



# Analysis of localized CP asymmetry in $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$

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**Abstract** An investigation about localized CP asymmetries for the processes of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$  is presented in this paper. The innovation of this paper is that there is a consideration of three-particle  $\rho^0(770)$ ,  $\omega(782)$  and  $\phi(1020)$  interferences effect. Generally,  $\omega(782)$  and  $\phi(1020)$  both can decay into  $\pi^+\pi^-$  pair where can cause extremely small contribution from isospin symmetry breaking. Nevertheless, our analysis shows that  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$  decay process can bring differential CP asymmetry about 65% (36%) because of isospin symmetry breaking. To be better compared with the data from experimental in the future, we integrate CP asymmetry over the invariant mass and obtain localized CP asymmetry value for the decay  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$ . We find that there is an evident signal about CP asymmetry at invariant mass value  $m(\pi^+\pi^-)$  below the mass of  $\rho^0(770)$  with the decay  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$ .

## 1 Introduction

The sources of CP asymmetry in particle physics have caused a great deal of attention since 1964 [1]. It is possible for CP asymmetry to occur during the decay of a hadron, during the mixing of neutral hadrons, or during the interference between the two processes [2]. The strong interaction and electromagnetic interaction all satisfy the CP transformation without deformation while CP symmetry breaks down only in the weak interaction, and it has been believed that the root is because there is a complex phase angle in the Cabibbo–Kobayashi–Maskawa (CKM) matrix [3]. The amplitude is connected at least twice interaction of a weak phase and a

strong phase in a b hadron decay [4]. The weak phase is relevant to the Cabibbo–Kobayashi–Maskawa (CKM) matrix while the strong phase is created by re-scattering or by other mechanism involved in the two-body decay process. The three-body decay process includes complex dynamic mechanism and phase space associated with resonance and non-resonance contributions. It is the intermediate resonance hadrons linked to Breit–Wigner formalism that give rise to the strong phases accounting for CP asymmetry in three-body decay processes.

Vector meson dominance model (VMD) predicts that the vacuum polarisation of the photon is entirely made up of vector mesons of  $\rho^0(770)$ ,  $\omega(782)$  and  $\phi(1020)$  [5]. The photon couples to the neutral vector meson which is dominated by a two-pion state when  $e^+e^-$  decay into the pair of  $\pi^+\pi^-$ . The transitions of  $\omega(782)$  and  $\phi(1020)$  decay to  $\pi^+\pi^-$  pair which originate in isospin breaking related to the mixings of  $\omega(782) - \rho^0(770)$  and  $\phi(1020) - \rho^0(770)$ . One can combine the intermediate state with the physical states from isospin states by the unitary matrix for the three hadrons. The dynamics mechanism can be obtained from the interference of  $\rho^0(770)$ ,  $\omega(782)$  and  $\phi(1020)$  mesons [6]. A new strong phase is formed with the help of the intermediate resonance hadrons, which may have an impact on the CP asymmetry of hadron decay.

The LHCb collaboration has gained a lot of attention to the CP asymmetries about  $B^\pm \rightarrow \pi^\pm\pi^+\pi^-$  and  $B^\pm \rightarrow K^\pm\pi^+\pi^-$  decay processes in recent years [7,8]. Experimental studies have found the above decay processes produce huge CP asymmetry, which reach 58% and 67% for the local regions, respectively [9,10]. Currently, there are many theoretical explanations for the CP asymmetry of these three-body decay processes, for example, based on resonance effects and end-state rescattering [11,12]. Especially for  $B^\pm \rightarrow K^\pm\pi^+\pi^-$  process, the huge CP asymmetry occurs when the invariant mass of  $\pi^+\pi^-$  is in region of the  $\rho^0(770)$  and  $f_0(980)$  resonance [10]. Although our pre-

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vious results show that when the invariant masses of  $\pi^+\pi^-$  in the resonance region of  $\rho^0(770)$  and  $\omega(782)$  have large CP asymmetry, we failed to compare with the experiment because we do not consider the unevenness of phase space distribution [13–17]. At the same time, end-state rescattering can produce new strong phase (which can even be relatively large), which may affect CP asymmetry [18, 19]. Based on the above considerations, we focus on the CP asymmetry in the resonance region of  $\rho^0(770)$ ,  $\omega(782)$  and  $\phi(1020)$  in this work so that we can make a comparison with the experimental data.

An observation of CP asymmetry has been made in localized phase space region from three-body decay of the B meson [20, 21]. Especially, quasi-two-body  $B \rightarrow PV$  resulting in three-body final states from vector meson decays are presented [22]. In view of vector mesons resonances, the different types of resonant contributions are allowed to estimate the strong phase. CP violations are measured in charm-less B decay which related to the  $\rho(770)^0 - \omega(782)$  mixing region. Including the  $\omega(782)$  contribution, the CP violation related to the vector resonance is measured to be  $A_{CP}(B^+ \rightarrow \rho(770)^0\pi^+ \rightarrow \pi^+\pi^+\pi^-) = -0.004 \pm 0.017$  and  $A_{CP}(B^+ \rightarrow \rho^0(770)K^+ \rightarrow K^+\pi^+\pi^-) = 0.150 \pm 0.019$ . The method comes from the approximation of a two-body interaction plus one spectator meson [23].

These processes improves our knowledge about the CP asymmetry by precise measurements. CP asymmetry measurements will soon be possible for the  $B_s$  decay with the data obtained by the upgraded LHCb. The three-body decay of  $B_s$  meson can provide new opportunities for searching CP asymmetry. In the case of three-body decays, intermediate states often dominate through quasi-two-body decay channels. There are several factorization techniques for calculating the hadron matrix elements in two-body hadron decays [24–27]. And by introducing a sudakov factor based on the QCD correction, the PQCD method can calculate decay amplitudes without endpoint divergence is more appreciate. Besides, the final state of this work can be handled as this quasi-two-body form  $\bar{B}_s^0 \rightarrow V\pi^0(K^0)$ , where  $V = \rho^0(770), \omega(782), \phi(1020)$  [28–30].

As presented in our work, we consider  $\rho^0(770)$ ,  $\omega(782)$  and  $\phi(1020)$  resonance effect for CP asymmetry, which relates with new complex strong phase. The new strong phase for the first order causes the isospin symmetry to shatter, which results in the CP asymmetry. We aim at the CP asymmetry in the decay process of  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  in perturbative QCD approach. While at the same time, the localized integrated CP asymmetry can be obtained to compare with the results of experiments in near future.

We organize the information as below. In Sect. 2, theoretical framework about three-particle mixing is introduced. Following that, in Sect. 3, we show the decay diagrams and

analytical formalization of the primary decay processes. Section 4 of this paper contains the precise calculations, while in Sect. 5 we show the numerical findings. Section 6 has a summary and a conclusion. Acknowledgement is given below.

