Double exotic state productions in pion and kaon induced reactions

Jing Liu^{1,a}, Dian-Yong Chen^{1,3,b}, Jun He^{2,3,c}

¹ School of Physics, Southeast University, Nanjing 210094, China

² Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing 210097, China

³ Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, China

Received: 3 August 2021 / Accepted: 21 October 2021 / Published online: 2 November 2021
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Abstract In the present work, we investigate the pion/kaon induced P_c/P_{cs} productions with Z_c state off proton target with an effective Lagrangian approach. Our estimations indicate that the cross sections depend on the model parameters and beam energies, in particular, the cross sections for $\pi p \rightarrow Z_c(3900)P_c(4312)$ and $\pi p \rightarrow Z_c(3900)P_c(4440)$ are similar and can reach up to 10 nb, while those for $\pi p \rightarrow Z_c(3900)P_c(4457)$ can be 30 nb. As for $Kp \rightarrow Z_{cs}(3985)P_c$, the cross sections are about 2.5 times smaller than those for $\pi p \rightarrow Z_c(3900)P_c$. The discussed processes in the present work, especially $\pi p \rightarrow Z_c(3900)P_c(4457)$ process, may be accessible by the high-energy pion and kaon beams in the facilities of J-PARC and COMPASS.

1 Introduction

The investigations of exotic states beyond the conventional hadrons are one of the most intriguing topics in hadron physics in the past two decades. Since the observations of $X(3872)$ in 2003 [1], a series of new hadron states have been observed (see Refs. [2–11] for recent reviews of experimental and theoretical status), which turns a new page of searching exotic states. Among these new hadron states, some of them are particular interesting, such as the series of Z_c/Z_{cs} [12–18] and P_c/P_{cs} states [19–22], which are good candidates of tetraquark [9, 23] and pentaquark states [24–26], respectively.

The tetraquark candidate $Z_c(3900)$ was first observed by the BESIII and Belle Collaboration in 2011 [12, 13] in the $J/\psi\pi^\pm$ invariant mass distributions of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ process. Later the CLEO-c confirmed the existence of $Z_c^\pm(3900)$ by using the data of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$

process at $\sqrt{s} = 4170$ MeV [27]. Moreover, the CLEO-c Collaboration also reported the neutral isospin partner of $Z_c^\pm(3900)$, i.e., $Z_c^0(3900)$ for the first time [27], which was further observed by the BESIII Collaboration [18]. Similar to $Z_c(3900)$, the BESIII Collaboration reported another tetraquark candidate $Z_c(4020)$ in the $h_c\pi$ invariant mass spectrum of $e^+e^- \rightarrow \pi^+\pi^-h_c$ [16] and $e^+e^- \rightarrow \pi^0\pi^0h_c$ [22] processes. Besides the hidden charm channel, these two tetraquark candidates have also been observed in the open charm channels [14, 17]. In the $D^*\bar{D}$ invariant mass spectrum of $e^+e^- \rightarrow (D^*\bar{D})^\pm\pi^\mp$ process, a state named $Z_c(3885)$ was reported by the BESIII Collaboration in 2013 [14], while in the $D^*\bar{D}^*$ invariant mass spectrum of $e^+e^- \rightarrow D^*\bar{D}^*\pi$, one state, $Z_c(4015)$ was also reported by the BESIII Collaboration in the same year [17]. As the strange partner of $Z_c(3900)$, $Z_{cs}(3985)$ was observed in the $D_s^-D^{*0} + D_s^{*-}D^0$ invariant mass spectrum of $e^+e^- \rightarrow K^+(D_s^-D^{*0} + D_s^{*-}D^0)$ process by BESIII Collaboration [28], which makes the charmonium-like tetraquark family abundant. More recently, the LHCb collaboration reported a structure, $Z_{cs}(4003)$, in the $J/\psi\phi$ invariant mass spectrum of $B^+ \rightarrow J/\psi\phi K^+$ [29] process, the mass of $Z_{cs}(4003)$ is consistent with that of $Z_{cs}(3985)$, but the width is much different. The measured resonance parameters of these tetraquark candidates are collected in Table 1. For comparison, we also present the thresholds near the observed masses of these tetraquark candidates.

