



Searching for doubly charmed tetraquark candidates T_{cc} and $T_{cc\bar{s}}$ in B_c decays

Yuan Li, Ying-Bo He, Xiao-Hai Liu^a, Baoyi Chen^b, Hong-Wei Ke^c

Center for Joint Quantum Studies and Department of Physics, School of Science, Tianjin University, Tianjin 300350, China

Received: 7 February 2023 / Accepted: 14 March 2023 / Published online: 28 March 2023
© The Author(s) 2023

Abstract In this work, we propose to search for the exotic doubly charmed meson T_{cc}^+ and its analog $T_{cc\bar{s}}^+$ in B_c^+ decays, which provide a good environment for the formation of the exotic state containing double charm quarks. Within the molecular scheme, the production of T_{cc}^+ and $T_{cc\bar{s}}^+$ through various rescattering processes with different intermediate states are investigated. For the moderate values of model parameters, the branching ratios of B_c^+ decaying into $T_{cc}^+\bar{D}^0$, $T_{cc}^+\bar{D}^{*0}$, $T_{cc\bar{s}}^+\bar{D}^0$ and $T_{cc\bar{s}}^+\bar{D}^{*0}$ are estimated to be of the order of 10^{-7} , 10^{-5} , 10^{-6} and 10^{-4} , respectively, which may be tested by future experiments.

1 Introduction

The LHCb collaboration recently reported the observation of a narrow doubly charmed tetraquark candidate T_{cc}^+ in the prompt production of proton-proton collisions [1, 2]. This is the first observation of the exotic state containing double charm quarks. The T_{cc}^+ is observed in the $D^0D^0\pi^+$ mass spectrum, and has mass of approximately 3875 MeV, which is just below the $D^{*+}D^0$ threshold. Using the Breit–Wigner parametrization, the location of the resonance peak relative to the $D^{*+}D^0$ threshold δ_m and the width Γ are determined to be

$$\begin{aligned}\delta_m &\equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) \\ &= -273 \pm 61 \pm 5_{-14}^{+11} \text{ keV}, \\ \Gamma &= 410 \pm 165 \pm 43_{-38}^{+18} \text{ keV},\end{aligned}\quad (1)$$

respectively.

The long-lived T_{cc}^+ particle has the quark content $cc\bar{u}\bar{d}$ and the spin-parity quantum number $J^P = 1^+$. The flavor

quantum number is absolutely exotic. In fact, the tetraquark state with double heavy quarks has been studied for many years. There are two popular theoretical pictures describing the underlying structures of such tetraquark states, i.e., the compact tetraquark picture and hadronic molecule picture. In the compact tetraquark picture, the doubly heavy state is usually thought to be composed of the compact diquark and anti-diquark [3–27], while in the hadronic molecule picture, it is composed of a pair of heavy mesons [28–40]. In Refs. [41–45], the compact tetraquark and hadronic molecule pictures are taken into account simultaneously. According to the LHCb measurements, the rather closeness of the T_{cc}^+ mass to the $D^{*+}D^0$ threshold strongly favors the molecular explanation concerning the T_{cc}^+ nature [46–61]. This is similar to the case of the famous $X(3872)$ state, which is widely supposed to be a weakly bound hadronic molecule composed of $D^*\bar{D}/D\bar{D}^*$. We refer to Refs. [62–73] for a review concerning the recent progress of the doubly heavy tetraquarks and many other exotic hadrons discovered in the last two decades.

The production mechanism of hadrons is closely related to their intrinsic structures. In theoretical study, the production of T_{cc}^+ in the γp scattering, pp collisions and heavy ion collisions have been discussed recently [74–77]. In 2008, Liu and Zhao have ever proposed an experimental scheme for searching for the doubly heavy mesons in the missing mass spectrum of two Λ_c final states in nucleon-nucleon collisions [78]. However, up to now the T_{cc}^+ is only observed in the prompt production of pp collisions. Searching for the T_{cc}^+ in more reactions is important for both confirming its existence and understanding its nature. In this work, we propose to search for the T_{cc}^+ and its analog $T_{cc\bar{s}}^+$ in the B_c decays.

This paper is organized as follows. After introduction, in Sect. 2 we present the formulae of the rescattering amplitudes corresponding to the $B_c^+ \rightarrow T_{cc}^+\bar{D}^0$ and $B_c^+ \rightarrow T_{cc}^+\bar{D}^{*0}$ processes. The pertinent numerical results and discussions are also given in Sect. 2. In Sect. 3, the similar processes

^a e-mail: xiaohai.liu@tju.edu.cn (corresponding author)

^b e-mail: baoyi.chen@tju.edu.cn

^c e-mail: khw020056@tju.edu.cn

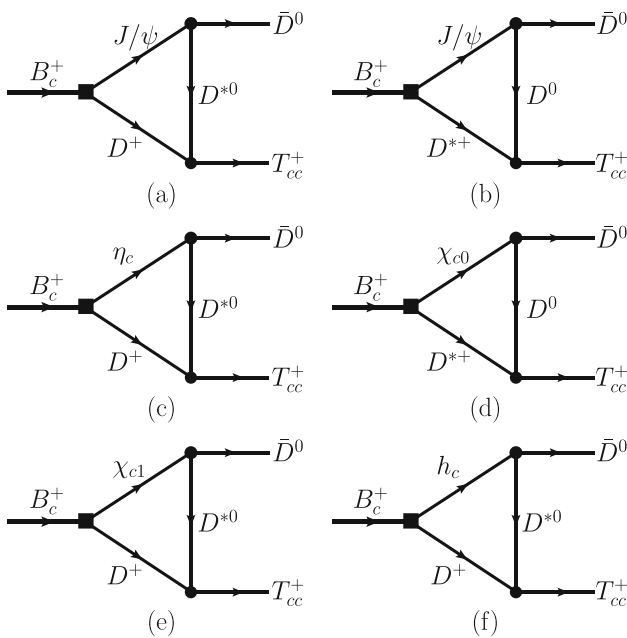


Fig. 1 Rescattering diagrams contributing to $B_c^+ \rightarrow T_{cc}^+ \bar{D}^0$. The boxes and dots represent the weak and strong vertices, respectively

$B_c^+ \rightarrow T_{ccs}^+ \bar{D}^0$ and $B_c^+ \rightarrow T_{ccs}^+ \bar{D}^{*0}$ are discussed. A summary is given in Sect. 4.

2 Production of T_{cc}^+ in B_c^+ decays

If the T_{cc}^+ is a hadronic molecule, we can expect that it may be produced from the rescattering processes illustrated in Figs. 1 and 2, i.e., the B_c^+ meson firstly decays into a charmonium and a charmed meson, the two particles then rescatter into the $\bar{D}^{(*)0}$ and the T_{cc}^+ via exchanging a charmed meson. In these rescattering diagrams, two charm quarks and one anti-charm quark are produced in the B_c^+ weak decay vertex, which creates a good environment for the formation of the double charm meson T_{cc}^+ . Actually in 2013, before the observation of the molecular candidate T_{cc}^+ , the production of the doubly charmed compact tetraquarks from the B_c or Ξ_{bc} decays has ever been suggested [79].

The B_c decays also provide a good environment for the formation of the charmonium or charmonium-like states. In Refs. [80–82], the production of tetraquark candidates $Z_c(3900)$, $Z_c(4020)$ and $X(3872)$ in B_c decays has ever been discussed. The contributions from rescattering processes in weak decays are also widely discussed in the literature [83–94].

2.1 Rescattering amplitudes

At the quark level, the weak decay process $B_c^+ \rightarrow M_{c\bar{c}} D^{(*)+}$ is induced by the $\bar{b} \rightarrow \bar{c} \bar{d}$ transition, where $M_{c\bar{c}}$ represents

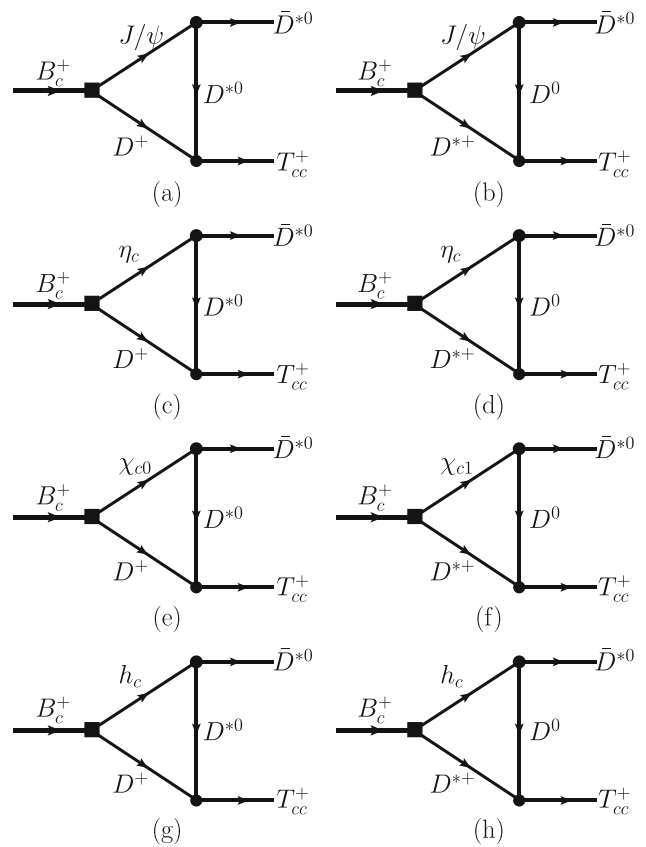


Fig. 2 Rescattering diagrams contributing to $B_c^+ \rightarrow T_{cc}^+ \bar{D}^{*0}$. The boxes and dots represent the weak and strong vertices, respectively

a charmonium state. The charm quark in B_c^+ is treated as a spectator. In the naive factorization approach the effective Hamiltonian governing the process reads

