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Primordial big bang nucleosynthesis and generalized uncertainty principle

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Abstract The Generalized Uncertainty Principle (GUP) naturally emerges in several quantum gravity models, predicting the existence of a minimal length at Planck scale. Here, we consider the quadratic GUP as a semiclassical approach to thermodynamic gravity and constrain the deformation parameter by using observational bounds from Big Bang Nucleosynthesis and primordial abundances of the light elements ^4He, D, ^7Li. We show that our result fits with most of existing bounds on β derived from other cosmological studies.

1 Introduction

Quantum Theory and General Relativity are the two best descriptions of Nature to date. On one hand, Quantum Mechanics governs the properties of matter at microscopic scales, laying the foundations of solid state physics. By contrast, General Relativity deals with large-scale phenomena in the Cosmos – from the solar system to the faraway galaxies – as well as with the evolution of the Universe as a whole. In spite of providing successful predictions in their respective domains, these two theories exhibit fatal inconsistencies when combined together. Much effort has been devoted to the construction of a unified formalism in the last decades, culminated with the development of a number of promising candidate models. Yet despite this striving, a definitive answer is still far from being reached, thus making the quantization of gravity a central open question in modern theoretical physics.

A distinctive signature of most approaches to quantum gravity (QG) is the emergence of a minimal measurable length at around Planck energy. Implications of this fundamental scale are often taken into account by deforming the Heisenberg Uncertainty Principle (HUP) [1–8], so as to accommodate a minimal uncertainty in position measure-

ments. The most common form of generalized uncertainty principle (GUP) is obtained by adding a term quadratic in the momentum over the standard Heisenberg limitation, i.e.

$$\Delta x \, \Delta p \gtrsim \hbar \left[1 + 4\beta \left(\frac{\Delta p}{m_p c} \right)^2 \right], \tag{1}$$

where the pre-factor has been set of order unity, as seen in [9–11]. Here, $m_p \simeq 10^{19}$ GeV denotes the Planck mass. To simplify the notation, henceforth we work in natural units $\hbar = 1 = c$.

The (dimensionless) deformation parameter β is not fixed by the theory, leaving room for an intensive research activity [12–28,47] (see Tables 1 and 2 for upper bounds of cosmological and quantum/gravitational origin, respectively). Debate also concerns the sign of β: although it is assumed to be positive in the original formulation of the GUP, arguments in favor of negative values are not missing [8,18,29,30].

One of the contexts in which the GUP has been studied most extensively is that of black holes (BH's). In particular, in [7] it has been shown that Eq. (1) inevitably affects Hawking temperature and the related BH evaporation process, with a non-trivial impact on the whole BH thermodynamics. Likewise, GUP-induced corrections enter the Bekenstein-Hawking entropy formula, resulting in a generalized Bekenstein bound [31] and a modified area law [10,32]. Remarkably, implications of the modified area law are also explored at cosmological level, because of the geometrical – and therefore universal – nature of this law, which can be applied to any causal horizon [33].

The tight interweaving of BH horizon thermodynamics and GUP has renewed the interest for *thermodynamic gravity*. In this approach, Einstein field equations are derived from the first law of thermodynamics, combined with the entropy area law [33]. An interesting consequence of this achievement is that one can recover the cosmological Friedmann equations by applying the first law of thermodynamics to the appar-

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ent horizon of the Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime [34–38]. This procedure has recently been proven to be quite general, being equally applicable in theories of gravity beyond General Relativity [39] and even in the presence of a modified entropy-area law [40]. Along this line, in [41] Friedmann equations have been derived from the GUP-modified expression of the entropy, obtaining generalized (i.e. β-dependent) relations. This indicates that GUP effects at high energies can affect the dynamics of the FLRW Universe at early times, albeit in a mild way. The resulting framework is often referred to as GUP Cosmology.

Besides the plethora of theoretical studies on the GUP, a research direction widely pursued in QG phenomenology is attempting to quantify the magnitude of GUP corrections by

Table 1 Upper bounds on the GUP parameter from cosmological analysis

$\|\beta\| \lesssim$	Physical framework	References
10^8	Baryogenesis	[42]
10^{59}	Full data cosmology	[43]
10^{81}	^4He, D abundances	[This work]
10^{81}	Type Ia supernovae	[44]
10^{81}	Baryon acoustic oscillations	[44]
10^{81}	Late-time cosmology	[43]
10^{82}	^7Li abundance	[This work]
10^{87}	Freeze-out temperature	[This work]

Table 2 Upper bounds on the GUP parameter from quantum and gravitational experiments.

