Study of the $D_s^+ \rightarrow a_0(980)\rho$ and $a_0(980)\omega$ decays

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Abstract We study $D_s^+ \rightarrow \rho^{0(+)}a_0^{+(0)}$, $D_s^+ \rightarrow \omega a_0^+$, and the resonant $D_s^+ \rightarrow \rho a_0$, $a_0 \rightarrow \eta\pi(KK)$ decays. In the final state interaction, where $D_s^+ \rightarrow (\eta^{(0)}\pi^+, K^+\bar{K}^0)$ are followed by the $(\eta^{(0)}\pi^+, K^+\bar{K}^0)$ to $\rho^{0(+)}a_0^{+(0)}$ rescatterings, we predict $\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)}a_0^{+(0)}) = (3.0 \pm 0.3 \pm 1.0) \times 10^{-3}$. Due to the cancellation of the rescattering effects and the suppressed short-distance W annihilation contribution, we expect that $\mathcal{B}(D_s^+ \rightarrow \omega a_0^+) \simeq \mathcal{B}(D_s^+ \rightarrow \pi^+\pi^0) < 3.4 \times 10^{-4}$. In our calculation, $\mathcal{B}(D_s^+ \rightarrow \rho^0(a_0^+ \rightarrow \eta\pi^+) = (1.6_{-0.3}^{+0.2} \pm 0.6) \times 10^{-3}$ agrees with the data, whereas $\mathcal{B}(D_s^+ \rightarrow \rho^+(a_0^0 \rightarrow K^+K^-)$ is 10 times smaller than the observation, which requires a careful examination.

1 Introduction

The two-body $D_s^+ \rightarrow PP$, PV decays with $P(V)$ denoting the strangeless pseudoscalar (vector) meson have no configurations from the W -boson emission processes because $\bar{s}$ in D_s^+ cannot be eliminated, as drawn in Fig. 1 for the topological diagrams T and C. Interestingly, it leads to a specific exploration for the annihilation mechanism applied to $D_s^+ \rightarrow \pi^+\pi^0, \pi^+\rho^0, \pi^+\omega$ [1–6].

In the short-distance W -boson annihilation (WA) $D_s^+(c\bar{s}) \rightarrow W^+ \rightarrow u\bar{d}$ process, $u\bar{d}$ can be seen to move in the opposite directions in the D_s^+ rest frame, such that there exists no orbital angular momentum between them. It indicates that $G(u\bar{d}) = G(\pi^+) = +1$ with G denoting the G -parity symmetry [3]. Since G -parity is a multiplicative quantum number, one obtains $G(\pi^+\rho^0, \pi^+\pi^0) = (+1, -1)$. Consequently, the WA $D_s^+ \rightarrow \pi^+\rho^0(\pi^+\pi^0)$ decay due to $G(u\bar{d}) = G(\pi^+\rho^0) [= -G(\pi^+\pi^0)]$ is a G -parity conserved (violated) process, which corresponds to the exper-

imental result $\mathcal{B}(D_s^+ \rightarrow \pi^+\rho^0) = (1.9 \pm 1.2) \times 10^{-4}$ [$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^0) < 3.4 \times 10^{-4}$] [7]. By contrast, although the WA $D_s^+ \rightarrow \pi^+\omega$ decay violates the G -parity symmetry, $\mathcal{B}(D_s^+ \rightarrow \pi^+\omega) = (1.9 \pm 0.3) \times 10^{-3}$ shows no suppression [7]. It is hence considered to receive the long-distance annihilation contribution [1,3].

The $D_s^+ \rightarrow SP$, SV decays can help to investigate the short and long-distance annihilation mechanisms [8–10], where S stands for a non-strange scalar meson. For example, the WA process for $D_s^+ \rightarrow a_0^{0(+)}\pi^{+(0)}$ violates G -parity [9,11], such that its branching fraction is expected as small as $\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^0)$. Nonetheless, one measures that $\mathcal{B}(D_s^+ \rightarrow a_0^{0(+)}\pi^{+(0)}) \simeq 100\mathcal{B}(D_s^+ \rightarrow \pi^+\rho^0)$ [7,12]. Clearly, it indicates the main contribution from the long-distance annihilation process [9]. Explicitly, the long-distance annihilation process for $D_s^+ \rightarrow a_0^{0(+)}\pi^{+(0)}$ starts with the $D_s^+ \rightarrow \eta^{(0)}\rho^+$ weak decay, followed by the $\eta^{(0)}$ and ρ^+ rescattering. With the $\pi^{+(0)}$ exchange, $\eta^{(0)}$ and π^+ are turned into $a_0^{+(0)}$ and $\pi^{0(+)}$, respectively. Since BESIII has recently reported the first observation of the branching fractions of $D_s^+ \rightarrow SV$, $S \rightarrow PP$ as [13,14]