As pentaquark candidates, $P_c(4380)$ and $P_c(4450)$ were first observed in the $J/\psi p$ invariant mass spectrum of $\Lambda_b \rightarrow K J/\psi p$ decay process by the LHCb Collaboration in 2015 [19]. The lower mass state is a broader one with a width $\Gamma = 205 \pm 18 \pm 86$ MeV, while the higher mass state is much narrower with a width to be $39 \pm 5 \pm 19$ MeV [19]. In 2019, the LHCb Collaboration reanalyzed the same process with a data sample to be nine times more than the one used in Ref. [19] and reported a new narrow pentaquark state $P_c(4312)$ [20], and the $P_c(4450)$ pentaquark structure was observed to consist of two narrow overlapping peaks,

^a e-mail: jingliu@seu.edu.cn

^b e-mail: chendy@seu.edu.cn (corresponding author)

^c e-mail: junhe@nynu.edu.cn

Table 1 A summary the experimental information of Z_c/Z_{cs} and P_c/P_{cs} states

State	Threshold	Mass (MeV)	Width (MeV)	Channel	Experiment
$Z_c^\pm(3900)$	$D^*\bar{D}$	$3899.0 \pm 3.6 \pm 4.9$ $3894.5 \pm 6.6 \pm 4.5$ 3901 ± 4	$46 \pm 10 \pm 20$ $63 \pm 24 \pm 26$ 58 ± 27	$e^+e^- \rightarrow \pi^+\pi^-J/\psi$	BESIII [12] Belle [13] CLEO-c [27]
$Z_c^0(3900)$		$3886 \pm 4 \pm 2$ $3894.8 \pm 2.3 \pm 3.2$	$37 \pm 4 \pm 8$ $29.6 \pm 8.2 \pm 8.2$	$e^+e^- \rightarrow \pi^0\pi^0J/\psi$	CLEO-c [27] BESIII [18]
$Z_c^\pm(3885)$		$3883.9 \pm 1.5 \pm 4.2$	$24.8 \pm 3.3 \pm 11.0$	$e^+e^- \rightarrow (D^*\bar{D})^\pm\pi^\mp$	BESIII [14]
$Z_c^\pm(4020)$	$D^*\bar{D}^*$	$4022.9 \pm 0.8 \pm 2.7$	$7.9 \pm 2.7 \pm 2.6$	$e^+e^- \rightarrow \pi^+\pi^-h_c$	BESIII [16]
$Z_c^0(4020)$		$4023.9 \pm 2.2 \pm 3.8$	–	$e^+e^- \rightarrow \pi^0\pi^0h_c$	BESIII [22]
$Z_c^\pm(4025)$		$4026.3 \pm 2.6 \pm 3.7$	$24.8 \pm 5.6 \pm 7.7$	$e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$	BESIII [17]
$Z_{cs}(3985)$	$D_s^-D^{*0}/D_s^{*-}D^0$	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$	$e^+e^- \rightarrow K^+(D_s^-D^{*0} + D_s^{*-}D^0)$	BESIII [28]
$Z_{cs}(4003)$		$4003 \pm 6^{+4}_{-14}$	$131 \pm 15 \pm 26$	$B^+ \rightarrow J/\psi\phi K^-$	LHCb [29]
$P_c^+(4380)$	$\Sigma_c^*\bar{D}$	$4380 \pm 8 \pm 9$	$205 \pm 18 \pm 86$	$\Lambda_b^0 \rightarrow J/\psi K^- p$	LHCb [19]
$P_c^+(4450)$	$\Sigma_c\bar{D}^*$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$		LHCb [19]
$P_c^+(4312)$	$\Sigma_c\bar{D}$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$		LHCb [20]
$P_c^+(4440)$	$\Sigma_c\bar{D}^*$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$		LHCb [20]
$P_c^+(4457)$	$\Sigma_c\bar{D}^*$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$		LHCb [20]
$P_{cs}^0(4459)$	$\Xi_c\bar{D}^*$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_b^- \rightarrow J/\psi K^- \Lambda$	LHCb [21]

$P_c(4440)$ and $P_c(4457)$ [20]. Later in 2020, the LHCb Collaboration reported an open-strange and hidden-charm pentaquark state, named $P_{cs}^0(4459)$ in $J/\psi\Lambda$ invariant mass spectrum of $\Xi_b^- \rightarrow K^- J/\psi\Lambda$ decay process [21]. The resonance parameters of these pentaquark states are also collected in Table 1.