$\|\beta\| \lesssim$	Physical framework	References
10^6	Harmonic oscillators	[25]
10^{21}	Scanning tunneling microscope	[13]
10^{21}	Equiv. princip. violation	[45]
10^{27}	Weak equiv. princip. violation	[46]
10^{33}	Gravity bar detectors	[22]
10^{36}	Lamb shift	[13,20]
10^{36}	Interferometry experiments	[47]
10^{39}	^{87}Rb Cold atom experiment	[48]
10^{50}	Landau levels	[13]
10^{60}	Gravitational waves	[49]
10^{69}	Perihelion precession	[15]
10^{71}	Pulsar periastron shift	[15]
10^{72}	Geodetic precession	[50]
10^{73}	Gravitational red-shift	[50]
10^{77}	Quasiperiodic oscillations	[51]
10^{78}	Light deflection	[15]
10^{78}	Shapiro time delay	[50]
10^{90}	BH shadow (M87*)	[52]

constraining the deformation parameter. This is particularly useful in that it paves the way for a low-energy investigation of QG, which could be somehow interfaced with experimental data. Nevertheless, to the best of our knowledge, situations where this kind of analysis is performed in GUP Cosmology are quite rare in the literature, as witnessed by the low number of bounds listed in Table 1. If on one hand this can be understood by observing that bounds of cosmological origin are less stringent than those obtained through quantum/gravitational experiments, on the other hand it should be acknowledged that these bounds can be derived with very high precision, due to the great and accurate amount of cosmological data available to date.

Starting from the above premises, the aim of this work is to explore the implications of GUP Cosmology on Big Bang Nucleosynthesis (BBN). BBN describes the sequence of nuclear reactions responsible for the synthesis of primordial light elements, such as Hydrogen H, its isotope Deuterium D, Helium isotopes ^3He and ^4He and Lithium isotope ^7Li [53–55]. It is believed to have taken place shortly after the Big Bang, when the Universe was cooled enough to form stable protons and neutrons. Since BBN drives the observed Universe, it is clear that primordial abundances must be very tightly constrained in order to reproduce the current chemical composition of the Universe. This fact promotes BBN as one of the best arena to constrain cosmological models. In particular, in what follows we shall fix the GUP parameter by requiring consistency between GUP Cosmology predictions and i) the existing upper bound on the variations of the freeze-out temperature, ii) the current estimates of the primordial abundances of ^4He, D and ^7Li. We show that the ensuing upper bound on β is consistent with most of existing constraints derived from other cosmological analysis. The results here discussed could contribute to the debate of fixing the most reliable scenario among cosmological models based on the GUP and also provide a possible explanation for the ^7Li puzzle.

The layout of the paper is as follows: in Sect. 2 we review the derivation of the modified Friedmann equations within GUP framework. Toward this end, we follow [56,57]. In Sects. 3 and 4 we constrain the GUP parameter based on observational data from BBN and primordial abundances, respectively. Section 5 is devoted to conclusions and outlook.

2 Modified Friedmann equations from GUP

In this section we summarize the main steps leading to the cosmological Friedmann equations and their generalization to the GUP framework. As usual, we assume that, for a homogeneous and isotropic $(1 + 3)$-dimensional FRW Universe, the line element is given by

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \quad a,b = \{0,1\},$$
$$(2)$$

where $h_{ab} = \text{diag}(-1, a^2/(1-kr^2))$ is the metric of a $(1+1)$-dimensional subspace, $x^a = (t,r)$, $\tilde{r} = a(t)r$, with $a(t)$ being the time-dependent scale factor, r is the comoving radius and k the (constant) spatial curvature. θ, ϕ are the angular coordinates.

One can think of the Universe as a physically bounded region of (apparent) horizon radius

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}, \qquad (3)$$

and temperature

$$T = -\frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right), \qquad (4)$$

where $H = \dot{a}(t)/a(t)$ is the Hubble parameter (the dot denotes time derivative). For our later purposes, we can roughly neglect the space curvature k, so that Eq. (3) reads $\tilde{r}_A \simeq 1/H$.

By describing the matter and energy content of the Universe as a perfect fluid, the energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \qquad (5)$$

where u_μ, ρ and p are the four-velocity, energy density and pressure of the fluid, respectively. The continuity equation

$$\dot{\rho} = -3H(\rho + p), \qquad (6)$$

holds true.