## 2 Three-particle mixing

Using a dominant model of vector meson (VMD), vector mesons of  $\rho^0(770), \omega(782), \phi(1020)$  can be obtained by the decay of photons formed by the polarization of positive and negative electron pairs in a vacuum. The momentum of this process is also transmitted through VMD model. Since the intermediate resonance state is not a physical field, we use the unitary matrix R to make a conversion:  $\rho_I^0(\omega_I, \phi_I) \rightarrow \rho^0(\omega, \phi)$ . In the two representations, one can get two expressions about R, where  $\langle \rho_I | \rho \rangle, \langle \omega_I | \omega \rangle$  and  $\langle \phi_I | \phi \rangle$  are equal to 1 and  $\langle \rho_I | \omega \rangle, \langle \rho_I | \phi \rangle$  and  $\langle \omega | \phi \rangle$  are equal to  $A_{\rho\omega}(s), B_{\rho\phi}(s)$  and  $C_{\omega\phi}(s)$  since they are all order  $\mathcal{O}(\lambda)$ , ( $\lambda \leq 1$ ) [6]. The isospin basis vector  $|I, I_3\rangle$  can be constructed by using the isospin field  $\rho_I^0(\omega_I, \phi_I)$ , where I and  $I_3$  refer to the isospin and the isospin third component, respectively. Therefore, we regard physical particle states as linear combinations of these essential vectors. We use the orthogonal normalization method to obtain the relationship between the physical state of the particle and the isospin basis vector, where we define the propagator  $D_{(s)}$ . We can get the physical states  $\rho^0, \omega$  and  $\phi$  as follows:

$$\rho^0 = \rho_I^0 - A_{\rho\omega}(s)\omega_I - B_{\rho\phi}(s)\phi_I, \quad (1)$$

$$\omega = A_{\rho\omega}(s)\rho_I^0 + \omega_I - C_{\omega\phi}(s)\phi_I, \quad (2)$$

$$\phi = B_{\rho\phi}(s)\rho_I^0 + C_{\omega\phi}(s)\omega_I + \phi_I. \quad (3)$$

Based on the physical representation, we take the method of diagonalization about  $W_I$  by the matrix R without considering higher order terms, where  $W_I$  is defined as the mass squared operator in an isospin field. Then we obtain the relationship  $A_{\rho\omega}(s), B_{\rho\phi}(s)$  and  $C_{\omega\phi}(s)$  with  $W_I$ . According to the representations of physics, we can describe the propagator of intermediate state particles from vector mesons. Considering the physics and isospin effect, we make these definitions respectively as follows, where  $D_{V_1V_2} = \langle 0|T V_1 V_2|0\rangle$  and  $D_{V_1V_2}^I = \langle 0|T V_1^I V_2^I|0\rangle$ .  $V_1$  and  $V_2$  of  $D_{V_1V_2}$  which refers to the meson of  $\rho^0, \omega$  or  $\phi$ . After taking the expressions of Eqs. (1), (2) and (3) into the definition of  $D_{V_1V_2}$ , we can find the form of  $D_{\rho\omega}, D_{\rho\phi}$  and  $D_{\omega\phi}$  are same. Actually, there is no three-particle mixing under the physical appearance so that  $D_{V_1V_2}$  is equal to zero. Besides, according to the expression for the physical state of the three-particle mixing, these parameters of  $\Pi_{\rho\omega}, \Pi_{\omega\phi}, \Pi_{\rho\phi}, A_{\rho\omega}, B_{\rho\phi}$  and  $C_{\omega\phi}$  take the leading order approximation. The product which any two of them is higher order and can be disregarded, which enables

us to get the formalisms of  $A_{\rho\omega}$ ,  $B_{\rho\phi}$  and  $C_{\omega\phi}$  as follows:

$$A_{\rho\omega} = \frac{\Pi_{\rho\omega}}{s_\rho - s_\omega}, \quad B_{\rho\phi} = \frac{\Pi_{\rho\phi}}{s_\rho - s_\phi}, \quad C_{\omega\phi} = \frac{\Pi_{\omega\phi}}{s_\omega - s_\phi}. \quad (4)$$

where  $s_\rho$ ,  $s_\omega$  and  $s_\phi$  refer to the propagators of  $\rho$ ,  $\omega$  and  $\phi$ .

We can write  $s_V + m_V^2 - im_V\Gamma_V = s$ , where the  $\sqrt{s}$  is the invariant mass of two pions. For the vector  $V$ ,  $s_V$  is the inverse propagator,  $m_V$  represents its mass and  $\Gamma_V$  refers to the decay rate. From above equations, we obtain  $\tilde{\Pi}_{\rho\omega}$  and  $\tilde{\Pi}_{\rho\phi}$  after definition about  $\tilde{\Pi}_{V_1V_2} = \frac{s_{V_1}\Pi_{V_1V_2}}{s_{V_1}-s_{V_2}}$  ( $\tilde{\Pi}_{\omega\phi}$  can be ignored in the next step of the calculation).

Wolfe and Maltman recently calculated the precise  $\rho - \omega$  mixing parameters [31,32]:

$$\begin{aligned} \Re\Pi_{\rho\omega}(m_\rho^2) &= -4470 \pm 250_{modl} \pm 160_{data} \text{MeV}^2, \\ \Im\Pi_{\rho\omega}(m_\rho^2) &= -5800 \pm 2000_{modl} \pm 1100_{data} \text{MeV}^2. \end{aligned} \quad (5)$$

Mixing parameters  $\rho - \phi$  near the  $\phi$  meson have been given as follows [33]:

$$F_{\rho\phi} = (0.72 \pm 0.18) \times 10^{-3} - i(0.87 \pm 0.32) 10^{-3}. \quad (6)$$

Different mixing parameters relate to the momentum dependence of  $\rho - \omega$  mixing and  $\rho - \phi$  mixing, which are  $\tilde{\Pi}_{\rho\omega}(s)$  and  $\tilde{\Pi}_{\rho\phi}(s)$  respectively. To absorb the contribution of  $\omega$  and  $\phi$  decay into  $\pi^+\pi^-$  pair, we anticipate discovering evidence of mixing in the resonance area of  $\omega$  and  $\phi$  where two pions are also generated by breaking isospin symmetry. We use  $\tilde{\Pi}_{\rho\omega}(s) = \Re\tilde{\Pi}_{\rho\omega}(m_\omega^2) + \Im m\tilde{\Pi}_{\rho\omega}(m_\omega^2)$  and  $\tilde{\Pi}_{\rho\phi}(s) = \Re\tilde{\Pi}_{\rho\phi}(m_\phi^2) + \Im m\tilde{\Pi}_{\rho\phi}(m_\phi^2)$  expressions and update the values as given in Table 1 [34–36].

### 3 CP asymmetry in

$$\bar{B}_s^0 \rightarrow \rho(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$$

For the convenience of representing decay process in our work, we replace  $\rho^0(770)$ ,  $\omega(782)$ , and  $\phi(1020)$  with  $\rho^0$ ,  $\omega$  and  $\phi$  respectively. And the main decay diagrams of  $\bar{B}_s^0 \rightarrow \rho(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  can be expressed in Fig. 1. One can see that the quasi-two-body decay of  $\bar{B}_s^0 \rightarrow \rho(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  is associated the diagrams (a) ~ (f) of Fig. 1. The main contribution depends on (a) diagram of the Fig. 1 since the decay rate of  $\rho \rightarrow \pi\pi$  is 100%. For simplify, we only present a few of the major graphs.

