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Explaining the CDF-II W-boson mass anomaly in the Georgi–Machacek extension models

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Abstract Original Georgi–Machacek model can preserve the custodial symmetry at tree level after the electroweak symmetry breaking. Unless additional $SU(2)_c$ custodial symmetry breaking effects are significant, the new physics contributions to Δm_W are always very small. Our numerical results show that ordinary GM model can contribute to Δm_W a maximal amount 0.0012 GeV, which can not explain the new CDF-II anomaly on W boson mass. We propose firstly to introduce small misalignment among the triplet VEVs to increase Δm_W, which can give large new physics contributions to Δm_W. Such slightly misaligned triplet VEVs from custodial symmetry preserving scalar potential can still be allowed. Our numerical results indicate that the resulting Δm_W can easily reach the 1σ range of CDF-II m_W data and the splitting among the triplet VEVs $\Delta v (\equiv v_\xi - v_\chi)$ can be as small as 0.8 GeV for $v_\chi \lesssim 15$ GeV. We also propose to introduce an additional custodial symmetry breaking source by extending the GM model with a low scale RH neutrino sector, which can adopt the leptogenesis mechanism and allow moderately large coupling strength h_{ij} even for triplet VEVs of order GeV. With low scale RH neutrino mass scale of order 10^2–10^4 TeV, the new physics contribution to Δm_W can reach 0.03 GeV and is much larger than that of ordinary GM model. Combining both custodial $SU(2)_c$ symmetry breaking effects, the small misalignment among the triplet VEVs and the moderately large h_{ij} couplings allowed with RH neutrino sector, the value of Δm_W can still easily reach the 1σ range of CDF-II m_W data, with a minimum splitting (among the triplet VEVs) approximately 0.7 GeV for $v_\chi \lesssim 15$ GeV.

Contents

1 Introduction

The standard model (SM) had already been corroborated by various contemporary collider experiments, including the discovery of 125 GeV SM-like Higgs boson by the Large Hadron Collider (LHC) [1,2]. Despite its immense success, it has many theoretical or aesthetical problems, such as the hierarchy problem related to the fundamental Higgs scalar, the origin of tiny neutrino masses, the origin of the baryon asymmetry in the universe and the measured W boson mass data recently reported by CDF-II experiment at the Fermilab [3]. In fact, the CDF-II experiment reported recently the most precise direct measurement of the W boson mass as

$$m_W = 80,433.5 \pm 6.4(stat) \pm 6.9(syst)$$
$$= 80,433.5 \pm 9.4 \, \text{MeV/c}^2, \quad (1.1)$$

using data corresponding to $8.8 \, \text{fb}^{-1}$ of integrated luminosity collected in proton–antiproton collisions at a 1.96 TeV center-of-mass energy. This measurement is in significant tension with the SM expectation [4]

$$m_W = 80,357 \pm 4(inputs) \pm 4(theory) MeV/c^2. \quad (1.2)$$

The data-driven techniques used by CDF experiment capture most of the higher order corrections and using higher order corrections result in a decrease in the W boson mass value reported by CDF-II by almost 10 MeV [5]. So, the deviation

of CDF-II m_W value from the SM expectation can be reduced at most from 7σ to 6σ, necessitating additional theoretical or experimental explanations.

Although the newly reported CDF-II measurement of m_W is still 2.4σ away from the combined results of ATLAS [6], LHCb [7] and LEP, such a discrepancy may be a hint for the new physics beyond the SM [8], especially if such an anomaly persists and gets confirmed by other experiments. Additional contributions to m_W may need TeV scale new particles that transform non-trivially under $SU(2)_L$ or tiny mixing between the ordinary W gauge boson and some heavy new gauge bosons etc, see various explanations in [9–34]. We should note that, the CDF-II measurement of m_W should better be seen as an additional constraint for predictive new physics models, for example, models with extended Higgs sector.

Electroweak symmetry breaking (EWSB) mechanism with extended Higgs sector is still allowed, given the uncertainties in Higgs boson coupling measurements. On the other hand, any extended Higgs sector must be carefully constructed to satisfy the stringent constraints from electroweak precision measurments, the most important one of which is the electroweak ρ parameter. We know that tree level relation $\rho_{tree} = 1$ is automatically satisfied by the Higgs sector of SM, which respects the custodial $SU(2)_c$ global symmetry. Georgi–Machacek (GM) model [35–68], which augments the SM Higgs sector with a complex $SU(2)_L$ triplet of hypercharge $Y = 1$ and a real $SU(2)_L$ triplet of $Y = 0$, can protect the relation $\rho_{tree} = 1$ with custodial symmetry preserving Higgs potential and vacuum alignment between the complex and real triplets. Therefore, a large triplet VEV v_Δ of order $\mathcal{O}(10)$ GeV is still allowed for the vacuum aligned triplets in the GM model, unlike that in the minimal Higgs triplet model (HTM) whose v_Δ is constrained to be much smaller because of its violation of $SU(2)_c$ at tree level. GM model can adopt the type-II neutrino seesaw mechanism to generate tiny neutrino masses.

However, we expect that the new physics contribution Δm_W in GM model should be small because of the tree-level custodial $SU(2)_c$ symmetry, which protects the ρ parameter from large deviations from unity. So, in order to increase Δm_W, relatively large custodial $SU(2)_c$ breaking effects should be introduced. We propose firstly to spoil the $SU(2)_c$ custodial symmetry with slightly misalignment among the two triplet VEVs, which can be natural because the global symmetry of the tree level scalar potential, after weakly gauged, is not fully respected by the 1-loop effective scalar potential. We also propose an alternative GM extension model, which augments the neutrino sector with low scale right-handed (RH) neutrinos and adopts the type I + II seesaw like mechanism similar to that in the Left-Right symmetric model. The presence of RH neutrino terms can not only be used to increase the value of neutrino-scalar couplings h_{ij}

and the masses of the light pseudo-Goldstone modes from $SU(2)_c$ to residual $U(1)$ breaking, but also be used to understand the Baryon Asymmetry of the Universe (BAU). Satisfying Sakharov's three conditions, leptogenesis mechanism naturally combine the neutrino seesaw mechanism and net baryon number generation by out-of-equilibrium L-violating decays of heavy Majorana neutrinos. Besides, low scale RH neutrino with large Yukawa couplings to leptons can have rich collider phenomenology.

This paper is organized as follows. In Sect. 2, we review the GM model and the GM extension model with slightly misaligned triplet VEVs. In Sect. 3, we show our numerical results on the new physics contributions to Δm_W in custodial preserving GM model and extended GM model with slightly misaligned triplet VEVs, respectively. In Sect. 4, we discuss the low scale RH neutrino extended GM models. Numerical results of Δm_W are given for the cases with (and without) slightly misaligned triplet VEVs, respectively. Section 5 contains our conclusions.

