



# Excited states of spin-(3/2) doubly-heavy baryons within the QCD sum rules method

H. I. Alrebdī<sup>1</sup>, R. F. Alnahdi<sup>2</sup>, T. Barakat<sup>3, a</sup>

<sup>1</sup> Department of Physics, College of Science, Princess Nourah Bint Abdulrahman University, P.O. Box 84428, Riyadh 11671, Saudi Arabia

<sup>2</sup> Department of Physics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

<sup>3</sup> Basic Sciences Unit, Faculty of Arts and Sciences, TED University, 06420 Ankara, Turkey

Received: 22 February 2022 / Accepted: 10 May 2022 / Published online: 16 May 2022  
© The Author(s) 2022

**Abstract** Within the framework of the QCD sum rules method, the masses and pole residues of the excited states  $2S$  and  $1P$  of the doubly-heavy spin-(3/2) baryons  $\Xi_{cc}^*$ ,  $\Xi_{bb}^*$ ,  $\Xi_{bc}^*$ ,  $\Omega_{cc}^*$ ,  $\Omega_{bb}^*$ , and  $\Omega_{bc}^*$  are investigated. The excited state masses and pole residues are predicted with reasonable accuracy. We expect that soon, the current predictions can be tested against future experimental data.

## 1 Introduction

Over the past few years, LHCb has discovered multiple heavily flavored baryons and many resonances containing one heavy bottom quark, as well as, additional baryons having one charm quark. The LHCb collaboration, for example, discovered five new narrow  $\Omega_c^0$  states,  $\Sigma_b(6097)$  excited state,  $\Xi_b(6227)$  bottom-strange state,  $\Lambda_b(6146)$ , and  $\Lambda_b(6152)$  resonances [1–8]. Additionally, the LHCb Collaboration has also identified four narrow peaks of  $\Omega_b^-$  excited states [9], and the CMS Collaboration has recently discovered a new state  $\Xi_b(6100)$  [10]. Moreover, many spin-(1/2) b-baryons ( $\Lambda_b^0$ ,  $\Sigma_b^{\pm,0}$ ,  $\Xi_b^{0,-}$ ,  $\Xi_b^{\prime,-}$ ,  $\Omega_b^0$ ), and spin-(3/2) b-baryons ( $\Lambda_b^0$ ,  $\Sigma_b^{*,\pm,0}$ ,  $\Xi_b^{*,0,-}$ ) were also discovered and their masses are precisely measured [11–15].

The SELEX Collaboration, on the other hand, has observed the doubly-heavy charmed baryon state  $\Xi_{cc}^+$  with spin-(1/2) and mass  $(3519 \pm 1)$  MeV in the  $pD^+K^-$  decay channel [16], and then later was confirmed by the same experiment in 2005 [17]. Whereas, the LHCb Collaboration have recently reported that the ground state mass of  $\Xi_{cc}^{++}$  is  $(3624.40 \pm 0.72 \pm 0.14)$  MeV [2]. These findings stimulated the search for a new class of long-lived baryons called doubly-heavy baryons.

The doubly-heavy baryons are hadrons containing two heavy quarks of ( $c$  and/or  $b$ ), and one light quark of ( $d$  or  $u$  or  $s$ ). The existence of such doubly-heavy baryons is considered a natural consequence of the quark model of hadrons.

Meanwhile, the LHCb has already begun the searching for the doubly-heavy bottom charmed baryon  $\Xi_{cb}$  and is still working to improve its signal sensitivity [18].

The existence of such experiments has prompted theorists and phenomenologists to investigate the structural features of the doubly-heavy flavor baryons. The charmed and bottom baryons, which are composed of one (two) heavy quark(s), are of particular interest for studying the dynamics of light quarks in the presence of heavy quarks and may provide an exceptional platform for testing quark model estimations and heavy quark symmetries.

Motivated by the possibility of observing these new doubly-heavy baryons shortly, the theoretical study of such baryons could shed light on the dynamics of the heavy quarks at the hadronic scale, and hence might be represented as a truthful investigation of the interaction between perturbative and non-perturbative QCD pictures. In the theory, the study of doubly-heavy baryons could provide crucial inputs for future experimental discoveries, as well as possibly clarifying and revealing secrets about the nature of other heavy baryons.

One of the most important physical properties of the doubly-heavy baryons is their masses and residues, which can be accessed experimentally. Based on this, many authors have created and established various phenomenology methods and models to estimate the mass spectrum of doubly-heavy baryons, such as the non-relativistic and relativistic quark models [19,20], the three-body Faddeev method [21], the QCD MIT bag model [22], effective field theory with the non-relativistic QCD potential method [23], the heavy quark spin symmetry method [24], the variational approach method [25], the full QCD sum rules method [26], and the lattice QCD method [27,28]. In addition to that, many other

<sup>a</sup> e-mail: [thabit.barakat@tedu.edu.tr](mailto:thabit.barakat@tedu.edu.tr) (corresponding author)

theoretical works inspired by the above mentioned discoveries also have been carried out to determine the characteristic properties like the masses, lifetimes, and the strong coupling constants of such baryons within various models [29–36].

Thus far, the QCD sum rules method is widely recognized as one of the most essential nonperturbative analytical methods for estimating the ground state masses of the heavy hadrons, and the numerical estimation of this method is extremely accurate. The success of this method made it as one of the most popular theoretical methods for testing the various properties of doubly-heavy baryons. The masses of spin-(1/2) doubly-heavy baryons, for example, were computed using the QCD sum rules approach in [37–41], whereas the ground state masses of spin-(3/2) doubly-heavy baryons were computed in [38–45]. The masses and pole residues of the ground state of spin-(3/2) doubly-heavy baryons  $\Xi_{QQq}^*$ ,  $\Xi_{QQ'q}^*$ ,  $\Omega_{QQq}^*$ , and  $\Omega_{QQ'q}^*$ , on the other hand, have been the focus of previous research [44]. Therefore, there is a lot of work to be done to have a full spectrum of spin-(3/2) doubly-heavy baryons.

Due to its numerical performance, we were tempted to extend the calculations of the QCD sum rules in [44]. However, this time to understand how the QCD sum rules method can be applied to derive the excited state masses and the excited state pole residues of spin-(3/2) doubly-heavy baryons. To do so, in the present work, we'll use the most common type of interpolation current for spin-(3/2) doubly-heavy baryons.

With this in mind, the form of this paper is arranged as follows: Sect. 2 presents the formulation of the QCD sum rules for spin-(3/2) doubly-heavy baryons. The analytical formulations of the QCD sum rules are written in such a way that the reader can use them without proceeding into their derivation. Section 3, focuses on the numerical analysis of the QCD sum rules for excited state masses and excited state pole residues of spin-(3/2) doubly-heavy baryons;  $2S$  and  $1P$  of  $\Xi_{cc}^*$ ,  $\Xi_{bb}^*$ ,  $\Xi_{bc}^*$ ,  $\Omega_{cc}^*$ ,  $\Omega_{bb}^*$ , and  $\Omega_{bc}^*$ . Finally, in the last section of this paper, we presented a summary of our findings and some concluding remarks.

## 2 QCD sum rules for doubly-heavy spin-(3/2) baryon states

One of the most important methods for non-perturbative QCD is the QCD sum rules method. In this method, hadrons are represented by their interpolating quark currents, and the major object of this method is the so-called correlation function  $\Pi(p^2)$ , which can be expressed in terms of the interpolating quark currents. The correlation function  $\Pi(p^2)$  is an analytic function of  $p^2$  that can be defined at both negative (space-like) and positive (time-like) values of  $p^2$ . When  $p^2$  is shifted from large negative values towards positive values,

the correlation function starts receiving contributions from the long-distance quark-gluon interactions. In this region, quarks start to form hadrons, and the phenomenological (or hadronic) side of the correlation function is investigated, and can be expressed in terms of hadronic degrees of freedom by inserting a complete set of intermediate hadronic states with the same quantum numbers as the current operators. In this representation, the correlation function leads to a mass-sum rule, which is in turn related to the physically observed hadrons.

On the other hand, when  $p^2 \ll 0$  (space-like), the correlation function is studied in terms of the quarks and gluons and their interactions with the QCD vacuum. This is referred to as theoretical (or QCD) representation. In this representation, the Wilson operator product expansion (OPE) method is usually used to separate the short-and long-distance quark-gluon interactions, by expressing the correlation function in terms of the QCD degrees of freedom, such as quark condensate, gluon condensate, etc.