$$\begin{aligned} \mathcal{B}_+(D_s^+ \rightarrow a_0^+\rho^0, a_0^+ \rightarrow \pi^+\eta) &= (2.1 \pm 0.8 \pm 0.5) \times 10^{-3}, \\ \mathcal{B}_0(D_s^+ \rightarrow a_0^0\rho^+, a_0^0 \rightarrow K^+K^-) &= (0.7 \pm 0.2 \pm 0.1) \times 10^{-3}, \end{aligned} \quad (1)$$

we are wondering which of the short and long-distance annihilation processes can be the dominant contribution. Hence, we propose to study $D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)}$, $D_s^+ \rightarrow a_0^+\omega$, and the resonant three-body $D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)}, a_0^{+(0)} \rightarrow [\eta\pi^{+(0)}, K^+\bar{K}^0(K^+K^-)]$ decays, in order to analyze the data in Eq. (1). We will also test if $\mathcal{B}(D_s^+ \rightarrow a_0^+\rho^0, a_0^+\rho^0)$ have nearly equal sizes as $\mathcal{B}(D_s^+ \rightarrow a_0^+\pi^0) \simeq \mathcal{B}(D_s^+ \rightarrow a_0^0\pi^+)$ that respects the isospin symmetry.

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Fig. 1 Topological diagrams for the Cabibbo-allowed D_s^+ weak decays

2 Formalism

Considering the short-distance WA processes, $D_s^+ \rightarrow PP(PV)$ and $D_s^+ \rightarrow SP(SV)$ both get an A term as the annihilation amplitude in Fig. 1. According to $\mathcal{B}(D_s^+ \rightarrow \pi^+\rho^0) \simeq 10^{-4}$ that receives the short-distance WA contribution [7], one regards A to give $\mathcal{B}(D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)})$ not larger than 10^{-4} . However, $\mathcal{B}_+$ in Eq. (1) suggests $\mathcal{B}(D_s^+ \rightarrow a_0^+\rho^0) \simeq 10^{-3}$. This strongly suggests that the main contribution to $D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)}$ is from the long-distance annihilation process. For $D_s^+ \rightarrow a_0^+\omega$, the WA contribution is suppressed with the G -parity violation, such that $A \simeq 0$. Therefore, we start with the triangle rescattering processes for $D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)}$ and $D_s^+ \rightarrow a_0^+\omega$ as the most possible main contributions.

See Fig. 2, the rescattering processes for $D_s^+ \rightarrow a_0^{+(0)}\rho^{0(+)}$ include both the weak and strong decays. In our case, the weak decays come from $D_s^+ \rightarrow \pi^+\eta^{(\prime)}, K^+\bar{K}^0$, and the amplitudes are given by [2–4,6]

For the strong decays $V, S \rightarrow PP$, the amplitudes are given by [9]

$$\begin{aligned}\mathcal{M}_{\rho^{0(+)} \rightarrow \pi^{+(0)}\pi^{-(+)}} &= g_\rho \epsilon \cdot (q_1 - q_2), \\ \mathcal{M}_{a_0^+ \rightarrow \eta^{(\prime)}\pi^+, K^+\bar{K}^0} &= g_{\eta^{(\prime)}}, g_K,\end{aligned}\quad (4)$$

where ϵ_μ is the polarization four vector of the ρ meson, and $q_{1,2}^\mu$ are the four momenta of $\pi^+\pi^-(\pi^0\pi^+)$, respectively. The $SU(3)$ flavor symmetry is able to relate different $V(S) \rightarrow PP$ decay channels [17,18], such that we obtain $g_\rho/2$ for $\mathcal{M}_{\rho^0 \rightarrow K^+K^-, \bar{K}^0K^0}$, $g_\rho/\sqrt{2}$ for $\mathcal{M}_{\rho^+ \rightarrow K^+\bar{K}^0}$, and $(-g_K/\sqrt{2})$ for $\mathcal{M}_{a_0^0 \rightarrow K^+K^-(\bar{K}^0K^0)}$, together with $(-g_\rho/2)$ for $\mathcal{M}_{\omega \rightarrow K^+K^-(\bar{K}^0K^0)}$.

