Eur. Phys. J. C (2023) 83:81
https://doi.org/10.1140/epjc/s10052-023-11204-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Early universe nucleosynthesis in massive conformal gravity

F. F. Faria[a]

Centro de Ciências da Natureza, Universidade Estadual do Piauí, Teresina, PI 64002-150, Brazil

Received: 5 July 2022 / Accepted: 5 January 2023 / Published online: 27 January 2023

Abstract We study the dynamics of the early universe in massive conformal gravity. In particular, we show that the theory is consistent with the observed values of the primordial abundances of light elements if we consider the existence of right-handed sterile neutrinos.

1 Introduction

It is well known that the standard ΛCDM cosmological model is consistent with most observations of the universe at both early and late times [1–4]. However, for this consistency to occur, a very small value for the cosmological constant (Λ) is required, which by far does not match with the huge value predicted by quantum field theory (see [5] for a nice review). This discrepancy between the cosmological and quantum values of Λ is known as the cosmological constant problem [6]. Another important problem of ΛCDM is that the primordial lithium abundance from the early universe nucleosynthesis predicted by it differs by about a factor of three from the observed abundance [7], which is known as the lithium problem. Despite several attempts over the years, no alternative cosmological model has succeeded in solving these two problems and being consistent with other cosmological observations at the same time.

One of such models comes from massive conformal gravity (MCG), which is a conformally invariant theory of gravity in which the gravitational action is the sum of the Weyl action with the Einstein-Hilbert action conformally coupled to a scalar field [8]. Among so many cosmological models, we chose the MCG model because it fits well with the Type Ia supernovae (SNIa) data without the cosmological constant problem [9]. In addition, the theory is free of the van Dam–Veltman–Zakharov (vDVZ) discontinuity [10], can reproduce the orbit of binaries by the emission of gravitational waves [11] and is consistent with solar system observations [12]. Furthermore, MCG is a power-counting renormalizable [13,14] and unitary [15] quantum theory of gravity.

In this paper, we want to see if the MCG cosmology is consistent with the observed primordial abundances of light elements without the lithium problem. In Sect. 2, we describe the MCG cosmological equations. In Sect. 3, we derive the matter energy-momentum tensor used in the theory. In Sect. 4, we study the dynamics of the early MCG universe. In Sect. 5, we compare the early universe nucleosynthesis of MCG with cosmological observations. In Sect. 6, we analyze the evolution of the baryon density of the MCG universe. Finally, in Sect. 7, we present our conclusions.

2 Massive conformal gravity

The total MCG action is given by[1] [10]

$$S = \int d^4x \sqrt{-g}\Big[\varphi^2 R + 6\partial^{\mu}\varphi\partial_{\mu}\varphi - \frac{1}{2\alpha^2}C^{\alpha\beta\mu\nu}C_{\alpha\beta\mu\nu}\Big] + \frac{1}{c}\int d^4x\mathcal{L}_m, \tag{1}$$

where φ is a scalar field called dilaton, α is a coupling constant,

$$C^{\alpha\beta\mu\nu}C_{\alpha\beta\mu\nu} = R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} - 4R^{\mu\nu}R_{\mu\nu} + R^2 + 2\left(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2\right) \tag{2}$$

is the Weyl tensor squared, $R^{\alpha}{}_{\mu\beta\nu} = \partial_{\beta}\Gamma^{\alpha}_{\mu\nu} + \cdots$ is the Riemann tensor, $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$ is the Ricci tensor, $R = g^{\mu\nu}R_{\mu\nu}$ is the scalar curvature, and $\mathcal{L}_m = \mathcal{L}_m(g_{\mu\nu}, \Psi)$ is the Lagrangian density of the matter field Ψ. It is worth noting that besides being invariant under coordinate transfor-

[1] This action is obtained from the action of Ref. [10] by rescaling $\varphi \to \left(\sqrt{32\pi G/3}\right)\varphi$ and considering $m = \sqrt{3/64\pi G\alpha}$.

[a] e-mail: felfrafar@hotmail.com (corresponding author)

mations, the action (1) is also invariant under the conformal transformations

$$\tilde{\Phi} = \Omega(x)^{-\Delta_\Phi}\Phi, \tag{3}$$

where $\Omega(x)$ is an arbitrary function of the spacetime coordinates, and Δ_Φ is the scaling dimension of the field Φ, whose values are -2 for the metric field, 0 for gauge bosons, 1 for scalar fields, and 3/2 for fermions.

