



# Non-Markovian speedup evolution of a center massive particle in two-dimensional environmental model

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**Abstract** A two-dimensional ray model is introduced to realize the non-Markovian speedup evolution of a center massive particle gravitationally coupled to a controllable environment (multilayer arrangement of the massive particles). By controlling the environment, for instance by choosing a judicious mass of the environmental particles or by changing the separation distance of each massive particle, two dynamical crossover behaviors from Markovian to non-Markovian and from no-speedup to speedup are achieved due to the gravitational interactions between the system particle and environmental particles. It is obvious that the critical mass of the environmental particles or the critical separation distance for these two dynamical crossover behaviors restrict each other directly. The larger the value of the mass of the environmental particles is, the smaller the value of the critical separation distance should be requested. In addition, it should be emphasized that the non-Markovian dynamics is the principal physical reason for the speedup evolution of the system massive particle. Particularly, the non-Markovianity of the dynamics process of the system massive particle in the even ray case has better correspondence with the quantum speed limit time than that in the singular ray case.

## 1 Introduction

The most fundamental physical theories, quantum theory and general relativity, claim to be universally applicable and have been confirmed to a high accuracy in their respective domains. So the unification of quantum mechanics with general relativity is one of the most fundamental problems in theoretical physics [1]. Yet, it is hard to merge them into a

unique corpus of laws. One possible route to that general theory is the quantization of gravity, with the same spirit as other field theories. However, there is a long-standing debate whether gravity should be quantized [2–6].

More recently the idea of tabletop detectors has been widely discussed, opening up the possibility of revealing the quantum nature of gravity. The BWV experiment [7, 8], which proposes the induced entanglement of two massive particles, has attracted much attention. This scheme is based on the theorem that local operation and classical communication (LOCC) [9] cannot produce quantum entanglement in quantum information theory. Due to gravity based on a local theory, if the gravitational interaction produces entanglement between two massive particles, then quantum nature of gravity can be verified. In this experiment, micro-diamond with an embedded nitrogen-vacancy center spin has been designed as the superposition of two massive particles at a certain distance parallel to their separation direction. The gravitational interaction between the particles could cause quantum entanglement. For the feasible detection of an entanglement, one requires the superposition of a mesoscopic particle. In Ref. [10], an experimental setup for realizing such a superposition is presented. Further, the origin of generating the quantum entanglement is discussed in [11]. These studies have in turn stimulated several studies on testing the quantum properties of gravity [12–19]. Among them, Nguyen and Bernards have proposed a setup similar to that of the BMV experiment [19], in which they assume the superposition of two separated massive particles in the direction perpendicular to their separation direction. Due to the symmetry of the configuration, the gravitational interaction between two massive particle can be simplified and the model is easy to achieve.

In view of the current experimental verification and theoretical research on the quantum nature of gravity, some efforts have been recently devoted to investigating the role

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played by gravity on the dynamics control of the massive particles [13, 14, 18–21]. For instance, by studying the entanglement dynamics between two particles due to gravity, the authors showed that the system entangles as long as the coupling between the particles is strong [19]. Then the dynamics of a gravity-induced entanglement for  $N$  massive particles was analyzed [20] by extending to  $N$  massive particles. The above studies mainly focus on the influence of the presence of gravity on entanglement and decoherence dynamics of the massive particles. However, how to protect the quantum properties of the system such as quantum entanglement and quantum coherence under the action of gravity has not been studied yet. Understanding how to protect the quantum properties of a system under gravity is crucial for detecting the quantum properties of gravity.

Currently, several studies have shown that non-Markovian effects [22–27] not only suppress the decay of the coherence or the entanglement of quantum systems [28–30], but also accelerate the quantum evolution speed of the systems [31–36]. A non-Markovian speedup evolution of an open system would be preferable to deal with the robustness of quantum simulators and computers against decoherence [37, 38]. That is to say, the non-Markovian accelerated evolution of the system could resist the decoherence of the quantum system in the environment. Therefore, how to control the dynamics of a massive particle as a quantum system due to gravity, in particular, to manipulate non-Markovian effects and to improve the quantum dynamical speed, becomes extremely significant for detecting the quantum properties of gravity.

In our two-dimensional ray model, a center massive particle  $m_o$  interacts with multiple environmental particles arranged in the surrounding ray structure. The distance between the particles before and after the same ray is  $r$ , the distance between the particles in the first layer is  $r$ , and the other particles are placed in the direction of the ray according to the position of the particles in the first layer. For this model, we demonstrate how the non-Markovian speedup dynamics control of a massive particle can be achieved by manipulating the parameters of the adjustable environmental particles due to gravity. To quantify the non-Markovian speedup dynamics of the system massive particle, here we apply the quantum speed limit time (QSLT) to define the border between no-speedup and speedup quantum evolution of the massive system [31, 39–46]. And a general measure for non-Markovianity defined by Breuer et al. [22] is used to distinguish the Markovian dynamics and the non-Markovian dynamics. By investigating the influences of the separation distance of each massive particle and the mass of the environmental particles on the non-Markovianity and QSLT, two dynamical crossover behaviors of the system, from Markovian dynamics to non-Markovian dynamics and from no-speedup evolution to speedup evolution, can be realized via the gravitational interactions between the system

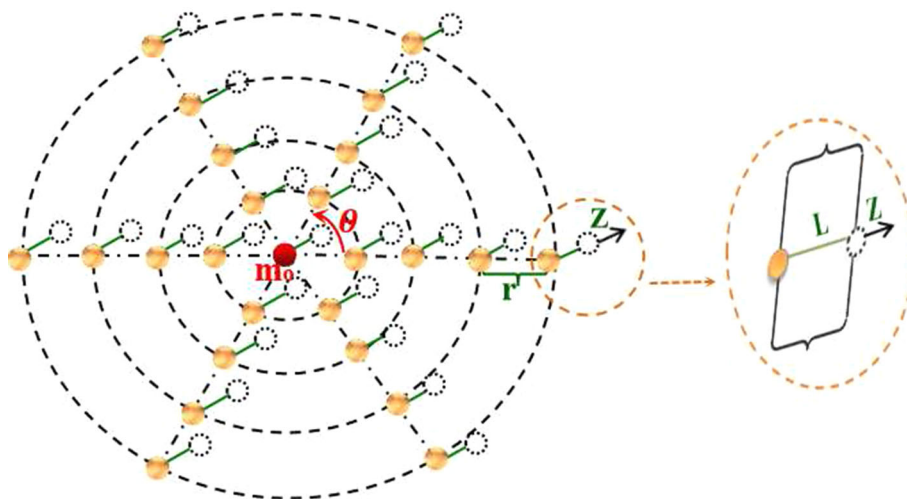
particle and each environmental particle. Additionally, we also have quantitatively analyzed the regions of the environment parameters for the occurrence of the non-Markovian speedup of the system particle. Our proposed scheme should be ideally performed in a zero-gravity environment. Because our model regards the system massive particle  $m_o$  and its environment particles as a closed system, the influence of external gravities on the whole closed system is eliminated as far as possible. When both the particles and the experimental setup, including the magnetic field gradient, are in perfect free fall, the equivalence principle prevents any observable effect of the other external gravitational fields.