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cd} [c_1 (\bar{b}c)_{V-A} (\bar{c}d)_{V-A} + c_2 (\bar{c}c)_{V-A} (\bar{b}d)_{V-A}], \tag{2}$$

where $(\bar{q}_1 q_2)_{V-A} \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, G_F is the Fermi constant, V_{cb} and V_{cd} are the CKM matrix elements, and c_1 and c_2 are the perturbatively calculable Wilson coefficients. Neglecting the contributions from the nonfactorizable, color-suppressed and annihilation terms, the decay amplitude of $B_c^+ \rightarrow M_{c\bar{c}} D^{(*)+}$ can be factorized as

$$A(B_c^+ \rightarrow M_{c\bar{c}} D^{(*)+}) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cd} a_1 \langle M_{c\bar{c}} | \bar{b} \gamma_\mu (1 - \gamma_5) c | B_c^+ \rangle \times \langle D^{(*)+} | \bar{c} \gamma^\mu (1 - \gamma_5) d | 0 \rangle, \tag{3}$$

where $a_1 = c_1 + c_2/N_c$, and N_c is the number of colors. The factorized amplitude can be expressed in terms of the form factors of the transition $B_c^+ \rightarrow M_{c\bar{c}}$ and the decay constant of $D^{(*)+}$. The form factors of B_c decaying into the lower charmonia have been well investigated in the literature [95–

[120]. To estimate the rescattering amplitudes, in this work we employ the relevant numerical results of Refs. [95, 96], where the form factors are calculated by means of the covariant light-front approach. The form factors of $B_c \rightarrow J/\psi$ and η_c induced by the vector and axial-vector currents are defined by

$$\langle \eta_c(k_1) | V_\mu | B_c(p_0) \rangle = \left(P_\mu - \frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q_\mu \right) \times F_1^{B_c \eta_c}(q^2) + \frac{m_{B_c}^2 - m_{\eta_c}^2}{q^2} q_\mu F_0^{B_c \eta_c}(q^2), \tag{4}$$

$$\langle J/\psi(k_1, \varepsilon_J^*) | V_\mu | B_c(p_0) \rangle = -\frac{1}{m_{B_c} + m_{J/\psi}} \times \epsilon_{\mu\nu\alpha\beta} \varepsilon_J^{*\nu} P^\alpha q^\beta V^{B_c J/\psi}(q^2), \tag{5}$$

$$\langle J/\psi(k_1, \varepsilon_J^*) | A_\mu | B_c(p_0) \rangle = i \left\{ (m_{B_c} + m_{J/\psi}) \varepsilon_J^{*\mu} \times A_1^{B_c J/\psi}(q^2) - \frac{P \cdot \varepsilon_J^*}{m_{B_c} + m_{J/\psi}} P_\mu A_2^{B_c J/\psi}(q^2) - 2m_{J/\psi} \frac{P \cdot \varepsilon_J^*}{q^2} q_\mu [A_3^{B_c J/\psi}(q^2) - A_0^{B_c J/\psi}(q^2)] \right\}, \tag{6}$$

where $P = p_0 + k_1$, $q = p_0 - k_1$, and ε_J is the polarization vector of J/ψ . The B_c decaying to the P -wave charmonia χ_{c0} , χ_{c1} and h_c form factors are defined by

$$\langle \chi_{c0}(k_1) | A_\mu | B_c(p_0) \rangle = \left(P_\mu - \frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2} q_\mu \right) \times F_1^{B_c \chi_{c0}}(q^2) + \frac{m_{B_c}^2 - m_{\chi_{c0}}^2}{q^2} q_\mu F_0^{B_c \chi_{c0}}(q^2), \tag{7}$$

$$\langle A(k_1, \varepsilon_A^*) | A_\mu | B_c(p_0) \rangle = -\frac{1}{m_{B_c} - m_A} \times \epsilon_{\mu\nu\alpha\beta} \varepsilon_A^{*\nu} P^\alpha q^\beta A^{B_c A}(q^2), \tag{8}$$

$$\langle A(k_1, \varepsilon_A^*) | V_\mu | B_c(p_0) \rangle = -i \left\{ (m_{B_c} - m_A) \varepsilon_A^{*\mu} \times V_1^{B_c A}(q^2) - \frac{P \cdot \varepsilon_A^*}{m_{B_c} - m_A} P_\mu V_2^{B_c A}(q^2) - 2m_A \frac{P \cdot \varepsilon_A^*}{q^2} q_\mu [V_3^{B_c A}(q^2) - V_0^{B_c A}(q^2)] \right\}, \tag{9}$$

where the state A represents the axial vector meson χ_{c1} or h_c .

The second matrix element in the right-hand side of Eq. (3) is parameterized as

$$\langle D^+(q) | \bar{c} \gamma^\mu (1 - \gamma_5) d | 0 \rangle = i f_D q^\mu, \tag{10}$$

$$\langle D^{*+}(q, \varepsilon_{D^*}^*) | \bar{c} \gamma^\mu (1 - \gamma_5) d | 0 \rangle = f_{D^*} m_{D^*} \varepsilon_{D^*}^{*\mu}, \tag{11}$$

with f_D and f_{D^*} the decay constants of D and D^* , respectively.

One of the strong vertices in the rescattering diagrams of Figs. 1 and 2 involves a charmonium and a pair of open charm

mesons. Taking into account the heavy quark spin symmetry, the effective Lagrangian describing these couplings is given by [84, 121, 122]

$$\mathcal{L}_\psi = i g_\psi \langle \mathcal{J} \bar{H}_{2a} \overleftrightarrow{\not{D}} \bar{H}_{1a} \rangle + \text{H.c.}, \tag{12}$$

$$\mathcal{L}_\chi = i g_\chi \langle \chi^\mu \bar{H}_{2a} \gamma_\mu \bar{H}_{1a} \rangle + \text{H.c.}, \tag{13}$$

with

$$\mathcal{J} = \frac{1 + \not{\psi}}{2} [\psi^\mu \gamma_\mu - \eta_c \gamma_5] \frac{1 - \not{\psi}}{2}, \tag{14}$$

$$\chi^\mu = \frac{1 + \not{\psi}}{2} \left[h_c^\mu \gamma_5 + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) \chi_{c0} + \frac{1}{\sqrt{2}} \epsilon^{\mu\alpha\beta\gamma} v_\alpha \gamma_\beta \chi_{c1\gamma} + \chi_{c2}^{\mu\alpha} \gamma_\alpha \right] \frac{1 - \not{\psi}}{2}, \tag{15}$$

$$H_{1a} = \frac{1 + \not{\psi}}{2} [\mathcal{D}_{a\mu}^* \gamma^\mu - \mathcal{D}_a \gamma_5], \tag{16}$$

$$H_{2a} = [\bar{\mathcal{D}}_{a\mu}^* \gamma^\mu + \bar{\mathcal{D}}_a \gamma_5] \frac{1 - \not{\psi}}{2}, \tag{17}$$

$$\bar{H}_{1a,2a} = \gamma^0 H_{1a,2a}^\dagger \gamma^0, \tag{18}$$

$$\mathcal{D}^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+}), \tag{19}$$

where $A \overleftrightarrow{\not{D}}_\mu B \equiv A(\partial_\mu B) - (\partial_\mu A)B$, H_{2a} is the charge conjugate field of H_{1a} , and a is the light flavor index. The S - and P -wave charmonia are collected into the fields \mathcal{J} and χ^μ , respectively. The pseudoscalar and vector charmed mesons are collected into the fields H_1 and H_2 . All of the heavy fields in the above equations contain a factor \sqrt{m} with m the corresponding heavy meson mass. The $\langle \dots \rangle$ in Eqs. (12) and (13) means the trace over Dirac matrices. According to the effective Lagrangian \mathcal{L}_ψ and \mathcal{L}_χ , we can obtain the corresponding vertex function $\mathcal{A}(M_{c\bar{c}} \rightarrow \bar{D}^{(*)0} D^{(*)0})$ in the rescattering diagram. The relevant vertex functions are collected in Appendix A. The coupling constants g_ψ and g_χ can be estimated by invoking the vector meson dominance arguments [83, 84]. The results are

$$g_\psi = \frac{\sqrt{m_\psi}}{2m_D f_\psi}, \tag{20}$$

$$g_\chi = \sqrt{\frac{m_{\chi_{c0}}}{3}} \frac{1}{f_{\chi_{c0}}}, \tag{21}$$

with f_ψ and $f_{\chi_{c0}}$ the decay constants of J/ψ and χ_{c0} , respectively.