Finally, equating the two sides of these representations and applying the double Borel transformations to the momentum of the state to suppress the contribution of the higher states and continuum, the QCD sum rules for the physical observables such as masses and residues can be estimated by matching the coefficients of the same structure in both representations of the correlation function.

Consequently, we start the analysis of spin-(3/2) doubly-heavy baryon states by taking into account the two-point correlation function  $\Pi_{\mu\nu}(p^2)$  between vacuum states, with no initial and final baryons:

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ \eta_\mu(x) \bar{\eta}_\nu(0) \} | 0 \rangle, \quad (1)$$

where  $\mathcal{T}$  represents the time ordering operator,  $\eta_\mu$  is the interpolating current of the doubly-heavy baryon with spin-(3/2) that injects quarks into the QCD vacuum at point  $x$ , and  $p$  is the four-momentum of the doubly-heavy baryon.

The conventional expression of the interpolating current, which may be coupled to both the positive parity as well as to negative parity spin-(3/2) doubly-heavy baryons is chosen to be of the following form:

$$\eta_\mu = \frac{1}{\sqrt{3}} \epsilon^{abc} \left\{ (q^{aT} C \gamma_\mu Q^b) Q'^c + (q^{aT} C \gamma_\mu Q'^b) Q^c + (Q^{aT} C \gamma_\mu Q'^b) q^c \right\}, \quad (2)$$

where  $q$  denotes the light quark,  $Q$  and  $Q'$  are the two heavy quarks.  $T$  stands for transposition,  $C$  for charge conjugation,  $a$ ,  $b$ , and  $c$  are for color indices.

The systematic procedure of the QCD sum rules method is started by saturating the above correlation function with full sets of the baryon states with equal quantum numbers as the interpolating current, and by summing on for the ground state and associated excited states, we obtain the physical

part or phenomenological representation of the correlation function as:

$$\Pi^{HAD}(p^2) = \sum_{i=1S, 2S, \text{ and } 1P} \frac{\langle 0|\eta_\mu(0)|B_i(p)\rangle\langle B_i(p)|\bar{\eta}_\mu(0)|0\rangle}{p^2 - m_{B_i}^2} + \dots, \tag{3}$$

where the summation here only includes the required hadronic states produced by the quark current  $\eta_\mu$ , and... includes contributions from higher states as well as the continuum.

The matrix elements of the interpolating current evolved into Eq. (3), on the other hand, are parameterized in terms of the three pole residues for the three different baryonic states, 1S, 2S, and 1P,

$$\begin{aligned} \langle 0|\eta_\mu(0)|B_{1S}(p)\rangle &= \lambda_g u_{\mu(1S)}(p, r), \\ \langle 0|\eta_\mu(0)|B_{2S}(p)\rangle &= \lambda u_{\mu(2S)}(p, r), \\ \langle 0|\eta_\mu(0)|B_{1P}(p)\rangle &= \tilde{\lambda} \gamma_5 u_{\mu(1P)}(p, r), \end{aligned} \tag{4}$$

where the  $\lambda$ 's are the pole residues of spin-(3/2) doubly-heavy baryon states, and  $u(p, r)$  is the Rarita–Schwinger spinor satisfying the identity,

$$\begin{aligned} \sum_r u_\mu(p, r) \bar{u}_\nu(p, r) &= \sum (\not{p} + m_{B_i}) \\ &\times \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2p_\mu p_\nu}{3m_{B_i}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3m_{B_i}} \right), \end{aligned} \tag{5}$$

where,  $m_{B_{1S}} \equiv m_g$ ,  $m_{B_{2S}} \equiv m$ , and  $m_{B_{1P}} \equiv \tilde{m}$  are to be the masses of spin-(3/2) doubly-heavy baryon states, as well as,  $\lambda_{B_{1S}} \equiv \lambda_g$ ,  $\lambda_{B_{2S}} \equiv \lambda$ , and  $\lambda_{B_{1P}} \equiv \tilde{\lambda}$  are to be the pole residues of spin-(3/2) doubly-heavy baryon states.

Before going further, a word of caution is in order. The above-mentioned interpolating current  $\eta_\mu$  of Eq. (2) couples to both the spin-(1/2) and spin-(3/2) baryons with positive and negative parities. However, because we are only interested in studying spin-(3/2) doubly-heavy baryon states of both positive and negative parities in this work. Therefore, the unwanted spin-(1/2) contributions of both parities should be removed from this study. It is clear from Eq. (5) that the unwanted spin-(1/2) structures are proportional to either  $\gamma_\mu \gamma_\nu$ ,  $p_\mu p_\nu$ , and  $(p_\mu \gamma_\nu - p_\nu \gamma_\mu)$ .

Overall, Eq. (5) shows that there are only two independent Lorentz structures left,  $\not{p} g_{\mu\nu}$  and  $g_{\mu\nu}$ , which are coupled only to spin-(3/2) doubly-heavy baryons with both positive and negative parities. On the whole, the hadronic side (HAD) therefore contains the (1S + 2S + 1P) spin-(3/2) doubly-heavy baryonic states, and the excited states are considered as (2S) and (1P) states,

$$\begin{aligned} \Pi^{HAD}(p^2) &= \lambda_g^2 \left( \frac{\not{p} + m_g}{m_g^2 - p^2} \right) g_{\mu\nu} + \lambda^2 \left( \frac{\not{p} + m}{m^2 - p^2} \right) g_{\mu\nu} \\ &+ \tilde{\lambda}^2 \left( \frac{\not{p} - \tilde{m}}{\tilde{m}^2 - p^2} \right) g_{\mu\nu} + \dots \end{aligned} \tag{6}$$

In this regard, the operator product expansion approach (OPE) in the deep Euclidean region  $-p^2 \ll m_Q^2$  on the other hand is used to determine the QCD side of the correlation function. The relevant correlation function in terms of the heavy (light) quark propagator may be obtained by putting the explicit expressions of the interpolating currents into the correlation function of Eq. (1) and then, by doing some contractions of all quark fields through the Wicks theorem, we obtain:

$$\begin{aligned} \Pi_{\mu\nu}^{QCD}(p^2) &= \frac{1}{3} \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{ip \cdot x} \langle 0 | \\ &\times \left\{ -S_Q^{cb'} \gamma_\nu \tilde{S}_Q^{aa'} \gamma_\mu S_q^{bc'} - S_Q^{ca'} \gamma_\nu \tilde{S}_q^{bb'} \gamma_\mu S_Q^{ac'} \right. \\ &- S_Q^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_q^{ac'} \\ &- S_Q^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_q^{ac'} - S_q^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_Q^{ac'} \\ &- S_q^{cb'} \gamma_\nu \tilde{S}_Q^{aa'} \gamma_\mu S_Q^{bc'} - S_Q^{cc'} Tr \left[ S_Q^{ba'} \gamma_\nu \tilde{S}_q^{ab'} \gamma_\mu \right] \\ &- S_q^{cc'} Tr \left[ S_Q^{ba'} \gamma_\nu \tilde{S}_Q^{ab'} \gamma_\mu \right] \\ &\left. - S_Q^{cc'} Tr \left[ S_q^{ba'} \gamma_\nu \tilde{S}_Q^{ab'} \gamma_\mu \right] \right\} |0\rangle, \end{aligned} \tag{7}$$

where  $\tilde{S}_{Q(q)}^{ij} \equiv C S_{Q(q)}^{ijT} C$ ,  $S_{Q(q)}^{ij}$  is the heavy (light) quark propagator in the coordinate space, and the explicit mathematical expressions of the heavy (light) quark propagators are,

$$\begin{aligned} S_Q(x) &= \frac{m_Q^2}{4\pi^2} \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} - \frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q \sqrt{-x^2}), \\ S_q(x) &= i \frac{\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12} \left( 1 - i \frac{m_q \not{x}}{4} \right) \\ &- \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left( 1 - i \frac{m_q \not{x}}{6} \right), \end{aligned} \tag{8}$$

where  $K_1$  and  $K_2$  are the modified Bessel functions of the second kind,  $\langle \bar{q}q \rangle$  is the quark condensates, and  $m_0^2$  is the conventional parametrization constant in the quark-gluon condensates.