The structure of this work is as follows. In Sect. 2, we give the global Hamiltonian for the “two-dimensional ray model”, and calculate the reduced density matrix of the system massive particle. In Sect. 3, we show the definition of QSLT and non-Markovianity for a dynamics process of a quantum system. In Sect. 4, we present the schematic diagram and conclusion analysis of the non-Markovianity and the QSLT in the two-dimensional singular ray case. In Sect. 5, the schematic diagram and conclusion analysis of the non-Markovianity and the QSLT in the two-dimensional even ray case are described. In Sect. 6, we control the non-Markovianity and the QSLT of the system massive particle by changing the number of particles in the first layer. In Sect. 7, the conclusion is put forward.

## 2 Model and dynamics

We mainly consider placing a multilayer adjustable environmental particles (the mass  $m_{n'}$  of the particle at coordinate position  $\mathbf{n}'$ , which is arranged in the  $(r_{n'}, \theta_{n'})$  polar coordinate plane as shown in Fig. 1) around a central system massive particle  $m_o$ . In order to exclude the effect of Casimir–Polder interaction on the gravitational interaction, we set the mass of the system particle  $m_o \sim 10^{-13}$  kg, the distance between the first layer of environmental particles and the system particle  $r \sim 250 \mu\text{m}$ . And other environmental particles are placed outward along the ray with the first layer of particles as reference. In this way, several rays are formed, and the distance between the environmental particles on the same ray is also set as  $r$ . And regardless of Casimir–Polder interaction between particles, the minimum distance between particles should not be less than  $r$ , so the first layer can only place six particles at most.

As shown in Fig. 1, when six particles are placed in the first layer, the distance between adjacent particles is equal to  $r$ , and the angle difference between adjacent rays equals to  $\pi/3$  (when the first layer is less than six, the distance between adjacent particles in the first layer can be greater than  $r$  (the angle difference can be greater than  $\pi/3$ ). And as long as the condition that the distance between two particles are not less



**Fig. 1** Schematic representation. The arrangement of environmental particles in two-dimensional polar coordinates ( $x$  is the number of branches of the rays, and here we take  $x = 6$  as an example). The central system massive particle with mass  $m_o$  is placed at polar coordinates  $(0, 0)$ , while the environmental particle at coordinate position  $\mathbf{n}'$  with mass  $m_{n'}$  is placed at coordinates  $(jr, \theta)$  ( $1 \leq j \leq N$ ,  $N$  is the number of environmental particles in each ray branch,  $r$  is the distance between the adjacent particles in each ray). The coordinate of the first particle to the right of  $m_o$  is fixed at  $(r, 0)$ . In addition, the total number of environmental particles  $n_x = Nx$ . This figure shows that each

particle is in a superposition of localized states along the  $z$ -axis separated by a distance  $L$  (Among them, the right is a local enlarged view of the model). In addition, the two-dimensional ray model we present is different from the traditional star model, that is, when the number of rays  $x$  is even, the two-dimensional ray model can be symmetric or asymmetric. Therefore, as long as the distance between the adjacent particles in the first layer meets the condition that the distance is not less than  $r$ , the distance between the adjacent particles in the first layer can also be non-equidistant

than  $r$  is satisfied, adjacent particles do not have to be placed at the same distance. The position of the particle  $m_{n'}$  on the  $(r_{n'}, \theta_{n'})$  is specified by  $(r_{n'}, \theta_{n'}) = (jr, \theta_{n'})$  with integers  $1 \leq j \leq N$  ( $N$  is the number of particles on each ray branch) and  $0 < \theta_{n'} < 2\pi$ . And the center system massive particle  $m_o$   $(0, 0)$  and the coordinate of the first particle on its right  $(r, 0)$  are fixed. Each particle is prepared to be in a spatially localized superposition state separated by a distance  $L$  along the direction of the  $z$ -axis.

The left and right paths of the particle  $m_{n'}$  at coordinate position  $\mathbf{n}'$  are represented as  $|\uparrow\rangle_{n'}$  and  $|\downarrow\rangle_{n'}$ . For this two-dimensional model, the initial state of the total system is chosen as

$$|\Psi(0)\rangle = |\psi(0)\rangle_o \otimes |\psi(0)\rangle_1 \otimes \dots \otimes |\psi(0)\rangle_{n'} \otimes \dots \otimes |\psi(0)\rangle_{n_x}, \tag{1}$$

where  $|\psi(0)\rangle_{n'} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_{n'} + |\downarrow\rangle_{n'})$  is the initial state of the massive particle  $m_{n'}$ ,  $n_x$  represents the total number of environmental particles in the two-dimensional model. The kinetic term is ignored in this work because all massive particles are in a zero-gravity environment and the wave packet of each particle does not spread. The initial state evolves under the gravitational interactions, then in this two-dimensional model, the corresponding Hamiltonian is

$$H = \sum_{n' \neq o} H_{o,n'} + \frac{1}{2} \sum_{n' \neq n''} H_{n',n''}, \tag{2}$$

$$H_{o,n'} = -\frac{\Delta_{o,n'}}{2} \sigma_z^{(o)} \otimes I^{(1)} \otimes \dots \otimes \sigma_z^{(n')} \otimes \dots \otimes I^{(n_x)}, \tag{3}$$

$$H_{n',n''} = -\frac{\Delta_{n',n''}}{2} I^{(1)} \otimes \dots \otimes \sigma_z^{(n')} \otimes \dots \otimes \sigma_z^{(n'')} \otimes \dots \otimes I^{(n_x)}, \tag{4}$$

where,  $H_{o,n'}$  represents the gravitational interaction between the central quantum system  $m_o$  and the environmental particle  $m_{n'}$  at coordinate position  $\mathbf{n}'$ , and  $H_{n',n''}$  represents the Newtonian gravitational potential energy between the ambient environmental particles at coordinates  $\mathbf{n}'$  and  $\mathbf{n}''$ . Therefore,  $\Delta_{o,n'}$  and  $\Delta_{n',n''}$  can be uniformly expressed as

$$\Delta_{o,n'} = \Delta_{o,j} = Gm_o m_{n'} \left( \frac{1}{jr} - \frac{1}{\sqrt{j^2 r^2 + L^2}} \right), \tag{5}$$

