



Gravitational matter creation, multi-fluid cosmology and kinetic theory

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Abstract A macroscopic and kinetic relativistic description for a decoupled multi-fluid cosmology endowed with gravitationally induced particle production of all components is proposed. The temperature law for each decoupled particle species is also kinetically derived. The present approach points to the possibility of an exact (semi-classical) quantum-gravitational kinetic treatment by incorporating back reaction effects for an arbitrary set of dominant decoupled components. As an illustration we show that a cosmology driven by creation of cold dark matter and baryons (without dark energy) evolves like Λ CDM. However, the complete physical emulation is broken when photon creation is added to the mixture thereby pointing to a crucial test in the future. The present analysis also opens up a new window to investigate the Supernova-CMB tension on the values of H_0 , as well as the S_8 tension since creation of all components changes slightly the CMB results and the expansion history both at early and late times. Finally, it is also argued that cross-correlations between CMB temperature maps and the Sunyaev–Zeldovich effect may provide a crucial and accurate test confronting extended CCDM and Λ CDM models.

1 Introduction

The late time accelerating stage of the universe is usually explained by assuming the existence of a dominant dark energy (DE) component, in addition to cold dark matter (CDM) and baryons. Its most popular candidate is the cosmological constant (Λ) or the rigid energy density of the current false vacuum state ($\rho_V = \Lambda/8\pi G$). The observational pillars providing convincing evidences for the so-called Λ CDM model include several independent astronomical observations [1, 2]. When combined with the primeval inflation for

describing the first stages of the early universe including the resulting scenario (inflation + Λ CDM) is widely known to be considerably simple and quite predictive.

Nevertheless, there are two old theoretical cosmological puzzles or mysteries plus at least two recent observational difficulties plaguing the Λ CDM model, namely: (i) the cosmological constant problem [3], (ii) the coincidence problem [4], (iii) the statistical observational discrepancy between measurements of the Hubble constant (H_0) from Supernovae (SNe) and other distance indicators at low [5] and intermediate redshifts [6–8] as compared with independent estimates at high redshifts based on the CMB angular power spectrum, and (iv) the so-called S_8 tension on the (σ_8, Ω_M) plane by confronting Planck + Λ CDM estimates with cosmic shear experiments [9], where σ_8 measures the current mass fluctuation in a scale of $8h^{-1}$ Mpc. Currently (both tensions H_0 and S_8 are the major observational anomalies plaguing the Λ CDM model (see below).

Many attempts to solve or allviate the theoretical puzzles gave rise to a plethora of dark energy possibilities including different kinds of running vacuum or decaying Λ -models, interactions in the dark sector and other noncanonical scalar fields [10–21]. There are also more fundamental approaches beyond Einstein's theory, like several extensions of Einstein's general relativity, among them: F(R), F(R,T) and Gauss-Bonnet type theories [22–27].

In the observational front, Riess and collaborators are now claiming for a statistical discrepancy of 5σ level between the local H_0 value and the one predicted by Planck + Λ CDM [28]. This means that CMB-SNe tension remains unsolved regardless of the realistic dark energy model in general relativity. Further, although statistically less significant (2.6σ to 3σ confidence levels) in comparison with the H_0 trouble, the S_8 estimates based on cosmic shear measurements from weak lensing collaborations, like Kilo-Degree Surveys (KiDS) and the Dark Energy Survey (DES) are providing val-

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ues for the parameter $S_8 = \sigma_8 \sqrt{\Omega_M/0.3}$ lower than the early-time probes [29–31]. Such observations are clearly opening the possibility to cosmic scenarios beyond Λ CDM model. Actually, some authors are claiming that solutions for the H_0 and S_8 tensions may require changes in the expansion history both at early and late-times (for more details see [32,33]).

In this context, the central interest here is to investigate a possible reduction of the dark sector by eliminating within general relativity, all possible species of dark energy, that is, we set $\Omega_{DE} \equiv 0$ from the very beginning, including the rigid vacuum itself. Therefore, even if early inflation was caused by the dominance of a vacuum state, its energy density was totally spent to create light particles forming the primeval thermal bath, as usually assumed in many spontaneously symmetry breaking models [34,35]. In addition, any subsequent phase transition was also unable to generate a sizable vacuum state potentially capable to accelerate the universe at late times. This means that the alluded discrepancy of the Λ -term and also the coincidence problem would be naturally solved.

In scenarios with $\Omega_{DE} = 0$ new challenges take place. For instance, some mechanism emulating the late time accelerating Λ CDM evolution must to be proposed at the level of the cosmic smooth expansion. Further, any new picture must also be successful in the perturbative approximation. In other words, although only slightly different from Λ CDM evolution, it needs to be as close as possible to the perturbed Λ CDM description.

The mechanism adopted here is the gravitationally induced particle creation by the expanding universe, a process already investigated in general relativity and also in alternative theories of gravity. Such investigations were carried out both from microscopic and macroscopic viewpoints. The former is based on methods and techniques from quantum field theory in curved spacetimes [36–42], while the latter rested upon the non-equilibrium irreversible thermodynamic approach [26,43–45]. Here we focus our attention on the irreversible macroscopic and its associated relativistic kinetic counterpart. The basic reasons are briefly outlined below.

Some early theoretical attempts [46–51] gave rise a decade ago to a new accelerating cosmology based on the “adiabatic” creation of cold dark matter (CCDM) [52]. In this general relativistic model with ($\Omega_{DE} = 0$), the cosmic smooth history is fully equivalent to the Λ CDM model. This very compelling aspect is not shared by any previous phenomenological matter creation models. In particular, the transition from a decelerating to the late-time accelerating stage happens at the same redshift. The evolution of perturbations was also discussed in such framework [53,54]. Under certain circumstances the CCDM dynamics is equivalence to the Λ CDM cosmology not only at the level of the Hubble flow but also for the evolving matter fluctuating field. Actually, it was demonstrated that the CCDM cosmology (without

creation of baryons) emulates perfectly the Λ CDM model in the linear and nonlinear levels [55,56]. Moreover, a kinetic approach for a single component based on a modified relativistic Boltzmann equation with matter creation was also proposed and the CCDM cosmology was kinetically recovered [57]. Later on, a model with creation of non-relativistic components (baryons + cold dark matter) with different creation rates was also proposed [58]. This scenario was also proved to be equivalent to Λ CDM also at a perturbative level, thereby confirming in a more general way the results of Ref. [55].

CCDM type models have also been tested through a Bayesian analysis applied to SNe Ia data and clusters. A joint analysis (without creation of photons) involving baryon acoustic oscillations (BAO) + cosmic microwave background (CMB) + SNe Ia data yielded $\Omega_m = 0.28 \pm 0.01 (1\sigma)$, where Ω_m is the matter density parameter. In particular, this implies that the model has no dark energy but the part of the matter that is effectively clustering is in good agreement with determinations from the large-scale structure [59].

It is also interesting that the simplest extensions of the original CCDM model by including baryons, mimicks exactly the observed accelerating Λ CDM cosmology with just one dynamical constant free parameter $\Gamma = \Gamma_b + \Gamma_{dm}$ describing the total creation rate of both components. Since the model is also equivalent to Λ CDM at the perturbative levels, it reinforces the idea that the “cosmic concordance model” may be only an effective cosmology. However, this macroscopic non-equilibrium treatment was not the most general one since the behavior of the CMB radiation with creation was separately discussed [60], and, as such, not properly inserted in the complete picture. In principle, the thermodynamic and kinetic results for massless particles remain valid even for dark photons and massless dark fermions [61].

Here we explore this kind of model one step further by discussing the general macroscopic formulation for a decoupled multi-fluid mixture endowed with “adiabatic and “non-adiabatic” matter creation of all components. It will be demonstrated here that the most interesting kinetic counterpart for applications to late time cosmology is the “adiabatic” case. Such multi-fluid formulations (macroscopic and kinetics) provide a detailed and coherent extension of the partial results discussed in the above quoted papers thereby suggesting a new route to investigate the H_0 and S_8 tensions, and, naturally, the CMB anisotropies and distortions. As far as we know, this is the first detailed study proposing the basic complete approach (macroscopic and kinetic) for a decoupled mixture with creation of all components.

The present article is planned as follows. In Sect. 2, the cosmic irreversible particle production process for a multi-fluid with different creation rates is thermodynamically and dynamically formulated. Corrections for the dynamic pressure and temperature law for each component are deduced

assuming different rates for the particle and entropy productions, but a special attention will be physically justified for the so-called “adiabatic” creation process. In Sect. 3, a Boltzmann equation for mixtures with “adiabatic” creation is proposed (some technicalities related to the extended Boltzmann equation are presented in the Appendix A). In Sect. 4, the counterpart of all non-equilibrium macroscopic results are kinetically derived. As an example of both consistent routes for a multifluid description (irreversible thermodynamics and kinetics), Sect. 5.1 is dedicated to a new extended CCDM cosmology including creation of baryons, CDM, CMB photons and neutrinos with different creation rates, whereas in Sects. 5.2 and 5.3 we focus on CMB effects (distortions and temperature anisotropies) with emphasis in observations relating distortions and CMB temperature maps. In particular, the cross-correlation of SZE and the integrated Sachs–Wolfe (ISW) effect, is suggested here as a crucial and accurate test for confronting CCDM and Λ CDM models. Finally, the article is closed in Sect. 6, by summarising the main results and conclusions for the relativistic extended accelerating model without dark energy ($\Omega_{DE} \equiv 0$) powered by gravitationally induced particle production.

2 Irreversible particle production: macroscopic multi-fluid formulation

For the sake of simplicity, let us consider that the spacetime geometry is described by a flat ($k = 0$) Friedman–Lemaître–Robertson–Walker (FLRW) metric:

$$ds^2 = dt^2 - a^2(t) [dx^2 + dy^2 + dz^2], \tag{1}$$

where $a(t)$ is the scale factor. For this metric, the non-null Christoffel’s symbols are:

$$\Gamma_{00}^0 = \Gamma_{jk}^i = 0, \quad \Gamma_{0j}^i = H\delta_j^i, \quad \Gamma_{ij}^0 = -Hg_{ij}, \tag{2}$$

where $\dot{H} = \dot{a}/a$ is the Hubble parameter.