These observations stimulated theorists great interests to investigate the nature and internal structure of these tetraquark and pentaquark candidates. The observed channels indicates that the most possible quark components of Z_c/Z_{cs} states should be $c\bar{c}q\bar{q}/c\bar{c}q\bar{s}$, while those of P_c/P_{cs} should be $c\bar{c}uud/c\bar{c}uds$, which means that these states can be naturally considered as multi-quark states. In the compact tetraquark scenario, the mass spectrum [30–41] and decay properties [36,42–45] of $Z_c^{(\prime)}/Z_{cs}$ have been investigated by various methods, such as QCD sum rule, constituent quark model. The estimations by QCD sum rule [46–48] and constituent quark model [39,49–52] supported the compact pentaquark interpretations of P_c/P_{cs} .

As presented in Table 1, all these observed tetraquark and pentaquark candidates are close to the thresholds of a pair of hadrons. Then, all these states can be considered as molecular states. In the $D^*\bar{D}^{(*)}/D_s\bar{D}^*$ molecular scenario, the mass spectrum of $Z_c^{(\prime)}/Z_{cs}$ were estimated by using QCD sum rule [37,47,48,53] and potential model with one-boson-exchange potential [54–57]. The decay [58,59] and production properties [60–66] were also investigated in an effective Lagrangian approach. As for P_c/P_{cs} , their properties, including mass spectrum [67–70], decay properties [60,71–73] and

production behaviors [74–76] have been investigated in the molecular scenarios. Besides the molecular interpretation, these tetraquark and pentaquark states were also considered as structures generated by some special kinematics mechanisms, such as cusp effect [77,78], initial single chiral partial emission mechanism [79,80], and triangle singularity [81].

As one of important aspects of evaluating the properties of the tetraquark and pentaquark candidates, the productions of these states have been widely investigated. For example, in Ref. [61], the production process $Y(4260) \rightarrow Z_c(3900)\pi$ was investigated in a molecular scenario, where $Y(4260)$ and $Z_c(3900)$ were considered as $D\bar{D}_1(2420) + h.c$ and $D\bar{D}^* + h.c$ molecular states, respectively. The productions of $Z_c^{(\prime)}$ states from B_c decays were predicted in Ref. [82]. Moreover, the productions of Z_c in the πp [63], γp [35] process have also been investigated in an effective Lagrangian approach. As for P_c/P_{cs} states, the observed production process has been investigated in Refs. [75,83] and the productions of these pentaquark states from πp scattering [76] and bottom baryon decay processes [81,84–86] were predicted. It worth to mention that in the high energy pion/kaon induced reaction, the contributions from s - and u -channels are much smaller than the t -channel contribution, which indicates that the s - and u -channels can usually be ignored. Considering the strong coupling between Z_c and $J/\psi\pi$, the process $\pi p \rightarrow Z_c p$ could occur via a J/ψ exchange while the J/ψ couples to the proton via vector meson dominance as indicated in Ref. [63]. The cross sections were predicted to be of order of 1 nb when considering $J/\psi\pi$ channel to be the

dominant one of Z_c . Actually, the exchanged J/ψ and proton target can strongly couples to the pentaquark candidate P_c states directly. Thus, one can construct a possible double exotic production process, i.e., $\pi p \rightarrow Z_c P_c$. In a same way, one can replace the pion beam by the kaon beam to construct a very similar double exotic production process $Kp \rightarrow Z_{cs} P_c$. In the present work, we adopt an effective Lagrangian approach to estimate the cross sections for these two kinds of double exotic production processes, which may be accessible by the high-energy pion and kaon beams in the facilities of J-PARC [87,88] and COMPASS [89].

This work is organized as follows. After the introduction, we present the effective Lagrangians used in the present work and the amplitudes in Sect. 2. The numerical results of the total cross section and the differential cross sections are present in Sect. 3 and we give a brief summary of this work in the last section.

2 Pion and kaon induced productions on a proton target

As indicated in Fig. 1, the double exotic production process $\pi p \rightarrow P_c Z_c$ occur via a J/ψ exchange. These diagrams can be estimated in an effective Lagrangian approach, where the relevant effective Lagrangians read [74,90],