In the diagram (a),  $\bar{B}_s^0$  meson decays into  $\pi^0(K^0)$  and  $\pi^+\pi^-$  pair which is produced directly by  $\rho^0$  resonance

effect. Meanwhile, it is known that  $\pi^+\pi^-$  pair can also exist by the resonance effect of  $\omega$  or  $\phi$  meson, where corresponding mixing parameters are involved. As shown in the diagram of (b),  $\rho^0$  meson decays into  $\pi^+\pi^-$  by  $\omega$  resonance. The mixing parameter  $\Pi_{\rho\omega}$  is generated during the  $\omega$  resonance, which as shown in the black dots of (b). Diagram of (c) is almost similar to diagram (b), but the differences are the resonance effect is  $\phi$  and the mixing parameter is  $\Pi_{\rho\phi}$ . In the diagram (d), the meson of  $\omega$  or  $\phi$  decay into  $\pi^+\pi^-$  compared with the  $\rho$  meson is extremely small. Furthermore, the contributions from the diagram (e) and diagram (f) are so tiny that can be neglected since  $\omega$  and  $\phi$  decay into  $\pi^+\pi^-$  through the resonance effect of  $\omega - \phi$  mixing. After considering above, we can see  $\bar{B}_s^0 \rightarrow \rho(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  decay process receives effectively diagrams contributions from (a) ~ (c).

The procedure of the decay amplitude  $A(\bar{A})$  is described as:

$$\begin{aligned} A_{total} &= \langle \pi^+\pi^-\pi^0(K^0) | \mathcal{H} | \bar{B}_s^0 \rangle \\ &= \langle \pi^+\pi^-\pi^0(K^0) | \mathcal{H}_T | \bar{B}_s^0 \rangle \\ &\quad + \langle \pi^+\pi^-\pi^0(K^0) | \mathcal{H}_P | \bar{B}_s^0 \rangle, \end{aligned} \quad (7)$$

where  $\langle \pi^+\pi^-\pi^0(K^0) | \mathcal{H}_T (\mathcal{H}_P) | \bar{B}_s^0 \rangle$  is the contribution of tree (penguin). Through above contribution, we can define:

$$r \equiv \left| \frac{\langle \pi^+\pi^-\pi^0(K^0) | H^P | \bar{B}_s^0 \rangle}{\langle \pi^+\pi^-\pi^0(K^0) | H^T | \bar{B}_s^0 \rangle} \right|. \quad (8)$$

The relative value  $r$  of the contributions made by the tree operator and the penguin operator are described below:

$$A = \langle \pi^+\pi^-\pi^0(K^0) | H^T | \bar{B}_s^0 \rangle \left[ 1 + r e^{i(\delta+\phi)} \right], \quad (9)$$

where  $\delta$  and  $\phi$  refer to the strong phase and weak phase, respectively.

We can get the physical information from diagrams (a), (b) and (c) in Fig. 1:

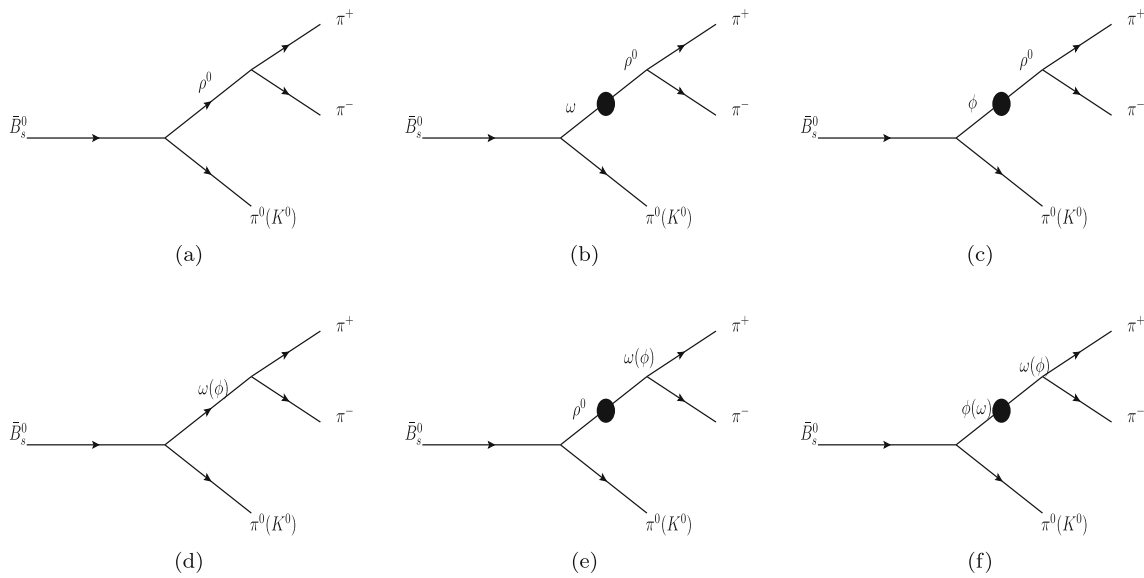
$$\begin{aligned} &\langle \pi^+\pi^-\pi^0(K^0) | H^T | \bar{B}_s^0 \rangle \\ &= \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} t_\omega + \frac{g_\rho}{s_\rho s_\phi} \tilde{\Pi}_{\rho\phi} t_\phi + \frac{g_\rho}{s_\rho} t_\rho, \end{aligned} \quad (10)$$

$$\begin{aligned} &\langle \pi^+\pi^-\pi^0(K^0) | H^P | \bar{B}_s^0 \rangle \\ &= \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega} p_\omega + \frac{g_\rho}{s_\rho s_\phi} \tilde{\Pi}_{\rho\phi} p_\phi + \frac{g_\rho}{s_\rho} p_\rho. \end{aligned} \quad (11)$$

where  $t_V (p_V)$  is the tree (penguin) amplitudes,  $g_\rho$  refers to the coupling constant and  $s_V$  represents the Breit–Wigner formalism of vector meson ( $V = \rho, \omega, \phi$ ). We define these

**Table 1** The value of the mixing parameter

$\Re\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -4760 \pm 440 \text{MeV}^2$	$\Im\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -6180 \pm 3300 \text{MeV}^2$
$\Re\tilde{\Pi}_{\rho\phi}(m_\phi^2) = 796 \pm 312 \text{MeV}^2$	$\Im\tilde{\Pi}_{\rho\phi}(m_\phi^2) = -101 \pm 67 \text{MeV}^2$



**Fig. 1** The diagrams of  $\bar{B}_s^0 \rightarrow \pi^+ \pi^- \pi^0(K^0)$  decay process

**Table 2** The relationships about  $\rho$  and  $\eta$  for  $\bar{B}_s^0 \rightarrow \rho(\omega, \phi) \pi^0(K^0)$  decay processes

$\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0$	$\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)K^0$
$-\sin\phi\sqrt{\rho^2 + \eta^2} = \eta$	$\sin\phi\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2} = \eta$
$-\cos\phi\sqrt{\rho^2 + \eta^2} = \rho$	$\cos\phi\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2} = \rho(1-\rho)$

equations of  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0(K^0)$  to obtain CP asymmetry according to the Wolfenstein parametrization [37,38].

