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Entropic uncertainty relation of two-qubit detectors coupled to scalar fields in an expanding spacetime

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Abstract The principles of quantum information provide new avenues to investigate the cosmos. The uncertainty principle is an important trait of the nonclassical world, and it characterizes a significant lower bound (LB) that can be used to estimate measurement results for two noncommuting observables. Subsequently, the uncertainty principle is generalized to a new version by using the entropy and the quantum memory, that is, the quantum-memory-assisted entropic uncertainty relations (Q-M-A-E-U-Rs). Here, considering two qubit detectors coupled to scalar fields, we explore the effects of different cosmic parameters on Q-M-A-E-U-Rs and reveal the influence of the Holevo quantity on the LB of a Q-M-A-E-U-R. It is revealed that an increase in the expansion rapidity of spacetime can enhance the entropic uncertainty and decrease the ability to accurately predict the measurement outcome. The volume expansion leads to the invariance of the entropic uncertainty. An increase in the particle mass of the scalar field causes degradation in entropic uncertainty. In addition, the influence of the Holevo quantity on the Q-M-A-E-U-R's LB can be ignored if one considers (σ_x, σ_y) as two complementary observables. Therefore, one can use the Adabi bound and Berta bound to equivalently predict the left-hand side (LHS) of the Q-M-A-E-U-R in this situation. However, when (σ_x, σ_z) are chosen as two noncommuting observables, the effect of the Holevo quantity on the LB of the uncertainty relation cannot be ignored, and the Adabi bound can always precisely achieve the LHS and predict entropic uncertainty.

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1 Introduction

The uncertainty principle (U-P) reveals an essential discrepancy between classical and quantum theories, and it is a significant backbone of the nonclassical theory [1]. Significantly for what follows, if one considers a pair of noncommuting observables, the U-P can be used to construct a lower bound (LB) for accurately estimating the measurement result [1–6]. Kennard [7] and Robertson [8] ascertained the U-P in terms of the standard deviation. However, the LB proposed by Kennard and Robertson is state dependent and is not an optimal prediction of the result. There have been numerous efforts to improve this principle [9–16]. Deutsch [17] obtained the U-P via the Shannon entropy, namely, the entropic uncertainty relations (E-U-Rs). Kraus reformed the uncertainty relation of Deutsch [18]. Subsequently, Maassen and Uffink verified the results of Kraus [19]. Of particular note is that a novel E-U-R of von Neumann entropy has been extensively implemented in quantum information science. Considering a bipartite system that contains particle A and particle B, Berta et al. [20] obtained a quantum-memory-assisted E-U-R (Q-M-A-E-U-R), and particle B was treated as quantum memory in their scenario. Subsequent works have attempted to ameliorate the Q-M-A-E-U-R [21–33]. To date, there has been valuable progress concerning experimental observations of Q-M-A-E-U-Rs [34–39]. Recently, explorations of entropic uncertainty in various systems have been widely carried out, including neutrino oscillations [40,41], the Unruh–Dewitt detector [42], the Ising model [43], and two two-level atoms [44].

In particular, concepts of quantum information have been used to cognize the nonclassical traits of spacetime, such as the nonclassical traits of metric expansion [45]. In an expanding spacetime, Parker et al. [46,47] revealed particle generation by virtue of quantum entanglement. The creation of entanglement between quantum field modes was investigated by Ball et al. [48]. Considering a Robertson–Walker universe, Fuentes et al. [49] and Moradi et al. [50] explored the entanglement of Dirac modes. Steeg et al. [51] demonstrated that entanglement in the field can be used to capture spacetime curvature. In a spacetime with time dependence, Lin et al. [52] explored the foundational issues regarding the entanglement of generated particles. In an expanding spacetime, Li et al. [53] examined the purity, mutual information, and concurrence of a qubit detector. Nevertheless, there are still few investigations regarding Q-M-A-E-U-Rs in various qubit detector models. Examples include two-qubit detectors coupled to scalar fields in an expanding spacetime [53], harmonic-oscillator particle detectors in relativistic quantum fields [54], an arbitrary number of detectors interacting with a quantum field moving in spacetime [55], a realistic detector model in which the detector has a finite size conveniently tailored by a spatial profile [56], and particle detectors subjected to the zero mode of a quantum field [57]. Relevant explorations can shed new light on expanding spacetime from the view of quantum information.

Enlighted by this, we ascertain the Q-M-A-E-U-R of two-qubit detectors coupled to scalar fields in an expanding spacetime and investigate the influences of different cosmic parameters on the measured uncertainty. In addition, we reveal the effect of the Holevo quantity on the Q-M-A-E-U-R's LB. Our results indicate that the enhancement of the expansion rapidity of spacetime gives rise to increases in the entropic uncertainty and LB. The entropic uncertainty and LB are very sensitive to the volume expansion if the cosmic volume is close to maximum, and the expansion of this volume reduces the ability to accurately estimate the measurement result. The enhancement of the mass of a scalar field particle is responsible for degradation of the entropic uncertainty, which leads to more precise predictions of the measurement outcomes. The influence of the Holevo quantity on the LB of the uncertainty can be almost ignored if one considers (σ_x, σ_y) as two complementary observables. Notably, when (σ_x, σ_z) are chosen as the two possible measurements, the effect of the Holevo quantity on the LB of the uncertainty should be considered because the Holevo quantity can tighten the LB and allow the entropic uncertainty to be precisely estimated in this situation.