$$\begin{aligned}\mathcal{L}_{\pi\psi Z_c} &= \frac{g_{\pi\psi Z_c}}{m_{Z_c}} (\partial_\mu \psi_\nu \partial^\mu \pi Z_c^\nu - \partial_\mu \psi_\nu \partial^\nu \pi Z_c^\mu), \\ \mathcal{L}_{K\psi Z_{cs}} &= \frac{g_{K\psi Z_{cs}}}{m_{Z_{cs}}} (\partial_\mu \psi_\nu \partial^\mu \pi Z_{cs}^\nu - \partial_\mu \psi_\nu \partial^\nu \pi Z_{cs}^\mu), \\ \mathcal{L}_{P_c\psi p}^{1/2-} &= g_{P_c\psi p}^{1/2-} \bar{p} \gamma_5 \gamma_\mu P_c \psi^\mu + H.c., \\ \mathcal{L}_{P_c\psi p}^{3/2-} &= \frac{-ig_{P_c\psi p}^{3/2-}}{2m_p} \bar{P}_c \mu \psi^{\mu\nu} \gamma_\nu p + H.c.,\end{aligned}\quad (1)$$

where $\psi^{\mu\nu} = \partial^\mu \psi^\nu - \partial^\nu \psi^\mu$. The concrete values of the relevant coupling constants will be discussed in the next section.

With the above effective Lagrangians, one can obtain the amplitudes corresponding to the diagrams in Fig. 1, which are,

$$\begin{aligned}\mathcal{M}^{1/2-} &= g_{P_c p \psi}^{1/2-} [\bar{u}(p_4, m_4) \gamma_\mu \gamma_5 u(p_2, m_2)] \\ &\quad \times g_{Z_c \pi \psi} [(-iq^\rho)(-p_1^\rho) g^{\nu\beta} - (-q^\rho)(-p_1^\nu) g^{\rho\beta}] \epsilon_3^\beta \\ &\quad \times \frac{-g^{\mu\nu} + q^\mu q^\nu / m_\psi^2}{q^2 - m_\psi^2} F(q^2, m_\psi), \\ \mathcal{M}^{3/2-} &= \frac{g_{P_c p \psi}^{3/2-}}{2m_2} [\bar{u}_\mu(p_4, m_4) (-iq^\mu) g^{\nu\eta} - (-iq^\nu) g^{\mu\eta} \gamma_\nu u(p_2, m_2)] \\ &\quad \times g_{Z_c \pi \psi} [(-iq^\rho)(-ip_1^\rho) g^{\sigma\xi} - (-iq^\rho) g^{\rho\xi} (-ip_1^\sigma)] \epsilon_3^\xi \\ &\quad \times \frac{-g^{\eta\sigma} + q^\eta q^\sigma / m_\psi^2}{q^2 - m_\psi^2} F(q^2, m_\psi),\end{aligned}\quad (2)$$

Fig. 1 A sketch diagram contributing to $\pi^- p \rightarrow Z_c(3900) P_c$ (diagram **a**) and $K^- p \rightarrow Z_{cs}(3985) P_c$ (diagram **b**), where P_c can be $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$, respectively

Table 2 The values of the coupling constants estimated by the partial widths of the corresponding process. The error of these coupling constants are resulted from the uncertainties of the measured widths

Coupling	$g_{P_c(4312)\psi p}^{1/2}$	$g_{P_c(4440)\psi p}^{1/2}$	$g_{P_c(4457)\psi p}^{3/2}$
Value	$0.11^{+0.03}_{-0.03}$	$0.14^{+0.03}_{-0.02}$	$0.08^{+0.04}_{-0.02}$
Coupling	$g_{Z_c\psi\pi}$	$g_{Z_{cs}\psi K}$	
Value	0.562 ± 0.026	$0.364^{+0.09}_{-0.08}$	

where $\mathcal{M}^{1/2-}$ and $\mathcal{M}^{3/2-}$ are the amplitudes of $\pi p \rightarrow Z_c P_c$ with the J^P quantum numbers of P_c state being $\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively. In particular, $\mathcal{M}^{1/2-}$ can be the amplitudes of $\pi p \rightarrow Z_c(3900) P_c(4312)$ and $\pi p \rightarrow Z_c(3900) P_c(4440)$, while the amplitude $\mathcal{M}^{3/2-}$ corresponds to the process $\pi p \rightarrow Z_c(3900) P_c(4457)$. In the amplitudes, a form factor, $F(q^2, m_\psi)$ is introduced to depict the internal structure and the off-shell effect of the exchanged J/ψ . In this work, we adopt the form factor in the monopole form, which is [91,92],

$$F(q^2, m_\psi) = \frac{m_\psi^2 - \Lambda^2}{q^2 - \Lambda^2}. \quad (3)$$

Here, Λ is the cutoff parameter and of order of 1 GeV, q and m_ψ are the momentum and the mass of the exchanged J/ψ meson, respectively.

