



Gravitational waves from preheating in Gauss–Bonnet inflation

K. El Bourakadi^{1,2,a}, M. Ferricha-Alami^{1,2}, H. Filali², Z. Sakhi^{1,2}, M. Bennai^{1,2}

¹ Physics and Quantum Technology Team, LPMC, Ben M'sik Faculty of Sciences, Casablanca Hassan II University, Casablanca, Morocco

² LPHE-MS Laboratory Department of Physics, Faculty of Science, Mohammed V University in Rabat, Rabat, Morocco

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Abstract We study gravitational wave production in an expanding Universe during the first stages following inflation, and investigate the consequences of the Gauss–Bonnet term on the inflationary parameters for a power-law inflation model with a GB coupling term. Moreover, we perform the analyses on the preheating parameters involving the number of e-folds N_{pre} , and the temperature of thermalization T_{th} , and show that it's sensitive to the parameters n , and γ , the parameter γ is proposed to connect the density energy at the end of inflation to the preheating energy density. We set a correlation of gravitational wave energy density spectrum with the spectral index n_s detected by the cosmic microwave background experiments. The density spectrum Ω_{gw} shows good consistency with observation for $\gamma = 10^3$ and 10^6 . Our findings suggest that the generation of gravitational waves (GWs) during preheating can satisfy the constraints from Planck's data.

1 Introduction

In the very early stages of the Universe's evolution, inflation is the leading paradigm that was proposed to resolve issues namely flatness and horizon problems that appear in the standard big bang cosmological model. During inflation, tensor modes are produced from the amplification of initial quantum fluctuations into classical perturbations outside the Hubble radius, due to the accelerated expansion of the universe [1]. They may cause a *B*-mode polarization of the cosmic microwave background *CMB* photons. As a result, observations *CMB* can be used to constrain the amplitude of the tensor perturbations and inflationary models can be strongly constrained using the combination of n_s and r . In the first stage following inflation, preheating is characterized in most models by an explosive and non-perturbative generation of non-thermal fluctuations of the inflaton and other bosonic

fields connected to it [3]. In chaotic models of inflation, the inflaton decays via parametric resonant particle creation [4], accompanied by violent dynamics of non-linear inhomogeneous structures of the scalar fields [5]. Preheating can accelerate the thermalization of our universe since the inflaton energy can be transferred rapidly into radiation matter. Thus, This period of a rapid particle production is highly inhomogeneous and generically generates gravitational waves with large energy densities [6,7]. As a consequence, the detection of GWs generated during preheating can help us to test inflation and understand the process of reheating. According to general relativity, the current universe should be penetrated by a diffuse gravitational wave background (GWB) coming from several sources like relic stochastic backgrounds from the early universe, phase transitions, inflation, turbulent plasmas, cosmic strings, etc. [8]. These backgrounds have very different spectral shapes and amplitudes that may, in the future, allow gravitational wave observatories like LIGO, LISA, BBO, or DECIGO [8] to disentangle their origin. Cosmological gravitational wave background could potentially carry original and pure information about the universe at early times. For low energy scale inflationary models, the frequencies of the gravity waves generated after inflation may occur in the range that can be detected in theory by direct detection tests, providing us with a channel for verifying inflation from the *CMB* data.

Another extended theory of inflation that has been studied is a scalar field coupled to the Gauss–Bonnet combination of quadratic curvature scalars R_{GB}^2 [9,10]. In this paper, we study whether a family of such theories, specifically the Gauss–Bonnet theory coupled with functions of a scalar field, may accurately predict inflationary dynamics compatible with current observational constraints on the parameters of these theories. It was also claimed that the temperature of reheating and the equation-of-state (EoS) parameter during reheating can be probed by looking at the spectrum of the GW background [11,12]. Therefore, in this work, we consider inflationary models with a Gauss–Bonnet (GB) term

^ae-mail: k.elbourakadi@yahoo.com (corresponding author)

to estimate the energy spectrum of the PGW and to provide constraints on the preheating parameters.

The paper is organized as follows. In Sect. 2, we develop the basic equations that describe a GB inflation model. In Sect. 3, we calculate the expression for the energy spectrum of these GWs. We further perform constraints on the preheating parameters in Sect. 4. In Sect. 5, we convert the spectra into physical variables and describe gravitational waves from Planck’s measurements point of view. We conclude in Sect. 6.