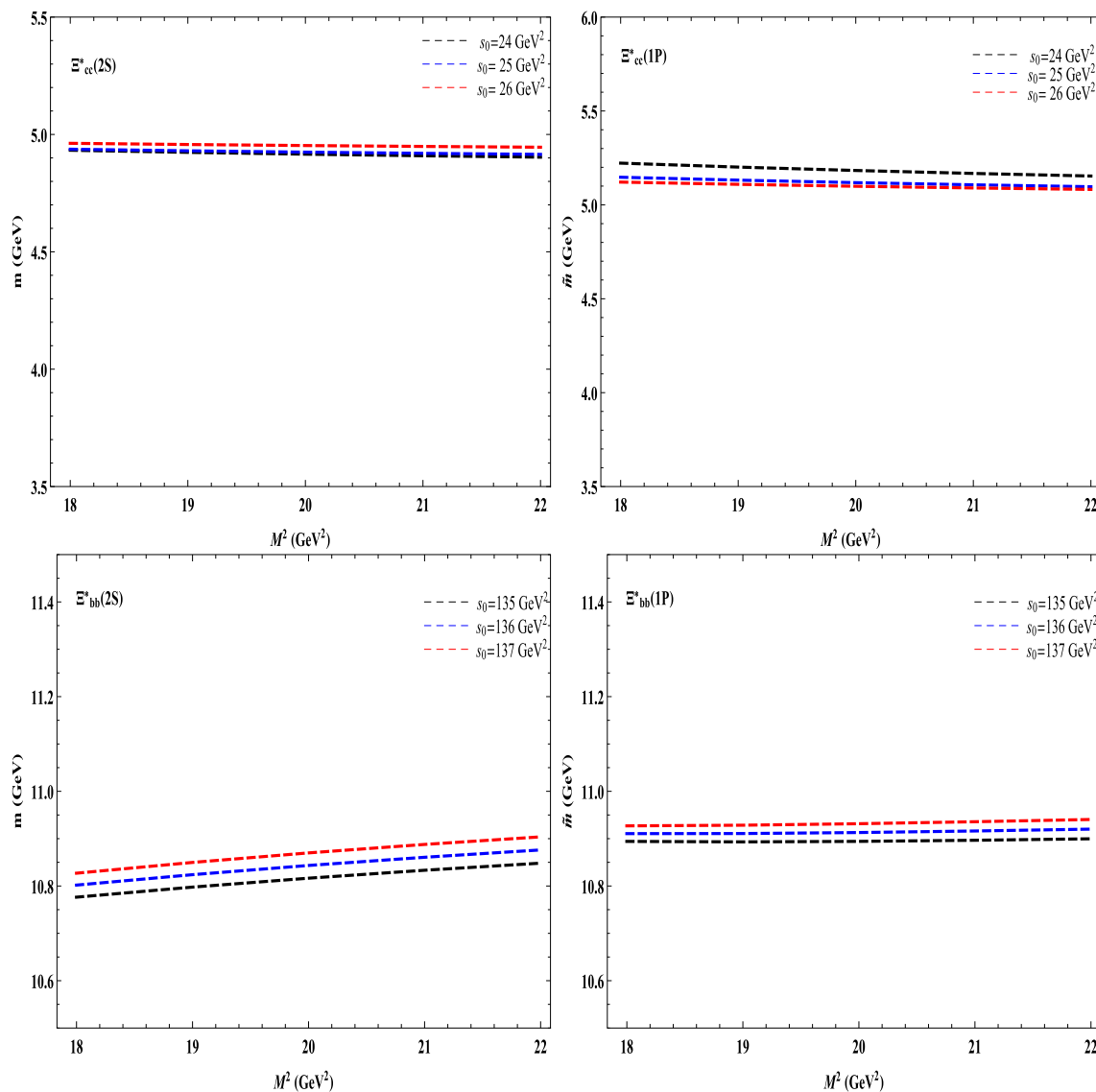
At this point, we make the necessary substitutions of the heavy (light) quark propagators into Eq. (7), the two-point correlation function can be then re-expressed in terms of some spectral densities  $\rho(s)$ 's with the two independent structures  $\not{p} g_{\mu\nu}$  and  $g_{\mu\nu}$ , respectively,

$$\Pi^{QCD}(p^2) = \Pi_1^{QCD}(p^2) \not{p} g_{\mu\nu} + \Pi_2^{QCD}(p^2) g_{\mu\nu}, \tag{9}$$

where,

$$\begin{aligned} \Pi_1^{QCD}(p^2) &= \int_{s_{min}}^{s_0} \frac{\rho_1(s)}{s - p^2} ds, \\ \Pi_2^{QCD}(p^2) &= \int_{s_{min}}^{s_0} \frac{\rho_2(s)}{s - p^2} ds. \end{aligned} \tag{10}$$

The spectral densities  $\rho_1(s)$  and  $\rho_2(s)$  in Eq. (10) are obtained from the imaginary part of the correlator function;  $\rho_i(s) =$



**Fig. 1** The dependency of the excited states 2S, 1P masses of the doubly-heavy spin-(3/2)  $\Xi_{cc}^*$ , and  $\Xi_{bb}^*$  baryons on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

$\frac{1}{\pi} \text{Im}\Pi_i(s)$ , where  $i = 1(2)$  is related to the structures  $\not{p}g_{\mu\nu}$  and  $g_{\mu\nu}$ . Further,  $s_0$  is the continuum threshold, which is not fully arbitrary and it is associated with the energy of the excited states,  $s_{min} = (m_Q + m_{Q'})^2$ , and finally, the spectral densities  $\rho_1(s)$  and  $\rho_2(s)$  are written as (see also [44]):

$$\rho_1(s) = \frac{1}{32\pi^4} \int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \left\{ \alpha \left[ 3xy(x+y)\alpha - 4(1-x-y)m_{Q'}m_Q - 4m_q(xm_{Q'} + ym_Q) \right] - \frac{\langle \bar{q}q \rangle}{24\pi^2} \int_{x_{min}}^{x_{max}} dx \left[ (1-x)(3xm_q - 4m_Q) - 4xm_{Q'} \right] \right\}$$

$$\rho_2(s) = \frac{1}{16\pi^4} \int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \left\{ \alpha \left[ (x+y)\alpha(xm_{Q'} + ym_Q) \right. \right.$$

(11)

$$\left. -m_q(xy\alpha + 3m_Qm_{Q'}) \right\} + \frac{\langle \bar{q}q \rangle}{12\pi^2} \int_{x_{min}}^{x_{max}} dx \times \left\{ (1-x) \left[ 2x(m_Q^2 + \beta - s) - m_Qm_q \right] + (xm_q + 3m_Q)m_{Q'} \right\},$$

(12)