$$\Delta_{n',n''} = \left( \frac{Gm_{n'} m_{n''}}{\sqrt{j^2 r^2 + j'^2 r'^2 - 2jj'r^2 \cos(\theta_{n'} - \theta_{n''})}} - \frac{Gm_{n'} m_{n''}}{\sqrt{j^2 r^2 + j'^2 r'^2 - 2jj'r^2 \cos(\theta_{n'} - \theta_{n''}) + L^2}} \right), \tag{6}$$

here, two particles are in different rays,  $|\theta_{n'} - \theta_{n''}| \geq \pi/3$ . When two particles are in the same ray,  $|\theta_{n'} - \theta_{n''}| = 0$ . For the initial states in Eq. (1), the evolution density matrix of the total system at time  $t$  can be obtained by  $\rho(t) = e^{-iHt/\hbar} |\Psi(0)\rangle \langle \Psi(0)| e^{iHt/\hbar}$ . In order to explore the dynamical behavior of the system massive particle, we should calculate the evolution-reduced density matrix of the particle  $m_o$  from the density matrix of the total system by tracing over the environmental particles. Therefore, in the basis of the system particle  $\{|\uparrow\rangle_o, |\downarrow\rangle_o\}$ , the evolution-reduced density matrix of the system particle  $m_o$  is

$$\rho_{m_o}(t) = \text{Tr}_{\mathbf{1}, \dots, n', \dots, n_x} [\rho(t)] = \frac{1}{2} \begin{pmatrix} 1 & \prod_{j=1}^N \cos^x \left( \frac{\Delta_{o,j} t}{\hbar} \right) \\ \prod_{j=1}^N \cos^x \left( \frac{\Delta_{o,j} t}{\hbar} \right) & 1 \end{pmatrix}, \tag{7}$$

here  $x$  represents the number of ray branches. From Eq. (7), what we need to emphasize is that the dynamical behavior of the system particle can only be influenced by the gravitational interactions between the system massive particle  $m_o$  and the environmental particles. The gravitational interactions between environmental particles have no effect on the dynamics of the system particle. In the next section, we would monitor the system massive particle dynamical behavior (such as the non-Markovianity and the QSLT) by manipulating the various parameters of the environmental particles.

### 3 Definition of QSLT and non-Markovianity

The QSLT effectually defines the bound of minimal evolution time for an actual dynamics process from the initial state  $\rho_s(0)$  to the target final state  $\rho_s(\tau)$ . And it is helpful to analyze the maximal evolutionary speed of the dynamics process [31,33,36]. To describe the dynamics speedup of the system massive particle  $m_o$  by manipulating the environmental particles, a proper QSLT could be effectually defined by the bound of minimal evolution time for an arbitrary initial state  $\rho_s(0)$  to the corresponding target final state  $\rho_s(\tau)$ , which can facilitate to analyze the maximal evolution speed of the quantum system. In Campaioli et al. [48], the authors have redefined a new bound from a geometric perspective by using the method of states of geometric distance in a generalized Bloch sphere. The QSLT is as follows

$$\tau_{QSL} = \frac{\|\rho_s(0) - \rho_s(\tau)\|_{hs}}{\overline{\|\dot{\rho}_s(t)\|}}, \tag{8}$$

with  $\overline{\|\dot{\rho}_s(t)\|} = \frac{1}{\tau} \int_0^\tau dt \|\dot{\rho}_s(t)\|$  and  $\|A\|_{hs} = \sqrt{\sum_i M_i^2}$ . Here  $M_i$  are the singular values of  $A$ , and  $\tau$  is set as the

actual evolution time of the dynamics process. The advantage of this definition is tighter and easier to calculate for almost all quantum evolution processes. The physical interpretation of  $\tau_{QSL}$  is as follows: If  $\tau_{QSL}/\tau$  is equal to 1, the dynamics evolution of the quantum state would not be speedup. That is to say, the evolutionary speed is already maximum. While for  $\tau_{QSL}/\tau < 1$ , the dynamics evolution of the quantum state may be speedup. Moreover, the smaller this  $\tau_{QSL}/\tau$  is, the greater the quantum speedup could be.

As noted in [31,33,34], the non-Markovian behavior in the dynamics process from  $\rho_s(0)$  to  $\rho_s(\tau)$  can be attributed to quantum speedup evolution of a quantum system. That is, the associated information back-flow from the environment to the system, can lead to faster quantum evolution, and hence, to a smaller  $\tau_{QSL}/\tau$ . To further analyze the connection between the non-Markovian behavior and quantum speedup process, in the following, we would try to explore the dynamics control from Markovianity to non-Markovianity of the system massive particle  $m_o$  by adjusting the environment particles, i.e., by tuning the particles' separation distance  $L$  or changing the mass of the particles  $m_{n'}$ , and the environmental particles' number  $n_x$ .

It is well known that quantum non-Markovianity is a multifaceted phenomenon and different methods for quantifying memory effects do not agree with each other in general [49–51]. However, for a single quantum system such as the considered single massive particle in our work, a measure of non-Markovianity  $\mathcal{N}(\Phi)$  (Breuer–Laine–Piilo method [22]) can be introduced. For a quantum process  $\Phi(t)$ ,  $\rho_s(t) = \Phi(t)\rho_s(0)$ , with  $\rho_s(0)$  and  $\rho_s(t)$  denoting the density operators at time  $t = 0$  and at any time  $t > 0$ , then the non-Markovianity  $\mathcal{N}(\Phi)$  is quantified by

$$\mathcal{N}(\Phi) = \max_{\rho_s^{1,2}(0)} \int_{\sigma > 0} dt \sigma[t, \rho_s^{1,2}(0)], \tag{9}$$

here  $\sigma[t, \rho_s^{1,2}(0)]$  is the rate of change of the trace distance,  $\sigma[t, \rho_s^{1,2}(0)] = \frac{d}{dt} \mathcal{D}(\rho_s^1(t), \rho_s^2(t))$  represents the information flow. The trace distance  $\mathcal{D}$  describing the distinguishability between the two states is defined as  $\mathcal{D}(\rho_s^1(t), \rho_s^2(t)) = \frac{1}{2} \|\rho_s^1 - \rho_s^2\|$ ,  $\|M\| = \sqrt{M^\dagger M}$  and  $0 < \mathcal{D} < 1$ . And  $\sigma[t, \rho_s^{1,2}(0)] \leq 0$  corresponds to all dynamical semigroups and all time-dependent Markovian processes. A non-Markovian evolution is defined as a process in which, for certain time intervals  $\sigma[t, \rho_s^{1,2}(0)] > 0$ , the information flows back into the system temporarily, originating from the appearance of quantum memory effects. We need to take the maximum over all initial states for calculating the non-Markovianity. In Refs. [22,47], by drawing a sufficiently large sample of random pairs of initial states, it was proven that the optimal state pair of the initial states can

be selected as  $\rho_s^1(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  and  $\rho_s^2(0) = \frac{1}{2}(|\uparrow\rangle_s - |\downarrow\rangle_s)(\langle\uparrow| - \langle\downarrow|)$ .

