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Metastable vacua from torsion and machine learning

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Abstract By implementing an error function on a Machine Learning algorithm we look for minimal conditions to construct stable Anti de Sitter and de Sitter vacua from dimensional type IIB String theory compactification on Kähler manifolds with torsion. This allows to have contributions to the scalar potential from the five-form flux and from D-branes wrapping torsional cycles, interpreted as non-BPS states. The former implies the possibility to construct stable AdS vacua while the latter constitutes a mechanism to uplift AdS to dS vacua. Particularly we consider $\widehat{D5}$ non-BPS states to uplift the stable AdS vacua to an (apparently) stable dS minimum. Both results – the generation of an AdS vacuum and the corresponding uplifting to a dS one – are restricted to a certain type of configurations, specifically with the number of $O3$ orientifolds bounded from below by the number of $D3$-branes and fluxes. Under these conditions, we report over 170 dS (classical) stable vacua. In all of them, the uplifted effective potential becomes very flat indicating the presence of possible sources of instabilities. We comment on their relationship with the Swampland Conjectures.

1 Introduction

The Swampland Program has received a lot of attention over the last few years. Its importance relies on the establishment of some criteria to separate effective quantum field theories −considered as consistent with Quantum Gravity, a.k.a. String Theory− from those which are not. The program focuses on different proposals commonly referred to as Conjectures which appear to rule out some of the string model engineering constructions so far presented in the literature. Some of those conjectures are involved in our work, such as the instability of non-SUSY Anti-de Sitter (AdS) vacua, the AdS scale separation, and the Refined de Sitter conjecture, which in turn seem to be interconnected [1–3]).

The refined dS conjecture establishes that the minima of the scalar potential coming from the dimensional reduction of the low energy theory in string theory have to be AdS otherwise they are tachyonic or not consistent with Quantum Gravity, at least in the asymptotic regions of moduli space [4–8]. Even more restrictive, the AdS conjecture establishes that the scale of the lightest moduli is not parametrically separated from the AdS scale, and thus any attempt to uplift an AdS to a dS vacuum shall result in their destabilization.[1]

Recently it has been argued that the use of non-BPS states, classified by K-theory, shall be an interesting corner to evade these restrictions [10,11], unless the total K-theory charge must cancel as pointed out in [12] and related to the cobordism conjecture in [13].[2] The minimal ingredients necessary to construct a dS vacua have been studied extensively in the last years [14,15,35]. However, their presence is limited by the selected scenario under study. For example, in a Type IIB flux compactification, it seems that the RR 5-form and the D5-brane contribution to the scalar potential upon dimensional reduction, are key ingredients to the construction of a dS vacuum. However, in the presence of $O3$-planes the RR potential C_6 is projected out and no $D5$-branes are allowed to be on top of them, or equivalently, cannot wrap internal 2-cycles, but due to the same orientifold action, the tachyon present between a pair of $D5$ and an anti $D5$ is also removed,

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[1] See [9] for the case in the deformed conifold.

[2] The incorporation of non-BPS states in the compactification can be realized in any region of the moduli space, in particular in the asymptotic one. In the present case, we shall study a simple scenario where quantum corrections to the Kähler potential have been neglected for which the constructed vacua reside in the asymptotic limit of large volumes.

making the pair to be stable while carrying a discrete topological charge Z_2. This is the non-BPS state $\widehat{D5}$ which does have a contribution on the effective scalar potential, as a $D5$-brane. For this reason, it is possible to construct a dS vacuum in its presence. On the other hand, if the total K-theory charge is trivial then the scalar potential contribution coming from the $\widehat{D5}$ vanishes, and the uplift to dS would not be possible, validating the RdS. The use of non-BPS states, typically constructed from a pair of stable branes and anti-branes in the presence of an orientifold plane, emulates the role played by non-perturbative contributions in KKLT scenarios by breaking the non-scale structure of the $\mathcal{N} = 1$ superpotential and providing a nice mechanism to stabilize all the moduli. However, their inclusion is not sufficient to guarantee the presence of apparently stable dS vacua but contributions to the effective scalar potential coming from the RR 5-form are necessary.

We are interested in two main aspects. First, in constructing a (meta)-stable dS vacuum by identifying the minimal set of ingredients the effective scalar potential must possess in the spirit of [14,15] and also find possible compactification scenarios where such conditions might be present. Second, in case we can construct a classical stable dS vacuum we want to look for possible sources of instabilities which in turn can be taken as evidence (or not) of the realization of the above-referred Swampland Conjectures .

In this work, we consider a compactification on a Kähler manifold admitting torsion, upon which there is a contribution of the torsional part of F_5 to the scalar potential allowing us to find AdS vacua (but not dS). For that to happen it is necessary that the number of orientifolds fixed points be greater than the number of $D3$-branes such that their contribution to the tadpole is negative, i.e., $\mathcal{N}_3 < 0$. Under this context, it is then possible to wrap $D5$-branes on torsional 2-cycles which we claim are precisely the $\widehat{D5}$ non-BPS states and that contribute with a positive amount of energy such that uplift the AdS vacuum to a dS one which has been constructed in a single step. However, we also show that a 2 step procedure is available if the value at which the internal modulus is stabilized is large (large volume limit) for which the uplifting is free of tachyons although the resulting potential is very flat, indicating that it can be easily perturbed.

As in the case of the AdS vacua, the realization of dS minima requires some extra conditions which come from having a positive value for the stabilized internal volume, namely that there are fluxes in RR and NS-NS sectors supported in more than two 3-cycles and that the number of orientifold 3-planes has a lower bound given by

$$\mathcal{N}_{O3} > 4\frac{(A_{H_3} A_{F_3})^{1/2}}{A_3} + 2\mathcal{N}_{\text{flux}},$$

where A_{H_3}, A_{F_3} and A_3 are the contributions – upon dimensional reduction – of 3-form fluxes and 3-dimensional

sources as $D3$-branes and $O3^-$-planes, while $\mathcal{N}_{\text{flux}}$ is the usual flux number entering into the $D3$-brane charge tadpole contribution.

These conditions were inferred after implementing a Machine Learning (ML) algorithm specifically designed to look for dS vacua. The use of ML algorithms and tools has been proven to be prolific (and in a more systematic way) to explore the vacua in string theory compactifications (see for instance [16–25]). For that, we implemented a hybrid algorithm to explore the minima of a scalar potential of the form[3]

$$\mathcal{V}_{\text{eff}} = \mathcal{V}_{\text{eff}}(H_3, F_3, F_5, \widehat{D5})$$

subject to the constraints of (1) having a positive value at the minimum, (2) zero value of its derivative with respect to each of the moduli, (3) positive definiteness of the mass matrix, and (4) positiveness of the contribution of the $\widehat{D5}$-brane. In the context of ML, these restrictions can be implemented through an objective function written as

$$\text{Error} = \sum_{i=1}^{N} \alpha_i \text{error}_i$$

where each of the error_i contributions takes into account every single restriction above mentioned with the α parameter a real value giving a weight to each error contribution. In the present work, we employ the hybrid algorithm including the Simulated Annealing (SA) as well as the Gradient Conjugate (GC). The SA algorithm is a heuristic method for solving optimization problems which, inspired by the annealing procedure of metalworking, is able to look for an approximate solution to the optimization problem. The GC algorithm on the other hand is a second-order deterministic iterative optimization algorithm designed to find local minima provided that the first derivative is known. The need to combine heuristic algorithms together with deterministic ones relies on the point that heuristic algorithms are not intended to provide great accuracy, instead are employed to find the parameters that are close to a global minimum. Once these parameters close to the global minimum are found, the deterministic algorithms improve the accuracy providing a good numerical approximation of the global minimum. Thus, at the first step, the SA algorithm shall look for interesting points in the error function whereas the GC shall improve the solution guaranteeing the zero value of the first derivative of the scalar potential. We describe in detail these algorithms in Appendix A.

