Eur. Phys. J. C (2021) 81:1111
https://doi.org/10.1140/epjc/s10052-021-09902-2

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

QCD sum rules analysis of weak decays of doubly heavy baryons: the $b \to c$ processes

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Received: 23 September 2021 / Accepted: 2 December 2021 / Published online: 16 December 2021

Abstract A comprehensive study of $b \to c$ weak decays of doubly heavy baryons is presented in this paper. The transition form factors as well as the pole residues of the initial and final states are respectively obtained by investigating the three-point and two-point correlation functions in QCD sum rules. Contributions from up to dimension-6 operators are respectively considered for the two-point and three-point correlation functions. The obtained form factors are then applied to a phenomenological analysis of semi-leptonic decays.

1 Introduction

Quark model has achieved brilliant success in the study of hadron spectroscopy. However, the existence of doubly heavy baryons had become a long-standing problem in experiments until LHCb reported the observation of Ξ_{cc}^{++} [1]. The discovery of the doubly charmed baryon has triggered many related theoretical researches on the masses, lifetimes, strong coupling constants, and decay widths of doubly heavy baryons. They are based on various model calculations [2–26], SU(3) symmetry analysis [27–29], effective theories [30–40], QCD sum rules [41–43] and light-cone sum rules [44–52]. For a recent review, see [53]. It is promising that more doubly heavy baryons will be discovered in the near future.

In [5], we performed an analysis of weak decays of doubly heavy baryons using the approach of light-front quark model. However, model-dependent parameters are inevitably introduced. In view of this, in [43], we investigated the weak decays of doubly heavy baryons to singly heavy baryons using QCD sum rules (QCDSR). QCDSR is a QCD-based approach to deal with the hadron parameters. It reveals a connection between hadron phenomenology and QCD vacuum via a few universal condensate parameters. However, the processes induced by the $b \to c$ transition were not considered in [43]. In particular, these processes are considered to be important for the search of other doubly heavy baryons. This work aims to fill this gap. Specifically, we will consider the following processes ($q = u/d$):

- the bb sector,

$$\begin{aligned}\Xi_{bb}(bbq) &\to \Xi_{bc}(bcq),\\ \Omega_{bb}(bbs) &\to \Omega_{bc}(bcs),\end{aligned}$$

- the bc sector,

$$\begin{aligned}\Xi_{bc}(bcq) &\to \Xi_{cc}(ccq),\\ \Omega_{bc}(bcs) &\to \Omega_{cc}(ccs).\end{aligned}$$

The transition matrix element can be parametrized by the so-called helicity form factors $f_{0,+,\perp}$ and $g_{0,+,\perp}$ [54]:

$$\begin{aligned}&\langle \mathcal{B}_2(P_2)|(V-A)_\mu|\mathcal{B}_1(P_1)\rangle = \bar{u}(P_2,s_2)\bigg[\frac{q_\mu}{q^2}(M_1-M_2)f_0(q^2)\\ &+\frac{M_1+M_2}{Q_+}\left((P_1+P_2)_\mu-(M_1^2-M_2^2)\frac{q_\mu}{q^2}\right)f_+(q^2)\\ &+\left(\gamma_\mu-\frac{2M_2}{Q_+}P_{1\mu}-\frac{2M_1}{Q_+}P_{2\mu}\right)f_\perp(q^2)\bigg]u(P_1,s_1)\\ &-\bar{u}(P_2,s_2)\gamma_5\bigg[\frac{q_\mu}{q^2}(M_1+M_2)g_0(q^2)\\ &+\frac{M_1-M_2}{Q_-}\left((P_1+P_2)_\mu-(M_1^2-M_2^2)\frac{q_\mu}{q^2}\right)g_+(q^2)\\ &+\left(\gamma_\mu+\frac{2M_2}{Q_-}P_{1\mu}-\frac{2M_1}{Q_-}P_{2\mu}\right)g_\perp(q^2)\bigg]u(P_1,s_1) \qquad (1)\end{aligned}$$

with $Q_\pm = (M_1 \pm M_2)^2 - q^2$. These form factors can be extracted using the three-point correlation functions in QCDSR.

The leading logarithmic corrections are also considered in this work. In some literatures, the anomalous dimensions of

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interpolating currents of baryons are incorrectly cited. Therefore, in [55], we calculated these anomalous dimensions at one-loop level.

It is worth noting that Heavy Quark Effective Theory (HQET) does not apply to the situation of doubly heavy baryons. However, the heavy quark limit can still be taken from the full theory results, as can be seen in [56–58]. Some efforts were made to develop the effective theory for doubly heavy baryons in [37].

The rest of this paper is arranged as follows. In Sect. 2, the QCDSR methods for the two-point and three-point correlation functions are briefly introduced, and corresponding numerical results are shown in Sect. 3. The obtained form factors are applied to phenomenology analysis in Sect. 4. A short summary is given in the last section.

2 QCD sum rules

2.1 The two-point correlation functions

The pole residue of the doubly heavy baryon $\mathcal{B}_{Q_1Q_2q_3}$ can be obtained by calculating the following two-point correlation function

$$\Pi(q)=i\int d^4x e^{iq\cdot x}\langle 0|T\left[J_{\mathcal{B}_{Q_1Q_2q_3}}(x)\bar{J}_{\mathcal{B}_{Q_1Q_2q_3}}(0)\right]|0\rangle. \tag{2}$$

The interpolating currents of doubly heavy baryons are

$$\begin{aligned}J_{\mathcal{B}_{QQq}}(y)&=\epsilon_{abc}(Q_a^TC\gamma^\mu Q_b)\gamma_\mu\gamma_5q_c,\\ J_{\mathcal{B}_{Q_1Q_2q}}(y)&=\epsilon_{abc}\frac{1}{\sqrt{2}}(Q_{1a}^TC\gamma^\mu Q_{2b}\\ &\quad+Q_{2a}^TC\gamma^\mu Q_{1b})\gamma_\mu\gamma_5q_c.\end{aligned} \tag{3}$$

At the hadron level, by inserting the complete set of baryons in Eq. (2), one can obtain

$$\Pi^{\rm had}(q)=\lambda_+^2\frac{\not{q}+M_+}{M_+^2-q^2}+\lambda_-^2\frac{\not{q}-M_-}{M_-^2-q^2}+\cdots, \tag{4}$$

where we have also considered the contribution from the negative-parity baryon, and $M_\pm$ ($\lambda_\pm$) are respectively the masses (pole residues) of positive- and negative-parity baryons. The pole residues are introduced as

$$\begin{aligned}\langle 0|J_+(0)|\mathcal{B}_+(p,s)\rangle&=u(p,s)\lambda_+,\\ \langle 0|J_+(0)|\mathcal{B}_-(p,s)\rangle&=(i\gamma_5)u(p,s)\lambda_-.\end{aligned} \tag{5}$$