Table 2, the corresponding decay process of  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0$  on the left side of the table is a pure annihilation decay process. These two expressions for  $\rho$  and  $\eta$  obtained from  $V_{tb}, V_{ts}^*$  and  $V_{ub}, V_{us}^*$ . The CKM matrix, whose elements are determined from experiments can be expressed in terms of the Wolfenstein parameters  $A, \rho, \lambda$  and  $\eta$ . Then we can get the expressions of  $V_{tb} = 1, V_{ts}^* = -A\lambda^2, V_{ub} = A\lambda^3(\rho - i\eta)$  and  $V_{us}^* = \lambda$ . Therefore, we can get the relationships  $\rho$  and  $\eta$  between  $\sin\phi$  and  $\cos\phi$  in  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0$  decay process since the weak phase  $\phi$  is related with  $\frac{V_{tb}V_{ts}^*}{V_{ub}V_{us}^*}$ . Similarly, the decay process of  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)K^0$  on the right side of the table is a tree-dominant decay process whose two expressions for  $\rho$  and  $\eta$  obtained are associated with  $V_{tb}, V_{td}^*$  and  $V_{ub}, V_{ud}^*$ . We can obtain the expressions of  $\sin\phi$  and  $\cos\phi$  in  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0$  decay process after simplifying  $V_{tb} = 1, V_{td}^* = A\lambda^3(1 - \rho + i\eta), V_{ub} = A\lambda^3(\rho - i\eta)$  and  $V_{ud}^* = 1 - \frac{1}{2}\lambda^2$ .

Therefore, the differential parameter of CP asymmetry is shown as below:

$$A_{CP} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}. \tag{12}$$

In the future, we need to compare our results with experimental data while taking into consideration the localized integrated direct CP asymmetry of decay processes. For the decay amplitude of  $\bar{B}_s^0 \rightarrow \rho^0\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  process, we consider the contributions of  $\bar{B}_s^0 \rightarrow \rho^0\pi^0(K^0)$  and  $\rho^0 \rightarrow \pi^+\pi^-$ .

The amplitude of the  $\bar{B}_s^0 \rightarrow \rho^0\pi^0(K^0)$  can be written as  $M_{\bar{B}_s^0 \rightarrow \rho^0\pi^0(K^0)}^\lambda = \alpha P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda)$ ,  $\alpha$  is the part of the decay amplitude which is independent of the helicity,  $P_{\bar{B}_s^0}$  is the momenta of  $\bar{B}_s^0, \epsilon$  is the polarization vector of  $\rho^0, \lambda$  is the helicity of vector meson. And the amplitude for  $\rho^0 \rightarrow \pi^+\pi^-$  can be written as follows:  $M_{\rho^0 \rightarrow \pi^+\pi^-}^\lambda = g_\rho \epsilon(\lambda) \cdot (p_1 - p_2)$ , where  $p_1$  is the momenta of  $\pi^+$  and  $p_2$  is the momenta of  $\pi^-$ . Hence, the amplitude of  $\bar{B}_s^0 \rightarrow \rho^0\pi^0(K^0) \rightarrow \pi^+\pi^-\pi^0(K^0)$  is [39,40]:

$$A = \frac{g_\rho \alpha}{s_\rho} P_{\bar{B}_s^0}^\mu \sum_{\lambda=\pm 1,0} \epsilon_\mu^*(\lambda) \epsilon_r(\lambda) \cdot (p_1 - p_2)^r = -\frac{g_\rho \alpha}{s_\rho} P_{\bar{B}_s^0}^\mu \left[ g_{\mu r} - \frac{(p_1 + p_2)_\mu (p_1 + p_2)_r}{m_\rho^2} \right] (p_1 - p_2)^r. \tag{13}$$

In the three body decay process, we obtain  $P_{\bar{B}_s^0} = p_1 + p_2 + p_3$  and  $m_{ij}^2 = p_{ij}^2$  since conservation of energy and

momentum. Thus, the amplitude can be written as:

$$A = \frac{g_\rho}{s_\rho} \cdot \frac{M_{\bar{B}_s^0 \rightarrow \rho^0 \pi^0(K)}^\lambda}{P_{\bar{B}_s^0} \cdot \epsilon^*} \cdot (\xi - s') = (\xi - s') \cdot \mathcal{M}. \quad (14)$$

Using this formula,  $\sqrt{s'}$  ( $\sqrt{s}$ ) is the high (low) invariance mass of the  $\pi^+ \pi^-$  pair. According to the equation above,  $\xi$  is equal to  $\frac{1}{2} (s'_{\max} + s'_{\min})$ , where  $s'_{\max}$  ( $s'_{\min}$ ) is the maximum (minimum) value of  $s'$  for fixed  $s$ .

Integrate  $A_{CP}$  within the invariant mass range, which may be quantified by experiments. Then we are able to estimate the localized integrated CP asymmetry in this location:

$$A_{CP}^\Omega = \frac{\int_{s_1}^{s_2} ds \int_{s'_1}^{s'_2} ds' (\xi - s')^2 (|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2)}{\int_{s_1}^{s_2} ds \int_{s'_1}^{s'_2} ds' (\xi - s')^2 (|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2)}. \quad (15)$$

By analyzing the kinematic of three body decay process in the region of  $\Omega (s_1 < s < s_2, s'_1 < s' < s'_2)$ , we are able to conclude that  $\xi = \frac{1}{2} (s'_{\min} + s'_{\max})$  is correlated with  $s$  based on the kinematics.  $\xi$  is taken to be a constant in this case because  $s$  varies only in minimal scale [7]. This not only results in the cancellation of  $\int_{s'_1}^{s'_2} ds' (\xi - s')^2$  but also  $A_{CP}^\Omega$  is independent of the high invariant mass of positive and negative meson pairs in this way. We consider the  $s$  is dependent between the values of  $s'_{\max}$  and  $s'_{\min}$  in our calculations. It is assumed that  $s'_{\min} < s' < s'_{\max}$  represents an integral interval of the high invariance mass of  $\pi^+ \pi^-$  while  $\int_{s'_{\min}}^{s'_{\max}} ds' (\xi - s')^2$  represents the factor that is dependent upon  $s$  [20].

### 4 Calculation

In this paper, we take the perturbative QCD (PQCD) method to work out, which is obtained by applying the  $k_T$  factorization formalism to the decay process of two-body. Hadrons have a transverse momentum  $k_T$ , which we must take into account to calculate its transverse momentum. Besides, it introduces an additional energy scale that produces double logarithms in the QCD corrections. As a result, the renormalization group method is a great way to get factors of Sudakov once the terms in this matrix have been resummed. Because of this, the distribution amplitude of mesons in the tiny transverse momentum zone is effectively suppressed in this form factor, which increases the accuracy of the PQCD approach even in this region. The PQCD method has been widely used in the reach of pure annihilation decays. As a result, we believe this approach is appropriate for dealing

with processes that are not factorizable and determining the contribution to the annihilation diagram [41].

