Purely leptonic decays of the ground charged vector mesons

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Abstract The study of the purely leptonic decays of the ground charged vector mesons is very interesting and significant in determining the CKM matrix elements, obtaining the decay constant of vector mesons, examining the lepton flavor universality, and searching for new physics beyond the standard model. These purely leptonic decays of the ground charged vector mesons are induced by the weak interactions within the standard model, and usually have very small branching ratios, $\mathcal{B}(\rho^- \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-13})$, $\mathcal{B}(K^{*-} \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-13})$, $\mathcal{B}(D_d^{*-} \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-10})$, $\mathcal{B}(B_u^{*-} \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-10})$, $\mathcal{B}(D_s^{*-} \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-6})$ and $\mathcal{B}(B_c^{*-} \rightarrow \ell^- \nu_\ell) \sim \mathcal{O}(10^{-6})$. Inspired by the potential prospects of LHCb, Belle-II, STCF, CEPC and FCC-ee experiments, we discussed the probabilities of experimental investigation on these purely leptonic decays. It is found that the measurements of these decays might be possible and feasible with the improvement of data statistics, analytical technique, and measurement precision in the future. (1) With the hadron-hadron collisions, the purely leptonic decays of ρ^- , K^{*-} , $D_{d,s}^{*-}$ and $B_{u,c}^{*-}$ mesons might be accessible at LHC experiments. (2) With the e^+e^- collisions, the purely leptonic decays of $D_{d,s}^{*-}$ and $B_{u,c}^{*-}$ mesons might be measurable with over 10^{12} Z^0 bosons available at CEPC and FCC-ee experiments. In addition, the $D_{d,s}^{*-} \rightarrow \ell^- \nu_\ell$ decays could also be studied at Belle-II and SCTF experiments.

1 Introduction

In the quark model [1–3], mesons are generally regarded as bound states of the valence quark q and antiquark $\bar{q}'$. The classifications of mesons are usually based on the spin-parity quantum number J^P of the $q\bar{q}'$ system. The spin J of meson is given by the relation $|L - S| \leq J \leq |L + S|$. The orbital angular momentum and total spin of the $q\bar{q}'$ sys-

tem are respectively L and S , where $S = 0$ for antiparallel quark spins, and $S = 1$ for parallel quark spins. By convention, quarks have a positive parity and antiquarks have a negative parity. Hence, the parity of meson is $P = (-1)^{L+1}$. The $L = 0$ states are the ground-state pseudoscalars with $J^P = 0^-$ and vectors with $J^P = 1^-$. Both quarks and leptons are fermions with spin $S = 1/2$. Mesons are composed of a pair of fermions – quark and antiquark, therefore, they could in principle decay into a pair of fermions, for example, lepton and antilepton. The experimental observation of the two-body purely leptonic decays of mesons could be a clear and characteristic manifestation of the quark model. These leptonic decays provide us with valuable opportunities to fully investigate the microstructure and properties of mesons. The study of two-body purely leptonic decays of mesons is very interesting and significant.

The valence quarks of the electrically charged mesons must have different flavors. Within the standard model (SM) of elementary particles, the purely leptonic decays of the charged mesons (PLDCM) are typically induced by the tree-level exchange of the gauge bosons W , the quanta of the weak interaction fields. Up to today, the masses of all the experimentally observed mesons are much less than those of W bosons. Consequently, the massive W bosons are virtual propagators rather than physical particles in the true picture of PLDCM. Phenomenologically, by integrating out the contributions from heavy dynamical degrees of freedom such as the W fields, PLDCM can be properly described by the low-energy effective theory in analogy with the Fermi theory for β decays. Considering the fact that leptons are free from the strong interactions, the corresponding effective Hamiltonian [4] for PLDCM could be written as the product of quark currents and leptonic currents,

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} [\bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2] [\bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell] + \text{h.c.}, \quad (1)$$

where the contributions of the W bosons are embodied in the Fermi coupling constant $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [1],

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and $V_{q_1 q_2}$ is the Cabibbo–Kobayashi–Maskawa (CKM) [5, 6] matrix element between the quarks in the charged mesons. The decay amplitudes can be written as,

$$\langle \ell \bar{\nu}_\ell | \mathcal{H}_{\text{eff}} | M \rangle = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} \langle \ell \bar{\nu}_\ell | \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell | 0 \rangle \times \langle 0 | \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 | M \rangle. \quad (2)$$

The leptonic part of amplitudes can be calculated reliably with the perturbative theory. The hadronic matrix elements (HMEs) interpolating the diquark currents between the mesons concerned and the vacuum can be parameterized by the decay constants.

With the conventions of Refs. [7, 8], the HMEs of diquark currents are defined as,

$$\langle 0 | \bar{q}_1(0) \gamma_\mu q_2(0) | P(k) \rangle = 0, \quad (3)$$

$$\langle 0 | \bar{q}_1(0) \gamma_\mu \gamma_5 q_2(0) | P(k) \rangle = i f_P k_\mu, \quad (4)$$

$$\langle 0 | \bar{q}_1(0) \gamma_\mu q_2(0) | V(k, \epsilon) \rangle = f_V m_V \epsilon_\mu, \quad (5)$$

$$\langle 0 | \bar{q}_1(0) \gamma_\mu \gamma_5 q_2(0) | V(k, \epsilon) \rangle = 0, \quad (6)$$

where the nonperturbative parameters of f_P and f_V are the decay constants of pseudoscalar P and vector V mesons, respectively; and m_V and ϵ_μ are the mass and polarization vector, respectively. To the lowest order, the decay widths are written as,

$$\Gamma(P \rightarrow \ell \bar{\nu}_\ell) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_P m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2, \quad (7)$$

$$\Gamma(V \rightarrow \ell \bar{\nu}_\ell) = \frac{G_F^2}{12\pi} |V_{q_1 q_2}|^2 f_V^2 m_V^3 \left(1 - \frac{m_\ell^2}{m_V^2}\right)^2 \left(1 + \frac{m_\ell^2}{2m_V^2}\right), \quad (8)$$

where m_P and m_ℓ are the masses of the charged pseudoscalar meson and lepton, respectively.

It is clearly seen from the above formula that the highly precise measurements of PLDCM will allow the relatively accurate determinations of the product of the decay constants and CKM elements, $|V_{q_1 q_2}| f_{P,V}$. Theoretically, the decay constants are nonperturbative parameters, and they are closely related with the $\bar{q}_1 q_2$ wave functions at the origin which cannot be computed from first principles. There still exist some discrepancies among theoretical results of the decay constants with different methods, such as the potential model, QCD sum rules, lattice QCD, and so on. If the magnitudes of CKM element $|V_{q_1 q_2}|$ are fixed to the values of Ref. [1], the decay constants $f_{P,V}$ will be experimentally measured, and be used to seriously examine the different calculations on the decay constants with various theoretical models. Likewise, if the decay constants $f_{P,V}$ are well known to sufficient precision, the magnitudes of the corresponding CKM element will be experimentally determined, and provide complementary information to those from other processes. Within SM, the $P \rightarrow \ell \bar{\nu}_\ell$ and $V \rightarrow \ell \bar{\nu}_\ell$ decays are

induced by the axial-vector current of Eq. (4) and vector current of Eq. (5), respectively; and the electroweak interactions assign the vector-minus-axial-vector ($V - A$) currents to the W bosons. The CKM elements determined from two different and complementary parts of the electroweak interactions, charged vector and axial-vector currents, could be independently examined. The latest CKM elements determined by PLDCM, such as $|V_{us}|$, $|V_{cd}|$ and $|V_{cs}|$, differ somewhat from those by exclusive and inclusive semileptonic meson decays [1]. The CKM elements extracted from various processes can be combined to test the electroweak characteristic charged-current $V - A$ interactions.

Within SM, the lepton-gauge-boson electroweak gauge couplings are generally believed to be universal and process independent, which is called lepton flavor universality (LFU). However, there are some hints of LFU discrepancies between SM predictions and experimental measurements, such as the ratios of branching fractions of semileptonic B decays $R(D^{(*)}) \equiv \mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell)$ with $\ell = e/\mu$ [1]. The LFU validity can be carefully investigated through the PLDCM processes. Beyond SM, some possible new heavy particles accompanied with novel interactions, such as the charged higgs bosons, would affect PLDCM and LFU, and might lead to detectable effects. So PLDCM provide good arenas to search for the smoking gun of new physics (NP) beyond SM.

By considering the angular momentum conservation and the final states including a left-handed neutrino or right-handed antineutrino, the purely leptonic decay width of charged pseudoscalar meson, Eq. (7), is proportional to the square of the lepton mass. This is called helicity suppression. While there is no helicity suppression for the purely leptonic decay of charged vector meson (PLDCV). From the analytical expressions of Eqs. (7) and (8), the decay width of pseudoscalar meson is suppressed by the factor m_ℓ^2/m_P^2 compared with that of vector meson. What's more, both the masses and the decay constants of vector mesons are relatively larger than those of corresponding pseudoscalar mesons, which would result in an enhancement of the decay widths for vector mesons. Of course, the vector mesons decay dominantly through the strong and/or electromagnetic interactions. The branching ratios for the PLDCV weak decays are usually very small, sometimes even close to the accessible limits of the existing and the coming experiments.