If we assume that the T_{cc}^+ is a pure isoscalar hadronic molecule, its wave function can be written as

$$|T_{cc}^+\rangle = \frac{1}{\sqrt{2}} \left(|D^{*+} D^0\rangle - |D^{*0} D^+\rangle \right). \tag{22}$$

The vertex functions in the rescattering diagrams for the $D^{*+} D^0$ or $D^{*0} D^+$ fusing into the T_{cc}^+ read

$$\mathcal{A}(D^{*+} D^0 \rightarrow T_{cc}^+) = + \frac{g_1}{\sqrt{2}} \varepsilon_{T_{cc}^+}^* \cdot \varepsilon_{D^{*+}}, \tag{23}$$

$$A(D^{*0}D^+ \rightarrow T_{cc}^+) = -\frac{g_2}{\sqrt{2}}\varepsilon_{T_{cc}^+}^* \cdot \varepsilon_{D^{*0}}. \tag{24}$$

In the molecular scheme, the effective coupling of the loosely bound state with its component is in correlation with the binding energy. Assuming that T_{cc}^+ is a pure molecule, for the single-channel case, the couplings take the form [54,64,123–125]

$$g_{1,2} = \frac{4m_{T_{cc}^+}\sqrt{\pi\kappa_{1,2}}}{\sqrt{\mu_{1,2}}}, \tag{25}$$

where $\kappa_1 = \sqrt{2\mu_1(m_{D^{*+}} + m_{D^0} - m_{T_{cc}^+})}$, $\kappa_2 = \sqrt{2\mu_2(m_{D^{*0}} + m_{D^+} - m_{T_{cc}^+})}$, and μ_1 (μ_2) is the reduced mass of $D^{*+}D^0$ ($D^{*0}D^+$). The thresholds of $D^{*+}D^0$ and $D^{*0}D^+$ are located above $m_{T_{cc}^+}$ around 0.3 and 1.7 MeV, respectively. Then the couplings are estimated to be $g_1 \simeq 4.23$ GeV and $g_2 \simeq 6.62$ GeV. The difference between g_1 and g_2 is relatively larger, while they are expected to be the same taking into account the isospin symmetry. Within a coupled-channel approach, the effective couplings for the $D^{*+}D^0$ and $D^{*0}D^+$ channels are found to be similar [53]. In our numerical calculation, we adopt the values given in the coupled-channel approach (Scheme I in Ref. [53]), i.e., $g_1 = g_2 = (5.64 \pm 0.16)$ GeV. One may notice that this value is similar to the average value g_{ave} estimated in the single-channel approach, i.e., $g_{ave} = (g_1 + g_2)/2 \simeq 5.43$ GeV.

The decay amplitude of $B_c^+ \rightarrow T_{cc}^+\bar{D}^{(*)0}$ via one of the rescattering diagrams in Figs. 1 and 2 can be expressed in a general form as follows

$$A(B_c^+ \rightarrow T_{cc}^+\bar{D}^{(*)+}) = -i \int \frac{d^4k_3}{(2\pi)^4} \frac{A(B_c^+ \rightarrow M_{c\bar{c}}D^{(*)+})}{(k_1^2 - m_1^2)} \times \frac{A(M_{c\bar{c}} \rightarrow \bar{D}^{(*)0}D^{(*)0})A(D^*D \rightarrow T_{cc}^+)}{(k_2^2 - m_2^2)(k_3^2 - m_3^2)} \mathcal{F}(k_3^2, m_3^2), \tag{26}$$

where k_1 , k_2 and k_3 (m_1 , m_2 and m_3) correspond to the momenta (mass) of intermediate mesons $M_{c\bar{c}}$, $D^{(*)+}$ and $D^{(*)0}$, respectively. The sum over polarization of intermediate state in Eq. (26) is implicit. For the intermediate spin-1 state, the sum over polarization reads $\sum_{\text{pol}} \varepsilon_\mu \varepsilon_\nu^* = -g_{\mu\nu} + k_\mu k_\nu/m^2$. A dipole form factor $\mathcal{F}(k_3^2, m_3^2)$ is introduced in Eq. (26), which takes the form

$$\mathcal{F}(k_3^2, m_3^2) = \left(\frac{\Lambda^2 - m_3^2}{\Lambda^2 - k_3^2} \right)^2. \tag{27}$$

This form factor is supposed to parameterize the off-shell effects of the intermediate state and to kill the ultraviolet divergence in the loop integrals. We employ a dipole form factor rather than a monopole form factor because the latter cannot kill the divergence appearing in some rescattering

amplitudes. The loop integral is performed by employing the program package *LoopTools* [126].

2.2 Numerical analysis

In this subsection, we give the results from explicit calculations of the rescattering amplitudes. First we list the input parameters used in the numerical calculation. In the weak decay amplitude $A(B_c^+ \rightarrow M_{c\bar{c}}D^{(*)+})$, the central values of the CKM matrix elements $|V_{cb}| = 0.0408$ and $|V_{cd}| = 0.221$ reported by the Particle Data Group (PDG) [62] are adopted, and the combination of Wilson coefficients a_1 is set to be 1.14 [118]. For the relevant decay constants, we use the following values: $f_D = 0.209$ GeV [62], $f_{D^*} = 0.245$ GeV [127], $f_\psi = 0.416$ GeV [62] and $f_{\chi_{c0}} = 0.510$ GeV [83]. Concerning the relevant particle mass, the PDG 2022 central values are used [62].

The numerical results of $B_c \rightarrow \eta_c, J/\psi, \chi_{c0,c1}$ and h_c form factors in Table II of Ref. [95] and Table I of Ref. [96] are employed. In our calculation of the loop integral, as an approximation we do not take into account the q^2 -dependence of the form factors. The values of these weak decay form factors are fixed at $q^2 = m_{D^{(*)+}}^2$, i.e., an on-shell approximation of the weak vertex function is adopted.

There is still a free parameter, i.e., the cutoff energy Λ , in the dipole form factor Eq. (27). Its explicit value should be determined from the experimental data. As an theoretical prediction, the empirical value of Λ is usually set to be larger than the mass of the exchanged particle, and it also depends on the formalism of the form factor. For instance, in Ref. [83], one obtains $\Lambda \approx 2.7$ GeV to roughly fit the experimental data of $\text{Br}(B^- \rightarrow K^- \chi_{c0})$, where the form factor is the monopole type and the exchanged particles are D and D^* . This cut-off energy is just around the typical values of the mass of the radially excited states of $D^{(*)}$ mesons. Taking into account the uncertainty induced by the cutoff Λ , we plot the branching ratio of $B_c^+ \rightarrow T_{cc}^+\bar{D}^{(*)0}$ via the rescattering processes as a function of Λ . The numerical results are displayed in Figs. 3 and 4, where we consider a relatively larger range 2.5–5 GeV for Λ . The uncertainties from the couplings g_1 and g_2 are also taken into account.

The thresholds of final states $T_{cc}^+\bar{D}^0$ and $T_{cc}^+\bar{D}^{*0}$ are around 5739.7 and 5881.7 MeV, respectively, which are not far from the B_c^+ mass 6274.5 MeV. Therefore the higher partial-wave amplitudes of the two channels are expected to be highly suppressed by the limited phase space.

From Figs. 3 and 4, we can see that the two branching ratios increase monotonically with Λ increasing. Within the cutoff range 2.5–5 GeV, the branching ratio of $B_c^+ \rightarrow T_{cc}^+\bar{D}^0$ increases from $\mathcal{O}(10^{-8})$ to $\mathcal{O}(10^{-7})$ with Λ increasing, while that of $B_c^+ \rightarrow T_{cc}^+\bar{D}^{*0}$ increases from $\mathcal{O}(10^{-6})$ to $\mathcal{O}(10^{-5})$ with Λ increasing. The branching ratio of $B_c^+ \rightarrow T_{cc}^+\bar{D}^{*0}$ is much larger than that of $B_c^+ \rightarrow T_{cc}^+\bar{D}^0$. This is because that

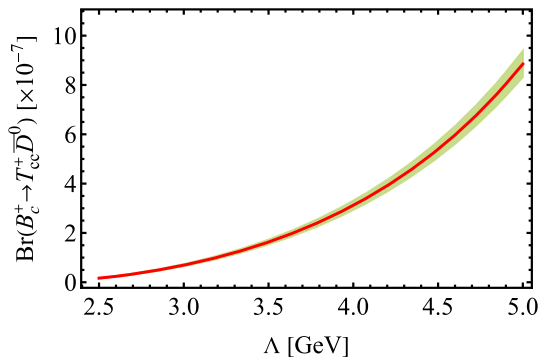


Fig. 3 Λ -dependence of the branching ratio of $B_c^+ \rightarrow T_{cc}^+ \bar{D}^0$ via the rescattering processes in Fig. 1. The band is obtained by taking into account the uncertainties of g_1 and g_2

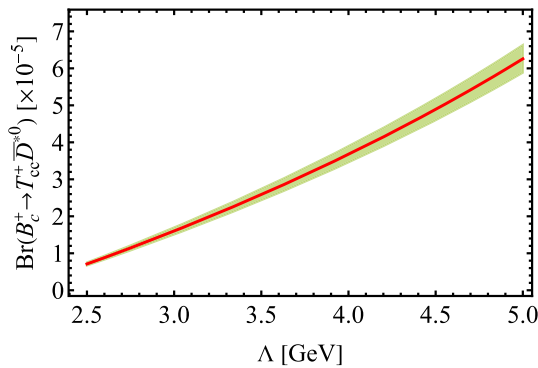


Fig. 4 Λ -dependence of the branching ratio of $B_c^+ \rightarrow T_{cc}^+ \bar{D}^{*0}$ via the rescattering processes in Fig. 2. The band is obtained by taking into account the uncertainties of g_1 and g_2

the S -wave decay is allowed in $B_c^+ \rightarrow T_{cc}^+ \bar{D}^{*0}$ but forbidden in $B_c^+ \rightarrow T_{cc}^+ \bar{D}^0$ to conserve the angular momentum. For the moderate cutoff Λ around 3 GeV, $\text{Br}(B_c^+ \rightarrow T_{cc}^+ \bar{D}^{*0})$ is of the order of 10^{-5} , and we can expect it may be detectable in future experiments.

