33]. For the baryonic $B$ decays, $\sqrt{t}>2 \mathrm{GeV}$ is also sufficient for the holding of the relations in Eqs. (10) and (11).

The four-body $B\left(p_{B}\right) \rightarrow \mathbf{B}\left(p_{\mathbf{B}}\right){\overline{\mathbf{B}^{\prime}}}^{\prime}\left(p_{\overline{\mathbf{B}}^{\prime}}\right) L\left(p_{L}\right) \bar{L}^{\prime}\left(p_{\bar{L}^{\prime}}\right)$ decay involves five kinematic variables in the phase space, that is, $s \equiv\left(p_{L}+p_{\bar{L}^{\prime}}\right)^{2} \equiv m_{L \bar{L}^{\prime}}^{2}, t$, and $\left(\theta_{\mathbf{B}}, \theta_{\mathbf{L}}, \phi\right)[34-36]$. As depicted in Fig. 3, the angle $\theta_{\mathbf{B}(\mathbf{L})}$ is between $\vec{p}_{\mathbf{B}}\left(\vec{p}_{L}\right)$ in the $\mathbf{B} \bar{B}^{\prime}\left(L \bar{L}^{\prime}\right)$ rest frame and the line of flight of the $\mathbf{B} \bar{B}^{\prime}$


Fig. 3 The angular variables $\theta_{\mathbf{B}}, \theta_{L}$ and $\phi$ depicted for the four-body $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ decays
$\left(L \bar{L}^{\prime}\right)$ system in the $B$ meson rest frame. The angle $\phi$ is from the $\mathbf{B} \bar{B}^{\prime}$ plane to the $L \bar{L}^{\prime}$ plane defined by the momenta of the $\mathbf{B} \bar{B}^{\prime}$ pair and $L \bar{L}^{\prime}$ pair in the $B$ meson rest frame, respectively. The partial decay width then reads [8,9]
$d \Gamma=\frac{|\overline{\mathcal{M}}|^{2}}{4(4 \pi)^{6} m_{B}^{3}} X \alpha_{\mathbf{B}} \alpha_{\mathbf{L}} d s d t d \cos \theta_{\mathbf{B}} d \cos \theta_{\mathbf{L}} d \phi$
where $X=\left[\left(m_{B}^{2}-s-t\right)^{2} / 4-s t\right]^{1 / 2}, \alpha_{\mathbf{B}}=\lambda^{1 / 2}\left(t, m_{\mathbf{B}}^{2}, m_{\overline{\mathbf{B}}^{\prime}}^{2}\right) / t$, and $\alpha_{\mathbf{L}}=\lambda^{1 / 2}\left(s, m_{L}^{2}, m_{\tilde{L}^{\prime}}^{2}\right) / s$, with $\lambda(a, b, c)=a^{2}+b^{2}+$ $c^{2}-2 a b-2 b c-2 c a$. For integration, the allowed ranges of the five variables are $\left(m_{L}+m_{\bar{L}^{\prime}}\right)^{2} \leq s \leq\left(m_{B}-\sqrt{t}\right)^{2}$, $\left(m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}\right)^{2} \leq t \leq\left(m_{B}-m_{L}-m_{\bar{L}^{\prime}}\right)^{2}, 0 \leq \theta_{\mathbf{B}, \mathbf{L}} \leq \pi$, and $0 \leq \phi \leq 2 \pi$. The partial decay width of $B\left(p_{B}\right) \rightarrow$ $\mathbf{B}\left(p_{\mathbf{B}}\right) \overline{\mathbf{B}}^{\prime}\left(p_{\overline{\mathbf{B}}^{\prime}}\right) M\left(p_{M}\right)$ involves two variables in the phase space, given by $[3,28]$
$d \Gamma=\frac{\beta_{\mathbf{B}}^{1 / 2} \beta_{t}^{1 / 2}}{\left(8 \pi m_{B}\right)^{3}}|\overline{\mathcal{M}}|^{2} d t d \cos \theta$
where $\beta_{\mathbf{B}}=\left[1-\left(m_{\mathbf{B}}+m_{\left.\overline{\mathbf{B}}^{\prime}\right)^{2}}\right)^{2} t\left[1-\left(m_{\mathbf{B}}-m_{\overline{\mathbf{B}}^{\prime}}\right)^{2} / t\right]\right.$, $\beta_{t}=\left[\left(m_{B}+m_{M}\right)^{2}-t\right]\left[\left(m_{B}-m_{M}\right)^{2}-t\right]$, and $\theta$ is the angle between the meson and baryon moving directions in the $\mathbf{B} \bar{B}^{\prime}$ rest frame. The allowed regions of the variables are $-1<\cos \theta<1$ and $\left(m_{\mathbf{B}}+m_{\overline{\mathbf{B}^{\prime}}}\right)^{2}<t<\left(m_{B}-m_{M}\right)^{2}$. For the global fit in the next section, we define the $C P$ asymmetry [4,37], and angular asymmetries of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ [3,26,28] and $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}[8,9]$, written as