2 GM model and its extension with slightly misaligned triplet VEVs

In the GM model [35,48], the Higgs sector contains the ordinary SM $SU(2)_L$ doublet Higgs field ϕ with hypercharge $Y = 1/2$ and two $SU(2)_L$ triplet Higgs fields: complex triplet Higgs χ with $Y = 1$ and real triplet ξ with $Y = 0$. These fields can be written in the form of $SU(2)_L \times SU(2)_R$ symmetry

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}, \qquad (2.1)$$

to track the symmetry preserved by the scalar potential, where the transformations of Φ and Δ under $SU(2)_L \times SU(2)_R$ are given as $\Phi \rightarrow U_L \Phi U_R^\dagger$ and $\Delta \rightarrow U_L \Delta U_R^\dagger$ with $U_{L,R} = \exp(i\theta_{L,R}^a T^a)$ and T^a being the $SU(2)$ generators. The global symmetry can be weakly gauged so as that the $SU(2)_L, U(1)_Y$ group can be associated with the T_L^a and T_R^3 generators of $SU(2)_L \times SU(2)_R$, respectively. The gauge invariant scalar potential without $\Delta \rightarrow -\Delta$ discrete Z_2 symmetry are given as [48]

$$\begin{aligned} V(\Phi, \Delta) &= \frac{1}{2}m_\Phi^2 \text{tr}\left[\Phi^\dagger \Phi\right] + \frac{1}{2}m_\Delta^2 \text{tr}\left[\Delta^\dagger \Delta\right] + \lambda_1 \left(\text{tr}\left[\Phi^\dagger \Phi\right]\right)^2 \\ &\quad + \lambda_2 \left(\text{tr}\left[\Delta^\dagger \Delta\right]\right)^2 \\ &\quad + \lambda_3 \text{tr}\left[\left(\Delta^\dagger \Delta\right)^2\right] + \lambda_4 \text{tr}\left[\Phi^\dagger \Phi\right] \text{tr}\left[\Delta^\dagger \Delta\right] \\ &\quad + \lambda_5 \text{tr}\left[\Phi^\dagger \frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right] \text{tr}\left[\Delta^\dagger T^a \Delta T^b\right] \end{aligned}$$

$$+ \mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right](P^\dagger \Delta P)_{ab}$$

$$+ \mu_2 \mathrm{tr}\left[\Delta^\dagger T^a \Delta T^b\right](P^\dagger \Delta P)_{ab}, \tag{2.2}$$

where σ^a are the Pauli matrices, T^a are the 3×3 matrix representation of the $SU(2)$ generators, and the similarity transformation, which rotates Δ into the Cartesian basis, is given by

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

The scalar potential can trigger successful EWSB with the scalar VEVs

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} v_\phi, \quad \langle \chi \rangle = v_\chi, \quad \langle \xi \rangle = v_\xi. \tag{2.3}$$

When the two triplet VEVs align, $v_\chi = v_\xi \equiv v_\Delta$, the custodial $SU(2)_c$ symmetry from the diagonal breaking of the global $SU(2)_L \times SU(2)_R$ symmetry of the potential[1] is preserved. The EWSB condition

$$v^2 = v_\phi^2 + 4v_\chi^2 + 4v_\xi^2 = v_\phi^2 + 8v_\Delta^2$$
$$= \frac{1}{\sqrt{2}G_F} \approx (246\,\mathrm{GeV})^2, \tag{2.4}$$

can be recast into s_H variable with $s_H = \sin\theta_H$ for

$$\tan\theta_H = 2\sqrt{2}v_\Delta/v_\phi. \tag{2.5}$$

The gauge boson masses at tree level can be given as

$$m_W = \frac{g_2^2}{4}\left(v_\phi^2 + 4v_\chi^2 + 4v_\xi^2\right),$$
$$m_Z^2 = \frac{g_2^2}{4\cos^2\theta_W}\left(v_\phi^2 + 8v_\chi^2\right), \tag{2.6}$$

which obviously predicts $\rho_{tree} \equiv m_W^2/M_Z^2 \cos^2\theta_W = 1$ when $v_\chi = v_\xi \equiv v_\Delta$. In fact, in multiple Higgs scenarios, the tree level ρ parameter can be expressed as the sum of contributions from Higgs multiplets $\phi_{T,Y}$ with the corresponding hypercharge $Y \equiv 2(Q - T_3)$, isospin T and the VEV $\langle \phi_{T,Y} \rangle = V_{T,Y}$ [69]

$$\rho_{tree} \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum\limits_{T,Y}\left[4T(T+1) - Y^2\right]|V_{T,Y}|^2 c_{T,Y}}{2\sum\limits_{T,Y} Y^2 |V_{T,Y}|^2}, \tag{2.7}$$

for

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation}, \\ \frac{1}{2}, & (T, Y = 0) \in \text{real representation}. \end{cases} \tag{2.8}$$

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[1] Global $SU(2)_L \times SU(2)_R$ is explicitly broken by the Yukawa and the hypercharge gauge interactions.

After introducing the Yukawa interactions between the lepton doublets and the Higgs triplet

$$\mathcal{L} \supseteq h_{ij} \overline{L_L^{ic}} i \tau_2 \chi L_L^j + \text{h.c.}, \tag{2.9}$$

tiny Majorana neutrino masses can be generated

$$m_\nu \approx h_{ij} v_\Delta. \tag{2.10}$$

For $v_\Delta \sim \mathcal{O}(10)$ GeV, very small coupling $h_{ij} \sim 10^{-13}$ is needed to give tiny neutrino mass, which is rather unnatural.

The scalar potential with $\Delta \to -\Delta$ symmetry, originally proposed in [36], can naturally eliminate the μ_1, μ_2 cubic terms in the Higgs potential (2.2) and lead to the custodial symmetry preserving minimum with $v_\chi = v_\xi \equiv v_\Delta$. In fact, such a Z_2 symmetric potential can not only preserves the $\rho = 1$ condition at tree-level but also offers the possibility that the triplets give large contribution to the EWSB. Detailed discussions on the vacuum structure of this Z_2 symmetric scalar potential are given in [68]. On the other hand, it had been shown that the custodial symmetry preserving minimum may not be safe from eventual tunneling to a deeper non-custodial vacuum or a charge breaking vacuum. In fact, it was shown in [55] that a sufficient (but not necessary) condition for the custodial symmetry preserving vacuum is given by $\lambda_3 > 0, \lambda_5 < 0, \mu_1 < 0, \mu_2 < 0$. Choices of inputs to generate non-custodial symmetry preserving vacuum from $SU(2)_L \times SU(2)_R$ symmetry preserving scalar potential can still be possible. Explicit expressions for the difference of the scalar potential values between real non-custodial vacuum V_{NC} and the custodial vacuum V_C in the Z_2 symmetric case had been calculated in [68], which can take either signs with proper choices of parameters. Adding the mass parameters μ_1, μ_2 related terms will enlarge the allowed parameters space for real non-custodial minimum. Therefore, it is possible that the true vacuum does not keep the custodial symmetry and takes only the real non-custodial one. In addition, the tree level scalar potential will receive loop corrections from custodial symmetry breaking hypercharge gauge and Yukawa interactions. The one-loop effective potential includes the tree-level scalar potential and the Coleman–Weinberg potential [70] is given by

$$V_{CW}^1(\varphi, \mu_R)$$
$$= \frac{1}{64\pi^2}\left[\sum_i n_i \left(m_i^2(\varphi)\right)^2 \left\{\log\left(\frac{m_i^2(\varphi)}{\mu_R^2}\right) - C_i\right\}\right], \tag{2.11}$$

where $m_i^2(\varphi)$ denotes field-dependent mass squared of mass eigenstate species i (with i runs over particle species), n_i counts degrees of freedom (with a minus sign for fermions), μ_R is the renormalization scale and C_i's are constants that depend on the renormalization scheme. The particle species include the W and Z bosons, the (field-dependent)

mass eigenstates of Higgs bosons, the would-be Nambu–Goldstone modes and the top quarks (light fermions with small Yukawa couplings can be neglected). See [56] for discussions and applications in GM model. Taking into such non-custodial symmetry preserving loop contributions, it is also natural for the one-loop effective potential to choose the real non-custodial minimum and slightly misaligned v_χ and v_ξ. Such custodial symmetry breaking vacuum with misaligned triplet VEVs is welcome to explain the new CDF-II data on m_W. Substitution the misaligned VEVs for triplets and the doublet VEVs

$$\Phi = \begin{pmatrix} \phi^{0*} + \frac{1}{\sqrt{2}} v_\phi & \phi^+ \\ \phi^- & \phi^0 + \frac{1}{\sqrt{2}} v_\phi \end{pmatrix},$$

$$\Delta = \begin{pmatrix} \chi^{0*} + v_\chi & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 + v_\xi & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 + v_\chi \end{pmatrix}, \quad (2.12)$$

we can obtain the low energy scalar potential, which preserves residual $U(1)$ instead of the custodial $SU(2)$ symmetry.