The variation of (1) with respect to $g^{\mu\nu}$ and φ gives the MCG field equations

$$\varphi^2 G_{\mu\nu} + 6\partial_\mu\varphi\partial_\nu\varphi - 3g_{\mu\nu}\partial^\rho\varphi\partial_\rho\varphi + g_{\mu\nu}\nabla^\rho\nabla_\rho\varphi^2 - \nabla_\mu\nabla_\nu\varphi^2 - \alpha^{-2}W_{\mu\nu} = \frac{1}{2c}T_{\mu\nu}, \tag{4}$$

$$\left(\nabla^\mu\nabla_\mu - \frac{1}{6}R\right)\varphi = 0, \tag{5}$$

where

$$W_{\mu\nu} = \nabla^\rho\nabla_\rho R_{\mu\nu} - \frac{1}{3}\nabla_\mu\nabla_\nu R - \frac{1}{6}g_{\mu\nu}\nabla^\rho\nabla_\rho R + 2R^{\rho\sigma}R_{\mu\rho\nu\sigma} - \frac{1}{2}g_{\mu\nu}R^{\rho\sigma}R_{\rho\sigma} - \frac{2}{3}RR_{\mu\nu} + \frac{1}{6}g_{\mu\nu}R^2 \tag{6}$$

is the Bach tensor,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{7}$$

is the Einstein tensor,

$$\nabla^\rho\nabla_\rho\varphi = \frac{1}{\sqrt{-g}}\partial^\rho\left(\sqrt{-g}\partial_\rho\varphi\right) \tag{8}$$

is the generally covariant d'Alembertian for a scalar field, and

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}} \tag{9}$$

is the matter energy-momentum tensor.

Before we proceed, it is important to note that both the symmetries of the theory allow us to introduce in (1) a quartic self-interacting term of the dilaton $\lambda\int\sqrt{-g}\varphi^4$ as well as interaction terms of the dilaton with the matter fields. In the case of the dilaton self-interaction term, we do not include it in the MCG action because this inclusion makes the flat metric no longer a solution of the field equations, which invalidates the S-matrix formulation. Although such a term is reintroduced in the effective action by quantum corrections, we can consider the renormalized value of the coupling constant λ equal zero so that the self-interacting term is present in the renormalized action only to cancel out the corresponding divergent term. In addition, we neglect the couplings between the dilaton and the matter fields because they make the field equation (5) no longer valid. This equation is fundamental to cancel non-renormalizable divergent terms that appear in the effective action [16].

At scales below the Planck scale, the dilaton field acquires a spontaneously broken constant vacuum expectation value φ_0 [17]. In this case, the field equations (4) and (5) become

$$\varphi_0^2 G_{\mu\nu} - \alpha^{-2}W_{\mu\nu} = \frac{1}{2c}T_{\mu\nu}, \tag{10}$$

$$R = 0. \tag{11}$$

In addition, for $\varphi = \varphi_0$, the MCG line element $ds^2 = (\varphi/\varphi_0)^2 g_{\mu\nu}dx^\mu dx^\nu$ reduces to

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu. \tag{12}$$

The full dynamics of the MCG universe can be described by (10)–(12) without loss of generality.

3 Dynamical perfect fluid

In order to find the MCG matter energy-momentum tensor, we consider the conformally invariant matter Lagrangian density [18]

$$\mathcal{L}_m = -\sqrt{-g}c\left[S^2R + 6\partial^\mu S\partial_\mu S + \lambda S^4 + \frac{i}{2}\hbar\left(\overline{\psi}\gamma^\mu D_\mu\psi - D_\mu\overline{\psi}\gamma^\mu\psi\right) - \hbar\mu S\overline{\psi}\psi\right], \tag{13}$$

where S is a scalar Higgs field,[2] λ and μ are coupling constants, $\overline{\psi} = \psi^\dagger\gamma^0$ is the adjoint fermion field, $D_\mu = \partial_\mu + [\gamma^\nu, \partial_\mu\gamma_\nu]/8 - [\gamma^\nu, \gamma_\lambda]\Gamma^\lambda{}_{\mu\nu}/8$ ($\Gamma^\lambda{}_{\mu\nu}$ is the Levi-Civita connection), and γ^μ are the general relativistic Dirac matrices, which satisfy the anti-commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

By varying (13) with respect to S, $\overline{\psi}$ and ψ, we obtain the field equations

$$12\nabla^\mu\nabla_\mu S - 2RS - 4\lambda S^3 + \hbar\mu\overline{\psi}\psi = 0, \tag{14}$$

$$i\gamma^\mu D_\mu\psi - \mu S\psi = 0, \tag{15}$$

$$iD_\mu\overline{\psi}\gamma^\mu + \mu S\overline{\psi} = 0. \tag{16}$$

Additionally, the substitution of (13) into (9) gives

[2] Although the Higgs field is actually a doublet, and it is more likely that we must have two more scalar fields to get the correct quantum phenomenology at low energies [19], considering only a scalar Higgs field will not change the classical results of the theory.