Inspired by the potential prospects of the future high-intensity and high-energy frontiers, along with the noticeable increase of experimental data statistics, the remarkable improvement of analytical technique and the continuous enhancement of measurement precision, the carefully experimental study of PLDCV might be possible and feasible. In this paper, we will focus on the PLDCV within SM to just provide a ready reference. The review of the purely leptonic

decays of charged pseudoscalar mesons can be found in Ref. [1].

2 $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays

The mass of the $\rho^\pm$ meson, $m_\rho = 775.11(34)$ MeV [1], is much larger than that of two-pion pair. The rate of the ρ meson decay into two pions via the strong interactions is almost 100%, which results in the very short lifetime $\tau_\rho \sim 4.4 \times 10^{-24}$ s [1]. The direct measurements of the electroweak properties of the ρ meson would definitely be very challenging. It is evident from Eq. (8) that the parameter of $|V_{ud}| f_\rho$ could be experimentally determined from the observations of decay widths for the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays (if it is not specified, the corresponding charge-conjugation processes are included in this paper), with the coupling constant G_F , the masses of lepton m_ℓ and meson m_ρ .

The precise values of the CKM element $|V_{ud}|$ in ascending order of measurement accuracy mainly come from β transitions between the super-allowed nuclear analog states with quantum number of both $J^P = 0^+$ and isospin $I = 1$, between mirror nuclei with $I = 1/2$, between neutron and proton, between charged and neutral pions [9]. These four results for $|V_{ud}|$ are basically consistent with one another. The result of the super-allowed $0^+ \rightarrow 0^+$ nuclear β transitions has an uncertainty a factor of about 10 smaller than the other results, and thus dominates the weighted average value [9]. The best value from super-allowed nuclear β transitions is $|V_{ud}| = 0.97370(14)$ [1], which is smaller compared with the 2018 value $|V_{ud}| = 0.97420(21)$ [10], as illustrated Fig. 1. This reduction of the value of $|V_{ud}|$ leads to a slight deviation from the first row unitarity requirement $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. The current precision of the CKM element $|V_{ud}|$ is about 0.01%. The latest value from the global fit in SM, $|V_{ud}| = 0.97401(11)$ [1], will be used in our calculation.

The decay constant f_ρ is a very important characteristics of the ρ meson. Compared with the CKM element $|V_{ud}|$, the present precision of decay constant f_ρ is still not very high and needs to be improved. Theoretically, the estimations from different methods are more or less different from each other and even calculations with the same method sometimes give the diverse results. Some theoretical estimations on the decay constant f_ρ are presented in Table 1. Experimentally, the decay constant f_ρ can be obtained from the 1-prong hadronic $\tau^\pm \rightarrow \rho^\pm \nu_\tau$ decay. The partial width for the $\tau \rightarrow V \nu_\tau$ decay is given by Ref. [37],

$$\Gamma(\tau \rightarrow V \nu) = \mathcal{S} \frac{G_F^2 m_\tau^3}{16\pi} |V_{q_1 q_2}|^2 f_V^2 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_V^2}{m_\tau^2}\right), \quad (9)$$

where the factor $\mathcal{S} = 1.0154$ includes the electroweak corrections [37–39]. With the mass $m_\tau = 1776.86(12)$ MeV and

Fig. 1 The values of the CKM element $|V_{ud}|$ from Particle Data Group (PDG)

lifetime $\tau_\tau = 290.3(5)$ fs [1], and branching ratio $\mathcal{B}(\tau \rightarrow \rho \nu) = 25.19(33)\%$ [1], one can easily extract the decay constant $f_\rho^{\text{exp}} = 207.7 \pm 1.6$ MeV, which agrees well with the latest numerical simulation result from lattice QCD $f_\rho = 208.5 \pm 5.5 \pm 0.9$ MeV [34]. The more accurate decay constant f_ρ^{exp} will be used in our calculation.

For the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays, one can obtain the PLDCV partial decay widths with Eq. (8) and the corresponding branching ratios with the full width $\Gamma_\rho = 149.1 \pm 0.8$ MeV [1],

$$\Gamma(\rho^- \rightarrow e^- \bar{\nu}_e) = 68.8 \pm 1.2 \mu\text{eV}, \quad (10)$$

$$\Gamma(\rho^- \rightarrow \mu^- \bar{\nu}_\mu) = 66.9 \pm 1.1 \mu\text{eV}, \quad (11)$$

$$\mathcal{B}(\rho^- \rightarrow e^- \bar{\nu}_e) = (4.6 \pm 0.1) \times 10^{-13}, \quad (12)$$

$$\mathcal{B}(\rho^- \rightarrow \mu^- \bar{\nu}_\mu) = (4.5 \pm 0.1) \times 10^{-13}, \quad (13)$$

where the uncertainties come from the uncertainties of mass m_ρ , decay constant f_ρ and CKM element $|V_{ud}|$, and additional decay width Γ_ρ for branching ratios. Clearly, the branching ratios are very small. Given the identification efficiency and pollution from background, the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays might be measured only with more than 10^{14} $\rho^\pm$ events available.

There are at least three possible ways to experimentally produce the charged ρ mesons in the electron-positron collisions, (a) the prompt pair production $e^+ e^- \rightarrow \rho^+ \rho^-$, (b) the pair production via V decay $1^{--} \rightarrow \rho^+ \rho^-$, and (c) the single production via V decay $1^{--} \rightarrow \rho^\pm h^\mp$. The cross section $\sigma(e^+ e^- \rightarrow \rho^+ \rho^-)$ has been determined by the BaBar group to be $19.5 \pm 1.6 \pm 3.2$ fb near the center-of-mass energy $\sqrt{s}$

Table 1 Decay constant f_ρ (in the unit of MeV) obtained from the diffractive vector meson production using deep inelastic scattering (DVP), QCD sum rules (SR), relativistic quark model (RQM), light-front quark model (LFQM), lattice QCD (LQCD) and so on

DVP	143 [11] ^a	147 [11] ^b	153 [11] ^c	161 [11] ^d	211 [14]
SR	201±5 [15] ^e	205±10 [15] ^f	194 [15] ^g	198±7 [18]	206±7 [19]
RQM	168.3 [20] ^h	151.3 [20] ⁱ	190.1 [20] ^j	175.4 [20] ^k	219 [22]
LFQM	246 [23] ^l	215 [23] ^m	205 [24] ^l	166 ⁺² ₋₄ [25] ⁿ	210±6 [25] ^o
LFQM	215±5 [26]	211±1 [27]	242 ⁺²³ ₋₂₄ [28]		
LQCD	239.4±7.3 [29,30]	210±15 [31]	239±18 [32]	199±4 [33]	208.5±5.6 [34]
Other	490 [22]	207 [35]	254 [36]		

^aWith light-cone wave functions and parameters of Ref. [12]

^bWith light-cone wave functions and parameters of Ref. [13]

^cWith Gaussian wave functions and parameters of Ref. [12]. ^dWith Gaussian wave functions and parameters of Ref. [13]

^eWith nonlocal condensates like functions. ^fWith Ball–Braun wave functions [16]

^gWith Chernyak–Zhitnitsky wave functions [17]

^hWith Gaussian spatial wave functions and adjusted parameters of Ref. [20]

ⁱWith Gaussian spatial wave functions and parameters of Ref. [21]

^jWith rational spatial wave functions and adjusted parameters of Ref. [20]

^kWith rational spatial wave functions and parameters of Ref. [21]

^lWith Coulomb plus linear potential model

^mWith Coulomb plus harmonic oscillator potential model

ⁿWith a dilation parameter $\kappa = 0.54$ GeV

^oWith a dilation parameter $\kappa = 0.68$ GeV

$= 10.58$ GeV [40]. Assuming the production cross section $\sigma \propto 1/s$ [41,42], it could be speculated that $\sigma(e^+e^- \rightarrow \rho^+\rho^-) \sim 230$ fb near $\sqrt{s} = 3.1$ GeV. There would be only about 10^6 $\rho^+\rho^-$ pairs with a data sample of 50 ab^{-1} [43,44] near $\sqrt{s} \approx m_{\Upsilon(4S)}$ at the Belle-II detector or a data sample of 10 ab^{-1} [45] near $\sqrt{s} \approx m_{J/\psi}$ with the future super-tau-charm factory like STCF or SCTF [46–48]. The charge ρ mesons can in principle be produced from the $\Upsilon(4S)$, J/ψ and ϕ decays. The branching ratios are

$$\mathcal{B}(\Upsilon(4S) \rightarrow \rho^+\rho^-) < 5.7 \times 10^{-6} [39], \quad (14)$$

$$\mathcal{B}(J/\psi \rightarrow \rho^+\rho^-) \sim 10^{-3} [1,47], \quad (15)$$

$$\mathcal{B}(J/\psi \rightarrow \rho^\pm \pi^\mp) \sim 10^{-2} [1], \quad (16)$$

$$\mathcal{B}(\phi \rightarrow \rho^\pm \pi^\mp) \sim 10^{-1} [1], \quad (17)$$

where the branching ratio $\mathcal{B}(J/\psi \rightarrow \rho^+\rho^-)$ is assumed to be the same order of magnitude as $\mathcal{B}(J/\psi \rightarrow K^{*+}K^{*-}) \sim 10^{-3}$ [1] from the phenomenological analysis based on the flavor- $SU(3)$ symmetry [49]. Now, there are 7.7×10^8 $\Upsilon(4S)$ events at Belle [50], 10^{10} J/ψ events [46] at BES-III, and 2.4×10^{10} ϕ events at KLOE/KLOE-2 [51] available. It is expected that only about 5×10^{10} $\Upsilon(4S)$ events [43,44] and 10^{13} J/ψ events at SCTF or STCF [46] could be accumulated. It is clearly seen that unless a very significant enhancement to branching ratios from some NP, the experimental data on the $\rho^\pm$ meson are too scarce to search for the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays at the electron-positron collisions in the near future, which result in the natural difficulties to understand the ρ meson.