2 The Gauss–Bonnet Model

We consider the following action that involves the Einstein–Hilbert term and the GB term coupled to a canonical scalar field ϕ through the coupling function $\xi(\phi)$ [9, 13],

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] - \int d^4x \sqrt{-g} \left[V(\phi) + \frac{1}{2} \xi(\phi) R_{GB}^2 \right] \tag{1}$$

where $R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the GB term and $\kappa^2 = 8\pi G = M_p^{-2}$. The model is hence specified by two arbitrary functions, the potential $V(\phi)$ and the Gauss–Bonnet coupling $\xi(\phi)$. The background dynamical equations for inflation with the GB term which couples to a scalar field ϕ in a spatially flat FRW Universe are

$$\frac{3H^2}{\kappa^2} = \frac{1}{2} \dot{\phi}^2 + V(\phi) + 12\dot{\xi}H^3, \tag{2}$$

$$-\frac{2\dot{H}}{\kappa^2} = \dot{\phi}^2 - 4\dot{\xi}H^2 - 4\dot{\xi}H(2\dot{H} - H^2), \tag{3}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + 12\xi_{,\phi}H^2(\dot{H} + H^2) = 0, \tag{4}$$

the dot represents a derivative with respect to the cosmic time t , $H = \dot{a}/a$ denotes the Hubble parameter, and $V_{,\phi} = \partial V/\partial\phi$, $\xi_{,\phi} = \partial\xi/\partial\phi$, ξ is a function on ϕ , and $\dot{\xi} = \xi_{,\phi}\dot{\phi}$.

The so-called slow-roll parameters are expressed in terms of the potential and the coupling functions as

$$\epsilon \approx \frac{Q}{2} \frac{V_{,\phi}}{V}, \tag{5}$$

$$\eta \approx -Q \left(\frac{V_{,\phi\phi}}{V_{,\phi}} - \frac{V_{,\phi}}{V} + \frac{Q_{,\phi}}{Q} \right), \tag{6}$$

$$\delta_1 \approx -\frac{4}{3} \xi_{,\phi} Q V, \tag{7}$$

$$\delta_2 \approx -Q \left(\frac{\xi_{,\phi\phi}}{\xi_{,\phi}} + \frac{V_{,\phi}}{V} + \frac{Q_{,\phi}}{Q} \right), \tag{8}$$

with $Q \equiv V_{,\phi}/V + (4/3)\xi_{,\phi}V$. The potential and the coupling function can also be used to define the e-folding number N at the horizon exit before the completion of inflation,

$$N_k \approx \int_{\phi_{end}}^{\phi_k} \frac{3V}{3V_{,\phi} + 4\xi_{,\phi}V^2} \kappa^2 d\phi = \int_{\phi_{end}}^{\phi_k} \frac{\kappa^2}{Q} d\phi, \tag{9}$$

where the subscripts “ k ” and “ end ” respectively indicate the moment when a mode k crosses the horizon and the end of inflation.

The spectral indices of scalar and tensor perturbations n_s , and tensor-to-scalar ratio r are calculated as [14]

$$n_s - 1 \simeq -2\epsilon - \frac{2\epsilon(2\epsilon + \eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}, \tag{10}$$

$$r \simeq 8(2\epsilon - \delta_1). \tag{11}$$

Choosing the form of the potential $V(\phi)$ and the coupling function $\xi(\phi)$ and using Eqs. (5–8), (10–11), the theoretical predictions of any particular inflation model can be verified using observational data [2].

2.1 Power-law model with inverse monomial coupling

Let us consider a power-law model of GB inflation with inverse monomial coupling. The inflaton potential and the coupling function are given by

$$V(\phi) = V_0(\kappa\phi)^n, \quad \xi(\phi) = \xi_0(\kappa\phi)^{-n}, \tag{12}$$

here, n is assumed to be positive, and V_0, ξ_0 are a dimensionless constants. This model has received a lot of attention [9, 14], were they establish an analytic relationship between the spectral index of curvature perturbations and the tensor-to-scalar ratio thanks to the specific choice of GB coupling. From Eqs. (5)–(8) and (9), the observable quantities in Eqs. (10–11) can be obtained in terms of N_k as

$$n_s - 1 = -\frac{2(n+2)}{4N_k+n}, \quad r = \frac{16n(1-\alpha)}{4N_k+n}, \tag{13}$$

where $\alpha \equiv 4V_0\xi_0/3$. For that case, we conclude that such a specific choice of GB coupling allows us to find an analytic relation between r, n_s in terms of N_k .

In Fig. 1, we consider the usual power-law inflation, where the inflaton field is coupled with the Gauss–Bonnet term through the inverse monomial model. It is apparent from these plots that increasing the value of α to 0.2 makes the decreasing function obtain from our model in certain regions consistent with the latest observations in certain regions for $n = 1, 2$, and 3.

