Resonant leptogenesis and TM_1 mixing in minimal type-I seesaw model with S_4 symmetry

Bikash Thapa^a, Ng. K. Francis^b

Department of Physics, Tezpur University, Tezpur 784028, India

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Abstract We present an S_4 flavour symmetric model within a minimal seesaw framework resulting in mass matrices that leads to TM_1 mixing. Minimal seesaw is realized by adding two right-handed neutrinos to the Standard Model. The model predicts Normal Hierarchy (NH) for neutrino masses. Using the constrained six-dimensional parameter space of the model, we have evaluated the effective Majorana neutrino mass, which is the parameter of interest in neutrinoless double beta decay experiments. The possibility of explaining baryogenesis via resonant leptogenesis is also examined within the model. A non-zero, resonantly enhanced CP asymmetry generated from the decay of right-handed neutrinos at the TeV scale is studied, considering flavour effects. The evolution of lepton asymmetry is discussed by solving the set of Boltzmann equations numerically and obtain the value of baryon asymmetry to be $|\eta_B| = 6.3 \times 10^{-10}$ with the choice of right-handed neutrino mass, $M_1 = 10$ TeV and mass splitting, $d \simeq 10^{-8}$.

1 Introduction

Neutrino oscillation experiments have determined that the mass of neutrinos are small but non-zero, and indicate flavour mixing [1–6]. Observations such as these pose a question about the origin of the tiny neutrino masses. The absence of the right-handed counterpart of the neutrinos within the standard model (SM) suggests that, unlike charged fermions, Dirac masses could not be the origin of neutrino masses. There are numerous frameworks beyond the standard model (BSM) that can explain the origin of neutrino masses, for instance, the seesaw mechanism [7–9], radiative seesaw mechanism [10], models based on extra dimensions [11, 12], and other models. The minimal seesaw mechanism, which is the extension of SM with two right-handed neutrinos,

can explain the origin of neutrino masses as well as that of the Baryon Asymmetry of the Universe (BAU) through leptogenesis [13]. Here, the generation of baryon asymmetry involves conversion of lepton asymmetry, obtained from the CP-violating decay of the heavy right-handed neutrinos, via the sphaleron processes [14]. It has been reported in Ref. [15] that the mass scale $\mathcal{O}(10^9)$ for the right-handed neutrino is needed to explain the observed BAU. However, this scale can be lowered if we have nearly degenerate mass of right-handed neutrinos. Such a case leads to resonantly enhanced CP-violating effects, and sufficient lepton asymmetry to account for BAU can be generated at relatively low masses (TeV scale). Such a situation is termed Resonant leptogenesis [16]. One may note that a recent study [17] based on the $SU(5) \times \mathcal{T}_{13}$ model shows viable GeV–TeV scale resonant leptogenesis in the type-I seesaw model with active-sterile mixing within the sensitivity of DUNE.

Further, the study of the origin of neutrino flavour mixing has received much attention through the years. Among various possibilities, tri-bimaximal mixing (TBM) [18] seemed the most plausible explanation; however, experimental observations at Daya Bay [19], RENO [20] and Double Chooz [21] suggest TBM requires corrections to incorporate $\theta_{13} \neq 0$. Trimaximal TM_1 [22–24] is one such attractive mixing scheme that can be obtained by multiplying TBM with a 23-rotation leaving the first column of the TBM mixing matrix intact in TM_1 mixing, and the relationships among the observables in such a scheme are given as [25]

$$\sin^2 \theta_{12} = \frac{1 - 3 \sin^2 \theta_{13}}{3 \cos^2 \theta_{13}}, \quad (1)$$

$$\cos \delta_{CP} = \frac{(1 - 5 \sin^2 \theta_{13})(2 \sin^2 \theta_{23} - 1)}{4 \sin \theta_{13} \sin \theta_{23} \sqrt{2(1 - 3 \sin^2 \theta_{13})(1 - \sin^2 \theta_{23})}}. \quad (2)$$

TM_1 mixing has proved to be compatible with the global data on neutrino oscillations, in a sense that it includes non-zero θ_{13} and agrees very well with the experiments on its predic-

^a e-mail: bikash2@tezu.ernet.in (corresponding author)

^b e-mail: francis@tezu.ernet.in

Table 1 Field content and their representations under $S_4 \times Z_3 \times Z_2$