With the above amplitudes, one can obtain the differential cross sections of $\pi p \rightarrow Z_c P_c$, which reads,

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \left(\frac{1}{2} |\mathcal{M}|^2 \right), \quad (4)$$

where $s = (p_1 + p_2)^2$ is the center mass energy and θ is the angle of the outgoing P_c states related to the pion beam, while the $\vec{p}_i$ and $\vec{p}_f$ are the momenta of initial pion beam and final P_c states in the center of mass frame, respectively.

3 Numerical results and discussion

3.1 Coupling constants

In the present work, an effective Lagrangian approach is adopt to estimate the cross sections for the double exotic states production process. The effective coupling constants in Eq. (1) can be determined by using the corresponding partial

widths. From the effective Lagrangians listed in Eq. (1), one can estimate the partial widths of $P_c(p_0) \rightarrow J/\psi(p_1)p(p_2)$ and $Z_c(p_0) \rightarrow J/\psi(p_1)\pi(p_2)$, which are,

$$\begin{aligned}\Gamma_{P_c \rightarrow J/\psi p}^{1/2-} &= g_{P_c J/\psi p}^2 \frac{|\vec{p}|}{8\pi m_0^2 m_1^2} [(m_2 + m_0)^2 - m_1^2] \\ &\quad \times [(m_2 - m_0)^2 + 2m_1^2], \\ \Gamma_{P_c \rightarrow J/\psi p}^{3/2-} &= g_{P_c J/\psi p}^2 \frac{|\vec{p}|}{192\pi m_0^4 m_2^2} [(3m_0^6 - m_0^4(m_1^2 + 5m_2^2) \\ &\quad + 12m_0^3 m_1^2 m_2 + m_0^2(m_2^4 - m_1^4) - (m_1^2 - m_2^2)^3], \\ \Gamma_{Z_c \rightarrow J/\psi \pi} &= g_{Z_c J/\psi \pi}^2 \frac{|\vec{p}|}{96\pi m_0^4} [2m_0^6 - m_0^4(3m_1^2 + 4m_2^2) \\ &\quad + 2m_0^2(3m_1^2 m_2^2 + m_2^4) + (m_1^3 - m_1 m_2^2)^2], \quad (5)\end{aligned}$$

respectively. Here $|\vec{p}| = \lambda(m_0^2, m_1^2, m_2^2)/2m_0$ and λ is the Källén function with the definition $\lambda(x, y, z) \equiv \sqrt{(x^2 + y^2 + z^2 - 2xy - 2xz - 2yz)}$.

As for $Z_c(3900)$, the partial width ratio of $D^* \bar{D}$ and $J/\psi \pi$ channels is measured to be [14],

$$\frac{\Gamma(Z_c \rightarrow \bar{D}^* D)}{\Gamma(Z_c \rightarrow J/\psi \pi)} = 6.2 \pm 1.1 \pm 2.7. \quad (6)$$

With the assumption that $Z_c(3900)$ dominantly decays into $D^* \bar{D}$ and $J/\psi \pi$ and considering the width of $Z_c(3900)$ [93], one can estimate the partial width of $Z_c(3900) \rightarrow J/\psi \pi$ channel, and then the coupling constant $g_{Z_c \psi \pi}$ can be evaluated by using Eq. (5). As for $Z_{cs}(3985)$, the experimental measurements are still not abundant. Here, we assume that the branching ratio of $Z_{cs}(3985) \rightarrow J/\psi K$ is the same as the one of $Z_c(3900) \rightarrow J/\psi \pi$ by considering the SU(3) symmetry [94].

As for P_c states, only the $J/\psi p$ decay mode have been observed experimentally at present, which indicates that the $J/\psi p$ should be one of important decay modes of P_c states. The branching ratios of $P_c \rightarrow J/\psi p$ were estimated from several percents to several tens percents depending on different models [75]. In the present work, the branching ratios of $P_c \rightarrow J/\psi p$ are assumed to be 10%. The coupling constants $g_{P_c \psi p}$ can be evaluated by using Eq. (5) with the measured widths of P_c states. All the estimated coupling

constants have been collected in Table 2. The error of the coupling constants are resulted from the uncertainties of the measured widths.