By assembling the weak and strong couplings in the rescattering processes, we derive that

$$\begin{aligned}\mathcal{M}(D_s^+ \rightarrow \rho^{0(+)}a_0^{+(0)}) &= \mathcal{M}_a + \mathcal{M}'_a + \mathcal{M}_b + \mathcal{M}_c, \\ \mathcal{M}(D_s^+ \rightarrow \omega a_0^+) &= \hat{\mathcal{M}}_b + \hat{\mathcal{M}}_c,\end{aligned}\quad (5)$$

with $\mathcal{M}_a^{(\prime)}$ and $\mathcal{M}_{b(c)}[\hat{\mathcal{M}}_{b(c)}]$ for Fig. 2a, b(c), respectively. More explicitly, $\mathcal{M}_{a,b,c}$ are given by [9,19,20]

$$\begin{aligned}\mathcal{M}_a &= \int \frac{d^4q_1}{(2\pi)^4} \frac{\mathcal{M}_\eta \mathcal{M}_{\rho^{0(+)} \rightarrow \pi^+\pi^{-(0)}} \mathcal{M}_{a_0^{+(0)} \rightarrow \eta\pi^{+(0)}} F_\pi(q_2^2)}{(q_1^2 - m_\pi^2 + i\epsilon)[(q_1 - p_2)^2 - m_\pi^2 + i\epsilon][(q_1 - p_1)^2 - m_\eta^2 + i\epsilon]}, \\ \mathcal{M}_b &= \int \frac{d^4q_1}{(2\pi)^4} \frac{\mathcal{M}_K \mathcal{M}_{\rho^{0(+)} \rightarrow K^+K^-(K^+\bar{K}^0)} \mathcal{M}_{a_0^{+(0)} \rightarrow \bar{K}^0K^{+(0)}} F_K(q_2^2)}{(q_1^2 - m_K^2 + i\epsilon)[(q_1 - p_2)^2 - m_K^2 + i\epsilon][(q_1 - p_1)^2 - m_K^2 + i\epsilon]}, \\ \mathcal{M}_c &= \int \frac{d^4q_1}{(2\pi)^4} \frac{\mathcal{M}_K \mathcal{M}_{\rho^{0(+)} \rightarrow \bar{K}^0K^{0(+)}} \mathcal{M}_{a_0^{+(0)} \rightarrow \bar{K}^0K^{+(0)}} F_K(q_2^2)}{(q_1^2 - m_K^2 + i\epsilon)[(q_1 - p_2)^2 - m_K^2 + i\epsilon][(q_1 - p_1)^2 - m_K^2 + i\epsilon]},\end{aligned}\quad (6)$$

$$\begin{aligned}\mathcal{M}_\eta(D_s^+ \rightarrow \pi^+\eta) &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\sqrt{2}A \cos \phi - T \sin \phi), \\ \mathcal{M}_{\eta'}(D_s^+ \rightarrow \pi^+\eta') &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\sqrt{2}A \sin \phi + T \cos \phi), \\ \mathcal{M}_K(D_s^+ \rightarrow K^+\bar{K}^0) &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C + A),\end{aligned}\quad (2)$$

where G_F is the Fermi constant, $V_{cs}^* V_{ud}$ the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements, and $V_{cs}^* V_{ud} \simeq 1$ presents the Cabibbo-allowed decay modes. In addition, (T, C, A) are the topological amplitudes, along with the mixing angle $\phi = 43.5^\circ$ from the $\eta - \eta'$ mixing matrix [15,16]:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \sqrt{1/2}(u\bar{u} + d\bar{d}) \\ s\bar{s} \end{pmatrix}. \quad (3)$$

with $q_2 = q_1 - p_2$ and $q_3 = q_1 - p_1$ following the momentum flows in Fig. 2. The form factor $F_M(q_2^2) \equiv (\Lambda_M^2 - m_M^2)/(\Lambda_M^2 - q_2^2)$ with the cutoff parameter Λ_M [$M = (\pi, K)$] is to avoid the overestimation with q_2 to $\pm\infty$ [21]. Substituting η' and ω for η in $\mathcal{M}_a$ and ρ^0 in $\mathcal{M}_{b(c)}$ leads to $\mathcal{M}'_a$ and $\hat{\mathcal{M}}_{b(c)}$, respectively.

As a consequence, we find that

$$\begin{aligned}\mathcal{M}(D_s^+ \rightarrow \rho^0 a_0^+) &= \mathcal{M}(D_s^+ \rightarrow \rho^+ a_0^0), \\ \mathcal{M}(D_s^+ \rightarrow \omega a_0^+) &= 0,\end{aligned}\quad (7)$$

where the first relation respects the isospin symmetry, whereas $\hat{\mathcal{M}}_b = -\hat{\mathcal{M}}_c$ due to $\mathcal{M}_{\omega \rightarrow K^+K^-} = -\mathcal{M}_{\omega \rightarrow \bar{K}^0K^0}$ cancels the rescattering contributions to $D_s^+ \rightarrow \omega a_0^+$, which causes $\mathcal{M}(D_s^+ \rightarrow \omega a_0^+) = 0$.