Field	$\bar{L}$	e_R	(μ_R, τ_R)	N_1	N_2	H	φ_l	ϕ_l	φ_ν	ϕ_ν	ψ	ξ	ζ
S_4	3_1	1_1	2	1_1	1_2	1_1	3_1	3_2	3_1	3_2	3_2	1_1	1_2
Z_3	1	ω^2	ω^2	1	1	1	ω	ω	1	1	1	1	1
Z_2	1	1	1	-1	1	1	1	1	-1	1	1	1	-1

tion on the mixing angles θ_{13} , θ_{23} and the Dirac phase, δ_{CP} . Over the years, many discrete symmetry-based studies have been done that gives rise to TM_1 mixing [25–31]. Herein, we propose a model constructed using S_4 discrete symmetry within the framework of the minimal seesaw model. The resulting mass matrix leads to TM_1 mixing and can simultaneously explain BAU via resonant leptogenesis. The choice of right-handed neutrino Majorana mass matrix, M_R , is such that the right-handed neutrinos have degenerate mass at dimension five-level and successful resonant leptogenesis is achieved by introducing the higher-order term. In other words, our work is based on the extension of the model presented in [32], such that it makes it suitable to study resonant leptogenesis within minimal seesaw scenario, and the orthogonality condition [24, 26] allows us to realize TM_1 mixing in the leptonic sector.

Similar study of resonant leptogenesis within minimal seesaw model and based on S_4 symmetry has been carried out in Ref. [33]. Unlike the present work, successful realization of resonant leptogenesis is made possible by radiatively generating the mass splitting among the heavy RHN and the non-zero off-diagonal terms in the Dirac Yukawa-coupling matrix for the models presented in [33]. In their analyses low energy CP violation in neutrino oscillations is absent as a result of the exact TBM structure of the mixing matrix hence, the only source of CP violation comes from the Majorana phases. With the discovery of non-zero θ_{13} [34–37] taken in consideration, we revisit the study of resonant leptogenesis in minimal seesaw model based on S_4 discrete symmetry that leads to TM_1 mixing. As mentioned earlier, in our work the mass splitting required to achieve successful resonant leptogenesis comes from symmetry-based breaking term.

This paper is structured as follows. In Sect. 2, we have presented the S_4 flavour symmetric minimal seesaw model followed by the features of the S_4 flavour group relevant to the construction of the model. Using the 3σ range of neutrino oscillation data as constraints, we defined the allowed region for the model parameters in Sect. 3. In Sect. 4, the framework for resonant leptogenesis is described and the Boltzmann equations, which govern the evolution of the lepton number density and baryon asymmetry parameter, are numerically solved. It also includes the numerical results on neutrinoless double beta decay within the model and we finally conclude our work in Sect. 5.

2 Model framework

The S_4 flavour symmetry has been widely used to explain the observed flavour mixing of neutrinos [22, 30, 32, 38–45]. S_4 group is a non-Abelian discrete group of permutations of 4 objects. It has 24 elements and 5 irreducible representations 1_1 , 1_2 , 2 , 3_1 and 3_2 . The product rules and Clebsch–Gordan coefficients are presented in Appendix A. In this work, we have considered the extension of the standard model (SM) with a discrete non-abelian group S_4 . Also, a $Z_3 \times Z_2$ group is introduced to avoid specific unwanted couplings and achieve desired structures for the mass matrices. The fermion sector includes, in addition to the SM fermions, two right-handed neutrinos N_1 and N_2 . Flavours φ_l , ϕ_l , φ_ν , ϕ_ν , χ , ψ , β and ρ forms the extension in the scalar sector. The charges carried by the various fields under different symmetry groups are presented in Table 1. Following the representations of the fields given in Table 1, we can write the invariant Yukawa Lagrangian

$$\begin{aligned}
 -\mathcal{L} \supset & \frac{y_{l1}}{\Lambda} \bar{L} H \varphi_l e_R + \frac{y_{l2}}{\Lambda} \bar{L} H \varphi_l (\mu_R, \tau_R) + \frac{y_{l3}}{\Lambda} \bar{L} H \phi_l \\
 & \times (\mu_R, \tau_R) + \frac{y_{\nu 1}}{\Lambda} \bar{L} \tilde{H} \varphi_\nu N_1 + \frac{y_{\nu 2}}{\Lambda} \bar{L} \tilde{H} \phi_\nu N_2 \\
 & + \frac{y_{\nu 3}}{\Lambda} \bar{L} \tilde{H} \psi N_2 + y_{N1} \bar{N}_1^c N_1 \beta + y_{N2} \bar{N}_2^c N_2 \beta \\
 & + y_{N3} \bar{N}_1^c N_2 \rho \frac{\beta\beta}{\Lambda^2} + h.c.,
 \end{aligned} \quad (3)$$