3 Primordial gravitational waves

The intense production of matter fields after inflation can promote substantial metric changes. However, we are only interested in the evolution of the transverse-traceless satisfied by the metric perturbation h_{ij} , and the generation of GWs during preheating. Therefore, GWs can be represented by

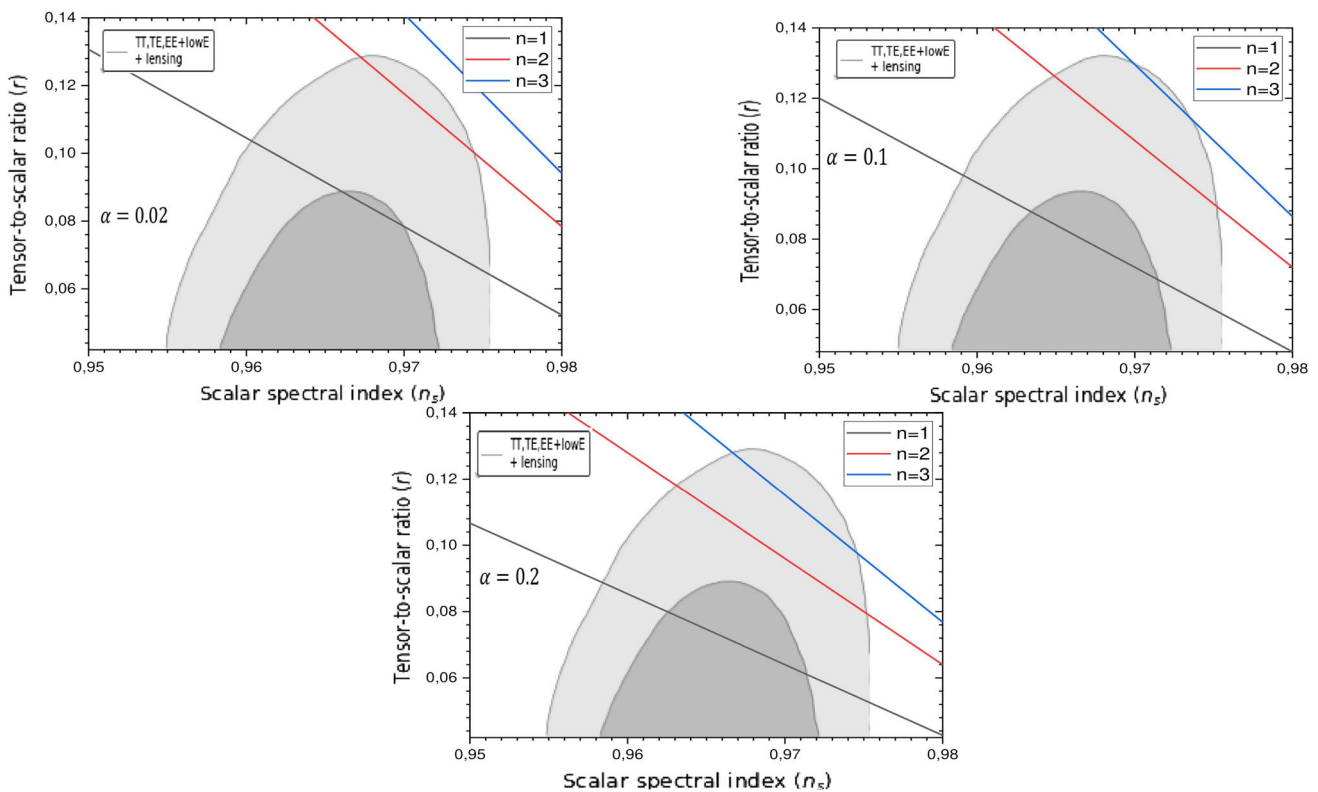


Fig. 1 r versus n_s plot for monomial inflation with a Gauss–Bonnet coupling function. Inner and outer shaded regions are 1σ and 2σ constraints from Planck respectively. We choose three different values of n , for the black line $n = 1$, the red line represents $n = 2$ and for the blue line, $n = 3$

the traceless part of the spatial metric perturbations in the FRW background [15, 16]:

$$ds^2 = g_{ij}dx^i dx^j = -dt^2 + a(t)^2 (\delta_{ij} + h_{ij}) dx^i dx^j. \tag{14}$$

The perturbation h_{ij} satisfies the transverse-traceless (TT) conditions: $\partial_i h_{ij} = h_{ii} = 0$, and has the equation of motion

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = 2\kappa^2 S_{ij}^{TT}, \tag{15}$$

the source term S_{ij}^{TT} is the transverse-traceless of the anisotropic stress S_{ij} .