Our work is then organized as follows: in Sect. 2 we present the most usual conditions for a type IIB compactification and specify the notation we use along the paper. In Sect. 3

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[3] Other sources are considered in the Appendix A such as fluxes, branes, and negative curvature. More exotic fluxes, as non-geometric have been considered in the literature (see [26] and references therein).

we show that it is possible to construct AdS vacua by compactification of type IIB string Theory on a Kähler manifold with torsion, such that the RR five-form has a torsional contribution to the effective scalar. For that, we implement a ML algorithm through the presence of an error function which allows to easily find a large number of stable and unstable vacua. In this case, we report 389 different AdS vacua whose existence relies upon the requirement that the number of $D3$-branes be less than one-half of the number of orientifolds $O3^-$-planes. However, no dS vacua were found under these conditions. In Sect. 4, once we take a compactification on a manifold with torsion, we also consider $D5$-branes wrapping torsional 2-cycles while fulfilling the aforementioned conditions on fluxes and the orientifold bound. Extra assumptions were taken, such as the nonexistence of torsional components of all 3-form fluxes. For this case, we report over 170 different dS stable vacua. In Sect. 5 we discuss the conditions upon which the AdS vacua can be lifted to dS ones and comment on the implications with respect to the Swampland Conjectures. In Sect. 6 we present our conclusions, while in the Appendix we describe some useful technical information in relation to the Machine Learning algorithm to be implemented in our search, particularly about the incorporation of the above-mentioned two algorithms: the Simulated Annealing and the Conjugate Gradient.

2 Contribution to the scalar potential

Let us review the standard dimensional reduction procedure to construct the effective scalar potential. Consider the type IIB superstring compactified on a manifold $\mathbb{X}_6$ in the presence of 3-form fluxes and 3-dimensional local sources. In order to explore the simplest possible scenario we are not including 7-branes or orientifold 7-planes to avoid the presence of extra moduli related to their wrapping on internal cycles [27] and also to have the possibility to add non-BPS states related to the action of the $O3$-plane. See appendix B for more scenarios. As usual, the action for the massless modes in the string frame is

$$S_{IIB} = S_G + S_\phi + S_{G_3} + S_{F_5} + S_{CS} + S_{loc}, \tag{1}$$

with

$$S_G = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G}\, e^{-2\phi} R, \tag{2}$$

$$S_\phi = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\phi} \left(4(\nabla\phi)^2 \right) - \frac{\|F_1\|^2}{2} \right], \tag{3}$$

$$S_{G_3} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(e^{-2\phi} \|H_3\|^2 - \|\hat{F}_3\|^2 \right), \tag{4}$$

$$S_{F_5} = -\frac{1}{8\kappa_{10}^2} \int d^{10}x \sqrt{-G}\, \|F_5\|^2, \tag{5}$$

$$S_{CS} = -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3, \tag{6}$$

$$S_{loc} = S_{DBI} + S_3 = T_3 \mathcal{N}_3 \int d^4x \sqrt{-g_4}\, e^{-2\phi} + \frac{1}{2}\mathcal{N}_3\, \mu_3 \int_{\Sigma_4} C_4, \tag{7}$$

where in terms of the string length l_s,

$$\kappa_{10}^2 = \frac{l_s^8}{4\pi}, \tag{8}$$

T_3 is the $D3$-brane tension, $\mathcal{N}_3 = \mathcal{N}_{D3} - \frac{1}{2}\mathcal{N}_{O3}$ counts the number of $D3$-branes minus the number of orientifold planes $O3^-$ with $\mu_3 = T_3 = \frac{2\pi}{l_s^4}$. We consider the DBI action at leading order in α' for $D3$-branes and $O3^-$-planes along the extended coordinates, where the RR fluxes are

$$\hat{F}_3 = F_3 - C_0 \wedge H_3,$$
$$F_5 = dC_4 - \frac{1}{2}C_2 \wedge H_3 - \frac{1}{2}B_2 \wedge dC_2. \tag{9}$$

Thus, the action S_{F_5} (before self-duality is imposed) can be written as

$$S_{F_5} = -\frac{5!}{8\kappa_{10}^2} \int F_5 \wedge *F_5$$
$$= \frac{15}{\kappa_{10}^2} \int \left[C_4 \wedge d * F_5 + \left(\frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge dC_2 \right) \wedge *F_5 \right]. \tag{10}$$

Due to the action of the orientifold planes $O3^-$, the RR and NS-NS potentials C_2 and B_2 are projected out and the equations of motion from $\delta S/\delta C_4 = 0$ give us the tadpole condition for the 3-dimensional sources

$$\mathcal{N}_3 + \frac{1}{l_s^4} \int F_3 \wedge H_3 = \mathcal{N}_3 + \mathcal{N}_{fluxes} = 0. \tag{11}$$

Therefore, the contribution from $S_{F_5} + S_{CS} + S_3$ to effective the scalar potential – in a compactification on a CY manifold – vanishes. As we shall see we are going to depart from a CY compactification into a more general setup such that S_{F_5} does have a contribution.

In order to construct the effective scalar potential $\mathcal{V}_{eff}$, we specify the ten-dimensional metric as

$$ds_{10}^2 = g_{\mu\nu}dx^\mu dx^\nu + h_{mn}dy^n dy^m,$$
$$= e^{-2\Omega}e^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{h}_{mn}dy^m dy^n, \tag{12}$$

where $e^{-2\Omega}$ is the conformal factor fixed as

$$e^{-2\Omega} = e^{-2\phi}\mathcal{V}_6 \tag{13}$$

to change into the Einstein frame, with $\mathcal{V}_6 = \int d^6y \sqrt{h_6}$. Notice we are not taking into account warping effects on the internal metric, thus, although the geometry and topology of the internal space are important to the specifics in the construction of a stable vacuum, in the following we shall

focus only on a description based on the universal moduli that appear in string compactifications, rendering our result generic.

In terms of the axionic moduli fields τ and s are given by

$$\tau = e^{-\phi}(\mathcal{V}_6)^{2/3}, \qquad s = e^{-\phi}, \tag{14}$$

so, the contributions for the action terms S_{G_3} and S_{DBI} are given by

$$S_{G_3} = \int d^4x \sqrt{-g_4} \left(\frac{A_{F_3}}{s\tau^3} + \frac{A_{H_3}s}{\tau^3} \right), \tag{15}$$

$$S_{\text{DBI}} = \int d^4x \sqrt{-g_4} \, \frac{A_3 \mathcal{N}_3}{\tau^3}, \tag{16}$$

where A_{F_3}, A_{F_3} and A_3 are the corresponding contributions not depending on τ and s where

$$S = C_0 + is, \quad \text{and} \quad T = \int C_4 + i\tau). \tag{17}$$

On the above, we have assumed that complex structure moduli z_i are fixed through 3-form fluxes, by $D_{z_i}\mathcal{W} = 0$,[4] where as usual

$$\mathcal{W} = \int (F_3 - SH_3) \wedge \Omega(z_i), \tag{18}$$

but $D_S\mathcal{W} \neq 0$. Therefore SUSY is broken at least by the axio-dilaton modulo S, and the fluxes we are turning on, have not $(1, 2)$-components. Together with the Kähler potential of the form

$$\mathcal{K} = -\log(-i(S - \bar{S})) - 3\log(-i(T - \bar{T})), \tag{19}$$

after the stabilization of the complex structure, the terms in the scalar potential which are proportional to $\|W\|^2$ in $\|D_T W\|^2$ cancel the gravitino mass term $-3\|W\|^2$, thus the flux contribution to the scalar potential reduces to

$$\mathcal{V}_{\text{fluxes}} = e^{\mathcal{K}} \|D_S W\|^2 K^{S\bar{s}} = \frac{\hat{f}^2 + s^2 h^2}{2s\tau^3}. \tag{20}$$

with

$$\hat{f} = \int \hat{F}_3 \wedge \Omega, \quad \text{and} \quad h = \int H_3 \wedge \Omega. \tag{21}$$

Comparing with expression (15),

$$A_{F_3} = \frac{\|\hat{f}\|^2}{2\kappa_{10}^2}, \quad \text{and} \quad A_{H_3} = \frac{\|h\|^2}{2\kappa_{10}^2}. \tag{22}$$

As known, by exploring different values for A_{F_3}, A_{H_3}, and A_3 we find that no stable vacuum is obtained. More ingredients are required.

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[4] A particular scenario where the stabilization of the complex structure can be carried out is in the well-known factorizable $\mathbb{T}^6$ which is parametrized by only one complex structure.