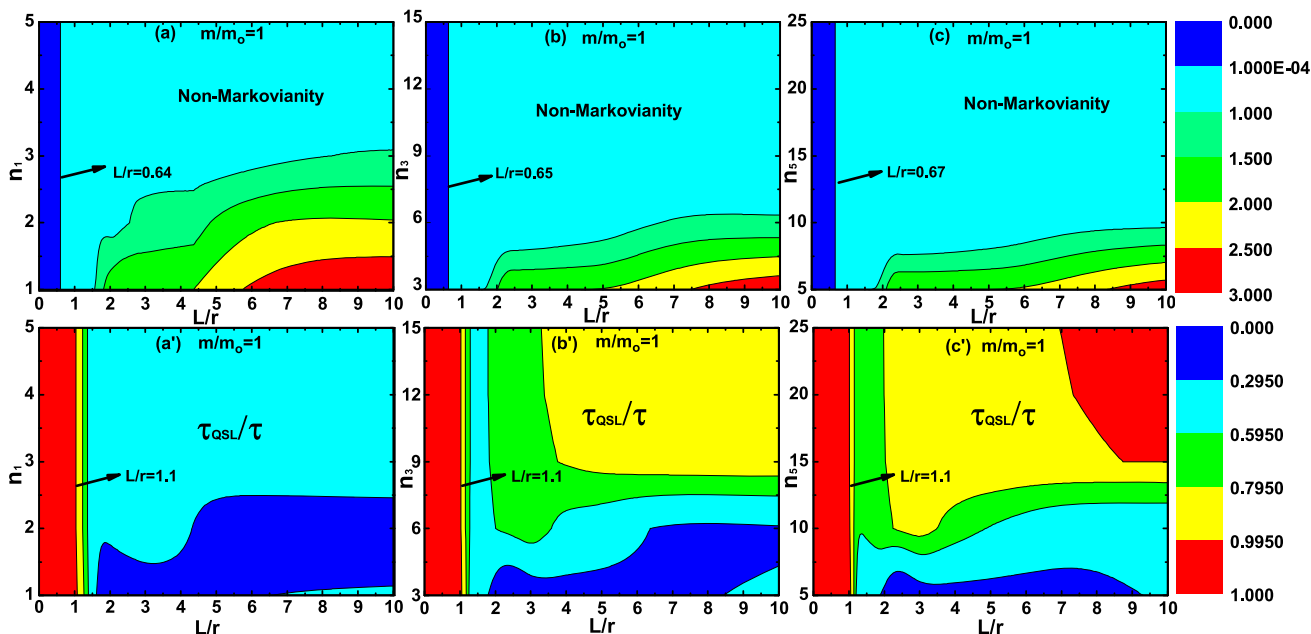
Therefore, by considering each environment massive particle prepared to be in a spatially localized superposition state  $\rho_{n'}^1(0) = \frac{1}{2}(|\uparrow\rangle_{n'} + |\downarrow\rangle_{n'})(\langle\uparrow| + \langle\downarrow|)$ , and choosing the optimal state pair of the initial states of the system particle as  $\rho_{m_o}^1(0) = \frac{1}{2}(|\uparrow\rangle_{m_o} + |\downarrow\rangle_{m_o})(\langle\uparrow| + \langle\downarrow|)$  and  $\rho_{m_o}^2(0) = \frac{1}{2}(|\uparrow\rangle_{m_o} - |\downarrow\rangle_{m_o})(\langle\uparrow| - \langle\downarrow|)$ , the trace distances  $\mathcal{D}(\rho_{m_o}^1(t), \rho_{m_o}^2(t))$  in the two-dimensional model can be respectively obtained

$$\mathcal{D}(\rho_{m_o}^1(t), \rho_{m_o}^2(t)) = \left| \prod_{j=1}^N \cos^x \left( \frac{\Delta_{o,j} t}{\hbar} \right) \right|. \tag{10}$$

The rate of change of the trace distance  $\sigma[t, \rho_s^{1,2}(0)]$  can be acquired from Eq. (10). Then one can readily verify the non-Markovianity  $\mathcal{N}(\Phi)$  of the dynamics process of the system particle  $m_o$ . The above is an illustration of the basic calculation of the non-Markovianity and the QSLT. In the following, by controlling the parameters (such as the mass of environmental particles  $m_{n'}$ , the number of environmental particles  $n_x$  and the separation distance  $L$  of each massive particle), the dynamical crossover behaviors from Markovian to non-Markovian and from no-speedup to speedup can be achieved in this two-dimensional model.