The above expanding FLRW geometry (1) is also assumed capable to produce all species of particle existing in the Universe. In principle, due to the time-varying gravitational field all the particle components are springing-up in the spacetime with different creation rates. As will be discussed next, this macroscopic statement is also in agreement with the standard quantum field theoretic approach with an advantage, namely: the back reaction effect on the geometry is introduced from the very beginning through a specific stress term.

Following standard lines, the non-equilibrium thermodynamic states of a relativistic comoving mixture may be characterised by $3N$ -independent macroscopic quantities associated to the distinct components: the energy–momentum tensor (EMT), $T_{(i)}^{\mu\nu}$, a particle current, $N_{(i)}^\mu$, and the entropy current, $S_{(i)}^\mu$, where the total quantities are summed over all

species:

$$T_T^{\mu\nu} = \sum_{i=1}^N T_{(i)}^{\mu\nu}, \quad N_T^\mu = \sum_{i=1}^N N_{(i)}^\mu, \quad S_T^\mu = \sum_{i=1}^N S_{(i)}^\mu. \tag{3}$$

where $i = 1, 2, \dots, N$ denotes the i -th fluid component in the mixture.

Now, irreversible particle creation requires a modification of the basic equilibrium equations. In order to clarify how the above basic thermodynamic fluxes are modified, we need to modify the EMT and the balance equations for the particle number density and entropy currents in agreement with the second law of thermodynamics. Let us first consider the possible corrections in the EMT. It can be written as:

$$T_{(i)}^{\mu\nu} = T_{(i)|E}^{\mu\nu} + \Delta T_{(i)}^{\mu\nu}, \tag{4}$$

where $T_{(i)|E}^{\mu\nu}$ describes the equilibrium states and $\Delta T_{(i)}^{\mu\nu}$ is the correction associated to the effects of particle production. The homogeneity and isotropy of the FLRW metric implies that the only possibility is a scalar process which in terms of components reads:

$$\Delta T_{0(i)}^0 = 0 \quad \text{and} \quad \Delta T_{j(i)}^i = P_{c(i)}\delta_j^i, \tag{5}$$

where Latin indexes in the second equality containing round brackets are not summed. Note also that the first condition (for each component) removes the ambiguity on the energy density for non-equilibrium states. It means that ρ_i is the same function of the thermodynamic variables in the absence of dissipation, and $P_{c(i)}$ is a dynamic pressure here describing macroscopically the emergence of particles into the space-time. Although similar, it cannot be confused with the scalar (collisional) process of the standard nonequilibrium fluid mechanics and kinetic theory, widely known as bulk viscosity (second viscosity). In a manifestly covariant description we can write

$$\Delta T_{(i)}^{\mu\nu} = P_{c(i)}(g^{\mu\nu} - u^\mu u^\nu) \equiv -P_{c(i)}h^{\mu\nu}, \tag{6}$$

where $h^{\mu\nu}$ is the projector onto the rest frame of u^μ . As happens in the nonequilibrium thermodynamics, the correction for each component, $\Delta T_{(i)}^{\mu\nu}$, works like a source term for the equilibrium EMT. It can be incorporated back for describing the whole process through a conserved EMT as required by the Einstein field equations. Finally, by extending the one-fluid description and assuming for a while that at late times all components filling the universe are decoupled even in presence of gravitationally induced matter creation, the EMT of each component takes the form:

$$T_{(i)}^{\mu\nu} = (\rho_i + p_i + P_{c(i)})u^\mu u^\nu - (p_i + P_{c(i)})g^{\mu\nu}. \tag{7}$$

with the energy conservation law for each component becoming

$$u_\mu T_{(i);v}^{\mu\nu} = \dot{\rho}_i + \Theta(\rho_i + p_i + P_{c(i)}) = 0, \tag{8}$$

where $\Theta = 3H$ is the scalar of expansion

Note that if P_{ci} is negligible (no particle production), the equilibrium energy conservation law is recovered. Such a condition will be more physically defined below.

In the presence of a gravitational particle source, the balance equation for the particle flux and entropy current are redefined in order to derive the creation pressure of each component. The particle flux is $N_{(i)}^\mu = n_i u^\mu$, and its divergence takes the form

$$N_{(i);\mu}^\mu \equiv \dot{n}_i + n_i \Theta = n_i \Gamma_{iN} \Leftrightarrow \frac{\dot{N}_i}{N_i} = \Gamma_{iN}, \tag{9}$$

where by definition the total number of particles in the comoving volume, $N_i = n_i a^3$, and $[\Gamma_{iN}]$ with dimension of $[time]^{-1}$ is the number particle creation rate of the i -th component. Naturally, when compared with Θ this new microscopic time scale quantifies the efficiency of the gravitational particle production. In particular, if $\Gamma_{iN} \ll \Theta$, the creation process according to (9) can safely be neglected.

In the same vein, the entropy current reads:

$$S_{(i)}^\mu = s_i u^\mu \equiv n_i \sigma_i u^\mu, \tag{10}$$

where σ_i is the specific entropy per particle. Now, by taking the 4-divergence of the first equality above, the irreversible creation process implies that:

$$S_{(i);\mu}^\mu = \dot{s}_i + s_i \Theta = s_i \Gamma_{iS} \Leftrightarrow \frac{\dot{S}_i}{S_i} = \Gamma_{iS}, \tag{11}$$

where we have also defined $S_i = s_i a^3$ and Γ_{iS} is the entropy creation rate, a new (irreversible) time scale (dimension $[\Gamma_{iS}] \equiv [time]^{-1}$). In addition, $\Gamma_{iS} \geq 0$ because the constraint defining the second law of thermodynamics must be satisfied.

It is worth noticing the difference between Eqs. (9) and (11). The first one is clearly related with the emergence of particles in the spacetime which must also affect the fluid entropy production. Under certain conditions that will be discussed below, the existence of this second time scale implies that the variation rate of the specific entropy may be different from zero. In fact, since $\sigma_i = S_i/N_i$, its time-comoving derivative combined with (9) and (11) yields

$$\dot{\sigma}_i = \sigma_i (\Gamma_{iS} - \Gamma_{iN}). \tag{12}$$

Hence, $\dot{\sigma}_i = 0$ only for two different situations: (i) equilibrium states when $\Gamma_{iS} = \Gamma_{iN} \equiv 0$, and (ii) nonequilibrium states ($\dot{S}_i, \dot{N}_i \neq 0$), but $\Delta\Gamma_i = 0$ so that $\Gamma_{iS} = \Gamma_{iN}$. Following the nomenclature introduced long ago for a one-component fluid [45, 64, 65], this case it will also dubbed here ‘‘adiabatic’’ creation and will be discussed separately (see subsection IIIC below). As we shall see later, $\Gamma_{iS} \geq \Gamma_{iN}$. Hence, in general $\dot{\sigma}_i \geq 0$ for matter creation models with the specific entropy produced coming from the ‘‘uncompensated heat’’ spent for thermalization of the created particles

for any decoupled component. It is also worth notice that for $\Gamma_{iN} \equiv 0$ but $\Gamma_{iS} \neq 0$, we are describing (for each component) the pure phenomenon of bulk viscosity due to the universe expansion. In this case, $\dot{\sigma}_i > 0$ and the CMB thermal spectrum is destroyed in the course of the expansion. However, this does not happens in the ‘‘adiabatic’’ case (see Sect. 3).

At this point, one may ask: *What about the creation pressure and the temperature law for this general case?* Such topics, including the ‘‘adiabatic case’’, will be separately discussed in the next subsections.

2.1 Creation pressure

To begin with, we first remark that even in the presence of a dissipative processes like matter creation, the local equilibrium hypothesis means that each component satisfies the local form of the Euler relation [63] the comoving time derivative of the Gibbs law

$$n_i T_i \dot{\sigma}_i = d\rho_i - \left(\frac{\rho_i + p_i}{n_i} \right) dn_i, \tag{13}$$

Now, by taking the comoving time derivative of the above expression, and combining the result with the balance equation for the particle number density we find:

$$n_i T_i \dot{\sigma}_i = \dot{\rho}_i + (\rho_i + p_i)\Theta - (\rho_i + p_i)\Gamma_{iN}. \tag{14}$$

Thus, in order to obtain the general form of the creation pressure, it is enough to consider the energy conservation law for each component (8) plus the variation rate of the specific entropy (12). The general form of the creation pressure reads:

$$P_{ci} = -(\rho_i + p_i) \frac{\Gamma_{iN}}{\Theta} - \frac{n_i \sigma_i T_i}{\Theta} (\Gamma_{iS} - \Gamma_{iN}). \tag{15}$$

Ultimately, the above quantity must be incorporated in the Einstein field equations endowed with creation of all components. It is immediate to see that the set of independent FLRW equations take the irreversible form:

$$\rho_T \equiv \sum_{i=1}^N \rho_i = 3M_P^2 H^2, \tag{16}$$

$$p_T \equiv \sum_{i=1}^N (p_i + P_{ci}) = -M_P^2 [2\dot{H} + 3H^2], \tag{17}$$

where ρ_T and p_T are, respectively, the total energy density and pressure while $M_P = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, and P_{ci} is given by (15). Note that the late time Λ CDM model is readily recovered by assuming $P_{ci} = 0$, $N = 5$ and a matter-energy content formed by (1) CDM, (2) baryons, (3) radiation, (4) neutrinos, and (5) a dark energy represented by a vacuum state with negative pressure, $p_v = -\rho_v$. As usual, the model is completed when such decoupled components are described by the equation of

state (EoS), $p_i = \omega_i \rho_i$, where ω_i is contained in the interval $[-1, 1]$.

Note also that creation pressure above is always negative. In this case one may choose $\Omega_{DE} = 0$ and, as such, this kind of scenario may provide a description of the present accelerating stage of the Universe without vacuum energy density, thereby reducing the dark sector. In single-fluid description (CCDM), some possibilities have been discussed in the literature (see Introduction).