The production cross sections of prompt J/ψ and J/ψ -from- b mesons in proton-proton collisions at $\sqrt{s} = 13$

TeV are measured by LHCb to be $15.0 \pm 0.6 \pm 0.7 \mu\text{b}$ and $2.25 \pm 0.09 \pm 0.10 \mu\text{b}$, respectively, [52,53]. It is expected that some 10^{12} J/ψ events could be accumulated at $\sqrt{s} = 13$ TeV with an integrated luminosity of 300 fb^{-1} at LHCb [54]. There are only about 10^{10} $\rho^\pm$ mesons from J/ψ decays available for prying into the $\rho^\pm$ PLDCV decays. At the same time, the inclusive cross-sections for prompt charm production at LHCb at $\sqrt{s} = 13$ TeV are measured to be $\mathcal{O}(1 \text{ mb})$ [55–57]. Analogically assuming the inclusive cross section of prompt $\rho^\pm$ meson production at LHCb at $\sqrt{s} = 13$ TeV is $\mathcal{O}(10 \text{ mb})$, some 3×10^{15} $\rho^\pm$ events would be accumulated with an integrated luminosity of 300 fb^{-1} at LHCb [54]. Optimistically assuming the reconstruction efficiency is about 10%, there would be about $\mathcal{O}(10^2)$ events of the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays at LHCb, and more events with the enhanced branching ratios from NP contributions. Even through it will be very challenging for experimental analysis due to the complex background in hadron-hadron collisions, there is still a strong presumption that the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays could be explored and studied at LHC in the future. In addition, it is expected that an integrated luminosity exceeding 10 ab^{-1} would be reached at the future HE-LHC experiments [58]. More experimental data at HE-LHC would make the study of the $\rho^- \rightarrow \ell^- \bar{\nu}_\ell$ decays indeed feasible in hadron-hadron collisions.

3 $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

The parameter product $|V_{us}| f_{K^*}$ could be experimentally determined from the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays using Eq. (8).

Like the $\rho^\pm$ meson, the mass of the $K^{*\pm}$ meson, $m_{K^{*\pm}} = 895.5(8)$ MeV, is above the threshold of $K\pi$ pair, and the partial branching ratio of the K^* meson decay into $K\pi$ pair via the strong interactions is almost 100% [1]. It is not hard to imagine that the very short lifetime $\tau_{K^*} \sim 1.4 \times 10^{-23}$ s would enable the measurements of the electroweak properties of the K^* meson to be very challenging or nearly impossible.

The CKM element $|V_{us}| \simeq \lambda$ up to the order of $\mathcal{O}(\lambda^6)$, where λ is a Wolfenstein parameter. The current precision of the CKM element $|V_{us}|$ from purely leptonic and semileptonic K meson decays and hadronic τ decays are 0.2%, 0.3% and 0.6%, respectively. It is seen from Fig. 2 that these three results for $|V_{us}|$ are not very consistent with one another. So if the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays could be measured, they would provide another determination and constraint to $|V_{us}|$. Probably due to the reduction of the value of $|V_{ud}|$, the latest value from the global fit in SM, $|V_{us}| = 0.22650(48)$ [1], is slightly larger than the 2018 value, to satisfy the first row unitarity requirement.

Some theoretical results on the decay constant f_{K^*} are presented in Table 2. Like the case of the decay constant f_ρ , the model dependence of theoretical estimations on the decay constant f_{K^*} is also obvious. Experimentally, the decay constant f_{K^*} can be obtained from the hadronic $\tau^\pm \rightarrow K^{*\pm} \nu_\tau$ decays. Using Eq. (9) and experimental data on branching ratio $\mathcal{B}(\tau \rightarrow K^* \nu) = 1.20(7)\%$ [1], one can obtain the decay constant $f_{K^*}^{\text{exp}} = 202.5^{+6.5}_{-6.7}$ MeV. The value of $f_{K^*}^{\text{exp}}$ is much less than that of LQCD results, and will be used in our calculation.

For the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays, the SM expectations on the partial decay widths and branching ratios are,

$$\Gamma(K^{*-} \rightarrow e^- \bar{\nu}_e) = 5.5 \pm 0.4 \mu\text{eV}, \quad (18)$$

$$\Gamma(K^{*-} \rightarrow \mu^- \bar{\nu}_\mu) = 5.3 \pm 0.4 \mu\text{eV}, \quad (19)$$

$$\mathcal{B}(K^{*-} \rightarrow e^- \bar{\nu}_e) = (1.2 \pm 0.1) \times 10^{-13}, \quad (20)$$

$$\mathcal{B}(K^{*-} \rightarrow \mu^- \bar{\nu}_\mu) = (1.2 \pm 0.1) \times 10^{-13}. \quad (21)$$

The decay width $\Gamma_{K^*} = 46.2 \pm 1.3$ MeV [1] is used in our calculation. It is apparent that more than 10^{14} $K^{*\pm}$ events are the minimum requirement for experimentally studying the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays.

Based on the U -spin symmetry, the production mechanism of the $K^{*\pm}$ mesons in electron-positron collisions is similar to that of the $\rho^\pm$ mesons. An educated guess is that the cross section $\sigma(e^+e^- \rightarrow K^{*+} K^{*-}) \sim 20$ fb and 230 fb near $\sqrt{s} \sim m_{\Upsilon(4S)}$ and $m_{J/\psi}$, respectively. The branching ratios of J/ψ decays are [1],

$$\mathcal{B}(J/\psi \rightarrow K^{*+} K^{*-}) = (1.00^{+0.22}_{-0.40}) \times 10^{-3}, \quad (22)$$

$$\mathcal{B}(J/\psi \rightarrow K^{*\pm} K^\mp) = (6.0^{+0.8}_{-1.0}) \times 10^{-3}, \quad (23)$$

$$\mathcal{B}(J/\psi \rightarrow K^{*\pm} K^\mp \pi^0) = (4.1 \pm 1.3) \times 10^{-3}, \quad (24)$$

$$\mathcal{B}(J/\psi \rightarrow K^{*\pm} K_S^0 \pi^\mp) = (2.0 \pm 0.5) \times 10^{-3}, \quad (25)$$

Fig. 2 The values of the CKM element $|V_{us}|$ from PDG

$$\mathcal{B}(J/\psi \rightarrow K^{*\pm} K_2^*(1430)^\mp) = (3.4 \pm 2.9) \times 10^{-3}, \quad (26)$$

$$\mathcal{B}(J/\psi \rightarrow K^{*\pm} K^*(700)^\mp) = (1.0^{+1.0}_{-0.6}) \times 10^{-3}. \quad (27)$$

It is approximately estimated that $\mathcal{B}(J/\psi \rightarrow K^{*\pm} X^\mp) \sim 1.8\%$. Hence, the experimental data on the $K^{*\pm}$ mesons at the e^+e^- collisions, which would be available by either the prompt $K^{*+} K^{*-}$ pair production at SuperKEKB and SCTF experiments or the production via 10^{13} J/ψ decay at SCTF, are far from sufficient for investigating the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays. If we assume that the inclusive cross section of prompt $K^{*\pm}$ meson production in pp collisions at the center-of-mass energy of 13 TeV is similar to that of $\rho^\pm$ mesons, about $\mathcal{O}(10\text{ mb})$, there would be some 3×10^{15} $K^{*\pm}$ events to be available with an integrated luminosity of 300 fb^{-1} at LHCb, which correspond to about $\mathcal{O}(10^2)$ events of the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays. It should be some glimmer of hope for observation and scrutinies of the $K^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

Table 2 The theoretical values of decay constant f_{K^*} (in the unit of MeV), where the legends are the same as those in Table 1

RQM	170.5 [20] ^a	153.1 [20] ^b	192.6 [20] ^c	177.6 [20] ^d	236 [22]
LFQM	256 [23] ^e	223 [23] ^f	224 [24] ^e	186 ⁺² ₋₃ [25] ^g	204 ⁺⁷ ₋₉ [25] ^h
LFQM	217±5 [26]	223±1 [27]	253±25 [28]		
LQCD	255.5±6.5 [29,30] ⁱ	257.7±6.4 [29,30] ^j	240±18 [59]		
other	226±28 [18]	508 [22]	241 [35]		

^aWith Gaussian spatial wave functions and adjusted parameters of Ref. [20]^bWith Gaussian spatial wave functions and parameters of Ref. [21]^cWith rational spatial wave functions and adjusted parameters of Ref. [20]^dWith rational spatial wave functions and parameters of Ref. [21]^eWith Coulomb plus linear potential model^fWith Coulomb plus harmonic oscillator potential model^gWith a dilation parameter $\kappa = 0.54$ GeV^hWith a dilation parameter $\kappa = 0.68$ GeVⁱThe K meson mass is used as input^jThe ϕ meson mass is used as input

at hadron-hadron collisions in the future, particularly at the planning HE-HLC.