Fig. 2 Triangle rescattering diagrams for $D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)}$ and $D_s^+ \rightarrow \omega a_0^{+(0)}$

To deal with the triangle loops in Eq. (6), the equation in Refs. [22–25] can be useful, given by

In the above equation, $1/D_{a_0}(s)$ presents the propagator for a_0 , and we define [26]

$$\int \frac{d^4 q_1}{i\pi^2} \frac{q_1^\mu}{(q_1^2 - m_1^2 + i\epsilon)[(q_1 - p_2)^2 - m_2^2 + i\epsilon][(q_1 - p_1)^2 - m_3^2 + i\epsilon]} = -p_\rho^\mu C_1(p_{a_0}^2, m_\rho^2, m_{D_s}^2, m_1^2, m_2^2, m_3^2) - p_{D_s}^\mu C_2(p_{a_0}^2, m_\rho^2, m_{D_s}^2, m_1^2, m_2^2, m_3^2). \quad (8)$$

By calculating the triangle rescattering processes, we obtain

$$\Gamma(D_s^+ \rightarrow \rho^{0+} a_0^{+(0)}) = \frac{|\mathbf{p}_2|^3}{8\pi m_\rho^2} |H|^2, \quad H = \frac{1}{8\pi^2} \left(g_\rho g_\eta \mathcal{M}_\eta C_\eta + g_\rho g_{\eta'} \mathcal{M}_{\eta'} C_{\eta'} + g_\rho g_K \mathcal{M}_K C_K \right), \quad (9)$$

with $C_{\eta^{(\prime)}} = C_{2,\eta^{(\prime)}} - C_{2,\eta^{(\prime)}}^*$ and $C_K = C_{2,K} - C_{2,K}^*$ as the integrated results of the C_2 terms in Eq. (8), where $C_{2,M}^{(*)}$ are defined by

$$\begin{aligned} C_{2,\eta}^{(*)} &= C_2(m_\pi, m_\pi(\Lambda_\pi), m_\eta), \\ C_{2,\eta'}^{(*)} &= C_2(m_\pi, m_\pi(\Lambda_\pi), m_{\eta'}), \\ C_{2,K}^{(*)} &= C_2(m_K, m_K(\Lambda_K), m_K). \end{aligned} \quad (10)$$

However, the C_1 terms have been disappearing due to $p_\rho^\mu \cdot \epsilon = 0$ in the amplitudes.

For the three-body decays $D_s^+ \rightarrow \rho^{0(+)} (a_0^{+(0)} \rightarrow \eta \pi (K \bar{K}))$, $K^+ \bar{K}^0 (K^+ K^-)$, we present [7]

$$\begin{aligned} \Gamma(D_s^+ \rightarrow \rho a_0, a_0 \rightarrow \eta \pi (K \bar{K})) \\ = \int \int \frac{1}{(2\pi)^3} \frac{|\mathbf{p}_2|^2}{32m_{D_s} m_\rho^2} |H|^2 \frac{g_{\eta(K)}^2}{|D_{a_0}(s)|^2} ds dt, \end{aligned} \quad (11)$$

with $s = (p_{\eta(K)} + p_{\pi(K)})^2$ and $t = (p_\rho + p_{\pi(K)})^2$, such that the theoretical results can be compared to the data in Eq. (1).

$$D_{a_0}(s) = s - m_{a_0}^2 - \sum_{\alpha\beta} [\text{Re}\Pi_{a_0}^{\alpha\beta}(m_{a_0}^2) - \Pi_{a_0}^{\alpha\beta}(s)], \quad (12)$$