where H is SM Higgs doublet and $\tilde{H} = i\sigma_2 H^*$, σ_2 being the 2nd Pauli matrices. The vacuum expectation values (vev) of the scalar fields are of the form [32]

$$\begin{aligned}
 \langle \varphi_l \rangle &= (v_{\varphi_l}, 0, 0), \quad \langle \phi_l \rangle = (v_{\phi_l}, 0, 0), \quad \langle \varphi_\nu \rangle = (0, -v_{\varphi_\nu}, v_{\varphi_\nu}), \\
 \langle \phi_\nu \rangle &= (v_{\phi_\nu}, v_{\phi_\nu}, v_{\phi_\nu}), \quad \langle \beta \rangle = v_\beta, \quad \langle \rho \rangle = v_\rho.
 \end{aligned} \quad (4)$$

As for the vev of ψ we choose $\langle \psi \rangle = (0, -v_\psi, v_\psi)$ following the orthogonality conditions $\langle \psi \rangle \cdot \langle \phi_l \rangle$ and $\langle \psi \rangle \cdot \langle \phi_\nu \rangle$. After electroweak and flavour symmetry breaking, we obtain the following structure for the charged lepton mass matrix

$$m_l = \frac{v_H}{\Lambda} \begin{pmatrix} y_{l1} v_{\varphi_l} & 0 & 0 \\ 0 & y_{l2} v_{\varphi_l} + y_{l3} v_{\phi_l} & 0 \\ 0 & 0 & y_{l2} v_{\varphi_l} - y_{l3} v_{\phi_l} \end{pmatrix}. \quad (5)$$

The charged lepton sector of the model is similar to that of [32] and we similarly assume that the Froggatt–Nielsen mechanism explains the observed mass hierarchy of the charged-leptons. Similarly, using the vev presented in Eq. (4) for the neutrino sector, we obtain the Dirac and Majorana mass matrix

$$m_D = \begin{pmatrix} 0 & b \\ a & b+c \\ -a & b-c \end{pmatrix}, \quad (6)$$

$$m_R = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix}, \quad (7)$$

where $a = y_{v1} \frac{v_H v_{\phi v}}{\Lambda}$, $b = y_{v2} \frac{v_H v_{\phi v}}{\Lambda}$, $c = y_{v3} \frac{v_H v_{\psi}}{\Lambda}$ with v_H being the vev of the SM Higgs. Taking $y_{N1} \simeq y_{N2} = y_N$ we have degenerate masses for the right-handed neutrinos, $M = y_N v_{\xi}$.¹

In the seesaw framework, the resultant light neutrino mass matrix is given by the well-known formula

$$m_\nu = -m_D m_R^{-1} m_D^T \quad (8)$$

$$= \frac{1}{M} \begin{pmatrix} b^2 & b(b+c) & b(b-c) \\ b(b+c) & a^2 + (b+c)^2 & -(a^2 - b^2 + c^2) \\ b(b-c) & -(a^2 - b^2 + c^2) & a^2 + (b-c)^2 \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} b^2 & b'(b'+c') & b'(b'-c') \\ b'(b'+c') & a'^2 + (b'+c')^2 & -(a'^2 - b'^2 + c'^2) \\ b'(b'-c') & -(a'^2 - b'^2 + c'^2) & a'^2 + (b'-c')^2 \end{pmatrix}, \quad (10)$$

with $a' = \frac{a}{\sqrt{M}}$, $b' = \frac{b}{\sqrt{M}}$ and $c' = \frac{c}{\sqrt{M}}$. In charged-lepton diagonal basis, the neutrino mixing matrix, U_ν , is the unitary matrix that diagonalizes the mass matrix in Eq. (10). The resulting U_ν matrix, which is determined entirely from the neutrino sector, is

$$U_\nu = U_{\text{TBM}} U_{23} = U_{\text{TM}_1}, \quad (11)$$

where U_{TBM} is the tri-bimaximal mixing (TBM) matrix, U_{23} is a unitary matrix whose (1,2), (1,3), (2,1), (3,1) entries are vanishing and the resulting matrix, U_{TM_1} , has its 1st column coinciding with that of TBM matrix. The diagonalization equation thus reads

$$U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3). \quad (12)$$

The light neutrino masses are given as,

¹ Here we obtain degenerate masses for the right-handed neutrino considering terms only upto dimension-5.