The most general $SU(2)_L \times U(1)_Y$ invariant tree-level scalar potential is given by [65]

$$\begin{aligned} V(\phi, \chi, \xi) =\ & m_\phi^2(\phi^\dagger\phi) + m_\chi^2 \mathrm{tr}(\chi^\dagger\chi) + m_\xi^2 \mathrm{tr}(\xi^2) \\ & + \mu_1 \phi^\dagger \xi \phi + \mu_2 [\phi^T(i\tau_2)\chi^\dagger\phi + \mathrm{h.c.}] \\ & + \mu_3 \mathrm{tr}(\chi^\dagger\chi\xi) + \lambda(\phi^\dagger\phi)^2 \\ & + \rho_1[\mathrm{tr}(\chi^\dagger\chi)]^2 + \rho_2 \mathrm{tr}(\chi^\dagger\chi\chi^\dagger\chi) + \rho_3 \mathrm{tr}(\xi^4) \\ & + \rho_4 \mathrm{tr}(\chi^\dagger\chi)\mathrm{tr}(\xi^2) + \rho_5 \mathrm{tr}(\chi^\dagger\xi)\mathrm{tr}(\xi\chi) \\ & + \sigma_1 \mathrm{tr}(\chi^\dagger\chi)\phi^\dagger\phi + \sigma_2 \phi^\dagger\chi\chi^\dagger\phi + \sigma_3 \mathrm{tr}(\xi^2)(\phi^\dagger\phi) \\ & + \sigma_4(\phi^\dagger\chi\xi\phi^c + \mathrm{h.c.}), \quad (2.13) \end{aligned}$$

where $\phi^c = i\tau_2\phi^*$. It was pointed out in [71] that one can introduce custodial symmetry breaking terms from the beginning to make the model consistent at loop levels. They noted that the minimal extension of the Higgs potential is to introduce explicitly custodial symmetry breaking mass term $m_\xi^2 \xi^\dagger\xi$ in addition to ordinary custodial preserving GM scalar potential, which acts as a special case of (2.13) and is also adopted in [72]. Such a choice can potentially avoid undesirable light masses for pseudo-Goldstone modes $H_5^\pm$ and spoil the bad mass relations $m_{H_5^{\pm\pm}}^2 \simeq -3m_{H_5^0}^2$,[2] which is observed in a large portion of parameter space and can potentially cause the breaking of the $U(1)_{EM}$ symmetry [72]. On the other hand, no disruptive jumping transition from the true stable vacuum in the aligned case to an unstable saddle point vacuum in the misaligned case should occur when one begins to slightly turn on small misalignment among the triplet VEVs

(unless the splitting within the misaligned triplet VEVs is big enough) from a custodial symmetry preserving tree level scalar potential (appended with custodial symmetry breaking loop level effective potential) by continuously varying the relevant parameters. So, it is reasonable that small viable parameter space can be found to get a stable vacuum (or a metastable one with long enough lifetime) with small splitting among the triplet VEVs from the custodial symmetry preserving scalar potential so as that such a vacuum is not bothered with the previously mentioned extra-light pseudo-Goldstone $H_5^\pm$ masses and negative $m_{H_5^{\pm\pm}}^2$ problems. So, our case with small (but non-vanishing) splitting among the misaligned triplet VEVs corresponds to a stable vacuum (or a metastable one with long enough lifetime), which connects smoothly to the aligned true stable vacuum before its disruptive transition by jump to the vacuum that bothered with the previous noted problems when the splitting among the misaligned triplet VEVs is too large.

The breaking of $SU(2)_L \times SU(2)_R$ global symmetry into residual $U(1)$ instead of custodial $SU(2)_c$ will lead to two additional light pseudo-Goldstone modes, which can be identified to be $H_5^\pm$ and face various phenomenological constraints. Such pseudo-Goldstone modes can be compatible with current collider exclusion bounds because the global symmetries $SU(2)_L \times SU(2)_R$ of the tree level potential are not fully respected by the electroweak gauge group and Yukawa couplings. Taking into account the loop corrections to the scalar potential, it is reasonable that the custodial symmetry breaking 1-loop effective scalar potential can give additional contributions to the masses of the pseudo-Goldstone modes. On the other hand, additional custodial $SU(2)_c$ breaking sources are still welcome to increase further the masses of the otherwise Goldstone modes so as that current collider constraints can be satisfied safely. One can simply introduce the soft custodial breaking triplet mass terms in the scalar potential so as the generalized GM model scalar potential is used [65]. Contributions from moderately large Yukawa couplings involving the neutrinos and χ triplet can also be adopted to push heavy such pseudo-Goldstone modes, which can potentially be advantageous to explain the CDF-II data on W boson mass in the GM framework. Moderately large Yukawa couplings for neutrinos can be well motivated when the RH neutrino sector is introduced to adopt a mixed type I + II like neutrino seesaw mechanism. The pseudo-Goldstone modes $H_5^\pm$ can receive quadratic divergence contributions from scalar, gauge loops and possible lepton loops from the neutrino Yukawa coupling term (2.9). Although additional mechanism from UV completion can be introduced to tame the quadratic divergence of the scalar self energy, the scalar masses can still receive large contributions from new heavy particles in the UV completed theory, for example, the sneutrino loops in the SUSY framework. In gen-

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[2] This mass relation will be changed by loop corrections to the scalar masses in a UV completed theory.

eral, with proper UV completion, the new pseudo-Goldstone modes $H_5^\pm$ from the breaking of custodial $SU(2)_c$ into residual $U(1)$ can be heavy enough to survive the collider exclusion bounds. Especially, when the coupling strengths h_{ij} are large, the UV completion scale can be as low as the TeV scale, which is still large enough to push heavy the possible Goldstone modes so as the the collider bounds can be passed.

3 Contributions to Δm_W with slightly misaligned triplet VEVs

If the mass scale of new physics is much higher than m_Z, it contributes to the electroweak precision observables only through virtual loops. The dominant loop contributions to the EW precision observables can be parameterized by the oblique parameters S, T and U, which characterize the influence of physics beyond the standard model on the experimentally measurable quantities in terms of their contributions to the usual gauge boson propagators. They can also be seen as the reparameterizations of the variables $\Delta\rho$, $\Delta\kappa$ and Δr, which absorb the radiative corrections into the quantities that characterize the deviations from the tree-level value of ρ, the weak mixing angle and the Fermi constant, respectively.