The problem setting of two-qubit detectors coupled to scalar fields is introduced in Sect. 2. For two-qubit detectors coupled to scalar fields in an expanding spacetime, the Q-M-A-E-U-R of the system is ascertained in Sect. 3. The influences of different cosmic parameters and the Holevo quantity on the Q-M-A-E-U-R and LB are explored in Sects. 4 and 5, respectively. Finally, conclusions are drawn.

2 Two-qubit detectors coupled to scalar fields

Considering a spacetime that is expanding in accordance with the Robertson–Walker metric, the line element is

$$\mathrm{d}s^2 = R^2(\eta)(d\eta^2 - \mathrm{d}x^2), \tag{1}$$

where η is the conformal time, ranging from $-\infty$ to $+\infty$, and $R^2(\eta)$ is the conformal scale factor [48,58], namely,

$$R^2(\eta) = 1 + \varepsilon(1 + \tanh(\sigma\eta)). \tag{2}$$

Here, ε and σ are positive real parameters, and these parameters control the total volume and rapidity , respectively, of the spacetime expansion. $R^2(\eta) = 1$ $(\eta \to -\infty)$ is the flat spacetime of the distant past, and $R^2(\eta) = 1+2\varepsilon$ $(\eta \to +\infty)$ is the flat spacetime of the far future [48,58].

We consider the Klein–Gordon equation $(\Box + m^2)\Phi = 0$ and a massively real scalar field $\Phi(x, \eta)$ that satisfies the equation $\Box\Phi = \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi)/\sqrt{-g}$ in this metric. Based on the translational invariance of $(\Box + m^2)\Phi = 0$, one can obtain [58,59]

$$\Phi(x, \eta) = (2\pi\omega_k)^{-1/2} e^{ikx} \xi_k(\eta), \tag{3}$$

which satisfies

$$\partial_\eta^2 \xi_k(\eta) + (k^2 + R^2(\eta)m^2)\xi_k(\eta) = 0. \tag{4}$$

μ_k^{in} and μ_k^{out} are the solutions of the equation for $\eta \to -\infty$ and $\eta \to +\infty$, respectively. With the introduction of the Bogoliubov coefficients α_k and β_k, the relation between μ_k^{in} and μ_k^{out} is [58,59]

$$\mu_k^{in}(x, \eta) = \alpha_k \mu_k^{out}(x, \eta) + \beta_k \mu_{-k}^{out*}(x, \eta). \tag{5}$$

Here, $|\alpha_k|^2 - |\beta_k|^2 = 1$.

$$\alpha_k = \left(\frac{\omega_{out}}{\omega_{in}}\right)^{\frac{1}{2}} \frac{\Gamma\left(1 - \frac{i\omega_{in}}{\sigma}\right)\Gamma\left(-\frac{i\omega_{out}}{\sigma}\right)}{\Gamma\left(-\frac{i\omega_+}{\sigma}\right)\Gamma\left(1 - \frac{i\omega_+}{\sigma}\right)}, \tag{6}$$

$$\beta_k = \left(\frac{\omega_{out}}{\omega_{in}}\right)^{\frac{1}{2}} \frac{\Gamma\left(1 - \frac{i\omega_{in}}{\sigma}\right)\Gamma\left(\frac{i\omega_{out}}{\sigma}\right)}{\Gamma\left(\frac{i\omega_-}{\sigma}\right)\Gamma\left(1 + \frac{i\omega_-}{\sigma}\right)}. \tag{7}$$

$\omega_{in} = (k^2 + m^2)^{1/2}$, $\omega_{out} = [k^2 + m^2(1 + 2\varepsilon)]^{1/2}$, and $\omega_\pm = (\omega_{out} \pm \omega_{in})/2$. To characterize the mixed level between $\mathbf{k}$ and $-\mathbf{k}$ (the "in" modes), one can define $\Im_k$, i.e.,

$$\Im_k = \left|\frac{\beta_k}{\alpha_k}\right|^2 = \frac{\sinh^2(\pi\omega_-/\sigma)}{\sinh^2(\pi\omega_+/\sigma)}. \tag{8}$$

One can use $|\beta_k|^2$ to indicate the average number of particles created in "out" modes $\mathbf{k}$, viz. [47]

$$|\beta_k|^2 = \frac{1}{\Im_k^{-1} - 1}. \tag{9}$$

Consequently, the average number approaches infinity (zero) for $\Im_k \to 1$ ($\Im_k \to 0$). For asymptotic regions, the particle content can be characterized by the corresponding creation $\hat{a}_k^{in} = \alpha_k^* \hat{a}_k^{out} - \beta_k^* \hat{a}_{-k}^{out\dagger}$ and annihilation operators [60] $\hat{a}_k^{in\dagger} = \alpha_k \hat{a}_k^{out\dagger} - \beta_k \hat{a}_{-k}^{out}$.