$$
\begin{equation*}
\mathcal{A}_{C P} \equiv \frac{\Gamma\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M\right)-\Gamma\left(\bar{B} \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \bar{M}\right)}{\Gamma\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M\right)+\Gamma\left(\bar{B} \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \bar{M}\right)} \tag{14}
\end{equation*}
$$

$\mathcal{A}_{F B, \theta_{i}} \equiv \frac{\Gamma\left(\cos \theta_{i}>0\right)-\Gamma\left(\cos \theta_{i}<0\right)}{\Gamma\left(\cos \theta_{i}>0\right)+\Gamma\left(\cos \theta_{i}<0\right)}$
where $\bar{B} \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \bar{M}$ represents the anti-particle decay.

## 3 Numerical results and discussions

In the numerical analysis, the CKM matrix elements in the Wolfenstein parameterization read [7]
$V_{u b}=A \lambda^{3}(\rho-i \eta), V_{u d}=1-\lambda^{2} / 2, V_{u s}=\lambda$,
$V_{c b}=A \lambda^{2}, V_{t b}=1, V_{t d}=A \lambda^{3}, V_{t s}=-A \lambda^{2}$

Table 1 The effective Wilson coefficients $c_{i}^{e f f}$
$(i=1,2, \ldots, 6)$ for $b$ and $\bar{b}$ decays

| $c_{i}^{\text {eff }}$ | $b \rightarrow d(\bar{b} \rightarrow \bar{d})$ | $b \rightarrow s(\bar{b} \rightarrow \bar{s})$ |
| :--- | :--- | :--- |
| $c_{1}^{\text {eff }}$ | $1.168(1.168)$ | $1.168(1.168)$ |
| $c_{2}^{\text {eff }}$ | $-0.365(-0.365)$ | $-0.365(-0.365)$ |
| $10^{4} c_{3}^{\text {eff }}$ | $238.0+12.7 i(257.4+46.1 i)$ | $243.3+31.2 i(240.9+32.3 i)$ |
| $10^{4} c_{4}^{\text {eff }}$ | $-497.0-38.0 i(-555.2-138.3 i)$ | $-512.8-93.7 i(-505.7-96.8 i)$ |
| $10^{4} c_{5}^{\text {eff }}$ | $145.5+12.7 i(164.7+46.1 i)$ | $150.7+31.2 i(148.4+32.3 i)$ |
| $10^{4} c_{6}^{\text {eff }}$ | $-633.8-38.0 i(-692.0-138.3 i)$ | $-649.6-93.7 i(-642.6-96.8 i)$ |

with $(\lambda, A, \rho, \eta)=(0.225,0.826,0.163 \pm 0.010,0.357 \pm$ 0.010).