where

$$\begin{aligned} \Pi_{a_0}^{\alpha\beta}(x) &= \frac{G_{\alpha\beta}^2}{16\pi} \left\{ \frac{m_{\alpha\beta}^+ m_{\alpha\beta}^-}{\pi x} \log \left[\frac{m_\beta}{m_\alpha} \right] - \theta[x - (m_{\alpha\beta}^+)^2] \right. \\ &\quad \times \rho_{\alpha\beta} \left(i + \frac{1}{\pi} \log \left[\frac{\sqrt{x - (m_{\alpha\beta}^+)^2} + \sqrt{x - (m_{\alpha\beta}^-)^2}}{\sqrt{x - (m_{\alpha\beta}^-)^2} - \sqrt{x - (m_{\alpha\beta}^+)^2}} \right] \right) \\ &\quad \left. - \rho_{\alpha\beta} \left(1 - \frac{2}{\pi} \arctan \left[\frac{\sqrt{-x + (m_{\alpha\beta}^+)^2}}{\sqrt{x - (m_{\alpha\beta}^-)^2}} \right] \right) \right. \\ &\quad \times (\theta[x - (m_{\alpha\beta}^-)^2] - \theta[x - (m_{\alpha\beta}^+)^2]) \\ &\quad \left. + \rho_{\alpha\beta} \frac{1}{\pi} \log \left[\frac{\sqrt{(m_{\alpha\beta}^+)^2 - x} + \sqrt{(m_{\alpha\beta}^-)^2 - x}}{\sqrt{(m_{\alpha\beta}^-)^2 - x} - \sqrt{(m_{\alpha\beta}^+)^2 - x}} \right] \right. \\ &\quad \left. \times \theta[(m_{\alpha\beta}^-)^2 - x] \right\}, \\ \rho_{\alpha\beta} &\equiv \left| \sqrt{x - (m_{\alpha\beta}^+)^2} \sqrt{x - (m_{\alpha\beta}^-)^2} \right| / x, \end{aligned} \quad (13)$$

with $G_{\alpha\beta} = (g_{\eta^{(\prime)}}, g_K)$ for $\alpha\beta = (\eta^{(\prime)}\pi, K \bar{K})$ and $m_{\alpha\beta}^\pm = m_\alpha \pm m_\beta$.

3 Numerical results

In the numerical analysis, we adopt $V_{cs} = V_{ud} = 1 - \lambda^2/2$ with $\lambda = 0.22453 \pm 0.00044$ in the Wolfenstein parameterization [7], along with $m_{a_0^0} = 0.987$ GeV [27, 28]. The topological parameters (T, C, A) have been extracted as [6]

$$(|T|, |C|, |A|) = (0.363 \pm 0.001, 0.323 \pm 0.030, 0.064 \pm 0.004) \text{ GeV}^3, \\ (\delta_C, \delta_A) = (-151.3 \pm 0.3, 23.0_{-10.0}^{+7.0})^\circ, \quad (14)$$

where $\delta_{C,A}$ are the relative strong phases. According to the extraction, we obtain $\mathcal{B}(D_s^+ \rightarrow \pi^+\eta, \pi^+\eta', K^+\bar{K}^0) = (1.8 \pm 0.2, 4.2 \pm 0.5, 3.1 \pm 0.5) \times 10^{-2}$, consistent with the experimental values of $(1.68 \pm 0.10, 3.94 \pm 0.25, 2.95 \pm 0.14) \times 10^{-2}$, respectively [7]. For $V, S \rightarrow PP$, the strong coupling constants read [7, 27, 28]

$$g_\rho = 6.0, (g_\eta, g_{\eta'}, g_K) = (2.87 \pm 0.09, -2.52 \pm 0.08, 2.94 \pm 0.13) \text{ GeV}. \quad (15)$$

Empirically, Λ_M of $\mathcal{O}(1.0 \text{ GeV})$ is commonly used to explain the data [29–31]; besides, it is obtained that $\Lambda_K - \Lambda_\pi = m_K - m_\pi$ [32]. Therefore, we are allowed to use $(\Lambda_\pi, \Lambda_K) = (1.25 \pm 0.25, 1.60 \pm 0.25) \text{ GeV}$, which result in

$$C_\eta = [(0.57 \pm 0.08) + i(0.17 \pm 0.09)] \text{ GeV}^{-2}, \\ C_{\eta'} = [(0.34 \pm 0.03) - i(0.27 \pm 0.07)] \text{ GeV}^{-2}, \\ C_K = [(0.85 \pm 0.03) - i(0.45 \pm 0.08)] \text{ GeV}^{-2}, \quad (16)$$

with $p_{a_0}^2 = m_{a_0}^2$. Subsequently, we predict

$$\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)}) = (3.0 \pm 0.3 \pm 1.0) \times 10^{-3}, \\ \mathcal{B}(D_s^+ \rightarrow \omega a_0^+) = 0, \quad (17)$$

where the first error in $\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)})$ takes into account the uncertainties from Λ_π and Λ_K , and the second one combines those from V_{cs}^* , V_{ud} , (T, C, A), and the strong coupling constants. For the resonant three-body decays, we obtain

$$\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)}(a_0^{+(0)} \rightarrow \eta)\pi^{+(0)}) \\ = (1.6_{-0.3}^{+0.2} \pm 0.6) \times 10^{-3}, \\ \mathcal{B}(D_s^+ \rightarrow \rho^+(a_0^0 \rightarrow K^+K^-, K^0\bar{K}^0) \\ = (0.9_{-0.1}^{+0.1} \pm 0.4, 0.7_{-0.1}^{+0.1} \pm 0.3) \times 10^{-4}, \\ \mathcal{B}(D_s^+ \rightarrow \rho^0(a_0^+ \rightarrow K^+\bar{K}^0) \\ = (1.5_{-0.3}^{+0.2} \pm 0.6) \times 10^{-4}, \quad (18)$$

where the sources of the two errors are the same as those in Eq. (17).