At the QCD level, the correlation functions are calculated using the operator product expansion (OPE) technique. Contributions from up to dimension-5 operators are considered in this work. The result can be formally written as

$$\Pi(q)=\not{q}\Pi_1(q^2)+\Pi_2(q^2). \tag{6}$$

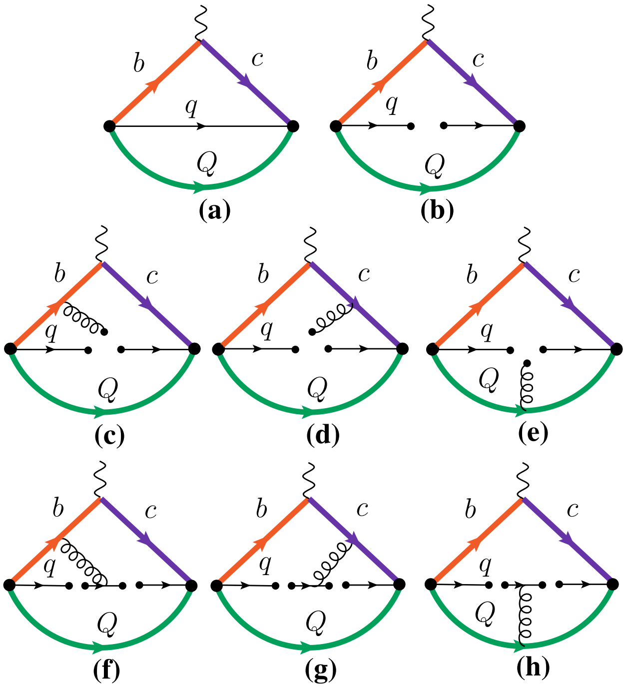

Fig. 1 Feynman diagrams in the calculation of the three-point correlation functions at the quark level

Table 1 Masses of doubly heavy baryons with $J^P=1/2^-$ [62,63]

Baryon	$\Xi_{cc}(\frac{1}{2}^-)$	$\Omega_{cc}(\frac{1}{2}^-)$	$\Xi_{bc}(\frac{1}{2}^-)$	$\Omega_{bc}(\frac{1}{2}^-)$	$\Xi_{bb}(\frac{1}{2}^-)$	$\Omega_{bb}(\frac{1}{2}^-)$
Mass	3.77	3.91	7.231	7.346	10.38	10.53

Π_i can be written in terms of dispersion relation for practical purpose

$$\Pi_i(q^2)=\int_0^\infty ds\frac{\rho_i(s)}{s-q^2}. \tag{7}$$

Assuming quark-hadron duality and performing the Borel transformation, one can obtain the following sum rule for $1/2^+$ baryon

$$(M_++M_-)\lambda_+^2e^{-M_+^2/T_+^2}=\int^{s_+}ds(M_-\rho_1(s)+\rho_2(s))e^{-s/T_+^2}, \tag{8}$$

where T_+^2 and s_+ are respectively the Borel parameter and continuum threshold parameter. From Eq. (8), one can obtain the squared mass for $1/2^+$ baryon

$$M_+^2=\frac{\int^{s_+}ds\ (M_-\rho_1+\rho_2)\ s\ e^{-s/T_+^2}}{\int^{s_+}ds\ (M_-\rho_1+\rho_2)\ e^{-s/T_+^2}}. \tag{9}$$

The leading logarithmic (LL) corrections are considered in this work. The Wilson coefficients of OPE should be multiplied by

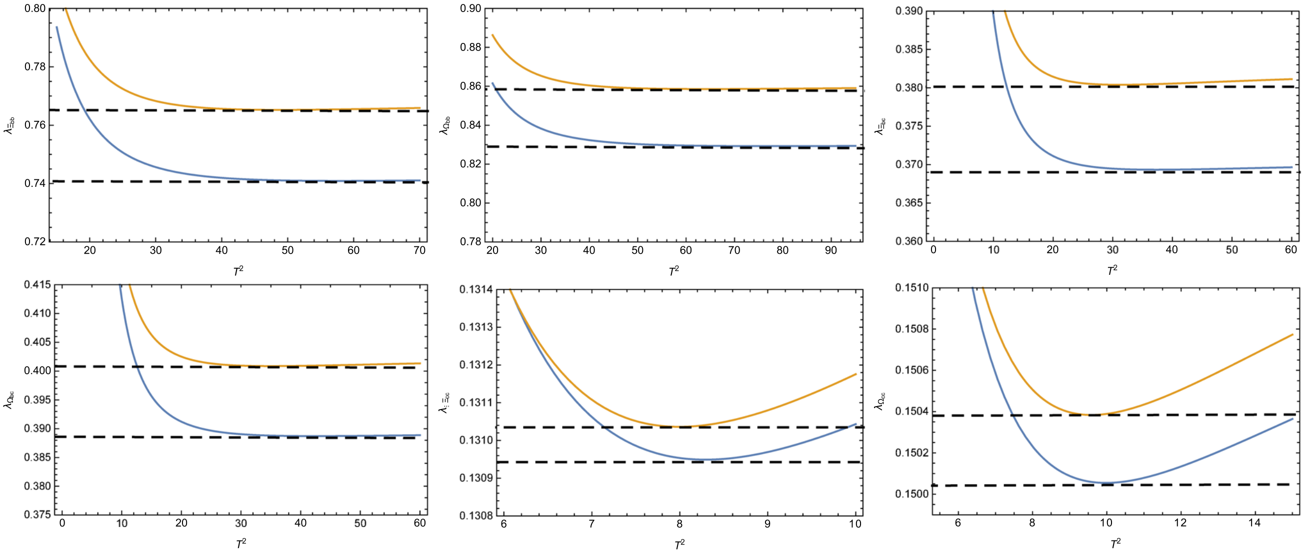

Fig. 2 The pole residues as functions of the Borel parameters. The blue and orange curves respectively correspond to our predictions with and without the LL corrections

Table 2 The pole residues

	This work (without the LL corrections)	This work (with the LL corrections)
$\lambda_{\Xi_{bb}}$	0.760	$0.736^{+0.000}_{-0.000}(T^2)^{+0.053}_{-0.052}(s_0)$
$\lambda_{\Omega_{bb}}$	0.854	$0.825^{+0.000}_{-0.000}(T^2)^{+0.060}_{-0.059}(s_0)$
$\lambda_{\Xi_{bc}}$	0.379	$0.369^{+0.000}_{-0.000}(T^2)^{+0.028}_{-0.027}(s_0)$
$\lambda_{\Omega_{bc}}$	0.400	$0.388^{+0.000}_{-0.000}(T^2)^{+0.030}_{-0.029}(s_0)$
$\lambda_{\Xi_{cc}}$	0.130	$0.130^{+0.000}_{-0.000}(T^2)^{+0.011}_{-0.011}(s_0)$
$\lambda_{\Omega_{cc}}$	0.150	$0.149^{+0.000}_{-0.000}(T^2)^{+0.013}_{-0.012}(s_0)$

$$\left(\frac{\log(\mu_0/\Lambda_{\mathrm{QCD}}^{(n_f)})}{\log(\mu/\Lambda_{\mathrm{QCD}}^{(n_f)})}\right)^{2\gamma_J-\gamma_O}, \tag{10}$$

where γ_J and γ_O are anomalous dimensions of the interpolating current and the local operator respectively. $\Lambda_{\mathrm{QCD}}^{(n_f)}$ is given by $\Lambda_{\mathrm{QCD}}^{(3)} = 223$ MeV and $\Lambda_{\mathrm{QCD}}^{(4)} = 170$ MeV [59,60]. The renormalization scale $\mu_0 \sim 1$ GeV, and μ is chosen as m_c for doubly charmed baryons and m_b for doubly bottom and bottom-charmed baryons. The masses and pole residues of doubly heavy baryons are also be considered in [61].

