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On angular features of axial channeling radiation in crystals

K. B. Korotchenko[1,a], **Y. P. Kunashenko**[2], **S. B. Dabagov**[3,4,5,b]

[1] National Research Tomsk Polytechnic University, Lenin Pr. 30, 634050 Tomsk, Russia
[2] Tomsk State Pedagogical University, Kievskaya Str. 60, 634061 Tomsk, Russia
[3] INFN-Laboratori Nazionali di Frascati, Via E. Fermi 54, 00044 Frascati, RM, Italy
[4] RAS-P.N. Lebedev Physical Institute, Leninsky Pr. 53, 119991 Moscow, Russia
[5] NR Nuclear University MEPhI, Kashirskoe Sh. 31, 115409 Moscow, Russia

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Abstract Channeling of light relativistic particles in crystals is accompanied by intense radiation emission known as channeling radiation. Typically all calculations of channeling radiation aim in getting the total radiation intensity and its dependence on the parameters of particles and crystals. In the same time, since the discovery, the angular behaviours of channeling radiation have been studied just in a few works, in which only a polar dependence of the radiation intensity near the forward direction is estimated. However, simple analysis of the interaction potential predicts very specific features to be observed in the angular distributions of channeling radiation, especially at axial regime. In this work, for the first time, the expressions for angular and spectral distributions of electromagnetic radiation at axial channeling of relativistic charged particles in thin crystals are analytically refined within the QED theory. Obtained results allows predicting complex structures of radiation intensity and polarisation distributions. The results obtained might be of special interests for experimental studies.

1 Introduction

The interaction of relativistic charged particles in aligned crystals remains of growing interest for theorists and experimentalists since the first discoveries related to the possibility of its use as a powerful source of electromagnetic radiation based on the phenomena of *coherent bremsstrahlung* (CB) and *channeling radiation* (CR) [1–9]. Both types of the radiation have been in detail studied within classical and quantum theoretical models as well as in many dedicated experiments that has formed rather adequate formalism of general common description of coherent radiation processes by charged particles in crystals (see, for instance, in [10]). However, even today some features of particles scattering in crystals can be the origin of fine peculiarities to be observed in the emitted electromagnetic radiation. In this work we examine the influence of the asymmetry in angular scattering of axially channeled electrons on the accompanying channeling radiation.

So far the theory of electromagnetic radiation by electrons in crystals has been developed for the planar channeling regime within the approximations of both classical and quantum mechanics revealing very details of the beam motion in the crystal field characterised by strong redistribution of the beam in a phase space [4,7,8,11–17]. Having known the behaviours of channeled beam evolution in the transverse space of motion, usually the calculations of radiation characteristics are limited just in getting the total radiation intensity. Its dependence on the angular distance from the forward direction, for which the maximum of radiation intensity is observed, is assumed to be azimuthally symmetric at a small distance and just formally estimated. However, new types of electromagnetic radiation can be observed at well defined angular parameters of channeled particles (for instance, see in [18]).

As regards the theory of CR at axial channeling the calculations are mostly performed within the limits of classical electrodynamics (CED), i.e. one calculates the trajectories of channeled particles in the potentials of crystal axes followed by the numerical integration of the derived CED expressions [2,19–23]. Some details of the beam dynamics can be additionally included in the calculations of axial CR ones applied a well known "quasi-classical theory" ([6] and Refs. therein).

In many papers the quantum electrodynamics (QED) estimations of CR at axial channeling have been obtained (see,

[a] e-mail: korotchenko@tpu.ru (corresponding author)
[b] e-mail: sultan.dabagov@lnf.infn.it

for instance, in [9,24]). These approximations are based on studying the transverse wave functions of channeled particles for the stationary Schrödinger equation and well describe CR for single crystal axes (*the single-string approximation*). However, taking into account the crystal periodicity in a plane perpendicular to the crystal axes allows the ψ -function of the transverse motion of a channeled particle to be presented as a sum of Bloch functions [25–28], which satisfy the Dirac equation (*the many-beam approximation*). The latter may result in appearance of some singularities for the angular and energy distributions of CR. For electrons and positrons the quantum mechanical description of the projectile motion at channeling conditions becomes mostly applicable for low and moderate particle energies (≤ 100 MeV), as well as for hyper-relativistic ones (≥ 10 GeV) [26].

