



New Tsallis holographic dark energy

Bramha Dutta Pandey^{1,2,a}, P. Suresh Kumar^{1,2,b}, Pankaj^{3,c}, Umesh Kumar Sharma^{1,d}

¹ Department of Mathematics, Institute of Applied Sciences and Humanities, GLA University Mathura, Mathura, Uttar Pradesh 281406, India

² IT Department (Math Section), University of Technology and Applied Sciences-Salalah, Salalah, Oman

³ IT Department (Math Section), University of Technology and Applied Sciences-HCT, Muscat, Oman

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Abstract Tsallis entropy is a generalization of the Boltzmann–Gibbs entropy in statistical theory which uses a parameter δ to measure the deviation from the standard scenario quantitatively. Using concepts of Tsallis entropy and future event horizon, we construct a new Tsallis holographic dark energy model. The parameters c and δ will be used to characterize various aspects of the model. Analytical expressions for various cosmological parameters such as the differential equation describing the evolution of the effective dark energy density parameter, the equation of state parameter and the deceleration parameter are obtained. The equation of state parameter for the current model exhibits the pure quintessence behaviour for $c > 1$, quinton behaviour for $c < 1$ whereas the Λ CDM model is recovered for $c = 1$. To analyze the thermal history of the universe, we obtained the expression for the deceleration parameter and found that for $z \approx 0.6$, the phase transits from deceleration to acceleration.

1 Introduction

Based on the observations carried out by Reiss and Perlmutter [1, 2] the current universe is in an accelerated expansion phase. The reason behind it can be considered as the existence of cosmological constant Λ . But the dynamic behaviour of Λ and computing the value of it in a quantum field theoretic way opens the paths of extended scenarios. One of which is to keep general relativity based on gravitation theory while considering new, exotic matter, which explains the dark energy (DE) concept [3–5]. The other is to extend the theory of gravity whose special case is general relativity with extended degree of freedom to get the explanation about the accelerated universe [6–9].

Using the holographic Principle (HP), one intriguing possibility for explaining the genesis and nature of dark energy can be obtained at a cosmological framework [10–12]. G. 't Hooft [10] presented the well-known concept of holographic principle based on investigations of black hole thermodynamics [13, 14]. This asserts that a hologram corresponding to a theory on the volume's border can be used to represent the whole information contained in a spatial volume. HP was used to solve the DE problem by proposing the concept of holographic dark energy (HDE) model [15]. According to HDE model, on the universe's edge, the reduced Plank's mass $M_p \equiv \frac{1}{\sqrt{8\pi G}}$, where G denotes the universal gravitational constant of Newton and universe's future event horizon L [15] are the physical quantities on which the dark energy density ρ_d depends. The DE model equipped with HP (HDE) supports the current cosmological observations [15–21] and extensively studied [22–26]. Also, observational data are in agreement with the concept of HDE [27–31].

Due to the long-range nature of gravity and the unpredictable structure of spacetime, many extended entropy formalisms have been employed to investigate gravitational and cosmic phenomena. In order to study gravitational and cosmic systems through the concepts of generalized statistical mechanics, the Tsallis's entropy [32, 33] plays a central role. Kaniadakis, on the other hand, presented generalized Boltzmann–Gibbs entropy through single parameter, known as Kaniadakis entropy [34, 35] and studied by Niki and Sharma [36, 37] using the concepts of future event horizon and apparent horizon respectively. This is the consequence of a unified and self-consistent relativistic statistical theory that retains the core properties of normal statistical theory. The usual Maxwell–Boltzmann statistics is continuously deformed by a single parameter leading to the extended statistical theory, whose limiting case is the standard statistical theory. In this manuscript we will apply the

^ae-mail: bdpandey05@gmail.com

^be-mail: sureshharinirahav@gmail.com

^ce-mail: pankaj.fellow@yahoo.co.in

^de-mail: sharma.umesh@gla.ac.in (corresponding author)

Tsallis entropy concept to formulate new Tsallis holographic dark energy (NTHDE) by considering the future event horizon as an IR cut-off and investigate its cosmic implications. The study of NTHDE carried out by [38–40] is based on the consideration of Hubble horizon as an IR cut-off which could not recover the standard HDE model for which Tsallis entropy should become the standard entropy but it is not. Large parameter values could represent the universe’s evolution. Such a consideration results in more deviation from the standard entropy. This difference is due to the Hubble horizon acting as an IR cut-off. As a result, in this paper, we develop a consistent formulation of NTHDE to get a well-defined extension of conventional HDE, which is the limiting case when the Tsallis entropy becomes the conventional Bekenstein–Hawking entropy.