4 $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

The mass of D_d^* mesons, $m_{D_d^*} = 2010.26(5)$ MeV, are just above the threshold of $D\pi$ pair. The D_d^* meson decays via the strong interactions are dominant, and the ratio of branching ratios [1], $\mathcal{B}(D_d^{*\pm} \rightarrow D_d^\pm \pi^0) / \mathcal{B}(D_d^{*\pm} \rightarrow D_u \pi^\pm) = 30.7(5)\% / 67.7(5)\% \sim 1/2$, basically agrees with the relations of isospin symmetry. It should be pointed out that the D_d^* strong decays are highly suppressed by the compact phase spaces because of $m_{D_d^*} - m_D - m_\pi < 6$ MeV. The branching ratio of the magnetic dipole transition is small, $\mathcal{B}(D_d^* \rightarrow D_d \gamma) = 1.6(4)\%$ [1]. Hence, the decay width of D_d^* mesons is narrow, $\Gamma_{D_d^*} = 83.4 \pm 1.8$ keV [1]. From the $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays, the parameter $|V_{cd}| f_{D_d^*}$ is expected to be experimentally determined.

Currently, the precise values of the CKM element $|V_{cd}|$ comes mainly from the leptonic and semileptonic D meson decays [1], as illustrated in Fig. 3. Because of the decay width of Eq. (7) being proportional to m_ℓ^2 , the $D^- \rightarrow e^- \bar{\nu}_e$ decay is helicity suppressed. And the $D^- \rightarrow \tau^- \bar{\nu}_\tau$ decay suffers from the complications caused by the additional neutrino in τ decays. The $D^- \rightarrow \mu^- \bar{\nu}_\mu$ decay is the most favorable mode for experimental measurement. For the values of $|V_{cd}|$ from the purely leptonic decay $D^- \rightarrow \mu^- \bar{\nu}$, the experimentally statistical uncertainties are dominant uncertainties. For the values of $|V_{cd}|$ from the semileptonic D meson decays, the theoretical uncertainties from the form factor controlled by nonperturbative dynamics are dominant uncertainties. It is clearly seen from Fig. 3 that the experimental uncertainties have not decreased significantly recently. Besides, $|V_{cd}|$ can also be determined from the neutrino-induced charm production data [1], but the relevant experimental data have not been updated after the measurements given by the CHARM-

Fig. 3 The values of the CKM element $|V_{cd}|$ from PDG

II Collaboration in 1999 [60]. According to the Wolfenstein parameterization of the CKM matrix, there is an approximate relation between its elements $|V_{cd}| = |V_{us}| = \lambda$ up to $\mathcal{O}(\lambda^4)$. However, the measurement precision of the CKM element $|V_{cd}|$ from both leptonic and semileptonic D meson decays is generally about an order of magnitude smaller than that of $|V_{us}|$ from leptonic and semileptonic K meson decays for the moment. The most precise values are from the global fit in SM, $|V_{cd}| = 0.22636(48)$ [1] with uncertainties $\sim 0.2\%$.

Table 3 The theoretical values of decay constant $f_{D_d^*}$ (in the unit of MeV), where the legends are the same as those in Table 1, NRQM is an abbreviation for nonrelativistic quark model

NRQM	223^{+23}_{-19} [61]	307 [62] ^a	253 [62] ^b	353.8 [63] ^a	290.3 [63] ^b
NRQM	391 [22]	290 [64] ^a	210 [64] ^b	332 [65,66]	
RQM	310 [22]	315 [65,66]	327±13 [67] ^c	252±10 [67] ^d	
LFQM	254 [68] ^e	228 [68] ^f	259.6±14.6 [69] ^g	306.3 ^{+18.2} _{-17.7} [69] ^h	253±7 [27]
LFQM	230 [24] ⁱ	226.6 ^{+5.9} _{-10.2} [25] ^j	230.1±6.2 [25] ^k	245 ⁺³⁵ ₋₃₄ [26]	230±29 [28]
LFQM	252.0 ^{+13.8} _{-11.6} [70] ^g	264.9 ^{+10.2} _{-9.5} [70] ^h	272 [71] ^l	260 [71] ^m	269 [71] ⁿ
LQCD	245±20 [75]	234±26 [76] ^o	278±16 [77]	223.5±8.7 [78]	234(6) [79]
SR	242 ⁺²⁰ ₋₁₂ [80–82]	263±21 [83]	252.2±22.7 [84]	250±11 [85]	
other	186 [71] ^p	273±13 [86]	341±23 [87]	237 [88]	

^aWithout QCD radiative corrections.^bWith QCD radiative corrections^cWith constituent quark masses for the light quarks m_s and m_d ^dWith current quark masses for the light quarks m_s and m_d ^eWith Coulomb plus linear potential model^fWith Coulomb plus harmonic oscillator potential model^gWith the Gaussian type wave functions^hWith the power-law type wave functionsⁱWith Coulomb plus linear potential model^jWith a dilation parameter $\kappa = 0.54$ GeV^kWith a dilation parameter $\kappa = 0.68$ GeV^lWith Martin potential model [72]^mWith Cornell potential model [73]ⁿWith logarithmic potential model [74]^oWith $m_{D^*}/f_{D^*} = 8.6 \pm 0.3^{+0.5}_{-0.9}$ [76] and $m_{D_d^*} = 2010.26(5)$ MeV [1]^pWith harmonic plus Yukawa potential model [63]

The information about the decay constant $f_{D_d^*}$ has not yet been obtained experimentally by now. Some theoretical results on $f_{D_d^*}$ are listed in Table 3. The theoretical discrepancies among various methods are obvious. In our calculation, as a conservative estimate, we will take the recent value $f_{D_d^*} = 230 \pm 29$ MeV [28] from the light front quark model, which agrees basically with the values $f_{D_d^*} = 234 \pm 6$ MeV [79] from the recent lattice QCD simulation.

After some simple computation with Eq. (8), we obtain the partial decay widths and branching ratios for the $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays as follows.

$$\Gamma(D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = 79^{+22}_{-19} \mu\text{eV}, \quad \text{for } \ell = e, \mu, \quad (28)$$

$$\Gamma(D_d^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = 5 \pm 1 \mu\text{eV}, \quad (29)$$

$$\mathcal{B}(D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = (9.5^{+2.9}_{-2.4}) \times 10^{-10}, \quad \text{for } \ell = e, \mu, \quad (30)$$

$$\mathcal{B}(D_d^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = (0.6 \pm 0.2) \times 10^{-10}. \quad (31)$$

These branching ratios are consistent with those of Ref. [89] if the different values of decay constants $f_{D_d^*}$ are considered. The relatively large uncertainties of branching ratios come from the uncertainties of mass $m_{D_d^*}$, width $\Gamma_{D_d^*}$, decay constant $f_{D_d^*}$ and the CKM element $|V_{cd}|$. To experimentally study the $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays, more than 10^{11} D_d^* events are needed. Due to the short lifetime of lepton $\tau^\pm$ and the lepton number conservation in $\tau^\pm$ decays, additional neutri-

nos will make the measurement of the $D_d^{*-} \rightarrow \tau^- \bar{\nu}_\tau$ decay to have a poor reconstruction efficiency and to be very challenging. Perhaps some 10^{12} or more D_d^* events are necessarily required to study the $D_d^{*-} \rightarrow \tau^- \bar{\nu}_\tau$ decay.

Above the open charm production threshold, there are several charmonium resonances and charmonium-like structures decaying predominantly into pairs of charmed meson final states. The studies of Belle [90], BaBar [91] and CLEO-c [92,93] collaborations have shown that there is a sharply peaked $D_d^+ D_d^{*-}$ structure and a broad $D_d^{*+} D_d^{*-}$ plateau just above threshold, as illustrated in Fig. 4. Assuming the exclusive cross sections near threshold $\sigma(e^+e^- \rightarrow D_d^+ D_d^{*-}) \sim 4$ nb and $\sigma(e^+e^- \rightarrow D_d^{*+} D_d^{*-}) \sim 3$ nb, there will be about 10^{11} $D_d^{*\pm}$ events corresponding to the total integrated luminosity of 10 ab^{-1} at future STCF, and about 5×10^{11} $D_d^{*\pm}$ events corresponding to a data sample of 50 ab^{-1} at SuperKEKB. In addition, about 10^{12} Z bosons will be produced on the on the schedule of the large international scientific project of Circular Electron Positron Collider (CEPC) [94] and 10^{13} Z bosons at Future Circular e^+e^- Collider (FCC-ee) [95]. Considering the branching ratio $\mathcal{B}(Z \rightarrow D^{*\pm} X) = (11.4 \pm 1.3)\%$ [1], the Z boson decays will yield more than 10^{11} $D_d^{*\pm}$ events at the tera-Z factories. So the $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays could be investigated at Belle-II, SCTF, CEPC and FCC-ee experiments.