For $\mathbf{k}$ and $-\mathbf{k}$, the "in" vacuum is [53]

$$|0\rangle_k^{in} \, |0\rangle_{-k}^{in} = \sum_n A_{n,k} \, |n\rangle_k^{out} \, |n\rangle_{-k}^{out} \tag{10}$$

with

$$A_{n,k} = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n \sqrt{1 - \Im_k}, \tag{11}$$

where n is the particle content. Subsequently, each pair of modes separates in opposite directions [47]. In light of Eq. (10), the state for mode $\mathbf{k}$ can be described as [53]

$$\begin{aligned}\rho_k^{out} &= \mathrm{Tr}_{-k}[|0\rangle_k^{in} \, |0\rangle_{-k}^{in} \, {}_{-k}^{in}\langle 0| \, {}_k^{in}\langle 0|] \\ &= (1 - \Im_k) \sum_n \Im_k^n |n\rangle_k \, \langle n|,\end{aligned} \tag{12}$$

where $|n\rangle_k \, \langle n|$ is shorthand for $|n\rangle_k^{out} \, {}_k^{out}\langle n|$.

In the far future of the expanding spacetime expressed by Eq. (2), for the Unruh–Wald qubit detector model [61] locally coupled to the scalar field, the Hamiltonian is given by [53]

$$\hat{H} = \hat{H}_\Phi + \hat{H}_q + \hat{H}_I. \tag{13}$$

The Klein–Gordon Hamiltonian of the scalar field is represented by $\hat{H}_\Phi$, and the Hamiltonian of the qubit detector is denoted by $\hat{H}_q = \Omega \hat{\Re}^\dagger \hat{\Re}$. Ω is the energy level difference between $|0\rangle$ and $|1\rangle$, and $\hat{\Re}^\dagger$ and $\hat{\Re}$ are creation and annihilation operators, respectively. $\hat{H}_I(t)$ stands for the following interaction:

$$\hat{H}_I(t) = \varsigma(t) \int_\Sigma A \sqrt{-g} dx, \tag{14}$$

$A = \Phi(x,t)[\psi(x)\hat{\Re} + \psi^*(x)\hat{\Re}^\dagger]$, where $\psi(x)$ represents a smooth function that is nonzero within a small volume around the qubit. $\varsigma(t)$ is the coupling constant corresponding to the qubit-field interrelation. $\sum$ is the spacelike Cauchy surface. The unitary transformation is [61]

$$U \approx 1 - i\hat{\Re}^\dagger \hat{a}(\Gamma^*) + i\hat{\Re}\hat{a}^\dagger(\Gamma^*). \tag{15}$$

$\Gamma(x) = -2i \int B \times C\sqrt{-g'} d^2x'$, $B = G_R(x; x') - G_A(x; x')$, $C = \varsigma(t') e^{i\Omega t'} \psi^*(x')$, $\hat{a}(\Gamma^*)|n\rangle_k = \sqrt{n}\mu_k \, |n-1\rangle_k$, $\hat{a}^\dagger(\Gamma^*)|n\rangle_k = \sqrt{n+1}\mu_k^* |n+1\rangle_k$, and $\mu_k = \left\langle \Gamma_q^*, \chi_k \right\rangle$ [61].

In the single-mode approximation, one can consider only the coupling between the detector qubit and the field mode $\mathbf{k}_0$ with energy Ω. The interaction between $\mathbf{k}_0$ and the detector qubit is [53]

$$\begin{aligned}&|n\rangle \otimes |0\rangle \to |n\rangle \otimes |0\rangle - i\sqrt{n}\mu \, |n-1\rangle \otimes |1\rangle, \\ &|n\rangle \otimes |1\rangle \to |n\rangle \otimes |1\rangle + i\sqrt{n+1}\mu^* \, |n+1\rangle \otimes |0\rangle.\end{aligned} \tag{16}$$

For a particle created in this expanding spacetime, its minimum energy is bounded by $m\sqrt{1+2\varepsilon}$. $\omega_{out} \geq m\sqrt{1+2\varepsilon}$, and $\omega_{out} = \sqrt{k^2 + m^2(1+2\varepsilon)}$. To ensure that the field modes can influence the qubit, $\Omega \geq m\sqrt{1+2\varepsilon}$ should hold. Hence, the volume of spacetime expansion is $\varepsilon \leq \varepsilon_{\max} = (\Omega^2/m^2 - 1)/2$ [53].

Here, we consider a two-entangled-qubit detector, where the surrounding environment is the particles created in the expanding spacetime. Let us investigate the influence of spacetime expansion on the two-qubit detector. The initially entangled state is $|\psi\rangle = \cos\theta \, |01\rangle + \sin\theta \, |10\rangle$. Each qubit can interact with a scalar field and cannot interact with the scalar field around the other qubit [53]. For the two-qubit detector, the state is [53]

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix} \tag{17}$$

with $\rho_{11} = \cos^2\theta \wp_4 + \sin^2\theta \wp_3$, $\rho_{22} = \cos^2\theta \wp_0 + \sin^2\theta \wp_5$, $\rho_{33} = \cos^2\theta \wp_6 + \sin^2\theta \wp_0$, $\rho_{44} = \cos^2\theta \wp_1 + \sin^2\theta \wp_2$, and $\rho_{23} = \rho_{32} = \cos\theta \sin\theta \wp_0$.