3.1 Gravitational wave energy density

The energy density power spectrum of GWs sourced by the inhomogeneous decay of the symmetry breaking field, can be defined as the energy density averaged over a volume V of several wavelengths size [3], This energy density carried by GWs can be calculated through the following Eq. [17]

$$\rho_{gw} = \frac{1}{4\kappa^2} \langle \dot{h}_{ij}(t, \mathbf{x}) \dot{h}_{ij}(t, \mathbf{x}) \rangle. \tag{16}$$

The strength of GW is characterized by their energy spectrum, which represents the abundance of gravity wave energy density today, is given as

$$h^2 \left(\frac{\rho_{gw,0}}{\rho_{c,0}} \right) = \int \frac{df}{f} h^2 \Omega_{gw,0}(f), \tag{17}$$

which can be rewritten as

$$h^2 \Omega_{gw,0}(f) = \frac{h^2}{\rho_{c,0}} \frac{d\rho_{gw,0}}{d \ln f}, \tag{18}$$

where f is the frequency and $\rho_{c,0} = 3H_0^2/(8\pi G)$ is the critical energy density today.

Next, we need to consider the evolution of the scale factor during preheating that is parameterized by an e -folds number N_{pre} and test its dependency on the evolution of the equation of state. In general, when the inflaton field oscillates around its minimum, the equation of state jumps from $\omega = 0$ to an intermediate value close to $\omega = 1/3$ during preheating [18, 19].

4 Preheating constraints

The process of preheating happens in the early stages of the Universe’s evolution. This is thought to be necessary because the universe cools as it expands. As a result, there must be a period immediately following inflation to allow it to thermally prepare for the next step, which we call preheating. Preheating occurs due to the interaction of massless scalar

field χ with the oscillating inflaton field, which causes it to grow exponentially fast, as a result of parametric resonance, followed by a stage of thermal equilibrium (reheating) [20]. To extract information about preheating we need to consider the phase between the time observable CMB scales crossed the horizon and the present time. Deferents eras occurred throughout this length of time which can be described by the following equation :

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{end}} \frac{a_{end}}{a_{pre}} \frac{a_{pre}}{a_{th}} \frac{a_{th}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_{eq}}{H_0} \frac{H_k}{H_{eq}}, \tag{19}$$

here, $a_0, a_k, a_{end}, a_{pre}, a_{th}$ end a_{eq} respectively correspond to the scale factor at present, time of horizon crossing, end of inflation, end of preheating, and the time of thermal equilibrium, and finally, the end of the matter and radiation equality era, whereas H_0 and H_{eq} are the Hubble constant at present time and the time of matter and radiation equality. Taking the number of e-folds N into consideration, we can rewrite Eq. (19) as

$$\ln\left(\frac{k}{a_0 H_0}\right) = -N_k - N_{pre} - N_{th} + \ln\frac{a_{th}}{a_0} + \ln\frac{H_k}{H_0}, \tag{20}$$

the number of e-folds between the time when a mode exits the horizon and the end of inflation is parametrized by $N_k = a_{end}/a_k$, and $N_{pre} = a_{pre}/a_{end}$ is the duration from the end of inflation to the end of preheating, finally, $N_{th} = a_{th}/a_{pre}$ is the number of e-folds between the end of preheating and the thermal equilibrium (end of reheating). Our goal is to calculate the preheating duration N_{pre} in terms of inflationary parameters. Considering that no entropy production occurred after the thermal equilibrium was completed, one can write [24]

$$\frac{a_{th}}{a_0} = \frac{T_0}{T_{th}} \left(\frac{43}{11g_*}\right)^{\frac{1}{3}}, \tag{21}$$

where T_0 is the current temperature of the Universe, and T_{th} is the thermal equilibrium temperature of reheating, the energy density ρ_{th} at the end of reheating is defined as:

$$\rho_{th} = \frac{\pi^2}{30} g_* T_{th}^4, \tag{22}$$

where g_* is the number of relativistic degrees of freedom at the end of reheating. Using the expressions $\rho_{th} \propto a_{th}^{-3(1+\omega)}$ and $\rho_{end} \propto a_{end}^{-3(1+\omega)}$ that respectively corresponds to the reheating and inflation energy densities. We assume that the energy density at the end of inflation and preheating energy density are related by a parameter γ

$$\rho_{end} = \gamma \rho_{pre} = \gamma a_{pre}^{-3(1+\omega)}, \tag{23}$$

then,

$$\frac{\rho_{end}}{\rho_{th}} = \gamma \left(\frac{a_{pre}}{a_{th}}\right)^{-3(1+\omega)}, \tag{24}$$

writing this in terms of e-foldings, one can obtain

$$\rho_{th} = \frac{\rho_{end}}{\gamma} e^{-3(1+\omega) N_{th}}, \tag{25}$$

the energy density of inflation ρ_{end} is determined by the potential at the end of inflation V_{end} and λ_{end} given as follows :

$$\rho_{end} = \lambda_{end} V_{end}, \tag{26}$$

the effective ratio of kinetic energy to potential energy λ_{end} is calculated from the GB field Eq. (4) [21]:

$$\lambda_{end} = \left(\frac{6}{6 - 2\epsilon - \delta_1 (5 - 2\epsilon + \delta_2)}\right)_{\phi=\phi_{end}}. \tag{27}$$