Table 2 The best-fit values, 1σ errors and 3σ ranges of neutrino oscillation parameters from NuFIT 5.0 [46]

Parameters	Best-fit $\pm 1\sigma$	3σ range
$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	$7.42^{+0.21}_{-0.20}$	6.82–8.04
$\Delta m_{31}^2 [10^{-3} \text{ eV}^2]$ (NH)	$2.517^{+0.026}_{-0.028}$	2.435–2.598
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	0.269–0.343
$\sin^2 \theta_{23}$	$0.573^{+0.016}_{-0.020}$	0.415–0.616
$\sin^2 \theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	0.02032–0.02410
δ_{CP}/π (NH)	$1.09^{+0.15}_{-0.13}$	0.667–2.05

$$m_1 = 0, \quad m_2 = \frac{1}{2} |(s - \sqrt{t + s^2})|, \quad m_3 = \frac{1}{2} |(s + \sqrt{t + s^2})|, \quad (13)$$

where $s = 2a'^2 + 3b'^2 + 2c'^2$ and $t = -24a'^2 b'^2$. It is evident from Eq. (13) that the model predicts normal hierarchy of light neutrino.

In the following sections, we have presented the numerical approaches and discussed baryogenesis via resonant leptogenesis, neutrinoless double beta decay within the context of our model.

3 Numerical analysis

In the previous section, we have shown how the S_4 model can be implemented in the minimal seesaw scenario, which results in mass matrices that lead to TM_1 mixing and normal hierarchy (NH) of masses for the neutrinos. Here, we perform numerical analysis to see the model's implication on leptogenesis and other phenomenological predictions. The mass matrix in Eq. (10) gives the effective neutrino mass matrix in terms of the complex model parameters a' , b' , and c' . We find the values of the model parameters by fitting the model to the current neutrino oscillation data. To do so, we use the 3σ interval [46] for the neutrino oscillation parameters (θ_{12} , θ_{23} , θ_{13} , Δm_{21}^2 , Δm_{31}^2) as presented in Table 2. A further constraint on the model parameters was applied on the sum of absolute neutrino masses $\sum_i m_i < 0.12 \text{ eV}$ from the cosmological bound [47].

In our analysis, the three complex parameters of the model are treated as free parameters and are allowed to run over the following ranges:

$$\begin{aligned} |a'| &\in [0.1, 0.2] \text{ eV}^{1/2}, & \phi_a &\in [-\pi, \pi], \\ |b'| &\in [0.03, 0.06] \text{ eV}^{1/2}, & \phi_b &\in [-\pi, \pi] \\ |c'| &\in [10^{-4}, 0.1] \text{ eV}^{1/2}, & \phi_c &\in [-\pi, \pi] \end{aligned}$$

where ϕ_a , ϕ_b , ϕ_c are the phases given by $\arg(a')$, $\arg(b')$, $\arg(c')$, respectively. Using relation $U^\dagger \mathcal{M} U = \text{diag}(m_1^2, m_2^2,$

Fig. 1 Left and right panel shows the correlation of $|a'|$ with $|b'|$ and $|c'|$ respectively along with the variation of $\sum m_i$. The cross mark indicate the best-fit values with $\chi^2 = \chi_{min}^2$, which corresponds to $\sum m_i = 0.0586$ eV

Fig. 2 Correlation between the phases ϕ_a , ϕ_b and ϕ_c . The cross mark indicate the best-fit values corresponding to χ_{min}^2

m_3^2), with $\mathcal{M} = m_\nu m_\nu^\dagger$ and U is a unitary matrix, we numerically diagonalize the effective neutrino mass matrix m_ν . The mixing angles, θ_{23} , θ_{13} are obtained using the relation

$$\sin^2 \theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2}, \quad \sin^2 \theta_{13} = |U_{13}|^2. \quad (14)$$