The W boson mass in the SM can be precisely calculated with the following set of input parameters:

$$G_F,\ m_Z,\ m_t,\ m_h,\ \alpha_{em}(0),\ \alpha_s(M_Z).$$

Using the central values of input parameters, the calculated W boson mass $m_W = 80.357$ GeV is 7σ away from the CDF-II data, necessitating additional contributions Δm_W from new physics. The oblique parameters can be used to calculate the loop contributions of new physics to the W-boson masses [80–85]

$$\Delta m_W = \frac{\alpha M_W}{2(c_W^2 - s_W^2)}\left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2}U\right),$$
(3.1)

with

$$\alpha S = 4s_W^2 c_W^2\left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right],$$

$$\alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},$$

$$\alpha U = 4s_W^2\left[\Pi'_{WW}(0) - c_W^2\Pi'_{ZZ}(0) - 2s_W c_W\Pi'_{Z\gamma}(0)\right.$$
$$\left. - s_W^2\Pi'_{\gamma\gamma}(0)\right],$$
(3.2)

in additional to possible tree-level shift of m_W as $\Delta m_W^{tree} \approx m_W^{SM}\Delta\rho/2$, which will be present if the tree level custodial

$SU(2)_c$ symmetry is violated. In our cases, we have

$$\Delta\rho \equiv \rho - 1 = \frac{v_\phi^2 + 4v_\chi^2 + 4v_\xi^2}{v_\phi^2 + 8v_\chi^2} - 1$$
$$= \frac{v_{EW}^2}{v_{EW}^2 + 4(v_\chi^2 - v_\xi^2)} - 1 \approx \frac{4(v_\xi^2 - v_\chi^2)}{v_{EW}^2},$$
(3.3)

with $v_{EW}^2 = v_\phi^2 + 4v_\chi^2 + 4v_\xi^2 \approx (246\ \text{GeV})^2$. So, if the CDF-II m_W anomaly is explained to 1σ range by pure tree-level custodial symmetry breaking contributions

$$\frac{1}{2}\frac{4(v_\xi^2 - v_\chi^2)}{v_{EW}^2}m_W \gtrsim 0.0671\ \text{GeV},$$
(3.4)

the misaligned triplet VEVs need to satisfy

$$v_\xi^2 - v_\chi^2 \gtrsim 25.27(\text{GeV})^2.$$
(3.5)

Taking into account the loop contributions, such a bound for misaligned triplet VEVs can be slightly weaken.

We plot in Fig. 1 the allowed values of S, T with $U = 0$[3] that can explain Δm_W up to $1\sigma - 3\sigma$ range of new CDF-II data by new physics contributions. The green box denotes the old fit 1σ constraints on S and T from various inputs combined with M_Z [87]. Recently, electroweak precision fit of the oblique parameters from the analysis of precision electroweak data, including the new CDF-II result of m_W, gives [88]

$$S = 0.06 \pm 0.10,\quad T = 0.11 \pm 0.12,\quad U = 0.13 \pm 0.09,$$
$$S = 0.14 \pm 0.08,\quad T = 0.26 \pm 0.06,\quad (\text{fix } U = 0)$$
(3.6)

with (and without) fix $U = 0$. Both box regions allow the choices of S, T parameters that can explain the CDF-II m_W anomaly to 2σ range.

GM-type models are constrained by the measurements of SM quantities, such as the determination of the $Zb\bar{b}$ coupling, the measurement of the Higgs boson signal strengths, the vacuum stability bounds and the unitary bounds. For example, the couplings of SM-like Higgs h to the SM fermions and weak gauge bosons $V = W, Z$ are modified in GM-type models, who is arguably the simplest custodially symmetric model whose κ_V values can be larger than unity and would be constrained by LHC measurements [89]. The deviations of the Higgs boson couplings to weak gauge bosons etc from the SM predictions can be parameterized by the κ_F, κ_W and κ_Z parameters

$$g_{h\bar{f}f} = \kappa_F g_{h\bar{f}f}^{SM},\quad g_{hVV} = \kappa_V g_{hVV}^{SM}\quad (V = W, Z).\quad (3.7)$$

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[3] Fixing $U = 0$ is motivated by the fact that U is suppressed by an additional factor M_{new}^2/M_Z^2 compared to S and T [86]. Such a choice can greatly improves the precision on S and (in particularly) T. The tree level $\Delta\rho$ can also be absorbed into the effective T parameter.
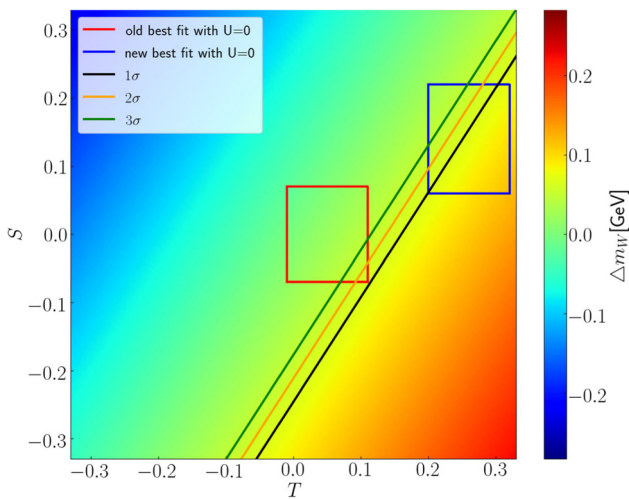
Fig. 1 Possible values of S, T with $U = 0$ that can give Δm_W up to $1\sigma - 3\sigma$ range of new CDF-II data. The box with red line denotes the range $S = 0.00 \pm 0.07$ and $T = 0.05 \pm 0.06$ [87] while the box with blue line denotes the range $S = 0.14 \pm 0.08$ and $T = 0.26 \pm 0.06$ [88]

Note that κ_W and κ_Z, which should take different values in the custodial symmetry breaking case with misaligned triplet VEVs, will turn to same values in the limit where the custodial symmetry is restored. The updated LHC constraints on κ_V, κ_F by ATLAS [90] and CMS [91] collaborations are given as

$$\kappa_W = 1.05 \pm 0.06, \quad \kappa_Z = 0.99 \pm 0.06, \quad \text{(ATLAS)}$$
$$\kappa_W = 1.02 \pm 0.08, \quad \kappa_Z = 1.04 \pm 0.07, \quad \text{(CMS)}, \quad (3.8)$$

Constraints from updated values of $\kappa_t, \kappa_b, \kappa_\tau, \kappa_\mu$ etc by LHC [90,91] can be similarly imposed

$$\kappa_t = 0.94 \pm 0.11, \quad \kappa_b = 0.89 \pm 0.11,$$
$$\kappa_\tau = 0.93 \pm 0.07, \quad \kappa_\mu = 1.06^{+0.25}_{-0.30}, \quad \text{(ATLAS)}$$
$$\kappa_t = 1.01 \pm 0.11, \quad \kappa_b = 0.99 \pm 0.16,$$
$$\kappa_\tau = 0.92 \pm 0.08, \quad \kappa_\mu = 1.12 \pm 0.21, \quad \text{(CMS)} \quad (3.9)$$

The code SARAH [92] is used to generate the input model files to link the SPheno [93] package, which has already implemented various updated collider constraints. We require the predicted Higgs mass to lie between 124 GeV and 126 GeV. Bounds from the stability of the electroweak vacuum (or the bounded from below condition), the perturbative unitary condition in [60] are imposed. We also impose the most stringent upper bound for the s_H parameter with $s_H \lesssim 0.2$ [63], although such a constraint can be relaxed in various parameter space regions. All the survived points need to pass the bounds in HiggsBounds [94], HiggsSignals [95,96] as well as GMCALC [97].