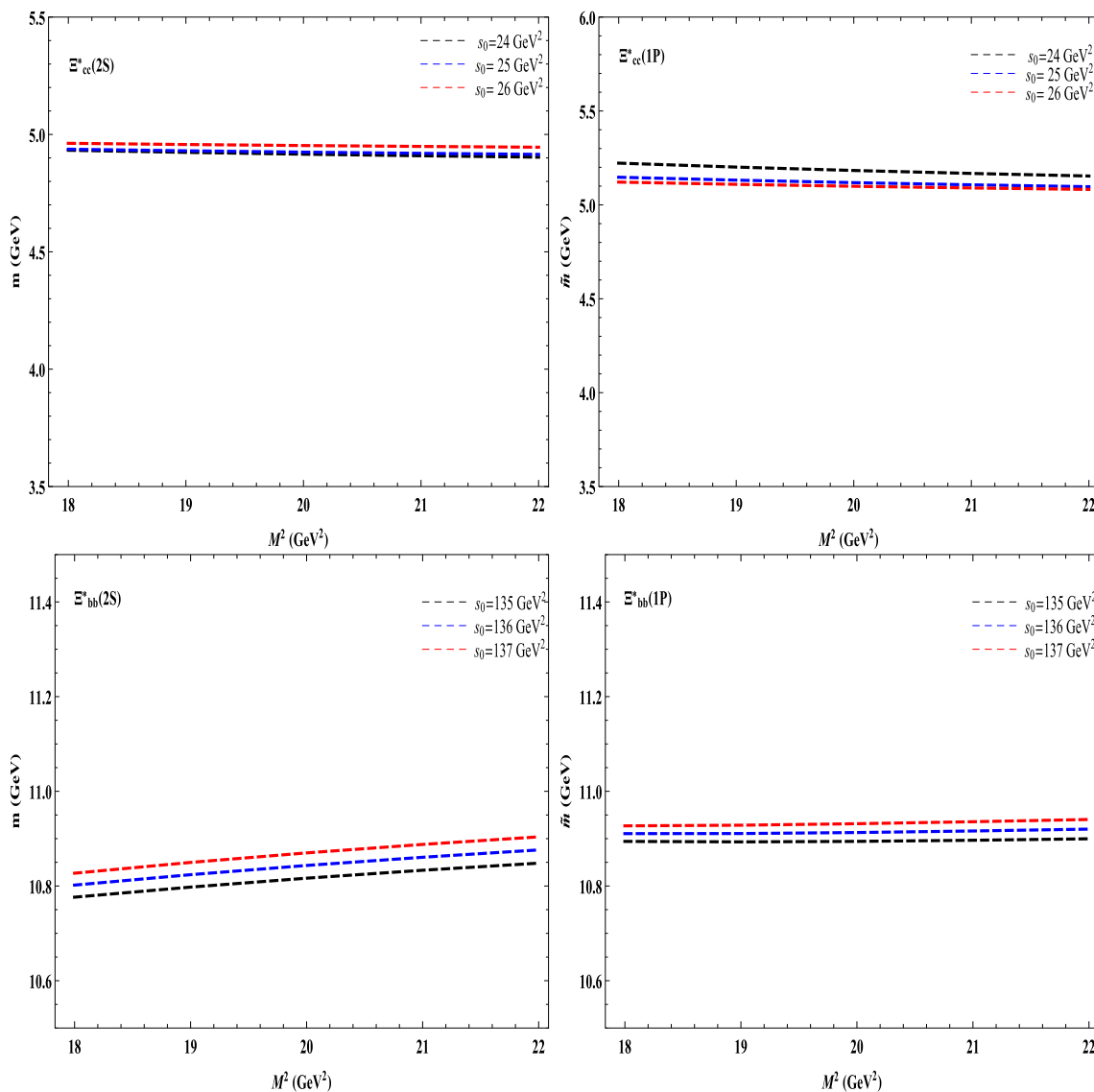
where,

$$\alpha = \frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y} - s,$$

$$\beta = \frac{m_Q^2}{x} + \frac{m_{Q'}^2}{(1-x)} - s,$$

$$y_{min} = \frac{xm_{Q'}^2}{sx - m_Q^2},$$

$$y_{max} = 1 - x,$$



**Fig. 2** The dependency of the excited states  $2S, 1P$  masses, and pole residues of the doubly-heavy of spin- $(3/2)$   $\Xi_{cc}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

$$x_{min} = \frac{1}{2s} \left[ s + m_Q^2 - m_{Q'}^2 - \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right],$$

$$x_{max} = \frac{1}{2s} \left[ s + m_Q^2 - m_{Q'}^2 + \sqrt{(s + m_Q^2 - m_{Q'}^2)^2 - 4m_Q^2 s} \right].$$

After calculating both the HAD and QCD sides of the correlation function, we matched the coefficients of the same structures from both sides of Eqs. (6) and (9), and finally applying the Borel transformation to the variable  $p^2$  to eliminate the subtraction terms and suppress the contributions from higher states and the continuum. As a result, the desired sum rules

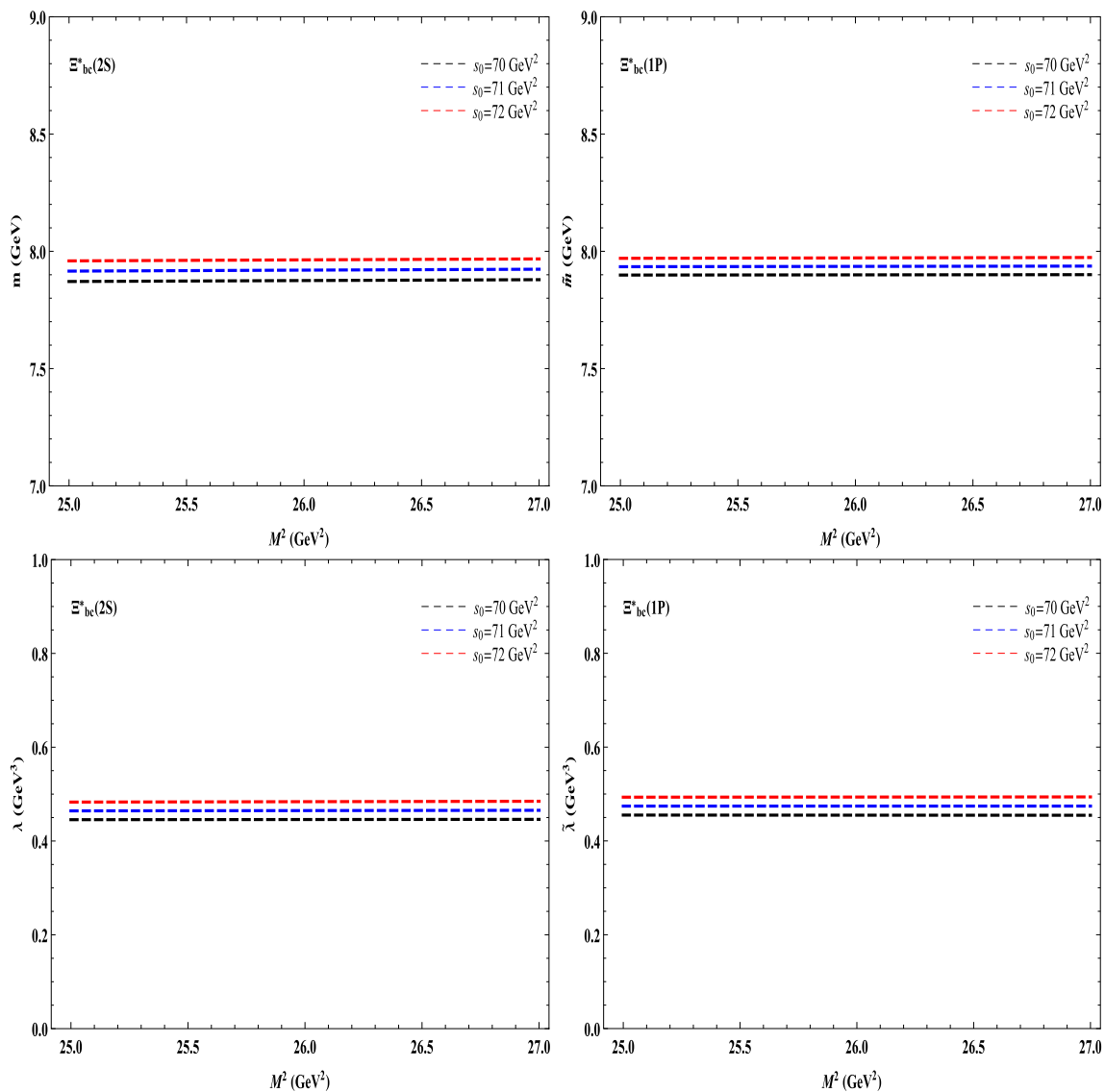
for the states  $1S + 2S + 1P$  are obtained:

$$\lambda_g^2 e^{-m_g^2/M^2} + \lambda^2 e^{-m^2/M^2} + \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} = \Pi_1^{QCD}(M^2),$$

$$m_g \lambda_g^2 e^{-m_g^2/M^2} + m \lambda^2 e^{-m^2/M^2} - \tilde{m} \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} = \Pi_2^{QCD}(M^2), \tag{13}$$

where  $M^2$  is the Borel mass parameter, and  $\Pi_1^{QCD}(M^2)$  and  $\Pi_2^{QCD}(M^2)$  are the invariant functions of  $\not{p}g_{\mu\nu}$  and  $g_{\mu\nu}$  structures, respectively.

The derived sum rules in Eq. (13) contain six unknown parameters for the ground state and excited state baryons. However, Eq. (13) is broken down into two sum rule equations, each containing six unknowns: three masses and three pole residues. To solve this system of equations linearly,



**Fig. 3** The dependency of the excited states  $2S, 1P$  masses, and pole residues of the doubly-heavy of spin-(3/2)  $\Xi_{bc}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