$$\begin{aligned}\frac{T_{\mu\nu}}{c} &= 12\partial_\mu S\partial_\nu S - 6g_{\mu\nu}\partial^\rho S\partial_\rho S \\ &+2g_{\mu\nu}\nabla^\rho\nabla_\rho S^2 - 2\nabla_\mu\nabla_\nu S^2 + 2S^2 G_{\mu\nu} - g_{\mu\nu} \\ &\times\left[\lambda S^4 + \frac{i}{2}\hbar\left(\overline{\psi}\gamma^\rho D_\rho\psi - D_\rho\overline{\psi}\gamma^\rho\psi\right) - \hbar\mu S\overline{\psi}\psi\right] \\ &+\frac{i}{4}\hbar\big(\overline{\psi}\gamma_\mu D_\nu\psi - D_\nu\overline{\psi}\gamma_\mu\psi \\ &+\overline{\psi}\gamma_\nu D_\mu\psi - D_\mu\overline{\psi}\gamma_\nu\psi\big). \end{aligned} \tag{17}$$

Then, using (14)–(16) and $\nabla_\mu\nabla_\nu S^2 = 2(S\nabla_\mu\nabla_\nu S+\partial_\mu S\partial_\nu S)$ in (17), we find the energy-momentum tensor

$$\begin{aligned}T_{\mu\nu} &= c\left(8\partial_\mu S\partial_\nu S - 2g_{\mu\nu}\partial^\rho S\partial_\rho S \right.\\ &\left.-4S\nabla_\mu\nabla_\nu S + g_{\mu\nu}S\nabla^\rho\nabla_\rho S\right) \\ &+2cS^2\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) + T^f_{\mu\nu},\end{aligned} \tag{18}$$

where

$$\begin{aligned}T^f_{\mu\nu} &= \frac{i}{4}c\hbar\big(\overline{\psi}\gamma_\mu D_\nu\psi - D_\nu\overline{\psi}\gamma_\mu\psi + \overline{\psi}\gamma_\nu D_\mu\psi \\ &-D_\mu\overline{\psi}\gamma_\nu\psi\big) - \frac{1}{4}g_{\mu\nu}c\hbar\mu S\overline{\psi}\psi\end{aligned} \tag{19}$$

is the fermion energy-momentum tensor.

Considering that, at scales below the electroweak scale, the Higgs field acquires a spontaneously broken constant vacuum expectation value S_0, and making some algebra, we find that (15) and (18) become

$$\left[D^\mu D_\mu - \left(\frac{mc}{\hbar}\right)^2\right]\psi = 0, \tag{20}$$

$$\begin{aligned}T_{\mu\nu}(S_0, g_{\mu\nu}) &= 2cS_0^2\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) \\ &+T^f_{\mu\nu}(S_0, g_{\mu\nu}),\end{aligned} \tag{21}$$

where

$$\begin{aligned}T^f_{\mu\nu}(S_0, g_{\mu\nu}) &= \frac{i}{4}c\hbar\big(\overline{\psi}\gamma_\mu D_\nu\psi - D_\nu\overline{\psi}\gamma_\mu\psi + \overline{\psi}\gamma_\nu D_\mu\psi \\ &-D_\mu\overline{\psi}\gamma_\nu\psi\big) - \frac{1}{4}g_{\mu\nu}mc^2\overline{\psi}\psi,\end{aligned} \tag{22}$$

with $m = \mu S_0\hbar/c$ being the fermion mass. In flat spacetime, is not difficult to see that (20) and (22) reduce to

$$\left[\partial^\mu\partial_\mu - \left(\frac{mc}{\hbar}\right)^2\right]\psi = 0, \tag{23}$$

$$\begin{aligned}T^f_{\mu\nu}(S_0, \eta_{\mu\nu}) &= \frac{i}{4}c\hbar\big(\overline{\psi}\gamma_\mu\partial_\nu\psi - \partial_\nu\overline{\psi}\gamma_\mu\psi \\ &+\overline{\psi}\gamma_\nu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma_\nu\psi\big) - \frac{1}{4}\eta_{\mu\nu}mc^2\overline{\psi}\psi,\end{aligned} \tag{24}$$

where now the Dirac matrices satisfy the anti-commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$.

