The explanation of some exotic states in the $cs\bar{c}\bar{s}$ tetraquark system

Xuejie Liu^{1,a}, Hongxia Huang^{2,b}, Jialun Ping^{2,c}, Dianying Chen^{1,d}, Xinmei Zhu^{3,e}

¹ School of Physics, Southeast University, Nanjing 210094, People's Republic of China

² Department of Physics, Nanjing Normal University, Nanjing 210023, People's Republic of China

³ Department of Physics, Yangzhou University, Yangzhou 225009, People's Republic of China

Received: 31 March 2021 / Accepted: 14 October 2021 / Published online: 27 October 2021

© The Author(s) 2021

Abstract Inspired by the recent observation of $\chi_{c0}(3930)$, $X(4685)$ and $X(4630)$ by the LHCb Collaboration and some exotic resonances such as $X(4350)$, $X(4500)$, etc. by several experiment collaborations, the $cs\bar{c}\bar{s}$ tetraquark systems with $J^{PC} = 0^{++}$, 1^{++} , 1^{+-} and 2^{++} are systematically investigated in the framework of the quark delocalization color screening model(QDCSM). Two structures, the meson–meson and diquark–antidiquark structures, as well as the channel-coupling of all channels of these two configurations are considered in this work. The numerical results indicate that the molecular bound state $D_s^- D_s^+$ with $J^{PC} = 00^{++}$ can be supposed to explain the $\chi_{c0}(3930)$. Besides, by using the stabilization method, several resonant states are obtained. Among these states, $X(4350)$, $X(4500)$ and $X(4700)$ can be explained as the compact tetraquark states with $J^{PC} = 00^{++}$, and the $X(4274)$ is possible to be a candidate of the compact tetraquark state with $J^{PC} = 1^{++}$. Apart from that, the $J^{PC} = 0^{++}$ resonance state with energy range 4028–4033 MeV, the two $J^{PC} = 2^{++}$ resonance states with energy range of 4394–4448 MeV and 4526–4536 MeV are possible to be new exotic states, which are indeed worthy of attention. More experimental tests are expected to check the existence of all these possible resonance states.

1 Introduction

The underlying theory of strong interaction is quantum chromodynamics (QCD), which is nonperturbative at low energy. Therefore, it is very difficult to be solved by the hadron spectrum model independently. Although the traditional quark

model can well describe the hadron spectrum, which is classified as mesons, composed of $q\bar{q}$ and baryons, composed of qqq [1,2], the emergence of plenty of states or resonant structures in experiments can not fit the hadron spectrum predicted by the naive quark model in last two decades. Those states are called exotic states. A lot of effort has been made to understand the nature of these states, but there is still controversy about their nature.

Actually, most of these states are in the proximity of di-hadron thresholds. To name a few famous examples, the observed $X(3872)$ [3] and $Z_c(3900)$ [4–6] are around the $D\bar{D}^*$ threshold and the $Z_c(4020)$ [7,8] are near the $D^*\bar{D}^*$ threshold, etc. Recently, more representative states were searched for experimentally, such as $Z_c(3985)$ [9], which are near the $\bar{D}_s D^*$ and $\bar{D}_s^* D$ thresholds.

Inspired by the results of recent LHCb experiment about the $\chi_{c0}(3930)$ which is just below the $D_s^- D_s^+$ threshold [10,11], we have a great interest in studying the tetraquark system composed of $cs\bar{c}\bar{s}$. Besides, several experiment collaborations have discovered some relevant resonance states. For instance, in 2009, the CDF Collaboration reported a new state $X(4140)$ in the $J/\psi\phi$ invariant mass distribution [12] and this structure was then observed by other collaborations, such as LHCb, CMS, D0, and BABAR in next few years [13–16]; in 2010, a narrow resonance $X(4350)$ was found in the $\gamma\gamma \rightarrow J/\psi\phi$ process by the Belle Collaboration [17], which favored the spin parity $J^{PC} = 0^{++}$ or 2^{++} ; in 2017, the CDF Collaboration observed the resonance state $X(4274)$ in the $B^+ \rightarrow J/\psi\phi K^+$ decay with 3.1σ significance [18]; in 2016, in the $B^+ \rightarrow J/\psi\phi K^+$ decay, the LHCb Collaboration confirmed the existence of the $X(4140)$ and $X(4274)$ [19]. Their quantum numbers are measured to be $J^{PC} = 1^{++}$; in the same process, the Collaboration observed two higher resonances, $X(4500)$ and $X(4700)$ with $J^{PC} = 0^{++}$ [20]; in 2021, an improved full amplitude analysis of the $B^+ \rightarrow J/\psi\phi K^+$ decay is performed by using 6 times larger signal yield than the previous analysis, the LHCb

^a e-mail: 1830592517@qq.com

^b e-mail: hxhuang@njnu.edu.cn (corresponding author)

^c e-mail: jlping@njnu.edu.cn (corresponding author)

^d e-mail: chendy@seu.edu.cn

^e e-mail: 49106929@qq.com

Collaboration discovered two new hadron states, which are $X(4685)$ and $X(4630)$ [21].