Firstly, we scan the parameter space of ordinary GM model with aligned triplet VEVs to survey if it can successfully account for the recent W-boson mass anomaly

reported by CDF-II experiment. The value of the T parameter, which quantifies the strength of weak-isospin breaking through the radiative corrections, is small in this model because of the tree-level custodial $SU(2)_c$ symmetry. So, we do not expect large new physics contributions to m_W in ordinary GM model. We show our numerical results in Fig. 2. It can be seen from the left panel that the survived points of ordinary custodial preserving GM model can not explain the CDF-II data, which can contribute to Δm_W a maximal amount 0.0012 GeV at one loop level. The corresponding heavy Higgs mass m_{h_2} and the doubly charged Higgs $m_{H^{++}}$ mass are also shown in the right panel with $m_{H^{++}}$ lying near 900 GeV for $\Delta m_W \sim 0.0012$ GeV. Such a heavy H^{++} can give very small contributions to the muon anomalous magnetic moment Δa_μ by the Barr–Zee diagrams, which can not [98] account for the recently reported muon $g - 2$ anomaly [99].

The T parameter can be increased in GM type models if additional $SU(2)_c$ custodial symmetry breaking effects are included. The most economical recipe without putting in by hand new $SU(2)_c$ breaking terms is to split slightly the triplet VEVs, that is, requiring misalignment between v_ξ and v_χ by a small amount. Such a small misalignment with real non-custodial vacuum from $SU(2)_L \times SU(2)_R$ invariant scalar potential can still be allowed and consistent, as discussed in previous sections. Besides, it had been shown in [65] that the effects of custodial symmetry breaking by loop effects of the $U(1)_Y$ hypercharge gauge interaction in the GM model are under control up to high energy close to the theory cutoff. So we expect that the full effective scalar potential only break softly the custodial symmetry and the misalignment among the triplet VEVs should not be large. With small misalignment among the triplet VEVs, the effective T parameter can be increased accordingly because of the slightly breaking of $SU(2)_c$ custodial symmetry.

We scan the parameter space of GM model with slightly misaligned triplet VEVs and show our numerical results in Fig. 3. From our numerical results, we can see in the upper left panel that the new physics contribution Δm_W is indeed correlated to the $SU(2)_c$ breaking parameter $\Delta v \equiv v_\xi - v_\chi$. New contribution to Δm_W can easily reach 0.067 GeV for Δv as small as 0.80 GeV with $v_\chi \lesssim 15$ GeV, explaining the CDF-II data to 1σ range. As expected, larger Δv can increase further the value of Δm_W, which can be welcome to explain the CDF-II anomaly. Besides, for fixed Δv, larger value of v_χ always lead to larger new physics contribution to Δm_W. On the other hand, to explain the CDF-II m_W data to 1σ range, larger value of v_χ needs smaller misalignment (so smaller value of Δv), which is also welcome to keep the stability of such vacuum with misaligned triplet VEVs. From our numerical results, we can see that the values of κ_Z versus κ_W are constrained to be larger than unity (see the upper right panel of Fig. 3), which can be seen to be consistent with
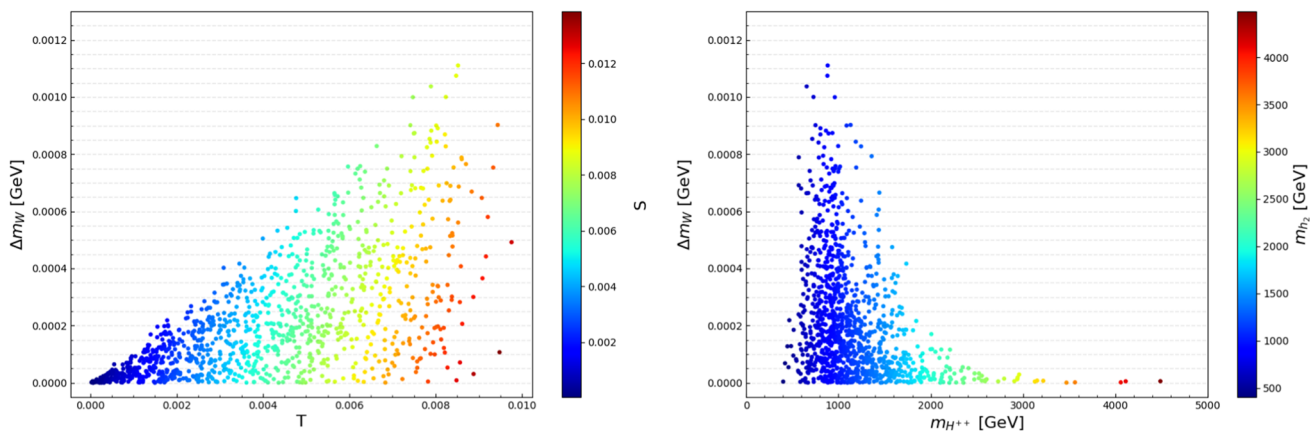
Fig. 2 We show the new physics contributions to Δm_W in ordinary custodial symmetry preserving GM model. The dependence of Δm_W on the mass of doubly charged scalar is also shown

the updated LHC constraints in (3.8). Besides, the allowed values of κ_t (in the 1σ range of Δm_W) should lie within the narrow range $0.990 \lesssim \kappa_t \lesssim 0.998$, which is also compatible with the LHC constraints in (3.9). It can be seen from the lower right panel of Fig. 3 that the mass of H^{++} Higgs is constrained to be larger than 600 GeV. Although the Barr–Zee type diagrams involving the H^{++} Higgs can contribute to Δa_μ, such a heavy H^{++} (larger than almost 400 GeV [98]) can not lead to large enough Δa_μ to account for the new muon $g - 2$ anomaly.

4 RH-neutrino extended GM model

Unlike Higgs Triplet Model (HTM) model, which extends the SM with one $SU(2)_L$ triplet, ordinary GM model always predict $\rho_{tree} = 1$ by the custodial $SU(2)_c$ symmetry. So, it is fairly interesting to seek alternative custodial $SU(2)_c$ breaking sources to explain the CDF-II m_W anomaly. As Yukawa couplings always break the custodial $SU(2)_c$ symmetry that preserved by the Higgs potential, we could introduce additional large $SU(2)_c$ breaking Yukawa coupling terms involving neutrinos. In ordinary GM model, the Yukawa couplings terms $h_{ij}\bar{L}_L^{ic}i\tau_2\chi L_L^j$ are responsible for neutrino masses generation, whose coupling strength should be rather tiny for $v_\Delta \sim \mathcal{O}(10)$ GeV. So we need to find ways to consistently increase the coupling strength h_{ij}. We find that large h_{ij} can be naturally allowed when an additional RH neutrino sector is introduced. The new RH neutrino sector in our extended GM model can be written as

$$-\mathcal{L} \supseteq y_{ij}^N \bar{L}_{L,i}\phi N_{R,j} + \frac{1}{2}(M_R)_{ij}N_{R,i}^T C N_{R,j}$$
$$+ h_{ij}\overline{L_L^{ic}}i\tau_2\chi L_L^j + \text{h.c.}, \quad (4.1)$$

with $N_{R,i}$ the RH neutrinos. After EWSB, it will lead to the neutrino mixing mass matrix of the form

$$\mathcal{M}_\nu = \begin{pmatrix} h_{ij}v_\Delta & (y_{ij}^N)^T v_\phi \\ y_{ij}^N v_\phi & (M_R)_{ij} \end{pmatrix}. \quad (4.2)$$