2.2 Temperature law

Let us now discuss how the temperature evolution law is modified by the particle creation (Γ_{iN}) and entropy production (Γ_{iS}) rates.

By taking the pair (n_i, T_i) as dependent variables, using the balance equation for the particle number density (9) and the thermodynamic identity:

$$T_i \left(\frac{\partial p_i}{\partial T_i} \right)_{n_i} = \rho_i + p_i - n_i \left(\frac{\partial \rho_i}{\partial n_i} \right)_{T_i}, \tag{18}$$

it is readily checked that the variation rate of the temperature takes the form:

$$\left(\frac{\partial \rho_i}{\partial T_i} \right)_{n_i} \dot{T}_i = -T_i \left(\frac{\partial p_i}{\partial T_i} \right)_{n_i} \frac{\dot{n}_i}{n_i} - P_{ci} \Theta - (\rho_i + p_i) \Gamma_{iN}, \tag{19}$$

or still, by inserting the second equality of (9) and the creation pressure (15):

$$\frac{\dot{T}_i}{T_i} = - \left(\frac{\partial p_i}{\partial \rho_i} \right)_{n_i} [\Theta - \Gamma_i] + \frac{n_i \sigma_i (\Gamma_{iS} - \Gamma_{iN})}{(\partial \rho_i / \partial T_i)_{n_i}}. \tag{20}$$

Note that whether $\Gamma_{iS} = \Gamma_{iN} \neq 0$, the form of the equilibrium temperature law

$$\frac{\dot{T}_i}{T_i} = \left(\frac{\partial p_i}{\partial \rho_i} \right)_{n_i} \frac{\dot{n}_i}{n_i}, \tag{21}$$

is readily recovered, as should be expected. For $\Gamma_{iS} \neq \Gamma_{iN}$, we obtain the temperature law for general “nonadiabatic” case, since $\dot{\sigma}_i \neq 0$ [see (12)]. Now, due to its physical importance, the “adiabatic” case, that is, $\Gamma_{iS} = \Gamma_{iN} \equiv \Gamma_i$ will be separately discussed next.

2.3 The “adiabatic” case

The “adiabatic” creation in the decoupled multi-fluid description is defined by $\dot{\sigma}_{(i)} = 0$, that is, $\Gamma_{iS} = \Gamma_{iN} \equiv \Gamma_i$ [see discussion below Eq. (12)]. In this case the creation pressure (15) becomes

$$P_{ci} = -(\rho_i + p_i) \frac{\Gamma_i}{\Theta} = -(\rho_i + p_i) \frac{\Gamma_i}{3H}, \tag{22}$$

where Γ_i is positive definite because the second law of thermodynamics. Actually, in this case the balance equation for the entropy boils down to:

$$S^{\mu}_{(i);\mu} = \dot{s}_i + s_i \Theta = s_i \Gamma_i \geq 0 \tag{23}$$

Hence, one may conclude from (9) that the Universe may only create matter ($\dot{N}_i > 0$). In addition, for the expanding Universe ($H > 0$), the associated creation pressure of each decoupled component is always negative. This generalises the original results of Prigogine et al. [43,44] for a single-fluid approach with irreversible particle creation. Of course, it also explains why a generic multi-fluid cosmology may accelerate at low redshifts mimicking (for non-relativistic components) the Λ CDM model (see Introduction).

Further, since $\sigma_i = S_i/N_i$, where $S_i = s_i a^3$ is the entropy in a comoving volume, and $N_i = n_i a^3$, the condition $\dot{\sigma}_i = 0$ also implies that

$$\dot{S}_i/S_i = \dot{N}_i/N_i \Leftrightarrow S_i = k_B N_i. \tag{24}$$

Therefore, the entropy growth associated to this gravitational particle production process is actually closely related with the quantum emergence of particles in the space-time. As we shall see, the created particles are in thermal equilibrium with the existing ones. An important point to keep in mind here is that the presence of the creation pressure in this macroscopic description is not the result of a collisional process as happens, for instance, with the standard bulk viscosity mechanism.

In the “adiabatic” case the temperature law is also directly obtained from (20) by taking $\Gamma_{iS} = \Gamma_{iN} = \Gamma_i$. Hence, the temperature law (20) reduces to

$$\frac{\dot{T}_i}{T_i} = - \left(\frac{\partial p_i}{\partial \rho_i} \right)_{n_i} (\Theta - \Gamma_i) = \left(\frac{\partial p_i}{\partial \rho_i} \right)_{n_i} \frac{\dot{n}_i}{n_i}, \tag{25}$$

thereby recovering the standard equilibrium relation in the limit $\Gamma_i \rightarrow 0$. In the nonrelativistic approximation [62], the EoS reads

$$\rho_i = n_i m_i + \frac{3}{2} n_i T_i = n_i m_i + \frac{3}{2} p_i, \tag{26}$$

with the temperature law (25) assuming the modified form:

$$\frac{\dot{T}_i}{T_i} = -2 \frac{\dot{a}}{a} + \frac{2}{3} \Gamma_i, \tag{27}$$

which for $\Gamma_i = 0$ also reduces to the standard equilibrium result, $T_i \propto a^{-2}$. By using the EoS, $p_i = \omega_i \rho_i$, we also see that the temperature law for arbitrary values of ω_i becomes:

$$\frac{\dot{T}_i}{T_i} = -3\omega_i \frac{\dot{a}}{a} + \omega_i \Gamma_i. \tag{28}$$

Hence, for $\Gamma_i = 3\beta_i H$, a simple integration of the above equation yields, $T_i = T_{0i} (1+z)^{3\omega_i(1-\beta_i)}$. In particular, for radiation (CMB) $\omega_i = 1/3$, $\beta_i \equiv \beta$, this expression reduces

to $T = T_0(1 + z)^{1-\beta}$, where β would be determined by the astronomical observations [75]. Later on, several authors investigated how β would be constrained by using the absorption lines of quasars at the redshift of the absorber [76, 77], as well as from Sunyaev–Zeldovich effect [78–87]. More recently, other relations for “adiabatic” production based on different phenomenological expressions for Γ_i have also been proposed and constrained by the existing observations [31, 60, 87–89]. Note also that for CMB or more generally for massless (bosonic or fermionic) particles ($\omega_i = 1/3$), the temperature evolution (28) can be rewritten as

$$\frac{\dot{T}_i}{T_i} = -\frac{\dot{a}}{a} + \frac{\Gamma_i}{3} \Leftrightarrow \frac{\dot{T}_i}{T_i} = -\frac{\dot{a}}{a} + \frac{\dot{N}_i}{3N_i}, \tag{29}$$

and a simple integration yields:

$$T_i(t)a(t)N_i(t)^{-1/3} = \text{const.} \rightarrow T_i = T_0(1 + z) \left(\frac{N_i(t)}{N_0} \right)^{1/3}. \tag{30}$$

Therefore, if the average number of photons $N_i(t)$ is constant (no photon creation), the standard CMB temperature law is recovered. In addition, since $N(t) \leq N_0$, it follows that the value of the temperature for a finite redshift is always smaller than the one predicted by the Λ CDM model.

Now, some comments are in line in order to stress the generality of the above temperature law. Firstly, it is widely believed that cosmological creation of photons in the expanding Universe is not allowed because the blackbody form of the CMB spectrum is destroyed [34, 90–92]. However, as discussed long ago and rigorously proved recently [60], the blackbody form of the CMB spectrum is still preserved when gravitational photon production occurs under “adiabatic” conditions. There is a twofold reason for that: (i) the second equality in (25) has the same equilibrium form. In fact, for $\omega_i = 1/3$, simple integration yields $n_i \propto T_i^3$. In addition, from $\dot{\sigma}_i = 0$, the Gibbs law (14) yields $\rho_i \propto n_i^{4/3}$ so that $\rho_i \propto T_i^4$, which are the same equilibrium relations for blackbody radiation, and (ii) as we shall see next section, the preservation of the equilibrium shape for any component in the mixture is also a direct consequence of the modified relativistic Boltzmann equation recently derived by incorporating matter creation under “adiabatic” conditions. This is an interesting point because a more detailed study of the CMB anisotropies and distortions requires the previous knowledge of the conditions under which a blackbody shape is preserved.

3 Modified Boltzmann equation and particle creation

From a kinetic viewpoint, the behavior of a decoupled multi-fluid mixture can properly be derived by following the evolution of each phase space density, $f_{(i)}(x_{(i)}^\mu, P_{(i)}^\mu)$. If one includes gravitational matter creation of all components, this

of course must be governed by a suitable modification of the Boltzmann equation.

Let us first recall that in the relativistic kinetic framework, the basic macroscopic quantities (fluxes) are microscopically defined taking the averaging over the distribution function [34, 68]. For each component we have:

$$N_{(i)}^\mu = \frac{g_{(i)}}{(2\pi)^3} \int f_{(i)} P_{(i)}^\mu \sqrt{g} \frac{d^3 P_{(i)}}{P_{(i)}^0}, \tag{31}$$

$$S_{(i)}^\mu = -\frac{g_{(i)}}{(2\pi)^3} \int [f_{(i)} \ln f_{(i)} - f_{(i)}] P_{(i)}^\mu \sqrt{g} \frac{d^3 P_{(i)}}{P_{(i)}^0}, \tag{32}$$

$$T_{(i)}^{\mu\nu} = \frac{g_{(i)}}{(2\pi)^3} \int f_{(i)} P_{(i)}^\mu P_{(i)}^\nu \sqrt{g} \frac{d^3 P_{(i)}}{P_{(i)}^0}, \tag{33}$$

where $g_{(i)}$ counts the internal degrees of freedom (spin states degeneracy) of a given component, g is the metric determinant, P^μ is the comoving momentum, and, as before, the indexes (i) denotes the i -th component in the mixture. Henceforth, unless explicitly stated, all repeated Latin scripts in round brackets are not summed.