In this work, we use the channel of  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0(K^0) \rightarrow \pi^+ \pi^- \pi^0(K^0)$  as an illustration of how the process conducts under the perturbative QCD. The formalism of  $t_V$  and  $p_V$  ( $V = \rho, \omega, \phi$ ), which are obtained from the tree (penguin) level contribution to the equation must be obtained to calculate the CP asymmetry.

Using CKM matrix elements of  $V_{ub}V_{us}^*$  ( $V_{ub}V_{ud}^*$ ) and  $V_{tb}V_{ts}^*$  ( $V_{tb}V_{td}^*$ ) as a basis for analysis, the amplitude in  $\bar{B}_s^0 \rightarrow \rho^0 \pi^0(K^0) \rightarrow \pi^+ \pi^- \pi^0(K^0)$  process can be written as

$$\begin{aligned} A(\bar{B}_s^0 \rightarrow \rho^0(\rho^0 \rightarrow \pi^+ \pi^-)\pi^0) &= \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_\rho \epsilon(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{\sqrt{2} s_{\rho^0}} \\ &\times \left\{ \frac{1}{2} \left[ A(\bar{B}_s^0 \rightarrow \pi^+ \rho^-) + A(\bar{B}_s^0 \rightarrow \rho^+ \pi^-) \right] \right\}. \end{aligned} \quad (16)$$

where

$$\begin{aligned} A(\bar{B}_s^0 \rightarrow \pi^+ \rho^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \left\{ \left[ f_{B_s} F_{ann}^{LL}(a_2) + M_{ann}^{LL}(c_2) \right] \right. \\ &\quad - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ f_{B_s} F_{ann}^{LL}(a_3 + a_9) \right. \\ &\quad \left. - f_{B_s} F_{ann}^{LR}(a_5 + a_7) + M_{ann}^{LL}(c_4 + c_{10}) \right. \\ &\quad \left. \left. - M_{ann}^{SP}(c_6 + c_8) + (\pi^+ \leftrightarrow \rho^-) \right\} \right\}, \\ A(\bar{B}_s^0 \rightarrow \rho^+ \pi^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \left\{ \left[ f_{B_s} F_{ann}^{LL}(a_2) + M_{ann}^{LL}(c_2) \right] \right\} \\ &\quad - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ f_{B_s} F_{ann}^{LL}(a_3 + a_9) \right. \\ &\quad \left. - f_{B_s} F_{ann}^{LR}(a_5 + a_7) + M_{ann}^{LL}(c_4 + c_{10}) \right. \\ &\quad \left. - M_{ann}^{SP}(c_6 + c_8) + (\rho^+ \leftrightarrow \pi^-) \right\}. \end{aligned}$$

where the amplitude of  $\bar{B}_s^0 \rightarrow \rho^0 \pi^0$  is composed of two parts  $\bar{B}_s^0 \rightarrow \pi^+ \rho^-$  and  $\bar{B}_s^0 \rightarrow \rho^+ \pi^-$ . And they are related to each other as  $2A(\bar{B}_s^0 \rightarrow \rho^0 \pi^0) = A(\bar{B}_s^0 \rightarrow \pi^+ \rho^-) + A(\bar{B}_s^0 \rightarrow \rho^+ \pi^-)$ .

$$\begin{aligned} A(\bar{B}_s^0 \rightarrow \rho^0(\rho^0 \rightarrow \pi^+ \pi^-)K^0) &= \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_\rho \epsilon(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{2s_{\rho^0}} \\ &\times \left\{ V_{ub} V_{ud}^* \left[ f_\rho F_{B_s \rightarrow K}^{LL}(a_2) + M_{B_s \rightarrow K}^{LL}(c_2) \right] \right. \\ &\quad \left. - V_{tb} V_{td}^* \left[ f_\rho F_{B_s \rightarrow K}^{LL} \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \times \left( -a_4 + \frac{3}{2}a_7 + \frac{1}{2}a_{10} + \frac{3}{2}a_9 \right) \\
 & + M_{B_s \rightarrow K}^{LR} \left( -c_5 + \frac{1}{2}c_7 \right) \\
 & + M_{B_s \rightarrow K}^{LL} \left( -c_3 + \frac{1}{2}c_9 + \frac{3}{2}c_{10} \right) - M_{B_s \rightarrow K}^{SP} \left( \frac{3}{2}c_8 \right) \\
 & + f_{B_s} F_{ann}^{LL} \left( -a_4 + \frac{1}{2}a_{10} \right) - f_{B_s} F_{ann}^{SP} \left( -a_6 + \frac{1}{2}a_8 \right) \\
 & + M_{ann}^{LL} \left( -c_3 + \frac{1}{2}c_9 \right) + M_{ann}^{LR} \left( -c_5 + \frac{1}{2}c_7 \right) \Big] \Big\}. \tag{17}
 \end{aligned}$$

It is possible to write the decay amplitudes of  $\bar{B}_s^0 \rightarrow \omega \pi^0(K^0) \rightarrow \pi^+ \pi^- \pi^0(K^0)$  as follows:

$$\begin{aligned}
 & A \left( \bar{B}_s^0 \rightarrow \omega (\omega \rightarrow \pi^+ \pi^-) \pi^0 \right) \\
 & = \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_{\omega\epsilon}(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{2\sqrt{2}s_\omega} \\
 & \times \left\{ V_{ub} V_{us}^* M_{ann}^{LL}(c_2) - V_{tb} V_{ts}^* \left[ M_{ann}^{LL} \left( \frac{3}{2}c_{10} \right) \right. \right. \\
 & \left. \left. - M_{ann}^{SP} \left( \frac{3}{2}c_8 \right) + (\pi^0 \leftrightarrow \omega) \right] \right\}. \tag{18}
 \end{aligned}$$

$$\begin{aligned}
 & A \left( \bar{B}_s^0 \rightarrow \omega (\omega \rightarrow \pi^+ \pi^-) K^0 \right) \\
 & = \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_{\omega\epsilon}(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{2s_\omega} \\
 & \times \left\{ V_{ub} V_{ud}^* \left[ f_\omega F_{B_s \rightarrow K^0}^{LL}(a_2) + M_{B_s \rightarrow K}^{LL}(c_2) \right] \right. \\
 & \left. - V_{tb} V_{td}^* \left[ f_\omega F_{B_s \rightarrow K^0}^{LL} \right. \right. \\
 & \times \left( 2a_3 + a_4 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \\
 & + M_{B_s \rightarrow K^0}^{LR} \left( c_5 - \frac{1}{2}c_7 \right) \\
 & + M_{B_s \rightarrow K^0}^{LL} \left( c_3 + 2c_4 - \frac{1}{2}c_9 + \frac{1}{2}c_{10} \right) \\
 & \left. - M_{B_s \rightarrow K^0}^{SP} \left( 2c_6 + \frac{1}{2}c_8 \right) \right. \\
 & \left. + f_{B_s} F_{ann}^{LL} \left( a_4 - \frac{1}{2}a_{10} \right) + f_{B_s} F_{ann}^{SP} \left( a_6 - \frac{1}{2}a_8 \right) \right. \\
 & \left. + M_{ann}^{LL} \left( c_3 - \frac{1}{2}c_9 \right) + M_{ann}^{LR} \left( c_5 - \frac{1}{2}c_7 \right) \right] \Big\}. \tag{19}
 \end{aligned}$$