From Refs. [18-21], we adopt $D(x)$ as

$$
\begin{align*}
D(x)= & D_{0}(x)+\frac{\alpha_{s}}{4 \pi} D_{1}(x), \\
D_{0}(x)= & \frac{x}{8}\left[-\frac{2+x}{1-x}+\frac{3 x-6}{(1-x)^{2}} \ln (x)\right] \\
D_{1}(x)= & -\frac{23 x+5 x^{2}-4 x^{3}}{3(1-x)^{2}}+\frac{x-11 x^{2}+x^{3}+x^{4}}{(1-x)^{3}} \ln (x) \\
& +\frac{8 x+4 x^{2}+x^{3}-x^{4}}{2(1-x)^{3}} \ln ^{2}(x) \\
& -\frac{4 x-x^{3}}{(1-x)^{2}} L_{2}(1-x)+8 x \frac{\partial D_{0}(x)}{\partial x} \ln \left(\mu^{2} / m_{W}^{2}\right), \tag{16}
\end{align*}
$$

where $L_{2}(1-x) \equiv \int_{1}^{x} \ln (t) /(1-t) d t$ and $\mu=m_{b}$. For $B \rightarrow p \bar{p} M(V)$ and $\bar{B}^{0} \rightarrow p \bar{p} D^{0(*)}$, we present $c_{i}^{\text {eff }}$ in Table 1, where $b$ and $\bar{b}$ decays are both considered, together with the decay constants $\left(f_{\pi}, f_{K}, f_{\rho}, f_{K^{*}}\right)=(130.2 \pm$ $1.2,155.7 \pm 0.3,210.6 \pm 0.4,204.7 \pm 6.1) \mathrm{MeV}[7,38]$ and $\left(f_{D}, f_{D^{*}}\right)=(208.9 \pm 6.5,252.2 \pm 22.7) \mathrm{MeV}[28,39]$. In the generalized edition of the factorization [30,37], $N_{c}$ is taken as the effective color number with $N_{c}^{(e f f)}=(2,3, \infty)$, in order that the non-factorizable QCD corrections can be estimated.

Using the minimum $\chi^{2}$-fit of

$$
\begin{equation*}
\chi^{2}=\sum\left(\frac{\mathcal{O}_{t h}^{i}-\mathcal{O}_{e x}^{i}}{\sigma_{e x}^{i}}\right)^{2}+\left(\frac{\left|V_{u b}\right|_{t h}-\left|V_{u b}\right|_{e x}}{\sigma_{\left|V_{u b}\right|_{e x}}}\right)^{2} \tag{17}
\end{equation*}
$$

we test if the observables of non-leptonic and semileptonic baryonic $B$ decays can both be interpreted, where $\mathcal{O}_{t h}^{i}$ stand for the theoretical calculations of $\mathcal{B}, \mathcal{A}_{C P}$ and $\mathcal{A}_{F B}$, while $\mathcal{O}_{e x}^{i}$ the experimental inputs in Table 2, together with $\sigma_{e x}^{i}$ the experimental errors. Since the $V_{u b}$ in Eq. (3) is for the exclusive baryonic $B_{(s)}$ decays, which can be different from that in the inclusive ones [40-42], we choose $\left|V_{u b}\right|_{e x}=(3.43 \pm 0.32) \times 10^{-3}$ determined from the $\bar{B}_{s}^{0}$ and baryonic $\Lambda_{b}$ decays [7] as our experimental input in Eq. (17).

With 16 experimental inputs from Table 2 and $\left|V_{u b}\right|_{\text {ex }}$, we fit ( $D_{\|}, D_{\Pi}, D_{2,3,4,5}$ ) and ( $\bar{D}_{\|}, \bar{D}_{\Pi}, \bar{D}_{2,3}$ ) in Eqs. (10) and (11), respectively, and $\left|V_{u b}\right|_{t h}$, which amount to 11 parame-