3.2 Cross sections for double exotic states production processes

With the above preparation, we can estimate the cross sections for the double exotic states production processes. The cross sections for $\pi p \rightarrow Z_c(3900)P_c$ are presented in Fig. 2, where diagrams (a), (b), and (c) correspond to $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ productions, respectively. The band in the diagrams are the uncertainties of the cross sections resulted from the relevant coupling constants. For comparison, we take the same model parameter Λ as the one used in the estimation of $\pi p \rightarrow Z_c(3900)p$ [63], and then we can compare the cross sections for double exotic production process with those for $\pi p \rightarrow Z_c(3900)p$ [63]. Our estimations indicate that the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ increase very fast near the thresholds and then become rather flat at higher pion momentum region. The cross sections can be greater than 10 nb when the momentum of pion is 70 GeV with $\Lambda = 1.0$ GeV. In particular, when including the effect of model parameter and the uncertainties resulted from the relevant coupling constants, we find the cross sections for $\pi p \rightarrow Z_c(3900)P_c(4312)$ is 1 ~ 13 nb when Λ varying from 1.0 to 2.0 GeV at $P_\pi = 70$ GeV. As for $\pi p \rightarrow Z_c(3900)P_c(4440)$ and $\pi p \rightarrow Z_c(3900)P_c(4457)$, the cross sections at $P_\pi = 70$ GeV are 2 ~ 20 nb and 6 ~ 72 nb, respectively. The cross sections for $Kp \rightarrow Z_{cs}(3985)P_c$ are presented in Fig. 3, where diagrams (a), (b), and (c) correspond to $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ productions, respectively. The behaviors of the cross sections are very similar to those of $\pi p \rightarrow Z_c(3900)P_c$, which show sharp increases near the threshold but the cross sections for $Kp \rightarrow Z_{cs}(3985)P_c$ are about 2.5 times smaller than those for $\pi p \rightarrow Z_c(3900)P_c$ due to the coupling constants $g_{Z_{cs} \psi K}$ is smaller than the one of $g_{Z_c \psi \pi}$.

As shown in Figs. 2 and 3, the energy and model parameter dependences of the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ for

Fig. 2 The cross sections for $\pi p \rightarrow Z_c(3900)P_c$ depending on the momentum of pion beam. Diagrams **a**, **b** and **c** correspond to cross sections for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ production, respectively. The bands indicate the uncertainties of the cross sections resulted from the relevant coupling constants

Fig. 3 The same as Fig. 2 but for $Kp \rightarrow Z_{cs}(3985)P_c$

Fig. 4 The ratios of cross sections depending on beam energy and model parameter Λ . Diagram **a** and **b** present the beam energy dependences for processes $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{cs}(3985)P_c$ with $\Lambda = 2.0$, respectively. Diagram **c** and **d** present the model parameter dependences for πp and Kp reactions with beam energy to be 55 GeV, respectively. The bands are the uncertainties resulted from the error of P_c/P_{cs} widths

$P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ are very similar, then one can define cross sections ratio as following,

$$\begin{aligned} R_{10} &= \frac{\sigma(\pi p \rightarrow Z_c(3900)P_c(4440))}{\sigma(\pi p \rightarrow Z_c(3900)P_c(4312))}, \\ R_{20} &= \frac{\sigma(\pi p \rightarrow Z_c(3900)P_c(4457))}{\sigma(\pi p \rightarrow Z_c(3900)P_c(4312))}, \end{aligned} \quad (7)$$

which are expected to be independent on the energy and the model parameter. In Fig. 4, the ratios of the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{cs}(3985)P_c$ depending

on the model parameter and beam energy are presented. The band in the diagrams indicate the uncertainties resulted from the error of P_c/P_{cs} widths, while the errors of Z_c/Z_{cs} widths are cancelled. As shown in Fig. 4c, d, the ratios are entirely independent on the model parameter Λ . By using the center values of the P_c width, our estimations indicate that the cross sections for $P_c(4457)$ productions from πp and Kp reactions are about 3 times larger than those of $P_c(4440)$ and $P_c(4312)$, while cross sections for $P_c(4440)$ and $P_c(4312)$ are almost the same.