The equilibrium states associated to the i -th decoupled component in the mixture is described by a distribution function. For a relativistic non-quantum weakly-interacting dilute gas, it assumes the form, $f_{(i)} = \exp(\alpha_i - \beta_i E_i)$, where $\alpha_i(t) = \mu_i/T$ defines the relativistic chemical potential and $\beta_i(t)$ is the inverse of temperature [66]. Such a form is a solution of the standard collisionless Boltzmann equation (CBE). It is widely known that when the mass shell condition, $g_{\mu\nu} P_{(i)}^\mu P_{(i)}^\nu \equiv m_{(i)}^2$, is imposed for the physical momentum in a flat geometry [$p_{(i)} = a(t)P_{(i)}$], $f_{(i)} = f_{(i)}(t, p_{(i)})$, the CBE can be written as [34, 66, 68]

$$\frac{1}{E_i} \mathcal{L}[f_{(i)}] \equiv \frac{\partial f_{(i)}(t, p_{(i)})}{\partial t} - H p_{(i)} \frac{\partial f_{(i)}(t, p_{(i)})}{\partial p_{(i)}} = 0 \tag{34}$$

where $\mathcal{L}[f_{(i)}]$ is the standard Liouville operator.

It is readily verified using (34) that the kinetic definitions (31)–(33) reproduce the *macroscopic equilibrium expressions* for $N_{(i)}^\mu, S_{(i)}^\mu, T_{(i)}^{\mu\nu}$ and also the equilibrium conservation equations in the FLRW metric, namely: $N_{(i)}^\mu{}_{;\mu} = 0, S_{(i)}^\mu{}_{;\mu} = 0$, and also the energy conservation law, $u_\mu T_{(i)}^{\mu\nu}{}_{;\nu} = 0$ (see calculations in [66–68]).

At this point, it is also natural to ask: *What happens in the presence of gravitationally induced matter creation?* In the next two sections, it will be shown that an appropriated (collisionless) modified Boltzmann equation (MBE) also reproduce all the results of Sect. 3 when gravitational matter creation occurs under “adiabatic” conditions (see also appendix A). Only in this case, the equilibrium shape of the distribution function is preserved both for massive and massless particles.

To begin with, let us first recall that the distribution function, $f_{(i)}(x_{(i)}^\mu, P_{(i)}^\mu)$, for each component, must be a solution

of the MBE. In the present context, a basic requisite is that particle and entropy productions must be included (see [57] for a single fluid), in agreement with the second equation in (24). For a decoupled self-gravitating mixture, its manifestly covariant expression in terms of the comoving momentum takes the form:

$$P_{(i)}^\mu \frac{\partial f_{(i)}}{\partial x_{(i)}^\mu} - \Gamma_{\alpha\beta}^\mu P_{(i)}^\alpha P_{(i)}^\beta \frac{\partial f_{(i)}}{\partial p_{(i)}^\mu} + \mathcal{P}_{Gi}(x_{(i)}^\mu, P_{(i)}^\mu) = C[f_{(i)}, f'_{(i)}], \tag{35}$$

where $P_{(j)}^\mu \equiv (E_{(j)}, P_{(j)}^i)$ is the four-momentum and the geodesic equation has been used to rewrite the second term. $C[f_{(i)}, f'_{(i)}]$ is the standard collisional term, including all possible interactions and sources of distortions changing the form of the equilibrium distribution. For cosmic background radiation (CMB), for instance, it also includes Compton scattering, double Compton and Bremsstrahlung emissions. The physical consequences of such a term have already been quite explored in the literature [34, 92] but, for a while, we consider $C[f_{(i)}, f'_{(i)}] \equiv 0$ thereby focusing our attention over the “adiabatic” gravitationally induced creation contribution.

The extra term in the left hand side (*l.h.s.*) of (35), $\mathcal{P}_{Gi}(x_{(i)}^\mu, P_{(i)}^\mu)$, is a non-collisional source term describing the gravitationally induced particle production process due to the expansion of the Universe. This process cannot be thought as a kind of particle injection whose momentum must be lately redistributed thereby repopulating the distribution, and, as such, provoking distortions in the equilibrium spectrum. Although being responsible for a dynamical creation pressure and modifying the temperature law, it does not change the equilibrium shape of the distribution function. This explain why it was written in the left hand side of the above modified Boltzmann equation. Its choice is dictated by two simple criteria [57, 60]: (i) $\mathcal{P}_{Gi} \propto \Gamma_{\lambda\nu}^i$ since $\mathcal{P}_{Gi}[f_{(i)}]$ should disappear in the absence of gravity when the Levi-Civita connections are identically null ($g_{\mu\nu} = \eta_{\mu\nu}$), and (ii) $\mathcal{P}_{Gi} \propto \Gamma_i / \Theta$. Such a condition is suggested by the macroscopic equations [see (9) and (11)] when $\dot{\sigma}_i = 0$ (“adiabatic” case). As in the macroscopic approach, Γ_i represents the creation rate of the *i*-th fluid component in the mixture.

Now, it is worth noticing that the constraint derived from the “mass shell” condition, $g_{\mu\nu} P_{(i)}^\mu P_{(i)}^\nu \equiv m_{(i)}^2$, has not been imposed in the above expression. By neglecting the standard collisional term, $C[f_{(i)}, f'_{(i)}] \equiv 0$, the mass shell Boltzmann equation in terms of the local momentum can be written as:

$$\frac{1}{E_i} \mathcal{L}[f_{(i)}] \equiv \frac{\partial f_{(i)}}{\partial t} - H p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} + \frac{\Gamma_i}{3} p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} = 0. \tag{36}$$

where $f_{(i)} = f_{(i)}(t, p_{(i)})$, where $p_{(i)}$ is the modulus of the momentum of the *i*-th decoupled component (for more details see Appendix A).

Note that the non-null creation rates Γ_i which is a consequence of the expanding Universe, contributes at the level of the Liouville operator like the Hubble parameter (with a changed sign), as should be physically expected for a purely gravitational effect. In addition, for $\Gamma_i \ll 3H$ such a term is negligible as previously determined based on the macroscopic approach (see Sect. 3) thereby reducing (36) to the standard collisionless Boltzmann equation without creation. In what follows the above equation will be justified by deriving kinetically the macroscopic balance equations with “adiabatic” creation (see also Appendix A for more details).

4 Kinetics versus thermodynamics: recovering the macroscopic results with “adiabatic” creation

Let us now show how the modified Boltzmann equation (50) allow us to recover the basic macroscopic balance equations for the particle and entropy fluxes, as well as, the energy–momentum tensor including the creation pressure. All the derivations are based on the kinetic definitions for an arbitrary number of decoupled components, as given in the previous section [see (31)–(33)].

4.1 Particle flux

To begin with, let us combine the flat FLRW geometry (1) and (31). As one may check, the only non-null component of the particle flux is the particle number density itself:

$$N_i^0 = n_i = \int f_{(i)} d^3 p_{(i)}. \tag{37}$$

Hence, the divergence of the particle flux becomes:

$$N_{(i); \mu}^\mu = \frac{1}{a^3} \frac{\partial}{\partial t} \left(a^3 \int f_{(i)} d^3 p_{(i)} \right), \tag{38}$$

and by expanding the product and using the collisionless Boltzmann equation (36) we find:

$$N_{(i); \mu}^\mu = 3Hn_i + H \left(1 - \frac{\Gamma_i}{3H} \right) \int_0^\infty p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} d^3 p_{(i)}. \tag{39}$$

Now, integrating by parts and using that for a well behaved distribution, the product $p^3 f(p)$ vanishes in the limits of integration, one finds:

$$\int_0^\infty p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} d^3 p_{(i)} = -3n_i,$$

and inserting the above result into (39), we recover the macroscopic evolution equation for the particle number density with “adiabatic” creation [cf. Eq. (9)]

$$N_{(i); \mu}^\mu = n_i \Gamma_i. \tag{40}$$

4.2 Entropy flux

Similarly, the balance equation for $S_{(i)}^\mu$ may be derived based on the previous approach for $N_{(i)}^\mu$. The only non-null component of the entropy flux $S_{(i)}^\mu$ also defines the entropy density [see kinetic definition in (32)]

$$S_{(i)}^0 = s_i = - \int [f_{(i)} \ln f_{(i)} - f_{(i)}] d^3 p_{(i)}, \tag{41}$$

while the divergence of the entropy flux reads:

$$S_{(i);\mu}^\mu = -\frac{1}{a^3} \frac{\partial}{\partial t} \left(a^3 \int [f_{(i)} \ln f_{(i)} - f_{(i)}] d^3 p_{(i)} \right). \tag{42}$$

Then, by expanding the product, and again using the collisionless Boltzman equation with creation (36) we find:

$$S_{(i);\mu}^\mu = 3Hs_{(i)} - H \left(1 - \frac{\Gamma_i}{3H} \right) \int p_{(i)} \frac{\partial f_{(i)}}{\partial p} \ln f_{(i)} d^3 p_{(i)}. \tag{43}$$

To proceed further we need to solve the integral in the last term. By adopting spherical coordinates and solving it by parts we obtain

$$\int_0^\infty p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} \ln f_{(i)} d^3 p_{(i)} = 3s_i,$$

where it was used that the product $p_{(i)}^3 [f_{(i)} \ln f_{(i)} - f_{(i)}]$ vanishes in the integration limits. Now, by inserting the above result into (43), the kinetic balance equation for the entropy of a decoupled mixture with creation

$$S_{(i);\mu}^\mu = s_i \Gamma_i, \tag{44}$$

is recovered. This result is clearly a consequence of the modified Boltzmann equation being also in perfect agreement with (11) appearing in the macroscopic approach when ‘‘adiabatic’’ conditions are assumed ($\Gamma_{iS} = \Gamma_{iN} = \Gamma_i$).