We can derive a total duration using Eqs. (20–22), and (25):

$$\begin{aligned} N_{pre} + \frac{1 - 3\omega}{4} N_{th} &= -\ln\left(\frac{k}{a_0 T_0}\right) - \frac{1}{3} \ln\left(\frac{11\bar{g}_*}{43}\right) \\ &\quad - \frac{1}{4} \ln\left(\frac{30\lambda_{end}}{\gamma\pi^2\bar{g}_*}\right) - \frac{1}{4} \ln\left(\frac{V_{end}}{H_k^4}\right) - N_k, \end{aligned} \tag{28}$$

this expression is not defined in the value of (EoS) $\omega = 1/3$. According to [2], a numerical values can be obtain : $M_p = \kappa^{-1} = 2.435 \times 10^{18} GeV$, $a_0 = 1$, $T_0 = 2.725K$, $\bar{g}_* \simeq 106.75$, $k = 0.05 Mpc^{-1}$, which reduces Eq. (28) to

$$\begin{aligned} N_{pre} &= \left[60.0085 - \frac{1}{4} \ln\left(\frac{3\lambda_{end}}{100\gamma\pi^2}\right) - \frac{1}{4} \ln\left(\frac{V_{end}}{H_k^4}\right) - N_k \right] \\ &\quad - \frac{1 - 3\omega}{4} N_{th}, \end{aligned} \tag{29}$$

with N_{th} , the reheating duration can be obtained from Eqs. (22) and (25)

$$N_{th} = \frac{1}{3(1 + \omega)} \ln\left(\frac{\lambda_{end} V_{end}}{\frac{\gamma\pi^2}{30} \bar{g}_* T_{th}}\right). \tag{30}$$

The duration of preheating N_{pre} is linked to the inflationary quantities through λ_{end} , V_{end} , N_k , and H_k . These quantities need to be calculated for the model we considered previously in this work. In addition to that, the preheating duration is also described by a parameter γ we defined previously, that connects the energy density at the end of inflation ρ_{end} to the preheating energy density ρ_{pre} . N_{th} can be calculated considering the final reheating thermalization temperature as [22] $T_{th} > 10^{12} GeV$. Inflation ended when the value of ω became larger than $-1/3$, in order to satisfy the condition of density energy dominance and preserve the causality ω must be smaller than 1, when reheating is finished the (EoS) reached 1/3, for this reason, we will test if the choice of

specific values of (EoS) parameter has effects on preheating duration.

Inflation ends when the slow-roll parameters ϵ, δ_1 become as $\epsilon(\phi_{end}) = 1, \delta_1(\phi_{end}) = 1$. One can calculate V_{end} and λ_{end} using Eqs. (5–8), and (27) which gives:

$$V_{end} = \frac{V_0}{\kappa^4} \left[\frac{n^2}{2}(1 - \alpha) \right]^{\frac{n}{2}}, \tag{31}$$

$$\lambda_{end} = -\frac{3n}{4\alpha(n + 1) - 2n}. \tag{32}$$

The Hubble parameter at the time of horizon from the slow-roll approximations $3H_k^2 \approx \kappa^2 V(\phi_k)$, is obtained by calculating $\phi_k(N_k)$ from Eq. (9) taking into account the large field inflation case ($\phi_k \gg \phi_{end}$)

$$\kappa\phi_k = \sqrt{\frac{n}{2}(1 - \alpha)(4N_k + n)}, \tag{33}$$

as a result

$$H_k^4 = \left(\frac{V_0}{3\kappa^2} \right)^2 \left[\frac{n}{2}(1 - \alpha)(4N_k + n) \right]^n. \tag{34}$$

It can be seen from previous results that $N_k, V_{end}, \lambda_{end}$ and H_k are all expressed in terms of spectral index n_s, α , and n . Hence N_{pre} can be obtained as a function of n_s from Eq. (29).

Figure 2 show the variation of the e-folds number during preheating as a function of spectral index n_s . We choose the three values $n = 1, 2$ and 3 . Each curve fall at a point that corresponds to an instantaneous preheating ($N_{pre} \rightarrow 0$), we should mention here that the preheating duration is independent of the choice of the (EoS) value ω . As depicted in the Figure, for $\gamma = 10^3, 10^6$, the case $n = 3$ completely lies outside the Planck bounds on $n_s = 0.9649 \pm 0.0042$ [2], in order to satisfy observations, n must be bounded as $n < 3$.