Fig. 4 The exclusive cross sections (in the unit of nb) as functions of $\sqrt{s}$ (in the unit of GeV) for $e^+e^- \rightarrow D^+D^{*-}$ in (a) and $e^+e^- \rightarrow D^{*+}D^{*-}$ in (b). The Belle, BaBar and CLEO-c data are from Refs. [90,91,93], respectively.

In hadron-hadron collisions, the inclusive cross sections for the $c\bar{c}$ pair and D^{*+} meson production are measured to be $\sigma(pp \rightarrow c\bar{c}X) = 2369 \pm 3 \pm 152 \pm 118 \mu\text{b}$ and $\sigma(pp \rightarrow D^{*+}X) = 784 \pm 4 \pm 87 \pm 118 \mu\text{b}$ at the center-of-mass energy of $\sqrt{s} = 13 \text{ TeV}$ by the LHCb group, with the transverse momentum p_T within the range of $1 \text{ GeV} < p_T < 8 \text{ GeV}$ [55–57]. Some $2 \times 10^{14} D_d^{\pm}$ events could be accumulated with the integrated luminosity 300 fb^{-1} at LHCb. The total cross sections of charm and D^{*+} production measured at $\sqrt{s} = 7 \text{ TeV}$ by the ALICE group are $\sigma_{c\bar{c}}^{\text{tot}} \simeq 8.5 \text{ mb}$ and $\sigma_{D^{*+}}^{\text{tot}} \simeq 2.11 \text{ mb}$, respectively [96]. The total cross sections of charm production measured at $\sqrt{s} = 7 \text{ TeV}$ by the ATLAS group are $\sigma_{c\bar{c}}^{\text{tot}} \simeq 8.6 \text{ mb}$ [97]. The D^{*+} production cross section at ATLAS should be very close to that at ALICE based on an educated guess. In addition, the D^* meson can also produced from b decays with the fragmentation fraction about $f(b \rightarrow D_d^*) \simeq 23\%$ [98]. The b -quark production cross sections at $\sqrt{s} = 13 \text{ TeV}$ determined by LHCb and ALICE are about $\sigma(pp \rightarrow b\bar{b}X) \simeq 495 \mu\text{b}$ [52,53] and $541 \mu\text{b}$ [99], respectively. So more than $10^{13} D_d^{\pm}$ events from b decays could be accumulated with the integrated luminosity 300 fb^{-1} at LHCb. All in all, the large cross section of D^* meson plus the high luminosity at hadron-hadron collisions result in the abundant $D_d^{\pm}$ events, and make the carefully experimental study of the $D_d^{*-} \rightarrow e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$ decays, even the $D_d^{*-} \rightarrow \tau^- \bar{\nu}_\tau$ decay, to be possible and practicable.

5 $D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

The $D_s^{\pm}$ mesons have explicitly nonzero quantum number of electric charges, charm and strange, $Q = C = S = \pm 1$. Considering the conservation of the charm and strange quantum number in the strong and electromagnetic interactions, and the mass of D_s^* mesons, $m_{D_s^*} = 2112.2(4) \text{ MeV}$ [1], being just above the threshold of $D_s\pi$ pair but below the threshold of DK pair, the $D_s^* \rightarrow D_s\pi$ decays are the only

allowable hadronic decay modes. However, the $D_s^* \rightarrow D_s\pi$ decays are highly suppressed due to four factors: (1) from the dynamical view, the $D_s^* \rightarrow D_s\pi$ decays are induced by the electromagnetic interactions rather than the strong interactions because of the isospin non-conservation between the initial and final states, (2) from the perspective of the conservation of angular momentum, the orbital angular momentum of final states should be $L = 1$, so the $D_s^* \rightarrow D_s\pi$ decays are induced by the contributions of the P -wave amplitudes, (3) from the phenomenological view, the $D_s^* \rightarrow D_s\pi$ decays are suppressed by the Okubo-Zweig-Iizuka rules [3,100,101] because the quark lines of pion disconnect from those of the $D_s^*D_s$ system, (4) from the kinematic view, the phase spaces of final states are very compact because of $m_{D_s^*} - m_{D_s} - m_{\pi^0} \sim 9 \text{ MeV}$. Hence, the branching ratio for the hadronic decay is very small $\mathcal{B}(D_s^* \rightarrow D_s\pi) = 5.8(7)\%$ [1]. And the branching ratio of the electromagnetic radiative decay is dominant, $\mathcal{B}(D_s^* \rightarrow D_s\gamma) = 93.5(7)\%$ [1]. Except for the $D_s\pi$, $D_s\gamma$ and $D_s e^+e^-$ final states, other decay modes of the D_s^* mesons have not yet been observed [1]. The $D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ weak decays are favored by the CKM element $|V_{cs}|$. The information about the $|V_{cs}|f_{D_s^*}$ can be experimentally obtained from the $D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays.

The direct determinations of the CKM element $|V_{cs}|$ come mainly from leptonic D_s decays and semileptonic D decays, as shown in Fig. 5. The uncertainties of $|V_{cs}|$ from the D_s leptonic decays, about 1%, are dominated by the experimental uncertainties. The uncertainties of $|V_{cs}|$ from the D semileptonic decays, about 4%, are dominated by the theoretical calculations of the form factors. It is worth noting that the recent CKM element $|V_{cs}|$ determined by the BES-III group from the $D_s^+ \rightarrow \mu^+ \nu_\mu$ and $D_s^+ \rightarrow \tau^+ \nu_\tau$ decays based on available 6.32 fb^{-1} data is $|V_{cs}| = 0.978 \pm 0.009 \pm 0.014$ [102], where the systematic (second) uncertainties has outweighed the statistical (first) one. This value is very close to the precise result from the global fit, $|V_{cs}| = 0.97320(11)$ [1] that will be used in this paper.

Fig. 5 The values of the CKM element $|V_{cs}|$ from PDG

By now, a relatively little information about the properties of the D_s^* mesons is available. For example, the quantum number of J^P , the decay constant $f_{D_s^*}$, and the width $\Gamma_{D_s^*}$ have not yet been determined or confirmed explicitly by experiments. It is generally thought that the J^P of the D_s^* mesons is consistent with 1^- from decay modes [105]. Some theoretical results on the decay constant $f_{D_s^*}$ are listed in Table 4. It can be seen that the theoretical results are various. The recent LQCD results on the decay constant from ETM [78], HPQCD [106] and χ QCD [79] groups are in reasonable agreement with each other within an error range. The latest decay constant $f_{D_s^*} = 274 \pm 7$ MeV from LQCD calculation [79] will be used for an estimation for PLDCV of the D_s^* mesons in this paper. The experimental upper limit of the decay width is $\Gamma_{D_s^*} < 1.9$ MeV at the 90% confidence level set by the CLEO collaboration in 1995 [105]. An approximate relation for the decay width, $\Gamma_{D_s^*} \simeq \Gamma(D_s^* \rightarrow \gamma D_s)$, is often used in theoretical calculation. The radiative transition process, $D_s^* \rightarrow \gamma D_s$, is a parity conserving decay. The parity and angular momentum conservation implies that the orbital angular momentum of final states $L = 1$. There are many theoretical calculation on the decay width $\Gamma_{D_s^*}$, for example,

Refs. [106–138]. The partial decay width for the magnetic dipole transition is generally written as [139],

$$\Gamma(V \rightarrow P\gamma) = \frac{4}{3} \alpha_{\text{em}} k_\gamma^3 \mu_{VP}^2, \quad (32)$$

with the definition of the magnetic dipole moment μ_{VP} and the momentum of photon k_γ in the rest frame of the vector meson,

$$\mu_{VP} = \langle P | \hat{\mu}_z | V \rangle = \langle P | \sum_i \frac{Q_i}{2m_i} \hat{\sigma}_z | V \rangle, \quad (33)$$

$$k_\gamma = \frac{m_V^2 - m_P^2}{2m_V}, \quad (34)$$

where Q_i and m_i are the electric charge in the unit of $|e|$ and mass of the constituent quark, respectively. With $m_d \approx 336$ MeV, $m_s \approx 490$ MeV, $m_c \approx 1500$ MeV and the

$$\mu_{D_d^* D_d} = \frac{1}{6} \left(\frac{2}{m_c} - \frac{1}{m_d} \right), \quad (35)$$

$$\mu_{D_s^* D_s} = \frac{1}{6} \left(\frac{2}{m_c} - \frac{1}{m_s} \right), \quad (36)$$

one can obtain $\Gamma(D_d^* \rightarrow \gamma D_d) \approx 1.8$ keV and $\Gamma(D_s^* \rightarrow \gamma D_s) \approx 0.36$ keV [139]. The theoretical value of partial decay width $\Gamma(D_d^* \rightarrow \gamma D_d)$ is roughly consistent with the corresponding experimental data $\Gamma(D_d^* \rightarrow \gamma D_d) = \Gamma_{D_d^*} \times \mathcal{B}(D_d^* \rightarrow \gamma D_d) = 1.33 \pm 0.33$ keV within 2σ regions [1]. For the moment, we will use $\Gamma_{D_s^*} = 0.36$ keV in the calculation to give an estimate of branching ratios for the $D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays.

$$\Gamma(D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = 2.4 \pm 0.1 \text{ MeV}, \quad \text{for } \ell = e, \mu, \quad (37)$$

$$\Gamma(D_s^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = 0.28 \pm 0.02 \text{ MeV}, \quad (38)$$

$$\mathcal{B}(D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = (6.7 \pm 0.4) \times 10^{-6}, \quad \text{for } \ell = e, \mu, \quad (39)$$

$$\mathcal{B}(D_s^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = (7.8 \pm 0.4) \times 10^{-7}. \quad (40)$$

If considering the experimental measurement efficiency, there are at least more than 10^7 D_s^* events to experimentally study the $D_d^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays. And more than 10^8 D_s^* events might be needed to explore the $D_s^{*-} \rightarrow \tau^- \bar{\nu}_\tau$ decay.