In Sect. 2, by formulating the NTHDE expression, the differential equation for specific DE density parameter Ω_d , expressions for deceleration parameter and equation of state (EoS) parameter are obtained analytically. Section 3 is devoted to studying cosmological behaviour. In Sect. 4, a discussion on the obtained results is carried out with a concluding summary.

2 New Tsallis holographic dark energy

NTHDE formulation will be established here. The DE density ρ_d , the entropy S of black hole with radius L and the largest theory’s distance L connected by the relation $\rho_d L^4 \leq S$ is the key idea for HDE formulation [15, 16]. For usual Bekenstein–Hawking entropy $S_{BH} \propto (4G)^{-1} A = \pi G^{-1} L^2$ with Newton’s gravitational constant G . The standard HDE $\rho_d = 3c^2 M_p^2 L^{-2}$ with model parameter c is the saturation of the above inequality. As a result, a modified HDE model is obtained by modifying the entropy.

If $k_B = 1$ and a distribution has W states with Gibbs and Shannon entropies, then the expression for each state is same and given by

$$S = - \sum_{i=1}^W P_i \ln(P_i). \tag{1}$$

The Von-Neumann entropy or the quantum mechanical equivalent of (1) is

$$S = -\text{Tr}[\rho \ln(\rho)]. \tag{2}$$

For classical systems, (2) supports Boltzmann’s proposal in phase space with state density ρ . The Bekenstein–Hawking entropy ($\equiv S_{BH} = \frac{A}{4}$) is obtained by applying (2) to a pure gravitational system where A is system’s area [41]. By assumption that the degrees of freedom are dispersed on the

horizon where no particular priority for each other is specified [42, 43], all P_i ’s are equal and $P_i = \frac{1}{W}$. Both (1) and (2) implies the Boltzmann’s entropy ($S = \ln(W)$) and hence we get the expression for horizon entropy [44]

$$S_{BH} = \frac{A}{4} = \ln(W) \rightarrow W = e^{\left(\frac{A}{4}\right)}. \tag{3}$$

The Tsallis entropy is defined by [45]

$$S_n^T = \frac{1}{1-n} \sum_{i=1}^W (P_i^n - P_i) = \frac{W^{1-n} - 1}{1-n}, \tag{4}$$

where $P_i = \frac{1}{W}$, n is an unknown parameter (non-extensive) and as $n \rightarrow 1$, $S_n^T \rightarrow S$. The parameter n may also have its roots in quantum features of gravity. Using (3), (4) and $1 - n = \delta$, we get

$$S_n^T = \frac{1}{1-n} \left[e^{(1-n)S_{BH}} - 1 \right],$$

$$S_\delta^T = \frac{2 e^{\left(\frac{\delta S_{BH}}{2}\right)}}{\delta} \sinh\left(\frac{\delta S_{BH}}{2}\right). \tag{5}$$

As $\delta \rightarrow 0$ the standard Bekenstein–Hawking entropy is recovered. As expected the usual Bekenstein–Hawking entropy is obtained as a limiting case of Tsallis entropy and hence $\delta \ll 1$ i.e. $\delta \in (-1, 1)$. The Eq. (5) in its expanded and truncated form is given by

$$S_\delta^T = S_{BH} + \frac{\delta S_{BH}^2}{2} + \frac{\delta^2 S_{BH}^3}{6} + \mathcal{O}(\delta^3). \tag{6}$$

Clearly, the first term of (6) is the standard entropy. Using (6) and $\rho_d L^4 \leq S$, we get

$$\rho_d = \frac{3c^2 M_p^2}{L^2} + \frac{3c_1^2 \delta M_p^4}{2} + \frac{3c_2^2 \delta^2 M_p^6 L^2}{6}, \tag{7}$$

where c, c_1, c_2 are constants. For $\delta = 0$, the Eq. (7) leads to standard HDE, i.e $\rho_d = \frac{3c^2 M_p^2}{L^2}$. By letting $\frac{3c_1^2 \delta}{2} = \delta_1$ and $\frac{3c_2^2 \delta^2}{6} = \delta_2^2$, Eq. (7) can be rewritten by absorbing c_1 and c_2 in δ as