2.2 The three-point correlation functions

The following three-point correlation functions are adopted to extract the transition form factors of $\mathcal{B}_{bQq} \to \mathcal{B}_{cQq}$

$$\Pi_\mu^{V,A}(P_1, P_2) = i^2 \int d^4x d^4y e^{-iP_1\cdot x+iP_2\cdot y} \langle 0|T\{J_{\mathcal{B}_{cQq}}(y) \times (V_\mu, A_\mu)(0)\bar{J}_{\mathcal{B}_{bQq}}(x)\}|0\rangle. \tag{11}$$

At the hadron level, the complete sets of baryon states are inserted to the correlation function to obtain for the vector current correlation function

Table 3 The masses

	This work (without the LL corrections)	This work (with the LL corrections)	Lattice QCD [64]
$m_{\Xi_{bb}}$	10.166	$10.152^{+0.007}_{-0.009}(T^2)^{+0.079}_{-0.080}(s_0)$	10.143
$m_{\Omega_{bb}}$	10.291	$10.279^{+0.006}_{-0.007}(T^2)^{+0.080}_{-0.081}(s_0)$	10.273
$m_{\Xi_{bc}}$	6.948	$6.935^{+0.009}_{-0.007}(T^2)^{+0.080}_{-0.081}(s_0)$	6.943
$m_{\Omega_{bc}}$	7.002	$6.998^{+0.006}_{-0.008}(T^2)^{+0.081}_{-0.081}(s_0)$	6.998
$m_{\Xi_{cc}}$	3.634	$3.629^{+0.010}_{-0.012}(T^2)^{+0.078}_{-0.079}(s_0)$	3.621 [1]
$m_{\Omega_{cc}}$	3.747	$3.743^{+0.009}_{-0.011}(T^2)^{+0.079}_{-0.080}(s_0)$	3.738

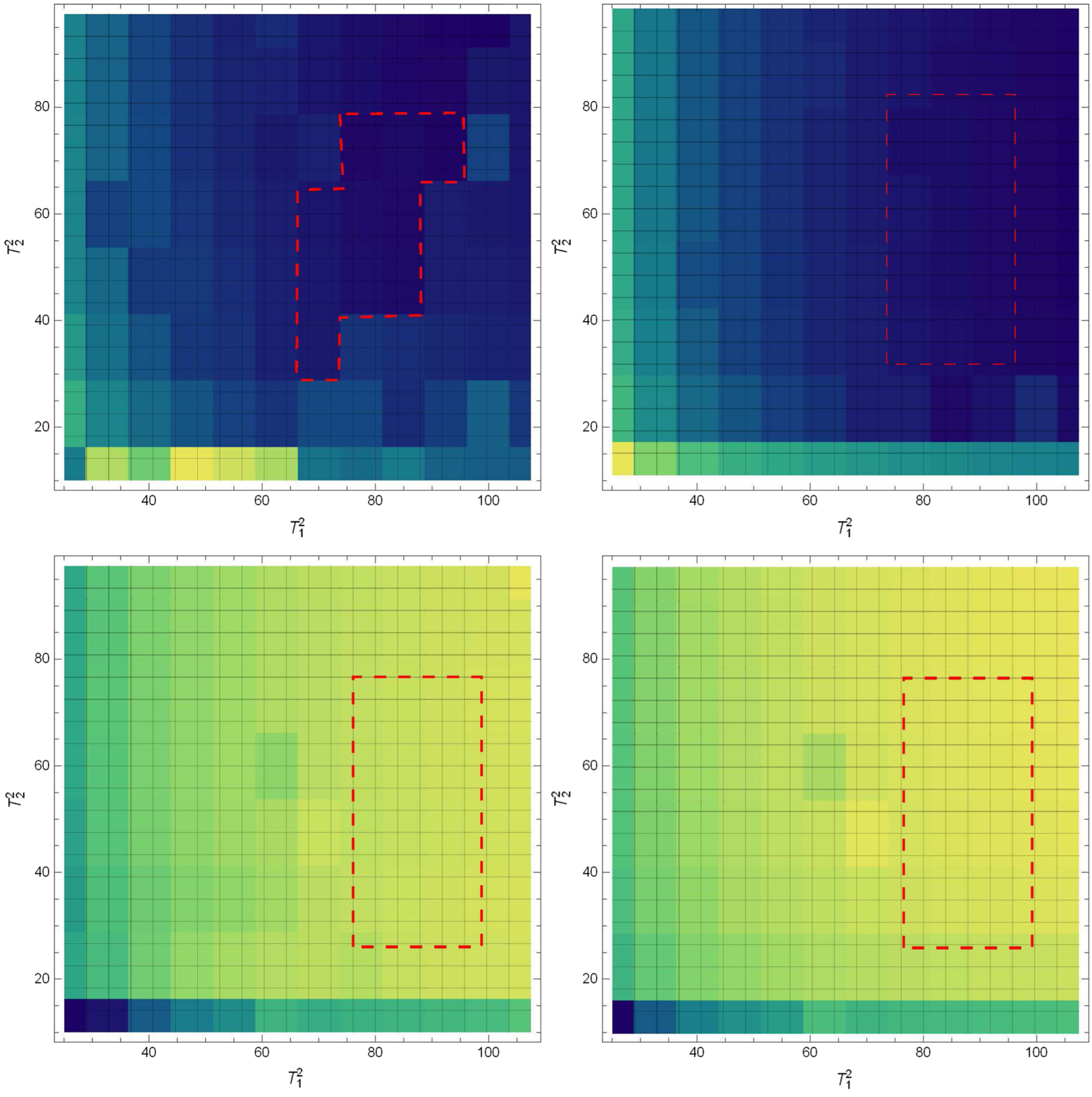

Fig. 3 The $\Xi_{bb} \to \Xi_{bc}$ form factors $f_{+,0}$ (top left), $f_\perp$ (top right), $g_{+,0}$ (bottom left) and $g_\perp$ (bottom right) at $q^2 = 0$: the dependence on the Borel parameters $T_{1,2}^2$

$$\begin{aligned}
\Pi_\mu^{V,\text{had}}&(P_1, P_2) \\
&= \frac{\lambda_f^+\lambda_i^+}{(P_2^2 - M_2^{+2})(P_1^2 - M_1^{+2})}(\not{P}_2 + M_2^+)\mathcal{V}_\mu^{++}(\not{P}_1 + M_1^+) \\
&+ \frac{\lambda_f^+\lambda_i^-}{(P_2^2 - M_2^{+2})(P_1^2 - M_1^{-2})}(\not{P}_2 + M_2^+)\mathcal{V}_\mu^{+-}(\not{P}_1 - M_1^-) \\
&+ \frac{\lambda_f^-\lambda_i^+}{(P_2^2 - M_2^{-2})(P_1^2 - M_1^{+2})}(\not{P}_2 - M_2^-)\mathcal{V}_\mu^{-+}(\not{P}_1 + M_1^+) \\
&+ \frac{\lambda_f^-\lambda_i^-}{(P_2^2 - M_2^{-2})(P_1^2 - M_1^{-2})}(\not{P}_2 - M_2^-)\mathcal{V}_\mu^{--}(\not{P}_1 - M_1^-) \\
&+ \cdots \qquad (12)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{V}_\mu^{ij} \equiv \frac{q_\mu}{q^2}(M_1^j - M_2^i) f_0^{ij}(q^2) + \frac{M_1^j + M_2^i}{Q_+}((P_1 + P_2)_\mu \\
-(M_1^{j2} - M_2^{i2})\frac{q_\mu}{q^2}) f_+^{ij}(q^2)
\end{aligned}$$