Table 2 Experimental data for the $B^{-} \rightarrow p \bar{p} \ell^{-} \nu_{\ell}$ and $B \rightarrow p \bar{p} M_{(c)}$ decays, where the notation $\dagger$ for $\mathcal{A}_{F B}$ denotes the contribution from $m_{p \bar{p}}<2.85 \mathrm{GeV}$, and $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)$ has combined the Belle and LHCb data in Eq. (1)

| Decay modes | Data |
| :--- | :--- |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{\nu}_{e}\right)$ | $8.2 \pm 3.8[14]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)$ | $5.2 \pm 0.4[14,15]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} \pi^{-}\right)$ | $1.62 \pm 0.20[7]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$ | $5.9 \pm 0.5[7]$ |
| $10^{6} \mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} \bar{K}^{0}\right)$ | $2.66 \pm 0.32[7]$ |
| $10^{2} \mathcal{A}_{C P}\left(B^{-} \rightarrow p \bar{p} \pi^{-}\right)$ | $0 \pm 4[7]$ |
| $10^{2} \mathcal{A}_{C P}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$ | $0 \pm 4[7]$ |
| $10^{2} \mathcal{A}_{F B}\left(B^{-} \rightarrow p \bar{p} \pi^{-}\right)$ | $(-40.9 \pm 3.4)^{\dagger}[43]$ |
| $10^{2} \mathcal{A}_{F B}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$ | $(49.5 \pm 1.4)^{\dagger}[43]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} \rho^{-}\right)$ | $4.6 \pm 1.3[7]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} K^{*-}\right)$ | $3.4 \pm 0.8[45]$ |
| $10^{6} \mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} \bar{K}^{* 0}\right)$ | $1.2 \pm 0.3[45]$ |
| $10^{2} \mathcal{A}_{C P}\left(B^{-} \rightarrow p \bar{p} K^{*-}\right)$ | $21 \pm 16[7]$ |
| $10^{4} \mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} D^{0}\right)$ | $1.04 \pm 0.07[7]$ |
| $10^{4} \mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} D^{* 0}\right)$ | $0.99 \pm 0.11[7]$ |

ters, such that the number of degrees of freedom denoted by d.n.f is counted as d.n.f $=16-11=5$. As a result, we obtain $\chi^{2} / n . d . f=1.86$ as a measure of the global fit, and extract that

$$
\begin{align*}
& \left(D_{\|}, D_{\Pi}\right)=(11.2 \pm 43.5,332.3 \pm 17.2) \mathrm{GeV}^{5} \\
& \left(D_{\|}^{2}, D_{\|}^{3}, D_{\|}^{4}, D_{\|}^{5}\right)=(47.7 \pm 10.1,442.2 \pm 103.4,-38.7 \\
& \quad \pm 9.6,80.7 \pm 27.2) \mathrm{GeV}^{4} \\
& \left(\bar{D}_{\|}, \bar{D}_{\Pi}, \bar{D}_{\|}^{2}, \bar{D}_{\|}^{3}\right) \\
& \quad=(-59.9 \pm 12.9,23.8 \pm 6.8,90.9 \pm 11.1,131.7 \\
& \quad \pm 330.7) \mathrm{GeV}^{4} \tag{18}
\end{align*}
$$

with $N_{c}^{\text {eff }}=2$ and $\infty$ for $B \rightarrow p \bar{p} M(V)$ and $B \rightarrow$ $p \bar{p} D^{0(*)}$, respectively. Using the parameters in Eq. (18), we calculate the branching fractions and angular asymmetries of $B^{-} \rightarrow p \bar{p} \ell \bar{v}, \Lambda \bar{p} \nu \bar{v}$ and $\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell \bar{v}, \Lambda \bar{\Lambda} \nu \bar{v}$, of which the results are compared with the experimental data

Table 3 Our calculations for the semileptonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ decays. For $\mathcal{B}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \ell \bar{v}_{\ell}\right)$, the values in the parentheses correspond to $\ell=(e, \mu, \tau)$, where the first and second errors come from $\left|V_{u b}\right|$ and
the form factors in Eq. (18), respectively. For $\mathcal{B}\left(B \rightarrow \mathbf{B}^{\prime} \nu \bar{v}\right)=$ $\Sigma_{\ell} \mathcal{B}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} \nu_{\ell} \bar{v}_{\ell}\right)$ and $\mathcal{A}_{F B}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}\right)$, the errors take into account the uncertainties of the form factors in Eq. (18)