### 4 Dynamical crossover in two-dimensional singular ray case

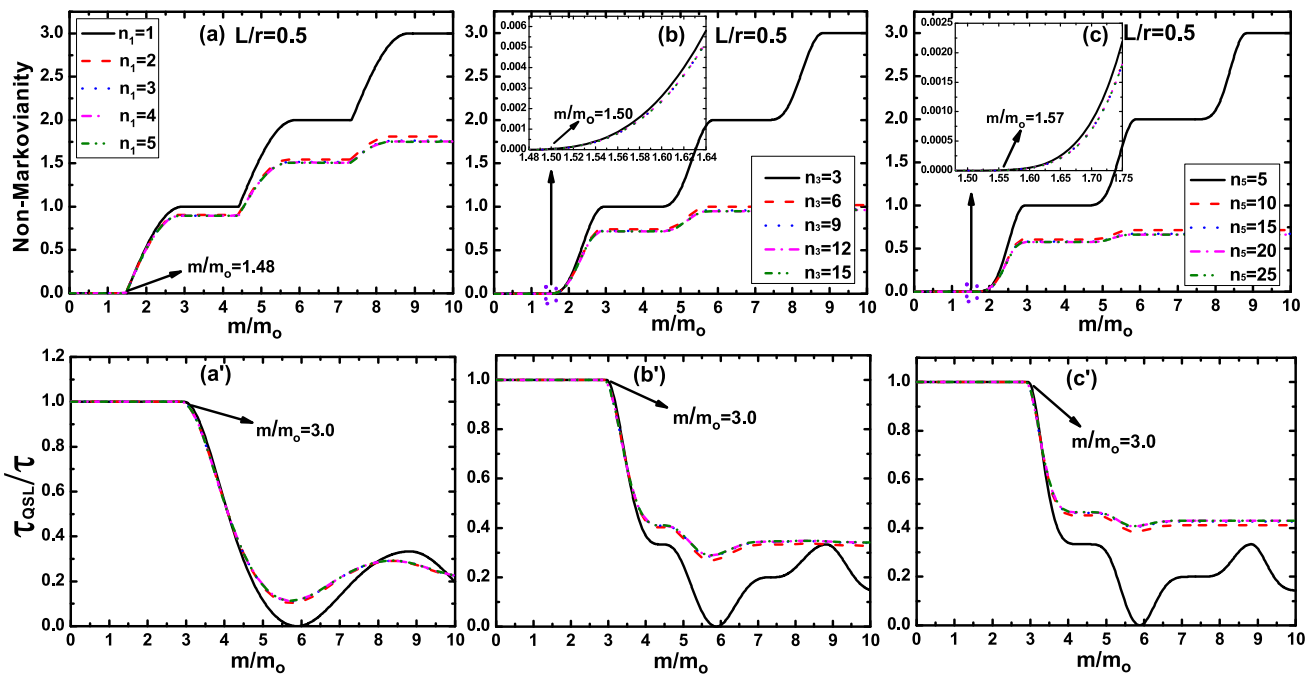
For two-dimensional singular ray case ( $x = 1, x = 3$  and  $x = 5$ ), we mainly explore the dynamic crossover behaviors of  $m_o$  coupled with the environmental particles with gravitational interactions. In Fig. 2, the relationship between the non-Markovianity  $\mathcal{N}(\varphi)$  and the QSLT  $\tau_{QSL}$  of the system particle dynamics process from  $\rho_{m_o}(0)$  to  $\rho_{m_o}(\tau)$  with the separation distance  $L$  and the number of environmental particles  $n_x$  under different structures, with the actual evolution time  $\tau = 0.4s$ ,  $r = 250\mu m$  and  $m_o = 10^{13}$  kg. For simplicity, here we consider that each environmental particle and the system massive particle have the same mass, that is  $m_o = m_{n'} = m$ . It is worth noting that a remarkable dynamical crossover behavior from Markovian dynamics to non-Markovian dynamics ( $\rho_{m_o}(0) \rightarrow \rho_{m_o}(\tau)$ ) can occur at a certain critical separation distance  $L_c = 0.64r$  (in the one-ray case  $x = 1$ ) of each environmental particle. When  $L < L_c$ , the dynamics process abides by Markovian behavior, and then the non-Markovianity would occur in the case  $L > L_c$ , as shown in Fig. 2a. And another dynamical crossover behavior from no-speedup to speedup of the dynamics process from  $\rho_{m_o}(0)$  to  $\rho_{m_o}(\tau)$  can also emerge at a different critical separation distance  $L_{c'} = 1.1r$ . In the case  $L < L_{c'}$ ,  $\tau_{QSL}/\tau = 1$  as shown in Fig. 2a', the quantum evolution process of the system particle cannot be accelerated. And when  $L > L_{c'}$ ,



**Fig. 2** For the two dimensional singular ray case ( $x = 1, x = 3$  and  $x = 5$ ), and the actual evolution time  $\tau = 0.4s$ ,  $r = 250\mu m$  and  $m = m_o = 10^{13}$  kg. **a-c** Non-Markovianity  $\mathcal{N}(\varphi)$  of the dynamics process of the system particle  $m_o$  as a function of  $L/r$  and the number of the

environment particles  $n_x$ . **a'-c'** The QSLT for the dynamics process of the system particle  $m_o$  from the initial state  $\rho_s(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  to the target final state  $\rho_s(\tau)$  as a function of  $L/r$  and the number of the environment particles  $n_x$





**Fig. 3** For the two dimensional singular ray case ( $x = 1, x = 3$  and  $x = 5$ ), and the actual evolution time  $\tau = 0.4s, r=250\mu m, L = 0.5r$  and  $m_o=10^{-13}$  kg. **a–c** Non-Markovianity  $N(\varphi)$  of the dynamics process of the system particle  $m_o$  as a function of  $m/m_o$  with different number of the environment particles  $n_x$ . **a'–c'** The QSLT

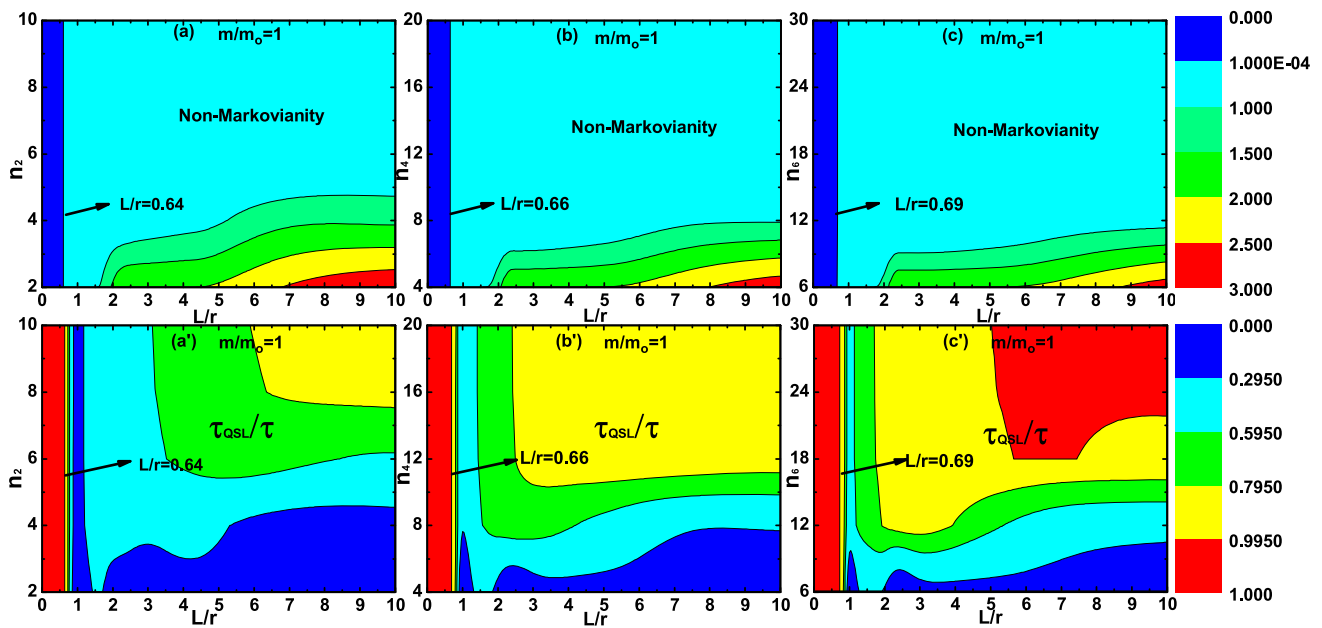
for the dynamics process of the system particle  $m_o$  from the initial state  $\rho_s(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  to the target final state  $\rho_s(\tau)$  a function of and  $m/m_o$  with different number of the environment particles  $n_x$

$\tau_{QSL}/\tau < 1$  can be acquired, the quantum dynamics process can be speedup. So the dynamical crossover behaviors from Markovian to non-Markovian and from no-speedup to speedup can be achieved by controlling the separation distance  $L$  of each environmental particle.