Table 4 The fitted results for the form factors. The parameter a and b satisfy a two-dimensional Gaussian distribution with $\rho = -0.999$

	$F(0)$	a	b
$f_+^{\Xi_{bb}\to\Xi_{bc}}$	$0.441^{+0.008}_{-0.008}(T_1^2, T_2^2)^{+0.020}_{-0.015}(s_1^0)^{+0.072}_{-0.037}(s_2^0)$	$1.200 \pm 0.460(T_1^2, T_2^2) \pm 0.600(s_1^0, s_2^0)$	$-10.1 \pm 6.1(T_1^2, T_2^2) \pm 7.9(s_1^0, s_2^0)$
$f_0^{\Xi_{bb}\to\Xi_{bc}}$	$0.441^{+0.008}_{-0.008}(T_1^2, T_2^2)^{+0.020}_{-0.015}(s_1^0)^{+0.072}_{-0.037}(s_2^0)$	$1.090 \pm 0.440(T_1^2, T_2^2) \pm 0.600(s_1^0, s_2^0)$	$-8.6 \pm 5.8(T_1^2, T_2^2) \pm 7.9(s_1^0, s_2^0)$
$f_\perp^{\Xi_{bb}\to\Xi_{bc}}$	$0.816^{+0.009}_{-0.007}(T_1^2, T_2^2)^{+0.045}_{-0.047}(s_1^0)^{+0.114}_{-0.092}(s_2^0)$	$1.030 \pm 0.480(T_1^2, T_2^2) \pm 0.710(s_1^0, s_2^0)$	$-2.7 \pm 6.3(T_1^2, T_2^2) \pm 9.3(s_1^0, s_2^0)$
$g_+^{\Xi_{bb}\to\Xi_{bc}}$	$-0.283^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.033}_{-0.027}(s_1^0)^{+0.048}_{-0.050}(s_2^0)$	$0.040 \pm 0.26(T_1^2, T_2^2) \pm 0.710(s_1^0, s_2^0)$	$-4.3 \pm 3.4(T_1^2, T_2^2) \pm 9.3(s_1^0, s_2^0)$
$g_0^{\Xi_{bb}\to\Xi_{bc}}$	$-0.283^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.033}_{-0.027}(s_1^0)^{+0.048}_{-0.050}(s_2^0)$	$-0.001 \pm 0.255(T_1^2, T_2^2) \pm 0.710(s_1^0, s_2^0)$	$-3.8 \pm 3.4(T_1^2, T_2^2) \pm 9.3(s_1^0, s_2^0)$
$g_\perp^{\Xi_{bb}\to\Xi_{bc}}$	$-0.287^{+0.003}_{-0.004}(T_1^2, T_2^2)^{+0.021}_{-0.016}(s_1^0)^{+0.038}_{-0.040}(s_2^0)$	$0.100 \pm 0.250(T_1^2, T_2^2) \pm 0.590(s_1^0, s_2^0)$	$-5.1 \pm 3.3(T_1^2, T_2^2) \pm 7.8(s_1^0, s_2^0)$
$f_+^{\Omega_{bb}\to\Omega_{bc}}$	$0.463^{+0.004}_{-0.008}(T_1^2, T_2^2)^{+0.041}_{-0.047}(s_1^0)^{+0.056}_{-0.070}(s_2^0)$	$0.310 \pm 0.480(T_1^2, T_2^2) \pm 0.740(s_1^0, s_2^0)$	$1.8 \pm 6.0(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$f_0^{\Omega_{bb}\to\Omega_{bc}}$	$0.462^{+0.004}_{-0.008}(T_1^2, T_2^2)^{+0.041}_{-0.047}(s_1^0)^{+0.056}_{-0.070}(s_2^0)$	$0.320 \pm 0.480(T_1^2, T_2^2) \pm 0.740(s_1^0, s_2^0)$	$1.7 \pm 5.9(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$f_\perp^{\Omega_{bb}\to\Omega_{bc}}$	$0.865^{+0.010}_{-0.009}(T_1^2, T_2^2)^{+0.013}_{-0.079}(s_1^0)^{+0.082}_{-0.124}(s_2^0)$	$1.646 \pm 0.600(T_1^2, T_2^2) \pm 2.500(s_1^0, s_2^0)$	$-9.8 \pm 7.4(T_1^2, T_2^2) \pm 31(s_1^0, s_2^0)$
$g_+^{\Omega_{bb}\to\Omega_{bc}}$	$-0.286^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.035}_{-0.057}(s_1^0)^{+0.065}_{-0.065}(s_2^0)$	$-0.560 \pm 0.170(T_1^2, T_2^2) \pm 0.740(s_1^0, s_2^0)$	$3.4 \pm 2.1(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$g_0^{\Omega_{bb}\to\Omega_{bc}}$	$-0.286^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.035}_{-0.057}(s_1^0)^{+0.065}_{-0.065}(s_2^0)$	$-0.600 \pm 0.170(T_1^2, T_2^2) \pm 0.740(s_1^0, s_2^0)$	$4.0 \pm 2.1(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$g_\perp^{\Omega_{bb}\to\Omega_{bc}}$	$-0.292^{+0.002}_{-0.002}(T_1^2, T_2^2)^{+0.019}_{-0.037}(s_1^0)^{+0.044}_{-0.042}(s_2^0)$	$-0.510 \pm 0.130(T_1^2, T_2^2) \pm 2.100(s_1^0, s_2^0)$	$2.8 \pm 1.7(T_1^2, T_2^2) \pm 26(s_1^0, s_2^0)$
$f_+^{\Xi_{bc}\to\Xi_{cc}}$	$0.671^{+0.007}_{-0.004}(T_1^2, T_2^2)^{+0.050}_{-0.052}(s_1^0)^{+0.142}_{-0.137}(s_2^0)$	$1.380 \pm 0.330(T_1^2, T_2^2) \pm 1.500(s_1^0, s_2^0)$	$-8.5 \pm 3.9(T_1^2, T_2^2) \pm 18(s_1^0, s_2^0)$
$f_0^{\Xi_{bc}\to\Xi_{cc}}$	$0.671^{+0.007}_{-0.004}(T_1^2, T_2^2)^{+0.050}_{-0.052}(s_1^0)^{+0.142}_{-0.137}(s_2^0)$	$1.310 \pm 0.330(T_1^2, T_2^2) \pm 2.000(s_1^0, s_2^0)$	$-7.7 \pm 4.0(T_1^2, T_2^2) \pm 24(s_1^0, s_2^0)$
$f_\perp^{\Xi_{bc}\to\Xi_{cc}}$	$0.815^{+0.006}_{-0.005}(T_1^2, T_2^2)^{+0.061}_{-0.063}(s_1^0)^{+0.175}_{-0.169}(s_2^0)$	$1.340 \pm 0.320(T_1^2, T_2^2) \pm 3.000(s_1^0, s_2^0)$	$-6.3 \pm 3.9(T_1^2, T_2^2) \pm 36(s_1^0, s_2^0)$
$g_+^{\Xi_{bc}\to\Xi_{cc}}$	$-0.435^{+0.002}_{-0.002}(T_1^2, T_2^2)^{+0.82}_{-0.083}(s_1^0)^{+0.121}_{-0.123}(s_2^0)$	$-0.310 \pm 0.130(T_1^2, T_2^2) \pm 0.820(s_1^0, s_2^0)$	$-1.6 \pm 1.6(T_1^2, T_2^2) \pm 9.8(s_1^0, s_2^0)$
$g_0^{\Xi_{bc}\to\Xi_{cc}}$	$-0.435^{+0.002}_{-0.002}(T_1^2, T_2^2)^{+0.077}_{-0.083}(s_1^0)^{+0.121}_{-0.123}(s_2^0)$	$-0.370 \pm 0.150(T_1^2, T_2^2) \pm 0.770(s_1^0, s_2^0)$	$-0.8 \pm 1.8(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$g_\perp^{\Xi_{bc}\to\Xi_{cc}}$	$-0.456^{+0.002}_{-0.002}(T_1^2, T_2^2)^{+0.040}_{-0.046}(s_1^0)^{+0.075}_{-0.080}(s_2^0)$	$-0.370 \pm 0.130(T_1^2, T_2^2) \pm 0.760(s_1^0, s_2^0)$	$-1.1 \pm 1.6(T_1^2, T_2^2) \pm 9.2(s_1^0, s_2^0)$
$f_+^{\Omega_{bc}\to\Omega_{cc}}$	$0.659^{+0.001}_{-0.000}(T_1^2, T_2^2)^{+0.050}_{-0.064}(s_1^0)^{+0.142}_{-0.137}(s_2^0)$	$0.960 \pm 0.130(T_1^2, T_2^2) \pm 1.400(s_1^0, s_2^0)$	$-3.8 \pm 1.6(T_1^2, T_2^2) \pm 18(s_1^0, s_2^0)$
$f_0^{\Omega_{bc}\to\Omega_{cc}}$	$0.659^{+0.001}_{-0.000}(T_1^2, T_2^2)^{+0.050}_{-0.064}(s_1^0)^{+0.142}_{-0.137}(s_2^0)$	$0.890 \pm 0.130(T_1^2, T_2^2) \pm 1.400(s_1^0, s_2^0)$	$-2.9 \pm 1.6(T_1^2, T_2^2) \pm 18(s_1^0, s_2^0)$
$f_\perp^{\Omega_{bc}\to\Omega_{cc}}$	$0.804^{+0.001}_{-0.001}(T_1^2, T_2^2)^{+0.058}_{-0.077}(s_1^0)^{+0.175}_{-0.169}(s_2^0)$	$1.510 \pm 0.170(T_1^2, T_2^2) \pm 2.400(s_1^0, s_2^0)$	$-9.0 \pm 2.1(T_1^2, T_2^2) \pm 34(s_1^0, s_2^0)$
$g_+^{\Omega_{bc}\to\Omega_{cc}}$	$-0.410^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.087}_{-0.079}(s_1^0)^{+0.270}_{-0.293}(s_2^0)$	$-0.630 \pm 0.280(T_1^2, T_2^2) \pm 0.750(s_1^0, s_2^0)$	$2.8 \pm 3.5(T_1^2, T_2^2) \pm 9.3(s_1^0, s_2^0)$
$g_0^{\Omega_{bc}\to\Omega_{cc}}$	$-0.410^{+0.003}_{-0.003}(T_1^2, T_2^2)^{+0.087}_{-0.079}(s_1^0)^{+0.270}_{-0.293}(s_2^0)$	$-0.960 \pm 0.130(T_1^2, T_2^2) \pm 0.750(s_1^0, s_2^0)$	$-3.8 \pm 1.6(T_1^2, T_2^2) \pm 9.3(s_1^0, s_2^0)$
$g_\perp^{\Omega_{bc}\to\Omega_{cc}}$	$-0.429^{+0.002}_{-0.002}(T_1^2, T_2^2)^{+0.049}_{-0.042}(s_1^0)^{+0.172}_{-0.190}(s_2^0)$	$-0.810 \pm 0.250(T_1^2, T_2^2) \pm 0.790(s_1^0, s_2^0)$	$4.8 \pm 3.2(T_1^2, T_2^2) \pm 9.8(s_1^0, s_2^0)$