To identify the internal structure of these resonance states, there are a large number of theoretical studies about the tetraquark state $c\bar{s}\bar{c}\bar{s}$. In Ref. [22], the $X(4140)$ resonance appeared as a cusp in the $J/\psi\phi$ channel and the $X(4274)$, $X(4500)$, $X(4700)$ were all defined as conventional charmonium states by using a nonrelativistic constituent quark model, respectively. In the relativized quark model, the resonance of $X(4140)$ was regarded as the $c\bar{s}\bar{c}\bar{s}$ tetraquark ground state, the $X(4700)$ was assigned as the 2S excited tetraquark state, $X(4500)$ was explained as the tetraquark composed of one 2S scalar diquark and one scalar antidiquark, and $X(4274)$ was a good candidate of the conventional χ_{c1} state [23]. It was argued that the cusp effects might explain the structure of the $X(4140)$, but failed to account for the $X(4274)$ [24]. In Ref. [25], the $X(4140)$ and $X(4270)$ with the tetraquark interpretation was consistent with $X(4350)$ while the interpretation of the $X(4500)$ and $X(4700)$ needed orbital or radial excitation in the simple color-magnetic interaction model. Maiani et al. proposed that the $X(4140)$ and the $X(4274)$ could be explained as the ground state 1S-multiplet of diquark–antidiquark tetraquarks while the $X(4500)$ and $X(4700)$ were radially excited 2S states [26]. Stancu argued that $X(4140)$ could be the strange partner of $X(3872)$ in a tetraquark interpretation within a simple quark model with the chromomagnetic interaction [27]. In Ref. [28], based on the diquark–antidiquark configuration within the framework of QCD sum rules, the $X(4500)$ and the $X(4700)$ were interpreted as the D-wave $c\bar{s}\bar{c}\bar{s}$ tetraquark states of $J^P = 0^+$. According to the calculation with multiquark color flux-tube model, Deng et al. pointed out that the $X(4500)$ and the $X(4700)$ were S-wave radial excited states $[cs][\bar{c}\bar{s}]$ [29]. Moreover, Yang explained the $X(4274)$ as the $c\bar{s}\bar{c}\bar{s}$ tetra-quark states with $J^{PC} = 1^{++}$, $X(4350)$ as a good candidate of the compact tetraquark state with $J^{PC} = 0^{++}$, and the $X(4700)$ as the 2S radial excited tetraquark state with $J^{PC} = 0^{++}$ [30]. In Refs. [31,32], the $X(4500)$ was observed as the first radial excited state of the axial-vector–diquark–axial-vector–antidiquark type scalar $c\bar{s}\bar{c}\bar{s}$ tetraquark state and the $X(4700)$ was assigned as the ground state vector–diquark–vector–antidiquark type scalar $c\bar{s}\bar{c}\bar{s}$ tetraquark state, but the results disfavored assigning the $X(4140)$ to the $J^{PC} = 1^{++}$ diquark–antidiquark type $c\bar{s}\bar{c}\bar{s}$ tetraquark state. A rescattering mechanism was used in Ref. [33] to understand the nature of $X(4140)$, $X(4350)$, $X(4500)$ and $X(4700)$, among which the $X(4140)$ and the $X(4700)$ could be simulated due to the $D_s^* D_s$ rescattering and the $\psi'\phi$ rescattering. However, this mechanism failed to generate the $X(4274)$ and $X(4500)$, which lead to the proposal that they might be the genuine resonances. In Ref. [34], the masses of the excited heavy tetraquarks with hidden charm were calculated within the relativistic diquark–

antidiquark picture, and the results showed that $X(3872)$, $Y(4260)$, $Y(4360)$, $Z(4248)$, $Z(4433)$ and $Y(4660)$ could be tetraquark states with hidden charm.

In this work, to see whether these exotic resonances can be described by $c\bar{s}\bar{c}\bar{s}$ tetraquark systems with $J^{PC} = 0^{++}$, 1^{++} , 1^{+-} and 2^{++} , we systematically study the properties of these exotic resonances by using the quark delocalization color screening model (QDCSM) [35], which was proposed particularly to study the similarities between nuclear and molecular forces. According to the characteristics of QDCSM, it can give a good description of the properties of the deuteron, nucleon–nucleon, and hyperon–nucleon interactions [36,37]. In the present calculation, two configurations, the meson–meson ($q\bar{q} - q\bar{q}$) and the diquark–antidiquark ($qq - \bar{q}\bar{q}$), are taken into account. Besides, to be more convincing, the channel coupling effect of $c\bar{s}\bar{c}\bar{s}$ tetraquark systems is also included.

This work is organized as follows. In Sect. 2, we present a review of the quark delocalization color screening model and the wave functions of the total system in the present work. The numerical results and a discussion for the tetraquarks are given in Sect. 3. Finally, the last section is devoted to a brief summary.

2 The quark delocalization color screening model (QDCSM) and wave functions

2.1 The quark delocalization color screening model (QDCSM)

The quark delocalization color screening model (QDCSM) is an extension of the native quark cluster model [38–41] and was developed with aim of addressing multiquark systems. The detail of QDCSM can be found in the Refs. [35,42,43]. Here, the general form of the four body complex Hamiltonian is given by

$$H = \sum_{i=1}^4 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^4 V(r_{ij}), \quad (1)$$

where the center-of-mass kinetic energy, T_{CM} is subtracted without losing generality since we mainly focus on the internal relative motions of the multiquark system. The interplay is of two body potential which includes color-confining, V_{CON} , one-gluon exchange, V_{OGE} , and Goldstone-boson exchange, V_χ , respectively,

$$V(r_{ij}) = V_{CON}(r_{ij}) + V_{OGE}(r_{ij}) + V_\chi(r_{ij}) \quad (2)$$

In this work, we focus on the low-lying positive parity $c\bar{s}\bar{c}\bar{s}$ tetraquark states of s -wave, and the spin-orbit and tensor

interactions are not included. The potential $V_{OGE}(r_{ij})$ can be written as

$$V_{OGE}(r_{ij}) = \frac{1}{4} \alpha_s^{q_i q_j} \lambda_i^c \cdot \lambda_j^c \times \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(r_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j} \right) \right] \quad (3)$$

where m_i and σ are the quark mass and the Pauli matrices, respectively. The λ^c is SU(3) color matrix. The $\alpha_s^{q_i q_j}$ is the quark-gluon coupling constant, which offers a consistent description of mesons from light to heavy-quark sector. It is associated with the quark flavor and determined by the mass difference of two mesons with the spin $S = 0$ and $S = 1$, respectively. For example, from the model Hamiltonian, the mass difference between D_s and D_s^* is determined by the color-magnetic interaction in Eq. (3), so the parameter α_s^{sc} is determined by fitting the mass difference between D_s and D_s^* .