In order to calculate the amplitude of  $\bar{B}_s^0 \rightarrow \phi \pi^0(K^0) \rightarrow \pi^+ \pi^- \pi^0(K^0)$ , we use the following formula:

$$A \left( \bar{B}_s^0 \rightarrow \phi (\phi \rightarrow \pi^+ \pi^-) \pi^0 \right)$$

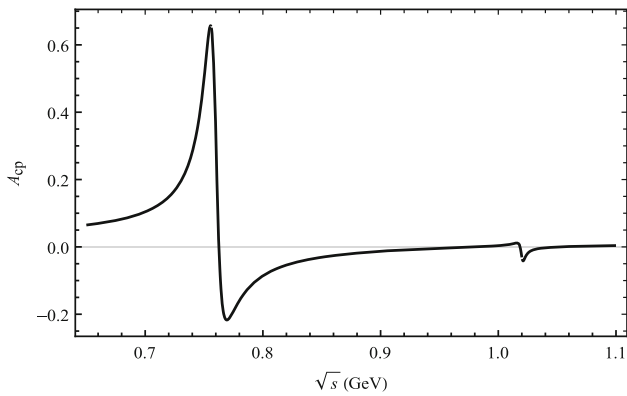
$$\begin{aligned}
 & = \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_{\phi\epsilon}(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{2s_\phi} \\
 & \times \left\{ V_{ub} V_{us}^* \left[ f_\pi F_{B_s \rightarrow \phi}^{LL}(a_2) + M_{B_s \rightarrow \phi}^{LL}(c_2) \right] \right. \\
 & \left. - V_{tb} V_{ts}^* \left[ f_\pi F_{B_s \rightarrow \phi}^{LL} \left( \frac{3}{2}a_9 - \frac{3}{2}a_7 \right) \right. \right. \\
 & \left. \left. + M_{B_s \rightarrow \phi}^{LL} \left( \frac{3}{2}c_8 + \frac{3}{2}c_8 \right) \right] \right\}. \tag{20}
 \end{aligned}$$

$$\begin{aligned}
 & A \left( \bar{B}_s^0 \rightarrow \phi (\phi \rightarrow \pi^+ \pi^-) K^0 \right) \\
 & = - \sum_{\lambda=0,\pm 1} \frac{G_F P_{\bar{B}_s^0} \cdot \epsilon^*(\lambda) g_{\phi\epsilon}(\lambda) \cdot (p_{\pi^+} - p_{\pi^-})}{\sqrt{2}s_\phi} \\
 & \times \left\{ V_{tb} V_{td}^* \left[ f_\phi F_{B_s \rightarrow K}^{LL} \left( a_3 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9 \right) \right. \right. \\
 & \left. \left. + f_K F_{B_s \rightarrow \phi}^{LL} \left( a_4 - \frac{1}{2}a_{10} \right) - f_K F_{B_s \rightarrow \phi}^{SP} \left( a_6 - \frac{1}{2}a_8 \right) \right. \right. \\
 & \left. \left. + M_{B_s \rightarrow K}^{LL} \left( c_4 - \frac{1}{2}c_{10} \right) + M_{B_s \rightarrow \phi}^{LL} \left( c_3 - \frac{1}{2}c_9 \right) \right. \right. \\
 & \left. \left. - M_{B_s \rightarrow K}^{SP} \left( c_6 - \frac{1}{2}c_8 \right) - M_{B_s \rightarrow \phi}^{LR} \left( c_5 - \frac{1}{2}c_7 \right) \right. \right. \\
 & \left. \left. + f_{B_s} F_{ann}^{LL} \left( a_4 - \frac{1}{2}a_{10} \right) - f_{B_s} F_{ann}^{SP} \left( a_6 - \frac{1}{2}a_8 \right) \right. \right. \\
 & \left. \left. + M_{ann}^{LL} \left( c_3 - \frac{1}{2}c_9 \right) - M_{ann}^{LR} \left( c_5 - \frac{1}{2}c_7 \right) \right] \right\}. \tag{21}
 \end{aligned}$$

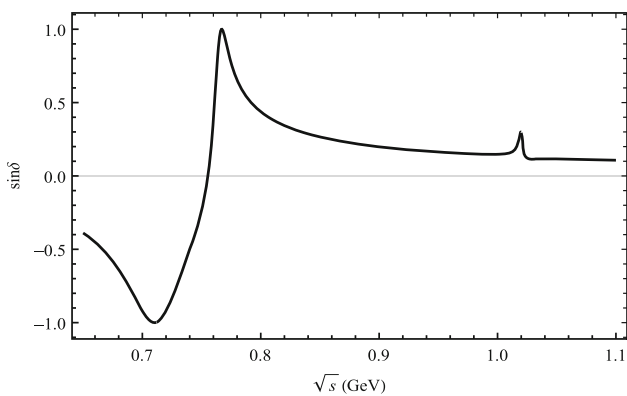
$g_V$  is the effective coupling constant of vector meson ( $V=\rho, \omega$  and  $\phi$ ).  $G_F = 1.16639 \times 10^{-5} GeV^{-2}$  is the Fermi constant, and  $a_i$  are related to the coefficients of Wilson  $c_i$ . Besides  $F_{B_s \rightarrow M_3}$  and  $M_{B_s \rightarrow M_3}$  represent the contributions of factorable emission diagrams and annihilation-type diagrams, respectively. LL, LR and SP refer to the contributions from  $(V - A) \otimes (V - A)$ ,  $(V - A) \otimes (V + A)$  and  $(S - P) \otimes (S + P)$  operators, respectively. There is a non-factorable emission diagram and an annihilation-type diagram in  $F_{ann}$  and  $M_{ann}$ , which indicate their respective contributions. The formalisms of the above expressions can be found in the literatures about perturbative QCD [22,42].

### 5 Numerical results

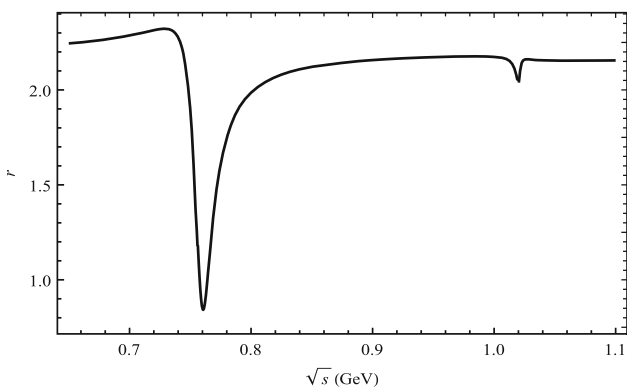
We investigate the CP asymmetry about  $\bar{B}_s^0 \rightarrow \pi^+ \pi^- \pi^0$  and  $\bar{B}_s^0 \rightarrow \pi^+ \pi^- K^0$  decay processes. According to Eqs. (9) and (12), we find the CP asymmetry is associated with the weak phase difference, strong phase difference and  $r$ . Our results are affected slightly by the variation of CKM matrix elements, which determine the weak phase. Thus, the results are presented based on the central parameter values of the CKM matrix elements. For the  $\bar{B}_s^0 \rightarrow \pi^+ \pi^- \pi^0$  decay process, Figs. 2, 3 and 4 show the results. As seen in Fig. 2, it is easy to see the CP asymmetry of  $\bar{B}_s^0 \rightarrow \pi^+ \pi^- \pi^0$  channel