4.3 Creation pressure and energy–momentum tensor

In order to obtain kinetically the creation pressure, let us multiply by E_i the modified Boltzmann equation (36). Now, by integrating the result over the momentum space, it follows that

$$\int_0^\infty E_{(i)} \frac{\partial f_{(i)}}{\partial t} d^3 p_{(i)} - H \left(1 - \frac{\Gamma_i}{3H} \right) \times \int_0^\infty E_{(i)} p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} d^3 p_{(i)} = 0. \tag{45}$$

A simple integration term by term yields:

$$\dot{\rho}_i + \Theta(\rho_i + p_i) - (\rho_i + p_i)\Gamma_i = 0, \tag{46}$$

which can be rewritten as:

$$\dot{\rho}_i + (\rho_i + p_i + P_{ci})\Theta = 0, \Leftrightarrow P_{ci} = -(\rho_i + p_i) \frac{\Gamma_i}{\Theta}. \tag{47}$$

As should be expected, the creation pressure above is exactly the same macroscopic expression for the ‘‘adiabatic’’ case [see Eq. (22)]. Therefore, the creation rate Γ_i for each decoupled component appearing in the modified Boltzmann equation (36), also modulates the noncollisional correction term that disappears in the special relativistic limit when $\Gamma_{\beta\gamma}^\alpha \equiv 0$.

Naturally, the above result implies that the energy conservation law can also be obtained from the kinetic definition of the EMT given by (33). In order to show that let us now calculate the divergence of the total energy–momentum tensor projected onto the four-velocity u_μ . Firstly, it should be recalled that unlike to what happens with the energy density, there is no constraint conditions to the kinetic pressure for states out of equilibrium [66]. Thus, it is also natural to assume the existence of a corrective (non-collisional) creation pressure term. Now, let us also assume that homogeneity and isotropy dictates the following form $\Delta T_{(i)k}^l = -P_{ci} \delta_k^l$, or equivalently, $\Delta T^{\mu\nu}_{(i)} = -P_{ci} h^{\mu\nu}$. However, for the sake of generality, we consider for a while that P_{ci} is an *unknown* creation pressure not necessarily equal to the value given by (47). Thus, the total EMT for each component is $T_{(i)}^{\mu\nu} = T_{(i)|E}^{\mu\nu} + \Delta T_{(i)}^{\mu\nu}$, being kinetically defined by the expression (33). In this case, we can write for the projected divergence:

$$u_\mu T_{(i);v}^{\mu\nu} \equiv u_\mu \left[\frac{1}{a^3} \frac{\partial}{\partial x^\nu} (a^3 T_{(i)}^{\mu\nu}) + \Gamma_{\alpha\beta}^\mu T_{(i)}^{\alpha\beta} \right], \tag{48}$$

Now, by summing over the repeated indices and using the last expression in (2) it becomes:

$$\begin{aligned} u_\mu T_{(i);v}^{\mu\nu} &\equiv \frac{1}{a^3} \frac{\partial}{\partial t} (a^3 T_{(i)|E}^{00}) + \Gamma_{ij}^0 (T_{(i)|E}^{ij} + \Delta T_{(i)}^{ij}) \\ &= \frac{1}{a^3} \frac{\partial f_i}{\partial t} \left(a^3 \int_0^\infty f_i E_i d^3 p_{(i)} \right) \\ &\quad + 3 \frac{\dot{a}}{a} (p_i + P_{ci}), \end{aligned} \tag{49}$$

or equivalently,

$$\begin{aligned} u_\mu T_{(i);v}^{\mu\nu} &= 3H(\rho_i + p_i + P_{ci}) \\ &\quad + H \left(1 - \frac{\Gamma_i}{\Theta} \right) \int_0^\infty E_i p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} d^3 p_{(i)}, \end{aligned} \tag{50}$$

and solving the integral by parts we find

$$\begin{aligned} u_\mu T_{(i);v}^{\mu\nu} &= 3H(\rho_i + p_i + P_{ci}) - 3H \left(1 - \frac{\Gamma_i}{\Theta} \right) (\rho_i + p_i), \\ &= 3H P_{ci} + (\rho_i + p_i)\Gamma_i = 0. \end{aligned} \tag{51}$$

Therefore, as required by the Einstein gravitational equations, the projected divergenceless total energy momentum-tensor ($u_\mu T_{(i);v}^{\mu\nu} = 0$), that is, the expression of the energy

conservation in the FLRW geometry, is obtained only when the creation pressure P_{ci} is given by the previously derived expression [see the second expression in Eq. (47)].

4.4 Temperature evolution law

Let us now proceed to calculate the temperature evolution for the decoupled fluid mixture endowed with “adiabatic” gravitational particle production based on our extended kinetic approach. In this section we assume that the distribution function for a non-quantum relativistic gas endowed with “adiabatic” creation is also given by the standard equilibrium form:

$$f_{(i)} = e^{\alpha_i(t) - \beta_i(t)E_i}, \tag{52}$$

where as before α_i is a scalar function and $\beta_i(t)$ can be interpreted as the inverse of temperature [see discussion right before Eq. (34)].

The main aim here is to show kinetically that such a form is preserved if and only if the temperature evolution law is modified in agreement with the general macroscopic law and a generic creation rate Γ_i [see Eq. (28)].

Now, by inserting (52) into the modified Boltzmann’s equation (50) we obtain:

$$\dot{\alpha}_i - \dot{\beta}_i E_i + \beta_i H \left(1 - \frac{\Gamma_i}{\Theta} \right) \frac{p^2}{E_i} = 0. \tag{53}$$

This equation has two extreme limits. The nonrelativistic limit ($m_i \gg T$, $E_i \simeq m_i + \frac{p^2}{2m_i}$) and the ultrarelativistic or negligible rest mass limit ($m_i \ll T$, $E_i \simeq p_{(i)}$). Let us now determine the solutions for such limits separately.

- The non-relativistic domain ($m_i \gg T$). In this limit, the above equation (53) takes the form:

$$\frac{\dot{\alpha}_i}{\dot{\beta}_i} - m_i = \frac{p^2}{m_i} \left[\frac{1}{2} - H \frac{\beta_i}{\dot{\beta}_i} \left(1 - \frac{\Gamma_i}{\Theta} \right) \right], \tag{54}$$

and it is readily checked that the solution $\alpha_i - m_i \beta_i = \text{constant}$, with the right hand side (*r.h.s*) providing the solution:

$$\frac{\dot{T}_i}{T_i} = -2 \frac{\dot{a}}{a} + \frac{2}{3} \Gamma_i, \tag{55}$$

which is the same macroscopic law as given by (27). As an illustration, let us consider the phenomenological law, $\Gamma_{(i)} = 3\beta_i H$, where $\beta_i = \text{constant}$. In this case, by choosing the present day scale factor, $a_0 = 1$, it is immediate to obtain from (55):

$$T_i = T_{0i} a^{-2(1-\beta_i)} \Leftrightarrow T_i = T_{0i} (1+z)^{2(1-\beta_i)}, \tag{56}$$

where in the second equality the redshift parameter was defined by $z \equiv a^{-1} - 1$. For $\beta_i = 0$ (no particle production), the usual equilibrium temperature law for a non-relativistic decoupled component is recovered. As one may check, in terms of the redshift, the general solution $T_i(z)$ for a nonrelativistic fluid endowed with an arbitrary “adiabatic” particle creation rate reads:

$$T_i = T_{0i} (1+z)^2 e^{\frac{1}{3} \int_0^z \Gamma_{(i)}(z') \frac{dt}{dz'} dz'}. \tag{57}$$

- The relativistic domain ($m_i \ll T$). In this case equation (53) the above equation becomes:

$$\frac{\dot{\alpha}_i}{\dot{\beta}_i} = E_i \left[1 - \left(1 - \frac{\Gamma_{(i)}}{\Theta} \right) \frac{\dot{a} \beta_i}{a \dot{\beta}_i} \right], \tag{58}$$

which leads solution $\dot{\alpha}_i = 0$ (null chemical potential) while $\beta_i = 1/T_i$ thereby recovering the non-equilibrium thermodynamic result [see Eq. (29)].

$$\frac{\dot{T}_i}{T_i} = -\frac{\dot{a}}{a} + \frac{\Gamma_i}{3}. \tag{59}$$

Again, for $\Gamma_i = 3\beta_i H$, the solution of the above equation reads:

$$T_i = T_{0i} a^{-(1-\beta_i)} \Leftrightarrow T_i = T_{0i} (1+z)^{(1-\beta_i)}, \tag{60}$$

a result to be compared with the non-relativistic solution (57). Different from the equilibrium case ($v_i = 0$) this is a non-linear law. As physically expected, for a given value of $z \neq 0$, the temperature is smaller than in the standard Λ CDM model. In the case of CMB, the current value of the temperature has been fixed with great precision by the FIRAS-COBE and recalibrated by the WMAP [69, 70]. It is also worth noticing that (56) has been extensively used in CMB studies related to Sunyaev–Zeldovich [71, 72] and excitation states of interstellar molecules like C, CN and CNO [75–85]. By fixing the constant at the present time, the general solution of the above equation can be written as:

$$T_i = T_{0i} \left(\frac{a_0}{a} \right) e^{\frac{1}{3} \int_{t_0}^{t_i} \Gamma_{(i)}(t') dt'}. \tag{61}$$

In the simplest but interesting case, the creation rate $\Gamma_{(i)}$ remains constant for a given cosmic time interval. This kind of situation may happens at the early inflation phase or at late times of the evolution. By defining $\Delta t = t_f - t_i$ one finds the general solution:

$$T_f = T_i \left(\frac{a_i}{a_f} \right) e^{\frac{\Gamma_{(i)}}{3} (t_f - t_i)}. \tag{62}$$

Now, in terms of the cosmic redshift, the general solution of the temperature law (60) for the CMB fluid endowed with “adiabatic” photon creation is given by

$$T = T_0(1+z)e^{\frac{1}{3}\int_0^z \Gamma_{(i)}(z') \frac{dt}{dz'} dz'}, \tag{63}$$

and, as should be expected, for $\Gamma_{(i)} = 0$, the same equilibrium result is recovered. It is also useful to show how the above temperature law (59) for massless particles is compatible with the radiation thermal equilibrium relations coming out from the kinetic approach. By eliminating $\Gamma_{(i)}$ from the balance equations (40) and (46) it follows that:

$$\frac{\dot{\rho}_i}{\rho_i + p_i} = \frac{\dot{n}_i}{n_i} \equiv \Gamma_i - \Theta \tag{64}$$

and since $n_i \propto T_{(i)}^3$, we see that for $p_i = \rho_i/3$, then, $\rho_i \propto n_i^{4/3}$, and also

$$\frac{\dot{\rho}_i}{\rho_i} = 4 \frac{\dot{T}_i}{T_i} \Rightarrow \rho_i \propto T_{(i)}^4. \tag{65}$$

The above equilibrium relations were first determined based on irreversible thermodynamics, but now it has been recovered from a kinetic approach. It means that under “adiabatic” conditions particles are created but the energy density and concentration as a function of the temperature are given by the same expressions obeyed by the states of equilibrium, only the time dependence of each one are different in comparison with the equilibrium evolution. Indeed, by using this result one may show that the spectrum of radiation is also preserved in the course of the cosmic evolution (see [75] for a preliminary deduction). A more rigorous deduction for a decoupled mixture with creation of massless quantum particles (bosons and fermions) and the associated spectrum will be discussed below.