5 Gravitational waves from preheating

Since we are interested in the correlation of gravity-wave energy density spectrum with current observations, We must translate the previous GW spectrum into current physical quantities. The present scale factor in comparison to the one when GW production stops can be expressed as [3, 15]

$$\frac{a_{end}}{a_0} = \frac{a_{end}}{a_{pre}} \left(\frac{a_{pre}}{a_{th}} \right)^{1 - \frac{3}{4}(1 + \omega)} \left(\frac{\bar{g}_*}{\bar{g}_0} \right)^{-1/12} \left(\frac{\rho_{r,0}}{\rho_*} \right)^{1/4}. \tag{35}$$

Supposing that GW production stops at the end of preheating, pre represents the time when GW production is finished, “0” and “ th ” represent the present and the time when thermal equilibrium is reached, respectively. While $\rho_{r,0}$ is the present radiation energy density and the total energy

density of the scalar field is represented by ρ_* . We define $\bar{g}_*/\bar{g}_0 \simeq 106.75/3.36 \simeq 31$. ω is the equation of state which in the Ref. [23] it has been shown that ω reaches $1/3$ just after preheating, that means that $(a_{pre}/a_{th})^{1 - 3/4(1 + \omega)} = 1$ since $\omega = 1/3$. From Eq. (35), the corresponding physical frequency today is given by

$$f = \frac{k}{2\pi a_0} = \frac{k_0}{a_{end}\rho_*^{1/4}} \times \left(4 \times 10^{10} \text{ Hz} \right), \tag{36}$$

let us denote $k_0 = k/a_0$. Knowing that the abundance of radiation today given as $\Omega_{r,0}h^2 = h^2\rho_{r,0}/\rho_{c,0}$, with h , is the present dimensionless Hubble constant and $\Omega_{gw,0}h^2 \propto 1/a_0^4$ [3]. Using Eq. (35) and Because GW decays like radiation with cosmic expansion, one can calculate the present GW spectra [15]

$$\Omega_{gw,0}h^2 = \Omega_{gw}(f) \frac{a_{end}}{a_{pre}} \left(\frac{\bar{g}_*}{\bar{g}_0} \right)^{-1/3} \Omega_{r,0}h^2. \tag{37}$$

The number of e-folds between the end of inflation to the time when preheating completed can be written as

$$\frac{a_{end}}{a_{pre}} = e^{-N_{pre}}, \tag{38}$$

to obtain the final form of gravity-wave energy density spectrum, given as follows

$$\Omega_{gw}(f) = \frac{\Omega_{gw,0}h^2}{\Omega_{r,0}h^2} \left(\frac{\bar{g}_*}{\bar{g}_0} \right)^{1/3} e^{4N_{pre}}. \tag{39}$$

From Fig. 3 the variation of Ω_{gw} as a function of N_{pre} for some fixed values of $\Omega_{gw,0}h^2$ are presented, we plotted the energy density spectrum Ω_{gw} as a function of the preheating duration N_{pre} , taking the present GW spectra to be $3.36 \times 10^{-7} \leq \Omega_{gw,0}h^2 \leq 1.85 \times 10^{-6}$, when $N_{pre} \rightarrow 0$ the GW energy density takes an initial value for all the cases with different $\Omega_{gw,0}h^2$. When we increase the present GW spectra, the initial values that correspond to $\Omega_{gw}(N_{pre} = 0)$ increases as well.

The variation of the GW density spectrum with respect to the spectral index n_s is shown in Fig. 4. Considering different values of the current GW spectra $\Omega_{gw,0}h^2$, we plot Ω_{gw} considering the expansion of the universe from the end of inflation up to later times of preheating, because of the higher final temperature of reheating $T_{th} > 10^{12} \text{ GeV}$, the duration N_{th} from Eq. (30) could be considered as instantaneous, which make preheating duration minimally dependent on the (EoS) parameter ω as observed in Eq. (29). We choose the most compatible case from the previous analysis which favors the case $n = 2$ and consider the two values of $\gamma : 10^3$, and 10^6 . It’s easy to see that the density spectrum curves with both values of γ are compatible with observations according to Planck’s results, the curves decrease away from the observation bound when the GW energy density became very negligible $\Omega_{gw} \rightarrow 0$. For the case $\gamma = 10^3$,