$$\wp_0 = D \sum_{n,m} \frac{\Im_A^n \Im_B^m}{\Re_n m}, \tag{18}$$

$$\wp_1 = D|\mu_A|^2 \sum_{n,m} \frac{n\Im_A^n \Im_B^m}{\Re_n m}, \tag{19}$$

$$\wp_2 = D|\mu_B|^2 \sum_{n,m} \frac{m\Im_A^n \Im_B^m}{\Re_n m}, \tag{20}$$

$$\wp_3 = D|\mu_A|^2 \sum_{n,m} \frac{(n+1)\Im_A^n \Im_B^m}{\Re_n m}, \tag{21}$$

$$\wp_4 = D|\mu_B|^2 \sum_{n,m} \frac{(m+1)\Im_A^n \Im_B^m}{\Re_n m}, \tag{22}$$

$$\wp_5 = D|\mu_A|^2|\mu_B|^2 \sum_{n,m} \frac{m(n+1)\Im_A^n \Im_B^m}{\Re_n m}, \tag{23}$$

$$\wp_6 = D|\mu_A|^2|\mu_B|^2 \sum_{n,m} \frac{n(m+1)\Im_A^n \Im_B^m}{\Re_n m}, \tag{24}$$

$$D = (1 - \Im_A)(1 - \Im_B), \tag{25}$$

$$\Re_n m = E_{nm} \sin^2\theta + F_{nm} \cos^2\theta, \tag{26}$$

$$\begin{aligned}E_{nm} &= 1 + m|\mu_B|^2 \\ &+(n+1)|\mu_A|^2 + m(n+1)|\mu_A|^2|\mu_B|^2,\end{aligned} \tag{27}$$

$$\begin{aligned}F_{nm} &= 1 + n|\mu_A|^2 + (m+1)|\mu_B|^2 \\ &+n(m+1)|\mu_A|^2|\mu_B|^2,\end{aligned} \tag{28}$$

and $\wp_0 + \cos^2\theta(\wp_1 + \wp_4 + \wp_6) + \sin^2\theta(\wp_2 + \wp_3 + \wp_5) = 1$.

3 Q-M-A-E-U-R of the system

One can characterize the Q-M-A-E-U-R by means of a guessing game. Bob sends particle A of an entangled state to Alice and keeps particle B to realize quantum storage. After Alice receives particle A, she implements one of two possible measurements on particle A and informs Bob of the measurement selection (X or Z). Based on the selected measurement, Bob can predict the measurement results within a minimal uncertainty limited by the LB of the Q-M-A-E-U-R. The Q-M-A-E-U-R is [20]

$$S(X|B) + S(Z|B) \geq \log_2\frac{1}{c} + S(A|B), \tag{29}$$

where $S(Q|B)=S(\rho_{QB}) - S(\rho_B)$ ($\rho_B = \mathrm{Tr}_A(\rho_{QB})$) represents the conditional von Neumann entropy of $\rho_{QB} = \sum_i \prod_i^A \rho_{AB} \prod_i^A$ ($\prod_i^A = |\varphi_i\rangle\langle\varphi_i|$). The eigenstates of the observable Q are denoted by $|\varphi_i\rangle$. The $|\phi_x\rangle$ ($|\psi_z\rangle$) are the eigenstates of the observable X (Z), and $c = max_{x,z}|\langle\phi_x|\psi_z\rangle|^2$. $S(\rho) = -\sum_i \lambda_i \log_2\lambda_i$, where the λ_i are the eigenvalues of ρ. $S(A|B) = S(\rho_{AB}) - S(\rho_B)$.

One can regard $\log_2(1/c) + S(A|B)$ (i.e., the LB of the entropic uncertainty) as the Berta bound and regard the entropic uncertainty (namely, the left-hand side of Eq. (29) as the LHS. Accordingly, by using the mutual information, Adabi et al. proposed an optimized LB for the Q-M-A-E-U-R, viz. [31]

$$S(X|B) + S(Z|B) \geq \log_2\frac{1}{c} + S(A|B) + \max\{0, \delta\}, \tag{30}$$

with $\delta = I(\rho_{AB}) - [I(\rho_{XB}) + I(\rho_{ZB})]$. Here, $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ is the mutual information and indicates the total correlation. $I(\rho_{QB}) = S(\rho_B) - \sum_i p_i S(\rho_i^B)$ is the Holevo quantity, and it denotes the upper limit on the information that can be acquired by Bob. $p_i = \mathrm{Tr}(\prod_i^A \rho_{AB} \prod_i^A)$ is the measurement probability corresponding to the measurement outcome, and $\rho_i^B = \mathrm{Tr}_A(\prod_i^A \rho_{AB} \prod_i^A)/\mathrm{Tr}(\prod_i^A \rho_{AB} \prod_i^A)$ is Bob's state. One can regard $\log_2(1/c) + S(A|B) + \max\{0, \delta\}$ of Eq. (30) as the Adabi bound.