5 Cosmology with creation of baryons, CDM, CMB photons and neutrinos

As remarked before (see introduction), a new scenario emulating the Λ CDM model, the so-called CCDM cosmology is based on the creation of cold dark matter alone [52]. Some attempts to consider creation of some dominant components (baryons + CDM) with different creation rates, were also discussed in the literature [50, 51, 58]. Now we show that the extended CCDM model macroscopically proposed in [58] which has an evolution equivalent to Λ CDM both at the background (cosmic history) and perturbative levels (linear and

nonlinear), can also be formulated in a natural way based on the kinetic theoretical formulation as developed in the previous section (see also Appendix A).

5.1 The extended CCDM model

For each decoupled component, the general ratio $\Gamma_i/\Theta = \alpha_i \rho_{co}/\rho_i$, now takes the following form:

$$\frac{\Gamma_b}{3H} = \alpha_b \frac{\rho_{co}}{\rho_b}, \quad \frac{\Gamma_{dm}}{3H} = \alpha_{dm} \frac{\rho_{co}}{\rho_{dm}}, \tag{66}$$

$$\frac{\Gamma_r}{3H} = \alpha_r \frac{\rho_{co}}{\rho_r}, \quad \frac{\Gamma_\nu}{3H} = \alpha_\nu \frac{\rho_{co}}{\rho_\nu}, \tag{67}$$

where $\alpha_b, \alpha_{dm}, \alpha_r$ and α_ν are constant parameters, while ρ_{co} is the present day value of the critical density. Similarly, for each component, the creation pressure $P_{ci} = -(\rho_i + p_i)\Gamma_i/3H$ reduces to:

$$P_{cb} = -\rho_b \Gamma_b/3H = -\alpha_b \rho_{co}, \tag{68}$$

$$P_{dm} = -\rho_{dm} \Gamma_{dm}/3H = -\alpha_{dm} \rho_{co}, \tag{69}$$

$$P_{cr} = -\frac{4}{3} \rho_r \Gamma_r/3H = -\frac{4}{3} \alpha_r \rho_{co}, \tag{70}$$

$$P_{c\nu} = -\frac{4}{3} \rho_\nu \Gamma_\nu/3H = -\frac{4}{3} \alpha_\nu \rho_{co}, \tag{71}$$

where for simplicity we have also assumed massless neutrinos. Note also that all creation pressures are modulated by its specific creation parameter, Γ_i , defined in (66)–(67).

At late times all these components are decoupled and radiation and neutrinos are subdominant in the deep matter phase. Although dynamically irrelevant at zero order, it is well known that CMB photons (and neutrinos) play an important role in the perturbative approach both from a thermodynamic and kinetic viewpoints. For a while we neglect such components. In this case, $P_{cT} = -(\alpha_b + \alpha_{dm})\rho_{co}$, so that the total creation pressure depends only on the effective creation rate parameter, $\alpha_{eff} = \alpha_{dm} + \alpha_b$.

Now, by combining Friedmann equation

$$8\pi G(\rho_{dm} + \rho_b) = 3H^2, \tag{72}$$

with the energy conservation law for both components one finds:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{meff}(1+z)^3 + \alpha_{eff}, \tag{73}$$

where $\Omega_{meff} = \Omega_{dm} + \Omega_b - \alpha_{eff} \equiv 1 - \alpha_{eff}$ is the clustered matter. Note that α_{eff} allows a reduction of the dark sector, thereby emulating the Λ CDM dynamics with $\alpha_{eff} = \alpha_b + \alpha_{dm}$. Actually, by integrating (73) we obtain:

$$a(t) = \left(\frac{1 - \alpha_{eff}}{\alpha_{eff}}\right)^{1/3} \sinh^{\frac{2}{3}}\left(\frac{3H_0\sqrt{\alpha_{eff}}}{2}t\right), \tag{74}$$

which is identical to that predicted by the standard flat Λ CDM model with only one new free dynamic parameter, α_{eff} . The analogy is perfect by identifying $\Omega_\Lambda \equiv \alpha_{eff}$. It is widely known that $a(t)$ is not directly observable. However, for $a(t) = a(t_0) = 1$, the age of the Universe today, t_0 , can be calculated. In this way, a lower bound on α_{eff} can be obtained when we compare it with the values of the oldest objects in our galaxy or even at high redshifts. Naturally, this can also be done using expression (73) for the Hubble parameter as discussed long ago for the Λ CDM model [73, 74].

Of course, when the creation of photons and neutrinos are not taken into account, such a reduced dark sector scenario mimics the cosmic concordance model from a dynamic viewpoint, and its background thermodynamic behavior is not modified. However, when the creation of CMB photons and neutrinos are added, the value of $H(z)$ as given by (73) is modified. In particular, at the level of the EFE, the new effective creation parameter $\bar{\alpha}_{eff}$ does not appear additively so that $\bar{\alpha} \neq \alpha_b + \alpha_{dm} + \alpha_r + \alpha_\nu$, and, as such, the model dynamics also require more than one free parameter. Of course, at the matter dominated phase the CMB temperature law depends only of α_r , as should be expected from (29) [see also the kinetic derivation (59)]. In this case, the equilibrium redshift and other relevant properties of the photon-baryon fluid are slightly modified.

An interesting effect to the large scale structure and CMB anisotropies (see Sect. 5.2) is related with the $\alpha_{eff} = \alpha_{dm} + \alpha_b$ driving the evolution of the non-relativistic matter density perturbation. In this case, the growing mode solution in the matter dominated phase in terms of the scale factor can be expressed as [58]:

$$\delta_{meff}(a) = C(\mathbf{x})aF\left(\frac{1}{3}, 1; \frac{11}{6}; -\frac{\alpha_{eff}a^3}{1 - \alpha_{eff}}\right), \quad (75)$$

where $C(\mathbf{x})$ is an integration constant $F = {}_2F_1(a, b, c, z)$ is the Gaussian hypergeometric function. As should be expected, if the net creation parameter $\alpha_{eff} = \alpha_{dm} + \alpha_b = \sum_i \alpha_i \rightarrow 0$, so that the results of the standard Einstein-de Sitter model are recovered ($\delta_{(m)}^{eff} \propto a, \Omega_{(m)}^{eff} \rightarrow 1$).

In Fig. 1, we display the evolution of the contrast density as a function of the dynamically relevant created components [58]. It is interesting that the best fit of the effective creation parameter, $\alpha_{eff} = \alpha_{dm} + \alpha_b$, from SNe Ia data is guaranteeing two nontrivial results, namely: the same evolution for the density contrast (blue line) and also the same value of the transition redshift as predicted by the flat Λ CDM model. In addition, the modified CMB temperature evolution from the same current value as given by the COBE data and recalibrated by WMAP, $T_0 = 2.72548 \pm 0.00057\text{K}$, points to a new physics close to Λ CDM model, potentially, modifying the relatively smaller value of H_0 as predicted by CMB. In other words, creation of photons satisfying the modified

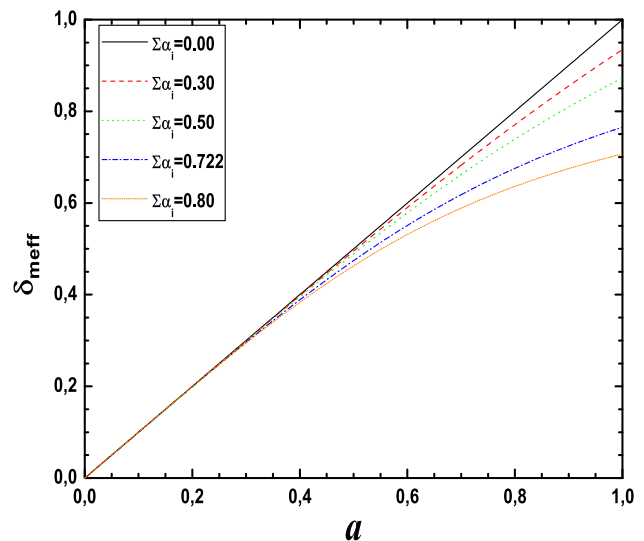


Fig. 1 Evolution of the effective matter density contrast in the extended CCDM model (for different values of α_{eff} as a function of the scale factor. The blue line is obtained for the best fit value from SNe Ia data to the unique effective free parameter, $\alpha_{eff} = \alpha_{dm} + \alpha_b$. It reproduces exactly the standard Λ CDM prediction for the nonrelativistic density contrast and also to the transition redshift. Note that $\delta_{(meff)}$ describes only that portion of the created nonrelativistic components (baryons + CDM), which is able to appear as clustered matter. This extended CCDM model is different from [52, 55] since it also includes the created subdominant components (CMB + neutrinos) thereby slightly changing the temperature law and others relevant properties of the photon-baryon fluid

temperature law suggests a crucial test in the thermal sector involving distortions, CMB temperature anisotropies and the value of H_0 itself, even considering that the same Λ CDM dynamics is preserved. At this point, it is natural to ask how distortions and CMB anisotropies would be investigated in this enlarged context emulating the Λ CDM dynamics but not its thermodynamics and kinetic approach.