$$+\left(\gamma_\mu-\frac{2M_2^j}{Q_+}P_{1\mu}-\frac{2M_1^i}{Q_+}P_{2\mu}\right)f_\perp^{ij}(q^2) \tag{13}$$

with $i, j=+,-$. In this step, both of the contributions from positive- and negative-parity baryons are considered. $M_{1(2)}^{+(-)}$ and $\lambda_{i(f)}^{+(-)}$ respectively denote the mass and pole residue of the baryon in the initial (final) state with positive (negative) parity, and $f_{0,+,\perp}^{ij}(q^2)$ are 12 form factors defined by:

$$\begin{aligned}
&\langle\mathcal{B}_f^+(p_2,s_2)|V_\mu|\mathcal{B}_i^+(p_1,s_1)\rangle=\bar u_{\mathcal{B}_f^+}(p_2,s_2)\mathcal{V}_\mu^{++}u_{\mathcal{B}_i^+}(p_1,s_1),\\
&\langle\mathcal{B}_f^+(p_2,s_2)|V_\mu|\mathcal{B}_i^-(p_1,s_1)\rangle\\
&\quad=\bar u_{\mathcal{B}_f^+}(p_2,s_2)\mathcal{V}_\mu^{+-}(i\gamma_5)u_{\mathcal{B}_i^-}(p_1,s_1),\\
&\langle\mathcal{B}_f^-(p_2,s_2)|V_\mu|\mathcal{B}_i^+(p_1,s_1)\rangle\\
&\quad=\bar u_{\mathcal{B}_f^-}(p_2,s_2)(i\gamma_5)\mathcal{V}_\mu^{-+}u_{\mathcal{B}_i^+}(p_1,s_1),\\
&\langle\mathcal{B}_f^-(p_2,s_2)|V_\mu|\mathcal{B}_i^-(p_1,s_1)\rangle\\
&\quad=\bar u_{\mathcal{B}_f^-}(p_2,s_2)(i\gamma_5)\mathcal{V}_\mu^{--}(i\gamma_5)u_{\mathcal{B}_i^-}(p_1,s_1).
\end{aligned}\tag{14}$$

At the quark level, the correlation functions in Eq. (11) are calculated using OPE technique. In this work, contributions from the perturbative term (dim-0), quark condensate term (dim-3), mixed quark-gluon condensate term (dim-5), and four-quark condensate term (dim-6) are considered, as can be seen in Fig. 1. The vector current correlation function is further written into the double dispersion relation

$$\Pi_\mu^V(P_1,P_2)=\int^\infty ds_1\int^\infty ds_2\frac{\rho_\mu^V(s_1,s_2,q^2)}{(s_1-P_1^2)(s_2-P_2^2)},\quad q=P_1-P_2, \tag{15}$$

where the spectral density functions $\rho_\mu^V(s_1,s_2,q^2)$ are obtained by taking discontinuities for s_1 and s_2. Our method is further illustrated by the calculation of perturbative diagram below.