3 Stable non SUSY AdS vacua from torsion

As suggested in the literature (see [14,15,27–36]), it is possible to find stable vacua by turning on different contributions to the scalar potential. Here we are interested in a non-vanishing contribution from S_{F_5} to $\mathcal{V}_{\text{eff}}$. For that, we shall take into account the presence of torsion in the internal manifold $\mathbb{X}_6$ which, as we shall argue, naturally comes into play in the presence of orientifold planes [37,38]. This implies that the Kähler 2-form J_2 is no longer closed, i.e., $dJ_2 \neq 0$ pointing out the necessity to compactify on generalized CY manifolds. By using the ML algorithm described in Appendix A, we find that AdS stable vacua are obtained under some specific conditions we shall describe in detail.

3.1 Effective scalar potential from torsion

Let us start by writing the action component S_{F_5} in (10) as

$$S_{F_5} = \frac{15}{2\kappa_{10}^2} \int \omega_5 \wedge *F_5. \tag{23}$$

where

$$\omega_5 = \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3. \tag{24}$$

As said, in generic compactifications on $\mathbb{X}_6$ with orientifold planes $O3^-$, 2-forms are divided on odd or even according to the orientifold action on them [39]. Since 2-form RR and NS-NS potentials are odd under an $O3^-$ action, and the fluxes F_3, H_3 are even and it follows that

$$\omega_5 \in \Omega_-^2(\mathbb{X}_6, \mathbb{Z}) \wedge H_+^3(\mathbb{X}_6, \mathbb{Z}). \tag{25}$$

Therefore, for a generic CY manifold, ω_5 does not contribute to $\mathcal{V}_{\text{eff}}$. Also notice that in the presence of orientifold $O3^-$-planes, the RR potential C_6 is projected out and it is not possible to have stable BPS $D5$-branes. The effective 4-dimensional scalar potential only receives contributions from the rest of the terms in the action S and from the Dirac-Born-Infeld action of extended objects wrapping internal cycles on $\mathbb{X}_6$, as D3-branes and orientifold planes $O3^-$.

However, in the presence of orientifold planes, it is natural and expected to have torsional cycles. For instance, in a IIB toroidal orientifold, the quotient space $\mathbb{T}^6/\mathbb{Z}_2$ contains torsional cycles of different dimensions (dual to torsional fluxes), meaning that there are cycles that after wrapping them a certain number of times, one ends up with a subspace of $\mathbb{T}^6$ with boundary. Since we are considering the presence of orientifold planes, we shall assume the existence of torsional cycles in generic Kähler manifolds. Under this context, we shall study whether or not ω_5 contributes to $\mathcal{V}_{\text{eff}}$ via torsional cycles.

The pth-cohomology group of a six-dimensional Kähler manifold is written as

$$
\begin{aligned}
H^p(\mathbb{X}_6; \mathbb{R}) &= H^p(\mathbb{X}_6; \mathbb{Z}) + \text{Tor } H^p(\mathbb{X}_6; \mathbb{Z}), \\
&= \mathbb{Z}^{b_p} + \left(\mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n} \right),
\end{aligned} \tag{26}
$$

where b_p is the Betti number for $H^p(\mathbb{X}_6, \mathbb{Z})$ and $k_i \in \mathbb{Z}$. Let us consider the case for $p = 3$. A 3-form in the torsional part can be decomposed as

$$
\pi_3^{\text{tor}} = \lambda^i \pi_{3,i}^{\text{tor}}, \tag{27}
$$

with $i = 1, \ldots, n$ according to (26) and $\lambda^i \in \mathbb{Z}$. In the case in which the set of integers λ^i has a greatest common divisor (gcd) κ, there exists a non-closed 2-form $\hat{\omega}_2$ such that $d\hat{\omega}_2 = \kappa \pi_3^{\text{tor}}$, i.e., $\pi_3^{\text{tor}} \in \mathbb{Z}_k$. The set of such 2-forms is denoted $\hat{\Omega}^2(\mathbb{X}_6)$. If $\lambda^i = \kappa^i k^i$ only for some i, then there exists $\hat{\omega}_i \in \hat{\Omega}_i^2(\mathbb{X}_6, \mathbb{Z})$ such that $d\hat{\omega}_i = k_i \pi_{3,i}^{\text{tor}}$. In this scenario, generic RR and NS-NS potentials are given by

$$
\begin{aligned}
C_2 &= c^a \omega_a + \tilde{c}^i \hat{\omega}_i, \\
B_2 &= b^a \omega_a + \tilde{b}^i \hat{\omega}_i
\end{aligned} \tag{28}
$$

where $\omega_a \in H_-^2(\mathbb{X}_6, \mathbb{Z})$, $\hat{\omega}_i \in \hat{\Omega}^2(\mathbb{X}_6)$ with $a = 1 \ldots h_-^{1,1}(\mathbb{X}_6)$ and $i = 1, \ldots n$. The presence of 2-forms $\hat{\omega}_i$ implies that the Kähler form J_2 can also be written as

$$
J_2 = t^a \omega_a + \tilde{t}^i \hat{\omega}_i, \tag{29}
$$

from which $dJ_2 = k_i \tilde{t}^i \pi_{3,i}^{\text{tor}}$. Hence for $\tilde{t}^i = \tau^i / k_i$, dJ_2 is non trivial in $H^3(\mathbb{X}_6, \mathbb{Z})$ and $\mathbb{X}_6$ is not a CY manifold but at least a Kähler manifold modulo k_i.

If now we restrict the compactification over a Kähler manifold with torsion as above, the contribution from S_{F_5} is not longer zero, but

$$
S_{F_5} = \frac{5!}{16\kappa_{10}^2} \int d\text{Vol}_4 \int \left(H_3 \tilde{c}^i - F_3 \tilde{b}^i \right) \wedge \hat{\omega}_i \wedge de^{4A(y)}, \tag{30}
$$

with $A(y)$ the warping factor in Eq. (12). Therefore, the contribution of F_5-form to the scalar potential, in the Einstein frame, is given by

$$
V_{F_5} \sim \frac{A_5}{\tau^4}, \tag{31}
$$

where $A_5 = A \bmod k_i$ for some A.

3.2 Conditions for finding stable AdS vacua

The above contribution to $\mathcal{V}_{\text{eff}}$ from S_{F_5} together with the contributions from 3-form fluxes, D3-branes and $O3^-$-planes, lead us to a scalar potential of the form

$$
\mathcal{V}_{\text{eff}} = \frac{A_{H_3} s}{\tau^3} + \frac{A_{F_3}}{s\tau^3} + \frac{A_{F_5}}{\tau^4} + \frac{A_3 \mathcal{N}_3}{\tau^3}, \tag{32}
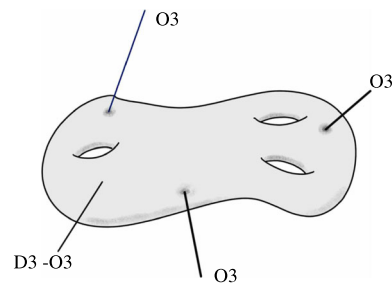$$

Fig. 1 Schematic picture of the compact space and the locii of the orientifold planes and $D3$-branes

which actually has some stable AdS minima if there is at least one negative contribution from the above terms. However, since the flux contribution A_{G_3} is positive definite[5] and A_{F_5} is defined modulo an integer, the only option left is that from the contribution of 3-dimensional sources, $\mathcal{N}_3$ must be negative. In the following, the numerators are selected numerically by the hybrid algorithm in order to find the conditions for a stable AdS vacuum.[6]

By restricting the flux configurations and local sources to satisfy that $\mathcal{N}_3 < 0$, the number of $O3^-$-planes must be larger than the number of $D3$-branes, implying that at some points in the internal space, there must be isolated orientifold planes, or in other words that there are no $D3$-branes of top of some of the $O3^-$-planes. This follows from the usual assumption that orientifold planes are immovable and from the fact that there is an attraction between $D3$-branes and $O3^-$-planes due to the RR $D3$-brane charge they carry. For instance, the most simple configuration involving the presence of $D3$-branes with $\mathcal{N}_3 < 0$ is to have 4 orientifold fixed points and a single $D3$-brane sitting at one of those points. In such case, $\mathcal{N}_3 = -1$ (see Fig. 1 for a schematic representation of this configuration).