$V_{CON}(r_{ij})$ is the confining interaction, which can be expressed as

$$V_{CON}(r_{ij}) = -a_c \lambda_i^c \cdot \lambda_j^c [f(r_{ij}) + V_{0_{q_i q_j}}], \quad (4)$$

where the $V_{0_{q_i q_j}}$ is determined by the mass shift of the absolute and experimental value of each meson, and it is also related to the quark flavor. $f(r_{ij})$ can be written as

$$f(r_{ij}) = \begin{cases} r_{ij}^2 & \text{if } i, j \text{ occur in the same cluster} \\ \frac{1-e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}} & \text{if } i, j \text{ occur in different cluster} \end{cases} \quad (5)$$

where the color screening parameter μ_{ij} is determined by fitting the deuteron properties, NN and NY scattering phase shifts, with $\mu_{qq} = 0.45$, $\mu_{qs} = 0.19$ and $\mu_{ss} = 0.08$, satisfying the relation $\mu_{qs}^2 = \mu_{qq}\mu_{ss}$, where q represents u or d quark. When extending to the heavy-quark case, we found that the dependence of the parameter μ_{cc} is not very significant in the calculation of the P_c states [44] by taking it from 0.0001 to 0.01. So here we take $\mu_{cc} = 0.01$. Then μ_{sc} and μ_{uc} are obtained by the relation $\mu^2 = \mu_{ss}\mu_{cc}$ and $\mu^2 = \mu_{uu}\mu_{cc}$, respectively.

The Goldstone-boson exchange interactions between light quarks appear because the dynamical breaking of chiral symmetry. For the $cs\bar{c}s$ system, the π and K exchange interactions do not appear because there is no up or down quarks herein. Only the following η exchange term works between the $s\bar{s}$ pair.

$$V_\chi(r_{ij}) = v_{ij}^\eta \left[(\lambda_i^8 \cdot \lambda_j^8) \cos \theta_P - (\lambda_i^0 \cdot \lambda_j^0) \sin \theta_P \right] \quad (6)$$

with

$$v_{ij}^\eta = \frac{g_{ch}^2}{4\pi} \frac{m_\chi^2}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} m_\chi \times \left\{ (\sigma_i \cdot \sigma_j) \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] \right\} \quad (7)$$

where $Y(x) = e^{-x}/x$ is the standard Yukawa function. The λ^a is the SU(3) flavor Gell–Mann matrix. The mass of the η meson is taken from the experimental value [45]. Finally, the chair coupling constant, g_{ch} , is determined from the πNN coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5} \right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2} \quad (8)$$

which assumes that flavor SU(3) is an exact symmetry, only broken by the different mass of the strange quark.

In this work, the model parameters, which are shown in Table 1, are fixed by masses of the corresponding mesons shown in Table 2 to keep the thresholds of tetraquark system correct. However, the mass of the mesons ω and η , which are predictions, are not reproduced accurately by the models. Then, we use these model parameters to investigate the double heavy $c\bar{c}s\bar{s}$ systems. Besides, in order to test the dependence of the results on the model parameters, three difference sets of parameters (QDCSM1, QDCSM2 and QDCSM3) are used in the calculations, the values of which are listed in Table 1.

In QDCSM, the quark delocalization is realized by specifying the single particle orbital wave function of QDCSM as a linear combination of left and right Gaussian, the single particle orbital wave functions used in the ordinary quark cluster model,

$$\psi_r(\mathbf{r}, s_i, \epsilon) = (\phi_R(\mathbf{r}, s_i) + \epsilon \phi_L(\mathbf{r}, s_i))/N(\epsilon), \quad (9)$$

$$\psi_l(\mathbf{r}, s_i, \epsilon) = (\phi_L(\mathbf{r}, s_i) + \epsilon \phi_R(\mathbf{r}, s_i))/N(\epsilon), \quad (10)$$

$$N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-s_i^2/4b^2}}, \quad (11)$$

$$\phi_R(\mathbf{r}, s_i) = \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{1}{2b^2}(\mathbf{r} - \frac{1}{2}s_i)^2}, \quad (12)$$

$$\phi_L(\mathbf{r}, s_i) = \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{1}{2b^2}(\mathbf{r} + \frac{1}{2}s_i)^2}, \quad (13)$$

The $s_i, i = 1, 2, \dots, n$, are the generating coordinates, which are introduced to expand the relative motion wave function [46–48]. The mixing parameter $\epsilon(s_i)$ is not an adjusted one but determined variationally by the dynamics of the multi-quark system itself. This assumption allows the multi-quark system to choose its favorable configuration in the interacting process. It has been used to explain the cross-over transition between the hadron phase and the quark-gluon plasma phase [49].