$$\rho_d = \frac{3c^2 M_p^2}{L^2} + \delta M_p^4 + \delta^2 M_p^6 L^2. \tag{8}$$

By considering the geometry of Friedmann–Robertson–Walker (FRW) model to be homogeneous, isotropic and flat with metric described by

$$ds^2 = \delta_{ij} dx^i dx^j a^2(t) - dt^2, \tag{9}$$

with a scaling factor $a(t)$ that varies with cosmic time. To investigate an HDE model the largest distance L of the theory is needed. According to Li and Hsu [15,46], $L \neq H^{-1}$ is the need for an HDE model to be consistent and standard. The Hubble horizon is expressed as $H^{-1} = \dot{a}(t)^{-1} a(t)$. Future event horizon as offered by Li [15] is expressed as

$$r_h = a(t) \int_t^\infty \frac{dy}{a(y)} = a(t) \int_{a(t)}^\infty \frac{da}{H [a(y)]^2}. \tag{10}$$

In [38,39], $L = H^{-1}$ is considered as IR-cutoff and the parameter δ is $\mathcal{O}(10^3)$. Such a high value of δ leads to high deviation from basic Bekenstein–Hawking entropy. We want to construct NTHDE consistently in this paper, thus we utilize the future event horizon r_h as L in (8) and get the NTHDE density as

$$\rho_d = \frac{3c^2 M_p^2}{r_h^2} + \delta M_p^4 + \delta^2 M_p^6 r_h^2. \tag{11}$$

Friedmann’s equations for a universe made up of perfect fluids such as DE and dark matter are expressed by

$$3M_p^2 H^2 = \rho_m + \rho_d, \tag{12}$$

$$-2M_p^2 \dot{H} = P_m + P_d + \rho_m + \rho_d, \tag{13}$$

where P_d represents NTHDE pressure, ρ_m represents dark matter energy density, and P_m represents dark matter pressure. The dark matter conservation equation is as follows:

$$\dot{\rho}_m + 3H(\rho_m + P_m) = 0. \tag{14}$$

The fractional DE and dark matter density parameters are defined as

$$\Omega_d = \frac{\rho_d}{3M_p^2 H^2}, \tag{15}$$

$$\Omega_m = \frac{\rho_m}{3M_p^2 H^2} \tag{16}$$

respectively. Using the Eq. (11) in (15), we get a fourth degree equation in r_h . By considering r_h to be positive and taking the limit $\delta \rightarrow 0$ the standard HDE $\int_x^\infty \frac{dx}{Ha} = \frac{c}{Ha\sqrt{\Omega_d}}$ is obtained. Hence such a value of r_h is considered and expressed by

$$r_h = \left(\frac{3H^2\Omega_d - \delta M_p^2 - \sqrt{(3H^2\Omega_d - \delta M_p^2)^2 - 12c^2\delta^2 M_p^4}}{2\delta^2 M_p^4} \right)^{\frac{1}{2}}. \tag{17}$$

Using Eqs. (10) and (17), we get

$$\int_x^\infty \frac{dx}{Ha} = \frac{1}{a} \left(\frac{3H^2\Omega_d - \delta M_p^2 - \sqrt{(3H^2\Omega_d - \delta M_p^2)^2 - 12c^2\delta^2 M_p^4}}{2\delta^2 M_p^4} \right)^{\frac{1}{2}}, \tag{18}$$

where $a = e^x$.

Now we consider the physically intriguing dust matter scenario for which the matter EoS parameter is zero. If we consider the present matter energy density to be ρ_{m_0} for current scale factor $a_0 = 1$, Eq. (14) gives

$$\rho_m = \frac{\rho_{m_0}}{a^3}. \tag{19}$$

Using Eq. (19) into (16) we get

$$\Omega_m = \frac{\Omega_{m_0} H_0^2}{H^2 a^3}, \tag{20}$$

where H_0 is the present value of the Hubble parameter.