In our previous works [14–17,29] we have developed a new method to figure the Bloch wave functions for planar and axial crystal orientations, the details of which is given in [30]. The present work aims in deducing the analytical expressions for spectral-angular CR distributions in thin crystals that demonstrate distinctive features of these dependences to be, in our opinion, of a special interest for future experimental studies.

2 CR probability matrix element

In QED the radiation probability is defined by the expression

$$dw_{if} = \frac{2\pi}{\hbar} \mid M_{if} \mid^2 \delta(E) d\varrho_f \,, \tag{1}$$

which, in our case, is determined by the wave functions of the quantum states for channeled projectile motion described by Dirac equation in so-called continuous potential formed by the system of crystal planes and axes. The potential is strongly related to the crystal orientation with respect to the projectile initial momentum. In this definition $d\varrho_f = 1/(2\pi)^3 d^3\kappa$ is the density of the final quantum states f, and M_{if} is the matrix element for the $i \to f$ transition accompanied by CR

$$M_{if} = -e \int \mathbf{A}^* \mathbf{J}_{fi} d^3\mathbf{r} \,, \tag{2}$$

where the vector $\mathbf{A} = \sqrt{2\pi\hbar c^2/\omega}\boldsymbol{\epsilon}_{\boldsymbol{\kappa}} e^{\mathbf{i}\boldsymbol{\kappa}\mathbf{r}}$ represents the wave function of a CR-photon with the polarisation vector $\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}$, $\mathbf{J}_{fi} = \Psi_f \boldsymbol{\gamma} \Psi_i$ is the current operator, namely, the probability density for the channeled particle transition $i \to f$ with the Dirac $\boldsymbol{\gamma}$-matrix.

The wave function of a channeled particle in the ith quantum state can be written in the form $\Psi_i = \sqrt{\left(E_{i\parallel} + mc^2\right)/2E_{i\parallel}} u_i \psi_i(\mathbf{r})$, where [31]

$$\psi_i(\mathbf{r}) = \phi_i(\mathbf{r}_\perp) \exp(\mathbf{i}\mathbf{p}_{i\parallel}\mathbf{r}_\parallel/\hbar), \; u_i = \begin{pmatrix} w \\ \frac{\boldsymbol{\sigma}\hat{\mathbf{p}}c}{c^2 m + E_{i\parallel}} \end{pmatrix} \tag{3}$$

Here w is the 2d-spinor normalised by the condition $w^+ w = 1$, $\boldsymbol{\sigma}$ are the Pauli matrixes. The transverse wave function $\phi_i(\mathbf{r}_\perp)$ describes the ith quantum state of a relativistic channeled particle of the mass γm (with the relativistic Lorentz factor $\gamma = E_\parallel / mc^2$) by the Schrödinger equation [4,5,8,31]

$$\hat{H}\phi_i(\mathbf{r}_\perp) = \left(\frac{\hat{\mathbf{p}}_\perp^{\,2}}{2m\gamma_i} + U(\mathbf{r}_\perp)\right)\phi_i(\mathbf{r}_\perp) = E_{i\perp}\phi_i(\mathbf{r}_\perp) \tag{4}$$

Besides, because of the crystal field periodicity, the transverse wave function $\phi_i(\mathbf{r}_\perp)$ can be represented by Bloch functions [25].

Successfully, having the longitudinal momentum of a channeled particle directed along the z-axis, which is coaxial with the crystal channeling chain, the matrix element for the CR probability is calculated as follows

$$\begin{aligned} M_{if} = -e\sqrt{\frac{2\pi\hbar c^2}{\omega}} \int (\alpha^0_{if}(\mathbf{r}_\perp)\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}) \\ \times e^{\mathbf{i}\kappa_\perp \mathbf{r}_\perp} e^{\mathbf{i}\kappa_z z} \exp(\mathbf{i}\Delta p_{if} z/\hbar) d\mathbf{r}_\perp dz \end{aligned} \tag{5}$$

with

$$\alpha^0_{if}(\mathbf{r}_\perp) = \frac{c}{E_{i\parallel}} \phi^*_f(\mathbf{r}_\perp)\hat{\mathbf{p}}_i \phi_i(\mathbf{r}_\perp) \tag{6}$$