The normalized plane wave solution to (23) is given by

$$\psi = \frac{1}{\sqrt{VE_k}}u_k e^{ik_\mu x^\mu}, \tag{25}$$

where V is the volume, $E_k = \sqrt{k^2c^2 + m^2c^4}$ is the energy, u_k is a spinor which satisfies $\left[\gamma^\mu k_\mu + mc/\hbar\right]u_k = 0$, and $k_\mu = (E_k/c\hbar, \vec{k}/\hbar)$ is the wave vector, with $\vec{k}$ being the momentum and $k = |\vec{k}|$. By substituting (25) and its adjoint into (24), and using $\overline{u}_k u_k = -mc^2$, we obtain

$$T^f_{\mu\nu}(S_0, \eta_{\mu\nu}) = \left(\frac{c^2\hbar^2}{VE_k}\right)k_\mu k_\nu + \left(\frac{m^2c^4}{4VE_k}\right)\eta_{\mu\nu}. \tag{26}$$

Incoherently adding to (26) the individual contributions of a set of six plane waves moving in the $\pm x$, $\pm y$ and $\pm z$ directions, all with the same E_k and k, we can write the energy-momentum tensor (26) in the perfect fluid form

$$T^f_{\mu\nu}(S_0, \eta_{\mu\nu}) = \left(\rho + \frac{p}{c^2}\right)u_\mu u_\nu + \eta_{\mu\nu}p + \eta_{\mu\nu}c^2\rho_\Lambda, \tag{27}$$

where

$$c^2\rho = \frac{6E_k}{V} \tag{28}$$

is the energy density of the fluid,

$$p = \frac{2k^2c^2}{VE_k} \tag{29}$$

is the pressure of the fluid,

$$c^2\rho_\Lambda = \frac{3m^2c^4}{2VE_k} \tag{30}$$

is the vacuum energy (dark energy) density, and u^μ is the four-velocity of the fluid, which is normalized to $u^\mu u_\mu = -c^2$. It follows from (28)–(30) that

$$p = 0, \quad \rho_\Lambda = \frac{1}{4}\rho, \tag{31}$$

for a non-relativistic perfect fluid ($k^2c^2 \ll m^2c^4$), and

$$p = \frac{1}{3}c^2\rho, \quad \rho_\Lambda = 0, \tag{32}$$

for a relativistic perfect fluid ($k^2c^2 \gg m^2c^4$).

In curved spacetime, the perfect fluid energy-momentum tensor (27) is generalized to

$$T^f_{\mu\nu}(S_0, g_{\mu\nu}) = \left(\rho + \frac{p}{c^2}\right)u_\mu u_\nu + g_{\mu\nu}p + g_{\mu\nu}c^2\rho_\Lambda. \tag{33}$$

Finally, the insertion of (33) into (21) gives the energy-momentum tensor of a dynamical perfect fluid

$$\begin{aligned}T_{\mu\nu}(S_0, g_{\mu\nu}) &= 2cS_0^2\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) \\ &+\left(\rho+\frac{p}{c^2}\right)u_\mu u_\nu + g_{\mu\nu}p + g_{\mu\nu}c^2\rho_\Lambda.\end{aligned} \tag{34}$$

Taking the trace of (34), and substituting into the trace of (10), whose left hand side is zero due to the field equation (11) and the tracelessness of the Bach tensor ($W = g^{\mu\nu}W_{\mu\nu} = 0$), we arrive at

$$T = g^{\mu\nu}T_{\mu\nu} = 3p - c^2\rho + 4c^2\rho_\Lambda = 0. \tag{35}$$

We can see from (31) and (32) that both non-relativistic and relativistic perfect fluids satisfies the tracelessness relation (35). For simplicity, we could isolate ρ_Λ in (35) and replace it in (34) as done in Ref. [9]. In this case, it is made clear that the vacuum energy density does not contribute directly to the dynamic evolution of the MCG universe, which solves the cosmological constant problem found in the ΛCDM model. However, here we will keep ρ_Λ so we don't miss any physical details during the calculations.

By substituting (34) into (10), and considering (11), we find

$$\begin{aligned}&\left(\varphi_0^2 - S_0^2\right) R_{\mu\nu} - \alpha^{-2}W_{\mu\nu}\\ &\quad= \frac{1}{2c}\left[\left(\rho + \frac{p}{c^2}\right)u_\mu u_\nu + g_{\mu\nu}p + g_{\mu\nu}c^2\rho_\Lambda\right],\end{aligned} \tag{36}$$

which is the field equation that we will use in the study of the dynamics of the early MCG universe in the next section. But before that, it is important to compare MCG with another conformally invariant theory of gravity called conformal gravity (CG),[3] whose action is given by [20]

$$S = -\frac{1}{2\alpha^2}\int d^4x\sqrt{-g}\left(C^{\alpha\beta\mu\nu}C_{\alpha\beta\mu\nu}\right) + \frac{1}{c}\int d^4x\mathcal{L}_m. \tag{37}$$

By varying (37) with respect to $g_{\mu\nu}$, we obtain the field equation

$$-\alpha^{-2}W_{\mu\nu} = \frac{1}{2c}T_{\mu\nu}, \tag{38}$$

where $T_{\mu\nu}$ is given by (18). We can easily see the difference between the two theories by comparing (38) with (10) and (11). Just to stay within the scope of this paper, it is worth noting that CG does not pass the early universe nucleosynthesis test [21].