Using Eq. (20) and the Friedmann equation $\Omega_d + \Omega_m = 1$ we get

$$\frac{1}{Ha} = \frac{\sqrt{a(1 - \Omega_d)}}{H_0 \sqrt{\Omega_{m_0}}}. \tag{21}$$

Substituting Eq. (21) into (18) we get

$$\int_x^\infty \frac{\sqrt{a(1 - \Omega_d)}}{H_0 \sqrt{\Omega_{m_0}}} dx = \frac{1}{a} \left(\frac{3H^2\Omega_d - \delta M_p^2 - \sqrt{(3H^2\Omega_d - \delta M_p^2)^2 - 12c^2\delta^2 M_p^4}}{2\delta^2 M_p^4} \right)^{\frac{1}{2}}. \tag{22}$$

Differentiating Eq. (22) with respect to ‘ x ’ we get

$$\Omega'_d = \Omega_d(1 - \Omega_d) \left[3 - \frac{2(\mathcal{I} - 2\delta^2 M_p^6 \mathcal{J})}{\mathcal{I} + \delta M_p^4} \times \left\{ 1 - \sqrt{3} \left(\frac{M_p^2 \Omega_d}{(\mathcal{I} + \delta M_p^4) \mathcal{J}} \right)^{\frac{1}{2}} \right\} \right], \tag{23}$$

where

$$\mathcal{I} = \frac{3 e^{-3x} H_0^2 M_p^2 \Omega_{m_0} \Omega_d}{1 - \Omega_d} - \delta M_p^4,$$

$$\mathcal{J} = \frac{\mathcal{I} - \sqrt{\mathcal{I}^2 - 12c^2\delta^2 M_p^8}}{2\delta^2 M_p^6}.$$

For flat spatial geometry and dust matter, the differential equation (23) describes the evolution of NTHDE. As a limit on considering $\delta \rightarrow 0$ we get $\mathcal{J} = \frac{3c^2}{\mathcal{I}}$, which implies (23) to recover the differential equation of standard HDE [47], i.e. $\Omega'_d = \Omega_d(1 - \Omega_d) \left(1 + \frac{2}{c}\sqrt{\Omega_d}\right)$ and can be solved analytically.

Now we will consider the EoS parameter for NTHDE defined by $w_d = \frac{P_d}{\rho_d}$. As the matter sector is conserved. The Eq. (14) and the Friedmann equations (12), (13) implies the DE sector to be conserved, i.e.

$$\dot{\rho}_d + 3H\rho_d(1 + w_d) = 0. \tag{24}$$

Differentiating (11) w.r.t. ‘ t ’ results

$$\dot{\rho}_d = -\frac{2M_p^2(3c^2 - \delta^2 M_p^4 r_h^4) r_h}{r_h^3}. \tag{25}$$

From Eq. (10) we get

$$\dot{r}_h = Hr_h - 1. \tag{26}$$

Using Eqs. (25) and (26) we get the expression for r_h in terms of ρ_d given by

$$r_h = \left[\frac{\rho_d - \delta M_p^4 - \sqrt{(\rho_d - \delta M_p^4)^2 - 12c^2\delta^2 M_p^8}}{2\delta^2 M_p^6} \right]^{\frac{1}{2}}. \tag{27}$$

Using Eqs. (15), (21) and (25) to (27), we get

$$w_d = -1 - 2 \left(\frac{M_p^6 \Omega_d}{3(\mathcal{I} + \delta M_p^4)^3} \right)^{\frac{1}{2}} \left(\frac{\delta^2 M_p^4 \mathcal{J}^2 - 3c^2}{\mathcal{J}^{\frac{3}{2}}} \right) \times \left[-1 + \left(\frac{(\mathcal{I} + \delta M_p^4) \mathcal{J}}{3M_p^2 \Omega_d} \right)^{\frac{1}{2}} \right]. \tag{28}$$

Clearly the standard HDE is recovered by letting $\delta \rightarrow 0$ i.e. as $\delta \rightarrow 0$, $w_d \rightarrow \frac{-1}{3} - \frac{2\sqrt{\Omega_d}}{3c}$. In general, we can highlight

that w_d can behave either like quintessence or quintom which shows the richness of the current model.

The parameter describing deceleration behaviour can be expressed as

$$q = -\frac{\dot{H}}{H^2} - 1 = \frac{3w_d\Omega_d + 1}{2}. \tag{29}$$

3 Cosmological evolution of NTHDE

In Sect. 2, we derived the differential equation describing the evolutionary behaviour of NTHDE density parameter, corresponding expressions for EoS and deceleration parameters. Now we will discuss the detailed cosmological behaviour for results obtained in the previous section. The numerical solution for the differential equation (23) reflects various evolutionary features of Ω_d for redshift z by the transformation $x = \ln\left(\frac{1}{1+z}\right)$ with initial condition $\Omega_d(x=0) = \Omega_d[0] \approx 0.7$. And hence by virtue of Friedmann equation $\Omega_{m0} \approx 0.3$.