Table 1 Model parameters

	QDCSM1	QDCSM2	QDCSM3
Quark masses			
m_u (MeV)	313	313	313
m_s (MeV)	536	536	536
m_c (MeV)	1728	1728	1728
Confinement			
b (fm)	0.29	0.3	0.315
a_c (MeV fm ⁻²)	101	101	101
$V_{0_{uu}}$ (MeV)	-2.3928	-2.2543	-2.0689
$V_{0_{us}}$ (MeV)	-1.9137	-1.7984	-1.6429
$V_{0_{uc}}$ (MeV)	-1.4175	-1.3231	-1.2052
$V_{0_{ss}}$ (MeV)	-1.3448	-1.2826	-1.2745
$V_{0_{sc}}$ (MeV)	-0.7642	-0.6739	-0.5452
$V_{0_{cc}}$ (MeV)	0.6063	0.7555	0.9829
OGE			
α_s^{uu}	0.2292	0.2567	0.3019
α_s^{us}	0.2655	0.2970	0.3484
α_s^{uc}	0.3437	0.3805	0.4405
α_s^{ss}	0.3856	0.3604	0.3360
α_s^{sc}	0.5969	0.6608	0.7649
α_s^{cc}	1.5101	1.6717	1.9353

Table 2 The masses (in MeV) of the ground mesons. Experimental values are taken from the Particle Data Group (PDG) [45]

	K	K^*	π	ρ	ω	η	η'
Exp.	495	892	139	770	782	548	958
QDCSM1	495	892	139	770	585	535	958
QDCSM2	495	892	139	770	598	538	958
QDCSM3	495	892	139	770	615	527	958
	ϕ	D_s	D_s^*	η_c	J/ψ	D	D^*
Exp.	1020	1968	2112	2983	3096	1865	2007
QDCSM1	1020	1968	2112	2983	3096	1865	2007
QDCSM2	1020	1968	2112	2983	3096	1865	2007
QDCSM3	1020	1968	2112	2983	3096	1865	2007

2.2 The wave function

In this work, we focus on the double heavy $c\bar{c}s\bar{s}$ system by using the resonance group method [50]. Figure 1 shows two kinds of configurations for this system, which are the meson-meson structures shown in Fig. 1a and b, and the diquark-antidiquark structure shown in Fig. 1c. For the purpose of solving a manageable 4-body problem, currently, the system calculation considers only these two structures. But an economic way is used to combine these two configurations to see the effect of the multi-channel coupling. Four fundamental degrees of freedom, which are color, spin, flavor, and orbit

are generally accepted by the QCD theory at the quark level. The multiquark system's wave function is an internal product of the color, spin, flavor, and orbit terms.

2.2.1 The color wave function

Plenty of color structures in multiquark systems will be available with respect those of conventional hadrons such as $q\bar{q}$ mesons and qqq baryons. In this section, the goal is to construct the colorless wave function of a 4-quark system. For the meson-meson configurations, the color wave functions of a $q\bar{q}$ cluster are listed.

Fig. 1 Two types of configurations in $c\bar{c}s\bar{s}$ tetraquarks. **a** and **b** is the meson–meson configuration, **c** is diquark–antidiquark

$$\begin{aligned}
 C_{[111]}^1 &= \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b}) \\
 C_{[21]}^2 &= r\bar{b}, C_{[21]}^3 = -r\bar{g} \\
 C_{[21]}^4 &= g\bar{b}, C_{[21]}^5 = -g\bar{g} \\
 C_{[21]}^6 &= g\bar{r}, C_{[21]}^7 = b\bar{r} \\
 C_{[21]}^8 &= \sqrt{\frac{1}{2}}(r\bar{r} - g\bar{g}) \\
 C_{[21]}^9 &= \sqrt{\frac{1}{6}}(-r\bar{r} - g\bar{g} + 2b\bar{b})
 \end{aligned} \quad (14)$$

where the subscript [111] and [21] stand for color-singlet ($\mathbf{1}_c$) and color-octet ($\mathbf{8}_c$), respectively. So, the $SU(3)$ color wave functions of color-singlet (two color-singlet clusters, $\mathbf{1}_c \otimes \mathbf{1}_c$) and hidden-color (two color-octet clusters, $\mathbf{8}_c \otimes \mathbf{8}_c$) channels are given respectively.

$$\chi_1^c = C_{[111]}^1 C_{[111]}^1 \quad (15)$$

$$\begin{aligned}
 \chi_2^c &= \sqrt{\frac{1}{8}}(C_{[21]}^2 C_{[21]}^7 - C_{[21]}^4 C_{[21]}^5 - C_{[21]}^3 C_{[21]}^6 \\
 &\quad + C_{[21]}^8 C_{[21]}^8 - C_{[21]}^6 C_{[21]}^3 + C_{[21]}^9 C_{[21]}^9 \\
 &\quad - C_{[21]}^5 C_{[21]}^4 + C_{[21]}^7 C_{[21]}^2)
 \end{aligned} \quad (16)$$

For the diquark–antidiquark structure, the color wave functions of the diquark clusters is given,

$$\begin{aligned}
 C_{[2]}^1 &= rr, C_{[2]}^2 = \sqrt{\frac{1}{2}}(rg + gr) \\
 C_{[2]}^3 &= gg, C_{[2]}^4 = \sqrt{\frac{1}{2}}(rb + br) \\
 C_{[2]}^5 &= \sqrt{\frac{1}{2}}(gb + bg), C_{[2]}^6 = bb
 \end{aligned}$$

$$\begin{aligned}
 C_{[11]}^7 &= \sqrt{\frac{1}{2}}(rg - gr), C_{[11]}^8 = \sqrt{\frac{1}{2}}(rb - br) \\
 C_{[11]}^9 &= \sqrt{\frac{1}{2}}(gb - bg)
 \end{aligned} \quad (17)$$

And the color wave functions of the antidiquark clusters can be written as:

$$\begin{aligned}
 C_{[22]}^1 &= r\bar{r}, C_{[22]}^2 = -\sqrt{\frac{1}{2}}(\bar{r}\bar{g} + \bar{g}\bar{r}) \\
 C_{[22]}^3 &= g\bar{g}, C_{[22]}^4 = \sqrt{\frac{1}{2}}(\bar{r}\bar{b} + \bar{b}\bar{r}) \\
 C_{[22]}^5 &= -\sqrt{\frac{1}{2}}(\bar{g}\bar{b} + \bar{b}\bar{g}), C_{[22]}^6 = \bar{b}\bar{b} \\
 C_{[211]}^7 &= \sqrt{\frac{1}{2}}(\bar{r}\bar{g} - \bar{g}\bar{r}), C_{[211]}^8 = -\sqrt{\frac{1}{2}}(\bar{r}\bar{b} - \bar{b}\bar{r}) \\
 C_{[211]}^9 &= \sqrt{\frac{1}{2}}(\bar{g}\bar{b} - \bar{b}\bar{g})
 \end{aligned} \quad (18)$$