Integration by z, this expression is deduced to much simpler expression

$$M_{if} = -e\sqrt{\frac{2\pi\hbar c^2}{\omega}} \delta_{\Delta p_{if} z/\hbar, \kappa_z} (\alpha_{if}(\boldsymbol{\kappa}_\perp)\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}) \tag{7}$$

with the angular operator

$$\alpha_{if}(\boldsymbol{\kappa}_\perp) = \int \alpha^0_{if}(\mathbf{r}_\perp) e^{\mathbf{i}\boldsymbol{\kappa}_\perp \mathbf{r}_\perp} d\mathbf{r}_\perp \tag{8}$$

Since the operator equation $\mathbf{i}\hbar\hat{\mathbf{p}}_\perp = -m\gamma[\hat{H}_\perp, \mathbf{r}_\perp]$, which defines the wave function $\phi_i(\mathbf{r}_\perp)$ for the ith quantum state of a channeled particle (9), the angular operator can be presented as a function of new matrix elements $X_{if\perp}$ and Z_{if}

$$\alpha_{if}(\boldsymbol{\kappa}_\perp) = \left(-\mathbf{i}\frac{\Omega_{if}}{c} X_{if\perp}, \beta Z_{if}\right), \tag{9}$$

where $\hbar\Omega_{if} = \varepsilon_i - \varepsilon_f$, ε_l is the transverse energy of the lth state of a channeled particle, $\beta = \mathrm{v}/c$,

$$X_{if\perp} = \int e^{\mathbf{i}\kappa_\perp \mathbf{r}_\perp}\phi_f^*(\mathbf{r}_\perp)\mathbf{r}_\perp\phi_i(\mathbf{r}_\perp)d\mathbf{r}_\perp,$$
$$Z_{if} = \int e^{\mathbf{i}\kappa_\perp \mathbf{r}_\perp}\phi_f^*(\mathbf{r}_\perp)\phi_i(\mathbf{r}_\perp)d\mathbf{r}_\perp \quad (10)$$

Applying well developed math techniques of the angular analysis for radiation polarisation [32,33], we can easily prove that the components of polarisation vector $\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}$ of Eq. (7) are defined by

$$\boldsymbol{\epsilon}_{\boldsymbol{\kappa}} = \frac{1}{\sqrt{2}}\Big(\Lambda\cos\Theta\cos\Phi + \mathbf{i}\sin\Phi\,, -\mathbf{i}\cos\Phi + \Lambda\cos\Theta\sin\Phi, -\Lambda\sin\Theta\Big)\,, \quad (11)$$

where Φ and Θ are the azimuthal and polar angles, respectively, and Λ is the radiation (photon) helicity.

3 Matrix transition elements for periodic potentials

Let define the wave function $\phi_i(\mathbf{r}_\perp)$ in a form of Bloch function in the cartesian coordinate system

$$\phi_{i,i_z}(x,y) = e^{-\frac{\mathbf{i}\pi i_z(x+y)}{5a_p}}\sum_{m_i,n_i} e^{-\mathbf{i}\left(\frac{4\pi m_i x}{a_p}+\frac{4\pi n_i y}{a_p}\right)}C_{i,i_z}^{m_i,n_i}\,, \quad (12)$$

where $C_{i,i_z}^{m_i,n_i}$ are the Fourier components of the wave function, and m_i, n_i are the Fourier components numbers for the ith energetic band of channeled electron transverse motion, a_p is the lattice constant. The i_z index corresponds to the internal point (numbered) within the ith energetic band.

The wave functions presented in the form (12) allow performing the calculations without the limits of dipole approximation. In this case for $\alpha_{if}(\mathbf{r}_\perp)$ we can get

$$\alpha_{if}(\mathbf{r}_\perp) = -C_{i,i_z}^{m_i,n_i} \times\left(\frac{\Omega_{if}}{c}F_x(\kappa_x,\kappa_y),\ \frac{\Omega_{if}}{c}F_y(\kappa_x,\kappa_y),\ -\beta F_z(\kappa_x,\kappa_y)\right), \quad (13)$$