The Figs. 1 and 2 shows the DE density parameter plots against the redshift z . In Fig. 1, we have considered $\delta = 0.2$ fixed with varying c values. Figure 2 is plotted by considering $c = 0.7$ fixed and varying δ . As we can see from both the graphs, the current model may give the universe’s needed thermal history, i.e. in the past matter dominated, current domination of 70% by DE and in future fully dominated by DE only.

Figures 3 and 4 represent EoS parameters for the NTHDE model. Which shows that the current value of w_d resides in the vicinity of -1 , which is consistent with the observational data. Now we’ll look at how the model parameters δ and c

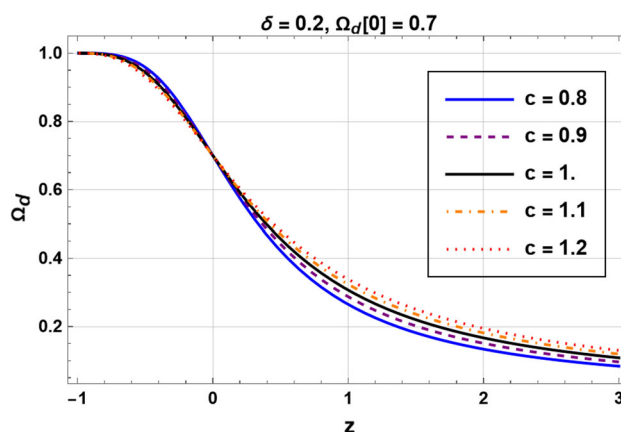


Fig. 1 NTHDE density parameter Ω_d with $\delta = 0.2$ and $c = 0.8$ to 1.2 is plotted w.r.t. redshift z by considering $\Omega_d[z=0] \approx 0.7$, $M_p^2 = 1$

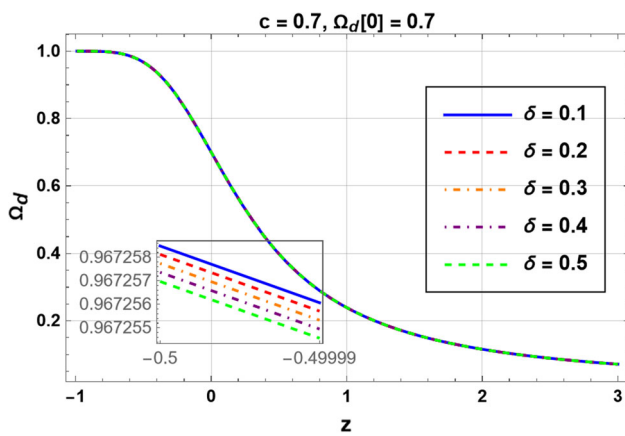


Fig. 2 NTHDE density parameter Ω_d with $c = 0.7$ and $\delta = 0.1$ to 0.5 is plotted w.r.t. redshift z by considering $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$

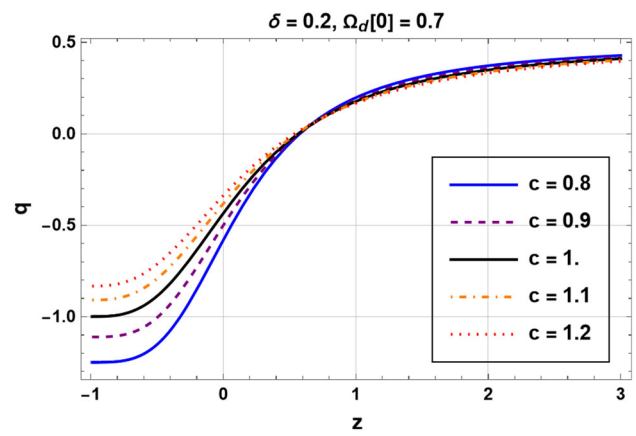


Fig. 5 The deceleration parameter q with $\delta = 0.2$ and $c = 0.8$ to 1.2 is plotted w.r.t. redshift z by considering $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$