Here, we use the normalized eigenstates $|\varphi_{ij}\rangle$ of the $\sigma_i (i = x, y, z)$ to construct the measurement operators $\prod_{ij}^A = |\varphi_{ij}\rangle\langle\varphi_{ij}|$, and implement the operators on particle A of ρ_{AB}. The postmeasurement states are $\rho_{\sigma_i B} = \sum_j \prod_{ij}^A \rho_{AB} \prod_{ij}^A$, namely,

$$\begin{aligned}\rho_{\sigma_x B} &= \tfrac{1}{2}\{[(\wp_4 + \wp_6)\cos^2\theta \\ &+(\wp_0 + \wp_3)\sin^2\theta]\,|00\rangle\langle 00| + |10\rangle\langle 10| \\ &+[(\wp_0 + \wp_1)\cos\theta^2 \\ &+(\wp_2 + \wp_5)\sin^2\theta](|01\rangle\langle 01| + |11\rangle\langle 11|) \\ &+\wp_0\cos\theta\sin\theta(|00\rangle\langle 11| + |01\rangle\langle 10| \\ &+|10\rangle\langle 01| + |11\rangle\langle 00|)\},\end{aligned} \tag{31}$$

$$\begin{aligned}\rho_{\sigma_y B} &= \tfrac{1}{2}\{[(\wp_4 + \wp_6)\cos^2\theta \\ &+(\wp_0 + \wp_3)\sin^2\theta]\,|00\rangle\langle 00| + |10\rangle\langle 10| \\ &+[(\wp_0 + \wp_1)\cos\theta^2 \\ &+(\wp_2 + \wp_5)\sin^2\theta](|01\rangle\langle 01| + |11\rangle\langle 11|) \\ &+\wp_0\cos\theta\sin\theta(-|00\rangle\langle 11| + |01\rangle\langle 10| \\ &+|10\rangle\langle 01| - |11\rangle\langle 00|)\},\end{aligned} \tag{32}$$

$$\begin{aligned}\rho_{\sigma_z B} &= (\wp_4\cos^2\theta + \wp_3\sin^2\theta)\,|00\rangle\langle 00| \\ &+(\wp_0\cos^2\theta + \wp_5\sin^2\theta)\,|01\rangle\langle 01| \\ &+(\wp_6\cos^2\theta + \wp_0\sin^2\theta)\,|10\rangle\langle 10| \\ &+(\wp_1\cos^2\theta + \wp_2\sin^2\theta)\,|11\rangle\langle 11|.\end{aligned} \tag{33}$$

The eigenvalues of these postmeasurement states can be calculated as shown below:

$$\lambda_{x1} = \lambda_{x2} = \lambda_{y1} = \lambda_{y2} = \frac{1}{4}\left\{1 - \sqrt{\Xi}\right\}, \tag{34}$$

$$\lambda_{x3} = \lambda_{x4} = \lambda_{y3} = \lambda_{y4} = \frac{1}{4}\left\{1 + \sqrt{\Xi}\right\}, \tag{35}$$

$$\begin{aligned}&\lambda_{z1} = \wp_4\cos^2\theta + \wp_3\sin^2\theta,\ \lambda_{z2} = \wp_0\cos^2\theta + \wp_5\sin^2\theta, \\ &\lambda_{z3} = \wp_6\cos^2\theta + \wp_0\sin^2\theta,\ \lambda_{z4} = \wp_1\cos^2\theta + \wp_2\sin^2\theta,\end{aligned} \tag{36}$$

and

$$\begin{aligned}\Xi = {\wp_0}^2 &+ 2\wp_0\cos 2\theta[(\wp_1 - \wp_4 - \wp_6)\cos^2\theta \\ &+(\wp_2 - \wp_3 + \wp_5)\sin^2\theta] \\ &+[(\wp_1 - \wp_4 - \wp_6)\cos^2\theta \\ &+(\wp_2 - \wp_3 + \wp_5)\sin^2\theta]^2.\end{aligned} \tag{37}$$

Thus, by considering two-entangled-qubit detectors coupled to scalar fields in an expanding spacetime, the Berta bound, Adabi bound and entropic uncertainty can each be investigated.

4 Ascertaining the influences of the Holevo quantity and various cosmic parameters on the Q-M-A-E-U-R considering (σ_x, σ_y)

Here, we consider (σ_x, σ_y) as two complementary observables and reveal the influences of various cosmic parameters on the Q-M-A-E-U-R. In addition, we explore the effect of the Holevo quantity on the LB of the Q-M-A-E-U-R. Figure 1 depicts the dependence of the LHS, Adabi bound, and Berta bound on the expansion rapidity σ. As revealed in Fig. 1a, the LHS of the Q-M-A-E-U-R obviously increases with increasing rapidity and eventually reaches a fixed value as the rapidity continues to increase. This finding indicates that the LHS of the Q-M-A-E-U-R is very sensitive to the rapidity σ when the rapidity is low. An increase in rapidity gives rise to dissipation of information and thus strengthens the entropic uncertainty. In other words, an increase in σ decreases the ability to accurately predict the measurement outcome. The traits of the Adabi bound and Berta bound are consistent with those