| Decay modes | This work | Data |
| :---: | :---: | :---: |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow p \bar{p} \ell^{-} \bar{v}_{\ell}\right)$ | $(5.3 \pm 1.1 \pm 1.7,5.4 \pm 1.1 \pm 1.7,7.6 \pm 1.5 \pm 3.9)$ | (8.2 $\pm 3.8$ [14], $5.2 \pm 0.4$ [14, 15],-) |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(B^{-} \rightarrow p \bar{p} \ell^{-} \bar{\nu}_{\ell}\right)$ | $(1.4 \pm 12.6,1.4 \pm 12.6,1.4 \pm 12.6)$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{L}}}\left(B^{-} \rightarrow p \bar{p} \ell^{-} \bar{v}_{\ell}\right)$ | $(-41.7 \pm 21.4,-41.2 \pm 20.4,-2.1 \pm 5.0)$ | - |
| $10^{6} \mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell^{-} \bar{\nu}_{\ell}\right)$ | $(2.1 \pm 0.4 \pm 0.5,2.1 \pm 0.4 \pm 0.5,1.7 \pm 0.3 \pm 0.9)$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell^{-} \bar{\nu}_{\ell}\right)$ | $(25.7 \pm 11.4,25.0 \pm 11.3,-3.5 \pm 2.7)$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{L}}}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \ell^{-} \bar{\nu}_{\ell}\right)$ | $(-44.1 \pm 10.8,-43.7 \pm 10.3,0.4 \pm 5.5)$ | - |
| $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)$ | $(3.5 \pm 1.0) \times 10^{-8}$ | $(0.4 \pm 1.3) \times 10^{-5}\left(<3 \times 10^{-5}\right)[16]$ |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)$ | $22.8 \pm 11.2$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{L}}}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)$ | $-40.9 \pm 8.3$ | - |
| $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{\nu}\right)$ | $(0.8 \pm 0.2) \times 10^{-8}$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{v}\right)$ | $24.4 \pm 11.8$ | - |
| $10^{2} \mathcal{A}_{F B, \theta_{\mathrm{L}}}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{v}\right)$ | $-40.1 \pm 8.0$ | - |

in Table 3. We also draw the $p \bar{p}$ invariant mass spectrum for $B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}$ in Fig. 4.

## 4 Discussions and conclusions

Since $\chi^{2} / n . d . f=1.86$ presents a reasonable fit, it indicates that the most recent data in Table 2 can be explained. It is interesting to note that $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} \pi^{-}, p \bar{p} \rho^{-}\right)$ [3,4] were once overestimated [7,43,44], and the relation of $\mathcal{A}_{F B}\left(B^{-} \rightarrow p \bar{p} \pi^{-}\right) \simeq \mathcal{A}_{F B}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$[3] was not verified by the measurements [43,44]. This is due to $F_{\mathbf{B B}^{\prime}}$ determined by the $B \rightarrow p \bar{p} K$ data [3], while $B \rightarrow p \bar{p} K$ are in fact the penguin dominated decays with $\hat{\mathcal{M}}_{6} \propto$ $\langle p \bar{p}|(S-P)^{b}|B\rangle$ to give the main contribution. To avoid the inconsistency unable to be solved at that time, one performed the extraction of Ref. [6] that excluded $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$, $\mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} \bar{K}^{0}\right)$, and $\mathcal{A}_{F B}\left(B^{-} \rightarrow p \bar{p} K^{-}\right)$, in order that the more associated tree dominated decays of $B \rightarrow p \bar{p}(\pi, \rho)$, $\bar{B}^{0} \rightarrow p \bar{p} D^{0(*)}$, and $B \rightarrow \mathbf{B} \overline{\mathbf{B}} L \bar{L}^{\prime}$ can be studied. However, it resulted in an unsatisfactory global fit not to accommodate the all data.

As $F_{\mathbf{B B}^{\prime}}$ determined in this work can be universal for the non-leptonic and semileptonic decay channels, we calculate $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}\right)=(5.3 \pm 2.0) \times 10^{-6}$ and $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}\right)=(5.4 \pm 2.0) \times 10^{-6}$ agreeing with the experimental values. Moreover, we revisit $B^{-} \rightarrow \Lambda \bar{p} v \bar{\nu}$, and obtain $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)=(3.5 \pm 1.0) \times 10^{-8} 20$ times smaller than the number of Ref. [9].