4 Discussions and conclusions

In the triangle loop, when the momentum flow approaches the mass shell for one of the three propagators, the integration with $i\epsilon \simeq im\Gamma$ gives rise to the imaginary parts in Eq. (16), where Γ is a very tiny decay width for $\pi, \eta^{(\prime)}$ or K . The off-shell integrations are responsible for the real parts in Eq. (16), for which we take $\mathcal{M}_a$ as our description. In principle, the integration allows a momentum flow from $-\infty$ to $+\infty$. However, when the exchange particle proceeds with q_2^2 around several GeV^2 , instead of the infinity, the integration with $q_2^2 \sim \pm\infty$ causes an overestimation [21]. We have accordingly introduced the form factor F_π in Eq. (6) to cut off the contribution from $q_2^2 \sim \pm\infty$. While q_1^2, q_2^2 and q_3^2 have been associated in the loop, F_π also works to cut off the contributions from the propagators of the rescattering particles π^+ and $\eta^{(\prime)}$. Note that the single cutoff form factor has also been commonly used elsewhere [33–35].¹

The smallness of the WA $D_s^+ \rightarrow MM$ decay can be traced back to its amplitude, given by [37–39]

$$\mathcal{M}_{WA}(D_s^+ \rightarrow MM) \propto f_{D_s} q^\mu \langle MM | \bar{u} \gamma_\mu (1 - \gamma_5) d | 0 \rangle \\ \simeq (m_u + m_d) \langle MM | \bar{u} \gamma_5 d | 0 \rangle, \quad (19)$$

where $q^\mu \langle MM | u \gamma_\mu d | 0 \rangle = 0$ corresponds to the conservation of the vector current (CVC); most importantly, $m_{u(d)} \simeq 0$ causes the chiral suppression of the WA $D_s^+ \rightarrow MM$ decay. According to the data, $\mathcal{B}(D_s^+ \rightarrow \pi^+ \rho^0) = (1.9 \pm 1.2) \times 10^{-4}$ indicates that $\mathcal{B}_{WA}(D_s^+ \rightarrow MM)$ should be around 10^{-4} . In addition, $\mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^0) < 3.4 \times 10^{-4}$ suggests that the G-parity violation suppresses the WA process even more. Therefore, since $D_s^+ \rightarrow \rho a_0, \rho \pi$ are both the G-parity conserved processes, it is reasonable to present that $\mathcal{B}_{WA}(D_s^+ \rightarrow \rho a_0) \simeq \mathcal{B}_{WA}(D_s^+ \rightarrow \rho \pi) \sim 10^{-4}$. As a theoretical support, we present

$$\mathcal{B}_{WA}(D_s^+(c\bar{s}) \rightarrow \rho a_0) \\ = R_f \frac{(V_{cs}^* V_{ud} f_{D_s})^2 \tau_{D_s}}{(V_{cb} V_{ud}^* f_{B_c})^2 \tau_{B_c}} \mathcal{B}_{WA}(B_c^+(c\bar{b}) \rightarrow \rho a_0) \sim 10^{-4}, \quad (20)$$

in agreement with our estimation, where $R_f = 3.1 \times 10^{-2}$ is mostly from the phase space factors, and $\mathcal{B}_{WA}(B_c^+ \rightarrow \rho a_0) \simeq 1.0 \times 10^{-5}$ is adopted from Ref. [40].

Disregarding the WA contributions, we predict $\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)}) = 3.0 \times 10^{-3}$ in Eq. (17). It is found that the $\eta\pi, \eta'\pi, K^+\bar{K}^0$ rescatterings and their interferences give 6%, 7%, 30% and 59% of $\mathcal{B}(D_s^+ \rightarrow a_0^{+(0)} \rho^{0(+)})$, respectively. By contrast, the $\rho\eta^{(\prime)}$ rescatterings from $D_s^+ \rightarrow \rho\eta^{(\prime)}$ dominantly contribute to $D_s^+ \rightarrow a_0^{+(0)} \pi^{0(+)}$, instead of

¹ Please also consult Refs. [36], where one considers three cutoff form factors.