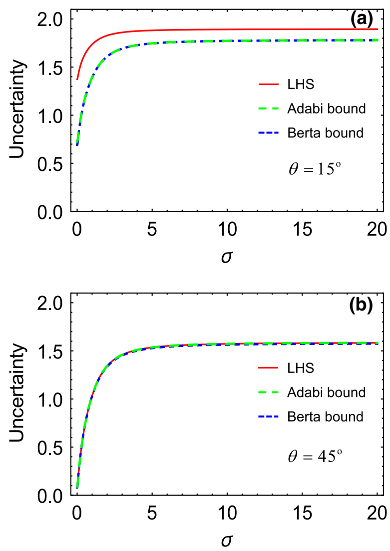

Fig. 1 The LHS, Adabi bound, and Berta bound with respect to the expansion rapidity σ when considering two complementary observables (σ_x, σ_y). $\varepsilon = 19999.5$, $\Omega = 2$, $\mu_A = \mu_B = 0.1$, and $m = 0.01$

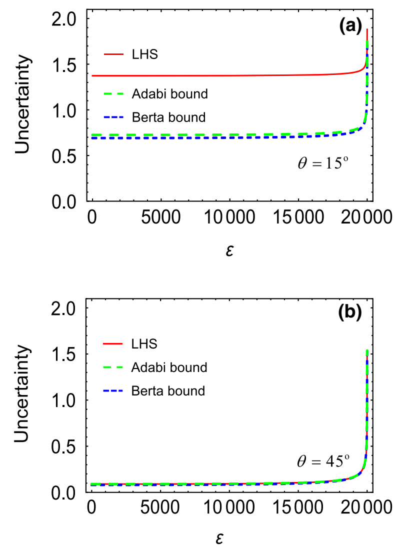

Fig. 2 The LHS, Adabi bound, and Berta bound with respect to the volume expansion ε when considering two complementary observables (σ_x, σ_y). $\sigma = 2$, $\Omega = 2$, $\mu_A = \mu_B = 0.1$, and $m = 0.01$

of the LHS. It should be emphasized that as seen in Fig. 1a, the Adabi bound overlaps with the Berta bound in the case of $\theta = 15^{\mathrm{o}}$ (initially nonmaximal entanglement state), and the two bounds cannot reach the LHS. These results reveal that the effect of the Holevo quantity on the LB of the Q-M-A-E-U-R can be ignored, and one can use the Adabi bound and the Berta bound to equivalently predict the LHS of the Q-M-A-E-U-R and the measurement outcomes. If one considers the initial state parameter $\theta = 45^{\mathrm{o}}$ (initially maximal entanglement state), as shown in Fig. 1b, the results are different from those in Fig. 1a. To be clearer, the LHS is close to zero if σ is very low. It can be conjectured that a low rapidity of expansion is conducive to obtaining a smaller measurement uncertainty and achieving a precise prediction of the measurement outcome. In addition, the Berta bound almost overlaps with the Adabi bound in the situation depicted in Fig. 1b. Both bounds, especially the Adabi bound, can always reach the LHS. The Holevo quantity does not need to be considered in this scenario. Therefore, the Berta bound and Adabi bound can both predict the entropic uncertainty very well.

Next, we direct our attention to ascertaining the effects of the Holevo quantity and volume expansion on the Q-M-A-E-U-R. Figure 2 illustrates the LHS, Adabi bound, and Berta bound with respect to the volume expansion ε. One can discover from Fig. 2a that the LHS is invariant with different values of ε, meaning that the volume expansion does not influence the precision of predicting the measurement outcome. However, the invariant characteristic of the LHS is broken as $\varepsilon \to \varepsilon_{\max} = 19999.5$, where the LHS greatly increases with increasing volume. That is, a volume expansion of $\varepsilon \to \varepsilon_{\max} = 19999.5$ inevitably gives rise to the disappearance of the ability to accurately predict Alice's measurement result. The characteristics of the Adabi bound and Berta bound are the same as those of the LHS, but the Adabi bound is a tighter bound. Of particular note is that the Berta bound overlaps with the Adabi bound when ε is close to $\varepsilon_{\max} = 19999.5$, and both bounds can almost reach the LHS when $\varepsilon \to \varepsilon_{\max} = 19999.5$. For an initial state with $\theta = 45^{\mathrm{o}}$, as shown in Fig. 2b, the LHS is almost equal to zero before ε expands to the maximum, and thus, Alice's results can be precisely predicted by Bob. Additionally, the Adabi bound is almost consistent with the Berta bound in Fig. 2b, and the influence of the Holevo quantity can be neglected in this situation. In general, both bounds, especially the Adabi bound, can always reach the LHS and predict the entropic uncertainty in the case of an initially maximal entanglement state.