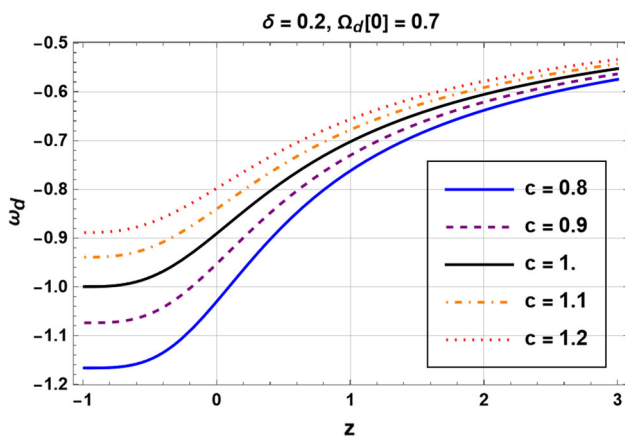


Fig. 3 The evolution of EoS parameter w_d of NTHDE with $\delta = 0.2$ and $c = 0.8$ to 1.2 is plotted w.r.t. redshift z by considering $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$

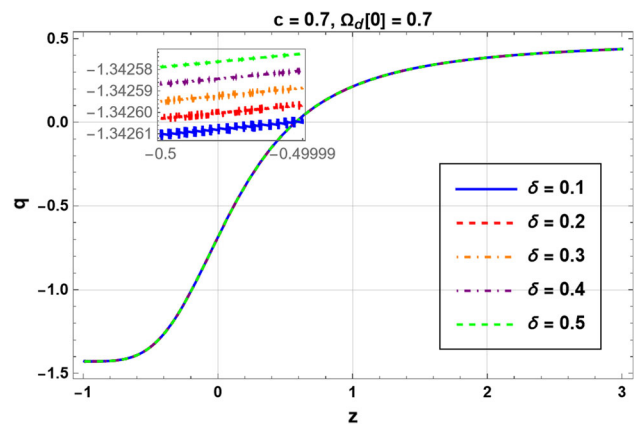


Fig. 6 The deceleration parameter q with $c = 0.7$ and $\delta = 0.1$ to 0.5 is plotted w.r.t. redshift z by considering $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$

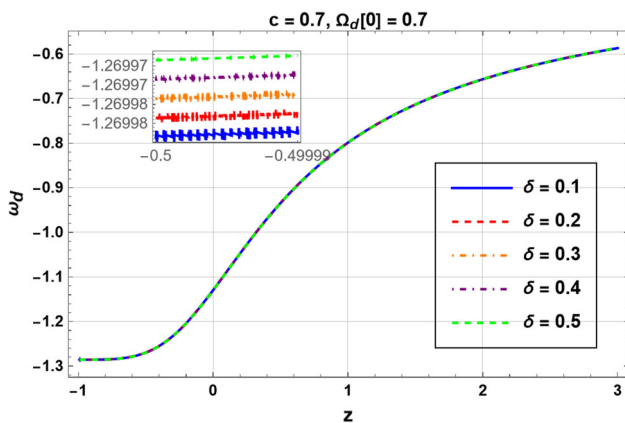


Fig. 4 The evolution of EoS parameter w_d of NTHDE with $c = 0.7$ and $\delta = 0.1$ to 0.5 is plotted w.r.t. redshift z by considering $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$

affect the DE's EoS parameter w_d . In Fig. 3 we have plotted w_d for $\delta = 0.2$ and different c values. As can be seen, for $c < 1$ values, w_d always enters the phantom regime in the

far future. While $c > 1$ completely lies in the quintessence region. $c = 1$ corresponds to the Λ CDM model. In addition, we show w_d for constant $c = 0.7$ and different δ values in Fig. 4. We have an intriguing pattern here with rising δ , w_d remains almost the same at times around the current ones. To get the far future value of w_d , i.e. for $z \rightarrow -1$ Eq. (28) indicates the combined dependence on δ and c . In conclusion NTHDE leads to some fascinating cosmic phenomenology where w_d shows behaviours like quintessence, or like quintom.

Figures 5 and 6 describe the deceleration parameter q behaviour against z . Figure 5 is plotted by fixing δ to be 0.2 and varying c values. While Fig. 6 is based on varying δ and fixed $c = 0.7$. It confirms the universe to enter an accelerated phase for $z \approx 0.6$. Which is in full agreement with the observational data supported by [1,2]. The inner plot of Fig. 6 shows a close-up of the outer plot in which the difference can be seen. They are not exactly identical but difference is very small. Similar is the case with the Figs. 2 and 4.