Based on the deep connection between gravity and thermodynamics [33], the Friedmann equations in the bulk of the Universe follow from the first law of thermodynamics

$$dE = TdS + WdV, \qquad (7)$$

applied on the boundary. Here, the total energy of the matter existing inside the apparent horizon of entropy S is given by $E = \rho V$, with $V = 4\pi\tilde{r}_A^3/3$ being the volume enclosed by the horizon. The work density W is related to the energy density and pressure by $W = -\frac{1}{2}T^{ab}h_{ab} = \frac{1}{2}(\rho - p)$.

In standard Cosmology the horizon entropy obeys the holographic principle

$$S = \frac{A}{4G}, \qquad (8)$$

where $A = 4\pi\tilde{r}_A^2$ is the horizon surface area (G denotes Newton's gravitational constant). With this as physical input,

it is a straightforward text-book exercise to show that Eq. (7) leads to the Friedmann equations for a flat Universe

$$H^2 = \frac{8}{3}\pi G\rho, \qquad (9)$$

$$\dot{H} = -4\pi G(\rho + p). \qquad (10)$$

Following [57], we now suppose that the general expression for the GUP-modified entropy-area law takes the form

$$S = \frac{f(A)}{4G}, \qquad (11)$$

$$\frac{dS}{dA} = \frac{f'(A)}{4G}, \qquad (12)$$

where the function $f(A)$ is to be determined ($f'(A)$ denotes the derivative of f respect to A). For the quadratic GUP model (1), this can be done by computing the minimal change of area $\Delta A_{min} = 8\pi\ell_p^2 E\Delta x$ of an apparent horizon absorbing a quantum particle of given energy $E \simeq \Delta p$ and finite size $\Delta x \simeq r_s = \sqrt{A/\pi}$ ($r_s = 2MG$ is the Schwarzschild radius). After some algebra, one gets [42,57]

$$\frac{dS}{dA} = \frac{\Delta S_{min}}{\Delta A_{min}} = \frac{1 + \sqrt{1 - \beta^*/A}}{8\ell_p^2}, \qquad (13)$$

where $\Delta S_{min} = \ln 2$ is the minimal increase in entropy, corresponding to one bit of information. Here, we have defined $\beta^* \equiv 16\pi\beta\ell_p^2$ and $\ell_p = 1/m_p = \sqrt{G}$ is the Planck length. Comparison with Eq. (12) allows us to identify

$$f'(A) = \frac{1 + \sqrt{1 - \beta^*/A}}{2}. \qquad (14)$$

It is easy to check that $f'(A) \to 1$ for vanishing β^*, consistently with the holographic relation (8). By plugging Eq. (14) into (12), and integrating over A, it is also possible to derive the explicit formula for the GUP-modified Bekenstein-Hawking entropy. The resulting expression is rather awkward to exhibit. Since we do not need it explicitly in the following analysis, we remand the interested reader to [42,57].

We have now all the ingredients to infer GUP effects on Friedmann equations. Indeed, by replacing Eqs. (12) and (14) into the first law of thermodynamics (7) and noticing that

$$dE = 4\pi\rho\tilde{r}_A^2 d\tilde{r}_A + \frac{4}{3}\pi\tilde{r}_A^3 d\rho \qquad (15)$$

on the horizon surface, we are led to

$$\frac{4\pi}{\tilde{r}_A^3}\left(1 + \sqrt{1 - \frac{\beta^*}{4\pi\tilde{r}_A^2}}\right)d\tilde{r}_A = -\frac{32}{3}\pi^2 G d\rho. \qquad (16)$$

After integrating the l.h.s. between $\tilde{r}_A$ and the minimal length-scale $\Delta x_{min} \simeq \sqrt{\beta^*/\pi}$ allowed by the GUP (1) and setting the integration constant $\rho(\Delta x_{min})$ in such a way that