4 Early universe

As usual, we consider that the geometry of the universe is described by the Friedmann–Lemaître–Robertson–Walker (FLRW) line element

$$\begin{aligned}ds^2 &= -c^2dt^2 + a(t)^2\\ &\quad\times\left(\frac{dr^2}{1-Kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right),\end{aligned} \tag{39}$$

where $a = a(t)$ is the scale factor and K = -1, 0 or 1 is the spatial curvature. By substituting (39) and the fluid four-velocity $u^\mu = (c, 0, 0, 0)$ into (36), we obtain[4]

$$\frac{\ddot{a}}{a} = -\frac{c}{6\left(\varphi_0^2 - S_0^2\right)}\left(c^2\rho - c^2\rho_\Lambda\right), \tag{40}$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{Kc^2}{a^2} = \frac{c}{2\left(\varphi_0^2 - S_0^2\right)}\left(p + c^2\rho_\Lambda\right), \tag{41}$$

where the dot denotes d/dt.

Subtracting (40) from (41), and considering that[5]

$$\varphi_0^2 = \frac{3c^3}{32\pi G} \gg S_0^2, \tag{42}$$

we obtain

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{9c^2}\left(c^2\rho + 3p + 2c^2\rho_\Lambda\right) - \frac{Kc^2}{a^2}. \tag{43}$$

The combination of (43) with (40) then gives the energy continuity equation

$$c^2\dot{\rho} + 3\frac{\dot{a}}{a}\left(c^2\rho + p\right) - c^2\dot{\rho}_\Lambda = 0, \tag{44}$$

which can also be obtained by the conservation law $\nabla^\mu T^f_{\mu\nu} = 0$, with $T^f_{\mu\nu}$ being the perfect fluid energy-momentum tensor (33).

Using either (31) or (32) in (44), we get

$$\dot{\rho} + 4\frac{\dot{a}}{a}\rho = 0, \tag{45}$$

which, consequently, is valid for both non-relativistic and relativistic dynamical perfect fluids. As usual, we can write the solution to (45) in the form

$$\rho = \rho_0\left(\frac{a_0}{a}\right)^4, \tag{46}$$

where, from now on, the subscript 0 denotes values at the present time t_0.

In the case of the early universe, which is composed by a very hot plasma dominated by relativistic particles (radiation), we find that (43) becomes

$$\dot{a}^2 = \frac{16\pi Ga_0^4}{9a^2}\rho_{r0} - Kc^2. \tag{47}$$

where we used (32) and (46), with ρ_r being the mass density of the radiation. Since a is small in the early universe, we can neglect the curvature term on the right hand side of (47) and write it in the approximate form

$$\dot{a}^2 = \frac{16\pi Ga_0^4}{9a^2}\rho_{r0}, \tag{48}$$

[3] Although the difference between the two theories is quite obvious, as we will readily show next, MCG is often confused with CG. Perhaps this is because CG is much older and known than MCG.

[4] It is worth noting that $W_{\mu\nu} = 0$ for the FLRW spacetime.

[5] This value of φ_0 is necessary for the theory to be consistent with solar system observations [12].

whose solution is given by

$$a(t) = \left(\frac{64\pi G a_0^4 \rho_{r0}}{9} \right)^{1/4} t^{1/2}. \tag{49}$$

Finally, inserting (49) into the Hubble constant

$$H = \frac{\dot{a}}{a}, \tag{50}$$

we obtain

$$H = \frac{1}{2t}, \tag{51}$$

which is the same relation between the Hubble constant and time that occurs in the early ΛCDM universe. However, since the MCG scale factor (49) is equal 0.9 times the value of the ΛCDM scale factor, the expansion of the early MCG universe is slower than the expansion of the early ΛCDM universe, which will give a difference in the values of the two Hubble constants, as we will show in the next section.

5 Nucleosynthesis

The abundances of light chemical elements in the early universe are mainly determined by one cosmological parameter, namely, the baryon-to-photon ratio $\eta = n_b/n_\gamma$, where n_b and n_γ are the number densities of baryons and photons in the universe. As usual, to find η we must first write the Hubble constant in function of temperature T using the Stefan-Boltzmann law

$$\rho_r = \left(\frac{g_* a_{\mathcal{B}}}{2c^2} \right) T^4, \tag{52}$$

where $a_{\mathcal{B}}$ is the radiation energy constant and g_* counts the number of relativistic particle species determining the energy density in radiation. Substituting (52) and (49) into (46), we obtain

$$t = \left(\frac{9c^2}{32\pi G g_* a_{\mathcal{B}}} \right)^{1/2} \frac{1}{T^2}. \tag{53}$$

It then follows from (51) and (53) that

$$H = \left(\frac{8\pi G g_* a_{\mathcal{B}}}{9c^2} \right)^{1/2} T^2, \tag{54}$$

which is equal 0.82 times the value of the ΛCDM Hubble constant.