Under these conditions, we implemented our ML algorithm described in Appendix A. With it, we were able to find 389 different stable AdS vacua. However, in spite of designing our algorithm such that finding dS vacua was favored over AdS, no dS one was found. Our results are shown in Fig. 2 where all found vacua, stable or not, are represented by black squares.

4 Stable dS vacua from non-BPS states

The presence of torsion opens up the possibility to consider wrapping D-branes on torsional cycles. The existence of tor-
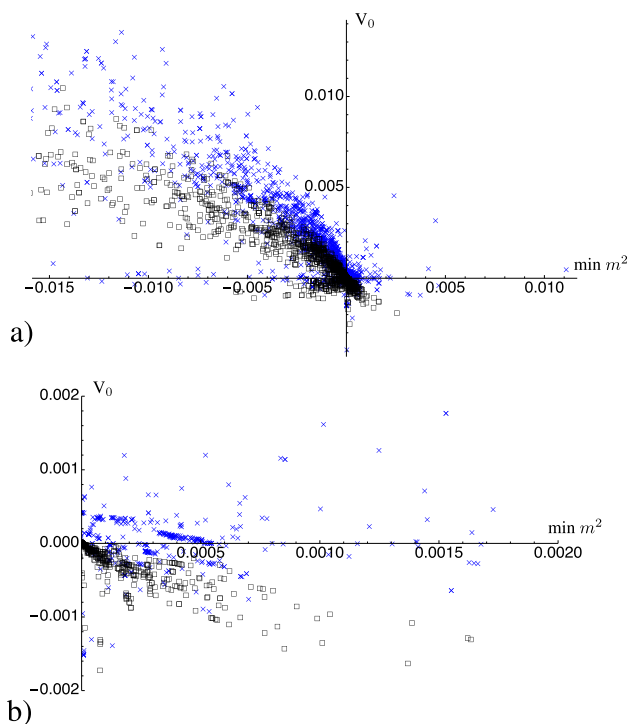
a)

b)

Fig. 2 Plot of the critical points found by the hybrid algorithm. The black squares correspond to the cases where F_5 contributions were taken into account without non-BPS states, whereas the blue crosses consider the presence of $\hat{D}5$ non-BPS states. On the second image, we present a zoom of the stable cases

sional cycles follows from the dual maps between homology and cohomology, where

$$\int_{\Sigma_2^{j,\text{tor}}} \hat{\omega}_i = \int_{\mathbb{X}_6} \hat{\omega}_i \wedge PD(\Sigma_2^{j,\text{tor}}) = \delta_i^j. \tag{33}$$

with

$$k_i \Sigma_2^{i,\text{tor}} = \partial \hat{\Pi}_3^i. \tag{34}$$

This last assertion means that the homology group $H_2(\mathbb{X}_6, \mathbb{R})$ also has a torsion component, i.e., $\Sigma_2^{i,\text{tor}} \in \text{Tor } H_2(\mathbb{X}_6, \mathbb{Z})$ and $\hat{\Pi}_3^i \in \hat{\Omega}_3(\mathbb{X}_6, \mathbb{Z})$. It follows then that $\text{Tor } H_2(\mathbb{X}_6, \mathbb{Z}) \sim \text{Tor } H^3(\mathbb{X}_6, \mathbb{Z})$. We shall follow the argument in which these states – D-branes wrapped on torsional cycles – are in fact related to the well-known non-BPS states constructed from K-theory [40].

The existence of non-BPS states in the presence of an orientifold plane $O3^-$ can be inferred by applying T-duality on the corresponding coordinates on a torus compactification of Type I string theory, which actually has non-BPS branes as $\widehat{D7}$, $\widehat{D8}$, $\widehat{D0}$ and $\widehat{D(-1)}$. Hence, by taking for instance a non-BPS $\widehat{D7}$-brane spanned on 4 coordinates on T^6 immersed in an $O9^-$-plane and applying T-duality on the compact coordinates, we get an extended $O3^-$-plane and a 5-brane wrapping a 2-dimensional space in the covering space. We expect this object to carry a topological $\mathbb{Z}_2$ charge as its T-

dual partner. Indeed, by computing the 2nd-homology group of $\mathbb{T}^6/\mathbb{Z}_2$ we see that there are torsional 2-cycles. Wrapping $D5$-branes of type IIB theory on such cycles seems to be the way to construct the aforementioned non-BPS states. Moreover, by computing the corresponding T-dual K-theory group one sees that stable non-BPS states are present, carrying discrete topological charge $\mathbb{Z}_2$ with three extended coordinates while the others are wrapped on the compact space.

For a more general compactification, one must compute the K-theory groups of intersecting sources, i.e., of configurations of branes intersecting orientifold planes wrapping cycles on a compact manifold. This is indeed a difficult task. However, ignoring the compact component of the space, it is possible to classify intersecting branes with orientifolds by the use of the Kasparov KK-theory [41,42]. Since the formulation is quite technical and it is beyond the scope of this work,[7] we just present the KK-theory group which classifies 5-branes fully intersecting an $O3^-$-plane, i.e., with 2 transversal coordinates to the orientifold plane and its relation to the orthogonal K-theory group. This is:

$$KKH^{-2}(\mathbb{R}^{2,0}, \mathbb{R}^{6,0}) = KO(\mathbb{S}^2) = \mathbb{Z}_2, \tag{36}$$

as expected.

Based on these results we are taking as valid the construction of stable non-BPS states by wrapping D-branes on torsional cycles of a Kähler manifold $\mathbb{X}_6$. In particular, we can construct a non-BPS $\widehat{D5}$-brane by wrapping a $D5$-brane on a torsional 2-cycle $\Sigma_2^{\text{tor}} \in H_2^{\text{tor}}(\mathbb{X}_6, \mathbb{Z})$, where Σ_2^{tor} is the cycle where the 2-form $\hat{\omega}_2$ is supported as in Eq. (33).

Summarizing, a compactification on a Kähler manifold $\mathbb{X}_6$ with torsional components in (co)-homology, leads us to the possibility to include D-branes wrapping torsional cycles. Here we shall consider the contribution to the effective scalar potential from non-BPS $\widehat{D5}$-branes. However, before that, we must discuss possible sources of instability on a configuration constructed with fluxes, $D3$-branes, $O3^-$-planes, and non-BPS states.

4.1 Consistency by adding non-BPS $\hat{D}5$-branes

As it is known [40], the non-BPS brane $\widehat{D7}$ in type I theory can be constructed by a pair of a $D7$ and $\bar{D}7$-branes, where the tachyon on the open sector string connecting the two branes is projected out by the orientifold $O9^-$. However, since in type I theory there are 32 $D9$-branes, there is also a tachyon from

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[7] The KK-theory group classifying Dd-branes on top of an Op^--plane, with $p = 3 \mod 4$ and $d > p$ is given by [42]

$$KKH^{-2}(\mathbb{R}^{d-s,r}, \mathbb{R}^{9-p,p+r-s}) = KO(\mathbb{S}^{2p-2s+d-3}), \tag{35}$$

where s are the number of coordinates of the Dd-brane overlapping the orientifold plane and r is the codimension of the Dd-brane inside the orientifold. For a $D5$-brane on top of an $O3^-$-plane with 2 transversal coordinates, $p = s = 3$, $r = 0$ and $d = 5$.

the open string between $D9$-branes and $D7$-branes, making the non-BPS $\widehat{D7}$-brane to be unstable [43].

In a T-dual version, upon compactification on a six-dimensional torus, the above configuration is mapped into $D3$-branes and $O3^-$-planes sitting at different points on $\mathbb{T}^6$ and $D5$-branes wrapping torsional 2-cycles on the compact space, corresponding to the non-BPS states $\widehat{D5}$. Therefore, by T-duality, it is expected that in a given fixed point in the internal space, a $\widehat{D5}$-brane coinciding with at least one $D3$-brane, would be unstable to decay into a field configuration while preserving its topological charge $\mathbb{Z}_2$. This instability is not present (at least locally) if, at the given fixed point, there are not $D3$-branes, a configuration we can have if there are more orientifolds than $D3$-branes, i.e., if $\mathcal{N}_3 = \mathcal{N}_{D3} - \frac{1}{2}\mathcal{N}_{O3} < 0$. In order to cancel the $D3$-brane charge tadpole, we then require a positive contribution from fluxes. These two characteristics, $\mathcal{N}_3 < 0$ and $\mathcal{N}_{\text{flux}} > 0$ are essential to guarantee the stability of adding non-BPS $\widehat{D5}$-branes. Notice that $\mathcal{N}_3 < 0$ is one of the conditions to assure the existence of stable AdS vacua without adding non-BPS states.