Eq. (9) is recovered for $\beta^* \to 0$, we obtain to the leading order in the deformation parameter[1]

$$H_\beta(\rho) = H(\rho)\left(1 + \frac{2\beta}{3}\pi G^2 \rho\right), \qquad (17)$$

with H being the standard Hubble parameter given by Eq. (9). This relation provides the first GUP-modified Friedmann equation. For later convenience, we recast it in the form

$$H_\beta(\rho) = H(\rho) Z_\beta(\rho), \qquad (18)$$

where we have separated out the β-dependence of H_β by defining

$$Z_\beta(\rho) = 1 + \frac{2\beta}{3}\pi G^2 \rho. \qquad (19)$$

In view of applying the above formalism to BBN, we can further manipulate Eq. (18) by using the relation

$$\rho = \frac{\pi^2 g(T)}{30} T^4, \qquad (20)$$

where $g(T)$ denotes the effective number of degrees of freedom. Equation (18) becomes

$$H_\beta(T) = H(T) Z_\beta(T), \qquad (21)$$

where

$$H(T) = \frac{2\pi}{3}\sqrt{\frac{\pi G g(T)}{5}} T^2, \qquad (22)$$

$$Z_\beta(T) = 1 + \frac{\beta}{45}\pi^3 G^2 g(T) T^4. \qquad (23)$$

In a similar fashion, one can derive the linearized second GUP-modified Friedmann equation to be [42]

$$\dot{H}_\beta = \dot{H}\left(1 + \beta G H^2\right), \qquad (24)$$

which still recovers Eq. (10) in the limit of vanishing β.

These GUP-corrected Friedmann equations form the basis on which variations of the Hubble parameter and of its time derivative in the early Universe will be studied.

3 Big bang nucleosynthesis in GUP cosmology

In this section we study the BBN within the framework of GUP Cosmology. We assume that the energy density of relativistic particles filling up the Universe is given by Eq. (20)

with $g(T) = g_* \simeq 10$ (henceforth we consider the radiation dominated era), the major contribution to the degrees of freedom being given by relativistic photons, $e^+ e^-$ pairs and the three neutrino species.

According to the standard BBN model, neutron and protons started to form only few thousandths of a second after the Big Bang, when the temperature dropped low enough. From the first hundredth of a second up to few minutes, the abundances of the first very light atomic nuclei were defined. In particular, the formation of the primordial ^4He took place at around $T \simeq 100$ MeV, while the energy and number density were still dominated by relativistic leptons (electrons, positrons and neutrinos) and photons. Due to their rapid collisions, such particles were in thermal equilibrium, so that $T_\nu = T_e = T_\gamma = T$ [54]. On the other hand, the smattering of protons and neutrons were kept in equilibrium owing to the following weak interactions with leptons

$$a) \quad \nu_e + n \longleftrightarrow p + e^-, \qquad (25)$$

$$b) \quad e^+ + n \longleftrightarrow p + \bar{\nu}_e, \qquad (26)$$

$$c) \quad n \longleftrightarrow p + e^- + \bar{\nu}_e. \qquad (27)$$

Within the framework outlined above, neutron abundance can be computed by estimating the conversion rate $\lambda_{pn}(T)$ of protons into neutrons and its inverse $\lambda_{np}(T) = e^{-Q/T}\lambda_{pn}(T)$, where $Q = m_n - m_p \simeq 1.29$ MeV is the difference between neutron and proton masses. Here λ_{np} is expressed as the sum of the rates associated to the three processes (25)–(27) separately, i.e.[2]

$$\lambda_{np}(T) = \lambda_a(T) + \lambda_b(T) + \lambda_c(T). \qquad (28)$$

In turn, the total weak interaction rate reads $\Lambda(T) = \lambda_{np}(T) + \lambda_{pn}(T)$.

Following [54], we further assume that, during the freeze-out period, the temperature T is low in comparison with the with the characteristic energies contributing to the rates for the decays (25)–(27). This allows us to estimate the lepton phase-space density functions by the "classical" Boltzmann weights, rather than the Fermi–Dirac distribution. The last requirement is that the electron mass m_e can be neglected with respect to the electron and neutrino energies. Under these conditions, one can show that [54,56]

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[1] Strictly speaking, we are expanding around $\epsilon \equiv \beta \ell_p^4 \rho$. We shall check *a posteriori* the degree of validity of this approximation (see Sect. 3).

[2] Notice that the integration over momentum appearing in the definition of λ_a, λ_b and λ_c might be affected in the GUP framework due to minimal-length effects. However, we expect these corrections not to spoil significantly the order of magnitude of the resulting rates, thus being negligible in first approximation.

$$\lambda_a(T) \simeq qT^5 + \mathcal{O}\left(\frac{\mathcal{Q}}{T}\right) = \lambda_b(T), \tag{29}$$

where $q \simeq 10^{-10} \, \text{GeV}^{-4}$. On the other hand, the contribution of the free-neutron decay process c) to the total rate is found to be negligible,[3] implying that the total rate $\lambda_{np}(T)$ is roughly twice that given in Eq. (29).

