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Letter

A calculation-free derivation of GR

S. Deser[1,2,a]

[1] Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125, USA
[2] Physics Department, Brandeis University, Waltham, MA 02454, USA

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Abstract I derive GR from the standard observational requirements plus a recent proof that a linear spin 2 field can only propagate on a Ricci-flat (or constant) background, of which it is a perturbation: it is an Einstein field.

Ingredients and history The gross structure observational ingredients are well-known: The gravitational field is long-range, hence massless. It bends light, hence not a scalar.
It is macroscopic, hence not spin 1/2 or 3/2. Attractive, so not s = 1, leaving spin 2 as the only candidate, higher spins being inconsistent except as free fields in flat space, as well as lacking matter sources. Separately, it was very recently shown [1] that a s = 2 field can only propagate on a Ricci-/Einstein-flat, or constant [if the excitation is (apparently) massive], background. Let us recapitulate the theory-side history: The initial attempt to derive GR was Kraichnan's [2], but we recently noted that it was unsatisfactory [3]. Gupta's later attempt was pure handwaving; Feynman's is also incomplete – he could not sum the resulting infinite series, but assumed general covariance, which is equivalent to guaranteeing GR ab initio. More precisely, he tried selfinteraction in second order form, and found of course an infinite series, that he could not sum, so he said "what else could it be", by invoking general covariance. Weinberg [4] did derive the equations in the sense of the interaction representation, but then the nonlinearity is hidden in the interaction term and the equations are not explicitly visible. The first (and only) correct explicit derivation was a short one [5,6] relying on the firstorder (Palatini) form of free spin 2, whose self-interaction (no covariance or any other assumptions!) led in one step to the full (cubic) GR, followed by a derivation in an arbitrary background [7] and finally one based on tree-level QM [8].

Proof At this point we merely invoke the result of [1], where all desired equations may be found: a spin 2 linear field must necessarily evolve on a Ricci-/Einsteinflat background, of which it is – most importantly – a perturbation. [It could be Riemann-flat as a special case, but then it would not be "dynamical enough", i.e., it would be a source-free linear field.] So, for consistency, (left side is covariantly conserved, so must the right side be) the source of the now full Einstein equations, namely the matter stress tensor, must also live in this Riemann space, whose metric is now dynamical. Had we allowed more generally the excitation to have a "mass" term, the resulting left side would instead be a perturbation of cosmological gravity, rather than just the Einstein tensor, with a constant proportional to m^2 [1]. So the only consistent alternative to GR is source-free, linear s = 2 in flat/(A)dS space, dull and unobservable, and no gravity! Some additional remarks: First all this is classical with no quantum notions, and there will also be improvements at higher Wilsonian levels – so our derivation is of the effective minimal low energy gravity, which could also include various small additions such as non-minimal terms. The Newtonian coupling "constant" G thus could have been a Brans-Dicke field. But such fine points, let alone the quantum theory, are not yet ripe for discussion! Finally, quadratic and higher order models, excluded both observationally and theoretically (dipole ghosts) are also not derivable from self-interaction arguments [9]. □

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[a] e-mail: deser@brandeis.edu (corresponding author)

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