Like the theoretical illustration in $B \rightarrow \mathbf{B} \overline{\mathbf{B}^{\prime}}$ and $B \rightarrow$ $\mathbf{B} \overline{\mathbf{B}}^{\prime} M[29,46]$, the gluon propagators of Fig. 1a-c play the key role in the $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ formation of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$, where two of them provide the valence quarks in $\mathbf{B} \bar{B}^{\prime}$, while the another


Fig. 4 The $p \bar{p}$ invariant mass spectrum of $B^{-} \rightarrow p \bar{p} \mu^{-} \bar{v}_{\mu}$, where the data points are from LHCb [15]
one speeds up the spectator quark in $B$. Accordingly, the approach of pQCD counting rules derives that $F_{\mathbf{B B}^{\prime}} \propto 1 / t^{3}$.

One can test the momentum dependence of $B^{-} \rightarrow$ $p \bar{p} \mu^{-} \bar{v}_{\mu}$, which is by scanning the partial branching fraction as a function of $\sqrt{t}=m_{p \bar{p}}$. In Fig. 4, as we draw the line to agree with the five data points [15]; particularly, those around the area of $\sqrt{t} \sim m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}$ for the threshold effect, it is shown that $F_{\mathbf{B B}^{\prime}}$ as a function of $1 / t$ can describe $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$.

By normalizing the prediction of the pQCD model [8], LHCb draws the $m_{p \bar{p}}$ spectrum of $B^{-} \rightarrow p \bar{p} \mu \bar{\nu}$ in Fig. 4 of Ref. [15], where the line is higher and narrower than our result. The difference is caused by the fact that the line of of Ref. [15] is chosen to more agree with the two data points around $m_{p \bar{p}} \sim 2.5 \mathrm{GeV}$. Subsequently, the peak should reach $17 \times 10^{-6}$ to be above the data point around $m_{p \bar{p}} \sim 2 \mathrm{GeV}$ for integrating over the partial branching fraction as large as $\mathcal{B} \simeq$ $5 \times 10^{-6}$. In comparison, our result prefers to agree with the threshold data points; however, requiring some broadening to give a sufficient branching fraction.

The decay channel $\bar{B}_{s}^{0} \rightarrow \Lambda \bar{p} K^{+}\left(\bar{\Lambda} p K^{-}\right)$is the first observation of a baryonic $\bar{B}_{s}^{0}$ decay [47], whose branching fraction $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{p} K^{+}+\bar{\Lambda} p K^{-}\right)=5.46 \times 10^{-6}$ is as large as those of the three-body baryonic $B^{-}\left(\bar{B}^{0}\right)$ decays. Hence, the semileptonic baryonic $\bar{B}_{s}^{0}$ decay is supposed to be compatible with $B^{-} \rightarrow p \bar{p} \ell \bar{\nu}_{\ell}$. In our prediction, we present

$$
\begin{equation*}
\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} e^{-} \bar{v}_{e}, p \bar{\Lambda} \mu^{-} \bar{v}_{\mu}\right)=(2.1 \pm 0.6,2.1 \pm 0.6) \times 10^{-6} \tag{19}
\end{equation*}
$$

which are accessible to the LHCb experiment, whereas $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{\nu}\right)=(0.8 \pm 0.2) \times 10^{-8}$ is relatively small.

Because of $m_{\tau} \gg m_{e, \mu}$ that strongly shrinks the phase space, it is anticipated that $\mathcal{B}\left(B \rightarrow \mathbf{B}^{\prime} \tau \bar{\nu}\right) \ll \mathcal{B}(B \rightarrow$ $\left.\mathbf{B} \overline{\mathbf{B}}^{\prime} e \bar{\nu}, \mathbf{B}^{\prime} \overline{\mathbf{B}}^{\prime} \mu \bar{\nu}\right)$. Nonetheless, the amplitude of Eq. (3) and the matrix elements of Eq. (6) result in