In order to describe the thermal history of the early MCG universe, we must compare the Hubble constant in the form (54) with the collision rate of particle interactions

$$\Gamma = n\sigma v, \tag{55}$$

where n is the number density of particles, σ is their interaction cross section and v is the average velocity of the particles. A specific temperature that is of particular importance for the outcome of the early universe nucleosynthesis (EUN) is the one at which the thermal equilibrium between neutrons and protons begins to break down, which happens when $H \sim \Gamma_\nu$, where

$$\Gamma_\nu \approx \frac{G_F^2}{c^6 \hbar^7} (k_{\mathcal{B}} T)^5 \tag{56}$$

is the collision rate of a neutrino with electrons or positrons, with G_F being the Fermi constant and $k_{\mathcal{B}}$ the Boltzmann constant.

By equating (54) with (56), and assuming that at the onset of the electron-positron annihilation the remaining relativistic particles are photons, electrons, positrons and left-handed neutrinos, for which $g_* = 10.75$, we obtain

$$k_{\mathcal{B}} T_{\text{eq}} = 0.75\,\text{MeV}. \tag{57}$$

We can see from (57) that the thermal equilibrium between neutrons and protons is maintained at temperatures above $T_{\text{eq}} = 8.7 \times 10^9$ K in the early MCG universe. At that time, the neutron-to-proton ratio was

$$\left(\frac{n_n}{n_p} \right)_{\text{eq}} = e^{-Q/k_{\mathcal{B}} T_{\text{eq}}} = 0.178, \tag{58}$$

where we used (57) and the neutron-proton energy difference $Q = 1.239$ MeV. Using (58), we can make a rough estimate that the final freeze-out neutron abundance is given by

$$X_n^\infty \sim X_n^{\text{eq}} = \frac{e^{-Q/k_{\mathcal{B}} T_{\text{eq}}}}{1 + e^{-Q/k_{\mathcal{B}} T_{\text{eq}}}} = 0.15. \tag{59}$$

Including the neutron decay in our calculation, we find

$$X_n(t) = X_n^\infty e^{-t/\tau_n} = 0.15\, e^{-t/\tau_n}, \tag{60}$$

where $\tau_n = 879.4$ s is the neutron mean lifetime [22].

The first light element formed in the early universe was deuterium (D), whose ratio to proton is approximately given by

$$\frac{n_D}{n_p} \approx 6.9\eta \left(\frac{k_{\mathcal{B}} T}{m_n c^2} \right)^{3/2} \exp\left(\frac{B_D}{k_{\mathcal{B}} T} \right), \tag{61}$$

where we used (58) and $B_D = 2.2$ MeV is the binding energy of deuterium. Noting that the EUN starts when $n_D \sim n_p$, it follows from (61) that

$$6.9\eta_{\text{EUN}} \left(\frac{k_{\mathcal{B}} T_{\text{EUN}}}{m_n c^2} \right)^{3/2} \exp\left(\frac{B_D}{k_{\mathcal{B}} T_{\text{EUN}}} \right) \approx 1, \tag{62}$$

where η_{EUN} and T_{EUN} are the baryon-to-photon ratio and temperature of the EUN. We can see from (62) that we need the value of T_{EUN} to find η_{EUN}. Fortunately, we can find such value from the primordial helium (^{4}He) abundance

$$Y_P \equiv \frac{4 n_{^4\text{He}}}{n_{\text{H}}} = \frac{2 X_n(t_{\text{EUN}})}{1 - X_n(t_{\text{EUN}})}, \tag{63}$$

where t_{EUN} is the time of the EUN.

The substitution of (60) and the observed value of the helium abundance $Y_P = 0.245$ [23] into (63) gives

$$t_{\mathrm{EUN}} \approx 279.7\,\mathrm{s}. \tag{64}$$

Then, by inserting (64) into (53), and considering that the electrons and protons are no longer relativistic after their annihilation, which gives $g_* = 3.36$, we obtain

$$T_{\mathrm{EUN}} \approx 8.8 \times 10^8\,\mathrm{K}. \tag{65}$$

Finally, using (65) in (62), we arrive at

$$\eta_{\mathrm{EUN}} \approx 5.12 \times 10^{-8}, \tag{66}$$

which produces abundances of other light elements besides helium orders of magnitude below the primordial abundances inferred from current observations [24]. However, this result does not automatically rule out MCG. If we consider that the theory has low energy ($\lesssim$ eV) right-handed sterile neutrinos,[6] then we must replace $g_* = 10.75$ by $g_* = 16.125$ prior to the electron-positron annihilation and $g_* = 3.36$ by $g_* = 5.04$ after the electron-positron annihilation due to the contribution of the sterile neutrinos to the relativistic energy content of the universe. These replacements lead to the standard value

$$\eta_{\mathrm{EUN}} \approx 6 \times 10^{-10}, \tag{67}$$

which is consistent with the observed abundances of all light elements with the exception of lithium.[7]