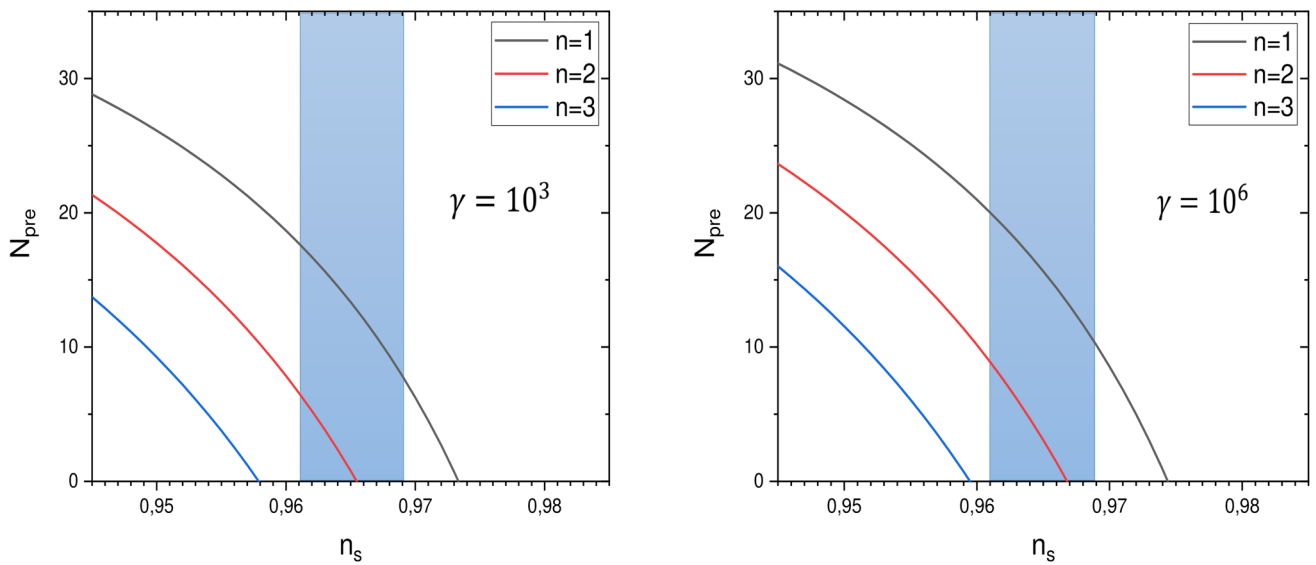
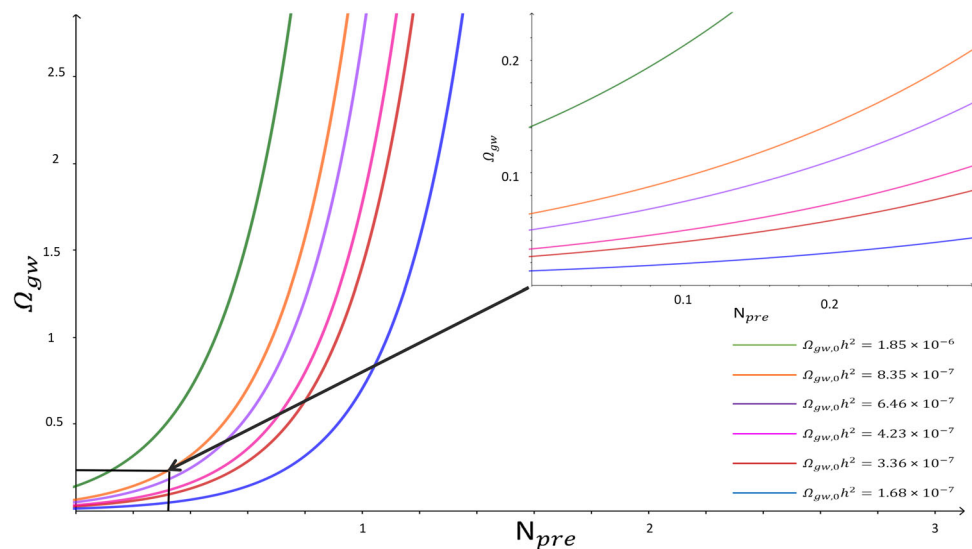


Fig. 2 The figure shows N_{pre} the length of preheating, as a function of n_s choosing the values of γ to be $\gamma = 10^3$ and $\gamma = 10^6$ for power-law potential with an inverse monomial coupling. Here, the vertical light

blue region represents Planck’s bounds on n_s [2]. The black line corresponds to $n = 1$, the red line corresponds to $n = 2$, and the blue line corresponds to $n = 3$

Fig. 3 The variation of the density spectra of GWs as a function of N_{pre} . For different values of gravity wave energy density $\Omega_{gw,0}h^2$



with $\Omega_{gw} \geq 4 \times 10^{-3}$ all the lines with different $\Omega_{gw,0}h^2$ tends towards the value of spectral index $n_s = 0.966$. However, the case where $\gamma = 10^6$ the curves converge to the value $n_s = 0.968$ when $\Omega_{gw} \geq 1.8 \times 10^{-5}$.