**Fig. 2** Plot of  $A_{CP}$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$



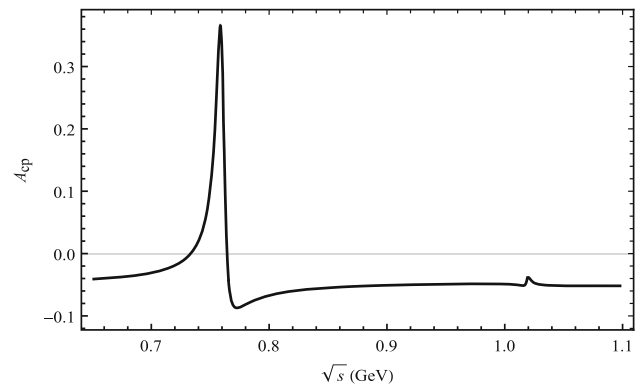
**Fig. 3** Plot of  $\sin\delta$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$



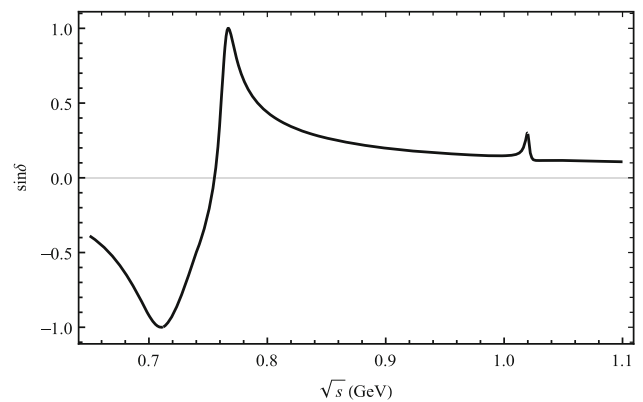
**Fig. 4** Plot of  $r$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$

changes when the invariant masses of the  $\pi^+\pi^-$  pair is surrounding the  $\omega$  resonance and the  $\phi$  resonance range, where the maximum CP asymmetry can reach 65%.

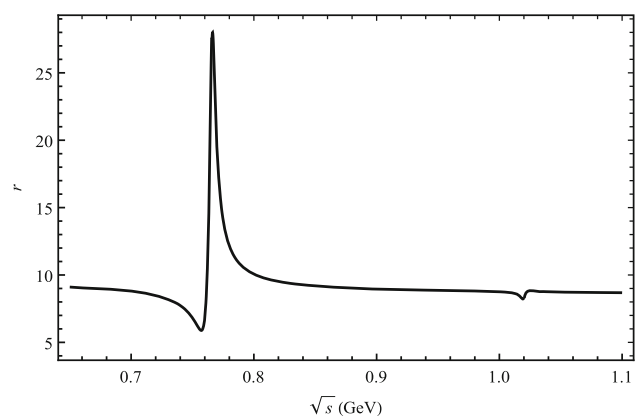
The values of  $\sin\delta$  and  $r$  as a function of  $\sqrt{s}$  are shown on the plots of in Figs. 3 and 4. If the mass invariant of the  $\pi^+\pi^-$



**Fig. 5** Plot of  $A_{CP}$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$



**Fig. 6** Plot of  $\sin\delta$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$



**Fig. 7** Plot of  $r$  as a function of  $\sqrt{s}$  corresponding to central parameter values of CKM matrix elements for the decay channel of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$

pair is in the area where the  $\omega$  resonance is located, then one can find that the  $\sin\delta$  and  $r$  vary sharply and  $\sin\delta$  and  $r$  vary slightly around the  $\phi$  resonance range which compared to the former. With regard to  $\bar{B}_s^0 \rightarrow \rho^0(\omega, \phi)\pi^0 \rightarrow \pi^+\pi^-\pi^0$  process, we obtain the CP asymmetries vary from 65% to

−23% (0% to −0.05%) when invariant mass of the  $\pi^+\pi^-$  pair is in  $\rho - \omega$  ( $\rho - \phi$ ) resonance range.

Figure 5 illustrates the sharp change in CP asymmetry for  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$  channel when the invariant masses of the  $\pi^+\pi^-$  pairs are around the  $\omega$  and  $\phi$  resonance range, where the maximum CP asymmetry can be achieved 36%. Then, we consider these two situations as follows. When  $\pi^+\pi^-$  pairs are around the region of  $\rho - \omega$  mixing, we find the CP asymmetry of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$  can reach 36%. While the CP asymmetry is just 2% when  $\pi^+\pi^-$  pairs is in the mixing of  $\rho - \phi$ . Similarly, we analyse the change map about  $\sin\delta$  and  $r$  at the resonance regions in Figs. 6 and 7. Apparent resonance can be found around region of  $\rho - \omega$  mixing comparing with slight effect from the  $\rho - \phi$  interference.

Concerning the three-body decay process, there are complex phase space problems from final states associated with the mechanism of dynamics. The subsequent  $V \rightarrow \pi^+\pi^-$  is described as the combination of the coupling constant and the momenta of double  $\pi$ . Integration of the phase space provides observation of CP asymmetry for detecting the structure of intermediate hadrons. The signals of CP asymmetry are fixed to low mass candidate regions of vectors. We scan the entire resonance regions to detect the large local CP asymmetry. Integrating CP asymmetry over the invariant mass of double  $\pi$ , the localized CP asymmetry values from −0.006 to −0.01 (−0.014 to −0.02) in  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$ . Related to the interferences of the vectors, the desired large CP asymmetry is not emerge, and cancel each other from the positive and negative values. The results are in the same order of magnitude as one percent. However, a clear evident of large CP asymmetry is found at invariant mass value  $m(\pi^+\pi^-)$  below the mass of  $\rho(770)^0$  with the decay  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$ . In the regions between 0.65 GeV and 0.78 GeV, CP asymmetry is consistent to the value of 0.13. From the 0.78 GeV to 1.1 GeV region, CP asymmetry changes the sign and reach −0.02.