The spectral density of the perturbative diagram in Fig. 1a can be obtained as

$$\begin{aligned}
\rho_\mu^{V,\mathrm{pert}}(P_1^2,P_2^2,q^2)&=\frac{1}{(2\pi i)^2}\int\frac{d^4p_bd^4p_cd^4p_Qd^4p_q}{(2\pi)^{16}}N_\mu\\
&\times(2\pi)^8\delta^4(p_b+p_Q+p_q-P_1)\delta^4(p_c+p_Q+p_q-P_2)\\
&\times(-2\pi i)^4\delta(p_b^2-m_b^2)\delta(p_c^2-m_c^2)\delta(p_Q^2-m_Q^2)\delta(p_q^2-m_q^2).
\end{aligned}\tag{16}$$

with

$$\begin{aligned}
N_\mu&=\mathrm{Tr}\left[(\not p_Q+m_Q)\gamma^\rho(\not p_c-m_c)(V_\mu,A_\mu)(\not p_b+m_b)\gamma^\nu\right]\\
&\times\frac{6}{\sqrt{2}}(\gamma_\rho\gamma_5(\not p_q+m_q)\gamma_5\gamma_\nu).
\end{aligned}\tag{17}$$

The integral in Eq. (16) can be written as a two-body phase space integral followed by a "triangle" phase space integral [43].

Equating Eq. (15) with Eq. (12), assuming quark-hadron duality, and performing the Borel transformation, one can obtain

$$\begin{aligned}
&\mathcal{B}\Pi_\mu^{V,\mathrm{pole}}(T_1^2,T_2^2)\\
&=\int^{s_1^0}ds_1\int^{s_2^0}ds_2\,\rho_\mu^V(s_1,s_2,q^2)e^{-s_1/T_1^2}e^{-s_2/T_2^2},
\end{aligned}\tag{18}$$

where the left-hand side denotes the Borel transformed four pole terms in Eq. (12) and $s_{1,2}^0$ are the continuum threshold parameters. Equating the coefficients of the same Dirac structures on both sides of Eq. (18), one can arrive at 12 equations, from which, one can further extract the form factors. More details can be found in [43,60].

In addition, similar as the situation of the two-point correlation function, the LL corrections are also considered.

3 Numerical results

The masses of negative-parity baryons are used in this work, and we adopt the results from [62,63], which are collected in Table 1.

3.1 Pole residues

Our predictions of pole residues and masses are respectively collected in Tables 2 and 3, and the pole residues as functions of the Borel parameters are plotted in Fig. 2. Both of the results without and with the LL corrections are shown. In Table 3, our predictions for the masses are also compared with those from Lattice QCD [64].

3.2 Form factors

We take the process of $\Xi_{bb}\to\Xi_{bc}$ as an example to illustrate the selection of Borel windows. In Fig. 3, the transition form factors of $\Xi_{bb}\to\Xi_{bc}$ are plotted as functions of the Borel parameters $T_{1,2}^2$. Relatively flat regions are selected as the working Borel windows.

We also consider the uncertainties of the form factors caused by the Borel parameters $T_{1,2}^2$ and the continuum threshold parameter $s_{1,2}^0$, as can be seen in Table 4. To access the q^2 dependence, we calculate the form factors at small q^2, and then fit the data with the following formula

$$f(q^2)=\frac{1}{1-q^2/(m_{\mathrm{pole}})^2}(a+bz(q^2)) \tag{19}$$

with

$$z(q^2)=\frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}. \tag{20}$$

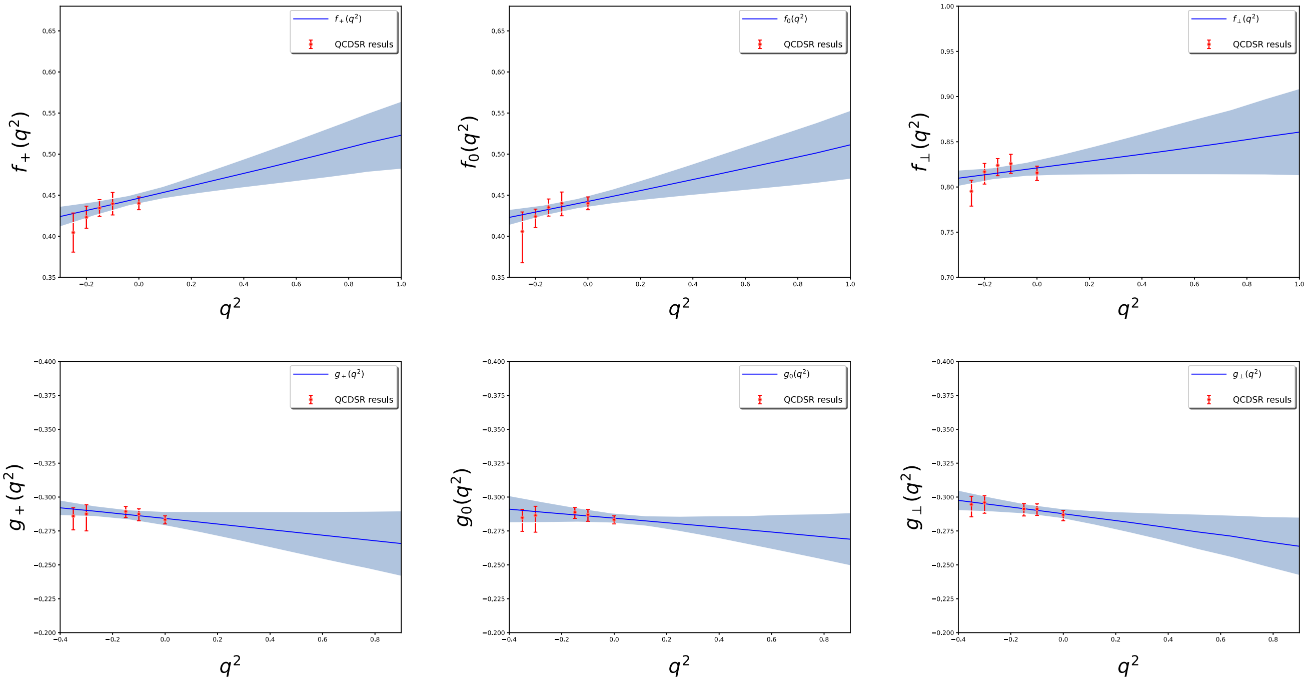

Fig. 4 One of the one loop Feynman diagram which exchange gluon

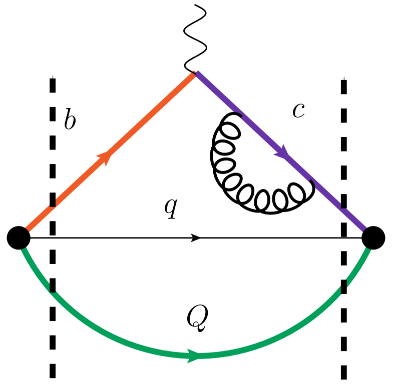

Fig. 5 The fitted results of the $\Xi_{bb} \to \Xi_{bc}$ form factors

Here $t_0 = q^2_{\max} = (m_{\mathcal{B}_i} - m_{\mathcal{B}_f})^2$ and $t_+ = (m_{\text{pole}})^2$ are chosen to be equal or below the location of any remaining singularity after factoring out the leading pole contribution [65]. The nonlinear least-χ^2 (lsq) method is used in our analysis [66]. The fitted results are shown in Table 4 and Fig. 4. The contributions of each local operator in the OPE are evaluated as