Under the above circumstances, we shall take a $D5$-brane and wrap it on a torsional 2-cycle $\Sigma_2^{\text{tor}} \in \text{Tor H}_2(\mathbb{X}_6, \mathbb{Z})$. Following [44], we argue that such a state is classified by the corresponding K-theory group on $\mathbb{X}_6$. Also, we shall consider the contribution of this non-BPS $\hat{D5}$-brane to the effective scalar from the DBI term. However, it is important to notice that its contribution must be measured as mod 2 since a pair of non-BPS branes with topological charge $\mathbb{Z}_2$ annihilate each other. This means that if the total discrete charge vanishes, the effective contribution from non-BPS branes is null [10,13]. Another important fact we must have in mind is that we are ignoring torsional components for 3-form fluxes, although there is no restriction for their presence.[8]

Hence, the effective contribution of a non-BPS brane $\widehat{D5}$ at leading order in α' is given by the DBI action,

$$S_{\hat{D5}} = -2T_5 \int d^6\xi \; e^{-\phi}\sqrt{-\widetilde{g}_6} \qquad (37)$$

where $\widetilde{g}_6$ is the determinant of the induced metric on the $\widehat{D5}$-brane worldvolume. Therefore, the corresponding effective scalar potential in the Einstein frame reads

$$V_{\hat{D5}} = \frac{A_{\widehat{D5}}}{s^{1/2}\tau^{5/2}}, \qquad (38)$$

where $2nA_{\widehat{D5}} = 0$ for $n \in \mathbb{Z}$.

4.2 Stable dS vacua with non-BPS states

In order to look for dS minima we shall employ a hybrid method which consists in applying a stochastic method known as Simulated Annealing followed by the conjugate gradient algorithm (see Appendix A). The effective scalar potential constructed by contributions from 3-form fluxes, 3-dimensional sources, a torsional component of F_5, and non-BPS $\widehat{D5}$-branes is

$$\mathcal{V}_{\text{eff}} = \frac{A_{H_3}s}{\tau^3} + \frac{A_{F_3}}{s\tau^3} + \frac{A_{F_5}}{\tau^4} + \frac{A_3\mathcal{N}_3}{\tau^3} + \frac{A_{\widehat{D5}}}{s^{1/2}\tau^{5/2}}. \qquad (39)$$

As discussed in [15] (see also [14]), it is expected that this anzats evades the no-go theorems and increases the possibility to find some stable dS vacua.

In Fig. 2 it is shown by blue crosses, the critical points found by the above-mentioned algorithm. Notice the presence of many stable dS vacua. In Table 1 we present the explicit values of the scalar potential contributions for some of these vacua.

5 Uplifting conditions by non-BPS states

In this section, we are interested in discussing the uplifting of AdS stable vacua to dS by the presence of non-BPS states as the $\widehat{D5}$-branes. As previously observed, a dimensional reduction in the presence of 3-form fluxes H_3 and F_3, as well as 3-dimensional sources as $D3$-branes and $O3^-$-planes together with a torsional F_5 form, leads us to the possibility to construct AdS stable vacua. For $A_{D5} = 0$, the minimum for $\mathcal{V}_{\text{eff}}$ is located at

$$s_0 = \left(\frac{A_{F_3}}{A_{H_3}}\right)^{1/2} \qquad \tau_0 = \frac{4}{3}\frac{A_{F_5}}{\Delta} \qquad (40)$$

for $\Delta = -(A_3\mathcal{N}_3 + 2A_{H_3}^{1/2}A_{F_3}^{1/2})$. Notice that in the case we are turning on a single flux G_3, meaning that we are considering a contribution to the superpotential along one single period, Δ reduces to zero due to the tadpole cancellation. Therefore, it is necessary to consider more than one flux in order to uplift the AdS vacua while keeping $\|A_3\mathcal{N}_3\| > 2(A_{H_3}A_{F_3})^{1/2}$ such that $\tau_0 > 0$. Therefore we require that two specific conditions must be taken:

1. $W = \int G_3 \wedge \Omega$ must be constructed from more than just one period.
2. $\mathcal{N}_{O3} > 4\frac{(A_{H_3}A_{F_3})^{1/2}}{A_3} + 2\mathcal{N}_{D3}$.

We shall restrict the rest of our analysis to such a case.

The minima of the AdS can be written in the function of the vacuum expectation value (vev) of the Kähler modulus as

Table 1 Selected vacua found by the hybrid SA+CG algorithm

min V	m_τ^2	m_s^2	τ	s	A_{F_3}	A_{H_3}	A_{F_5}	$A_3\mathcal{N}_3$	$A_{\widehat{D5}}$
1.309×10^{-6}	0.000546	0.003728	3.695	2.363	0.7628	0.2486	0.9769	-1.704	0.4231
5.676×10^{-6}	0.0005166	0.003874	3.727	2.298	0.7566	0.2575	0.9755	-1.708	0.4125
6.980×10^{-5}	0.0006562	0.007878	2.917	2.111	0.7463	0.2189	0.3022	$-1.135.$	0.1851
9.561×10^{-5}	0.0003757	0.003277	3.855	2.373	0.7638	0.2495	0.9778	-1.702	0.4238
4.039×10^{-4}	9.460×10^{-7}	0.004265	4.258	2.170	1.260	0.3864	0.6982	-2.067	0.3677

$$\mathcal{V}_{AdS} = -\frac{1}{3}\frac{A_{F_5}}{\tau_0^4}, \tag{41}$$

thus, the larger τ_0, the smaller value for the AdS vacua, which is compatible with the KKLT scenario. The eigenvalues can be written in terms of the vev's as

$$m_s^2 = \frac{2A_{H_3}^{1/2}}{s_0\tau_0} \quad \text{and} \quad m_\tau^2 = \frac{4A_{F_5}}{\tau_0^6}, \tag{42}$$

and we see that for large values of τ_0, the smallest eigenvalue is always in the τ direction.

Now, to uplift from stable AdS to dS vacua it is necessary to add energy associated with the non-BPS states $\widehat{D5}$ as in Eq. (38), which changes the vev's of the moduli shifting its numerical values to greater values. In this case, the Kähler modulus modify to

$$\tau = \frac{4(-A_{F_3} + A_{H_3}\tilde{s}^2)^2}{A_{\widehat{D5}}^2\tilde{s}}, \tag{43}$$

where $\tilde{s}$ is an algebraic number that vanishes a polynomial of degree 6 and at the limit of $A_{\widehat{D5}} \ll 1$ can be written as

$$s = s_0 + \frac{1}{2\sqrt{3}}\left(\frac{A_{F_5}}{A_{F_3}^{1/2}A_{H_3}^{3/2}}\right)^{1/2}\frac{1}{(\Delta)^{1/2}}A_{\widehat{D5}} + \mathcal{O}\left(A_{\widehat{D5}}^2\right)$$

$$\tau = \tau_0 + \frac{1}{2^6}\frac{\tau_0^2}{A_{F_3}^{3/2}A_{H_3}^{1/2}}A_{\widehat{D5}} + \mathcal{O}\left(A_{\widehat{D5}}^2\right). \tag{44}$$

Notice from this and from Eq. (40) that for $\Delta > 0$ this branch of solution takes real values. In this context, one also can express the effective potential at leading terms in $A_{\hat{D5}}$ as

$$\mathcal{V}_{\text{eff}} = V_{AdS} + \frac{1}{s_0^{1/2}\tau_0^{5/2}}A_{\widehat{D5}} + \mathcal{O}\left(A_{\widehat{D5}}^2\right) \tag{45}$$

where the uplifting from AdS to dS depends on how deep is the AdS vacuum.