6 Baryon density

Another important cosmological parameter that is determined by η is the baryon mass density ρ_b of the universe. In order to find the relation between these two parameters in the MCG universe, we start from the definitions of the baryon and photon number densities

$$n_b = \frac{\rho_b}{m_N}, \tag{68}$$

$$n_\gamma = 2\zeta(3)\frac{8\pi}{c^3}\left(\frac{k_{\mathcal{B}}T}{h}\right)^3 \approx 2 \times 10^7 T^3, \tag{69}$$

where m_N is the nucleons mass. The combination of (68), (69) and (52), with $g_* = 2$, then gives the relation

$$\eta = \frac{a_{\mathcal{B}}}{2 \times 10^7 m_N c^2}\frac{\rho_b}{\rho_\gamma}T, \tag{70}$$

which is valid for any cosmological model. Noting that both ρ_b and ρ_γ obey (46) in MCG, we can write (70) in the form

$$\eta = \frac{a_{\mathcal{B}}}{2 \times 10^7 m_N c^2}\frac{\rho_{b0}}{\rho_{\gamma 0}}T, \tag{71}$$

which means that the baryon-to-photon ratio evolves over time in the MCG universe,[8] different to what happens in the ΛCDM universe where η is constant after the EUN.

Using the current temperature of the universe $T_0 = 2.73$ K in (52), with $g_* = 2$, we find

$$\rho_{\gamma 0} = 4.65 \times 10^{-31}\ \mathrm{kg/m^3}. \tag{72}$$

In addition, the use of (67) in (62), with 6.9 replaced by 6.5 due to the different value of (58) which leads to (67), gives

$$T_{\mathrm{EUN}} \approx 7.56 \times 10^8\ \mathrm{K}. \tag{73}$$

Finally, substituting (67), (72) and (73) into (71), we obtain the current baryon mass density

$$\rho_{b0} = 1.46 \times 10^{-36}\ \mathrm{kg/m^3}. \tag{74}$$

Since ρ_r and ρ_b evolve at the same rate in MCG, it follows from (72) and (74) that radiation always dominates the MCG universe.

In fact, the scale factor is big at late times such that we can neglect the density term on the right hand side of (47), which makes the late MCG universe curvature dominated. In this case, we must impose $K = -1$, which gives the approximated solution

$$a(t) = ct \tag{75}$$

in the late MCG universe. It is not difficult to show that for an open universe with the scale factor (75) such as the late MCG universe, we have the luminosity distance

$$d_L(z) = \frac{c}{H_0}\left[\frac{(1+z)^2 - 1}{2}\right], \tag{76}$$

which fits well to SNIa data[9] [8]. We intend to check if (75) provides good fits to other low redshift data in future works.

Just to finish, it is important to note that the evolution of the baryon-to-photon ratio (71) causes the number of baryons N_b to decrease over time in the MCG universe. We can see this explicitly by substituting (46) and $V \sim a^3$ in

$$N_b = n_b V = \frac{\rho_b V}{m_N}, \tag{77}$$

[6] The existence of such neutrinos is allowed by the symmetries of the theory and may be responsible for the small masses of the left-handed neutrinos found in nature [25].

[7] It is possible that the decay of the sterile neutrinos solves the inconsistency between the predicted and observed values of the lithium abundance [26].

[8] It would be important to check if (71) at the time of recombination is consistent with the value of η measured by cosmic microwave background (CMB) anisotropies. However, a theory for the growth of inhomogeneities in MCG has not yet been developed due to the complexity generated by the contribution of the Bach tensor in (10). Therefore, we will leave this analysis for future works.

[9] It is worth noting that the density term has not been neglected in Ref. [8], which in practice does not change the SNIa data fitting.

which gives

$$N_b \sim \frac{\rho_{b0} a_0^4}{m_N a}. \tag{78}$$

Using (75), we find that the number of baryons evolves over time according to

$$N_b \sim \left(\frac{\rho_{b0} c^3 t_0^4}{m_N} \right) t^{-1} \tag{79}$$

in the late MCG universe.