where we have used the following nominations

$$F_x(\kappa_x,\kappa_y) = 16a_p\sin\left(\frac{\kappa_y a_p}{4}\right)(-1)^{m_f+m_i+n_f+n_i} \times\frac{\left(\cos\left(\frac{\kappa_x a_p}{4}\right)\left(\kappa_x a_p + 4\pi(m_f-m_i)\right) - 4\sin\left(\frac{\kappa_x a_p}{4}\right)\right)}{\left(\kappa_x a_p + 4\pi(m_f-m_i)\right)^2\left(\kappa_y a_p + 4\pi(n_f-n_i)\right)}\,,$$
$$F_y(\kappa_x,\kappa_y) = 16a_p\sin\left(\frac{\kappa_x a_p}{4}\right)(-1)^{m_f+m_i+n_f+n_i} \times\frac{\left(\cos\left(\frac{\kappa_y a_p}{4}\right)\left(\kappa_y a_p + 4\pi(n_f-n_i)\right) - 4\sin\left(\frac{\kappa_y a_p}{4}\right)\right)}{\left(\kappa_y a_p + 4\pi(n_f-n_i)\right)^2\left(\kappa_x a_p + 4\pi(m_f-m_i)\right)}\,,$$
$$F_z(\kappa_x,\kappa_y) = \frac{64\sin\left(\frac{\kappa_x a_p}{4}\right)\sin\left(\frac{\kappa_y a_p}{4}\right)(-1)^{m_f+m_i+n_f+n_i}}{\left(\kappa_x a_p+4\pi(m_f-m_i)\right)\left(\kappa_y a_p+4\pi(n_f-n_i)\right)} \quad (14)$$

At such definitions we will be able to perform precise analytical calculations of CR via rather routine work. However, it will not essentially change the quantitative data for the forward CR. On the contrary, huge difference between the energy of emitted CR photon and the longitudinal projectile energy suggests all transformations to be done within the dipole approach, i.e.

$$\kappa_x a_p \ll 1\,,\ \sin\left(\frac{\kappa_x a_p}{4}\right) \Rightarrow \frac{\kappa_x a_p}{4}\,,$$
$$\kappa_y a_p \ll 1\,,\ \sin\left(\frac{\kappa_y a_p}{4}\right) \Rightarrow \frac{\kappa_y a_p}{4} \quad (15)$$

Hence, the above introduced matrix elements can be rewritten in the following way

$$\alpha_{if}(\mathbf{r}_\perp) = -\left(\frac{\Omega_{if}}{c}\kappa_y,\ \frac{\Omega_{if}}{c}\kappa_x,\ -\beta\kappa_x\kappa_y\right)\langle xy\rangle_{if} \quad (16)$$

with

$$\langle xy\rangle_{if} = \int\phi_f^*(x,y)xy\phi_i(x,y)dxdy = -C_{i,i_z}^{m_i,n_i}C_{f,f_z}^{m_f,n_f}\frac{(-1)^{m_f+m_i+n_f+n_i}a_p^2}{16\pi^2(m_f-m_i)(n_f-n_i)} \quad (17)$$

Finally, substituting in the formula (7) for the matrix element of CR probability M_{if} the polarisation vector $\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}$ from Eq. (11) and the transition matrix element α_{if} from Eq. (16), we can get the square of the scalar function $(\alpha_{if}\cdot\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}^*)$

$$|\,(\alpha_{if}\cdot\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}^*)\,|^2 = \frac{\omega^2\langle xy\rangle_{if}^2}{8c^2}F(\Theta,\Phi,\omega) \quad (18)$$

with the angular function F

$$F(\Theta,\Phi,\omega) = \sin^2\Theta\Big(\beta^2\omega^2\sin^4\Theta\sin^2(2\Phi) + \Omega_{if}^2(3+2\sin^2\Theta\cos(4\Phi)+\cos(2\Theta)) - 4\beta\Omega_{if}\omega\cos\Theta\sin^2\Theta\sin^2(2\Phi)\Big) \quad (19)$$

4 Angular and spectral distributions

As a result of performed calculations, for the radiation probability (1) we can use the following expression

$$dw_{if} = \frac{\alpha\hbar}{2\pi}\,|\,(\alpha_{if}\cdot\boldsymbol{\epsilon}_{\boldsymbol{\kappa}}^*)\,|^2\,\delta(\Delta E_{if}-\hbar\omega)\omega d\omega\,, \quad (20)$$

where $\Delta E_{if} \simeq \Omega_{if} + \beta\omega\cos\Theta$.