Then we observe horizontally, as shown in Fig. 2a–c, the more complex the structure of the two-dimensional ray model is (the larger the  $x$  value is), the more critical point  $L_c$  for the transformation from Markovian behavior to non-Markovian behavior increases. For example,  $L_c = 0.64r$  when the environment particle is a two-dimensional one-ray structure. While for the environment particles are placed in the two-dimensional five-ray structure,  $L_c = 0.67r$ . Then, we make a transverse comparison of a', b' and c' in Fig. 2 and find that under the singular two-dimensional ray structure, the critical value  $L_{c'} = 1.1r$  would not change if  $x$  value is changed. In addition, we find that the critical value  $L_c$  (or  $L_{c'}$ ) in all pictures will not change in their respective structures (when  $x$  value is unchanged), forming the critical line  $L/r$  as shown in Fig. 2. Thus, we can infer that the change of particle number in the same structure will not affect the change of critical value.

Furthermore, we also find that the dynamics process of the system particle would abide by the Markovian no-speedup behavior in the condition of the shorter separation

distance  $L$ , such as  $L = 0.5r$ . In order to achieve the non-Markovian speedup dynamics process of the system particle in  $L = 0.5r$ , the role of the environmental particles' mass  $m$  on the dynamics behavior has been cleared in Fig. 3. In the two-dimensional three-ray case, by adjusting the mass of environmental particles satisfying  $m > 1.50m_o$ , the evolution of the system particle can transfer from Markovian behavior to the non-Markovian behavior. We find that in this two-dimensional three-ray case, the  $m_c$  critical point becomes slightly larger compared to the two-dimensional one-ray case. The same change also occurs in the the two-dimensional five-ray case, as shown in Fig. 3c, where the transition point  $m_c = 1.57m_o$ . At the same time, we know from Fig. 3a'–c' that in the case of two-dimensional singular rays, when  $L = 0.5r$  and  $m > 3m_o$ , the dynamics of the system can also achieve dynamic crossover behavior from no-speedup to speedup. Except for the separation distance  $L$  and the environmental particles' mass  $m$ , the environmental particles' number  $n_x$  can also be related to the dynamics behavior of the system particle. In the two-dimensional singular ray case, the transition critical point causing the non-Markovian behavior (Figs. 2a–c and 3a–c) and the transition critical point at which the dynamics process of the system particle starts to accelerate (Figs. 2a'–c' and 3a'–c'), are both independent of the environmental particles' number  $n_x$ . From this, we



**Fig. 4** For the two dimensional even ray case ( $x = 2, x = 4$  and  $x = 6$ ), and the actual evolution time  $\tau = 0.4s, r=250\mu m$  and  $m=m_o=10^{-13} kg$ . **a-c** Non-Markovianity  $N(\varphi)$  of the dynamics process of the system particle  $m_o$  as a function of  $L/r$  and the number of the

environment particles  $n_x$ . **a'-c'** The QSLT for the dynamics process of the system particle  $m_o$  from the initial state  $\rho_s(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  to the target final state  $\rho_s(\tau)$  as a function of  $L/r$  and the number of the environment particles  $n_x$

can simply infer that the change of the critical point is only related to the separation distance  $L$  and the mass  $m$  of the environment particle. However,  $n_x$  plays an important role in controlling non-Markovian behavior or speedup behavior of the system. Through verification, it is found that the smaller  $n_x$ , the greater the non-Markovianity and the smaller QSLT appear. And then when the number of environmental particles  $n_x$  reaches a certain value, the non-Markovianity and the QSLT will not be affected by  $n_x$ . It can be observed more clearly from Fig. 3 that the color dashed lines tend to coincide with the increase of particle number  $n_x$  in their respective structures.

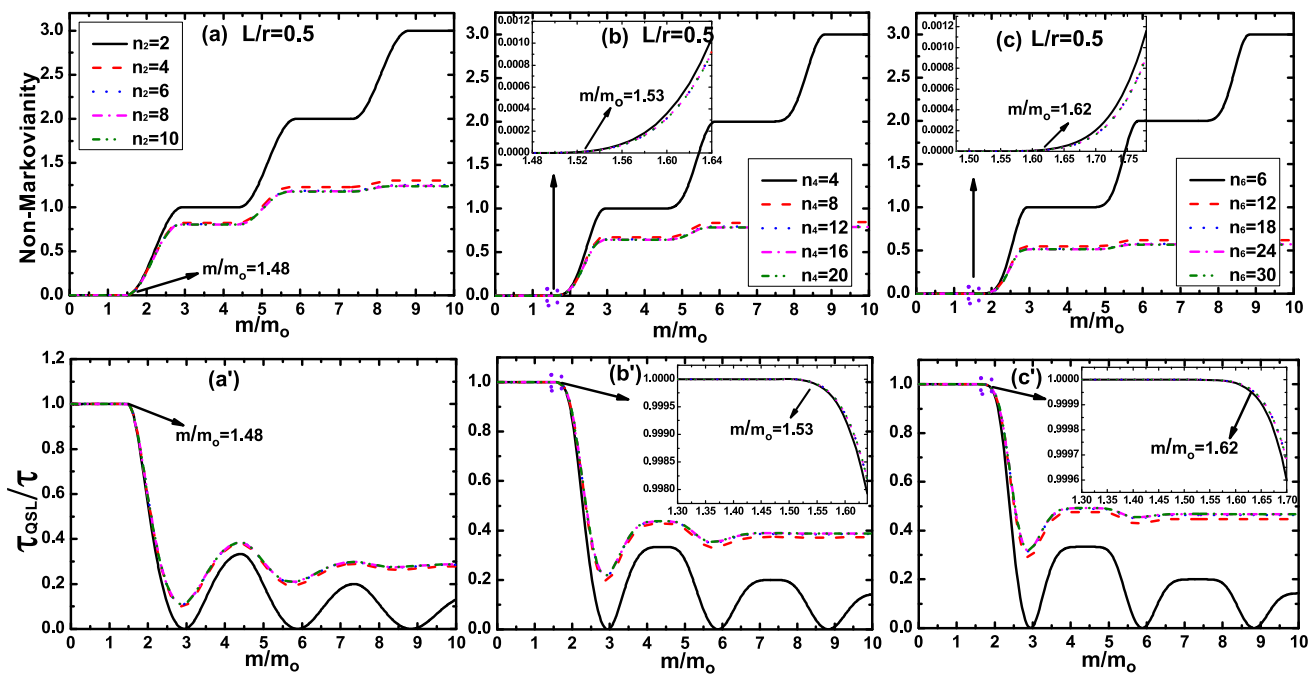
### 5 Dynamical crossover in two-dimensional even ray case