For simply, we choose $(M_R)_{ij} = M_{R,i}\delta_{ij}$ up to possible phases. So, the neutrino mass can be given as

$$m_\nu \approx h_{ij}v_\Delta - v_\phi^2(y_{ij}^N)^T M_{R,j}^{-1}(y_{jk}^N), \quad (4.3)$$

for $M_R \gg v_\phi \gg v_\Delta$, which is in fact a mixed type I + II like neutrino seesaw mechanism. Tiny neutrino mass of order 10^{-3} eV requires the cancelation among the two terms

$$h_{ij}v_\Delta \approx \frac{[(y^N)^T y^N]_{ij}v_\phi^2}{M_{R,i}}. \quad (4.4)$$

So, each term should take the following form

$$M_{ij}^{(I,II)} = V_{PMNS}^* \text{diag}(m_1, m_2 e^{-i\phi_1}, m_3 e^{-i\phi_1})_{ij} V_{PMNS}^{-1},$$

with $m_i \simeq v^2/M_{R,i}$ and V_{PMNS} the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [73], to guarantee that the form of neutrino mass matrix can be kept after large fine-tuned cancelations. For $y^N \sim \mathcal{O}(1)$, $M_{R,i} \sim \mathcal{O}(1)$ TeV and $h_{ij} \sim \mathcal{O}(1)$, tiny neutrino mass requires $v_\Delta \sim \mathcal{O}(10)$ GeV, which is naturally allowed by GM model.

From previous discussions, it can be seen that the coupling h_{ij} should take the following form

$$h_{ij} = 2\sqrt{2}(V_{PMNS}^T)_{im}^{-1}\left(\frac{v(1-s_H^2)}{s_H M_{R;m}}\right)\delta_{mn}(V_{PMNS})_{nj}^{-1}. \quad (4.5)$$

However, the coupling strength $|h_{ij}| \sim \mathcal{O}(1)$ will be constrained stringently by the lepton flavor violation (LFV) processes, especially when it is not very small. In fact, the main LFV signatures of the seesaw mechanism stem from muon
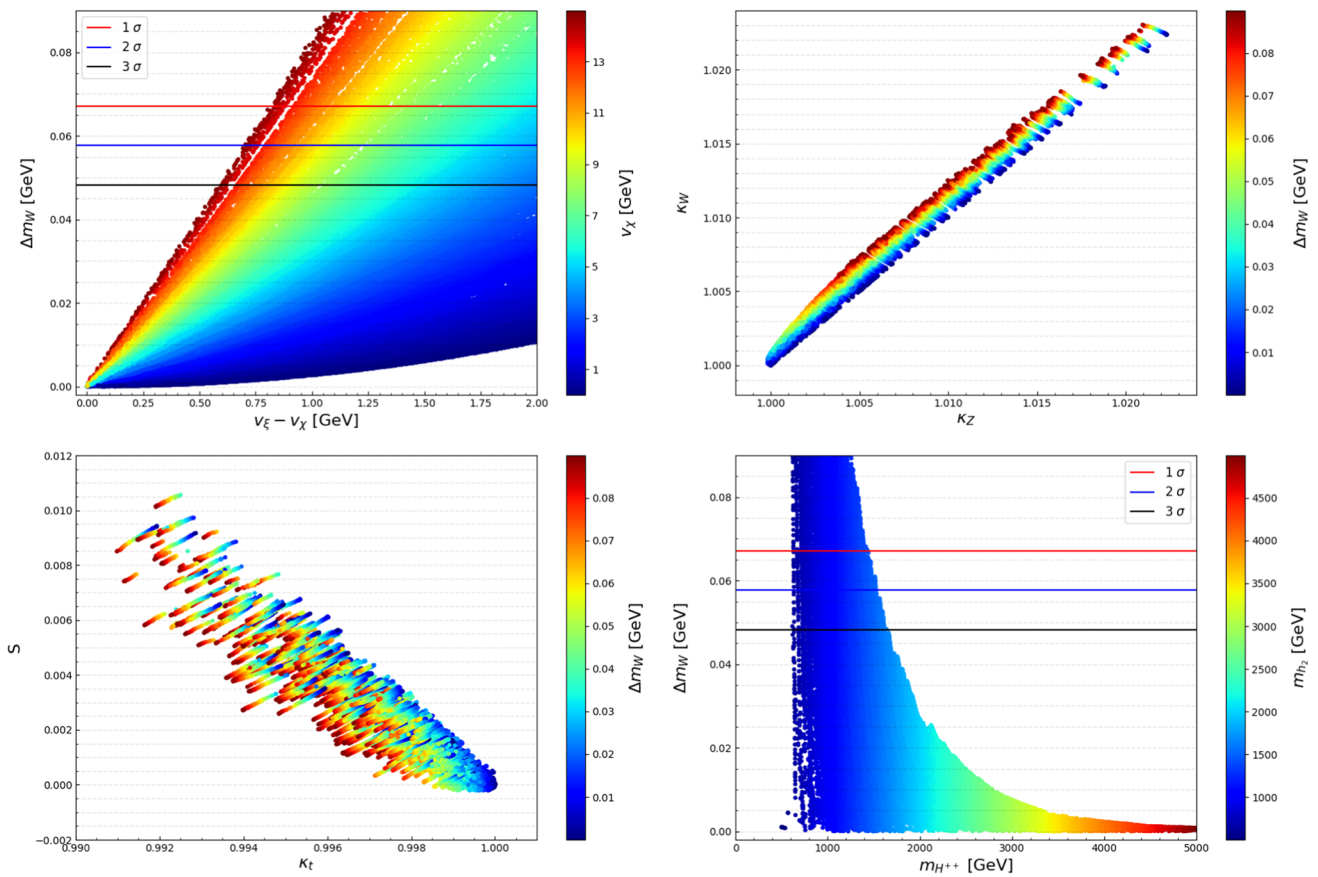
Fig. 3 We show the new physics contribution Δm_W in the GM model with a small misalignment among the triplet VEVs. The dependences of Δm_W on the mass of doubly charged scalar, the values of $\kappa_W, \kappa_Z, \kappa_t$ and the S parameter are also shown in the panels

decay, with the current bounds [100]

$$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}, \quad BR(\mu \to 3e) < 10^{-12}.$$

Other LFV process, such as $\mu - e$ conversion and LFV decays involving τ lepton, are subdominant [101,102]. For scalar triplets masses of order 1 TeV, the typical magnitude of $|h_{ij}|$ is constrained to be less than $\mathcal{O}(10^{-2})$ by the $BR(\mu \to e\gamma)$ bound

$$BR(\mu \to e\gamma) \simeq \frac{\alpha_{EM}}{192\pi G_F^2} |(h^\dagger h)_{e\mu}|^2 \left(\frac{1}{M_{H^{\pm\pm}}^2} + \frac{8}{M_{H^\pm}^2}\right)^2. \tag{4.6}$$

Due the correlation between h_{ij} and $M_{R,i}$ via Eq. (4.5), the scale of $M_{R,i}$ (for simply, we set $M_{R,i} \approx M_R$) should typically be heavier than 50 TeV for $s_H < 0.2$ and $M_{H^{\pm\pm}} \simeq m_\Delta$ at 1 TeV.