After integration of $dI_{if} = \hbar\omega dw_{if}$ over the CR photon frequencies ω, we get the angular distribution of CR by axially channeled electrons

$$\frac{dI}{do} = \sum_{i,f}P_{\vartheta,i_z}\frac{\alpha\hbar\,\Omega_{if}^6\langle xy\rangle_{if}^2}{16c^2\pi}F_{ang}(\Theta,\Phi) \quad (21)$$

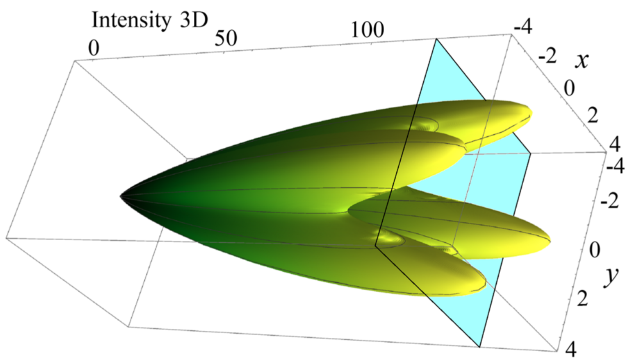

Fig. 1 CR angular distribution for 10 MeV electrons channeled in Si ⟨100⟩ crystal (the arbitrary units have been used)

where

$$F_{ang}(\Theta,\Phi)=\frac{\sin^2\Theta}{(1-\beta\cos\Theta)^7}$$
$$\times\Big[(3+\cos 2\Theta+2\sin^2\Theta\cos 4\Phi(1-\beta\cos\Theta)^2$$
$$+\beta\left(\beta-4(1-\beta\cos\Theta)\frac{\cot\Theta}{\sin\Theta^2}\right)\sin\Theta^4\sin 2\Phi^2\Big] \quad (22)$$

Here we have taken into account the initial population P_{ϑ,i_z} of the ith energy band of the transverse electron motion (ϑ is the angle of electron momentum relative to the crystal axes). All further numerical calculations were carried out for $\vartheta=\vartheta_c/5$, where ϑ_c is the critical channeling angle. It is important to underline that the populations of the above-barrier levels, which describe a quasi-channeling, is much smaller than those of the sub-barrier ones. Therefore, the contribution of these levels to the total radiation intensity is extremely small.

Figure 1 shows in arbitrary units an example of the CR angular distribution calculated for 10 MeV electrons channeled along the ⟨100⟩ axes of a Si crystal[1]. This distribution exhibits an emphasised structural dependence, which is tightly correlated to the crystal ⟨100⟩ orientation and does not accordingly reveal an absolute maximum at the forward direction. For clarity, Fig. 2 shows a section of the angular distribution by the plane indicated in Fig. 1.

Further integration of $dI_{if}=\hbar\omega dw_{if}$ over allowed Θ and Φ angles results in the formula for spectral distribution of CR by axially channeled electrons

$$\frac{dI}{d\omega}=\sum_{i,f}P_{\vartheta,i_z}\frac{\alpha\hbar\,\langle xy\rangle_{if}^2}{16c^4\beta^5}F_{sp}\left(\frac{\omega-\Omega_{if}}{\beta\omega}\right)$$
$$\times\mathcal{H}(\omega_1)\mathcal{H}(\omega_2)\,, \quad (23)$$

[1] The energy of 10 MeV in the numerical calculations for electrons was chosen only because at this energy the electrons do not have very many sub-barrier transverse levels (about 23 levels), while this energy is sufficient for channeling.

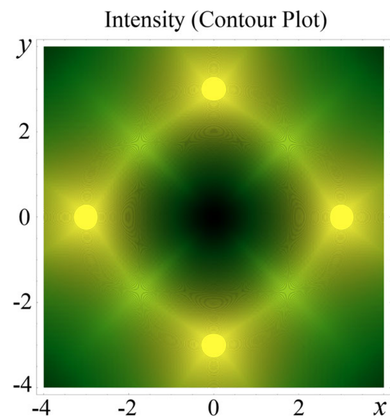

Fig. 2 Arbitrary contour (level) plot of the CR angular distribution for 10 MeV electrons channeled in Si ⟨100⟩ in a randomly selected transverse plane shown in Fig. 1