In this section, we have discussed the dynamic crossover behaviors of the system massive particle  $m_o$  in the two-dimensional even ray case ( $x = 2, x = 4$  and  $x = 6$ ). For example, in the two-dimensional four-ray case, by setting the mass of the system particle  $m_o$  equals to each environmental particle, i.e.,  $m_o = m_{n'} = m$ , the results can be obviously obtained. When  $L < 0.66r$ , the system dynamics process ( $\rho_{m_o}(0)$  to  $\rho_{m_o}(\tau)$ ) shows the Markovian behavior. By increasing  $L$  and making  $L > 0.66r$ , the system dynamics process would display the non-Markovian behavior. Therefore,  $L_c = 0.66r$  is the transition point from Markovian behavior to non-Markovian behavior (as shown

Fig. 4b). In addition, the critical value  $L_c = 0.64r$  in the two-dimensional two-ray case is slightly smaller. And the critical value is  $L_c = 0.69r$  in the case of two-dimensional six-ray, as shown in Fig. 4a, c. It can be seen that this result in the two-dimensional even ray case is consistent with that in the two-dimensional singular ray case. That is to say, the critical value of dynamic crossing behavior from Markovian to non-Markovian increases with the increase of structure complexity.

Moreover, when  $L = 0.5r, m = m_o$  satisfies, the evolution of the particle  $m_o$  is always Markovian behavior. Figure 5b describes the dynamics crossover behavior from Markovian to non-Markovian by adjusting the mass  $m$  of the environmental particles in the two-dimensional four-ray case, the transition point of this dynamics behavior of the system is  $m_c = 1.53m_o$ . Meanwhile, as shown in Fig. 5a, b, the critical points for the two-dimensional two-ray case and the two-dimensional four-ray case are  $m_c = 1.48m_o$  and  $m_c = 1.62m_o$ , respectively. At the same time, combined with the two-dimensional singular ray case, we draw a definitive conclusion that the more complex the structure (the larger the  $x$  value), the more difficult it is to reach the intersection point of Markovian to non-Markovian dynamic behavior.

While for the transition point for the dynamical crossover behavior from no-speedup to speedup of the dynamics process of the system particle as shown in Figs. 4b' and 5b' in the two-dimensional four-ray case, the critical separation distance and the environmental particles' mass satisfy  $L_c =$



**Fig. 5** For the two dimensional even ray case ( $x = 2, x = 4$  and  $x = 6$ ), and the actual evolution time  $\tau = 0.4s, r=250\mu m, L = 0.5r$  and  $m_o=10^{-13}$  kg. **a–c** Non-Markovianity  $N(\varphi)$  of the dynamics process of the system particle  $m_o$  as a function of  $m/m_o$  with different number of the environment particles  $n_x$ . **a'–c'** The QSLT for the

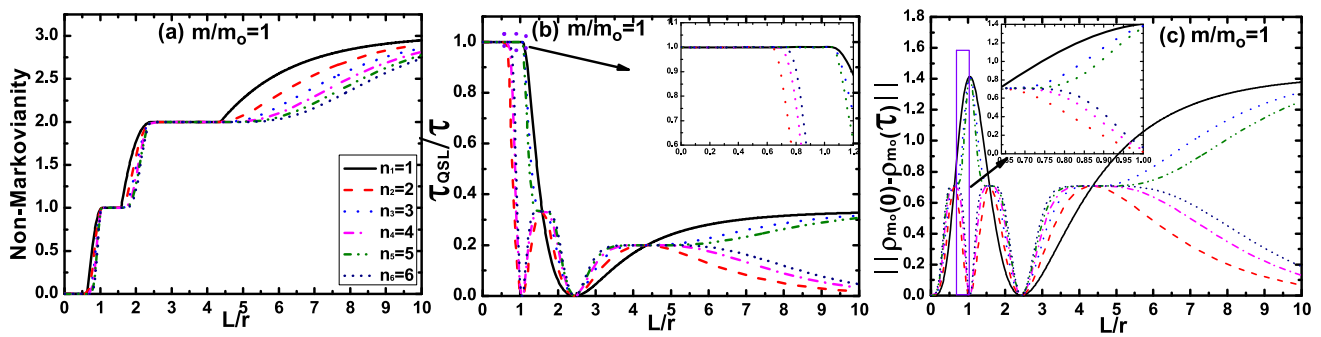
dynamics process of the system particle  $m_o$  from the initial state  $\rho_s(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  to the target final state  $\rho_s(\tau)$  as a function of  $m/m_o$  with different number of the environment particles  $n_x$

$0.66r$  and  $m_c = 1.53m_o$ . Obviously, in this two-dimensional four-ray case, two dynamics crossover behaviors would be converted at the same transition point. Similarly, the two-dimensional two-ray case and the two-dimensional six-ray case have the same conclusion, as shown in Figs. 4a', c' and 5a', c'. As noted in Refs. [31–33], the non-Markovianity in the dynamics process ( $\rho_{m_o}(0)$  to  $\rho_{m_o}(\tau)$ ), and the associated information backflow from the environment, can lead to faster quantum evolution and hence to smaller QSLT. So we can conclude that in the two-dimensional even ray case, the dynamical crossover behavior from no-speedup to speedup is strictly related to the dynamics crossover behavior from Markovian to non-Markovian. This conclusion is markedly different from that in the two-dimensional singular ray case. So we can easily get a strong rule: there is an obvious one-to-one correspondence between the non-Markovianity and the quantum speedup of the dynamics process of the system massive particle in the case of even number rays. However, the critical points increase with the complexity of the structure, for example, the critical points of the two dimensional six-ray case are  $L_c = 0.69r$  and  $m_c = 1.62m_o$  (Figs. 4b' and 5b').

As we work further, according to two-dimensional ray model, we have observed that the dynamical crossover behaviors for a massive particle can be manipulated by the appro-

priate parameters of the controllable environment. Generally in the realizable experiments, the parameters such as the environmental particles' number  $n_x$ , the mass  $m$  of the environmental particles and the separation distance  $L$  for each particle can be monitored. Then we need to emphasize that, when the number of the rays  $x$  is fixed, the change of environmental particles' number could not affect the value of transition point, but would affect the value of the non-Markovianity and the QSLT. In addition, the non-Markovianity tends to a stable value with the increase of separation distance. Finally, the greater the environmental particles' mass  $m$  satisfies, the greater the non-Markovianity is achieve, and the greater degree of dynamic acceleration will also occur. The physical reason for the conclusion can be explained as: for the models in our work, the greater the gravitational interaction between the system particle and the environmental particles, can lead to the stronger the information back-flow from the environmental particles to the system particle. And the larger non-Markovianity of dynamics process leads to the greater acceleration of the system particle' quantum state evolution. It is worth emphasizing that, the remarkable conclusion declares that our mechanism to control the non-Markovianity of a massive particle can also be used to accelerate the evolution speed of this massive particle.