We should note again that, in ordinary GM model, the coupling h_{ij} should be tiny (of order 10^{-13}) for $v_\Delta \sim \mathcal{O}(10)$ GeV. With the augmentation of the RH neutrino sector, the allowed value of h_{ij} can be greatly increased. On the other hand, to get the observed tiny neutrino masses, the price of large fine-tuning (FT) had to be paid for the cancelation in

Eq. (4.3). The larger the M_R value, consequently a smaller h_{ij} coupling term (hence a smaller $SU(2)_c$ breaking source), the smaller the FT is needed.

The introduction of RH neutrinos can be advantageous in cosmology. Although the realization of electroweak baryogenesis can be possible in GM type model after adding additional CP violation sources [56], the idea that the baryon asymmetry is induced by the interactions responsible for neutrino masses (the leptogenesis) is fairly attractive. In leptogenesis, one-loop non-vanishing CP asymmetry requires flavor "breaking", or more exactly at least two sources of flavor "breaking", i.e. two heavy states with unequal couplings to leptons and/or scalar bosons. So, pure type-II seesaw model (and also in GM model) with a single scalar triplet, which is the only one that can give 2 or 3 light neutrino masses from a single heavy state, gives a vanishing asymmetry, even though it a priori satisfies all Sakharov conditions [74,75]. Therefore, to successfully adopt the leptogenesis mechanism, it is attractive to reintroduce heavy Majorana neutrinos, which can be responsible for both the observed baryon asymmetry in the universe and the generation of the neutrino masses via mixed type I + II like seesaw mechanism. It has been found in [76,77] that, for leptogenesis in the framework of the type-

I seesaw model, a lower bound of about 10^8–10^9 GeV on the RH neutrino scale can be derived if the heavy singlet neutrinos have an hierarchical mass spectrum. To obtain this lower bound, the size of the leptonic asymmetry between the heavy Majorana neutrino decay $N \to L\phi$ and its respective CP conjugate plays a key role. It has been shown in [76] that the leptonic CP asymmetry is not only analytically well-behaved but also can be of order unity if two of the heavy Majorana neutrinos have mass differences comparable to their decay widths. Due to this resonant effect, the allowed RH neutrino mass scale from the requirement of generating sufficient lepton number asymmetry in resonant leptogenesis scenario can be significantly lower. Even TeV scale RH neutrino masses (M_1, $M_2 \sim \mathcal{O}(1)$ TeV) are still allowed to obtain sufficient baryon asymmetry. So, with $\mathcal{O}(100)$ TeV Majorana RH neutrino masses, leptogenesis with type-I seesaw can already generate the required baryon asymmetry in the universe. On the other hand, our RH neutrino extended GM model in fact leads to a mixed type I + II neutrino seesaw mechanism. With both RH neutrinos $N_{R,j}$ and a single scalar triplet χ, the corresponding leptogenesis can work well and have new CP asymmetry sources [78], such as $\epsilon^{\chi}_{N_k}$ from the N_k decay and ϵ_{χ} from the χ triplet decay. Because of large cancelations between the type-I and type-II seesaw contributions to the neutrino masses, both (type-I and type-II) contributions to lepton asymmetry by leptogenesis are important [79]. In addition to the contributions to leptogenesis from the decay of the RH neutrino singlets to leptons and Higgs scalar, the contributions from the decay of the triplet to two leptons can also generate sizeable lepton asymmetry.

GM extension model with RH neutrinos can allow moderately large Yukawa coupling strength $|h_{ij}| \sim \mathcal{O}(10^{-2})$ with large FT in the cancelation between type I and type II type contributions for neutrino masses to generate tiny neutrino masses of order 10^{-3} eV. So the introduction of RH neutrinos are crucial to allow large $h_{ij} L_L^T \chi L_L$ coupling. Larger values of h_{ij} coupling will lead to larger tree level $SU(2)_c$ symmetry breaking effects, which can be welcome to increase Δm_W by loop contributions. We show the contributions to Δm_W with moderately large h_{ij} terms in the upper panels of Fig. 4 for our RH neutrino extended GM model. We can see from the upper left panel of Fig. 4 that, although the new physics contribution Δm_W in the RH neutrino extended GM model is much larger than that in the ordinary custodial symmetry preserving GM model (with aligned triplet VEVs), this RH neutrino extension model without misalignment among the triplet VEVs can still not explain the new CDF-II anomaly on W-boson mass. Lower scale of M_R, which corresponds to larger value of h_{ij}, can possibly increase the value of Δm_W, with the maximal contribution about 0.03 GeV. However, most of such points are ruled out by LFV constraints. Therefore, large h_{ij} term with the introduction of the RH neutrino sector can greatly increase the new physics contributions to

Δm_W in comparison to ordinary GM model, but it is still insufficient to explain the new CDF-II m_W anomaly.

Although merely introducing small misalignment among the triplet VEVs can already explain the CDF-II m_W anomaly up to 1σ range, it is still interesting to see how the presence of both custodial breaking sources can affect the degree of misalignment needed to explain the anomaly. That is, we would like to combine both the effects of moderately large h_{ij} coupling and small misalignment among the triplet VEVs. As discussed previously, real custodial symmetry breaking vacuum can be the true/metastable minimum of the $SU(2)_L \times SU(2)_R$ invariant scalar potential. The pseudo-Goldstone modes $H_5^{\pm} \sim (\chi^{\pm} - \xi^{\pm})/\sqrt{2}$, which correspond to the breaking of custodial $SU(2)_c$ to residual $U(1)$, can receive large radiative contributions to their masses even for a TeV UV-completion scale, as the h_{ij} coupling can be large after the introduction of RH neutrinos.

It can be seen in the middle and lower panels of Fig. 4 that large new physics contributions to Δm_W can still be easily given in this RH neutrino extended GM model (accompanied with small misalignment among the triplet VEVs). Besides, the splitting needed to explain the CDF-II m_W anomaly up to 1σ range is smaller than that without RH neutrino sector. For example, from the middle left panel, it is obvious that to explain the CDF-II W boson mass anomaly to 1σ range, the minimal splitting among the triplet VEVs should be larger than 0.70 GeV for $v_{\chi} \lesssim 15$ GeV. The corresponding values of κ_W, κ_Z and the $m_{H^{++}}$, m_{h_2} masses are also shown in these panels, which lie within the LHC bounds. The 1σ ranges of CDF-II m_W data require κ_W, κ_Z to be larger than unity and smaller than 1.022 (and 1.020), respectively. The mass range for $m_{H^{++}}$ allows lighter (than 400 GeV) H^{++}, which can possibly explain the muon $g - 2$ anomaly to 3σ range by Barr–Zee type contributions. The combination of both custodial symmetry breaking effects can give constructive contributions to Δm_W. The loose correlations between the RH neutrino sector and the needed splitting of the triplet VEVs are shown in the M_R versus Δv plot (the lower right panel).