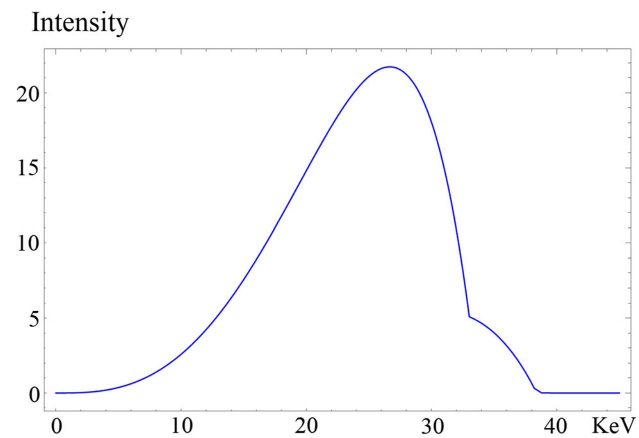

Fig. 3 CR spectral distribution for 10 MeV electrons axially channeled in Si ⟨100⟩ (the intensity is given in arbitrary units)

where $\mathcal{H}(...)$ is the Heaviside function with the following arguments

$$\omega_1=\omega-\frac{\Omega_{if}}{1+\beta}\,,\quad \omega_2=\frac{\Omega_{if}}{1-\beta}-\omega \quad (24)$$

and

$$F_{sp}(\omega)\ =(\omega(1+\beta)-\Omega_{if})(\Omega_{if}-(1-\beta)\omega)$$
$$\times(1-\beta^2)^2\omega^4+2(3\beta-1)\omega^2\Omega_{if}^2+\Omega_{if}^4 \quad (25)$$

An example of the total spectral distribution of CR calculated as the sum over all transitions between the energy bands of the transverse motion for 10 MeV electrons channeled in Si ⟨100⟩ is shown in Fig. 3. The calculations demonstrate that, in contrast to the CR spectrum at planar channeling, the emission at axial channeling along the Si ⟨100⟩ axis give out a smooth function up to ∼33 keV energy. This is might be a consequence of the much greater depth of the potential well of the crystal axes in comparison to that of the planes.

The maximum radiation intensity is predicted at the photon energy ∼26.8 keV.

5 Axial CR polarisation

The dependence of the degree of polarisation CR on the inlet direction of relativistic electrons into the Si crystal relative to the ⟨100⟩ axis was calculated in [35] at the axial channeling regime. A similar method has been applied to evaluate both the degree of polarisation and the angular directions of polarisation for a new type of radiation, i.e. diffracted channeling radiation (DCR), as a function of the emission angles [36].[2]

Let's present the circular polarisation vector β_Λ of a photon with the helicity Λ as

$$\beta_\Lambda = \frac{1}{\sqrt{2}}(\beta_2 + \mathbf{i}\Lambda\beta_3)\,, \tag{26}$$

where β_2 and β_3 are the vectors of the photon linear polarisation [32]. Then, for the radiation intensity I_Λ with the polarisation β_Λ proportional to the square of the matrix element of the transition probability $i \to f$, i.e. $I_\Lambda \propto |\alpha_{if}\beta_\Lambda|^2$, we obtain

$$I_\Lambda \propto \frac{1}{2}\left((\alpha_{if}\beta_2)^2 + (\alpha_{if}\beta_3)^2\right) \tag{27}$$

Thus, the radiation intensity does not depend on the photon helicity Λ, and we defiine a real value of α_{if} for CR in the case of axial channeling. This means that such radiation cannot disclose a circular polarisation [37] but only a linear one with the degree of polarisation defined as follows

$$P = \frac{|Me_1|^2 - |Me_2|^2}{|Me_1|^2 + |Me_2|^2}\,, \tag{28}$$

where e_1 and e_2 are the components of the polarisation vector β_Λ perpendicular to the unit vector $\boldsymbol{\kappa}$ in the direction of photon emission. As known [32], the vectors of the photon linear polarisation β_2 and β_3 satisfy this condition by default.

Taking into account that in Eq. (27)

$$\alpha_{if}\beta_2 = \sqrt{2}\,\Re(\alpha_{if}\beta_\Lambda),\ \ \alpha_{if}\beta_3 = \sqrt{2}\Lambda\,\Im(\alpha_{if}\beta_\Lambda), \tag{29}$$

where $\Re(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)$ and $\Im(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)$ are the real and imaginary parts of the complex function $(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)$, we reduce the formulas of the degree of polarisation for CR photons at axial channeling

$$P_{CR} = \frac{|\,\Re(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)\,|^2 - |\,\Im(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)\,|^2}{|\,\Re(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)\,|^2 + |\,\Im(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)\,|^2}\,, \tag{30}$$

[2] Diffracted channeling radiation (DCR) till now has not been observed.