**Fig. 6** The different structures by changing the number of particles in the first layer: **a** Non-Markovianity  $N(\varphi)$  of the dynamics process of the system particle  $m_o$  as a function of  $L/r$ . **b** The QSLT for the dynamics process of the system particle  $m_o$  from the initial state

$\rho_s(0) = \frac{1}{2}(|\uparrow\rangle_s + |\downarrow\rangle_s)(\langle\uparrow| + \langle\downarrow|)$  to the target final state  $\rho_s(\tau)$  a function of  $L/r$ . **c** The geometric distance  $\|\rho_{m_o}(0) - \rho_{m_o}(\tau)\|_{h_S}$  for different number of particles in the first layer as a function of  $L/r$ . Here the actual evolution time  $\tau = 0.4s$ ,  $r=250\mu m$  and  $m_o=10^{-13} kg$

### 6 Relationship between non-Markovianity and QSLT

In the above research content of this work, we have mentioned that under the fixed  $x$  value (i.e. in the same structure), the change of the environmental particle number has almost negligible effect on the critical value of dynamical crossover behaviors. Therefore, we give the non-Markovianity and the QSLT diagram corresponding to different ray number  $x$  in the first layer under other conditions being the same. We find that, the non-Markovianity decreases with the complexity of the structure. The difference between the critical points of the system particle  $m_o$  from Markovian behavior to non-Markovian behavior is not obvious (see Fig. 6a), but the difference between the corresponding quantum acceleration critical points is obvious (see Fig. 6b). Two interesting conclusions can be drawn: one is that the two-dimensional even-ray case accelerates more easily than the two-dimensional singular ray case. The other is that the two-dimensional two-ray case is the easiest to accelerate the system particle dynamics process, and this symmetrical structure is the best choice. However, we should note that this two-ray condition is not only linear, but also two-dimensional ray state (the angle between the two ambient particles is less than 180 degrees). In addition, we find that when  $1.6 \leq L/r \leq 4.35$  and there is only one particle in the environment, the system accelerates more easily, which is also the non-Markovianity corresponding to the QSLT in this stage.

We simply describe  $\|\rho_s(0) - \rho_s(\tau)\|_{h_S}$  changing with  $L/r$  to explore the reasons on the QSLT. In the small figure of Fig. 6c, in the singular-ray case  $\|\rho_s(0) - \rho_s(\tau)\|_{h_S}$  increases monotonically, while in the even-ray case  $\|\rho_s(0) - \rho_s(\tau)\|_{h_S}$  decreases monotonically, showing obvious shunt. Other conditions remain the same, and when  $n_2 = 2$ ,  $\|\rho_s(0) - \rho_s(\tau)\|_{h_S}$  has the minimum value, which also corresponds to Fig. 6b when  $n_2 = 2$ , it is easiest to accelerate. In addition, we can also find that when  $L/r \gtrsim 1$ , Fig. 6b, c can be well corre-

lated. So  $\|\rho_s(0) - \rho_s(\tau)\|_{h_S}$  is indeed one of the reasons to affect the QSLT.

### 7 Conclusions

In summary, the non-Markovian dynamics speedup control of a massive particle has been achieved by manipulating the parameters of the environment particles due to gravity. By considering two-dimensional ray model consisting of a massive particle gravitationally interacting with multiple massive particles as a environment, two dynamical crossover behaviors of the system particle, from Markovian dynamics to non-Markovian dynamics and from no-speedup evolution to speedup evolution, have been realized by adjusting the separation distance of each massive particle, the number and the mass of the environment particles.

We mainly consider the parameters (such as the environmental particles' number  $n_x$ , the environmental particles' mass  $m$ , the system particle' mass  $m_o$ , the separation distance  $L$  and the distance  $r$  between two adjacent particles) in the total system to manipulate the dynamical crossover behaviors of the system particle. In addition, when only one of these variables is regulated, and the other parameters are fixed. We have summarized the following physical conclusions: Firstly, with the increase of the separation distance  $L$  of each particle or the mass  $m$  of the environmental particle, the non-Markovianity of the dynamic process increases, the information back-flow capacity increases, and the quantum evolution process is easier to accelerate. Secondly, the quantum evolution process of the system particle is easier to accelerate in the even ray case than in the singular ray case. And in the even ray case, the dynamical crossover behavior from no-speedup to speedup is strictly related to the dynamics crossover behavior from Markovian to non-Markovian. They are one-to-one correspondence, and quantum acceleration is

caused by the non-Markovianity. However, in the singular ray case, the quantum speedup of the dynamics process is not only affected by the non-Markovianity. The geometric distance  $\|\rho_s(0) - \rho_s(\tau)\|_{h_s}$  must also be an important factor affecting the QSLT. So the non-Markovianity is not decisive for the speedup dynamics process. Thirdly, it is accurate that the number of environmental particles does not affect the change of critical value at the same  $x$  value (or in the same structure). When the structure is not changed (the  $x$  value is constant), the total number of particles increases to the second layer and above (for example,  $n_4 = 12$  and  $n_4 = 16$ , the dynamics crossing point is  $m/m_o = 1.53$ , as shown in Fig. 5b), and the critical value almost remains unchanged. This is because the distances between the environmental particles (in the second layer and above layers) and the system particle  $m_o$  are large, and the gravitational interactions between the system particle  $m_o$  and these environmental particles are weak, which has almost no effect on the change of the critical value.

Additionally, we make the explanations on our proposed scheme for the control of a massive particle evolution due to gravity in the two-dimensional ray model. (a) In terms of selection of actual physical system, the micro-size diamonds with an embedded nitrogen-vacancy center spin could be candidates for the massive system particle and environmental particles [7]. (b) When the model structure is the same (that is, the  $x$  value is the same), the change of the number of ambient particles will not affect the change of the critical value. Therefore, our most basic experimental environment can be determined, that is, the quantum system and the nearest ambient particles distributed around it constitute a basic experimental system. It should be noted that the number of the nearest ambient particles should not exceed six. According to the optimal result of our scheme (when  $x = 2$  and there are two environmental particles), it should be noted that the distance between environmental particles should be greater than or equal to  $250 \mu\text{m}$  (excluding the influence of Casimir–Polder interaction), and the size of system particles and environmental particles should be set at  $250 \mu\text{m}$ . (c) Both our work and the previous relevant study [12] mainly consider an ideal zero-gravity environment, in which the equivalence principle prevents any observable effects of other external gravitational fields. Of course, when the zero-gravity environment does not satisfy, the external gravitational fields [52] would affect the dynamic behavior of the system particle. In addition, for the sake of simplicity and applicability in theoretical analysis, we also choose the Newtonian approximation method. If the relativistic effect is taken into account, the existence of gravitational time dilation has a great influence on quantum coherence of the system particle [53–56]. The gravitational time dilation may affect the dynamical crossover behaviors in our scheme. Therefore, we would introduce the relativistic

effect into our model for further research to the control of quantum dynamics of the system massive particle.

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