3 Production of $T_{cc\bar{s}}^+$ in B_c^+ decays

Inspired by the observation of T_{cc}^+ , one may guess some other analogs can also exist, such as the doubly heavy states with the strange quark. In this work, we are interested in a T_{cc}^+ analog named $T_{cc\bar{s}}^+$. We assume $T_{cc\bar{s}}^+$ is a hadronic molecule composed of $D_s^{*+} D^0 / D_s^+ D^{*0}$, and the wave function is defined as

$$|T_{cc\bar{s}}^+\rangle = \frac{1}{\sqrt{2}} (|D_s^{*+} D^0\rangle - |D^{*0} D_s^+\rangle). \tag{28}$$

The $T_{cc\bar{s}}^+$ mass relative to the lower $D^{*0} D_s^+$ threshold $\tilde{\delta}_m$ is defined as

$$\tilde{\delta}_m \equiv (m_{D^{*0}} + m_{D_s^+}) - m_{T_{cc\bar{s}}^+}. \tag{29}$$

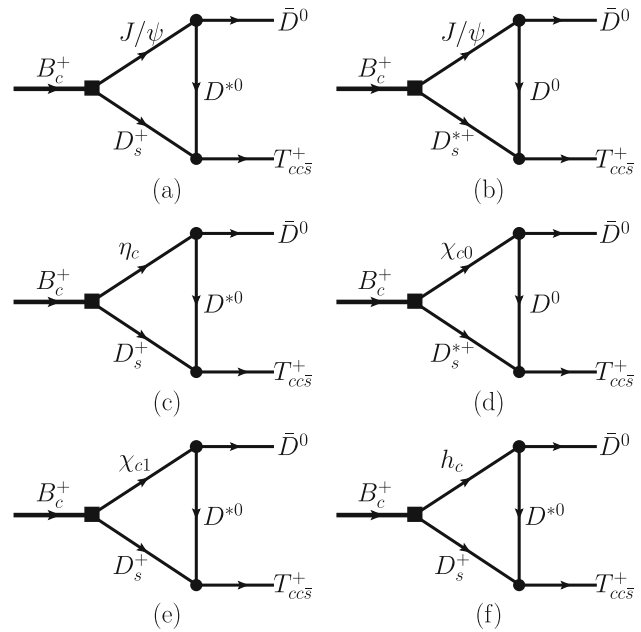


Fig. 5 Rescattering diagrams contributing to $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0$

The relation between T_{cc}^+ and $T_{cc\bar{s}}^+$ is similar to that between $Z_c(3900)$ and $Z_{c\bar{s}}(3985)$, which are widely supposed to be the hadronic molecules composed of $D\bar{D}^*/D^*\bar{D}$ and $D_s\bar{D}^*/D_s^*\bar{D}$. In Ref. [40], the doubly heavy systems composed of a pair of heavy mesons have been systematically studied in a quasi-potential Bethe–Salpeter equation approach. The authors predict that the bound state with $J^P = 1^+$ can be found from the $D_s^* D - D_s D^*$ coupled channel interactions, i.e., the hadronic molecule $T_{cc\bar{s}}^+$ we defined here may exist. In Ref. [40], the binding energy is estimated to be about 400 keV at the cutoff of 2 GeV. Although the prediction concerning the $T_{cc\bar{s}}^+$ may be model-dependent, it is still worth searching for it in future experiments.

In this work, we are mainly interested in the production of the $T_{cc\bar{s}}^+$ state. In the B_c^+ decays, the production mechanism of $T_{cc\bar{s}}^+$ is rather similar with that of T_{cc}^+ . Replacing the $D^{(*)+}$ in Figs. 1 and 2 with $D_s^{(*)+}$, the rescattering diagrams contributing to the $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0$ and $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0}$ are illustrated in Figs. 5 and 6, respectively.

In the rescattering diagrams, the effective Hamiltonian governing the weak process $B_c^+ \rightarrow M_{c\bar{c}} D_s^{(*)+}$ reads

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cs} [c_1 (\bar{b}c)_{V-A} (\bar{c}s)_{V-A} + c_2 (\bar{c}c)_{V-A} (\bar{b}s)_{V-A}], \tag{30}$$

with $|V_{cs}| = 0.975$ [62]. The decay constants in the factorized amplitudes are taken as $f_{D_s} = 0.247$ GeV [62] and $f_{D_s^*} = 0.272$ GeV [127]. Notice that $B_c^+ \rightarrow M_{c\bar{c}} D_s^{(*)+}$ is a Cabibbo-favored process compared with $B_c^+ \rightarrow M_{c\bar{c}} D^{(*)+}$.

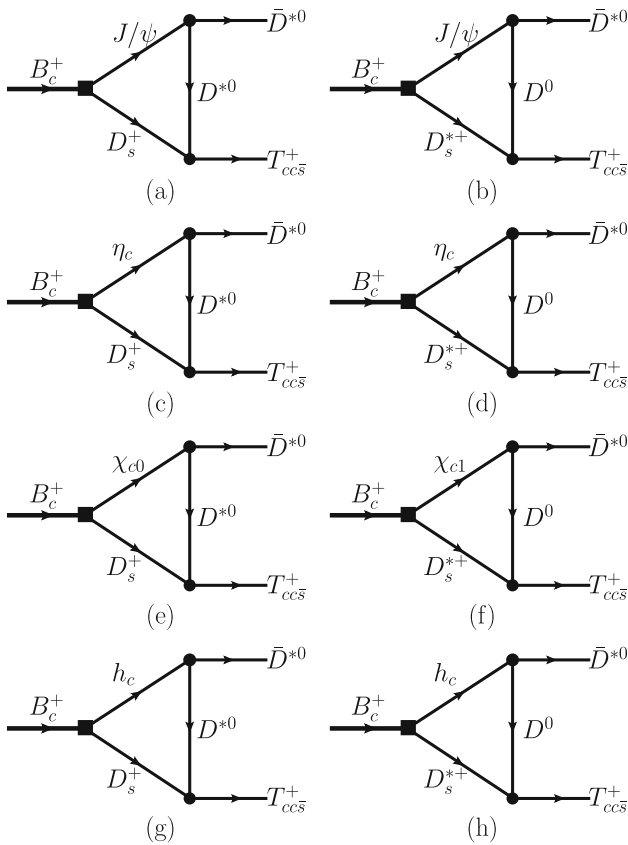


Fig. 6 Rescattering diagrams contributing to $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0}$

Therefore the branching ratio of $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0}$ via the rescattering processes is also expected to be larger.

Being similar to Eqs. (23) and (24), the vertex functions in the rescattering diagrams for the $D_s^{*+} D^0$ or $D^{*0} D_s^+$ fusing into the $T_{cc\bar{s}}^+$ read

$$A(D_s^{*+} D^0 \rightarrow T_{cc\bar{s}}^+) = + \frac{g'_1}{\sqrt{2}} \varepsilon_{T_{cc\bar{s}}^+}^* \cdot \varepsilon_{D_s^{*+}}, \tag{31}$$

$$A(D^{*0} D_s^+ \rightarrow T_{cc\bar{s}}^+) = - \frac{g'_2}{\sqrt{2}} \varepsilon_{T_{cc\bar{s}}^+}^* \cdot \varepsilon_{D^{*0}}. \tag{32}$$

For the single-channel case, the couplings can be determined as

$$g'_{1,2} = \frac{4m_{T_{cc\bar{s}}^+} \sqrt{\pi \kappa'_{1,2}}}{\sqrt{\mu'_{1,2}}}, \tag{33}$$

with $\kappa'_1 = \sqrt{2\mu'_1(m_{D_s^{*+}} + m_{D^0} - m_{T_{cc\bar{s}}^+})}$, $\kappa'_2 = \sqrt{2\mu'_2(m_{D^{*0}} + m_{D_s^+} - m_{T_{cc\bar{s}}^+})}$, and μ'_1 (μ'_2) the reduced mass of $D_s^{*+} D^0$ ($D^{*0} D_s^+$). In the numerical calculation, we take into account the U/V spin symmetry and assume that $g'_1 = g'_2$. The average value g'_{ave} of g'_1 and g'_2 estimated in the single-channel approach are adopted. Concerning the mass of $T_{cc\bar{s}}^+$, we choose several typical values $\tilde{\delta}_m = 100$ keV,

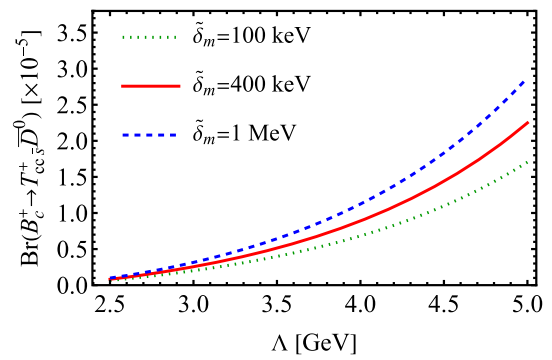


Fig. 7 Λ -dependence of the branching ratio of $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0$ via the rescattering processes in Fig. 5

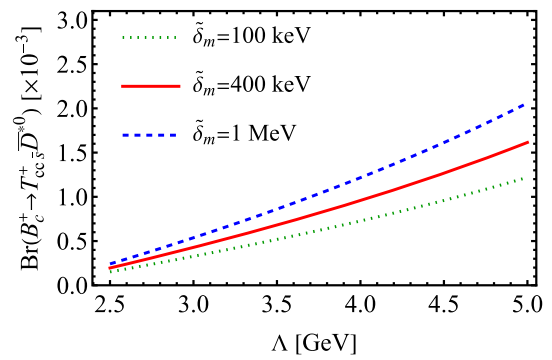


Fig. 8 Λ -dependence of the branching ratio of $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0}$ via the rescattering processes in Fig. 6

400 keV and 1 MeV to estimate the couplings, which give $g'_{ave} \simeq 5.21, 6.03$ and 6.86 GeV, respectively.