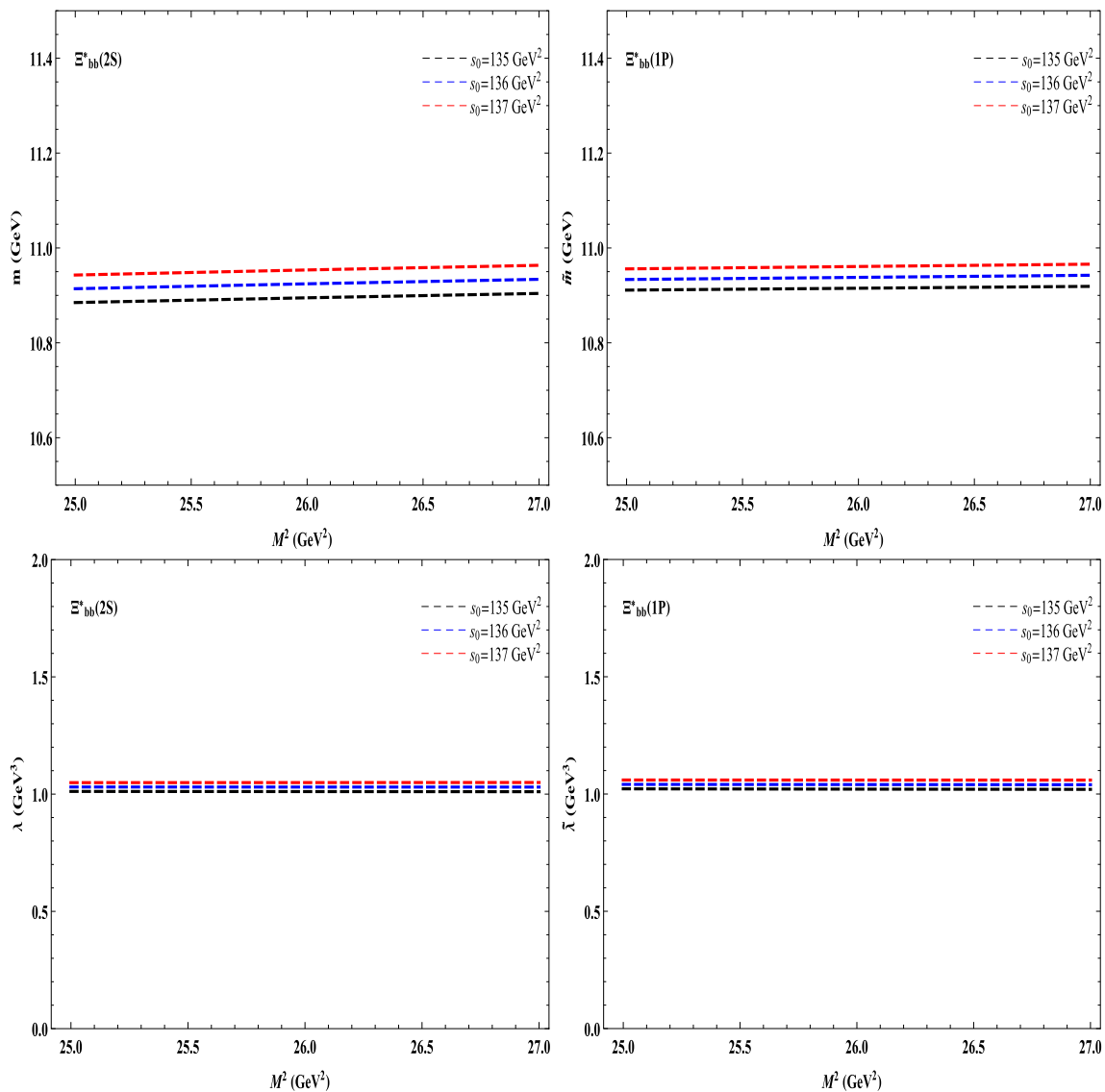
we'll need four more equations. The remaining four equations can be found by differentiating all sides of the sum rules in Eq. (13) with respect to  $\frac{d}{d(-1/M^2)}, \frac{d^2}{d^2(-1/M^2)}$ , and thus the six equations with the six unknowns are obtained simultaneously,

$$\begin{aligned}
 & m_g^2 \lambda_g^2 e^{-m_g^2/M^2} + m^2 \lambda^2 e^{-m^2/M^2} + \tilde{m}^2 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 &= \Pi_1'^{QCD}(M^2), \\
 & m_g^3 \lambda_g^2 e^{-m_g^2/M^2} + m^3 \lambda^2 e^{-m^2/M^2} - \tilde{m}^3 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 &= \Pi_2'^{QCD}(M^2), \\
 & m_g^4 \lambda_g^2 e^{-m_g^2/M^2} + m^4 \lambda^2 e^{-m^2/M^2} + \tilde{m}^4 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 &= \Pi_1''^{QCD}(M^2),
 \end{aligned}$$

$$\begin{aligned}
 & m_g^5 \lambda_g^2 e^{-m_g^2/M^2} + m^5 \lambda^2 e^{-m^2/M^2} - \tilde{m}^5 \tilde{\lambda}^2 e^{-\tilde{m}^2/M^2} \\
 &= \Pi_2''^{QCD}(M^2),
 \end{aligned} \tag{14}$$

where,  $\Pi_i'^{QCD}(M^2) = \frac{d\Pi_i^{QCD}(M^2)}{d(-1/M^2)}$ , and  $\Pi_i''^{QCD}(M^2) = \frac{d^2\Pi_i^{QCD}(M^2)}{d^2(-1/M^2)}$ .

The whole set of six equations above makes the system's solution extremely difficult. Rather, to solve this system of equations, we picked up an appropriate value of  $s_0$  at which the physical part of the correlation function is saturating only the ground state, and the other states,  $2S$  and  $1P$  are still remain in the continuum zoom. To put it differently, the initial guess of  $s_0$  solved the system of equations and reproduced



**Fig. 4** The dependency of the excited states  $2S$ ,  $1P$  masses, and pole residues of the doubly-heavy of spin-(3/2)  $\Xi_{bb}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

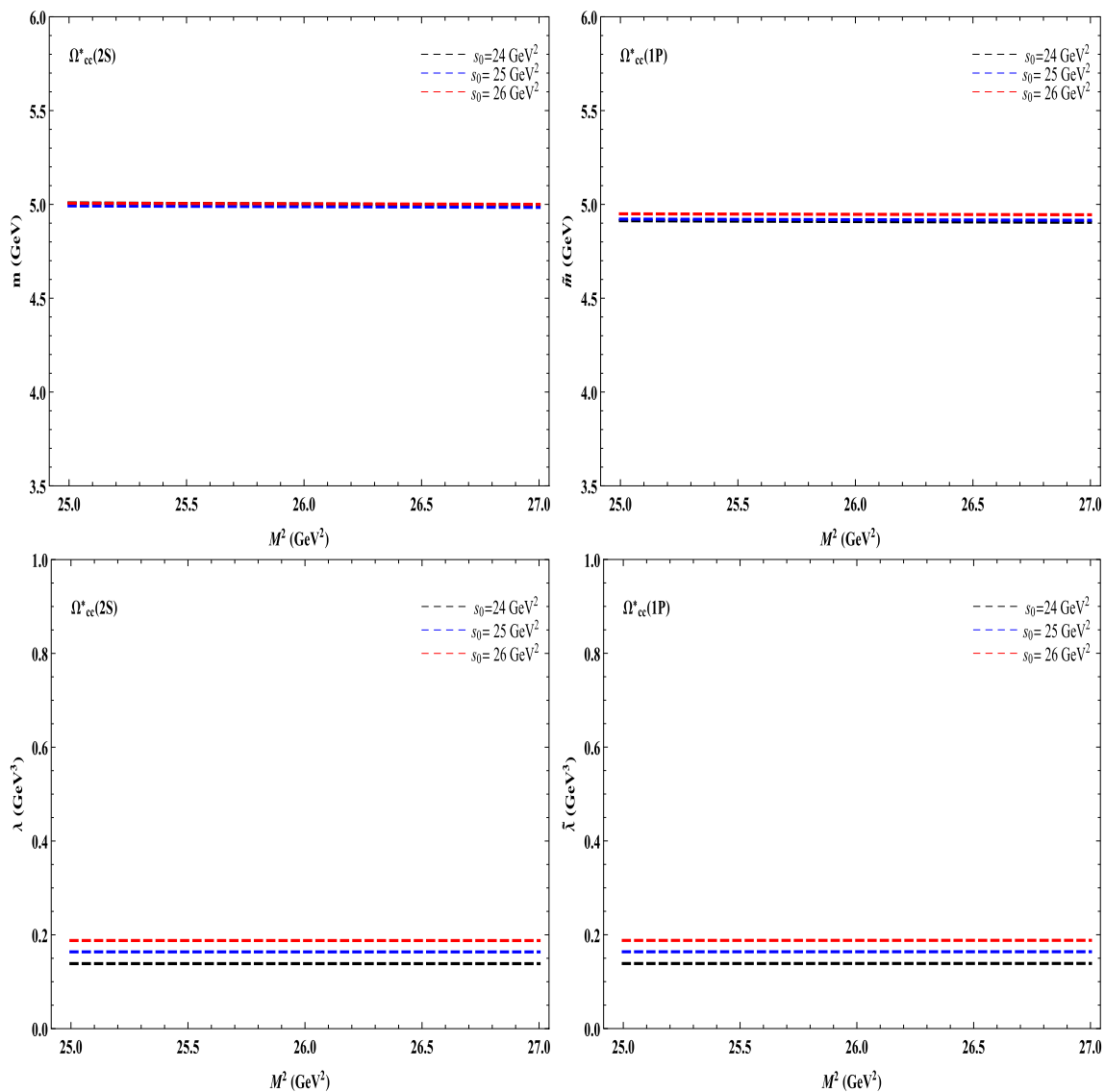
only the ground state masses and the pole residues of the spin-(3/2) doubly-heavy baryons  $m_g$  and  $\lambda_g$ .