The ^4He mass fraction of the total baryonic mass is now estimated as [53,56]

$$Y_p \equiv \gamma \frac{2x(t_f)}{1 + x(t_f)}, \tag{30}$$

where $\gamma = e^{-(t_n - t_f)/\tau} \simeq 1$ depends on the (relatively short) time between freeze-out (t_f) and nucleosynthesis (t_n) and on the neutron mean lifetime $\tau \simeq 877 \, \text{s}$. It can be as the fraction of neutrons that decay into protons in the interval $t \in [t_f, t_n]$. $x(t_f) = e^{-\mathcal{Q}/T(t_f)}$ is the neutron-to-proton equilibrium ratio.

Deviations from Y_p due to the variation of the freeze-out temperature T_f can be quantified as [56]

$$\delta Y_p = Y_p \left[\left(1 - \frac{Y_p}{2\gamma}\right) \log\left(\frac{2\gamma}{Y_p} - 1\right) - \frac{2t_f}{\tau}\right] \frac{\delta T_f}{T_f}, \tag{31}$$

where δT_n has been set to zero, since T_n is fixed by the D-binding energy [58,59].

The mass fraction of ^4He has been recently determined to a high degree of precision by making use of infrared and visible ^4He emission lines in extragalactic HII regions, obtaining [60]

$$Y_p = 0.2449, \quad |\delta Y_p| \lesssim 10^{-4}. \tag{32}$$

Insertion of these values into Eq. (31) gives

$$\left|\frac{\delta T_f}{T_f}\right| \lesssim 10^{-4}, \tag{33}$$

where we have set $t_f \simeq 1 \, \text{s}$ and $t_n \simeq 20 \, \text{s}$.

Following [56], we can compute the GUP-modified freeze-out temperature T_f by equating Eqs. (21) and (28). With the further definition $\delta T_f = T_f - T_{0f}$, where $T_{0f} \simeq 0.6 \, \text{MeV}$ [56], we get

$$\left|\frac{\delta T_f}{T_f}\right| = \left|1 - \frac{2\beta\pi^4 G^2 g_* \sqrt{\pi G g_*/5}}{135q} T_{0f}\right|. \tag{34}$$

The GUP parameter can be fixed by demanding consistency between Eqs. (33) and (34). A straightforward numerical evaluation leads to

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[3] For $T \gtrsim 1 \, \text{MeV}$ this contribution is 3 orders of magnitude lower than the rate (29) [54].

$$\beta \sim \mathcal{O}(10^{87}). \tag{35}$$

This means that our linearized approximation is well-justified, since for this value of β we have $\epsilon \sim \mathcal{O}(10^{-2})$ (see footnote 1).

By comparison with bounds in Table 1, we see that the result (35) provides us with a weak cosmological constraint on β. The gap becomes even wider if compared with bounds from gravitational/quantum experiments (see Table 2), thus emphasizing the quite negligible rôle of GUP on cosmic scales.

4 Primordial ^4He, D, ^7Li abundances in GUP cosmology

Let us now constrain the GUP by a slightly different approach. The basic idea is to study GUP-induced deviations from standard Cosmology on the primordial abundances of Helium isotope ^4He, Deuterium D and Lithium isotope ^7Li. This will be done by replacing the standard Z-factor entering primordial abundances with the β-dependent Z-factor appearing in Eq. (21).

In this regard, we observe that in the ordinary Cosmology based on General Relativity, one simply has $Z = 1$. Deviations of Z from unity may arise due to either modified descriptions of gravity or the presence of additional light particles such as neutrinos, in which case one has [61]

$$Z_\nu = \left[1 + \frac{7}{43}(N_\nu - 3)\right]^{1/2}, \tag{36}$$

where N_ν is the number of neutrino generations. However, since we aim to focus on the effects of the GUP on BBN, hereafter we assume $N_\nu = 3$, ruling out the possibility that in our framework departures of Z from unity are originated by degrees of freedom of additional particles. Given the very tight observational constraints on the allowed primordial abundances, we expect in this way to infer reliable bounds on the deformation parameter of GUP.