In the electron-positron collisions, the cross sections of $D_s^+ D_s^{*-}$ and $D_s^{*+} D_s^{*-}$ production have been experimentally studied by the Belle [140], BaBar [141] and CLEO-c [92, 93] groups, as illustrated in Fig. 6. Assuming the exclusive cross sections near threshold $\sigma(e^+ e^- \rightarrow D_s^+ D_s^{*-}) \sim 1.0$ nb and $\sigma(e^+ e^- \rightarrow D_s^{*+} D_s^{*-}) \sim 0.2$ nb, there will be about 10^{10} $D_s^{*\pm}$ events corresponding to a data sample of 10 ab^{-1} at STCF, and about 5×10^{10} $D_s^{*\pm}$ events corresponding to a data sample of 50 ab^{-1} at SuperKEKB. In addition, considering the branching ratio $\mathcal{B}(Z \rightarrow c\bar{c}) = (12.03 \pm 0.21)\%$ [1] and the fragmentation fraction $f(c \rightarrow D_s^*) \simeq 5.5\%$ [142], there will be more than 6×10^9 (and 6×10^{10}) $D_s^{*\pm}$ events corresponding

Table 4 The theoretical values of decay constant $f_{D_s^*}$ (in the unit of MeV), where the legends including the footnotes are the same as those in 3

NRQM	326^{+21}_{-17} [61]	344 [62] ^a	275 [62] ^b	382.1 [63] ^a	303.5 [63] ^b
NRQM	447 [22]	310 [64] ^a	212 [64] ^b	384 [65,66]	
RQM	315 [22]	335 [65,66]	362±15 [67] ^c	288±11 [67] ^d	272 [103,104]
LFQM	290 [68] ^e	268 [68] ^f	338.7±29.7 [69] ^g	391.0±28.9 [69] ^h	314±6 [27]
LFQM	260 [24] ⁱ	254.7 ^{+6.3} _{-6.7} [25] ^j	289.7 ^{+6.3} _{-4.5} [25] ^k	272 ⁺³⁹ ₋₃₈ [26]	253±32 [28]
LFQM	318.3 ^{+15.3} _{-12.6} [70] ^g	330.9 ^{+9.9} _{-9.0} [70] ^h	303 [71] ^l	291 [71] ^m	302 [71] ⁿ
LQCD	272±16 ⁺³ ₋₂₀ [75]	254±17 [76] ^o	311±9 [77]		
LQCD	268.8±6.6 [78]	274±6 [106]	274±7 [79]		
SR	293 ⁺¹⁹ ₋₁₄ [80–82]	308±21 [83]	305.5±27.3 [84]	270±19 [85]	
Other	240 [71] ^p	307±18 [86]	375±24 [87]	242 [88]	

^aWithout QCD radiative corrections^bWith QCD radiative corrections^cWith constituent quark masses for the light quarks m_s and m_d ^dWith current quark masses for the light quarks m_s and m_d ^eWith Coulomb plus linear potential model^fWith Coulomb plus harmonic oscillator potential model^gWith the Gaussian type wave functions^hWith the power-law type wave functionsⁱWith Coulomb plus linear potential model^jWith a dilation parameter $\kappa = 0.54$ GeV^kWith a dilation parameter $\kappa = 0.68$ GeV^lWith Martin potential model [72]^mWith Cornell potential model [73]ⁿWith logarithmic potential model [74]^oWith $m_{D_s^*}/f_{D_s^*} = 8.3 \pm 0.2 \pm 0.5$ [76] and $m_{D_s^*} = 2112.2(4)$ MeV [1]^pWith harmonic plus Yukawa potential model [63] **Fig. 6** The exclusive cross sections (in the unit of nb) as functions of $\sqrt{s}$ (in the unit of GeV) for $e^+e^- \rightarrow D_s^- D_s^{*+}$ in (a) and $e^+e^- \rightarrow D_s^{*-} D_s^{*+}$ in (b). The Belle, BaBar and CLEO-c data are from Ref. [93, 140, 141], respectively

to 10^{12} [94] (and 10^{13} [95]) Z bosons at the future CEPC (and FCC-ee). So the $D_s^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays (with $\ell = e, \mu$ and τ) could be measured at Belle-II, SCTF, CEPC and FCC-ee experiments.

In hadron-hadron collisions, the inclusive cross sections for the $c\bar{c}$ pair production are $\sigma(pp \rightarrow c\bar{c}X) \simeq 2.4$ mb at the center-of-mass energy of $\sqrt{s} = 13$ TeV at LHCb [55–57], $\sigma_{c\bar{c}}^{\text{tot}} \simeq 8.5$ mb and 8.6 mb at $\sqrt{s} = 7$ TeV at ALICE [96] and

ATLAS [97], respectively. With the fragmentation fraction $f(c \rightarrow D_s^*) \simeq 5.5\%$ [142], there will be about 4×10^{13} $D_s^{*\pm}$ events corresponding a data sample of 300 fb^{-1} at LHCb, and more $D_s^{*\pm}$ events available at ALICE and ATLAS. So the $D_s^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$ decays could be measured precisely at LHCb, ALICE and ATLAS experiments.

6 $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

The experimental information about the B_u^* mesons are very scarce. The already known information about the B_u^* mesons are their quark composition $b\bar{u}$ with the quark model assignment, the isospin $I = 1/2$, the spin-parity quantum number $J^P = 1^-$ and the mass $m_{B_u^*} = 5324.70(21)$ MeV [1]. Due to the mass difference $m_{B_u^*} - m_{B_u} = 45$ MeV $< m_\pi$, the electromagnetic radiative transition $B_u^* \rightarrow B_u \gamma$ certainly will be the important and dominant decay mode. The photon in the $B_u^* \rightarrow B_u \gamma$ decay is very soft, with the momentum $k_\gamma \sim 45$ MeV in the center-of-mass of the B_u^* mesons. No signal event of the $B_u^* \rightarrow B_u \gamma$ decay has yet been found. The $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays offer a complementary decay modes of the B_u^* meson. It can be seen from Eq. (8) that the information about $|V_{ub}| f_{B_u^*}$ could be obtained, however, the partial width for the purely leptonic decays $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ are highly suppressed by the CKM element of $|V_{ub}|^2 \sim \mathcal{O}(\lambda^6)$.

The precise determinations of the CKM element $V_{ub} = |V_{ub}| e^{-i\gamma}$ are very central and important to verify the CKM picture of SM, where γ is the angle of the unitarity triangle of $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. The experimental determination of $|V_{ub}|$ from the inclusive $B \rightarrow X_u \ell \bar{\nu}_\ell$ decay is complicated mainly by the large backgrounds from the CKM-favored $B \rightarrow X_c \ell \bar{\nu}_\ell$ decay. The experimental extraction of $|V_{ub}|$ from exclusive $B \rightarrow \pi \ell \bar{\nu}_\ell$ decay is subject to the form factors calculated with the lattice QCD or QCD sum rules. The latest values obtained from inclusive and exclusive determinations are [1]

$$|V_{ub}| \times 10^3 = 4.25 \pm 0.12_{\text{exp}}^{+0.15}_{-0.14} \pm 0.23_{\text{theo}} \Delta_{BF} \\ = 4.25_{-0.29}^{+0.30} \quad (\text{inclusive}), \quad (41)$$

$$|V_{ub}| \times 10^3 = 3.70 \pm 0.10_{\text{exp}} \pm 0.12_{\text{theo}} \\ = 3.70 \pm 0.16 \quad (\text{exclusive}). \quad (42)$$

It is clearly seen that (1) the difference between inclusive and exclusive determinations of $|V_{ub}|$ is obvious. (2) The best determinations of $|V_{ub}|$ are from exclusive semileptonic decays, with a precision of about 4%. The experimental errors for the exclusive semileptonic decays are expected to decrease from the current 2.7% to 1.2% based on a dataset of 50 ab^{-1} at Belle-II experiments [43, 44]. (3) The theoretical uncertainties are larger than experimental ones. Moreover, $|V_{ub}|$ can also be experimentally determined from the leptonic decay $B_u \rightarrow \tau \bar{\nu}_\tau$ and the semileptonic hyperon decay $\Lambda_b \rightarrow p \mu \bar{\nu}_\mu$. The constraint from the global fit gives $|V_{ub}| = (3.61_{-0.09}^{+0.11}) \times 10^{-3}$ [1], which will be used in our calculation.