6 Conclusion

After we review the basic equations that describe GB inflation, we discuss the power-law model of inflation with inverse monomial GB coupling. The expression of the observational parameters n_s , and r were calculated, we computed these parameters as functions of inflation e -folds N_k for the power-law potential with an inverse monomial model. We review the basics of Primordial GWs, then the energy density carried by

these waves was calculated. We derived the preheating duration as functions of inflationary Gauss–Bonnet parameters in Eq. (29), and consider the thermalization temperature as $T_{th} > 10^{12}$ GeV. We numerically estimated the preheating parameters using our analytic results. Knowing that it’s independent of the choice of the (EoS), the duration of preheating is plotted as a function of the spectral index for the model we considered previously, and showed that it’s sensitive to the parameters γ and n . We finally calculated the gravity-wave energy density spectrum as a function of the durations N_{pre} , which is a possible way to study GW density spectrum according to recent Planck’s results. Assuming the density parameter $\Omega_{gw,0}h^2$ to be $3.36 \times 10^{-7} \leq \Omega_{gw,0}h^2 \leq 1.85 \times 10^{-6}$, we chose $n = 2$ and $\gamma : 10^3, 10^6$, and found

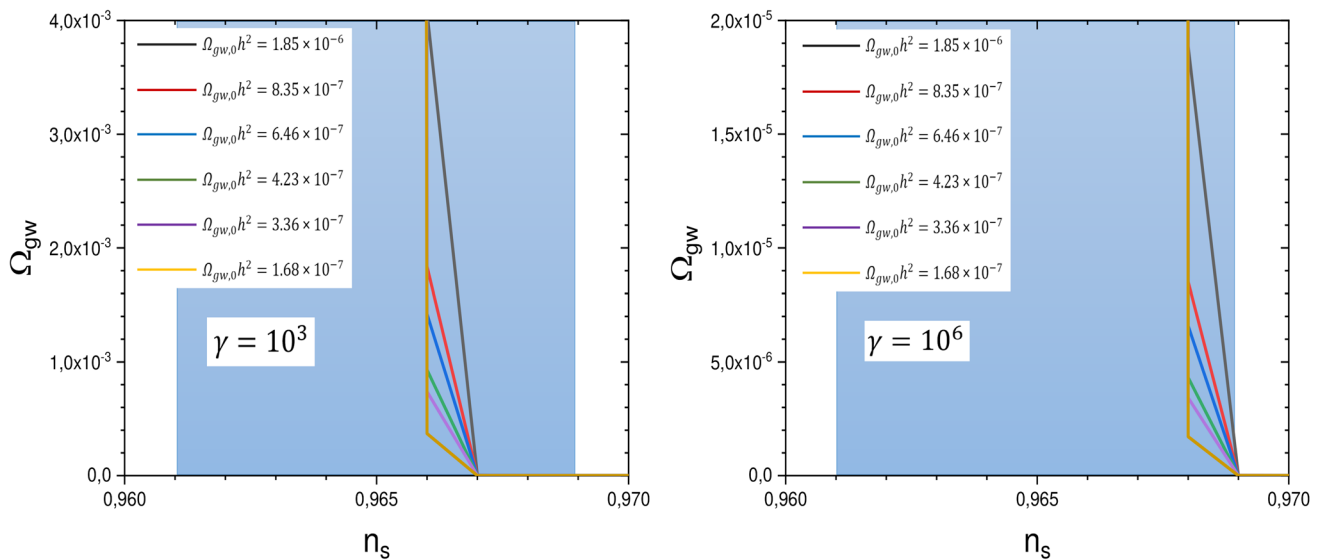


Fig. 4 The variation of the density spectra of GWs as a function of n_s . For different values of gravity wave energy density $\Omega_{\text{gw},0}h^2$, we choose $3.36 \times 10^{-7} \leq \Omega_{\text{gw},0}h^2 \leq 1.85 \times 10^{-6}$. The black line corresponds to $\Omega_{\text{gw},0}h^2 = 1.85 \times 10^{-6}$, red line corresponds to

$\Omega_{\text{gw},0}h^2 = 8.35 \times 10^{-7}$, blue line corresponds to $\Omega_{\text{gw},0}h^2 = 6.46 \times 10^{-7}$, green line corresponds to $\Omega_{\text{gw},0}h^2 = 4.23 \times 10^{-7}$, purple line corresponds to $\Omega_{\text{gw},0}h^2 = 3.36 \times 10^{-7}$, and yellow line corresponds to $\Omega_{\text{gw},0}h^2 = 1.68 \times 10^{-7}$

that both cases where $\gamma = 10^3, 10^6$ show good consistency with observation. We conclude that the GB term appears to be important not only during inflation but also during later phases such as preheating, regardless of whether the process is instant or takes a certain number of e-folds to complete, once we determine the final temperature of thermalization T_{th} , other preheating parameters are determined using a variety of inflation models. As a result, it would be interesting to investigate the physics of preheating in the context of PGW.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: Our results are based on the observational parameters provided by the recent Planck's data which we choose to constrain our model of preheating in addition to the GWs produced during this stage.]

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