4.1 ^4He abundance

In order to estimate ^4He primordial abundance, we follow the approach of [62], recently revived in [56]. Let us summarize here the sequence of nuclear reactions responsible for the production of this element. The first step consists in generating deuterium D from a neutron and a proton. After that, deuterium is converted into ^3He and tritium T. In short

$$n + p \rightarrow D + \gamma, \tag{37}$$

$$D + D \rightarrow {}^3He + n, \tag{38}$$

$$D + D \rightarrow T + p. \tag{39}$$

The last step of the chain leads to the production of ^4He due to the following processes

$$D + T \rightarrow {}^4He + n, \tag{40}$$
$$D + {}^3He \rightarrow {}^4He + p. \tag{41}$$

According to [63,64], the numerical best fit constrains the primordial ^4He abundance to be

$$Y_p = 0.2485 \pm 0.0006 + 0.0016 \left[(\eta_{10} - 6) + 100 \, (Z - 1) \right], \tag{42}$$

where in our case we have to set $Z = Z_\beta$ given by Eq. (23). Here, we have adopted the usual definition of the baryon density parameter [63,64]

$$\eta_{10} \equiv 10^{10} \eta_B \simeq 6, \tag{43}$$

where η_B is the baryon to photon ratio. Notice that, by setting $Z = 1$, we recover the standard ^4He abundance $Y_p = 0.2485 \pm 0.0006$ predicted by BBN model.

Now, as discussed in [56,62], consistency between observational data on ^4He abundance and Eq. (42) with $\eta_{10} = 6$ allows us to fix [65]

$$\delta Z \equiv Z - 1 \lesssim \mathcal{O}(10^{-2}). \tag{44}$$

By using the expression (23) for Z_β, we then obtain

$$\beta \lesssim \mathcal{O}(10^{81}), \tag{45}$$

assuming $T \simeq 10 \, \text{MeV}$ and

$$\beta \lesssim \mathcal{O}(10^{89}), \tag{46}$$

assuming $T \simeq 0.1 \, \text{MeV}$.

Let us focus on the most stringent bound (45). Except for the constraint of [42] (which is however computed by referring to the much earlier baryogenesis epoch[4]) we notice that the result $\beta \lesssim \mathcal{O}(10^{81})$ perfectly fits with other cosmological bounds obtained via Type Ia supernovae [44] and baryon acoustic oscillations measurements [44]. It also agrees with

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[4] We point out that the gap between the bound on β from GUP baryogenesis [42] and other cosmological bounds from different stages of the evolution of the Universe could be a hint for the need of a GUP model with a time (or equivalently energy) dependent deformation parameter. Of course, such a running behavior might not be described through a simply (i.e. monotonically) decreasing function of time, but rather by a more complicated function. And indeed this should be the case in order to cure the above inconsistency. In this regard, we mention that a similar time-dependence of the deformed commutator occurs in Maguejo–Smolin Doubly Special Relativity [8], which predicts that the generalized commutator should vanish at Planck scale, while approaching the conventional HUP at low energies.

late-time observational data from Early-Type Galaxies as Cosmic Chronometers, the H0 Lenses in COSMOGRAIL's Wellspring, the "Mayflower" sample of Gamma Ray Bursts and the latest Planck 2018 release for Cosmic Microwave Background radiation [43]. Furthermore, Eq. (45) provides us with a more stringent bound than Eq. (35).

Again, we can see that for the values of β in Eqs. (45), (46), the approximation $\epsilon \ll 1$ works well, since we have $\epsilon \sim \mathcal{O}(10^{-3})$.

4.2 D abundance

Deuterium D is generated form the process (37). Following the same analysis as above, D primordial abundance can be ascertained from the numerical best fit of [66], giving

$$y_{D_p} = 2.6 \, (1 \pm 0.06) \left(\frac{6}{\eta_{10} - 6 \, (Z - 1)} \right)^{1.6}. \tag{47}$$

As before, the values $\eta_{10} = 6$ and $Z = 1$ yield the standard BBN prediction $y_{D_p} = 2.6 \pm 0.16$.

Observational constraints on D abundance combined with Eq. (47) allow us to set $\delta Z \lesssim \mathcal{O}(10^{-2})$ [56,62,65], which is consistent with the constraint from ^2He abundance (see Eq. (44)). Therefore, one still gets the bounds (45)–(46) for $T \simeq (0.1 \div 10) \, \text{MeV}$.