The color wave functions of the diquark–antidiquark structure shown in Fig. 1c are χ_3^c (color sextet–antisextet clusters, $\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$) and χ_4^c (color-triplet–antitriplet cluster, $\mathbf{3}_c \otimes \bar{\mathbf{3}}_c$).

$$\begin{aligned}
 \chi_3^c &= \sqrt{\frac{1}{6}}(C_{[2]}^1 C_{[22]}^1 - C_{[2]}^2 C_{[22]}^2 + C_{[2]}^3 C_{[22]}^3 \\
 &\quad + C_{[2]}^4 C_{[22]}^4 - C_{[2]}^5 C_{[22]}^5 + C_{[2]}^6 C_{[22]}^6)
 \end{aligned} \quad (19)$$

$$\chi_4^c = \sqrt{\frac{1}{3}}(C_{[11]}^7 C_{[211]}^7 - C_{[11]}^8 C_{[211]}^8 + C_{[11]}^9 C_{[211]}^9) \quad (20)$$

2.2.2 The flavor wave function

For the flavor degree of freedom, since the quark content of the tetraquark systems are two heavy quarks and two strange quarks, the isoscalar sector is $I = 0$. The flavor wave functions denoted as F_I^i , with the superscript I referring to isoscalar, can be written as

$$\begin{aligned}
 F_0^1 &= c\bar{c}s\bar{s} \\
 F_0^2 &= c\bar{s}s\bar{c} \\
 F_0^3 &= cs\bar{c}\bar{s}
 \end{aligned} \quad (21)$$

2.2.3 The spin wave function

For the spin, the total spin S of tetraquark states ranges from 0 to 2. All of them are considered. The wave functions of two body clusters are

$$\begin{aligned}
 \chi_{11} &= \alpha\alpha, \\
 \chi_{10} &= \sqrt{\frac{1}{2}}(\alpha\beta + \beta\alpha) \\
 \chi_{1-1} &= \beta\beta \\
 \chi_{00} &= \sqrt{\frac{1}{2}}(\alpha\beta - \beta\alpha)
 \end{aligned} \quad (22)$$

Then, the total spin wave functions S_s^i are obtained by considering the coupling of two subcluster spin wave functions with SU(2) algebra, and the total spin wave functions of four-quark states can be read as

$$\begin{aligned} S_0^1 &= \chi_{00}\chi_{00} \\ S_0^2 &= \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}) \\ S_1^3 &= \sqrt{\frac{1}{2}}(\chi_{00}\chi_{11} + \chi_{11}\chi_{00}) \\ S_1^4 &= \sqrt{\frac{1}{2}}(\chi_{00}\chi_{11} - \chi_{11}\chi_{00}) \\ S_1^5 &= \chi_{00}\chi_{11} \\ S_1^6 &= \chi_{11}\chi_{00} \\ S_1^7 &= \sqrt{\frac{1}{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11}) \\ S_2^8 &= \chi_{11}\chi_{11} \end{aligned} \quad (23)$$

2.2.4 The orbital wave function

Among the different methods to solve the Schrödinger-like 4-body bound state equation, we use the resonating group method (RGM) [50], which is one of the most extend tools to solve eigenvalue problems and scattering problems. The total orbital wave functions can be constructed by coupling the orbital wave function of two internal cluster and the relative motion wave function between two clusters.

$$\psi^L = \psi_1(R_1)\psi_2(R_2)\chi_L(R) \quad (24)$$

where R_1 and R_2 are the internal coordinates for the cluster 1 and cluster 2. $R = R_1 - R_2$ is the relative coordinate between the two clusters 1 and 2. The ψ_1 and ψ_2 are the internal cluster orbital functions of the clusters 1 and clusters 2, and $\chi_L(R)$ is the relative motion wave function between two clusters, which is expanded by the gaussian bases

$$\begin{aligned} \chi_L(R) &= \sqrt{\frac{1}{4\pi}} \left(\frac{3}{2\pi b^2} \right) \sum_{i=1}^n C_i \\ &\times \int \exp \left[-\frac{3}{4b^2} (R - s_i)^2 \right] Y_{LM}(\hat{s}_i) d\hat{s}_i \end{aligned} \quad (25)$$

where n is the number of gaussian bases, which is determined by the stability of the results.

Finally, to fulfill the Pauli principle, the complete wave function is written as

$$\psi = A[\psi^L S_s^j]_{JM_J} F_I^i \chi_k^c \quad (26)$$

where A is the antisymmetry operator of double-heavy tetraquarks. In this work, the operator A is defined as $A = 1$ due to the absence of any homogeneous quarks in the $c\bar{c}s\bar{s}$ system.

3 Results and discussions

The low-lying S -wave states of $c\bar{c}s\bar{s}$ tetraquark are systematically investigated herein. The parity for $c\bar{c}s\bar{s}$ tetraquark is positive under our assumption that the total orbital angular momenta L is 0. Accordingly, the total angular momenta, J , can take values of 0, 1, 2. The value of isospin can only be 0 for the $c\bar{c}s\bar{s}$ tetraquark system. Two structures of $c\bar{c}s\bar{s}$ tetraquark, meson-meson and diquark-antidiquark structures, are investigated. In each structure, all possible states are considered, which are listed in Table 3. The F_I^i ; S_s^j ; χ_k^c shows the necessary basis combination in flavor (F_I^i), spin (S_s^j) and color (χ_k^c) degrees of freedom. For meson-meson structure, only the color singlet-singlet (1×1) is taken into account because of the effect of hidden color channel coupling is considered in QDCSM [42,43].