In this regard, we chose the initial  $s_0$  values:  $s_0 = (22-26) \text{ GeV}^2$  for the  $cc$ ,  $s_0 = (68-72) \text{ GeV}^2$  for  $bc$ , and  $s_0 = (133-137) \text{ GeV}^2$  for  $bb$  doubly-heavy baryons. At the same time, we allowed the Borel mass parameter  $M^2$  to be varied from  $5 \text{ GeV}^2$  up to  $30 \text{ GeV}^2$ . From there, we can see that the values of the ground state masses ( $m_g$ ) and the corresponding pole residues ( $\lambda_g$ ) of spin-(3/2) doubly-heavy baryons  $\Xi_{cc}^*$ ,  $\Xi_{bb}^*$ ,  $\Xi_{bc}^*$ ,  $\Omega_{cc}^*$ ,  $\Omega_{bb}^*$ , and  $\Omega_{bc}^*$  have begun to appear and converge quickly within  $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ . The obtained results for ( $m_g$ ) and ( $\lambda_g$ ) with the corresponding  $s_0$  values are also given in [44].

By doing so, we reduced the number of equations and the number of unknowns to four: two excited state masses  $m, \tilde{m}$ ,

and two excited state pole residues  $\lambda, \tilde{\lambda}$ , while we are still keeping the states  $2S$  and  $1P$  in the continuum zoom. To put it differently, we will solve the resulting four equations algebraically in terms ( $m_g$ ) and ( $\lambda_g$ ).

While solving the remaining four equations algebraically, we gradually increased the values of continuum zoom  $s_0$ , and we are still keeping the Borel mass parameter  $M^2$  running within the interval,  $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ . Bearing in mind, the results of the excited state masses and the excited state pole residues of the spin-(3/2) doubly-heavy baryons in Eq. (14) should also not depend on the Borel mass parameter  $M^2$ . In fact, within the interval,  $5 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$  we observed that there are different working regions where the results are supposedly acceptable and depend only moderately on the Borel mass parameter. For example, in Fig. 1,



**Fig. 5** The dependency of the excited states  $2S$ ,  $1P$  masses, and pole residues of the doubly-heavy of spin-(3/2)  $\Omega_{cc}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

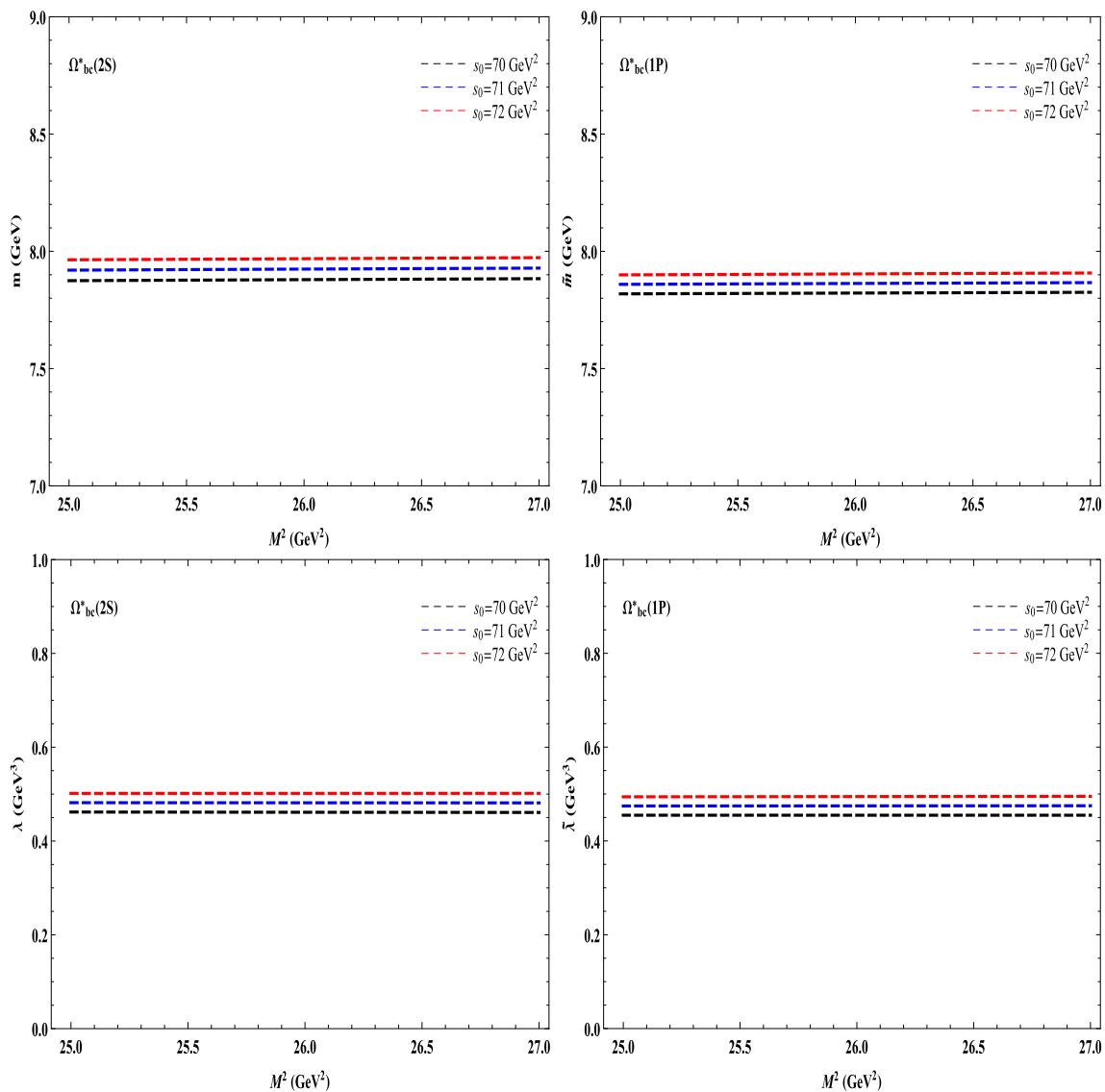
if we set the working region to be varied in the interval  $18 \text{ GeV}^2 \leq M^2 \leq 22 \text{ GeV}^2$ , clearly from this figure, we can still see that the excited state masses are rather stable with the variations of the Borel mass parameter.

However, we noticed that the excited state masses and the corresponding pole residues of the spin-(3/2) doubly-heavy baryons began to appear and rapidly converge within the interval  $25 \text{ GeV}^2 \leq M^2 \leq 27 \text{ GeV}^2$ . The obtained results for the excited states  $2S$  and  $1P$  are presented in a series of Figs. 2, 3, 4, 5, 6 and 7 with respect to  $M^2$ , along with the appropriate  $s_0$  values, so that the reader might, if so motivated, repeat our calculations.