Following the rather similar procedures as that in Sect. 2, we calculate the rescattering contributions and the numerical results are presented in Figs. 7 and 8. For the $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0$ process, varying the Λ from 2.5 to 5 GeV, the branching ratio increases from $\mathcal{O}(10^{-6})$ to $\mathcal{O}(10^{-5})$ for all of the three $\tilde{\delta}_m$ values. For the moderate cutoff Λ around 3 GeV, $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0)$ is of the order of 10^{-6} . Compared with $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0$, the $B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0}$ process has a larger branching ratio. The argument is similar as that in Sect. 2. For all of the three $\tilde{\delta}_m$ values, $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0})$ increases from $\mathcal{O}(10^{-4})$ to $\mathcal{O}(10^{-3})$ with Λ increasing. For the moderate cutoff Λ around 3 GeV, $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0})$ is of the order of 10^{-4} . This is a sizable branching ratio. If the $T_{cc\bar{s}}^+$ state truly exists, it is very likely that we can find it in the B_c decays.

4 Summary

In this work, we study the production of the doubly charmed state T_{cc}^+ and its analog $T_{cc\bar{s}}^+$ in B_c decays, which provide

a good environment for the formation of the exotic meson containing double charm quarks. The T_{cc}^+ or $T_{cc\bar{s}}^+$ is produced from a charmonium and a charmed meson rescattering via exchanging another charmed meson. The contributions from various rescatterings with different intermediate states are taken into account. The calculation of rescattering amplitudes is performed under the ansatz that T_{cc}^+ and $T_{cc\bar{s}}^+$ are weakly bound hadronic molecules. For the moderate cutoff energy, the branching ratios $\text{Br}(B_c^+ \rightarrow T_{cc}^+ \bar{D}^0)$ and $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0})$ are estimated to be of the order of 10^{-7} and 10^{-5} , respectively. The $T_{cc\bar{s}}^+$ production in the B_c^+ decay is a Cabibbo-favored process. For the moderate cutoff energy $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^0)$ and $\text{Br}(B_c^+ \rightarrow T_{cc\bar{s}}^+ \bar{D}^{*0})$ are estimated to be of the order of 10^{-6} and 10^{-4} , respectively.

We should also mention that even if T_{cc}^+ , $T_{cc\bar{s}}^+$ and some other analogs are not hadronic molecules, the production mechanism of the doubly charm mesons proposed in this paper still works. Correspondingly, the couplings between the tetraquark states and open charm mesons need to be modified.

The predicted relatively sizable branching ratios suggest that in future experiments one may search for the T_{cc}^+ and its analogs in the B_c decay processes discussed here. Besides, investigating different production mechanisms of these exotic doubly charmed states is also crucial in revealing their underlying structures.

Acknowledgements This work is supported by the National Natural Science Foundation of China (NSFC) under Grants No. 11975165, No. 12235018, No. 12175165 and No. 12075167.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data generated during this study are included in this published article.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

Funded by SCOAP³. SCOAP³ supports the goals of the International Year of Basic Sciences for Sustainable Development.

Appendix A: The vertex functions $\mathcal{A}(M_{c\bar{c}} \rightarrow \bar{D}^{(*)0} D^{(*)0})$ in the rescattering amplitudes

Here we give the vertex functions $\mathcal{A}(M_{c\bar{c}}(k_1) \rightarrow \bar{D}^{(*)0}(p_1) D^{(*)0}(k_3))$ corresponding to a charmonium cou-

pling with a pair of open charm mesons:

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}^0 D^0) &= 2g_\psi(k_3 - p_1) \cdot \varepsilon_J m_D \sqrt{m_\psi}, \end{aligned} \tag{A1}$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}^0 D^{*0}) &= -4i g_\psi \varepsilon_{\mu\nu\alpha\beta} k_1^\mu p_1^\nu \varepsilon_{D^*}^{\alpha} \varepsilon_J^\beta \sqrt{\frac{m_D m_{D^*}}{m_\psi}}, \end{aligned} \tag{A2}$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}^{*0} D^0) &= -4i g_\psi \varepsilon_{\mu\nu\alpha\beta} k_1^\mu p_1^\nu \varepsilon_{D^*}^{\alpha} \varepsilon_J^\beta \sqrt{\frac{m_D m_{D^*}}{m_\psi}}, \end{aligned} \tag{A3}$$

$$\begin{aligned} \mathcal{A}(J/\psi \rightarrow \bar{D}^{*0} D^{*0}) &= 2g_\psi m_{D^*} \sqrt{m_\psi} [-(k_3 - p_1) \cdot \varepsilon_J \varepsilon_{D^*}^* \cdot \varepsilon_{D^*}^* \\ &\quad + (k_3 - p_1) \cdot \varepsilon_{D^*}^* \varepsilon_{D^*}^* \cdot \varepsilon_J + (k_3 - p_1) \cdot \varepsilon_{D^*}^* \varepsilon_{D^*}^* \cdot \varepsilon_J], \end{aligned} \tag{A4}$$

$$\begin{aligned} \mathcal{A}(\eta_c \rightarrow \bar{D}^0 D^{*0}) &= 2g_\psi(k_3 - p_1) \cdot \varepsilon_{D^*}^* \sqrt{m_{\eta_c} m_D m_{D^*}}, \end{aligned} \tag{A5}$$

$$\begin{aligned} \mathcal{A}(\eta_c \rightarrow \bar{D}^{*0} D^0) &= -2g_\psi(k_3 - p_1) \cdot \varepsilon_{D^*}^* \sqrt{m_{\eta_c} m_D m_{D^*}}, \end{aligned} \tag{A6}$$

$$\begin{aligned} \mathcal{A}(\eta_c \rightarrow \bar{D}^{*0} D^{*0}) &= -4i g_\psi \varepsilon_{\mu\nu\alpha\beta} k_1^\mu p_1^\nu \varepsilon_{D^*}^{\alpha} \varepsilon_{D^*}^{\beta} \frac{m_{D^*}}{\sqrt{m_{\eta_c}}}, \end{aligned} \tag{A7}$$

$$\begin{aligned} \mathcal{A}(h_c \rightarrow \bar{D}^{*0} D^{*0}) &= 2i g_\chi \varepsilon_{\mu\nu\alpha\beta} k_1^\mu \varepsilon_{h_c}^\nu \varepsilon_{D^*}^{\alpha} \varepsilon_{D^*}^{\beta} \frac{m_{D^*}}{\sqrt{m_{h_c}}}, \end{aligned} \tag{A8}$$

$$\begin{aligned} \mathcal{A}(h_c \rightarrow \bar{D}^{*0} D^0) &= -2g_\chi \varepsilon_{h_c} \cdot \varepsilon_{D^*}^* \sqrt{m_{h_c} m_D m_{D^*}}, \end{aligned} \tag{A9}$$

$$\begin{aligned} \mathcal{A}(h_c \rightarrow \bar{D}^0 D^{*0}) &= 2g_\chi \varepsilon_{h_c} \cdot \varepsilon_{D^*}^* \sqrt{m_{h_c} m_D m_{D^*}}, \end{aligned} \tag{A10}$$

$$\begin{aligned} \mathcal{A}(\chi_{c0} \rightarrow \bar{D}^0 D^0) &= -2\sqrt{3} g_\chi m_D \sqrt{m_{\chi_{c0}}}, \end{aligned} \tag{A11}$$

$$\begin{aligned} \mathcal{A}(\chi_{c0} \rightarrow \bar{D}^{*0} D^{*0}) &= \frac{2}{\sqrt{3}} g_\chi \varepsilon_{D^*}^* \cdot \varepsilon_{D^*}^* m_{D^*} \sqrt{m_{\chi_{c0}}}, \end{aligned} \tag{A12}$$

$$\begin{aligned} \mathcal{A}(\chi_{c1} \rightarrow \bar{D}^{*0} D^0) &= 2\sqrt{2} i g_\chi \varepsilon_{\chi_{c1}} \cdot \varepsilon_{D^*}^* \sqrt{m_{\chi_{c1}} m_D m_{D^*}}, \end{aligned} \tag{A13}$$

$$\begin{aligned} \mathcal{A}(\chi_{c1} \rightarrow \bar{D}^0 D^{*0}) &= 2\sqrt{2} i g_\chi \varepsilon_{\chi_{c1}} \cdot \varepsilon_{D^*}^* \sqrt{m_{\chi_{c1}} m_D m_{D^*}}. \end{aligned} \tag{A14}$$