$$
\begin{equation*}
i \bar{u}\left(g_{3} \gamma_{5}-f_{3}\right) v m_{\ell} \bar{u}_{\ell} \gamma_{\mu}\left(1+\gamma_{5}\right) v_{\bar{v}} \tag{20}
\end{equation*}
$$

in $\mathcal{M}\left(B \rightarrow \mathbf{B}^{\prime} \overline{\mathbf{}}^{\prime} \bar{\nu}\right)$, where $m_{\tau}$ is able to enhance the decay. We thus obtain

$$
\begin{align*}
\mathcal{B}\left(B^{-} \rightarrow p \bar{p} \tau^{-} \bar{\nu}_{\tau}\right) & =(7.6 \pm 4.2) \times 10^{-6} \\
\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} \tau^{-} \bar{\nu}_{\tau}\right) & =(1.7 \pm 1.0) \times 10^{-6} \tag{21}
\end{align*}
$$

as large as their counterparts. Likewise, the mass effect can be found in $\mathcal{M}(B \rightarrow \tau \bar{\nu}) \propto m_{\tau} \bar{u}_{\tau}\left(1+\gamma_{5}\right) v_{\bar{v}}[48,49]$ and $\mathcal{M}\left(B \rightarrow \mathbf{B}_{c} \overline{\mathbf{B}}^{\prime}\right) \propto m_{c}\left\langle\mathbf{B}_{c} \overline{\mathbf{B}}^{\prime}\right| \bar{c}\left(1+\gamma_{5}\right) q|0\rangle[50]$, where $m_{\tau}$ and $m_{c}$ alleviate the decays from helicity suppression.

We study the angular asymmetries of the semileptonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ decays. While $\mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(B^{-} \rightarrow p \bar{p} \ell^{-} \bar{\nu}_{\ell}\right)$ are around several percents, $\mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} e^{-} \bar{\nu}_{e}, p \bar{\Lambda} \mu^{-} \bar{\nu}_{\mu}\right)$ and $\mathcal{A}_{F B, \theta_{\mathbf{B}}}\left(\bar{B}_{s}^{0} \rightarrow \Lambda \bar{\Lambda} \nu \bar{\nu}\right)$ can be around $25 \%$. Like the three-body baryonic $B$ decays [3,26,28], this implies a theoretical sensitivity for $F_{\mathbf{B E}^{\prime}}$ to be confirmed by future measurements.

In summary, we have investigated the semiletonic $B^{-}\left(\bar{B}_{s}^{0}\right)$ $\rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} L \bar{L}^{\prime}$ decays with $L \bar{L}^{\prime}=\left(\ell \bar{v}_{\ell}, \nu \bar{\nu}\right)$. We have newly extracted the $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime}$ transition form factors with the global fit that includes the data of $B \rightarrow p \bar{p} M(V), \bar{B}^{0} \rightarrow$ $p \bar{p} D^{0(*)}$ and $B \rightarrow p \bar{p} e^{-} \bar{v}_{e}, p \bar{p} \mu^{-} \bar{v}_{\mu}$ decays. In our demonstration, $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} e^{-} \bar{v}_{e}, p \bar{p} \mu^{-} \bar{v}_{\mu}\right)$ once overestimated to be as large as $10^{-4}$ has been reduced to be around $5 \times 10^{-6}$, in agreement with the current data. We have also presented $\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \nu \bar{\nu}\right)=(3.5 \pm 1.0) \times 10^{-8}$. It has been found that $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow p \bar{\Lambda} e^{-} \bar{v}_{e}, p \bar{\Lambda} \mu^{-} \bar{\nu}_{\mu}, p \bar{\Lambda} \tau^{-} \bar{\nu}_{\tau}\right)=$ $(2.1 \pm 0.6,2.1 \pm 0.6,1.7 \pm 1.0) \times 10^{-6}$ and $\mathcal{B}\left(\bar{B}_{s}^{0} \rightarrow\right.$ $\Lambda \bar{\Lambda} \nu \bar{\nu})=(0.8 \pm 0.2) \times 10^{-8}$ can be promising for future measurements.

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