However, it is important to analyze whether the uplifting would be stable or not. For that, we shall study under which conditions there are tachyons. Let us start by establishing the required stability criteria for the AdS vacua. Since we are interested only in their presence, we shall take the mass matrix as

$$(M_{AdS}^2)_{ij} = \partial_{ij}\mathcal{V}_{\text{AdS}}, \tag{46}$$

with $i, j = s, \tau$. The eigenvalues λ_{AdS} are given by

$$\lambda_{AdS} = \frac{1}{2}\text{tr } M_{AdS}^2 \pm \alpha \tag{47}$$

where $\alpha = \sqrt{(\text{tr } M_{AdS}^2)^2 - 4\det M_{AdS}^2}$. According to Silvester's criterium, a stable minimum exists always that $\text{tr } M_{AdS}^2 > 0$ and α be real. Notice that large values for the eigenvalues λ_{AdS} indicate that it is difficult to destabilize the minimum. On the contrary, small values of λ_{AdS} correspond to very flat potentials from which it is easy to escape. Following this line of reasoning, we want to show that by adding non-BPS states $\widehat{D5}$ the eigenvalues related to an AdS vacuum become smaller.

For that, let us consider adding the contribution from non-BPS states $\mathcal{V}_{\widehat{D5}}$, such that

$$(M^2)_{ij} = \partial_{ij}\left(\mathcal{V}_{AdS} + \mathcal{V}_{\widehat{D5}}\right) = (M_{AdS}^2)_{ij} + (M_{\widehat{D5}}^2)_{ij} \tag{48}$$

One realizes that the eigenvalues for each of the moduli decrease as we add the $A_{\widehat{D5}}$ term. To clearly show this, lets us split $\text{tr } M^2$ and $\det M^2$ in terms of the contributions of $A_{\widehat{D5}}$ as

$$\text{tr } M^2 = \text{tr } M_{AdS}^2 + f(A_{\widehat{D5}}),$$
$$\det M^2 = \det M_{AdS}^2 + g(A_{\widehat{D5}}), \tag{49}$$

where $f(A_{\widehat{D5}})$ and $g(A_{\widehat{D5}})$ are positive definite homogeneous functions of degree 1 on $A_{\widehat{D5}}$. If the added potential is of the form

$$\mathcal{V}_{\widehat{D5}} \sim \frac{1}{s^m\tau^n}, \tag{50}$$

with $n, m > 0$, which indeed is our case. Thus, by adding the $A_{\widehat{D5}}$ terms, there is a contribution $\delta\lambda$ to the eigenvalues λ_{AdS} as

$$\lambda = \lambda_{AdS} + \delta\lambda. \tag{51}$$

In this context, we say that if $\delta\lambda < 0$, the eigenvalues of the mass matrix decrease due to the contribution of the non-BPS states. Indeed, the change in the eigenvalues can be written explicitly as

$$\delta\lambda = \frac{1}{2}(f \pm \alpha)\left(1 - \sqrt{1+\gamma}\right) \tag{52}$$

where

$$\gamma = \frac{2f \left(\mathrm{tr}\, M_{AdS}^2 \pm \alpha \right) + 4g}{(f \mp \alpha)^2}. \tag{53}$$

Since f and g are positive functions and $\alpha < \mathrm{tr}\, M^2$, then γ is positive definite. In consequence the term $\left(1 - \sqrt{1+\gamma}\right)$ shall be negative. This in general implies that

$$\frac{2(\delta\lambda)}{(f \pm \alpha)} \le 0. \tag{54}$$

Finally, putting f and α in terms of the determinant and trace of the mass matrix we find that

$$f \pm \alpha = \mathrm{tr}\, M - \mathrm{tr}\, M_{AdS}^2 \pm \sqrt{(\mathrm{tr}\, M_{AdS}^2)^2 - 4 \det M_{AdS}^2} > 0, \tag{55}$$

and $\delta\lambda < 0$.

Adding non-BPS states drives two important features in the effective potential. On one hand, uplifts the value of $\mathcal{V}_{AdS}$ to a dS one, but on the other hand, since the contribution to the energy at the minimum is positive, the scalar potential becomes very flat increasing the probabilities for this vacuum to be destabilized. We show this behavior, for one case, in Fig. 3.

5.1 Comments about some Swampland conjectures

We have described a way to construct a dS vacuum by adding the contribution to the scalar potential from a non-BPS $\widehat{D5}$-brane to a non-SUSY AdS vacuum ($D_S W \ne 0$).[9] However, as recently studied, there are some constraints around the construction of both states. First of all, it has been argued that a non-supersymmetric AdS vacuum is at most metastable in the context of the Swampland program [45,46]. Second of all, it is expected a constraint on the AdS scale with respect to the lightest moduli mass, and finally, in case of uplifting the non-SUSY vacuum to a dS one, the final vacuum is at most, metastable. Let us comment on these three points and how they are manifested in our setup.

As mentioned, one way to assure the construction of an AdS vacuum by considering the contribution of F_5 in a manifold with torsion implies the stabilization of the complex structure by $D_U \mathcal{W} = 0$ while keeping $D_S \mathcal{W} \ne 0$. Therefore, the AdS vacuum is non-SUSY. According to the Swampland conjectures, such an AdS vacuum must be at most metastable. In our case, the source for instabilities could come from two places: first, from our assumption of not considering torsional components of 3-form fluxes, which usually drives some topological transitions as pointed out in [11]. Second,

since the contribution from F_5 is based on the existence of torsional cycles, it is possible that the total discrete charge must vanish following the recent proposal about having zero global charges in Quantum Gravity and its relation to K-theory by cobordisms as proposed in [13]. We believe that both aspects are in fact related.

The second point concerns the AdS scale which it is also conjectured to satisfy a relation of the form

$$m_{\mathrm{mod}} R_{\mathrm{AdS}} \gg c' \tag{56}$$

where $c' \sim 1$ and $R_{\mathrm{AdS}} \sim \|V\|^{1/2}$ in order to keep a robust realization of a dS vacuum. Recent studies argue that effective models which support such a parametric hierarchy are in fact in the Swampland. Again, in our case, the above two factors can be expressed in terms of each of the contributions to the scalar potential, for which we obtain that

$$m_{\mathrm{mod}} R_{\mathrm{AdS}} = \frac{3\sqrt{3}}{2} \frac{2 A_{H_3}^{1/2} A_{F_3}^{1/2} + A_3 \mathcal{N}_3}{A_{F_5}}. \tag{57}$$

As all the constants A_i for $i = \{H_3, F_3, D3, O3\}$ are of the same order, the energy added by F_5, for a $\mathbb{Z}_k$ discrete torsion, vanishes up to a multiple of k. Hence, unless k is too large, the quotient (57) is slightly larger than order 1, and by taking $k = 2$, $m_{\mathrm{mod}} R_{AdS} \gtrsim c'$.

In this context, it is possible to add energy for the uplifting in such a way we stay in a region where stability can be (parametrically) controlled. Indeed, in our model, the AdS vacua contain tachyons neither in the axio-dilaton nor along Kähler directions. Besides, the scale of the AdS is smaller than the energy coming from the lightest moduli violating the AdS conjecture. Thus, adding a non-BPS state whose energy contribution scales a $s^{-1/2}\tau^{-5/2}$ generates a flattering effect accordingly.