5.2 Extended CCDM cosmology and CMB distortions: a case for SZE

To begin with we investigate whether the zero-order spectrum is preserved. Although CMB distortions have already been partially investigated in a recent separated paper [60], this is needed because the underlying connections with the present framework were not properly discussed. Photons and neutrinos (massive or massless) have currently different temperatures. Now, the interest for distortions in this framework will be illustrated with the Sunyaev–Zeldovich effect (SZE). In the next subsection, its cross-correlation with CMB anisotropies will be discussed as a possible crucial test for CCDM and Λ CDM cosmologies.

Let us now consider an arbitrary decoupled massless component (bosonic or fermionic) at temperature $T_i, i = r, \nu$. The general relativistic equilibrium distribution function for

a massless dilute quantum gas with zero chemical potential takes the form:

$$f_{(i)}^0(t, p) = \frac{1}{e^{\frac{p}{T_i}} - \xi_i} \tag{76}$$

where p is the physical momentum and for bosons ($\xi_i = +1$), while for fermions ($\xi_i = -1$). In this section we are following closely the notation of the textbook [92] for CMB photons. Our basic aim here is to demonstrate that both quantum equilibrium distributions above are preserved in the course of the expansion when the creation of massless particles happens under “adiabatic” conditions. Of course, $f_{(i)}^0$ is also the solution without creation since the collisional term is also identically zero for equilibrium states.

Now, by assuming “adiabatic” creation with a rate Γ_i , the modified collisionless equation (36) for each component can be rewritten as:

$$\frac{\partial f_{(i)}^0}{\partial t} + \left(-H + \frac{\Gamma_i}{3}\right) p \frac{\partial f_{(i)}^0}{\partial p} = 0. \tag{77}$$

The first term in the equation above can be rewritten as:

$$\frac{\partial f_{(i)}^0}{\partial t} = \frac{\partial f_{(i)}^0}{\partial T_i} \dot{T}_i, \tag{78}$$

while the equilibrium distribution form (76) implies that:

$$\frac{\partial f_{(i)}^0}{\partial T_i} = -\frac{p}{T_i} \frac{\partial f_{(i)}^0}{\partial p} \Rightarrow \frac{\partial f_{(i)}^0}{\partial t} = -\frac{\dot{T}_i}{T_i} p \frac{\partial f_{(i)}^0}{\partial p}, \tag{79}$$

where in the second equality above the result in (78) has been used. Therefore, by inserting the above derivative into the MBE (77) we obtain:

$$\left[-\frac{\dot{T}_i}{T_i} - \frac{\dot{a}}{a} + \frac{\Gamma_i}{3}\right] p \frac{\partial f_{(i)}^0}{\partial p} = 0, \tag{80}$$

thereby providing the temperature law for quantum massless particles:

$$\frac{\dot{T}_i}{T_i} = -\frac{\dot{a}}{a} + \frac{\Gamma_i}{3} \Leftrightarrow T_i(t)a(t)N_i(t)^{-\frac{1}{3}} = constant. \tag{81}$$

Note that the above equation is the same temperature law deduced before for a dilute ultra-relativistic gas ($T_i \gg m_i$) of non-quantum point particles [see Eq. (59)]. Note also that in the second equality above $\Gamma_i = \dot{N}_i/N_i$ has been used. It thus follows that the equilibrium spectrum (76) for massless quantum particles is preserved in the course of the expansion regardless of the value of $\xi_i = \pm 1$. The price to pay is that the temperature law for each component (CMB photons or neutrinos) is modified by creation rate of the massless decoupled component. Thus, an interesting question here is how such a preserved blackbody spectrum preserved will be slightly distorted in the course of the expansion? Let us discuss that with a simple example.

Spectral distortions may be provoked by collisional processes like Compton scattering (C), double Compton (DC), and Bremsstrahlung (BR). Such processes usually involve a redistribution of photons over frequencies and sometimes readjustments on the photon number [95–98]. In order to exemplify that let us discuss the CMB distortions provoked by the inverse Compton scattering at low redshifts. In cosmology there are two important processes. The first is widely known as the (thermal) Sunyaev–Zeldovich effect (SZE) after their seminal papers [71, 72]. This means that the modified collisionless Boltzmann equation with creation (36) must be used in its complete form, that is, by including the required sources of spectral distortion. In the case of SZE, for instance, such a study must start based on the extended equation including the Compton scattering:

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} - Hp \frac{\partial f}{\partial p} + \frac{\Gamma}{3} p \frac{\partial f}{\partial p} = \frac{df}{dt}|_c, \tag{82}$$

where the new contribution in the right-hand-side is the corresponding collisional term.

The thermal SZE is the spectral distortion of CMB caused by inverse Compton scattering of photons when transverse the hot electrons of an ionized gas ($T_e \sim 10^{7-8} K$) across the line of sight. The total number of photons in such elementary processes is conserved and its effect is quite simple, namely: CMB photons are up-scattered by the hot electrons thereby depopulating the Rayleigh–Jeans low frequency region of the spectrum. As a result, the scattered photons move to the high energy side of the photon distribution. By assuming an initial perfect blackbody CMB spectrum, the net effect after scattering is that the spectrum is slightly distorted. In the standard treatment (no photon creation) the SZE is simplified because it does not depend on the redshift. However, such a condition is violated in the present framework because the standard Λ CDM temperature law is not obeyed [see, for instance, the kinetic law (81)]. Therefore, if properly studied based on the above modified Boltzmann equation with creation, the SZE may become a crucial test confronting Λ CDM and the present extended CCDM cosmology.

It should also be recalled that the motion of clusters relative to the Hubble flow also produce a “kinematic SZ effect” (KSZE). This effect is usually much smaller than the thermal SZE effect, but it is also quite relevant in the present context since the KSZE can be used to determine the behavior of clusters and the Hubble constant itself. Naturally, a detailed investigation of CMB secondary distortions based on the SZE by taking into account “adiabatic” photon creation process and inverse Compton scattering as described in (82) is beyond the scope of this paper, and will be discussed in a forthcoming communication.

5.3 Extended CCDM and CMB anisotropies: the case for ISW

As discussed in the previous sections, “adiabatic” creation by the smooth universe cannot be described as a source of CMB distortions because the Planckian spectrum is preserved. The only net effect of such a creation process to CMB is a modification of the temperature redshift relation. This means that the perturbation of the photon distribution function may be written in the standard way [92]

$$f(t, \mathbf{r}, p, \hat{p}) = \left[\exp \left\{ \frac{p}{T(t)[1 + \Theta(t, \mathbf{r}, \hat{p})]} \right\} - 1 \right]^{-1}$$

where $T(t)$ is the zero order-temperature with photon creation and $\Theta = \delta T/T(t, \mathbf{r}, \hat{p})$ is the small fractional dimensionless temperature perturbation observed in the direction of the unit vector \hat{p} on the sky at the time t and position \mathbf{r} . In addition, since the created photons share the same temperature of the already existing ones, all collision terms in the presence of “adiabatic” creation must be proportional to Θ and other perturbatively small quantities. This happens because the induced creation contribution by the expanding universe is not equivalent to a collisional term. Thus, at zero-order the Boltzmann equation is also isotropic as happens for any FLRW metric, as for instance, Λ CDM model.

In a point of fact, some authors already discussed CMB temperature anisotropies in models with creation of CDM, but not in the general framework presented here (see Sect. 5), which is dynamically equivalent to Λ CDM. In [88], for instance, it was assumed that baryons, and photons are conserved as in the standard Λ CDM treatment and the influence of neutrinos was also not considered. Three different phenomenological expressions of the creation rate Γ_{dm} were assumed. In their simplest model (MI), the creation rate was defined by $\Gamma_{dm} = 3\alpha_{dm}H$ (the authors used β instead of α_{dm}). The effects on CMB TT power spectrum (and also to CMB EE) were obtained and compared with the theoretical predictions of the Λ CDM (see Figs. 4 and 5 in Ref. [88]). As should be expected, due to the excess of CDM in comparison to baryons, significant deviations from Λ CDM were obtained for $\alpha_{dm} \geq 0.05$.

Nevertheless, although physically interesting their results cannot be considered definitive by the following reasons (i) the unperturbed model (cosmic history), although presenting a transition from acceleration to a decelerating regime, does not reproduce the Λ CDM dynamics, and (ii) the complete hierarchy of the perturbed Boltzmann equations for all components with creation were not considered. Note that the first condition is somewhat desirable because of the recognised successes of Λ CDM for many cosmic probes. Implicitly, it also means that any realistic cosmology must be as close as possible to Λ CDM model, but being slightly different in

order to point out a new route to handle the tensions and also shed some light in the theoretical puzzles (coincidence and Λ problems).

In this context, let us now highlight some new physical results predicted by the extended CCDM model and its comparison with Λ CDM. To begin with, we stress that the scale factor $a(t)$, as given by (74) has the same expression of the Λ CDM model. In addition, the evolution of the density contrast is also the same of Λ CDM [see Eq. (75) and Fig. 1]. Both results plus the modification of the temperature, in principle, are very significant to CMB anisotropies. To show that consider now the perturbed metric in the potential conformal Newtonian gauge. The temperature anisotropy provided by the change in the Newtonian potential along the line of sight since the LSS until the present day is often referred to as integrated Sachs–Wolfe (ISW) effect.

Now, in order to understand easily the forward step given here, we first assume that CMB photons are not created. In this way, the ISW effect assumes the standard expression [92,93]

$$\Theta_{ISW} \equiv \frac{\delta T(\hat{\mathbf{p}})}{T_0} = 2 \int_{\eta_{LSS}}^{\eta_0} d\eta \frac{\partial \Phi(\mathbf{r}, \eta)}{\partial \eta}, \quad (83)$$

where T_0 is the present day temperature and the limits of integration ranges from the recombination (LSS) to the present time, respectively, whereas $\Phi(\mathbf{r}, \eta)$ is the time-varying gravitational potential along the photon path. Recalling that only low redshifts are important to the above integral, we have also ignored the suppression factor caused by the Thomson scattering [94].