$$\begin{aligned} f_{+,0}^{\Xi_{bb}\to\Xi_{bc}}(0) &= 0.332(73.309\%)_{dim0} + 0.101(22.300\%)_{dim3} \\ &\quad +0.014(3.063\%)_{dim5} - 0.006(1.329\%)_{dim6}, \\ f_{\perp}^{\Xi_{bb}\to\Xi_{bc}}(0) &= 0.630(75.700\%)_{dim0} + 0.194(23.326\%)_{dim3} \\ &\quad -0.003(0.310\%)_{dim5} - 0.006(0.663\%)_{dim6}. \end{aligned} \tag{21}$$

It can be seen that the OPE has excellent convergence and the perturbative term dominates. In addition, we have investigated the heavy quark limit of our full QCD results for the form factors, and close results are obtained. However, it is seems that the leading order correction in α_s of perturbative term should not be neglect. To estimate the contribution of the leading order correction in α_s, the one loop Feynman diagram which exchange gluon in Fig. 5 is calculated. The contribution of this diagram in form factors $F_{+,0}$ are

$$\begin{aligned} f_{+,0}^{\Xi_{bb}\to\Xi_{bc}}(0) &= 0.332(73.309\%)_{dim0} + 0.0002(0.048\%)_{\alpha_s} \\ &\quad +0.101(22.300\%)_{dim3} \\ &\quad +0.014(3.063\%)_{dim5} - 0.006(1.329\%)_{dim6}. \end{aligned} \tag{22}$$

Considering the 34 diagrams for full calculations, the contribution of leading order in α_s correction is expected to be $\sim 5\%$, which is close to the contribution of dimension-5. And it is consistent with the contribution of leading order in α_s correction of two-point correction functions [67] which is 11%. The correction indeed should not be neglect. But estimating the correction of leading order in α_s is a great challenge for us. Therefore, this correction is still not included in this work.

4 Phenomenological applications

The weak decays of doubly heavy baryons induced by $b \to c\, l^- \bar{\nu}$ can be calculated using the low energy effective Hamiltonian

$$\mathcal{H}_{\text{eff}}(b \to c\, l^- \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma^\mu(1-\gamma_5)b][\bar{l}\gamma_\mu(1-\gamma_5)\nu_l]. \tag{23}$$

Table 5 Decay widths of semilepton decay of double heavy baryons which induced by $b \to c$

Channel	Decay width (10^{-14} GeV)
$\Gamma(\Xi_{bb} \to \Xi_{bc})$	$1.955 \pm 0.685(T_1^2, T_2^2) \pm 1.673(s_1^0, s_2^0)$
$\Gamma_L(\Xi_{bb} \to \Xi_{bc})$	$1.728 \pm 0.658(T_1^2, T_2^2) \pm 1.363(s_1^0, s_2^0)$
$\Gamma_T(\Xi_{bb} \to \Xi_{bc})$	$0.227 \pm 0.175(T_1^2, T_2^2) \pm 0.342(s_1^0, s_2^0)$
$\Gamma(\Omega_{bb} \to \Omega_{bc})$	$3.005 \pm 0.780(T_1^2, T_2^2) \pm 4.932(s_1^0, s_2^0)$
$\Gamma_L(\Omega_{bb} \to \Omega_{bc})$	$1.854 \pm 0.670(T_1^2, T_2^2) \pm 2.363(s_1^0, s_2^0)$
$\Gamma_T(\Omega_{bb} \to \Omega_{bc})$	$1.151 \pm 0.382(T_1^2, T_2^2) \pm 2.562(s_1^0, s_2^0)$
$\Gamma(\Xi_{bc} \to \Xi_{cc})$	$4.174 \pm 0.796(T_1^2, T_2^2) \pm 4.933(s_1^0, s_2^0)$
$\Gamma_L(\Xi_{bc} \to \Xi_{cc})$	$3.260 \pm 0.730(T_1^2, T_2^2) \pm 4.272(s_1^0, s_2^0)$
$\Gamma_T(\Xi_{bc} \to \Xi_{cc})$	$0.914 \pm 0.279(T_1^2, T_2^2) \pm 0.66(s_1^0, s_2^0)$
$\Gamma(\Omega_{bc} \to \Omega_{cc})$	$4.799 \pm 1.095(T_1^2, T_2^2) \pm 4.385(s_1^0, s_2^0)$
$\Gamma_L(\Omega_{bc} \to \Omega_{cc})$	$2.762 \pm 0.676(T_1^2, T_2^2) \pm 2.931(s_1^0, s_2^0)$
$\Gamma_T(\Omega_{bc} \to \Omega_{cc})$	$2.037 \pm 0.867(T_1^2, T_2^2) \pm 1.454(s_1^0, s_2^0)$

The helicity amplitudes are defined as follows:

$$\begin{aligned} HV^{\lambda}_{\lambda',\lambda_W} &= \langle \mathcal{B}_f(\lambda')|\bar{c}\gamma^{\mu} b|\mathcal{B}_i(\lambda)\rangle \epsilon^*_{\mu}(\lambda_W), \\ HA^{\lambda}_{\lambda',\lambda_W} &= \langle \mathcal{B}_f(\lambda')|\bar{c}\gamma^{\mu}\gamma_5 b|\mathcal{B}_i(\lambda)\rangle \epsilon^*_{\mu}(\lambda_W), \\ H^{\lambda}_{\lambda',\lambda_W} &= HV^{\lambda}_{\lambda',\lambda_W} - HA^{\lambda}_{\lambda',\lambda_W}. \end{aligned} \tag{24}$$

The differential decay widths can be shown as:

$$\begin{aligned} \frac{d\Gamma}{dq^2} &= \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}, \\ \frac{d\Gamma_T}{dq^2} &= \frac{G_F^2|V_{cb}|^2|P'||p_1|}{16(2\pi)^3 M_{\mathcal{B}}^2\sqrt{q^2}} \frac{2(m_l^2-q^2)(m_l^2+2q^2)}{3q^2} \\ &\quad \times\left(|H^{\frac{1}{2}}_{\frac{1}{2},1}|^2 + |H^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2\right), \\ \frac{d\Gamma_L}{dq^2} &= \frac{G_F^2|V_{cb}|^2|P'||p_1|}{16(2\pi)^3 M_{\mathcal{B}}^2\sqrt{q^2}} \frac{2(m_l^2-q^2)}{3q^2} \\ &\quad \times\Big((m_l^2+2q^2)(|H^{\frac{1}{2}}_{-\frac{1}{2},0}|^2 + |H^{-\frac{1}{2}}_{\frac{1}{2},0}|^2) \\ &\quad +3m_l^2(|H^{\frac{1}{2}}_{-\frac{1}{2},t}|^2 + |H^{-\frac{1}{2}}_{\frac{1}{2},t}|^2)\Big). \end{aligned} \tag{25}$$