Some theoretical results on the decay constant $f_{B_u^*}$ are collected in Table 5. The recent result $f_{B_u^*} = 185.9 \pm 7.2$ MeV from LQCD calculation [78] will be used in this paper. There are many theoretical calculation on the decay width $\Gamma_{B_u^*}$, for example, Refs. [119–138]. With the formula of Eq. (32), the quark mass $m_u = 336$ MeV [139] and $m_b = 4.78$ GeV [1],

and the magnetic dipole momentum

$$\mu_{B_u^* B_u} = \frac{1}{6} \left(\frac{2}{m_u} - \frac{1}{m_b} \right), \quad (43)$$

one can obtain $\Gamma_{B_u^*} \simeq \Gamma(B_u^* \rightarrow \gamma B_u) \simeq 820$ eV. The partial decay width and branching ratios for the $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays are

$$\Gamma(B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = 0.25_{-0.03}^{+0.04} \mu\text{eV}, \quad \text{for } \ell = e, \mu, \quad (44)$$

$$\Gamma(B_u^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = 0.20_{-0.02}^{+0.03} \mu\text{eV}, \quad (45)$$

$$\mathcal{B}(B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = (3.0 \pm 0.4) \times 10^{-10}, \quad \text{for } \ell = e, \mu, \quad (46)$$

$$\mathcal{B}(B_u^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = (2.5 \pm 0.4) \times 10^{-10}. \quad (47)$$

It is expected that there should be at least more than 10^{11} B_u^* events available for experimental study of the $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays.

The experimental study has shown that the exclusive cross sections for the final states of $B\bar{B}^*$, $B^*\bar{B}^*$ and $B\bar{B}^*\pi$ will have a large share of the total $b\bar{b}$ cross sections above the open bottom threshold, for example [1],

$$\mathcal{B}(Z_b(10610) \rightarrow B^+ \bar{B}^{*0} + B^{*+} \bar{B}^0) = 85.6_{-2.9}^{+2.1} \%, \quad (48)$$

$$\mathcal{B}(Z_b(10650) \rightarrow B^{*+} \bar{B}^{*0}) = 74_{-6}^{+4} \%, \quad (49)$$

$$\mathcal{B}(\Upsilon(5S) \rightarrow B\bar{B}^* + c.c.) = 13.7 \pm 1.6 \%, \quad (50)$$

$$\mathcal{B}(\Upsilon(5S) \rightarrow B^* \bar{B}^*) = 38.1 \pm 3.4 \%, \quad (51)$$

$$\mathcal{B}(\Upsilon(5S) \rightarrow B\bar{B}^* \pi + B^* \bar{B} \pi) = 7.3 \pm 2.3 \%, \quad (52)$$

$$\mathcal{B}(\Upsilon(5S) \rightarrow B^* \bar{B}^* \pi) = 1.0 \pm 1.4 \%. \quad (53)$$

There are about 36×10^6 $\Upsilon(5S)$ events corresponding to the dataset of 121 fb^{-1} at Belle experiments at the disposal [43, 44]. About 1.5×10^{10} $\Upsilon(5S)$ events with a dataset 50 ab^{-1} at Belle-II are an outside estimate. Assuming the inclusive branching ratio $\mathcal{B}(\Upsilon(5S) \rightarrow B_u^* X) \simeq 30\%$, there will be some 4.5×10^9 B_u^* events at most at Belle-II. And it is more important that the vast majority of the data will be taken at $\Upsilon(4S)$ resonance rather than $\Upsilon(5S)$ mesons at Belle-II experiments, and $\Upsilon(4S)$ lies below the $B\bar{B}^*$ threshold. So the probability of direct observation of the $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays at Belle-II experiments should be very tiny. Considering about 10^{13} Z bosons at FCC-ee [95] and branching ratio $\mathcal{B}(Z \rightarrow b\bar{b}) = 12.03 \pm 0.21\%$ [1], and assuming the fragmentation fraction $f(b \rightarrow B_u^*) \sim 20\%$ [146], there will be more than 4×10^{11} B_u^* events to search for the $B_u^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays. The b -quark production cross sections at the center-of-mass energy $\sqrt{s} = 13$ TeV is about $\sigma(pp \rightarrow b\bar{b}X) \simeq 495 \mu\text{b}$ at LHCb [52, 53]. There will be more than 5×10^{13} B_u^* events with a dataset of 300 fb^{-1} at LHCb and fragmentation fraction $f(b \rightarrow B_u^*) \sim 20\%$. Hence, the $B_u^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$ decays could be investigated at FCC-ee and LHCb experiments in the future.

Table 5 The theoretical values of decay constant $f_{B_u^*}$ (in the unit of MeV), where the legends are the same as those in Table 3

NRQM	280 [22]	151^{+15}_{-13} [61]	234.7 [63] ^a	225.1 [63] ^b	
NRQM	196 [64] ^a	182 [64] ^b	213 [65,66]	242.37 [143] ^a	232.47 [143] ^b
RQM	219 [22]	195 [65,66]	252 ± 10 [67] ^c	193 ± 8 [67] ^d	
LFQM	188 [24] ^e	$198.7^{+4.9}_{-11.3}$ [25] ^f	$193.1^{+4.3}_{-4.6}$ [25] ^g	196^{+28}_{-27} [26]	$172^{+24}_{-23} \pm 6$ [28]
LFQM	204 [68] ^h	193 [68] ⁱ	225 ± 38 [69] ^j	249^{+44}_{-42} [69] ^k	205 ± 5 [27]
LFQM	$201.9^{+43.2}_{-41.4}$ [70] ^j	$220.2^{+49.1}_{-46.2}$ [70] ^k	227 [71] ^l	211 [71] ^m	228 [71] ⁿ
LQCD	$196 \pm 24^{+39}_{-2}$ [75]	190 ± 28 [76] ^o	185.9 ± 7.2 [78]	175 ± 6 [144]	
SR	210^{+10}_{-12} [80–82]	213 ± 18 [83]	209 ± 8 [85]	209^{+23}_{-22} [138]	181.8 ± 13.7 [145]
Other	138 [71] ^p	200 ± 10 [86]	238 ± 18 [87]	164 [88]	

^aWithout QCD radiative corrections^bWith QCD radiative corrections^cWith $f_{B_u^*} = f_{B_d^*}$, and the constituent quark masses for the light quarks s and d ^dWith $f_{B_u^*} = f_{B_d^*}$, and the current quark masses for the light quarks s and d ^eWith Coulomb plus linear potential model^fWith a dilation parameter $\kappa = 0.54$ GeV^gWith a dilation parameter $\kappa = 0.68$ GeV^hWith Coulomb plus linear potential modelⁱWith Coulomb plus harmonic oscillator potential model^jWith the Gaussian type wave functions^kWith the power-law type wave functions^lWith Martin potential model [72]^mWith Cornell potential model [73]ⁿWith logarithmic potential model [74]^oWith $m_{B^*}/f_{B^*} = 28 \pm 1^{+3}_{-4}$ [76] and $m_{B^*} = 5324.70(21)$ MeV [1]^pWith harmonic plus Yukawa potential model [63]

7 $B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays

According to the conventional quark-model assignments, the B_c^* mesons consist of two heavy quarks with different flavor numbers $B = C = -Q = \pm 1$. Up to today, the experimental information of the B_c^* meson is still very limited. For example, the potential candidate of the B_c^* meson has not yet been determined. It is generally believed that the mass of the B_c^* meson should be in the region between $m_{B_c} = 6274.47 \pm 0.27 \pm 0.17$ MeV recently measured by LHCb [147] and $m_{B_c(2S)} = 6872.1 \pm 1.3 \pm 0.1 \pm 0.8$ MeV obtained by LHCb [148] (or $6871.0 \pm 1.2 \pm 0.8 \pm 0.8$ MeV given by CMS [149]), where the B_c , B_c^* and $B_c(2S)$ particles correspond to the sibling isoscalar states with quantum numbers of $n^{2S+1}L_J = 1^1S_0$, 1^3S_1 and 2^1S_0 , respectively. So the branching ratios for the strong decays $B_c^* \rightarrow BD$ are zero, because B_c^* meson is below the BD pair threshold. The experimental particle physicists are earnestly looking for and identifying the B_c^* meson, a long-expected charming beauty. For the moment, almost all of the information available about the properties of B_c^* meson (such as the mass, decay constant, lifetime, decay modes and so on) come from theoretical estimates. There are too many estimations on the B_c^* meson mass with various theoretical models, for example, in Refs. [150–207]. The recent result from lattice QCD calculation, $m_{B_c^*} = 6331 \pm 7$ MeV [192], which are basically

consistent with other estimations, will be used in this paper. Clearly, it is foreseeable that the isospin violating decay $B_c^* \rightarrow B_c \pi$ is explicitly forbidden by the law of energy conservation, because of $m_{B_c^*} - m_{B_c} \simeq 57$ MeV $< m_\pi$. Hence, the electromagnetic radiative transition $B_c^* \rightarrow B_c \gamma$ should be the dominant decay mode. In addition, the photon in the magnetic dipole transition $B_c^* \rightarrow B_c \gamma$ is very soft in the rest frame of the B_c^* meson. This might be one main reason why the unambiguously experimental identification of the B_c^* meson is very challenging. As an important complementary decay modes, the B_c^* meson has very rich weak decay channels, which could be approximately classified into three classes: (1) the valence b quark weak decay accompanied by the spectator c quark, (2) the valence c quark weak decay accompanied by the spectator b quark, and (3) the b and c quarks annihilation into a virtual W boson. The purely leptonic decays $B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ belong to the third case, which are favored by the CKM element $|V_{cb}|$. And the information of $|V_{cb}| f_{B_c^*}$ could be obtained from the $B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays.