It is widely known that for nonrelativistic matter in the Einstein–de Sitter cosmology, $a(t) \propto t^{2/3}$, the Newtonian gravitational potential is time-independent and the ISW is identically null. In contrast, the late time dominance of the vacuum energy density in the Λ CDM cosmology gives rise to a time-varying potential, and, as such, the ISW effect is different from zero [93,99,100] and have also been observed by different groups [101–103]. Hence, since the extended CCDM model driven by non-relativistic matter density plus its creation pressure is fully equivalent to Λ CDM, this means that the ISW effect is exactly the same of the standard cosmology whether photons are not created as assumed in [88].

On the other hand, some reported observational results for the ISW are in contradiction with the Λ CDM prediction. For example, an excess signal of ISW effect has been reported by several authors based on cross-correlation between WMAP and catalogues of quasars, clusters, supervoids and other surveys [101–103] and confronted with the Λ CDM results. Recently, new constraints were derived cross-correlating Planck’s temperature maps with AGN and radio sources catalogues thereby obtaining a very positive detection of the ISW signal at 5σ of significance level. In particular, this means that in the near future with more data and an improved treatment

of systematics for different surveys, potentially, may provide an additional difficulty to the standard Λ CDM cosmology.

In the same vein, we recall that some recent studies are based on cross-correlations of CMB, Gamma-Ray background and the SZE effect. In our view, given the above results and regardless of the present status of Λ CDM concerning the quoted analyses (see Sultun [101] for a short and nice review in the observational front), it seems interesting to propose a crucial test involving the extended CCDM model and Λ CDM cosmology. The reason is very simple. As we have seen, the extended CCDM cosmology has the same dynamics, but its thermodynamics is slightly different from Λ CDM. In particular, the SZE effect is not independent of the redshift as occurs in Λ CDM model (see discussion on the previous subsection). Moreover, the Boltzmann equation with photon production in CCDM model means that the first order perturbed Boltzmann equation for photons is also slightly modified in comparison with the standard Λ CDM treatment. On the other hand, since the analysis of the SZE is also modified by “adiabatic” photon creation, the cross-correlation between CMB and SZE, is the interesting statistical tool for a sharp test confronting CCDM and Λ CDM.

6 Conclusion

In this paper we have investigated the *thermodynamic* and *kinetic* properties of an arbitrary decoupled multi-fluid mixture endowed with gravitationally induced particle production of all components, in principle, with different creation rates Γ_i . The main results derived here may be summarised in the following statements:

1. For each component, the efficiency of the phenomenon depends on the ratio Γ_i/H . Of course, for a given component, the process is negligible whether $\Gamma_i \ll H$. The irreversible macroscopic results are valid for any FLRW geometry and also for values of Γ_{iN} and $\Gamma_{iS} \geq 0$. However, the kinetic counterpart was deduced only for the flat case ($k = 0$) and adiabatic creation defined by $\Gamma_{iN} = \Gamma_{iS}$.
2. The whole process is irreversible but the gain of entropy in the “adiabatic” case (the most interesting one from a physical point of view), depends only on the created particles ($S_i = k_B N_i$). This happens because $\Gamma_{iS} = \Gamma_{iN} = \Gamma_i$ so that $\dot{\sigma}_i = 0$ [see discussion in Sect. 3 right below Eq. (12)]. For each decoupled component, this means that both the total entropy and the number of particles increase but the specific entropy (per particle), $\sigma_i = S_i/N_i$, remains constant.
3. The multi-fluid approach developed here is in fact a quasi-zero-order description, in the sense that the relativistic distributions has the same form of equilibrium. In par-

ticular, the CMB blackbody spectrum with creation is not destroyed in the course of the expansion. Therefore, at zero order, the extended CCDM cosmology with creation of CDM, baryons, photons and neutrinos (see Sect. 5a) is now dynamically described by $H(t)$ and the different creation rates Γ_i . These quantities Γ_i affect the temperature law of each component. The extra bonus of the extended CCDM cosmology is that dark energy is not required anymore ($\Omega_{DE} = 0$) thereby solving naturally the coincidence and Λ -problem. Particularly, the transition from a decelerating to an accelerating regime in the matter dominated phase is provided by the negative creation pressure of the baryonic and CDM components [see Eqs. (17) and (47)].

4. All the macroscopic results obtained in the irreversible macroscopic approach for the decoupled multi-fluid were kinetic treatment.
5. When photon creation is neglected it was shown that the ISW effect of the extended CCDM model is the same of Λ CDM cosmology. However, this result is modified when CMB photons are created because (i) the temperature is modified, and (ii) the perturbed Boltzmann equation for CMB photons acquire an additional term. In particular, this means that the standard Sunyaev–Zeldovich effect is not independent of the redshift as happens in the Λ CDM model.
6. The present analysis also open a new window to investigate the H_0 and S_8 tensions in virtue of twofold reasons: (i) The unperturbed model CCDM model has the same Λ CDM dynamics (linear and nonlinear levels) powered by non-relativistic matter, (ii) The creation of the remaining components (CMB photons and neutrinos) changes slightly the expansion history at early and late times (see discussions in Sect. 5B). Its physical consequences at the level of the H_0 and S_8 tensions will be discussed with more detail in a subsequent paper.

Finally, we also emphasise an interesting aspect related to the spectral distortions and CMB anisotropies in the presence of “adiabatic” photon creation. As discussed in Sects. 5.2 and 5.3, the predictions of the extended CCDM cosmology with $\Omega_{DE} = 0$ must not only be compared with the observations but also confronted with the ones of the λ CDM model. In principle, the rationale and soundness of gravitationally induced particle production requires much more work and analysis based on the upcoming data, in particular, for prospecting the main consequences for the angular power spectrum and CMB distortions, as well as their cross correlations with different surveys (subsections Vb and Vc). As argued there, since the analysis of the SZE is also modified by “adiabatic” photon creation, the cross-correlation between CMB temperature maps and SZE (and other surveys) seems

to be the interesting statistical tool for a crucial and accurate test confronting the extended CCDM and Λ CDM models.

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Appendix A: Boltzmann equation and “adiabatic” creation

Let us discuss with more detail how the standard collisionless relativistic Boltzmann equation is modified in the presence of “adiabatic” matter creation. The main aim here is to derive the “mass shell” Boltzmann equation (36) by starting from the covariant form (35):

$$\mathcal{L}(f_i) \equiv P_{(i)}^\mu \frac{\partial f_{(i)}}{\partial x_{(i)}^\mu} - \Gamma_{\alpha\beta}^\mu P_{(i)}^\alpha P_{(i)}^\beta \frac{\partial f_{(i)}}{\partial P_{(i)}^\mu} + \mathcal{P}_{Gi}(x_{(i)}^\mu, P_{(i)}^\mu) = 0, \tag{A1}$$

The undefined quantity, \mathcal{P}_{Gi} , is assumed proportional to both terms Γ_i/Θ and $\Gamma_{\alpha\beta}^\mu P_{(i)}^\alpha P_{(i)}^\beta \frac{\partial f_{(i)}}{\partial P_{(i)}^\mu}$ [see discussion below (35)].

Like the expansion itself (second term), the form adopted above for \mathcal{P}_{Gi} reflects the fact that “adiabatic” matter creation is also a purely gravitational effect. Thus, (A1) takes the form:

$$P_{(i)}^\mu \frac{\partial f_{(i)}}{\partial x_{(i)}^\mu} - \left(1 - B \frac{\Gamma_i}{\Theta}\right) \Gamma_{\alpha\beta}^\mu P_{(i)}^\alpha P_{(i)}^\beta \frac{\partial f_{(i)}}{\partial P_{(i)}^\mu} = 0, \tag{A2}$$

where $B > 0$ is a pure number of the order of unity. It must be determined in such a way that all the “adiabatic” balance equations with creation are kinetically reproduced. Note also that the “mass shell” constraint, $g_{\mu\nu} P_{(i)}^\mu P_{(i)}^\nu = m_{(i)}^2$ implies that $f_{(i)} \equiv f_{(i)}(t, P_{(i)}^\mu)$ with the above equation

reducing to:

$$P_{(i)}^0 \frac{\partial f_{(i)}}{\partial t} - 2H P_{(i)}^0 \left(1 - B \frac{\Gamma_i}{\Theta}\right) P_{(i)}^i \frac{\partial f_{(i)}}{\partial P_{(i)}^i} = 0, \tag{A3}$$

where we have replaced the values $\Gamma_{0j}^i = \Gamma_{j0}^i = H\delta_j^i$ from (2). In addition, from spatial homogeneity and isotropy condition and mass shell condition, the distribution function of each decoupled component is a function of the time and energy $P_{(i)}^0 = E_i$ (or, equivalently, the modulus of the momentum of the i -th component $P = P_{(i)}$). Thus, we may rewrite the above expression as:

$$\frac{1}{E_i} \mathcal{L}(f_i) \equiv \frac{\partial f_{(i)}}{\partial t} - 2H \left(1 - B \frac{\Gamma_i}{\Theta}\right) P_{(i)} \frac{\partial f_{(i)}}{\partial P_{(i)}} = 0. \tag{A4}$$

Following standard lines, let us rewrite the above equation in terms of the physical momentum, $p_{(i)} \equiv a(t)P_{(i)}^i$: In this case, the time derivatives of the distribution function $f_i(t, P_{(i)})$ and $f_i(t, p_{(i)})$ are related by:

$$\frac{\partial f_{(i)}}{\partial t}(t, P_{(i)}) = \frac{\partial f_{(i)}}{\partial t}(t, p_{(i)}) + H p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}}. \tag{A5}$$

Now, inserting such results into (A4) it follows that:

$$\frac{1}{E_i} \mathcal{L}[f_{(i)}] \equiv \frac{\partial f_{(i)}}{\partial t} - H \left(1 - 2B \frac{\Gamma_i}{\Theta}\right) p_{(i)} \frac{\partial f_{(i)}}{\partial p_{(i)}} = 0. \tag{A6}$$

Finally, by comparing with the Liouville operator (36) from which the balance equations for “adiabatic” creation were kinetically calculated thereby reproducing all the macroscopic results [cf. Eqs. (9) and (23) in Sect. 3 with the corresponding Eqs. (40) and (44) in Sect. 4], we may conclude that the only possible value of the undetermined pure number in (A2) is $B = 1/2$.

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