Here $|P'|$ is the magnitude of three-momentum of $\mathcal{B}_f$ in the rest frame of $\mathcal{B}_i$, and $|p_1|$ is that of lepton in the rest frame of W boson. The helicity amplitudes in Eq. (25) are related to the form factors as follows:

$$\begin{aligned} HV^{\frac{1}{2}}_{-\frac{1}{2},0} &= HV^{-\frac{1}{2}}_{\frac{1}{2},0} = -if_+(M_1+M_2)\sqrt{\frac{Q_-}{q^2}}, \\ HV^{\frac{1}{2}}_{-\frac{1}{2},t} &= HV^{-\frac{1}{2}}_{\frac{1}{2},t} = -if_0(M_1-M_2)\sqrt{\frac{Q_+}{q^2}}, \end{aligned}$$

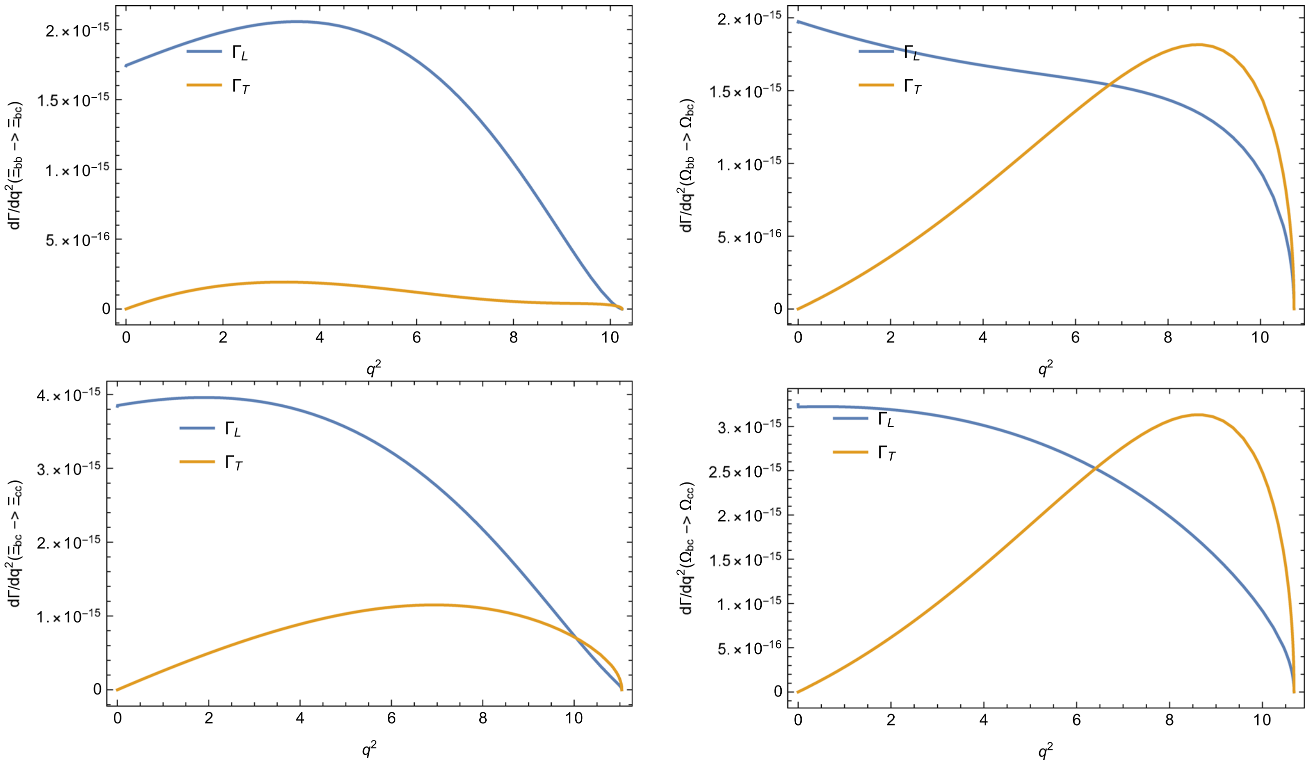

Fig. 6 Differential decay widths of $\Xi_{bb} \to \Xi_{bc}$ (top left), $\Omega_{bb} \to \Omega_{bc}$ (top right), $\Xi_{bc} \to \Xi_{cc}$ (bottom left), and $\Omega_{bc} \to \Omega_{cc}$ (bottom right)

$$HV^{\frac{1}{2}}_{\frac{1}{2},1} = HV^{-\frac{1}{2}}_{-\frac{1}{2},-1} = -if_\perp\sqrt{2Q_-}, \tag{26}$$

and

$$\begin{aligned} HA^{\frac{1}{2}}_{-\frac{1}{2},0} &= -HA^{-\frac{1}{2}}_{\frac{1}{2},0} = ig_+(M_1 - M_2)\sqrt{\frac{Q_+}{q^2}}, \\ HA^{\frac{1}{2}}_{-\frac{1}{2},t} &= -HA^{-\frac{1}{2}}_{\frac{1}{2},t} = ig_0(M_1 + M_2)\sqrt{\frac{Q_-}{q^2}}, \\ HA^{\frac{1}{2}}_{\frac{1}{2},1} &= -HA^{-\frac{1}{2}}_{-\frac{1}{2},-1} = -ig_\perp\sqrt{2Q_+}. \end{aligned} \tag{27}$$

Our predictions of the decay widths are given in Table 5. The differential decay widths are plotted in Fig. 6.

5 Summary

In this work, we have investigated the $b \to c$ decay form factors of doubly heavy baryons in QCD sum rules. For completeness, we have also performed the analysis of pole residues, and as by-products, the masses of doubly heavy baryons. Our predictions for the masses are in good agreement with those of Lattice QCD and experimental data. On the OPE side, contributions from up to dimension-5 and dimension-6 operators are respectively considered for the two-point and three-point correlation functions. We have also considered the leading logarithmic corrections for the Wilson coefficients of OPE, and it turns out that these corrections are small. The obtained form factors are then used to predict the corresponding semi-leptonic decay widths, which are considered to be helpful to search for other doubly heavy baryons at the LHC.

Acknowledgements The authors would like to thank Prof. Wei Wang for constant help and encouragement. Z.-X. Zhao is supported in part by scientific research start-up fund for Junma program of Inner Mongolia University, scientific research start-up fund for talent introduction in Inner Mongolia Autonomous Region, and National Natural Science Foundation of China under Grant no. 12065020.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The data used to support the findings of this study are available from the author upon request.]

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