The current values of the CKM element $|V_{cb}|$ come mainly from inclusive and exclusive semileptonic decays of B meson to charm [1]. The average values obtained from inclusive $b \rightarrow c \ell \bar{\nu}_\ell$ decays and exclusive $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ decays are $|V_{cd}| \times 10^3 = 42.2(8)$ and $39.5(9)$, respectively [1]. The lepton flavor non-universality in the ratio $R(D^{(*)})$ complicate the determination of $|V_{cb}|$. In addition, $|V_{cb}|$ can also be

Table 6 The theoretical values of decay constant $f_{B_c^*}$ (in the unit of MeV), where the legends are the same as those in Table 3

NRQM	562 [65,66]	434.64 [193]	544.3 [204]		
RQM	503 [65,66]	510±80 [164] ^a	456±70 [164] ^b	460±60 [164] ^c	
LFQM	391 ⁺⁴ ₋₅ [24]	440 ⁺⁵¹ ₋₅₂ [26]	387 [69] ^d	423 [69] ^e	465±7 [27]
LFQM	473.4±18.2 [70] ^d	487.6±19.2 [70] ^e	398 [208] ^f	551 [208] ^g	474±42 [209]
LQCD	422±13 [144]	387±12 [210]			
SR	384±32 [188] ^h	415±31 [188] ⁱ	300±30 [199]	442±44 [201] ^j	387±15 [201] ^k
Other	453±20 [86]	418±24 [87]	471 [194]		

^aWith Martin potential model [72]^bWith Coulomb plus linear potential model^cObtained from the scaling relation^dWith the Gaussian type wave functions^eWith the power-law type wave functions^fWith Coulomb plus linear potential model^gWith Coulomb plus harmonic oscillator potential model^hWith the current quark massⁱWith the pole quark mass^jWith inputs from the inverse Laplace-type model^kWith inputs from Heavy Quark Symmetry

obtained from the PLDCM $B_c^- \rightarrow \ell \bar{\nu}_\ell$ decays, although none of the measurements has reached a competitive level of precision due to either the serious helicity suppression for $B_c^- \rightarrow e \bar{\nu}_e$, $\mu \bar{\nu}_\mu$ decays or other additional neutrinos from τ decay for $B_c^- \rightarrow \tau \bar{\nu}_\tau$ decay. The global SM fit value is $|V_{cb}| = 40.53^{+0.83}_{-0.61} \times 10^{-3}$ [1], which will be used in this paper.

Both valence quarks of the B_c^{*-} mesons are regarded as heavy quarks. Their Compton wave lengths $\sim 1/m_{b,c}$ are much shorter than a typical hadron size. The spin-flavor symmetry in the heavy quark limit would lead to an approximation between decay constants $f_{B_c} \approx f_{B_c^*}$. Some theoretical results on the decay constant $f_{B_c^*}$ are collected in Table 6. The recent lattice QCD calculation $f_{B_c^*} = 387 \pm 12$ MeV [210] will be used in this paper. As it is well known that the magnetic momentum of both b and c quarks are inversely proportional to their mass. The magnetic dipole momentum

$$\mu_{B_c^* B_c} = \frac{1}{6} \left(\frac{2}{m_c} - \frac{1}{m_b} \right), \quad (54)$$

should be very small. With the quark mass $m_c = 1.5$ GeV and $m_b = 4.78$ GeV, one can obtain $\Gamma_{B_c^*} \simeq \Gamma(B_c^* \rightarrow \gamma B_c) \simeq 60$ eV using the formula of Eq. (32).

The partial decay width and branching ratios for the $B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays are estimated to be,

$$\Gamma(B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = 225^{+25}_{-21} \mu\text{eV}, \quad \text{for } \ell = e, \mu, \quad (55)$$

$$\Gamma(B_c^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = 198^{+22}_{-18} \mu\text{eV}, \quad (56)$$

$$\mathcal{B}(B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell) = (3.8^{+0.4}_{-0.3}) \times 10^{-6}, \quad \text{for } \ell = e, \mu, \quad (57)$$

$$\mathcal{B}(B_c^{*-} \rightarrow \tau^- \bar{\nu}_\tau) = (3.3^{+0.4}_{-0.3}) \times 10^{-6}. \quad (58)$$

To experimentally investigate the $B_c^{*-} \rightarrow \ell^- \bar{\nu}_\ell$ decays, there should be at least more than 10^7 B_c^* events available.

More than 10^{12} Z bosons are expected at the future e^+e^- colliders of CEPC [94] and FCC-ee [95]. Considering the branching ratio $\mathcal{B}(Z \rightarrow b\bar{b}) = 12.03 \pm 0.21\%$ [1] and fragmentation fraction $f(b \rightarrow B_c^*) \sim 6 \times 10^{-4}$ [211–213], there will be more than 10^8 B_c^* events to search for the $B_c^{*-} \rightarrow e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$, $\tau^- \bar{\nu}_\tau$ decays. In addition, the B_c^* production cross sections at LHC are estimated to be about 100 nb for pp collisions at $\sqrt{s} = 13$ TeV, about 8 mb for p -Pb collisions at $\sqrt{s} = 8.16$ TeV and some 920 mb for Pb-Pb collisions at $\sqrt{s} = 5.02$ TeV, respectively [214]. There will be more than 3×10^{10} B_c^* events corresponding to a dataset of 300 fb^{-1} at LHCb for pp collisions. Hence, the $B_c^{*-} \rightarrow e^- \bar{\nu}_e$, $\mu^- \bar{\nu}_\mu$, $\tau^- \bar{\nu}_\tau$ decays are expected to be carefully measured at LHCb experiments in the future.

8 Summary

The mass of the charged vector mesons are generally larger than that of the corresponding ground pseudoscalar mesons. The vector mesons decay mainly through the strong or/and electromagnetic interactions. These facts will inevitably result in that the branching ratios of the vector meson weak decays are often very tiny. Inspired by the potential prospects of existing and coming high-luminosity experiments, more and more experimental data will be accumulated, and higher measurement precision level will be reached. The probabilities of experimental investigation on the purely leptonic decays of charged vector mesons are discussed in this paper. We found that (1) for both $\rho^\pm$ and $K^{*\pm}$ mesons, their widths are large due to the dominance of strong decay. Their PLDCV branching ratios are estimated at the order of $\mathcal{O}(10^{-13})$. Although extremely complicated and difficult, the PLDCV

Table 7 The probabilities of experimental investigation on PLDCV, where $\mathcal{B}$ denotes the branching ratio, and the symbol $\star$ denotes that the PLDCV process might be experimentally accessible in the future

Decay modes	$\mathcal{B}$	Belle-II	SCTF/STCF	CEPC	FCC-ee	LHCb
$\rho^- \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu$	$\mathcal{O}(10^{-13})$					$\star$
$K^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu$	$\mathcal{O}(10^{-13})$					$\star$
$D_s^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$	$\mathcal{O}(10^{-10})$	$\star$	$\star$	$\star$	$\star$	$\star$
$D_s^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$	$\mathcal{O}(10^{-6})$	$\star$	$\star$	$\star$	$\star$	$\star$
$B_u^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$	$\mathcal{O}(10^{-10})$			$\star$	$\star$	$\star$
$B_c^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$	$\mathcal{O}(10^{-6})$			$\star$	$\star$	$\star$

decays $\rho^\pm, K^{*\pm} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu$ might be measurable due to the huge data of the $\rho^\pm$ and $K^{*\pm}$ mesons at LHCb. (2) The PLDCV D_s^* decays are favored by the CKM element $|V_{cs}|$. Their branching ratios are about $\mathcal{O}(10^{-6})$. The PLDCV decays $D_{d,s}^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$ could be carefully studied at the Belle-II, SCTF or STCF, CEPC, FCC-ee, LHCb experiments. (3) For the B_u^* mesons below the $B\pi$ thresholds and the B_c^* mesons below both BD and $B_c\pi$ thresholds, they decay predominantly through the magnetic dipole transitions. The branching ratios of the PLDCV B_c^* decays favored by the CKM element $|V_{cb}|$ could reach up to $\mathcal{O}(10^{-6})$. The PLDCV decays $B_{u,c}^{*-} \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$ might be searched for at the CEPC, FCC-ee, LHCb experiments. Our rough estimations and findings are summed in Table 7. We wish that our investigation could provoke physicists' researching interest in PLDCV and offer a ready reference for the future experimental analysis.

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