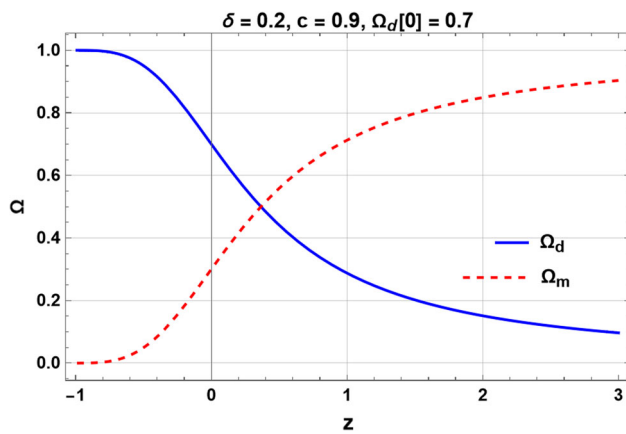


Fig. 7 The NTHDE density parameter Ω_d and dark matter density parameter Ω_m is depicted. The graph is plotted against the redshift z by considering $\delta = 0.2$, $c = 0.9$, $\Omega_d[z = 0] \approx 0.7$, $M_p^2 = 1$. Where the present time corresponds to $z = 0$

4 Conclusive remarks

In the present work we formulated the HDE model in which Tsallis entropy, a one-parameter generalization of Boltzmann–Gibbs entropy, is used. Such a concept is derived from a consistent relativistic statistical theory. A parameter δ is used to distinguish deviations from conventional entropy expressions. The consistent NTHDE model is obtained by applying IR cutoff in terms of future event horizon and the Tsallis entropy, to the standard HDE model. The parameter δ is responsible for such an extension with usual HDE as a limiting case $\delta \rightarrow 0$. We derived the differential equation to describe the evolutionary behaviour of dark energy density parameter Ω_d which investigates possible cosmic applicability of NTHDE. On considering today's universe to be dominated 70% by DE, Fig. 7, clearly indicates the full domination of the universe by DE in the far future. In addition, the analytical formulations of the deceleration parameter and the EoS parameter are obtained. As per the observation from NTHDE's EoS parameter, the parameters c and δ describe the diversified behaviour of the model i.e. pure quintessence for $c > 1$, quintom for $c < 1$ (in near or far future) and Λ CDM for $c = 1$. The trend shown by the deceleration parameter q for the model, possesses interesting cosmological descriptions such as the universe's thermal history from dark matter to DE. The transition from decelerated to accelerated phase happens at $z \approx 0.6$. Finally, because of consistent formulation and versatile behaviour, the NTHDE leads to standard HDE as a limiting case, which is the biggest advantage of the model.

Indeed, the use of such entropies is in the early stages [48]. The existence of long-range interactions in systems is a basic reason to use such entropies in describing the systems [45]. General relativity (GR) is not the final form of the grav-

itational theory. GR satisfies Bekenstein entropy, and thus, one may expect that other entropies should be satisfied by the final form of the gravitational theory. Such attempts can at least help us get some estimations about the final form of gravitational field equations. It seems that there is a connection between the quantum aspects of gravity and non-extensivity [49–56]. There are various works claiming that various problems are solved (at least, solved better) by considering such entropies which may be a clue to understand the thermodynamics of spacetime, gravity, and related phenomena [48, 57–63]. HDE is a great hypothesis to reconcile quantum field theory and gravity, a hope to solve the DE problem. Therefore, such papers may at least help us find a proper mathematical model for the density profile of DE, a result which is so vital to overcome the problems such as the nature and behavior of DE, DM, spacetime, and the final form of gravitational theory. Indeed, one may consider such entropies as well as the Loop Quantum Gravity entropy as the sub-classes of a general entropy [64].

As the origin and behavior of DE are not completely known, and moreover, due to the weakness of GR in describing DE, and also since HDE based Bekenstein entropy is not capable to describe DE, we think that it is too soon to confine ourselves to small values of δ . More observations and studies are still needed to get and apply this limitation.

In order for the NTHDE to be a successful alternative to describe the DE, the model parameters must be constrained. Such constraints can be obtained using the observational data from the Hubble parameter, CMB, BAO, and SNIa. The phase-space can be analyzed to understand the global dynamics of the DE.

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