In the above process, we take the intermediate value of CKM when we obtain these diagrams. Although it is a value with a range, we consider the middle value after we look at the range of values it takes and find it has little effect on the result. But there are still uncertainties we must consider when we calculate the amplitude in this work under the perturbative QCD method. The first error refer to the variation of the Wolfenstein parameters, which can be expressed in terms of the Wolfenstein parameters meet the following relationships  $\lambda = 0.22650 \pm 0.00048$ ,  $A = 0.790_{-0.012}^{+0.017}$ ,  $\bar{\rho} = 0.141_{-0.017}^{+0.016}$  and  $\bar{\eta} = 0.357 \pm 0.011$  [37, 38]. The second error comes from hadronic parameters: decay constant, form factor, and the wave function of  $B_s$  meson. In this work, we take  $f\rho = 215.6 \pm 5.9 MeV$ ,  $f_\rho^T = 165 \pm 9 MeV$ ,  $f\omega = 196.5 \pm 4.8 MeV$ ,  $f_\omega^T = 145 \pm 10 MeV$ ,  $f\phi = 231 \pm 4 MeV$  and  $f_\phi^T = 200 \pm 10 MeV$  these decay constants [43–45].  $F_{B_s \rightarrow \pi} = 0.23_{-0.04}^{+0.05}$  and  $F_{B_s \rightarrow K} = 0.24_{-0.04}^{+0.05}$  are taken as

form factors for the decays process of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$  respectively. Besides, we take  $\omega_b = 0.5 \pm 0.05 GeV$  and  $f_{B_s} = 0.23 \pm 0.03 GeV$  which can influence the wave function of  $B_s$  meson [22]. The third error corresponds to the choice of the hard scales, which vary from 0.75t to 1.25t, characterizing the size of the next-to-leading order QCD contributions. For these parameters of hadronic, they all have a range of values. Then we take the maximum, the minimum, and the intermediate values respectively. We calculate the  $A_{CP} = 65.6_{-0.17-0.33-0.26}^{+1.01+0.25+0.68} \%$  for  $\bar{B}_s^0 \rightarrow \rho^0\pi^0$  decay process and  $A_{CP} = 36.5_{-0.11-0.16-0.25}^{+3.23+0.15+0.05} \%$  for  $\bar{B}_s^0 \rightarrow \rho^0K^0$  decay process. In this way, the numerical results are more intuitive.

## 6 Summary and conclusion

The results of this study illustrate the  $\rho - \omega - \phi$  interference caused by the breaking of isospin. Resonance contributions of  $\rho - \omega$ ,  $\rho - \phi$  and  $\omega - \phi$  can be used to generate a new strong phase. In the processes of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$  and  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$ , a large CP asymmetry is found to occur in the resonance range. It is possible to reach a maximum CP asymmetry about 65% in  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$  process. A maximum CP asymmetry of 36% can occur in  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$  channel.

After integration from 0.65 GeV to 1.1 GeV for the decay processes of  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0$  and  $\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0$ , we obtain the local CP asymmetry as follows:

$$A_{CP}^{\Omega}(\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0) = -0.008 \pm 0.002, \quad (22)$$

$$A_{CP}^{\Omega}(\bar{B}_s^0 \rightarrow \pi^+\pi^-K^0) = -0.017 \pm 0.003. \quad (23)$$

The three-body decay process of bottom and charm mesons is formulated appropriately by the chain decay of quasi-two-body. We use  $B \rightarrow RP_3$  decay process as the case in analyses of quasi-two-body decay. During the progress, R is the state of intermediate resonance state which can further decay to two hadrons  $P_{1,2}$ , and  $P_3$  refers to another final hadron. This process can be factorized by using the narrow width approximation (NWA), which is also known as the factorization relation. As a result,  $B \rightarrow RP_3$  can be written as follows:  $\mathcal{B}(B \rightarrow RP_3 \rightarrow P_1P_2P_3) = \mathcal{B}(B \rightarrow RP_3)\mathcal{B}(B \rightarrow P_1P_2)$  due to the branching ratio. In quasi-two-body decay processes with small widths  $\omega$  and  $\phi$ , the effects can be safely ignored. In light of the large decay rate of  $\rho(770)$ , it makes sense to carry out a correction. According to the QCD factorization approach, the correction factor for the quasi-two-body decays process of  $B^- \rightarrow \rho(770)\pi^- \rightarrow \pi^+\pi^-\pi^-$  is at level 7%. As a measure of the degree of approximation of  $\Gamma(B \rightarrow RP_3)\mathcal{B}(B \rightarrow P_1P_2) = \eta_R\Gamma(B \rightarrow RP_3 \rightarrow P_1P_2P_3)$ , the parameter  $\eta_R$  is introduced [46, 47]. The for-



malisms of decay amplitudes contain the Breit-Wigner shapes which depend on the parameters of invariant mass  $m_{\pi^+\pi^-}$  associated with the Dalitz plot. We integrate over invariant mass  $m_{\pi^+\pi^-}$  in order to obtain localized value of CP asymmetry by quasi-two-body approximation. In the present manuscript, we neglect the effect of this correction within our range of accuracy. According to the QCD factorization approach, the correction is around at level of 7%. This is one of the sources of error in our results.

CP asymmetry measurements in the decay of B mesons have become more accurate thanks to the large number of data collected by the LHC in recent years. Theoretical developments using different methods have already led to many predictions of CP asymmetry. The LHCb experiments focus on the contribution to B Physics and the search of new physics. During the last few years, the LHC has made several upgrades and increased its luminosity by a factor of five.

Based on the amplitude analysis of  $B^+ \rightarrow \pi^+\pi^-\pi^-$  decay, LHCb Collaboration reports different sources of CP asymmetry. The hadronic structure of intermediate states is sensitive to observation of CP asymmetry. Contradicting the predictions of the theory, large CP asymmetry connected with the  $\rho$  and  $\omega$  interference is found, and cancels when intergration with the phase spaces. However, there is a evident signal about CP asymmetry at invariant mass value  $m(\pi^+\pi^-)$  below the mass of  $\rho(770)^0$  from the decay  $B^+ \rightarrow \pi^+\pi^-\pi^-$  [40,48,49]. The search for direct CP asymmetry in charmless  $B_s$  decay may be measured in the near future. The interferences  $\rho - \omega$ ,  $\rho - \phi$  and  $\omega - \phi$  resonances relate to the quasi-two-body decay  $\bar{B}_s^0 \rightarrow \pi^+\pi^-\pi^0(K^0)$ . Analysis of CP asymmetry indicates that the resonance of  $\rho(770)^0$  presents the dominant contribution. In fact, the results does not avoid the interference of  $\rho(770)^0$  and  $\omega(782)$  in experiments [23]. Hence, the measurement of CP asymmetry includes the effect of  $\omega(782)$  meson when one concerns the contribution of  $\rho(770)^0$  meson. Although, the mass of  $\phi(1020)$  is away from the mass of  $\rho(770)^0$  and  $\omega(782)$  hadrons. With regard to the SU(3) symmetry of flavour and Isospin symmetry, we can classify the  $\rho(770)^0$ ,  $\omega(782)$  and  $\phi(1020)$  hadrons associated with the properties in low mass region of vector interference. The results manifest that main contribution is from the resonances of  $\rho(770)^0$  and  $\omega(782)$  in comparison to the interferences of  $\rho(770)^0 - \phi(1020)$  and  $\omega(782) - \phi(1020)$  as we expected.

The CP asymmetry can be presented in the regions of  $\rho - \omega$  and  $\rho - \phi$  regions by reconstructing the  $\rho$ ,  $\omega$  and  $\phi$  mesons when the invariant masses of  $\pi^+\pi^-$  are at the resonant regions. Hopefully, our predictions will guide future experiments in the right direction.

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