For two complementary observables (σ_x, σ_y), to ascertain the influences of the cosmic parameters and the Holevo quantity on the entropic uncertainty, we present the LHS, Adabi bound, and Berta bound with respect to the particle mass m of the scalar field in Fig. 3. Our results in Fig. 3a reveal that the LHS, Adabi bound, and Berta bound degenerate with increasing m, indicating that a larger mass of the scalar field particle can enable more precise predictions of measurement outcomes. The Adabi bound and Berta bound approach the LHS at m =0; subsequently, the discrepancy between the Adabi bound (or Berta bound) and the LHS increases as the mass increases. In addition, the Adabi bound is tighter than the Berta bound when m is large, that is, the influence of the Holevo quantity is notable at large m. In Fig. 3b, both the Adabi bound and the Berta bound overlap with the LHS and can accurately predict the measurement outcome. The results

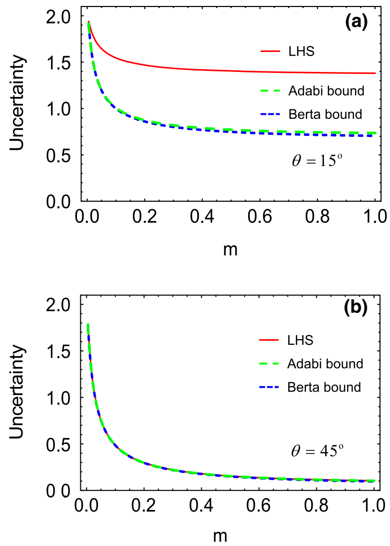

Fig. 3 The LHS, Adabi bound, and Berta bound with respect to the particle mass m of the scalar field when considering two complementary observables (σ_x, σ_y). $\varepsilon = \varepsilon_{\max} = (4/m^2 - 1)/2$, $\sigma = 2$, $\Omega = 2$, and $\mu_A = \mu_B = 0.1$

are similar to those in Figs. 1b and 2b. Both bounds are tight bounds for the entropic uncertainty, and the Holevo quantity need not be considered in the case of $\theta = 45^{\mathrm{o}}$.

5 Exploring the influences of the Holevo quantity and various cosmic parameters on the Q-M-A-E-U-R considering (σ_x, σ_z)

It deserves to be emphasized that if we choose (σ_x, σ_z) as the two possible measurements, the results are obviously different from those obtained when considering (σ_x, σ_y) as two complementary observables. For clarity, we present the LHS, Adabi bound, and Berta bound with respect to the rapidity σ for the case of two complementary observables (σ_x, σ_z) in Fig. 4. One can discover from Fig. 4a ($\theta = 15^{\mathrm{o}}$) that the Adabi bound can reach the LHS of the Q-M-A-E-U-R and precisely predict the entropic uncertainty for various σ. This result is different from that obtained when considering (σ_x, σ_y) as the two possible measurements in Fig. 1a. In contrast, the Berta bound can only reach the LHS at low σ. The effect of the Holevo quantity is responsible for the fact that the Adabi bound is a tighter bound. Additionally, it is found that the Adabi bound can always overlap with the LHS in the case of an initially maximal entanglement state, as shown in Fig. 4b. In comparison, the difference between the Berta bound and the LHS (or Adabi bound) is considerable at high σ. Hence, whether $|\psi\rangle$ is maximally entangled or non-maximally entangled, the Adabi bound can always overlap with the LHS of the Q-M-A-E-U-R and precisely predict the entropic uncertainty at different expansion rapidities. That is,

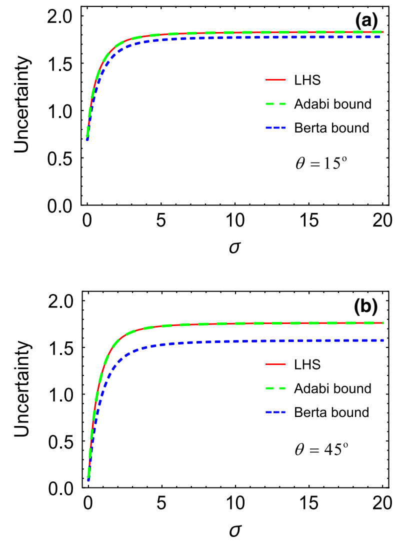

Fig. 4 The LHS, Adabi bound, and Berta bound with respect to the expansion rapidity σ when considering two complementary observables (σ_x, σ_z). $\varepsilon = 19999.5$, $\Omega = 2$, $\mu_A = \mu_B = 0.1$, and $m = 0.01$

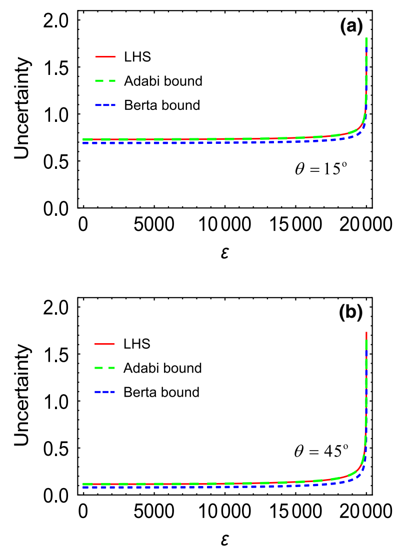

Fig. 5 The LHS, Adabi bound, and Berta bound with respect to the volume expansion ε when considering two complementary observables (σ_x, σ_z). $\sigma = 2$, $\Omega = 2$, $\mu_A = \mu_B = 0.1$, and $m = 0.01$

the Adabi bound does not rely on the selection of $|\psi\rangle$ when (σ_x, σ_z) are considered as two complementary observables. The Berta bound is only effective in predicting the entropic uncertainty if the expansion rapidity is low, and the Holevo quantity cannot be ignored in the case of (σ_x, σ_z).