References

1. R. Aaij et al. [LHCb], Nat. Phys. **18**(7), 751–754 (2022). <https://doi.org/10.1038/s41567-022-01614-y>. arXiv:2109.01038 [hep-ex]
2. R. Aaij et al. [LHCb], Nat. Commun. **13**(1), 3351 (2022). <https://doi.org/10.1038/s41467-022-30206-w>. arXiv:2109.01056 [hep-ex]
3. J.L. Ballot, J.M. Richard, Phys. Lett. B **123**, 449–451 (1983). [https://doi.org/10.1016/0370-2693\(83\)90991-7](https://doi.org/10.1016/0370-2693(83)90991-7)

4. S. Zouzou, B. Silvestre-Brac, C. Gignoux, J.M. Richard, Z. Phys. C **30**, 457 (1986). <https://doi.org/10.1007/BF01557611>
5. J. Carlson, L. Heller, J.A. Tjon, Phys. Rev. D **37**, 744 (1988). <https://doi.org/10.1103/PhysRevD.37.744>
6. L. Heller, J.A. Tjon, Phys. Rev. D **35**, 969 (1987). <https://doi.org/10.1103/PhysRevD.35.969>
7. C. Deng, H. Chen, J. Ping, Eur. Phys. J. A **56**(1), 9 (2020). <https://doi.org/10.1140/epja/s10050-019-00012-y>. arXiv:1811.06462 [hep-ph]
8. X. Yan, B. Zhong, R. Zhu, Int. J. Mod. Phys. A **33**(16), 1850096 (2018). <https://doi.org/10.1142/S0217751X18500963>. arXiv:1804.06761 [hep-ph]
9. J.M. Richard, A. Valcarce, J. Vijande, Phys. Rev. C **97**(3), 035211 (2018). <https://doi.org/10.1103/PhysRevC.97.035211>. arXiv:1803.06155 [hep-ph]
10. Q. Meng, E. Hiyama, A. Hosaka, M. Oka, P. Gubler, K.U. Can, T.T. Takahashi, H.S. Zong, Phys. Lett. B **814**, 136095 (2021). <https://doi.org/10.1016/j.physletb.2021.136095>. arXiv:2009.14493 [nucl-th]
11. Q. Meng, M. Harada, E. Hiyama, A. Hosaka, M. Oka, Phys. Lett. B **824**, 136800 (2022). <https://doi.org/10.1016/j.physletb.2021.136800>. arXiv:2106.11868 [hep-ph]
12. Q. Qin, Y.F. Shen, F.S. Yu, Chin. Phys. C **45**(10), 103106 (2021). <https://doi.org/10.1088/1674-1137/ac1b97>. arXiv:2008.08026 [hep-ph]
13. F.S. Navarra, M. Nielsen, S.H. Lee, Phys. Lett. B **649**, 166–172 (2007). <https://doi.org/10.1016/j.physletb.2007.04.010>. arXiv:hep-ph/0703071
14. J.M. Dias, S. Narison, F.S. Navarra, M. Nielsen, J.M. Richard, Phys. Lett. B **703**, 274–280 (2011). <https://doi.org/10.1016/j.physletb.2011.07.082>. arXiv:1105.5630 [hep-ph]
15. M.L. Du, W. Chen, X.L. Chen, S.L. Zhu, Phys. Rev. D **87**(1), 014003 (2013). <https://doi.org/10.1103/PhysRevD.87.014003>. arXiv:1209.5134 [hep-ph]
16. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **99**(11), 114016 (2019). <https://doi.org/10.1103/PhysRevD.99.114016>. arXiv:1903.11975 [hep-ph]
17. G.Q. Feng, X.H. Guo, B.S. Zou, arXiv:1309.7813 [hep-ph]
18. E. Braaten, L.P. He, A. Mohapatra, Phys. Rev. D **103**(1), 016001 (2021). <https://doi.org/10.1103/PhysRevD.103.016001>. arXiv:2006.08650 [hep-ph]
19. J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Chin. Phys. C **45**(4), 043102 (2021). <https://doi.org/10.1088/1674-1137/abde2f>. arXiv:2008.00737 [hep-ph]
20. E.J. Eichten, C. Quigg, Phys. Rev. Lett. **119**(20), 202002 (2017). <https://doi.org/10.1103/PhysRevLett.119.202002>. arXiv:1707.09575 [hep-ph]
21. T. Guo, J. Li, J. Zhao, L. He, Phys. Rev. D **105**(1), 014021 (2022). <https://doi.org/10.1103/PhysRevD.105.014021>. arXiv:2108.10462 [hep-ph]
22. X.Z. Weng, W.Z. Deng, S.L. Zhu, Chin. Phys. C **46**(1), 013102 (2022). <https://doi.org/10.1088/1674-1137/ac2ed0>. arXiv:2108.07242 [hep-ph]
23. Y. Kim, M. Oka, K. Suzuki, Phys. Rev. D **105**(7), 074021 (2022). <https://doi.org/10.1103/PhysRevD.105.074021>. arXiv:2202.06520 [hep-ph]
24. S.S. Agaev, K. Azizi, H. Sundu, Nucl. Phys. B **975**, 115650 (2022). <https://doi.org/10.1016/j.nuclphysb.2022.115650>. arXiv:2108.00188 [hep-ph]
25. M. Karliner, J.L. Rosner, Phys. Rev. D **105**(3), 034020 (2022). <https://doi.org/10.1103/PhysRevD.105.034020>. arXiv:2110.12054 [hep-ph]
26. K. Azizi, U. Özdem, arXiv:2301.07713 [hep-ph]
27. J.B. Wang, G. Li, C.S. An, C.R. Deng, J.J. Xie, Eur. Phys. J. C **82**(8), 721 (2022). <https://doi.org/10.1140/epjc/s10052-022-10673-7>. arXiv:2204.13320 [hep-ph]
28. A.V. Manohar, M.B. Wise, Nucl. Phys. B **399**, 17–33 (1993). [https://doi.org/10.1016/0550-3213\(93\)90614-U](https://doi.org/10.1016/0550-3213(93)90614-U). arXiv:hep-ph/9212236
29. N.A. Tornqvist, Z. Phys. C **61**, 525–537 (1994). <https://doi.org/10.1007/BF01413192>. arXiv:hep-ph/9310247
30. T.E.O. Ericson, G. Karl, Phys. Lett. B **309**, 426–430 (1993). [https://doi.org/10.1016/0370-2693\(93\)90957-J](https://doi.org/10.1016/0370-2693(93)90957-J)
31. D. Janc, M. Rosina, Few Body Syst. **35**, 175–196 (2004). <https://doi.org/10.1007/s00601-004-0068-9>. arXiv:hep-ph/0405208
32. G.J. Ding, J.F. Liu, M.L. Yan, Phys. Rev. D **79**, 054005 (2009). <https://doi.org/10.1103/PhysRevD.79.054005>. arXiv:0901.0426 [hep-ph]
33. R. Molina, T. Branz, E. Oset, Phys. Rev. D **82**, 014010 (2010). <https://doi.org/10.1103/PhysRevD.82.014010>. arXiv:1005.0335 [hep-ph]
34. N. Li, Z.F. Sun, X. Liu, S.L. Zhu, Phys. Rev. D **88**(11), 114008 (2013). <https://doi.org/10.1103/PhysRevD.88.114008>. arXiv:1211.5007 [hep-ph]
35. H. Xu, B. Wang, Z.W. Liu, X. Liu, Phys. Rev. D **99**(1), 014027 (2019). Erratum: Phys. Rev. D **104**(11), 119903 (2021). <https://doi.org/10.1103/PhysRevD.99.014027>. arXiv:1708.06918 [hep-ph]
36. L. Tang, B.D. Wan, K. Maltman, C.F. Qiao, Phys. Rev. D **101**(9), 094032 (2020). <https://doi.org/10.1103/PhysRevD.101.094032>. arXiv:1911.10951 [hep-ph]
37. S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, A. Hosaka, Phys. Rev. D **86**, 034019 (2012). <https://doi.org/10.1103/PhysRevD.86.034019>. arXiv:1202.0760 [hep-ph]
38. M.Z. Liu, T.W. Wu, M. Pavon Valderrama, J.J. Xie, L.S. Geng, Phys. Rev. D **99**(9), 094018 (2019). <https://doi.org/10.1103/PhysRevD.99.094018>. arXiv:1902.03044 [hep-ph]
39. Z.M. Ding, H.Y. Jiang, J. He, Eur. Phys. J. C **80**(12), 1179 (2020). <https://doi.org/10.1140/epjc/s10052-020-08754-6>. arXiv:2011.04980 [hep-ph]
40. Z.M. Ding, H.Y. Jiang, D. Song, J. He, Eur. Phys. J. C **81**(8), 732 (2021). <https://doi.org/10.1140/epjc/s10052-021-09534-6>. arXiv:2107.00855 [hep-ph]
41. Y. Tan, W. Lu, J. Ping, Eur. Phys. J. Plus **135**(9), 716 (2020). <https://doi.org/10.1140/epjp/s13360-020-00741-w>. arXiv:2004.02106 [hep-ph]
42. J. Vijande, A. Valcarce, N. Barnea, Phys. Rev. D **79**, 074010 (2009). <https://doi.org/10.1103/PhysRevD.79.074010>. arXiv:0903.2949 [hep-ph]
43. Y. Yang, C. Deng, J. Ping, T. Goldman, Phys. Rev. D **80**, 114023 (2009). <https://doi.org/10.1103/PhysRevD.80.114023>
44. G. Yang, J. Ping, J. Segovia, Phys. Rev. D **101**(1), 014001 (2020). <https://doi.org/10.1103/PhysRevD.101.014001>. arXiv:1911.00215 [hep-ph]
45. K. Azizi, U. Özdem, Phys. Rev. D **104**(11), 114002 (2021). <https://doi.org/10.1103/PhysRevD.104.114002>. arXiv:2109.02390 [hep-ph]
46. R. Chen, Q. Huang, X. Liu, S.L. Zhu, Phys. Rev. D **104**(11), 114042 (2021). <https://doi.org/10.1103/PhysRevD.104.114042>. arXiv:2108.01911 [hep-ph]
47. A. Feijoo, W.H. Liang, E. Oset, Phys. Rev. D **104**(11), 114015 (2021). <https://doi.org/10.1103/PhysRevD.104.114015>. arXiv:2108.02730 [hep-ph]
48. F.L. Wang, X. Liu, Phys. Rev. D **104**(9), 094030 (2021). <https://doi.org/10.1103/PhysRevD.104.094030>. arXiv:2108.09925 [hep-ph]
49. H. Ren, F. Wu, R. Zhu, Adv. High Energy Phys. **2022**, 9103031 (2022). <https://doi.org/10.1155/2022/9103031>. arXiv:2109.02531 [hep-ph]
50. C. Deng, S.L. Zhu, Phys. Rev. D **105**(5), 054015 (2022). <https://doi.org/10.1103/PhysRevD.105.054015>. arXiv:2112.12472 [hep-ph]