### 3 Numerical analysis

Using the QCD sum rules Eqs. (13) and (14), the masses and pole residues of the excited states of spin-(3/2) doubly-heavy baryons  $\Xi_{cc}^*$ ,  $\Xi_{bb}^*$ ,  $\Xi_{bc}^*$ ,  $\Omega_{cc}^*$ ,  $\Omega_{bb}^*$ , and  $\Omega_{bc}^*$  are calculated. Usually, in numerical calculations, the QCD sum rules approach involves a set of fixed input parameters. We used  $\overline{MS}$  values for the quark masses:  $\bar{m}_b(\bar{m}_b) = (4.16 \pm 0.03) \text{ GeV}$ ,  $\bar{m}_c(\bar{m}_c) = (1.28 \pm 0.03) \text{ GeV}$ , and  $m_s(2 \text{ GeV}) = (102 \pm 8) \text{ MeV}$  [46,47]. On the other hand, the values of the quark condensates are taken as  $\langle \bar{u}u \rangle(1 \text{ GeV}) = \langle \bar{d}d \rangle(1 \text{ GeV}) = -(246_{-19}^{+28} \text{ MeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$ , and  $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$  [48]. Moreover, the QCD sum rules technique also includes two auxiliary parameters whose





**Fig. 6** The dependency of the excited states  $2S$ ,  $1P$  masses, and pole residues of the doubly-heavy of spin- $(3/2)$   $\Omega_{bc}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

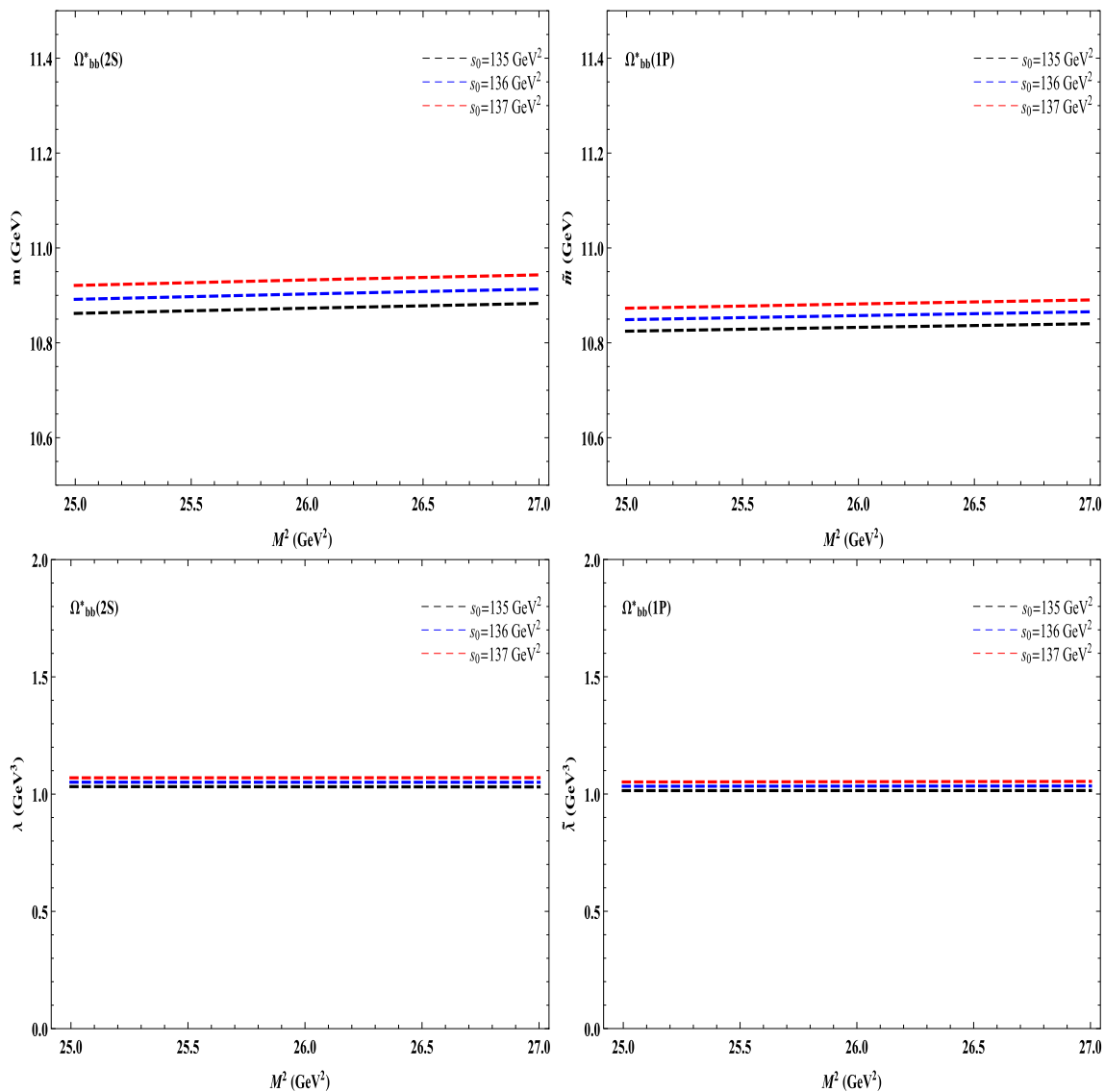
working regions must be determined, specifically, the continuum threshold  $s_0$ , and the Borel parameter  $M^2$ .

The physical working regions for these parameters should, however, be managed in such a way that the involved physical quantities are roughly independent of these parameters. The working regions of  $s_0$  and  $M^2$  are obtained by the sum rules analysis using the conventional criteria of the QCD sum rules approach, means that, the perturbative contributions must be greater than the condensate contributions, and the pole contributions must be greater than the continuum contributions. As a result, the bounds of  $M^2$  are obtained such that  $M^2$  must be large enough to satisfy the convergence criterion of OPE. Therefore, the working regions of the continuum threshold  $s_0$  and the Borel mass  $M^2$  must be found with the condition

that the masses and pole residues are stable with respect to variations of these parameters.

In Figs. 2, 3, 4, 5, 6 and 7, we showed our numerical findings for the masses and the pole residues of spin- $(3/2)$  doubly-heavy excited state baryons. The masses and the pole residues of the excited states  $2S$  and  $1P$  of spin- $(3/2)$  doubly-heavy baryons are represented as a function of  $M^2$  for different values of  $s_0$ . The expected masses and the pole residues of the excited states  $2S$  and  $1P$  showed good stability with respect to the obtained physical working regions of  $M^2$ , and  $s^0$ .

As discussed in the previous section, according to our findings, these requirements are met when  $M^2$  is changed from  $25 \text{ GeV}^2 \leq M^2 \leq 27 \text{ GeV}^2$  for all the considered spin- $(3/2)$  doubly-heavy baryons, while for example in Fig. 1,  $s_0$  is modified in the range:  $s_0 = (22 - 26) \text{ GeV}^2$  to saturate



**Fig. 7** The dependency of the excited states 2S, 1P masses, and pole residues of the doubly-heavy of spin-(3/2)  $\Omega_{bb}^*$  baryon on the Borel mass  $M^2$  at different values of the continuum threshold  $s_0$

**Table 1** The masses  $m$  and  $\tilde{m}$  of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons  $\Xi_{cc}^*$ ,  $\Xi_{bc}^*$ , and  $\Xi_{bb}^*$  (in units of GeV)

Baryons	$\Xi_{cc}^*$		$\Xi_{bc}^*$		$\Xi_{bb}^*$	
$J^P$	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	$4.95 \pm 0.05$	$5.15 \pm 0.05$	$7.91 \pm 0.05$	$7.95 \pm 0.05$	$10.92 \pm 0.025$	$10.95 \pm 0.025$
Ref. [49]	3.99	3.85	7.27	7.15	10.62	10.51

**Table 2** The masses  $m$  and  $\tilde{m}$  of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons  $\Omega_{cc}^*$ ,  $\Omega_{bc}^*$ , and  $\Omega_{bb}^*$  (in units of GeV)

Baryons	$\Omega_{cc}^*$		$\Omega_{bc}^*$		$\Omega_{bb}^*$	
$J^P$	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	$5.00 \pm 0.05$	$4.95 \pm 0.05$	$7.90 \pm 0.05$	$7.85 \pm 0.05$	$10.90 \pm 0.025$	$10.85 \pm 0.025$
Ref. [50]	4.10	3.97	7.48	7.37	10.74	10.64

**Table 3** The pole residues  $\lambda$  and  $\tilde{\lambda}$  of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons  $\Xi_{cc}^*$ ,  $\Xi_{bc}^*$ , and  $\Xi_{bb}^*$  (in units of  $\text{GeV}^3$ )

Baryons	$\Xi_{cc}^*$		$\Xi_{bc}^*$		$\Xi_{bb}^*$	
$J^P$	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	$0.175 \pm 0.025$	$0.185 \pm 0.025$	$0.475 \pm 0.025$	$0.480 \pm 0.025$	$1.050 \pm 0.05$	$1.060 \pm 0.05$

**Table 4** The pole residues  $\lambda$  and  $\tilde{\lambda}$  of the excited states (2S) and (1P) of spin-(3/2) doubly-heavy baryons  $\Omega_{cc}^*$ ,  $\Omega_{bc}^*$ , and  $\Omega_{bb}^*$  (in units of  $\text{GeV}^3$ )

Baryons	$\Omega_{cc}^*$		$\Omega_{bc}^*$		$\Omega_{bb}^*$	
$J^P$	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)	$\frac{3}{2}^+$ (2S)	$\frac{3}{2}^-$ (1P)
Our work	$0.160 \pm 0.025$	$0.165 \pm 0.025$	$0.475 \pm 0.025$	$0.480 \pm 0.025$	$1.080 \pm 0.05$	$1.075 \pm 0.05$

the masses and the pole residues of  $\Xi_{cc}^*(2S)$ , and  $\Xi_{cc}^*(1P)$  baryon excited states.