$D_s^+ \rightarrow K^* K$ [9]. Since $D_s^+ \rightarrow \omega a_0^+$ has no rescattering effects; besides, the WA contribution is suppressed by the G -parity violation, we anticipate that $\mathcal{B}(D_s^+ \rightarrow \omega a_0^+) \simeq \mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^0) < 3.4 \times 10^{-4}$ [7].

In Eq. (18), $\mathcal{B}(D_s^+ \rightarrow \rho^0(a_0^+ \rightarrow \eta)\pi^+) = (1.6_{-0.3}^{+0.2} \pm 0.6) \times 10^{-3}$ is able to explain the data [see Eq. (1)], demonstrating the sufficient long-distance annihilation contribution. It is confusing that $\mathcal{B}(D_s^+ \rightarrow a_0^0 \rho^+, a_0^0 \rightarrow K^+ K^-)$ is 10 times smaller than $\mathcal{B}_0$ in Eq. (1). For clarification, we take the approximate form of the resonant branching fraction: $\mathcal{B}(D_s^+ \rightarrow SV, S \rightarrow PP) \simeq \mathcal{B}(D_s^+ \rightarrow SV)\mathcal{B}(S \rightarrow PP)$, together with the isospin relation: $\mathcal{B}(D_s^+ \rightarrow \rho^0 a_0^+) \simeq \mathcal{B}(D_s^+ \rightarrow \rho^+ a_0^0)$, such that $\mathcal{B}_0/\mathcal{B}_+$ is reduced as $\mathcal{B}(a_0^0 \rightarrow K^+ K^-)/\mathcal{B}(a_0^+ \rightarrow \eta\pi^+) \simeq 1/3$, disagreeing with $\mathcal{B}(a_0^0 \rightarrow K^+ K^-)/\mathcal{B}(a_0^+ \rightarrow \eta\pi^+) = 1/10$ from the experimental extraction [8]. Therefore, we conclude that there exists a possible contradiction between the observations in Eq. (1).

In our reasoning, the contradiction might be caused by $D_s^+ \rightarrow \rho^+ f_0, f_0 \rightarrow K^+ K^-$ with $f_0 \equiv f_0(980)$, which can be mistaken as $D_s^+ \rightarrow \rho^+ a_0^0, a_0^0 \rightarrow K^+ K^-$ [41]. First, since $D_s^+ \rightarrow \rho^+ f_0$ is an external W -boson emission process, its branching fraction can be of order 10^{-3} . Second, a_0 and f_0 are both scalar mesons, and have nearly the same masses and overlapped decay widths. As a result, it is possible that one cannot distinguish between the resonant signals of $D_s^+ \rightarrow \rho^+(f_0, a_0), (f_0, a_0) \rightarrow K^+ K^-$ in the $K^+ K^-$ invariant mass spectrum. For a careful examination, we suggest a measurement of $\mathcal{B}(D_s^+ \rightarrow \rho^0(a_0^+ \rightarrow K^+ \bar{K}^0)/\mathcal{B}(D_s^+ \rightarrow \rho^+(a_0^0 \rightarrow K^+ K^-))$, which will be observed around 2 if there exists no resonant $f_0 \rightarrow K^+ K^-$ decay to be involved in $\mathcal{B}(D_s^+ \rightarrow \rho^+ K^+ K^-)$.

In summary, we have studied $D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)}, D_s^+ \rightarrow \omega a_0^0$, and the resonant $D_s^+ \rightarrow \rho a_0, a_0 \rightarrow \eta\pi(KK)$ decays. In the final state interaction, where $D_s^+ \rightarrow \eta^{(\prime)}\pi^+(K^+ \bar{K}^0)$ is followed by the $\eta^{(\prime)}\pi^+(K^+ \bar{K}^0)$ to ρa_0 rescattering, we have predicted $\mathcal{B}(D_s^+ \rightarrow \rho^{0(+)} a_0^{+(0)}) = (3.0 \pm 0.3 \pm 1.0) \times 10^{-3}$. Because of the non-contribution of the rescattering effects and the suppressed short-distance W annihilation, it has been expected that $\mathcal{B}(D_s^+ \rightarrow \omega a_0^+) \simeq \mathcal{B}(D_s^+ \rightarrow \pi^+ \pi^0) < 3.4 \times 10^{-4}$. For the resonant three-body decay, $\mathcal{B}(D_s^+ \rightarrow \rho^0(a_0^+ \rightarrow \eta)\pi^+) = (1.6_{-0.3}^{+0.2} \pm 0.6) \times 10^{-3}$ has been shown to agree with the data. We have presented $\mathcal{B}(D_s^+ \rightarrow \rho^+(a_0^0 \rightarrow K^+ K^-))$ 10 times smaller than the observation, indicating that a more careful examination is needed.