51. M. Albaladejo, Phys. Lett. B **829**, 137052 (2022). <https://doi.org/10.1016/j.physletb.2022.137052>. arXiv:2110.02944 [hep-ph]
52. L.R. Dai, R. Molina, E. Oset, Phys. Rev. D **105**(1), 016029 (2022). Erratum: Phys. Rev. D **106**(9), 099902 (2022). <https://doi.org/10.1103/PhysRevD.105.016029>. arXiv:2110.15270 [hep-ph]
53. M.L. Du, V. Baru, X.K. Dong, A. Filin, F.K. Guo, C. Hanhart, A. Nefediev, J. Nieves, Q. Wang, Phys. Rev. D **105**(1), 014024 (2022). <https://doi.org/10.1103/PhysRevD.105.014024>. arXiv:2110.13765 [hep-ph]
54. L. Meng, G.J. Wang, B. Wang, S.L. Zhu, Phys. Rev. D **104**(5), 051502 (2021). <https://doi.org/10.1103/PhysRevD.104.L051502>. arXiv:2107.14784 [hep-ph]
55. X.Z. Ling, M.Z. Liu, L.S. Geng, E. Wang, J.J. Xie, Phys. Lett. B **826**, 136897 (2022). <https://doi.org/10.1016/j.physletb.2022.136897>. arXiv:2108.00947 [hep-ph]
56. M.J. Yan, M.P. Valderrama, Phys. Rev. D **105**(1), 014007 (2022). <https://doi.org/10.1103/PhysRevD.105.014007>. arXiv:2108.04785 [hep-ph]
57. S. Fleming, R. Hodges, T. Mehen, Phys. Rev. D **104**(11), 116010 (2021). <https://doi.org/10.1103/PhysRevD.104.116010>. arXiv:2109.02188 [hep-ph]
58. Q. Xin, Z.G. Wang, Eur. Phys. J. A **58**(6), 110 (2022). <https://doi.org/10.1140/epja/s10050-022-00752-4>. arXiv:2108.12597 [hep-ph]
59. S.S. Agaev, K. Azizi, H. Sundu, JHEP **06**, 057 (2022). [https://doi.org/10.1007/JHEP06\(2022\)057](https://doi.org/10.1007/JHEP06(2022)057). arXiv:2201.02788 [hep-ph]
60. M.J. Zhao, Z.Y. Wang, C. Wang, X.H. Guo, Phys. Rev. D **105**(9), 096016 (2022). <https://doi.org/10.1103/PhysRevD.105.096016>. arXiv:2112.12633 [hep-ph]
61. H.W. Ke, X.H. Liu, X.Q. Li, Eur. Phys. J. C **82**(2), 144 (2022). <https://doi.org/10.1140/epjc/s10052-022-10092-8>. arXiv:2112.14142 [hep-ph]
62. R.L. Workman et al. [Particle Data Group], PTEP **2022**, 083C01 (2022). <https://doi.org/10.1093/ptep/ptac097>
63. H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. **86**(2), 026201 (2023). <https://doi.org/10.1088/1361-6633/aca3b6>. arXiv:2204.02649 [hep-ph]
64. F.K. Guo, C. Hanhart, U.G. Meißner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. **90**(1), 015004 (2018). Erratum: Rev. Mod. Phys. **94**(2), 029901 (2022). <https://doi.org/10.1103/RevModPhys.90.015004>. arXiv:1705.00141 [hep-ph]
65. Y.S. Kalashnikova, A.V. Nefediev, Phys. Usp. **62**(6), 568–595 (2019). <https://doi.org/10.3367/UFNe.2018.08.038411>. arXiv:1811.01324 [hep-ph]
66. N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.P. Shen, C.E. Thomas, A. Vairo, C.Z. Yuan, Phys. Rep. **873**, 1–154 (2020). <https://doi.org/10.1016/j.physrep.2020.05.001>. arXiv:1907.07583 [hep-ex]
67. F.K. Guo, X.H. Liu, S. Sakai, Prog. Part. Nucl. Phys. **112**, 103757 (2020). <https://doi.org/10.1016/j.pnpnp.2020.103757>. arXiv:1912.07030 [hep-ph]
68. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rep. **639**, 1–121 (2016). <https://doi.org/10.1016/j.physrep.2016.05.004>. arXiv:1601.02092 [hep-ph]
69. A. Esposito, A. Pilloni, A.D. Polosa, Phys. Rep. **668**, 1–97 (2017). <https://doi.org/10.1016/j.physrep.2016.11.002>. arXiv:1611.07920 [hep-ph]
70. S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. **90**(1), 015003 (2018). <https://doi.org/10.1103/RevModPhys.90.015003>. arXiv:1708.04012 [hep-ph]
71. R.F. Lebed, R.E. Mitchell, E.S. Swanson, Prog. Part. Nucl. Phys. **93**, 143–194 (2017). <https://doi.org/10.1016/j.pnpnp.2016.11.003>. arXiv:1610.04528 [hep-ph]
72. A. Ali, J.S. Lange, S. Stone, Prog. Part. Nucl. Phys. **97**, 123–198 (2017). <https://doi.org/10.1016/j.pnpnp.2017.08.003>. arXiv:1706.00610 [hep-ph]
73. Y.R. Liu, H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Prog. Part. Nucl. Phys. **107**, 237–320 (2019). <https://doi.org/10.1016/j.pnpnp.2019.04.003>. arXiv:1903.11976 [hep-ph]
74. Y. Huang, H.Q. Zhu, L.S. Geng, R. Wang, Phys. Rev. D **104**(11), 116008 (2021). <https://doi.org/10.1103/PhysRevD.104.116008>. arXiv:2108.13028 [hep-ph]
75. E. Braaten, L.P. He, K. Ingles, J. Jiang, Phys. Rev. D **106**(3), 3 (2022). <https://doi.org/10.1103/PhysRevD.106.034033>. arXiv:2202.03900 [hep-ph]
76. Y. Hu, J. Liao, E. Wang, Q. Wang, H. Xing, H. Zhang, Phys. Rev. D **104**(11), L111502 (2021). <https://doi.org/10.1103/PhysRevD.104.L111502>. ((hep-ph))
77. L.M. Abreu, H.P.L. Vieira, F.S. Navarra, Phys. Rev. D **105**(11), 116029 (2022). <https://doi.org/10.1103/PhysRevD.105.116029>. arXiv:2202.10882 [hep-ph]
78. X.H. Liu, Q. Zhao, J. Phys. G **36**, 015003 (2009). <https://doi.org/10.1088/0954-3899/36/1/015003>. arXiv:0805.1119 [hep-ph]
79. A. Esposito, M. Papinutto, A. Pilloni, A.D. Polosa, N. Tantalo, Phys. Rev. D **88**(5), 054029 (2013). <https://doi.org/10.1103/PhysRevD.88.054029>. arXiv:1307.2873 [hep-ph]
80. Q. Wu, D.Y. Chen, X.J. Fan, G. Li, Eur. Phys. J. C **79**(3), 265 (2019). <https://doi.org/10.1140/epjc/s10052-019-6784-6>. arXiv:1902.05737 [hep-ph]
81. W. Wang, Y.L. Shen, C.D. Lu, Eur. Phys. J. C **51**, 841–847 (2007). <https://doi.org/10.1140/epjc/s10052-007-0334-3>. arXiv:0704.2493 [hep-ph]
82. W. Wang, Q. Zhao, Phys. Lett. B **755**, 261–264 (2016). <https://doi.org/10.1016/j.physletb.2016.02.012>. arXiv:1512.03123 [hep-ph]
83. P. Colangelo, F. De Fazio, T.N. Pham, Phys. Lett. B **542**, 71–79 (2002). [https://doi.org/10.1016/S0370-2693\(02\)02306-7](https://doi.org/10.1016/S0370-2693(02)02306-7). arXiv:hep-ph/0207061
84. P. Colangelo, F. De Fazio, T.N. Pham, Phys. Rev. D **69**, 054023 (2004). <https://doi.org/10.1103/PhysRevD.69.054023>. arXiv:hep-ph/0310084
85. H.Y. Cheng, C.K. Chua, A. Soni, Phys. Rev. D **71**, 014030 (2005). <https://doi.org/10.1103/PhysRevD.71.014030>. arXiv:hep-ph/0409317
86. H.Y. Cheng, C.K. Chua, A. Soni, Phys. Rev. D **72**, 014006 (2005). <https://doi.org/10.1103/PhysRevD.72.014006>. arXiv:hep-ph/0502235
87. C.D. Lu, Y.L. Shen, W. Wang, Phys. Rev. D **73**, 034005 (2006). <https://doi.org/10.1103/PhysRevD.73.034005>. arXiv:hep-ph/0511255
88. Q. Wu, D.Y. Chen, W.H. Qin, G. Li, Eur. Phys. J. C **82**(6), 520 (2022). <https://doi.org/10.1140/epjc/s10052-022-10465-z>. arXiv:2111.13347 [hep-ph]
89. X.H. Liu, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, Eur. Phys. J. C **80**(12), 1178 (2020). <https://doi.org/10.1140/epjc/s10052-020-08762-6>. arXiv:2008.07190 [hep-ph]
90. J.J. Han, H.Y. Jiang, W. Liu, Z.J. Xiao, F.S. Yu, Chin. Phys. C **45**(5), 053105 (2021). <https://doi.org/10.1088/1674-1137/abec68>. arXiv:2101.12019 [hep-ph]
91. Y.H. Ge, X.H. Liu, H.W. Ke, Eur. Phys. J. C **82**(10), 955 (2022). <https://doi.org/10.1140/epjc/s10052-022-10923-8>. arXiv:2207.09900 [hep-ph]
92. X.H. Liu, Phys. Lett. B **766**, 117–124 (2017). <https://doi.org/10.1016/j.physletb.2017.01.008>. arXiv:1607.01385 [hep-ph]
93. X.H. Liu, U.G. Meißner, Eur. Phys. J. C **77**(12), 816 (2017). <https://doi.org/10.1140/epjc/s10052-017-5402-8>. arXiv:1703.09043 [hep-ph]
94. Q. Wu, Y.K. Chen, G. Li, S.D. Liu, D.Y. Chen, arXiv:2302.01696 [hep-ph]
95. W. Wang, Y.L. Shen, C.D. Lu, Phys. Rev. D **79**, 054012 (2009). <https://doi.org/10.1103/PhysRevD.79.054012>. arXiv:0811.3748 [hep-ph]