As seen from Eqs. (1) and (2), TM_1 mixing gives correlations among the mixing angles and CP phases. These relations are assumed to calculate the observables θ_{12} and δ_{CP} . The points in the 6-dimensional parameter space which corresponds to the observables that satisfy the 3σ bound on neutrino oscillations are taken to be the allowed region and the best-fit values for the model parameters ($|a'|$, $|b'|$, $|c'|$, ϕ_a , ϕ_b , ϕ_c) correspond to the minimum of the following χ^2 function

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{model} - \lambda_i^{expt}}{\Delta \lambda_i} \right)^2, \quad (15)$$

where λ_i^{model} is the i^{th} observable predicted by the model, λ_i^{model} stands for the i^{th} experimental best-fit value (Table 2) and $\Delta \lambda_i$ is the 1σ range of the i^{th} observable.

The allowed regions for the various model parameters are given in Figs. 1 and 2. The best-fit values for $|a'|$, $|b'|$, $|c'|$, ϕ_a , ϕ_b and ϕ_c obtained using the function defined in Eq. (15) are (0.155, 0.054, 0.09, 0.379π , 0.139π , 0.087π), denoted by the cross mark. Correspondingly, the best-fit values for the neutrino oscillation parameters are: $\sin^2 \theta_{12} = 0.318$, $\sin^2 \theta_{23} = 0.592$, $\sin^2 \theta_{13} = 0.02225$, $\sin \delta_{CP} = -0.454$, $\Delta m_{21}^2 = 7.25 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.51 \times 10^{-3} \text{ eV}^2$. We have shown correlation of $\sin \delta_{CP}$ with $\sum m_i$ in Fig. 3.

4 Resonant leptogenesis

The mechanism of leptogenesis, first proposed by Fukugita and Yanagida [13], is one of the popular mechanisms that can explain the observed baryon asymmetry of the universe (BAU). In the simplest scenario of thermal leptogenesis with a hierarchical mass spectrum of right-handed neutrinos, there

Fig. 3 Shows the correlation of $\sum m_i$ with $\sin \delta_{CP}$

is a lower bound on the mass of the lightest right-handed neutrino, $M_1 \simeq 10^9$ GeV [15]. Although one can lower this limit if their masses are nearly degenerate, the scenario is popularly known as resonant leptogenesis [16,48]. In such a situation, one-loop self-energy contribution is enhanced resonantly, and the flavour-dependent asymmetry produced from the decay of right-handed neutrino into lepton and Higgs is given by [49–53]:

$$\varepsilon_{i\alpha} = \frac{\Gamma(N_i \rightarrow l_\alpha + H) - \Gamma(N_i \rightarrow \bar{l}_\alpha + \bar{H})}{\sum_\alpha \Gamma(N_i \rightarrow l_\alpha + H) + \Gamma(N_i \rightarrow \bar{l}_\alpha + \bar{H})} \quad (16)$$

$$= \sum_{i \neq j} \frac{\text{Im} \left\{ (Y_v^*)_{\alpha i} (Y_v)_{\alpha j} \left[(Y_v^\dagger Y_v)_{ij} + \xi_{ij} (Y_v^\dagger Y_v)_{ji} \right] \right\}}{(Y_v^\dagger Y_v)_{ii} (Y_v^\dagger Y_v)_{jj}} \times \frac{\xi_{ij} \zeta_j (\xi_{ij}^2 - 1)}{(\xi_{ij} \zeta_j)^2 + (\xi_{ij}^2 - 1)^2}. \quad (17)$$

where $\xi_{ij} = M_i/M_j$ and $\zeta_j = (Y_v^\dagger Y_v)_{jj}/(8\pi)$ with $Y_v = m_D/v$.

In our model, we have two right-handed neutrinos with exactly degenerate masses, $M_1 = M_2 = M$. However, successful leptogenesis requires a tiny mass splitting between the two right-handed neutrinos, which is introduced by adding a higher dimension term in the model (Eq. 3). Such term leads to a minor correction in the Majorana mass matrix of Eq. (7), and the resultant structure of the mass matrix may be written as

$$m_R = \begin{pmatrix} M & \varepsilon \\ \varepsilon & M \end{pmatrix}, \quad (18)$$

where $\varepsilon = y_{N_3} v_\rho \frac{v_\beta^2}{\Lambda^2}$ is a parameter that quantifies the tiny difference between masses required for leptogenesis.² The mass matrix in Eq. (18) is diagonalized using a (2×2) matrix of the form

$$U_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad (19)$$

with real eigenvalues $M_1 = M - \varepsilon$ and $M_2 = M + \varepsilon$. In the basis where the charged-lepton and Majorana mass matrix are diagonal, the Dirac mass matrix (Eq. 6) takes the form

$$m'_D = \frac{1}{\sqrt{2}} \begin{pmatrix} -b & b \\ a - (b+c) & a + (b+c) \\ -a - (b-c) & -a + (b-c) \end{pmatrix}. \quad (20)$$