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References

1. S. Fajfer, A. Prapotnik, P. Singer, J. Zupan, Phys. Rev. D **68**, 094012 (2003)
2. B. Bhattacharya, J.L. Rosner, Phys. Rev. D **81**, 014026 (2010)
3. H.Y. Cheng, C.W. Chiang, Phys. Rev. D **81**, 074021 (2010)
4. F.S. Yu, X.X. Wang, C.D. Lu, Phys. Rev. D **84**, 074019 (2011)
5. H.N. Li, C.D. Lu, Q. Qin, F.S. Yu, Phys. Rev. D **89**, 054006 (2014)
6. H.Y. Cheng, C.W. Chiang, Phys. Rev. D **100**, 093002 (2019)
7. P.A. Zyla et al., [Particle Data Group], PTEP. **2020**, 083C01 (2020)
8. H.Y. Cheng, C.W. Chiang, Phys. Rev. D **81**, 074031 (2010)
9. Y.K. Hsiao, Y. Yu, B.C. Ke, Eur. Phys. J. C **80**, 895 (2020)
10. X.Z. Ling, M.Z. Liu, J.X. Lu, L.S. Geng, J.J. Xie, Phys. Rev. D **103**, 116016 (2021)
11. N.N. Achasov, G.N. Shestakov, Phys. Rev. D **96**, 036013 (2017)
12. M. Ablikim et al., BESIII, Phys. Rev. Lett. **123**, 112001 (2019)
13. M. Ablikim et al., BESIII, Phys. Rev. D **104**, 071101 (2021)
14. M. Ablikim et al., BESIII, Phys. Rev. D **104**, 032011 (2021)
15. T. Feldmann, P. Kroll, B. Stech, Phys. Rev. D **58**, 114006 (1998)
16. T. Feldmann, P. Kroll, B. Stech, Phys. Lett. B **449**, 339 (1999)
17. N.A. Tornqvist, Ann. Phys. **123**, 1 (1979)
18. Fayyazuddin and Riazuddin, *A Modern Introduction To Particle Physics*, 3rd edn. (World Scientific, Singapore, 2011)
19. Y. Yu, Y.K. Hsiao, Phys. Lett. B **820**, 136586 (2021)
20. Y.K. Hsiao, Y. Yu, Phys. Rev. D **104**, 034008 (2021)
21. M.C. Du, Q. Zhao, Phys. Rev. D **104**, 036008 (2021)
22. G.T. Hooft, M.J.G. Veltman, Nucl. Phys. B **153**, 365 (1979)
23. G. Passarino, M.J.G. Veltman, Nucl. Phys. B **160**, 151 (1979)
24. T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999)
25. A. Denner, S. Dittmaier, Nucl. Phys. B **734**, 62 (2006)
26. N.N. Achasov, A.V. Kiselev, Phys. Rev. D **70**, 111901 (2004)
27. M. Ablikim et al., BESIII, Phys. Rev. D **95**, 032002 (2017)
28. D.V. Bugg, Phys. Rev. D **78**, 074023 (2008)
29. N.A. Tornqvist, Z. Phys. C **61**, 525 (1994)
30. X.Q. Li, D.V. Bugg, B.S. Zou, Phys. Rev. D **55**, 1421 (1997)
31. Q. Wu, D.Y. Chen, X.J. Fan, G. Li, Eur. Phys. J. C **79**, 265 (2019)
32. H.Y. Cheng, C.K. Chua, A. Soni, Phys. Rev. D **71**, 014030 (2005)
33. Q. Wu, D.Y. Chen, T. Matsuki, Eur. Phys. J. C **81**, 193 (2021)
34. Q. Wu, D.Y. Chen, Phys. Rev. D **100**, 114002 (2019)
35. G. Li, X.H. Liu, Z. Zhou, Phys. Rev. D **90**, 054006 (2014)

36. Y. Cheng, Q. Zhao, [arXiv:2106.12483](#) [hep-ph]
37. Y.K. Hsiao, C.Q. Geng, Phys. Rev. D **91**, 077501 (2015)
38. Y.K. Hsiao, S.Y. Tsai, C.C. Lih, E. Rodrigues, JHEP **04**, 035 (2020)
39. X. Huang, Y.K. Hsiao, J. Wang, L. Sun, [arXiv:2109.02897](#) [hep-ph]
40. X. Liu, Z.J. Xiao, Phys. Rev. D **82**, 054029 (2010)
41. Private communications with the group members at BESIII