To further explore the influence of the Holevo quantity on the bound of the Q-M-A-E-U-R under different levels of the volume expansion, Figure 5 displays the LHS, Adabi bound, and Berta bound with respect to ε when considering two complementary observables (σ_x, σ_z). Our results in Fig. 5 reveal that the Adabi bound can perfectly overlap with the

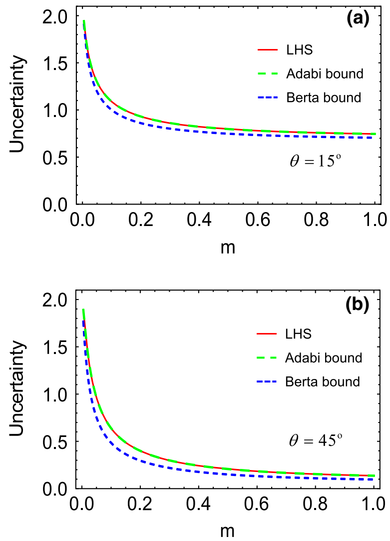

Fig. 6 The LHS, Adabi bound, and Berta bound with respect to the particle mass m of the scalar field when considering two complementary observables (σ_x, σ_z). $\varepsilon = \varepsilon_{\max} = (4/m^2 - 1)/2$, $\sigma = 2$, $\Omega = 2$, and $\mu_A = \mu_B = 0.1$

LHS for various initial states (the degree of overlap between the Adabi bound and the LHS is not dependent on the initial state), including an initially nonmaximal entanglement state (in Fig. 5a) and an initially maximal entanglement state (in Fig. 5b). This enables the Adabi bound to accurately predict the entropic uncertainty. The Berta bound can only reach the LHS when the expansion ε is at its maximum. Consequently, the Holevo quantity needs to be considered in the process of the volume expansion, except for $\varepsilon = \varepsilon_{\max} = 19999.5$.

Finally, we probe the effect of the Holevo quantity on the Q-M-A-E-U-R's LB under different masses of the scalar field particle when considering two complementary observables (σ_x, σ_z), as exhibited in Fig. 6. One can see that the Berta bound overlaps with the LHS at m=0. At higher m, however, the Berta bound is ineffective in precisely predicting the LHS of the Q-M-A-E-U-R if we choose the state parameter $\theta = 15^{\mathrm{o}}$ or $\theta = 45^{\mathrm{o}}$. The discrepancy between the Berta bound and the LHS initially strengthens and then decreases as m increases. In contrast, under the influence of the Holevo quantity, the Adabi bound can always reach the LHS in the cases of $\theta = 15^{\mathrm{o}}$ and $\theta = 45^{\mathrm{o}}$. Thus, the Adabi bound can precisely predict the entropic uncertainty when (σ_x, σ_z) are selected as the two possible measurements, and it is a tighter bound on the Q-M-A-E-U-R for a two-qubit detector coupled to a scalar field in an expanding spacetime.

6 Conclusions

We examine the Q-M-A-E-U-R of a model in which a two-qubit detector is coupled to a scalar field in an expanding spacetime. We ascertain the influences of the cosmic expansion rapidity, the cosmic volume expansion, and the mass of the scalar field particle on the measured uncertainty. Additionally, the influence of the Holevo quantity on the LB of the Q-M-A-E-U-R is explored. The results indicate that an enhancement in expansion rapidity results in an increase in entropic uncertainty. Low expansion rapidity is favourable for obtaining little uncertainty and precise prediction of the measurement outcome. The entropic uncertainty is not greatly susceptible to volume expansion unless the volume expansion is close to maximum. However, $\varepsilon \to \varepsilon_{\max}$ leads to disappearance of the ability to accurately predict the measurement result. The entropic uncertainty degrades with an increase in the particle mass of the scalar field. A larger mass can enable more precise prediction of the measurement outcome. If one considers (σ_x, σ_y) as the two possible measurements, the influence of the Holevo quantity on the LB of the Q-M-A-E-U-R is relevant to the choice of the initial state. The Adabi bound is tighter than the Berta bound in the case of a state parameter of $\theta = 15^{\mathrm{o}}$. However, the two bounds are almost equivalent and can perfectly predict the LHS of the Q-M-A-E-U-R in the case of $\theta = 45^{\mathrm{o}}$. On the other hand, if one chooses (σ_x, σ_z) as the two complementary observables, then the effect of the Holevo quantity on the LB of the Q-M-A-E-U-R does not depend on the choice of the initial state. In the cases of both $\theta = 15^{\mathrm{o}}$ and $\theta = 45^{\mathrm{o}}$, the Adabi bound always overlaps with the LHS and can elegantly predict the entropic uncertainty of a two-qubit detector coupled to a scalar field in an expanding spacetime. However, the Berta bound cannot precisely predict the LHS. For this reason, the Holevo quantity cannot be ignored for different initial states when (σ_x, σ_z) are chosen as two noncommuting observables.

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