Finally, according to the Swampland conjectures, a source of instabilities is expected to affect the uplifted dS vacuum. They could come from the fact that the pair $\widehat{D5} - D3$ (dual to the $\widehat{D7}$-$D9$) is unstable [47,48] and although a decay into a final state does not dilute the discrete charge, it is canceled out by requiring a vanishing K-theory charge [12,43]. However, in our case, $\widehat{D5}$-branes come from $D5$-branes wrapping torsional 2-cycles around an $O3^-$-plane with no $D3$-branes. Hence, at least locally, there are no instabilities at such points. Thus, the non-BPS states are stable and the only decay channel is through tunneling leading to the decompactification limit [49] probably described by a topological transition driven by torsional 3-form fluxes, as suggested in [11]. The presence of NS-NS 3-form fluxes triggers the appearance of instantonic branes transforming branes into fluxes. In our case, the non-BPS $\widehat{D5}$ transforms into discrete 1-form fluxes via the nucleation of a D7-brane [11], followed by the physical realization of the Atiyah Hirzebruch Spectral Sequence connecting cohomology to twisted K-theory as described in
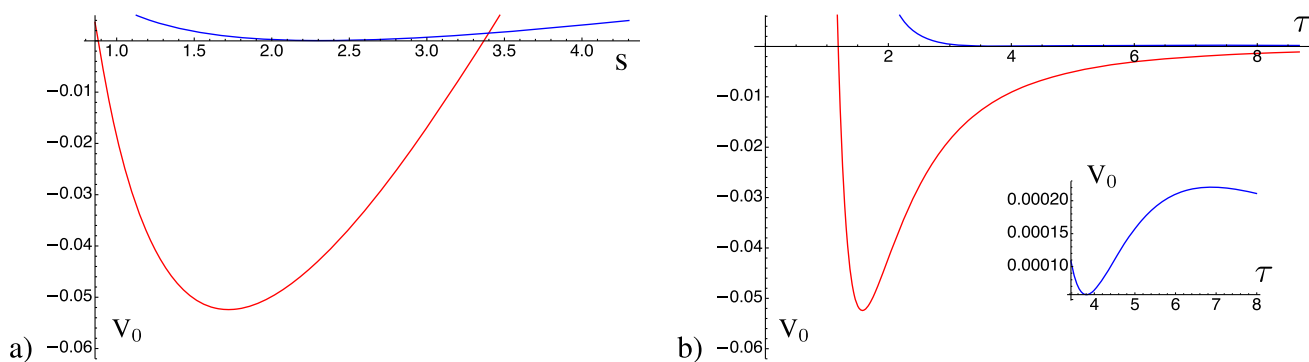
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[9] As implied by the AdS conjecture, only the AdS vacua which are not too far from the mass scale of the lightest modulus can be uplifted, thus to check if our constructions are consistent with the AdS conjecture, it is explored an uplift of the AdS to dS.

Fig. 3 Plots for the uplift mechanism by employing the $A_{\widehat{D5}}$ contribution. In red we show the effective potential in an AdS minimum defined by the contributions $A_{F_3} = 0.77046$, $A_{H_3} = 0.24018$, $A_{O_3} = -1.6974$, $A_{F_5} = 0.97955$ and with moduli vev's given by $s = 1.7911$, $\tau = 1.5603$. By adding non-BPS $\widehat{D5}$-branes with $A_{\widehat{D5}} = 0.43564$, it

is observed that the uplift reduces the mass of the scalar field while its expectation value moves to the right as $s = 2.4473$, $\tau = 3.8434$. Notice that the uplift of the Kähler moduli produces a nearly flat direction, which is compatible with the KKLT scenario

the paper of Maldacena et al. [50]. A detailed study of this process is reserved for future work.

6 Conclusions and final comments

As expected, the incorporation of F_5 fluxes by contribution to the effective scalar potential $\mathcal{V}_{\text{eff}}$ seems to be fundamental to finding classical stable dS vacua in an orientifolded flux compactification of string theory. However, since in a Calabi–Yau manifold F_5 does not contribute to $\mathcal{V}_{\text{eff}}$, we need to consider other internal manifolds, such as the considered in [51,52].

As shown in [38] a Kähler manifold admitting torsion is a suitable example in which F_5 contributes to $\mathcal{V}_{\text{eff}}$. Moreover, these types of manifolds allow wrapping $D5$-branes on torsional cycles, by which one can construct non-BPS states actually classified by K-theory, with a non-zero contribution to $\mathcal{V}_{\text{eff}}$.

Under these circumstances and by implementing a novel ML algorithm we were able to find more than 200 dS critical points for $\mathcal{V}_{\text{eff}}$ out of which 170 are stable.

We also find that there are certain specific conditions that our configurations of branes and fluxes must fulfill in order to generate a stable dS vacuum by uplifting an AdS one. First, to obtain a stable AdS it is necessary to turn on the torsional part of F_5 and to have a configuration of branes and orientifolds such that the number of $O3$-planes or fixed points is larger than the number of $D3$-branes implying that $\mathcal{N}_{flux} > 0$. Second, for these vacua to be uplifted to dS by incorporating the non-BPS states $\widehat{D5}$ we ought to have that:

1. The RR and NS-NS 3-form fluxes are supported in more than a single cycle,

2. $\mathcal{N}_{O3} > 4\frac{\sqrt{A_{H_3} A_{F_3}}}{A_3} + 2\mathcal{N}_{flux}$,

where A_{H_3} and A_{F_3} are the contributions to $\mathcal{V}_{\text{eff}}$ (independent of moduli) from the fluxes H_3 and F_3, while $\mathcal{N}_3 A_3$ is the corresponding from 3-dimensional sources, with $\mathcal{N}_3 = \mathcal{N}_{D3} - \frac{1}{2}\mathcal{N}_{O3}$. Under these conditions, it is possible to obtain that all mass eigenvalues are positive under the uplifting by non-BPS states. We observe, that

1. Getting a small positive value for $V_{\min}$ seems to be a natural consequence by uplifting AdS vacua with small deep. There are two consequences of this: the resulting uplifted potential is very flat while the probability for destabilization of the dS vacua increases since at the limit for large volume, the potential goes to zero, indicating the presence of a barrier potential between the dS local vacuum and the runaway region for the Kähler moduli.
2. We believe that this possibility of the scalar potential to become unstable could be generated by extra mechanisms or topological transitions driven by torsional components on the 3-form fluxes as suggested in [11] and in consequence, establishing a rich scenario where Swampland conjectures can be tasted.

Appendix A: Machine Learning algorithm

In this section, we want to show with some detail the characteristics of our Machine Learning algorithms and how they can help us to find stable vacua in a flux string compactification. We make use of two specific algorithms called the Simulated Annealing (SA) and the Conjugate Gradient (CG).

A.1 Simulated Annealing

The SA algorithm is one of the most preferred heuristic methods for solving optimization problems. SA was introduced by inspiring the annealing procedure of metalworking. In a general manner, the SA algorithm adopts an iterative movement according to a variable parameter that imitates the annealing transaction of the metals. Thus, by taking the objective function as "Error", the SA takes the probability distribution with support ΔError used to replace a new solution as

$$\mathbb{P}\left[\Delta\text{Error}\right] = \exp\left(\frac{\Delta\text{Error}\log(i+1)}{10}\right), \tag{A1}$$

for ΔError the change in the error function depends on an arbitrary number of parameters such as moduli vev's and numerical coefficients that depend on fluxes as well as the non-BPS states, and i the current iteration. Thus as ΔError or the iteration i becomes large the probability to replace a new solution decreases. The SA takes an initial value ϕ_1 and checks if

$$\text{Error}\left(\phi_1\right) \leq \text{Error}\left(\phi_{\text{best}}\right), \tag{A2}$$

if true, ϕ_{best} is replaced by ϕ_1, otherwise it is replaced with a probability $\mathbb{P}\left[\Delta\text{Error}\right]$. A schematic picture of the SA algorithm is shown in Fig. 4.

A.2 Conjugate gradient

Conjugate gradient (CG) is a second-order iterative optimization algorithm designed to find a local minimum provided that the first derivative is known (another alternative that gives us a similar result is the Powells algorithm). The main idea consists in to take repeated steps in conjugate directions of
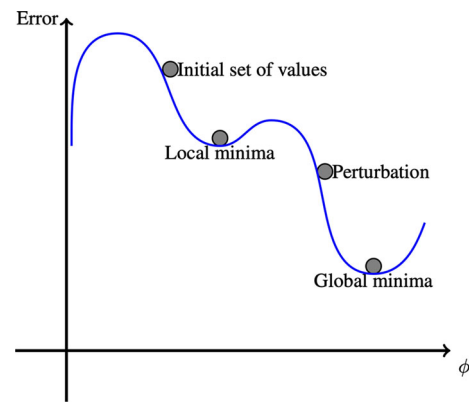