The energy of $c\bar{c}s\bar{s}$ tetraquark system with $J^{PC} = 0^{++}$, 1^{++} , 1^{+-} and 2^{++} for both the meson-meson and diquark-antidiquark structures, as well as the channel coupling of these two structures are listed in Tables 4, 5, 6 and 7, respectively. In those tables, the first column represents the index of every possible channel; the second column lists the corresponding physical channels; the third column indicates the theoretical threshold of every channel; the fourth column (E_{sc}) is the energy of every single channel; the fifth column (E_{cc}) shows the energy by channel coupling of one certain configuration; the last column (E_{mix}) is the lowest energy of the system by coupling all channels of both two configurations. In this work, although three different sets of parameters are used to calculate the $c\bar{c}s\bar{s}$ tetraquark system under QDCSM, since the results are similar as seen in Tables 4, 5, 6 and 7, here we mainly describe the results of QDCSM2 in detail to save space.

The $J^{PC} = 0^{++}$ system: Four possible meson-meson channels, $\eta' \eta_c$, $D_s^+ D_s^-$, $J/\psi \phi$, $D_s^{*+} D_s^{*-}$, and two diquark-antidiquark channels, $((cs)(\bar{c}\bar{s}))(3 \times \bar{3})$ and $((cs)(\bar{c}\bar{s}))(6 \times \bar{6})$, are studied in QDCSM2. All results with the $J^{PC} = 0^{++}$ are given in Table 4. We can see that the energy of every single channel for the meson-meson structure is higher than the corresponding theoretical threshold, which indicates that there is no any bound state. For the diquark-antidiquark configuration, all the masses are higher than the lowest energy of $D_s^+ D_s^-$ in our model calculation, and the minimum energy is 4292 MeV. Then, we perform the channel-coupling calculation on both the meson-meson and diquark-antidiquark structure, respectively. The energy of the meson-meson structure is 3938 MeV, almost the same as the lowest single channel $D_s^+ D_s^-$, which indicates that the effect of the channel coupling is quite weak and no bound state is found for the meson-meson structure. For the diquark-antidiquark structure, although the coupling is rather stronger than the

Table 3 All possible channels for all quantum numbers

$J^{PC} = 0^{++}$			$J^{PC} = 1^{++}$			$J^{PC} = 1^{+-}$			$J^{PC} = 2^{++}$		
Index	$F_L^i; S_s^j; \chi_k^c$ [i;j;k]	Channels	Index	$F_L^i; S_s^j; \chi_k^c$ [i;j;k]	Channels	Index	$F_L^i; S_s^j; \chi_k^c$ [i;j;k]	Channels	Index	$F_L^i; S_s^j; \chi_k^c$ [i;j;k]	Channels
1	[1,1,1]	$\eta_c \eta'$	1	[1,7,1]	$J/\psi \phi$	1	[1,5,1]	$\eta_c \phi$	1	[1,8,1]	$J/\psi \phi$
2	[2,1,1]	$D_s^+ D_s^-$	2	[2,4,1]	$D_s^+ D_s^{*-}$	2	[1,6,1]	$J/\psi \eta'$	2	[2,8,1]	$D_s^{*+} D_s^{*-}$
3	[1,2,1]	$J/\psi \phi$	3	[3,3,3]	$(cs)(\bar{c}\bar{s})$	3	[2,3,1]	$D_s^+ D_s^{*-}$	3	[3,8,3]	$(cs)(\bar{c}\bar{s})$
4	[2,2,1]	$D_s^{*+} D_s^{*-}$	4	[3,3,4]	$(cs)(\bar{c}\bar{s})$	4	[2,7,1]	$D_s^{*+} D_s^{*-}$	4	[3,8,4]	$(cs)(\bar{c}\bar{s})$
5	[3,1,3]	$(cs)(\bar{c}\bar{s})$				5	[3,4,3]	$(cs)(\bar{c}\bar{s})$			
6	[3,1,4]	$(cs)(\bar{c}\bar{s})$				6	[3,4,3]	$(cs)(\bar{c}\bar{s})$			
7	[3,2,3]	$(cs)(\bar{c}\bar{s})$				7	[3,7,3]	$(cs)(\bar{c}\bar{s})$			
8	[3,2,4]	$(cs)(\bar{c}\bar{s})$				8	[3,7,4]	$(cs)(\bar{c}\bar{s})$			

Table 4 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 0^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$\eta' \eta_c$	3942	3944	3938	3930	3944	3938	3930	3944	3938	3930
2	$D_s^+ D_s^-$	3936	3938			3938			3938		
3	$J/\psi \phi$	4117	4119			4119			4119		
4	$D_s^{*+} D_s^{*-}$	4224	4226			4226			4226		
5	$(cs)(\bar{c}\bar{s})$		4354	4168		4350	4163		4341	4138	
6	$(cs)(\bar{c}\bar{s})$		4553			4535			4499		
7	$(cs)(\bar{c}\bar{s})$		4469			4464			4453		
8	$(cs)(\bar{c}\bar{s})$		4310			4292			4252		

Table 5 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 1^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$J/\psi \phi$	4117	4119	4082	4082	4119	4082	4082	4119	4082	4082
2	$D_s^+ D_s^{*-}$	4080	4082			4082			4082		
7	$(cs)(\bar{c}\bar{s})$		4341	4311		4360	4328		4393	4365	
8	$(cs)(\bar{c}\bar{s})$		4377			4397			4426		