Besides the cross sections, the differential cross sections depending on the scattering angle related to the outgoing P_c and the incident pion/kaon beam are also estimated. In Fig. 5, we present the differential cross sections for $\pi p \rightarrow Z_c(3900)P_c$ depending on $\cos \theta$, where diagrams (a), (b), and (c) correspond to $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ production, respectively. Here, the pion beam momentum is set to be 40, 50, and 60 GeV, respectively. Moreover, the cross sections uncertainties resulted from the relevant coupling constants will only affect the magnitude of differential cross sections, but have no influence on the relative distributions of $\cos \theta$, thus in Fig. 5, we only present the results estimated with the center values of the coupling constants. From the diagrams, one can find that the differential cross sections reach their maximum at the forward angle limit, i.e., $\cos \theta = 1$. For the case of $P_\pi = 40$ GeV, the beam energy is just a bit larger than the threshold of the reaction, thus the differential cross sections do not strongly depend on the scattering angle. With the P_π increasing, more produced pentaquark or tetraquark states are concentrated in the forward angle area. Moreover, the $\cos \theta$ dependence of the differential cross sections are very similar for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ due to the similarity of these double exotic production process.

The differential cross sections for $Kp \rightarrow Z_{cs}(3985)P_c$ are presented in Fig. 6, where the kaon beam momentum are also set to be 40, 50, and 60 GeV, respectively. Our estimations indicate that the scattering angle dependences of the cross sections are almost the same as those of $\pi p \rightarrow Z_c(3900)P_c$ due to the similarity between these two kinds of double exotic production processes.

Fig. 5 The same as Fig. 2 but for differential cross sections depending on $\cos\theta$

Fig. 6 The same as Fig. 5 but for $Kp \rightarrow Z_{cs}(3985)P_c$ processes, where diagrams **a**, **b** and **c** correspond to $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ production, respectively

4 Summary

As the typical tetraquark and pentaquark candidates, Z_c/Z_{cs} and P_c/P_{cs} have stimulated theorists' great interests, their properties have been investigated from various aspects, such as mass spectra, decay behaviors and production processes, which are expected to reveal the inner structure of these multiquark states.

In the present work, we evaluate the experimental potential of the double exotic production in the πp and Kp scattering processes. The cross sections for $\pi p \rightarrow Z_c(3900)P_c$ and $Kp \rightarrow Z_{cs}(3985)P_c$ are evaluated. Our estimations indicate that the cross sections depend on the model parameter and the beam energy. In particular, at $P_\pi = 70$ GeV the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ can reach up to 13, 20, and 72 nb for $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, at $\Lambda = 1.0$ GeV, respectively. Moreover, the beam energy and model parameter dependences of the cross sections are very similar for three P_c states, thus the ratios of these cross section are almost independent on the beam energy and model parameter, which can be tested by further experimental measurements. In addition, the beam energy and model parameter dependences of the cross sections for $Kp \rightarrow Z_{cs}(3985)P_c$ are almost the same as those of $\pi p \rightarrow Z_c(3900)P_c$. Besides the cross sections, the differential cross sections for the considered processes are also evaluated and our estimations indicate that more

produced pentaquark or tetraquark states are concentrated in the forward angle area.

At the end of the present work, we would like to discuss the experimental potential of these double exotic production processes by comparing the cross sections for $\pi p \rightarrow Z_c(3900)P_c$ with those for $\pi p \rightarrow Z_c(3900)p$ [63]. In Ref. [63], the cross section for $\pi p \rightarrow Z_c(3900)p$ at $P_\pi = 70$ GeV is of order of 100 nb with the assumption that $J/\psi\pi$ should be the dominant decay channel of $Z_c(3900)$. However, the experimental measurements indicated that the branching fraction of $Z_c(3900) \rightarrow J/\psi\pi$ should be of order 10% as indicated in Eq. (6), thus the cross sections for $\pi p \rightarrow Z_c(3900)p$ should be about 10 nb at $P_\pi = 70$ GeV, which is the same order as the cross sections for $\pi p \rightarrow Z_c(3900)P_c(4312)/P_c(4440)$ and about several times smaller than the one of $\pi p \rightarrow Z_c(3900)P_c(4457)$. Thus, if the experimental measurement in COMPASS or J-PARC could detect $Z_c(3900)$ in the $\pi p \rightarrow Z_c(3900)p$ process, the double exotic production process, especially $\pi p \rightarrow Z_cP_c(4457)$, should be more easily accessible in these facilities.

Acknowledgements This work is supported in part by the National Natural Science Foundation of China (NSFC) under Grant nos. 11775050 and 11675228.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This is a theoretical study and no external data are associated with this work.]

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Funded by SCOAP³.

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