Fig. 4 In a schematic view, the SA algorithm starts at an arbitrary point in the parameter space of the Error function, then it lets to find in a random manner the best solution leading to a local minimum. Once the local minimum is reached, the algorithm shall find alternative paths that find a better minimum by perturbing the solution with a probability $\mathbb{P}\left[\Delta\text{Error}\right]$ avoiding getting stuck in a local minimum

the scalar potential at a given point of the moduli space since this is the direction of the steepest descent. Conversely, stepping in the direction of the gradient leads to a local minimum of the scalar potential. Thus, if the Error function near the global minima is approximated by

$$\text{Error}(\phi^{i+1}) = -b_a\phi_a^i + \frac{1}{2}\phi_a^i\phi_b^i A_{ab} \tag{A3}$$

the residual is defined as

$$r_a\left(\phi\right) = b_a^i - A_{ak}\phi_k^i, \tag{A4}$$

implies that $\partial_a\text{Error}(\phi^{i+1}) = -r_a^i$ vanishes at an extremum. Now, in order to move to the minima of the error function, the changes in the gradient have to follow the direction along

$$A_{ab}\partial_a\text{Error}(\phi^i)\partial_b\text{Error}(\phi^i) = 0. \tag{A5}$$

This implies that the directions $\partial_a\text{Error}(\phi^i)$ and $\partial_b\text{Error}(\phi^i)$ have to be conjugated. Thus, the CG moves through a conjugate direction leading to a local minimum for convex problems. By starting with an initial vector ϕ_a^0 the conjugate gradient method finds two sequences of vectors as

$$\begin{aligned} \phi_a^{i+1} &= \phi_a^i - s\partial_a\text{Error}^i \\ \partial_a\text{Error}^{i+1} &= -\partial_a\text{Error}^{i+1} + \gamma\partial_a\text{Error}^i \end{aligned} \tag{A6}$$

where $A_{ab}\partial_a\text{Error}^i\partial_b\text{Error}^j = 0$ for $j < i$, s is an small parameters and

$$\gamma = \frac{g_a^{i+1}g_a^{i+1}}{g_a^i g_a^i} \tag{A7}$$

is chosen in order to guarantee that the gradients in successive iteration steeps are conjugated.

A.3 Error functions

The objective function can be written as

$$\text{Error} = \sum_{i=1} \alpha_i \cdot \text{error}_i \tag{A8}$$

for $\alpha_i \in \mathbb{R}$ and in a range of $(0, 10^4)$.[10] These parameters are employed to give a penalty to regions on the moduli space that are not of interest. For instance, if we are looking for dS vacua, the error induced by finding a AdS is weighted by this factor, forcing the algorithm to look for another direction. Thus, in the present work, we are interested in finding dS vacua free of negative mass square moduli. Thus our penalty functions shall require (1) to avoid tachyons, (2) to avoid AdS vacua, (3) to require that A_{D5} to be positive. In order to penalize these constraints the following errors are employed

- As we are looking for the extrema of the scalar potential, the first error contribution is related to the derivative of the scalar potential. This is applied as

$$\text{error}_1 = (\partial_j V)^2 \tag{A9}$$

- The second contribution of the errors is defined by proposing a continuous function that penalizes the error function each time that the parameter space is in a AdS vacua. This is,

$$\text{error}_2 = \|V\| - V \tag{A10}$$

- The third contribution to the error function is proposed in order to avoid tachyons in the spectrum. For the simple case of two real moduli, the positive mass square moduli require that $\text{tr}\, m_{ij}^2 > 0$ as well as $\det m^2 - \frac{1}{4}(\text{tr}\, m)^2 > 0$. Thus the third contribution of the error is defined as

$$\text{error}_3 = \|\text{tr}\, m_{ij}\| - \text{tr}\, m_{ij}. \tag{A11}$$

as well as

$$\text{error}_4 = \|\det m_{ij}^2 - \frac{1}{4}\text{tr}\, m_{ij}^2\| + \det m_{ij}^2 - \frac{1}{4}\text{tr}\, m_{ij}^2. \tag{A12}$$

- The fifth contribution to the error is associated with a penalization of the error function each time the algorithm moves into the region of $A_{\widehat{D5}} < 0$. This requirement is implemented in the algorithm as

$$\text{error}_5 = \|A_{\widehat{D5}}\| - A_{\widehat{D5}}. \tag{A13}$$

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[10] Notice that this way to implement penalty functions is known as regularization in machine learning and is equivalent to implementing Lagrange multipliers in an approximate manner, this is inside the bounds of the convergence criteria.

Appendix B: More generic vacua

The implementation of our ML algorithms allows looking for stable vacua in more generic conditions. Here we present numerical results by considering extra terms in the scalar potential without wondering whether they can be constructed or not in a consistent scenario. Specifically, we incorporate the contributions to the scalar potential from $O5$ and $O7$ planes and the internal curvature R_6 besides the usual 3-form fluxes, the $O3$-plane, $D3$-branes and the non-BPS $\widehat{D5}$-branes. Our results are shown in Fig. 5 where we have plotted each vacuum in the function of the energy value at the extreme point and the value of the minimal mass eigenvalue. We observe that most of the vacua are unstable but some are actually dS and stable.

All the cases explored in this landscape contain a contribution of the curvature R_6 of the internal space. In order to check the landscape of critical points we employ different configurations with different content of fluxes and O-planes. Some particular comments for each case follow:

- For the case with F_3, H_3, $O3$, and $O5$, the algorithm is able to find stable dS minima. However, almost all the critical points are unstable.
- For the case of F_3, H_3, $O5$, and $O7$, the algorithm was able to find a few stable dS minima.
- For the case of F_3, H_3, $O3$, and $O5$ the algorithm was not able to find any dS minima.
- For the case of F_3, H_3, F_5, $O3$ and $\widehat{D5}$ the algorithm was able to find several dS minima. In particular, it is observed an abundance of dS is superior to all the other cases. Besides, as the $\widehat{D5}$ contribution is removed (black squares), the algorithm was not able to find any dS minima. This suggests that non-BPS states play an important role in stabilizing the vacua.

Finally and just for the sake of comparison, we want to show that the implementation of penalty constraints in the ML algorithms really impacts the number of stable vacua we find. Let us look for critical points with the same algorithm and by considering the same content the fluxes as in the body of the paper, i.e., F_3, H_3, and F_5, as well as $O3$-planes and non-BPS $\widehat{D5}$-branes (no curvature term). In this case, we realize that

- As we remove the constraints the algorithm finds a lot of critical points but just 6 stable dS against 529 stable AdS. This case is similar to the one obtained by employing the GA+NN classification of our previous work.
- As we implement the penalty functions, the algorithm is able to find 203 stable dS and 170 stable AdS. This is shown in Fig. 6.
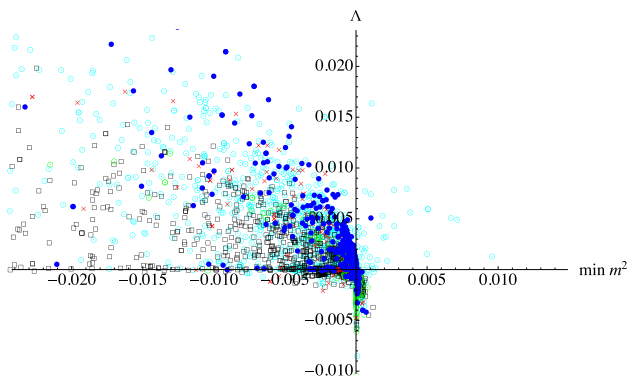
Fig. 5 Landscape found by the hybrid method. The red points are found by using a scalar potential with O3, the blue dots are the vacua found by using O7, the green circles are the critical points found by using O5, the cyan circles are the points found by employing RR F_5 fluxes and $\widehat{D5}$ and the black squares are found by using RR F_5 fluxes but not $\widehat{D5}$
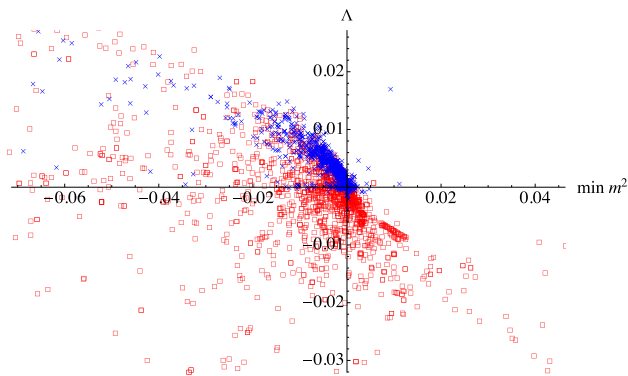

Fig. 6 Plot obtained by using non-penalty constraints (red points) and penalty constraints (blue points)

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