Table 6 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 1^{+-}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$\eta_c \phi$	4004	4006	4006	4006	4006	4006	4006	4006	4006	4006
2	$J/\psi \eta'$	4055	4057			4057			4057		
3	$D_s^+ D_s^{*-}$	4080	4082			4082			4082		
4	$D_s^{*+} D_s^{*-}$	4224	4226			4226			4226		
5	$(cs)(\bar{c}\bar{s})$		4434	4306		4431	4313		4424	4302	
6	$(cs)(\bar{c}\bar{s})$		4517			4504			4474		
7	$(cs)(\bar{c}\bar{s})$		4484			4480			4469		
8	$(cs)(\bar{c}\bar{s})$		4397			4379			4341		

Table 7 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 2^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$J/\psi\phi$	4117	4122	4122	4119	4122	4121	4119	4122	4121	4119
2	$D_s^{*+}D_s^{*-}$	4224	4228			4229			4229		
3	$(cs)(\bar{c}\bar{s})$		4404	4399		4427	4421		4453	4444	
4	$(cs)(\bar{c}\bar{s})$		4409			4431			4454		

meson–meson structure, the energy is still higher than the theoretical threshold of the lowest channel $D_s^+D_s^-$.

However, the lowest energy of 3930 MeV is obtained by coupling all channels of two structures, which is 6 MeV lower than the threshold of the lowest channel $D_s^+D_s^-$, which means that there is a bound state for the $J^{PC} = 0^{++}$ $c\bar{c}s\bar{s}$ tetraquark system with mass of 3930 MeV. In addition, to explore the structure of this bound state, we calculate the proportion of each channel, and find that the percentage of the $D_s^+D_s^-$ state is about 85%, while the percentages of the other seven channels are much smaller. This means that the largest contribution to forming this bound state comes from the $D_s^+D_s^-$ channel, so this bound state tends to be a molecular state. Moreover, this value is in proximity to the $\chi_{c0}(3930)$ observed by the LHCb collaboration. So we can explain the $\chi_{c0}(3930)$ as a molecular state $D_s^+D_s^-$ in present quark model calculation. This result is consistent with the work of Ref. [51], in which the lattice QCD calculation with $m_\pi \simeq 280$ MeV indicated the existence of the scalar $D_s^+D_s^-$ bound state, which might correspond to the $\chi_{c0}(3930)$ observed by the LHCb collaboration [10, 11]. Also, in Ref. [52], two pole positions of $D_s^+D_s^-$ system were obtained by solving the Bethe–Salpeter equation, which explained the properties of new exotic resonance $\chi_{c0}(3930)$.

The $J^{PC} = 1^{++}$ system: from Table 3, there are two meson–meson channels and two diquark–antidiquark channels. Table 5 lists the calculated masses of these channels and also their coupling results. The energy range of every single-channel of the meson–meson structure is about 4.0–4.2 GeV, and the mass of the diquark–antidiquark channel is around 4.4 GeV. All these single channels are unbound. By coupling the channels with the same configuration, the lowest masses are located at 4082 MeV for the meson–meson structure and 4328 MeV for the diquark–antidiquark structure, both of which are still above the threshold of the lowest channel $J/\psi\phi$, indicating that no any bound state exists in the meson–meson structure or the diquark–antidiquark structure. Meanwhile, the lowest energy is still 4082 MeV by the full channel coupling calculation, which means that the effect of all channel coupling is very minor here, and there is no any bound state in the $J^{PC} = 1^{++}$ system.

The $J^{PC} = 1^{+-}$ system: the four meson–meson channels and four diquark–antidiquark channels are all listed in Tabel 6, this results show that the eigenvalues of the single channel calculation no matter meson–meson configurations or diquark–antidiquark configurations is higher than the lowest threshold 4004 MeV, which means that there is no bound state, then when considering the calculation of the channel coupling, the fact that the effect of the channel coupling is minimal, so the bound states can not be found for the $J^{PC} = 1^{+-}$ system.

The $J^{PC} = 2^{++}$ system: Table 7 shows that there are two channels ($J/\psi\phi$ and $D_s^{*+}D_s^{*-}$) of the meson–meson structure and two channels of the diquark–antidiquark configurations for the $J^{PC} = 2^{++}$ system. The situation is similar to the $J^{PC} = 1^{++}$ case. The energy of each channel is above the threshold of the corresponding channel. Meanwhile, the channel coupling cannot help too much, the lowest energy is still higher than the threshold of the lowest channel $J/\psi\phi$. Therefore, there is no any bound state in the $J^{PC} = 2^{++}$ system at present calculation.

Although there is no any bound state for the $J^{PC} = 1^{++}$ and $J^{PC} = 2^{++}$ system, some resonance states are still possible in the $c\bar{c}s\bar{s}$ tetraquark system. The colorful subclusters diquark and antidiquark cannot fall apart directly due to the color confinement, so it is possible for them to be resonance states. To find out if there is any resonance state, a stabilization method, which has proven to be a valuable tool for estimating the energies of the stable states of electron–atom, electron–molecule, and atom–diatom complexes [53], is employed to find the genuine resonances. In this method, with the scaling of the distance between two clusters, the continuum state will fall off towards its threshold, while a resonance state will tend to be stable if it does not couple to the open channels, or a resonance acts as an avoid-crossing structure if it couples to the open channels and the structure will appear repeatedly with the increment of the scaling. This method has been successfully applied to the pentaquark systems [54, 55] and the fully-heavy tetraquark systems [56]. Here, we calculate the energy eigenvalues of the $c\bar{c}s\bar{s}$ tetraquark systems by taking the value of the distance (S) between two clusters from 4.0 fm to 9.0 fm to see if there is any stable state. The results of the $c\bar{c}s\bar{s}$ tetraquark

Fig. 2 The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $J^{PC} = 00^{++}$ in QDCSM2

Fig. 3 The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $J^{PC} = 01^{++}$ in QDCSM2

systems in QDCSM2 with $J^{PC} = 0^{++}, 1^{++}, 1^{+-}$ and 2^{++} are shown in Figs. 2, 3, 4 and 5, respectively.