96. X.X. Wang, W. Wang, C.D. Lu, Phys. Rev. D **79**, 114018 (2009). <https://doi.org/10.1103/PhysRevD.79.114018>. arXiv:0901.1934 [hep-ph]
97. Z.Q. Zhang, Z.J. Sun, Y.C. Zhao, Y.Y. Yang, Z.Y. Zhang, arXiv:2301.11107 [hep-ph]
98. R. Zhu, Y. Ma, X.L. Han, Z.J. Xiao, Phys. Rev. D **95**(9), 094012 (2017). <https://doi.org/10.1103/PhysRevD.95.094012>. arXiv:1703.03875 [hep-ph]
99. W. Wang, R. Zhu, Int. J. Mod. Phys. A **34**(31), 1950195 (2019). <https://doi.org/10.1142/S0217751X19501951>. arXiv:1808.10830 [hep-ph]
100. J. Harrison et al. HPQCD, Phys. Rev. D **102**(9), 094518 (2020). <https://doi.org/10.1103/PhysRevD.102.094518>. arXiv:2007.06957 [hep-lat]
101. M.A. Nobes, R.M. Woloshyn, J. Phys. G **26**, 1079–1094 (2000). <https://doi.org/10.1088/0954-3899/26/7/308>. arXiv:hep-ph/0005056
102. M.A. Ivanov, J.G. Korner, P. Santorelli, Phys. Rev. D **71**, 094006 (2005). Erratum: Phys. Rev. D **75**, 019901 (2007). <https://doi.org/10.1103/PhysRevD.75.019901>. arXiv:hep-ph/0501051
103. J.F. Sun, D.S. Du, Y.L. Yang, Eur. Phys. J. C **60**, 107–117 (2009). <https://doi.org/10.1140/epjc/s10052-009-0872-y>. arXiv:0808.3619 [hep-ph]
104. R. Dhir, R.C. Verma, Phys. Rev. D **79**, 034004 (2009). <https://doi.org/10.1103/PhysRevD.79.034004>. arXiv:0810.4284 [hep-ph]
105. V.V. Kiselev, A.K. Likhoded, A.I. Onishchenko, Nucl. Phys. B **569**, 473–504 (2000). [https://doi.org/10.1016/S0550-3213\(99\)00505-2](https://doi.org/10.1016/S0550-3213(99)00505-2). arXiv:hep-ph/9905359
106. M.A. Ivanov, J.G. Korner, P. Santorelli, Phys. Rev. D **63**, 074010 (2001). <https://doi.org/10.1103/PhysRevD.63.074010>. arXiv:hep-ph/0007169
107. F. Zuo, T. Huang, Chin. Phys. Lett. **24**, 61–64 (2007). <https://doi.org/10.1088/0256-307X/24/1/017>. arXiv:hep-ph/0611113
108. V.V. Kiselev, A.E. Kovalsky, A.K. Likhoded, Nucl. Phys. B **585**, 353–382 (2000). [https://doi.org/10.1016/S0550-3213\(00\)00386-2](https://doi.org/10.1016/S0550-3213(00)00386-2). arXiv:hep-ph/0002127
109. D.S. Du, Z. Wang, Phys. Rev. D **39**, 1342 (1989). <https://doi.org/10.1103/PhysRevD.39.1342>
110. D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **68**, 094020 (2003). <https://doi.org/10.1103/PhysRevD.68.094020>. arXiv:hep-ph/0306306
111. E. Hernandez, J. Nieves, J.M. Verde-Velasco, Phys. Rev. D **74**, 074008 (2006). <https://doi.org/10.1103/PhysRevD.74.074008>. arXiv:hep-ph/0607150
112. Z. Rui, H. Li, G. X. Wang, Y. Xiao, Eur. Phys. J. C **76**(10), 564 (2016). <https://doi.org/10.1140/epjc/s10052-016-4424-y>. arXiv:1602.08918 [hep-ph]
113. H.W. Ke, T. Liu, X.Q. Li, Phys. Rev. D **89**(1), 017501 (2014). <https://doi.org/10.1103/PhysRevD.89.017501>. arXiv:1307.5925 [hep-ph]
114. G. Aad et al. [ATLAS], JHEP **08**, 087 (2022). [https://doi.org/10.1007/JHEP08\(2022\)087](https://doi.org/10.1007/JHEP08(2022)087). arXiv:2203.01808 [hep-ex]
115. R. Aaij et al. [LHCb], Phys. Rev. D **87**(11), 112012 (2013). <https://doi.org/10.1103/PhysRevD.87.112012>. arXiv:1304.4530 [hep-ex]
116. T. Aaltonen et al. [CDF], Phys. Rev. D **87**(1), 011101 (2013). <https://doi.org/10.1103/PhysRevD.87.011101>. arXiv:1210.2366 [hep-ex]
117. P. Colangelo, F. De Fazio, Phys. Rev. D **61**, 034012 (2000). <https://doi.org/10.1103/PhysRevD.61.034012>. arXiv:hep-ph/9909423
118. M.A. Ivanov, J.G. Korner, P. Santorelli, Phys. Rev. D **73**, 054024 (2006). <https://doi.org/10.1103/PhysRevD.73.054024>. arXiv:hep-ph/0602050
119. S. Dubnicka, A.Z. Dubnickova, A. Issadykov, M.A. Ivanov, A. Liptaj, Phys. Rev. D **96**(7), 076017 (2017). <https://doi.org/10.1103/PhysRevD.96.076017>. arXiv:1708.09607 [hep-ph]
120. C.H. Chang, Y.Q. Chen, Phys. Rev. D **49**, 3399–3411 (1994). <https://doi.org/10.1103/PhysRevD.49.3399>
121. F.K. Guo, C. Hanhart, G. Li, U.G. Meissner, Q. Zhao, Phys. Rev. D **83**, 034013 (2011). <https://doi.org/10.1103/PhysRevD.83.034013>. arXiv:1008.3632 [hep-ph]
122. R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli, Phys. Rep. **281**, 145–238 (1997). [https://doi.org/10.1016/S0370-1573\(96\)00027-0](https://doi.org/10.1016/S0370-1573(96)00027-0). arXiv:hep-ph/9605342
123. S. Weinberg, Phys. Rev. **130**, 776–783 (1963). <https://doi.org/10.1103/PhysRev.130.776>
124. S. Weinberg, Phys. Rev. **131**, 440–460 (1963). <https://doi.org/10.1103/PhysRev.131.440>
125. S. Weinberg, Phys. Rev. **137**, B672–B678 (1965). <https://doi.org/10.1103/PhysRev.137.B672>
126. T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. **118**, 153–165 (1999). [https://doi.org/10.1016/S0010-4655\(98\)00173-8](https://doi.org/10.1016/S0010-4655(98)00173-8). arXiv:hep-ph/9807565
127. D. Becirevic, P. Boucaud, J.P. Leroy, V. Lubicz, G. Martinelli, F. Mescia, F. Rapuano, Phys. Rev. D **60**, 074501 (1999). <https://doi.org/10.1103/PhysRevD.60.074501>. arXiv:hep-lat/9811003