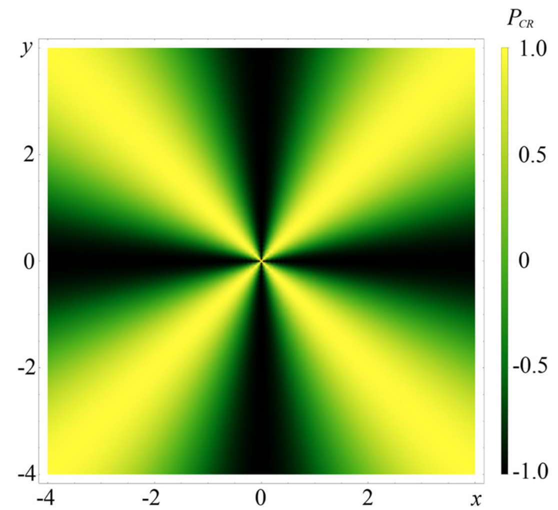

Fig. 4 Arbitrary contour plot of the degree of polarisation for CR by 10 MeV electrons channeled along Si ⟨100⟩ for a selected transverse plane shown in Fig. 1

and the polarisation directions

$$\tan\phi_{CR} = \Lambda\frac{\Im(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)}{\Re(\alpha_{if}\cdot\boldsymbol{\epsilon}^*_\kappa)} \tag{31}$$

Successfully, using the matrix element for the transition probability M_{if} from Eq. (12) and Eqs. (21), (28) we can calculate the degree and directions of polarisation for CR by axially channeled relativistic electrons. For the degree of polarisation we obtain

$$P_{CR} = \frac{\sin^2 2\Phi(3+\beta(\cos 2\Theta - 4\cos\Theta)^2 - 16\cos^2 2\Phi(1-\beta\cos\Theta)^2}{\sin^2 2\Phi(3+\beta(\cos 2\Theta - 4\cos\Theta)^2 + 16\cos^2 2\Phi(1-\beta\cos\Theta)^2}, \tag{32}$$

while for the directions of CR polarisation -

$$\phi_{CR} = \arctan\left(\frac{4(\beta\cos\Theta - 1)\cot 2\Phi}{\Lambda(\beta(\cos 2\Theta + 3) - 4\cos\Theta)}\right) \tag{33}$$

The angular dependence of the degree of polarisation calculated by Eq. (32) is presented in Fig. 4.

Comparison of Figs. 2 and 4 indicates that CR exhibits a degree of polarisation close to the maximum $P_{CR} = -1$ in the regions of maximum intensity.

The map of the angular directions of polarisation CR generated by 10 MeV electrons channeled along Si ⟨100⟩ axes has been drawn according the formula (33) and shown in Fig 5. The calculated scheme points out that the maximum radiation intensities can be observed when the CR polarisation planes are directed along the y-axis.

One can underline that, using the equation $\Lambda^2 = 1$, the scalar function $(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)$ can be written in the form

$$(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa) = e^{\mathbf{i}\varphi}\sqrt{\Re(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)^2 + \Im(\alpha_{if}\boldsymbol{\epsilon}^*_\kappa)^2}\,, \tag{34}$$

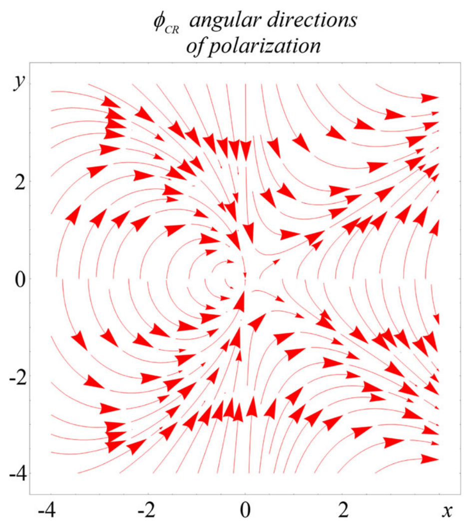

Fig. 5 The map of angular directions of polarisation CR for photons with positive helicity $\Lambda = 1$ (for $\Lambda = -1$ the directions will be opposite) generated by 10 MeV electrons channeled along the $\langle 100 \rangle$ axes in Si for selected transverse plane is shown in Fig. 1

where φ is the main value of the function

$$\varphi = \arctan\left(\Lambda \frac{\Im(\alpha_{if} \cdot \boldsymbol{\epsilon}^*_{\kappa})}{\Re(\alpha_{if} \cdot \boldsymbol{\epsilon}^*_{\kappa})}\right), \tag{35}$$

that allows the emission polarisation to be analysed in a typical way. Indeed, in this case the emission intensity does not depend on the emitted photons helicity Λ. The latter essentially simplifies its calculation.