5 Conclusions

GM model, which extends the SM with new triplets and preserves the tree level custodial symmetry, can hardly account for the new CDF-II anomaly on W-boson mass in its original form. As expected, unless additional $SU(2)_c$ custodial symmetry breaking effects are significant, the new physics contributions to Δm_W in GM type model are always very small. Our numerical results show that ordinary GM model can contribute to Δm_W a maximal amount 0.0012 GeV, which can not explain the new CDF-II data on W boson mass.
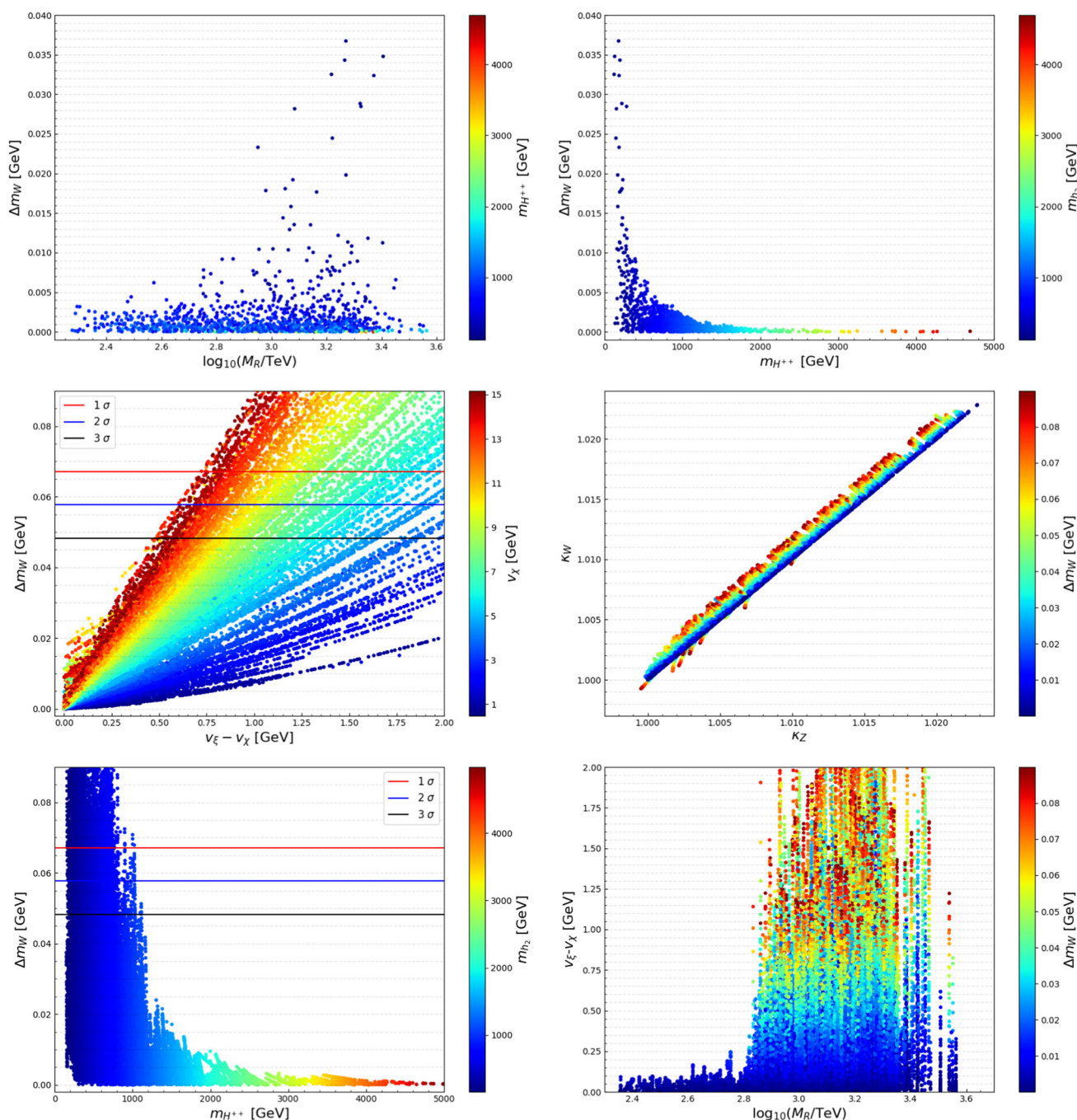
Fig. 4 We show the new physics contribution Δm_W in the RH neutrino extended GM model without (the upper panels) and with (the middle and the lower panels) small misalignment among the triplet VEVs. The corresponding ranges of κ_W, κ_Z, M_R and the doubly charged scalar masses $m_{H^{++}}$ etc are also shown

We propose firstly to introduce small misalignment among the triplet VEVs in GM model to break the custodial $SU(2)_c$ symmetry so as to that the new physics contributions to Δm_W can be increased. Such slightly misaligned triplet VEVs from tree level custodial symmetry preserving scalar potential (appended with custodial symmetry breaking loop level effective potential) can still be allowed, because no disruptive jumping transition from the true stable vacuum in the aligned case to an unstable saddle point vacuum in the misaligned case should occur when one begins to slightly turn on small misalignment among the triplet VEVs (unless the splitting within the misaligned triplet VEVs is big enough) from a custodial preserving tree level scalar potential (with custodial symmetry breaking loop level contributions) by continuously

varying the relevant parameters. Our numerical results indicate that the resulting Δm_W can easily reach the 1σ range of CDF-II m_W data and the splitting among the triplet VEVs $\Delta v = v_\xi - v_\chi$ can be as small as 0.8 GeV for $v_\chi \lesssim 15$ GeV.

In ordinary GM model, the neutrino masses can be generated by the h_{ij} term via the type II seesaw like mechanism with a very tiny h_{ij} coupling strength for triplet VEVs of order GeV. We also propose to extend the GM model with a low scale RH neutrino sector, which can be used to generate tiny neutrino masses by a mixed type I + II like neutrino seesaw mechanism and realize successfully leptogenesis mechanism, as type-II leptogenesis with a single triplet alone does not work. Given the tiny neutrino masses, the h_{ij} coupling strength can be much larger (even of order 10^{-2}) in this case. Larger h_{ij} term means a larger $SU(2)_c$ breaking source. With low scale RH neutrino mass scale of order $10^2 \sim 10^4$ TeV, the new physics contribution to Δm_W can reach 0.03 GeV and is much larger than that of ordinary custodial symmetry preserving GM model. However, such a Δm_W value is insufficient to explain the CDF-II m_W anomaly

It is interesting to combine both $SU(2)_c$ breaking effects, the small misalignment among the triplet VEVs and moderately large h_{ij} couplings. Moderately large h_{ij} term, allowed by the extension with a RH neutrino sector, can give large contributions to the masses of the pseudo-Goldstone modes $H_5^\pm$ even with low UV completion scale. As moderately large h_{ij} term acts as an additional $SU(2)_c$ breaking source other than the $U(1)_Y$ hypercharge (and Yukawa) interactions, we expect that the full scalar potential including the custodial symmetry breaking loop corrections should easily lead to a custodial symmetry breaking minimum with small misalignment among the triplet VEVs. In this case, our numerical results show that the value of Δm_W can still easily reach the 1σ range of CDF-II m_W data, with a minimum splitting (among the triplet VEVs) approximately 0.7 GeV for $v_\chi \lesssim 15$ GeV.

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