From this point onward, we will take $Y_v = m'_D/v$, which is relevant for calculating CP asymmetry that arises during the decay of right-handed neutrinos in out-of-equilibrium way. Taking the best-fit values of the model parameters obtained in the previous section, we solve the following coupled Boltzmann equations describing the evolution, with respect to $z = M_1/T$, of RH neutrino density, N_{N_i} and lepton number density for three flavours, $N_{\alpha\alpha}$ corresponding to $\alpha = e, \mu, \tau$ [16,52].

$$\frac{dN_{N_i}}{dz} = -D_i (N_{N_i} - N_{N_i}^{\text{eq}}) \quad (21)$$

$$\frac{dN_{\alpha\alpha}}{dz} = -\sum_{i=1}^2 \varepsilon_{i\alpha} D_i (N_{N_i} - N_{N_i}^{\text{eq}}) - \frac{1}{4} \left\{ \sum_{i=1}^2 (rz)^2 D_i \mathcal{K}_2(rz) + W_{\Delta L=2} \right\} N_{\alpha\alpha}. \quad (22)$$

The above Eqs. (21) and (22) are flavor-diagonal and we have not shown the effects from the off-diagonal terms of the matrix of number density. The following equation gives the equilibrium number density of N_i ,

$$N_{N_i}^{\text{eq}} = \frac{45 g_N}{4\pi^4 g_*} z^2 \mathcal{K}_2(z), \quad (23)$$

with $\mathcal{K}_{1,2}(z)$ being the modified Bessel function. The parameter, D_i , sometimes called the decay parameter is defined as

$$D_i = \frac{z}{H(z=1)} \frac{\Gamma_{N_i}}{N_{N_i}^{\text{eq}}}, \quad (24)$$

² ε is assumed to be real.

Fig. 4 Variation of lepton number density for three flavours and baryon asymmetry parameter, η_B as a function of z

Fig. 5 The predicted values of $|m_{ee}|$ with respect to $\sum m_i$

which gives the total decay rate with respect to Hubble rate and $W_{\Delta L=2}$ denotes the washout coming from $\Delta L = 2$ scattering process.³

We take $M_1 = 10$ TeV and $d = (M_2 - M_1)/M_1 \simeq 10^{-8}$ in order to estimate the value of BAU. We made the calculations related to baryon asymmetry using the ULYSSES package [54]. Figure 4 shows the evolution of three flavoured lepton number density, $N_{\alpha\alpha}$ and baryon asymmetry, η_B as a function of $z = M_{N_1}/T$. The asymptotic value suggests that the obtained value of baryon asymmetry is $|\eta_B| \approx 6.3 \times 10^{-10}$.

The effective Majorana mass relevant for the neutrinoless double beta decay ($0\nu\beta\beta$) is the (1, 1) element of the effective neutrino mass matrix (Eq. 10), $|m_{ee}| = |(m_\nu)_{11}|$. Figure 5 shows the predicted values of $|m_{ee}|$ with respect to $\sum m_i$ for the allowed region of parameter space. We have also shown the sensitivity reach of some experiments such as

nEXO [55], KamLAND-Zen [56], NEXT [57], AMoRE-II [58]. It shows that $|m_{ee}|$ ranges from 2.6 to 3.6 meV and probing such small parameters by ($0\nu\beta\beta$) experiments would be quite difficult.

5 Conclusion

We have explored the S_4 symmetric flavour model in the context of minimal Type-I seesaw mechanism leading to TM_1 mixing pattern in the leptonic sector. In order to achieve TM_1 mixing, we extended the scalar sector further by adding a flavon ψ and its vev is chosen such that it follows the orthogonality conditions (i.e., $\langle\psi\rangle\cdot\langle\phi_l\rangle$ and $\langle\psi\rangle\cdot\langle\phi_\nu\rangle$). The resulting effective neutrino mass matrix predicts NH for masses of the neutrinos and $0.0576 \text{ eV} < \sum m_i < 0.0599 \text{ eV}$. An allowed region for the model parameters is derived numerically such that the predictions on the mixing angles, CP phase, and mass squared differences lie within the 3σ bound of current oscillation data. Among various points within the 6-dimensional parameter space, the best-fit value is obtained through chi-squared analysis. Using the obtained parameter space, we evaluated the effective Majorana neutrino mass, $|m_{ee}|$ and we found that it is relatively small, and difficult to probe at the $0\nu\beta\beta$ experiments.