6 Conclusions

Since the first works dedicated to the interaction of light relativistic particles in aligned crystals, the influence of the crystal structure on the interaction potential has been mostly studied in order to define various scattering mechanisms of the beams in crystals. These results obtained within the classical and quantum approaches allowed the life-time in the channeling bound state[3] to be evaluated with a scope to calculate the collimation or radiation abilities of *a beam-aligned crystal system* (see, for instance, in [38]). For such calculations carried out for highly relativistic energies, in general, a complex field structure revealed at the fine diffraction pictures of particles scattering in crystals is not of a decisive importance. It is even less weight for the case of planar channeling, while at axial channeling we can observe very reach schemes of particles angular scattering. The latter should essentially contribute to the formation of a structured angular distribution of channeled particles for various quantum levels of the transverse bound motion that can be registered as fine angular and polarisation peculiarities in CR at axial channeling of MeV electrons.

In our work the use of the Bloch wave function formalism applied to the transverse motion of channeled particles has permitted performing analytical calculations of CR at axial channeling of relativistic electrons in thin crystals taking into account a strong angular redistribution of the beam[4]. The method has become rather precise being applied for the interaction potential formed by the crystal axes constructed by fitting the measured electron form-factors.

Evaluating the possibility of observing the features of the angular distribution of channeled radiation calculated by us, it should be noted that it is usually believed that for low-energy particles the angular distributions of radiation will be quickly smeared by above-barrier particles arising due to incoherent scattering. An estimate of the dechanneling length for 10 MeV electrons at axial channeling in Si $\langle 100 \rangle$ gives the value $L_d = 0.215$ µm (see the expression in [18]). Obviously, this is rather short distance for 10 MeV electrons. However, during the time of flight of even this distance, electrons in any case generate CR, which can be registered experimentally; the emission of dechanneled electrons in the direction of the channeling axes is essentially suppressed. As seen from our expressions, the angular distribution profile is prescribed by the function $F_{ang}(\Theta, \Phi)$. The intensity of CR can be increased if to use the electrons of higher energies, which are characterised by larger dechanneling length; for instance, we get $L_d = 1.1$ µm for 50 MeV electrons channeled in Si $\langle 100 \rangle$. As seen, even for a bit higher electron energy we deal with very thin crystals (~ 1 µm) that proves the feasibility of recording the picture similar of one presented in Fig. 1.

Reported calculations based on the newly deduced formulas for the radiation intensity show that the CR angular distributions have a much more complex structure in comparison with the results obtained in the approximation of a single crystal axes. This structure is associated with the crystal symmetry in general and in the transverse plane as well. So, for example, for a Si $\langle 100 \rangle$ crystal, four peaks should be observed in the angular distributions, located symmetrically about the crystal axis. The CR angular distribution geometry uniquely demonstrates an explicit dependence of the CR probability on the azimuthal angle.

The radiation angular distribution in the case of axial channeling has no axial-symmetry in the plane perpendicular to the crystal axis. Axially symmetric angular distribution is

[3] Typically, at channeling we deal with a number of transverse bound levels, which define the motion of the beams in the field of aligned crystallographic planes and axes. In general, the beam channeling can be characterised as a bound motion in the crystal field.

[4] The calculations were performed without taking into account inelastic scattering that is valid for a thin crystal. In thick crystals, on the contrary, inelastic events can essentially contribute to the scattering process resulting in evident smoothing the structured pictures (*blurring*).

typically obtained in a single-string approximation (no interference between separated axes), while the influence of adjacent axes leads to a more complex structure. The estimates can be performed in dipole approximation. However, the Bloch-function technique enables getting precise analytical expressions for the matrix elements of CR. But, it should be underlined that the obtained matrix elements contain additional components for the wave vector of CR-photons, which make extra extended the analytical expressions and, in turn, significantly increase the determination time for the radiation distribution.

A fundamentally new result of our work points out that the radiation itself in the case of axial channeling in a crystal does not exhibit circular polarisation, although the polarisation of each individual photon is circular. Accordingly, the tilt angles of the polarisation planes depend on the photons helicity. It is also notable that even the numerical results are obtained for 10 MeV electrons, our conclusions should be valid for any crystals and particles at a wide range of energies (up to ∼1 GeV).

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