It follows from the energy continuity equation (44) that

$$\dot{\rho}_b + 3H\rho_b = \dot{\rho}_\Lambda. \tag{80}$$

By comparing (80) with the standard adiabatic conservation equation, and noting that $\dot{\rho}_\Lambda < 0$, we conclude that the decrease in the number of baryons (79) is due to the decay of the baryons into dynamic vacuum,[10] which clearly leads to a violation of the conservation of the quantum numbers. However, we can see from (79) that the variation of the number of baryons should only be significant on cosmological time scales, which makes the decay of baryons into vacuum not observable in the laboratory.

On the other hand, the non-conservation of baryons can have an important impact on the evolution of inhomogeneous structures of the universe from the end of recombination until today. Due to the decrease in the amount of baryons in the MCG universe, it is expected that the formation of structures happen much later than is observed or not happen at all. However, the evolution of cosmological structures does not depend only on baryons but also on dark matter, whose existence is necessary in MCG to explain the galaxy rotation curves and the deflection of light by galaxies [12]. Therefore, although the theory possibly has an extra scalar field that is a good candidate for dark matter [16], much still has to be studied to find out if the evolution of cosmological structures predicted by MCG is consistent with observations or not.

7 Final remarks

Here we have shown that the abundances of light elements, including lithium, predicted by the early MCG cosmology are consistent with the observed values provided the theory has right-handed sterile neutrinos, which is allowed by the symmetries of the theory. Even though we still need to check the existence of such neutrinos in experiments like the Mini Booster Neutrino Experiment (MiniBooNE) [27], this result is quite encouraging for us to continue with the study of the theory.

In addition, it was shown in this paper that the baryon-to-photon ratio of the MCG universe evolves over time. Although further studies are needed to verify whether this evolution is consistent with the value of the baryon-to-photon ratio determined by the CMB anisotropies, who knows it solves other early universe problems found in the ΛCDM model such as the baryon asymmetry problem. We intend to study this and other MCG cosmological predictions in future works.

[10] This decaying process can be accounted by the Yukawa interaction $\mu S\overline{\psi}\psi$ in (19).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

Funded by SCOAP^3. SCOAP^3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

1. A.G. Riess et al., Astron. J. **116**, 1009 (1998)
2. S. Perlmutter et al., ApJ **517**, 565 (1999)
3. N. Aghanim et al. [Planck Collab.], Planck 2018 results. VI. Cosmological parameters. Astron. Astrophys. **641**, A6 (2020)
4. N. Aghanim et al. [Planck Collab.], Planck 2018 results. VI. Cosmological parameters. Astron. Astrophys. **652**, C4 (2021)
5. S.E. Rugh, H. Zinkernagel, Stud. Hist. Philos. Sci. B **33**, 663 (2002)
6. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
7. R.H. Cyburt, B.D. Fields, K.A. Olive, T.-H. Yeh, Rev. Mod. Phys. **88**, 015004 (2016)
8. F.F. Faria, Adv. High Energy Phys. **2014**, 520259 (2014)
9. F.F. Faria, Mod. Phys. Lett. A **36**, 2150115 (2021)
10. F.F. Faria, Adv. High Energy Phys. **2019**, 7013012 (2019)
11. F.F. Faria, Eur. Phys. J. C **80**, 645 (2020)
12. F.F. Faria, Mod. Phys. Lett. A **37**, 2250033 (2022)
13. F.F. Faria, Eur. Phys. J. C **76**, 188 (2016)
14. F.F. Faria, Eur. Phys. J. C **77**, 11 (2017)
15. F.F. Faria, Eur. Phys. J. C **78**, 277 (2018)
16. F.F. Faria, arXiv:1903.04893 [hep-th]
17. N. Matsuo, Gen. Relativ. Gravit. **22**, 561 (1990)
18. P.D. Mannheim, Gen. Relativ. Gravit. **22**, 289 (1990)
19. A.J. Helmboldt, P. Humbert, M. Lindner, J. Smirnov, JHEP **2017**, 113 (2017)
20. P.D. Mannheim, Prog. Part. Nucl. Phys. **56**, 340 (2006)
21. L. Knox, A. Kosowsky, arXiv:9311006 [astro-ph]
22. M. Tanabashi et al. [Particle Data Group], Phys. Rev. D **98**, 030001 (2018)
23. E. Aver et al., JCAP **03**, 027 (2021)

24. P.A. Zyla et al. [Particle Data Group], PTEP **2020**, 083C01 (2020)
25. K.A. Meissner, H. Nicolai, Phys. Lett. B **648**, 312 (2007)
26. L. Salvati et al., JCAP **08**, 022 (2016)
27. A.A. Aguilar-Arevalo et al. [MiniBooNE Collaboration], Phys. Rev. Lett. **121**, 221801 (2018)