Furthermore, we investigated baryogenesis via flavoured resonant leptogenesis. The right-handed neutrinos are degenerate at dimension 5 level, and hence a tiny splitting was generated by including higher dimension term. We have taken the splitting parameter, $d \simeq 10^{-8}$ and thus, obtained a non-zero, resonantly enhanced CP asymmetry from the out-of-equilibrium decay of right-handed Majorana neutrinos. The analysis of the evolution of particles and asymmetry is done by solving the Boltzmann equations. Here, the best-fit values for the model parameters is considered as inputs, and the Boltzmann equations are solved numerically to estimate baryon asymmetry. It was found that the model can explain the observed value of BAU with particular choice of RHN mass scale, $M_1 = 10$ TeV and mass splitting, $d \simeq 10^{-8}$.

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³ One may find explanation of such processes in [16].

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Funded by SCOAP³.

Appendix A: S_4 group

The irreducible representations of S_4 follow the following Kronecker products,

$$\begin{aligned} 1_1 \otimes \eta &= \eta, & 1_2 \otimes 1_2 &= 1, & 1_2 \otimes 2 &= 2, & 1_2 \otimes 3_1 &= 3_2, \\ 1_2 \otimes 3_2 &= 3_1 \oplus 2 = 1_1 \oplus 1_2 \oplus 2, & 2 \otimes 3_1 &= 2 \oplus 3_2 = \\ 3_1 \oplus 3_2, & 3_1 \otimes 3_1 &= 3_2 \oplus 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2, & 3_1 \otimes 3_2 &= \\ &= 1_2 \oplus 2 \oplus 3_1 \oplus 3_2. \end{aligned}$$

Now, we write the Clebsch–Gordan coefficients in particular basis [42].

For 1-dimensional representations:

$$\begin{aligned} 1_1 \otimes \eta &= \eta \otimes 1_1 = \eta \\ 1_2 \otimes 1_2 &= 1_1 \sim \alpha\beta \\ 1_2 \otimes 2 &= 2 \sim \begin{pmatrix} \alpha\beta_1 \\ -\alpha\beta_2 \end{pmatrix} \\ 1_2 \otimes 3_1 &= 3_2 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix} \\ 1_2 \otimes 3_2 &= 3_1 \sim \begin{pmatrix} \alpha\beta_1 \\ \alpha\beta_2 \\ \alpha\beta_3 \end{pmatrix}. \end{aligned}$$

For 2-dimensional representations:

$$2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \quad \text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1_2 \sim \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 \\ \alpha_1\beta_1 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_1 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \end{cases}$$

$$2 \otimes 3_2 = 3_1 \oplus 3_2 \quad \text{with} \quad \begin{cases} 3_1 \sim \begin{pmatrix} \alpha_1\beta_2 - \alpha_2\beta_3 \\ \alpha_1\beta_3 - \alpha_2\beta_1 \\ \alpha_1\beta_1 - \alpha_2\beta_2 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_1\beta_2 + \alpha_2\beta_3 \\ \alpha_1\beta_3 + \alpha_2\beta_1 \\ \alpha_1\beta_1 + \alpha_2\beta_2 \end{pmatrix}. \end{cases}$$

For 3-dimensional representations:

$$3_1 \otimes 3_1 = 3_2 \oplus 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2$$

$$\text{with} \quad \begin{cases} 1_1 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \end{cases}$$

$$3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2$$

$$\text{with} \quad \begin{cases} 1_2 \sim \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 2 \sim \begin{pmatrix} \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \\ -\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \end{pmatrix} \\ 3_1 \sim \begin{pmatrix} \alpha_2\beta_3 - \alpha_3\beta_2 \\ \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 \end{pmatrix} \\ 3_2 \sim \begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \end{cases}$$

where α_i and β_i denotes the elements of the first and second elements, respectively.

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