4.3 ^7Li abundance

It is well-known that the η_{10} parameter which successfully fits the abundances of ^4He, D and other light elements is somehow inconsistent with observations of ^7Li. In fact, the ratio of the predicted value of ^7Li abundance to the observed one lies in the interval [2.4, 4.3] according to the standard cosmological theory [61,67]. Quite unexpectedly, neither BBN nor any alternative model are able to fit this so low abundance ratio. This puzzle is referred to as *Lithium problem*.

Once more, we can constrain deviations of Z from unity by demanding consistency between the numerical best fit expression for ^7Li abundance [66]

$$y_{Li_p} = 4.82 \, (1 \pm 0.1) \left[\frac{\eta_{10} - 3 \, (Z - 1)}{6} \right]^2 \tag{48}$$

and observational bounds. In this case one has [56,62,65]

$$\delta Z \lesssim \mathcal{O}(10^{-1}). \tag{49}$$

Notice that this constraint is one order higher than the corresponding value in Eq. (44).

Thus, from Eq. (23) we obtain

$$\beta \lesssim \mathcal{O}(10^{82}), \tag{50}$$

for $T \simeq 10$ MeV and

$$\beta \lesssim \mathcal{O}(10^{90}), \tag{51}$$

for $T \simeq 0.1$ MeV. Also in this case, the approximation $\epsilon \ll 1$ is satisfied, being $\epsilon \sim \mathcal{O}(10^{-2})$.

As predictable, the overlap between the bound on β from ^7Li abundance on one hand and ^4He, D abundances on the other is only partial, though non-vanishing. This discloses the possibility that the ^7Li puzzle might be successfully addressed within the framework of GUP-modified Cosmology for a suitable choice of the GUP parameter. Investigation along this direction requires further attention and will be developed elsewhere.

5 Discussion and conclusions

Merging General Relativity and Quantum Theory is one of the hottest topics in modern theoretical physics. A phenomenological approach to endow Quantum Mechanics with gravity effects is to modify the Heisenberg Uncertainty Principle in such a way as to reproduce a minimal observable length at Planck scale – Generalized Uncertainty Principle. Although the natural domain of GUP is high-energy physics, the best – and, for the time being, unique - arena to quantify the magnitude of GUP corrections is low-energy regime. In this vein, it should be understood the large number of attempts to constrain the GUP deformation parameter via optomechanical/interferometry experiments on one hand and gravitational/cosmological measurements on the other (see [68,69] for a review).

Starting from the well-established connection between the first law of thermodynamics and the cosmological Friedmann equations, in this work we have investigated the implications of GUP on Big Bang Nucleosynthesis and the related abundances of primordial light elements. We emphasize that GUP enters the Friedmann equations through a non-trivial modification of the entropy area law (see Eq. (11)), which in turn affects the standard density/temperature dependence of Hubble constant and of its time derivative. GUP-corrected Friedmann equations are given in Eqs. (21) and (24) to the leading order in the deformation parameter.

Consistency with observational data on (1) variations of the freeze-out temperature T_f and (2) primordial abundances of ^4He, D and ^7Li has allowed us to infer various constraints on the GUP parameter β, the most stringent being $\beta \lesssim \mathcal{O}(10^{81})$ derived from the analysis of the ^4He and D abundances. It is worth noticing that such bound fits with those found in [43,44] from similar cosmological studies, although it is less stringent than constraints inferred via gravitational or quantum experiments. This somehow indicates the negligible, though non-vanishing, rôle of the GUP on

cosmological scales. In this sense, it would be interesting to study implications of the Extended Uncertainty Principle (EUP) [70–73], which naturally emerges in spacetime with a maximal length (horizon-like) scale, such as (anti)-de Sitter background. Besides this aspect, another important result of this work is the possibility that the ^7Li problem could be solved in the framework of modified GUP Cosmology.

A further direction to explore is the study of effects of other GUP formulations on BBN cosmological model. Indeed, as argued at the end of the previous section, higher-order GUP corrections terms might be relevant, particularly in the study of the ^7Li problem.

Finally, we mention that a similar analysis has been carried out in [56] in the context of non-extensive Tsallis Cosmology, which is a generalization of the ordinary Cosmology based on Tsallis non-additive definition of horizon entropy [74]. In light of this extension, it is worth investigating whether a connection between GUP and Tsallis frameworks can be established, so as to map the GUP parameter and Tsallis non-extensivity index into each other. Work along these and other directions is presently under active consideration and will be presented in future works.

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