Also, for example in Figs. 2 and 3, considering the heaviest baryons, namely the doubly-heavy baryons including the heavy quark ( $b$ ), the values of the continuum zoom  $s_0$  should be increased to a sufficiently value in order to saturate again the excited states  $2S$  and  $1P$ .

Finally, in Tables 1, 2, 3 and 4 we displayed our predictions along with the other theoretical approaches for comparison. Our findings are in line with the findings of other methodologies.

### 4 Conclusion

In the context of the QCD sum rules method, we numerically calculated the excited state masses and the pole residues  $2S$  and  $1P$  of spin-(3/2) doubly-heavy  $\Xi_{cc}^*$ ,  $\Xi_{bb}^*$ ,  $\Xi_{bc}^*$ ,  $\Omega_{cc}^*$ ,  $\Omega_{bb}^*$ , and  $\Omega_{bc}^*$  baryons.

The originality of the present work is that the contributions of  $1S$ ,  $2S$ , and  $1P$  states in Eqs. (13) and (14) are considered simultaneously.

On the whole, there should be a total of six unknowns: three masses and three pole residues. However, we only have two equations for two independent Lorentz structures  $\not{p}g_{\mu\nu}$  and  $g_{\mu\nu}$ . So, we get the remaining four equations by doing a differentiation for all sides of the sum rule equations with respect to  $\frac{d}{d(-1/M^2)}$ ,  $\frac{d^2}{d^2(-1/M^2)}$ , and then we solve these equations simultaneously.

We have used a slightly different method to solve this system of equations. We set a suitable value of  $s_0$  and then we saturated the physical part of the correlation function with ground state contributions only. The spectroscopic parameters were then used as input parameters for the remaining set of equations. As a result, we have worked out a system of four equations as an alternative to six equations. Then, we have calculated the masses and the pole residues of the excited states  $2S$  and  $1P$ .

From figures, we have noticed that the masses of the  $1P$  states is slightly exceeding the masses of the  $2S$  states in the doubly-heavy baryons that are containing one light ( $u$  or  $d$ ) quarks. Whereas, the masses of the  $1P$  states is slightly less than the masses of the  $2S$  states in the doubly-heavy baryons that are containing the strange quark. On the other hand, from figures, if one look at the values of the pole residues of the  $2S$  states and that of the  $1P$  states in the same region of  $s_0$  values, one can notice that the values of the pole residues are of the same order to each other, and this is due to the reason that we have mentioned in Sect. 2. That is, the interpolating current in this work have been chosen in such a way that this current can couple to doubly-heavy baryons of both positive and negative parities.

By comparing the QCD sum rules results in this work with the results of the hyper-central constituent quark models [49, 50], we have noticed that the  $2S$  and  $1P$  mass states of the doubly-heavy baryons are exceeding the mass states of the doubly-heavy baryons obtained within the potential models. Finally, we hope that future LHC experimental results can be compared to the current estimates.

**Acknowledgements** This research was funded by Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2022R106), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors' comment: All data underlying the results are available as part of the article and no additional source data are required.]

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copy-

right holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.  
Funded by SCOAP<sup>3</sup>.

## References

1. R. Aaij et al. (LHCb), Phys. Rev. Lett. **118**, 182001 (2017)
2. R. Aaij et al. (LHCb), Phys. Rev. Lett. **119**, 112001 (2017)
3. R. Aaij et al. (LHCb), Phys. Rev. Lett. **121**, 052002 (2018)
4. R. Aaij et al. (LHCb), Phys. Rev. Lett. **121**, 072002 (2018)
5. R. Aaij et al. (LHCb), Phys. Rev. Lett. **122**, 012001 (2019)
6. R. Aaij et al. (LHCb), Phys. Rev. Lett. **123**, 152001 (2019)
7. A.M. Sirunyan et al. (CMS), Phys. Lett. B **803**, 135345 (2020)
8. R. Aaij et al. (LHCb), JHEP **06**, 136 (2020)
9. R. Aaij et al. (LHCb), Phys. Rev. Lett. **124**, 082002 (2020)
10. A.M. Sirunyan et al. (CMS), Phys. Rev. Lett. **122**, 021801 (2019)
11. R. Aaij et al. (LHCb), JHEP **05**, 161 (2016)
12. R. Aaij et al. (LHCb), Phys. Rev. Lett. **119**, 062001 (2017)
13. R. Aaij et al. (LHCb), Phys. Rev. D **99**, 052006 (2019)
14. P.A. Zyla et al. (Particle Data Group), PTEP **2020**, 083C01 (2020)
15. H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. **80**, 076201 (2017)
16. M. Mattson et al. (SELEX Collaboration), Phys. Rev. Lett. **89**, 112001 (2002)
17. A. Ocherashvili et al. (SELEX Collaboration), Phys. Lett. B **628**, 18 (2005)
18. R. Aaij et al. (LHCb), JHEP **11**, 095 (2020)
19. W. Roberts, M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008)
20. A.P. Martynenko, Phys. Lett. B **663**, 317 (2008)
21. J.D. Bjorken, Preprint FERMILAB-Conf-85-069 (1985)
22. P. Hasenfratz, R.R. Horgan, J. Kuti, J.M. Richard, Phys. Lett. B **94**, 401 (1980)
23. N. Brambilla, T. Roesch, A. Vairo, Phys. Rev. D **72**, 034021 (2005)
24. J.M. Flynn, E. Hernández, J. Nieves, Phys. Rev. D **85**, 014012 (2012)
25. Y. Jia, JHEP **10**, 073 (2006)
26. J.R. Zhang, M.Q. Huang, Phys. Lett. B **674**, 28 (2009)
27. T.W. Chiu, T.H. Hsieh, Nucl. Phys. A **755**, 471 (2005)
28. S. Meinel, Phys. Rev. D **82**, 114514 (2010)
29. M. Padmanath, R.G. Edwards, N. Mathur, M. Peardon, Phys. Rev. D **91**, 094502 (2015)
30. Z.-F. Sun, Z.-W. Liu, X. Liu, S.-L. Zhu, Phys. Rev. D **91**, 094030 (2015)
31. H.I. Alrebdi, T.M. Aliev, K. Şimşek, Phys. Rev. D **102**, 074007 (2020)
32. K. Azizi, A.R. Olamaei, S. Rostami, Eur. Phys. J. C **80**, 1196 (2020)
33. T.M. Aliev, T. Barakat, K. Şimşek, Eur. Phys. J. A **57**, 160 (2021)
34. F.-S. Yu, H.-Y. Jiang, R.-H. Li, C.-D. Lü, W. Wang, Z.-X. Zhao, Chin. Phys. C **42**, 051001 (2018)
35. S. Rahmani, H. Hassanabadi, H. Sobhani, Eur. Phys. J. C **80**, 31 (2020)
36. D.-L. Yao, Phys. Rev. D **97**, 034012 (2018)
37. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385 (1979)
38. E. Bagan, M. Chabab, S. Narison, Phys. Lett. B **306**, 350 (1993)
39. J.R. Zhang, M.Q. Huang, Phys. Rev. D **78**, 094007 (2008)
40. Z.G. Wang, Eur. Phys. J. A **45**, 267 (2010)
41. T.M. Aliev, K. Azizi, M. Savci, JHEP **04**, 042 (2013)
42. Z.G. Wang, Eur. Phys. J. C **68**, 459 (2010)
43. N. Alomayrah, T. Barakat, Eur. Phys. J. A **56**, 76 (2020)
44. T.M. Aliev, K. Azizi, M. Savci, J. Phys. G **40**, 065003 (2013)
45. Z.G. Wang, Eur. Phys. J. A **47**, 81 (2011)
46. A. Khodjamirian, Ch. Klein, Th. Mannel, Y.M. Wang, JHEP **1109**, 106 (2011)
47. C.A. Dominguez, Mod. Phys. Lett. A **26**, 697 (2011)
48. A. Khodjamirian, Ch. Klein, Th. Mannel, N. Offen, Phys. Rev. D **80**, 114005 (2009)
49. Z. Shah, K. Thakkar, A.K. Rai, P.C. Vinodkumar, Eur. Phys. J. C **77**, 129 (2017)
50. Z. Shah, K. Thakkar, A.K. Rai, P.C. Vinodkumar, Eur. Phys. J. C **76**, 530 (2016)