For $J^{PC} = 0^{++}$ system in Fig. 2, it is obvious that the lowest horizontal line locates at the energy of 3930 MeV, which represents the bound state of this system. We mark it with a red line. Then, there are three horizontal lines, which stand for the thresholds of $\eta'\eta_c$, $J/\psi\phi$ and $D_s^{*+}D_s^{*-}$. We mark them with black lines. Besides, around the energy of 4033 MeV, there appears the avoid-crossing structure and the structure appears repeatedly with the increment of the scaling S , which indicates that there is a resonance state with

the resonance energy of 4033 MeV. We mark it with the green line in Fig. 2. Moreover, there are other three similar structures, which appear around the energy of 4381 MeV, 4524 MeV, and 4630 MeV, which corresponds to three resonance states respectively. By comparing with the experimental results, we find that the energy of 4381 MeV is close to the $X(4350)$, and the quantum number $J^{PC} = 0^{++}$ is consistent with the reported data by the Belle Collaboration [17]. So we explain the $X(4350)$ as a compact tetraquark resonance state with $J^{PC} = 0^{++}$ in present calculation. Our result is also agrees with the results of the Born–Oppenheimer approach,

Fig. 4 The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $J^{PC} = 01^{+-}$ in QDCSM2

Fig. 5 The stabilization plots of the energies of the $c\bar{c}s\bar{s}$ with $J^{PC} = 02^{++}$ in QDCSM2

in which a mass of 4370 MeV was obtained [57]. Besides, in Ref. [30], $X(4350)$ was also a good candidate of the compact tetraquark state with $J^P = 0^{++}$ in the chiral quark model. Similarly, the resonance energy of 4524 MeV is close to the $X(4500)$, and the quantum number $J^P = 0^{++}$ is also consistent with the reported result of the LHCb Collaboration [20]. So $X(4500)$ is possible to be a compact tetraquark resonance state with $J^{PC} = 0^{++}$ in present calculation. In addition to the $X(4350)$ and $X(4500)$, another resonance state with energy around 4630 MeV is obtained. Although the mass is

very close to the $X(4630)$, the quantum number $J^{PC} = 0^{++}$ is different from the reported one $J^P = 0^-$ [21]. However, the mass is also close to the state $X(4700)$, and the quantum number is also fit to the experimental data of the LHCb Collaboration [20]. So we prefer to assign the resonance state with energy 4630 MeV to be the exotic state $X(4700)$.

For $c\bar{c}s\bar{s}$ system with $J^{PC} = 1^{++}$ in Fig. 3, the first two horizontal lines located at the corresponding physical threshold of two channels, which are $D_s^+ D_s^{*-}$ and $J/\psi \eta'$. Obviously, a resonant state is obtained at the energy around 4324

Table 8 The energies (in MeV) of the resonance states for the $c\bar{c}s\bar{s}$ system

QDCSM1					
$J^{PC} = 0^{++}$	3930*	4028	4401	4531	4632
$J^{PC} = 1^{++}$	4307				
$J^{PC} = 1^{+-}$	—				
$J^{PC} = 2^{++}$	4394	4536			
QDCSM2					
$J^{PC} = 0^{++}$	3930*	4033	4381	4524	4630
$J^{PC} = 1^{++}$	4324				
$J^{PC} = 1^{+-}$	—				
$J^{PC} = 2^{++}$	4418	4527			
QDCSM3					
$J^{PC} = 0^{++}$	3930*	4031	4378	4501	4648
$J^{PC} = 1^{++}$	4341				
$J^{PC} = 1^{+-}$	—				
$J^{PC} = 2^{++}$	4448	4526			

(* stands for bound state, — means no resonance state exists)

MeV. Although the mass is very close to the $X(4350)$, the quantum number is not quite applicable. However, the LHCb Collaboration claimed the existence of the $X(4274)$ and the measured quantum number was $J^P = 1^+$ [19]. Therefore, we tend to use the resonance state around 4324 MeV to explain the $X(4274)$ state in this work. This is similar to the result of Ref. [58], in which a resonance state with energy near 4.3 GeV is considered as the $X(4274)$. However, for the $J^{PC} = 1^{+-}$ system, as shown in Fig. 4, there is no any resonance state.

For the last system $J^{PC} = 2^{++}$ in Fig. 5, the first two horizontal lines represent obviously the thresholds of two channels: $J/\psi\phi$ and $D_s^{*+}D_s^{*-}$. Another two horizontal lines stand for two resonance states, the energy of which is about 4418 MeV and 4527 MeV, respectively. One may note that the energy of 4527 MeV is also very close to the mass of the $X(4500)$, but the quantum number is not consistent with the experimental data. So these two resonance state maybe some new exotic states.

Meanwhile, we do the same calculation for the $c\bar{c}s\bar{s}$ system with another two sets of parameters in QDCSM. All possible resonance states with the corresponding energies are listed in Table 8. We find that the results are qualitatively consistent for the different sets of parameters. There are four resonance states and one bound state with $J^{PC} = 0^{++}$. The energy of the bound state is about 3930 MeV, which can be used to explain the $\chi_{c0}(3930)$ observed by the LHCb collaboration. The energy ranges of the four resonance states are 4028–4033 MeV, 4378–4401 MeV, 4500–4530 MeV, and 4630–4648 MeV, respectively. Combining the above anal-

ysis, $X(4500)$, $X(4350)$ and $X(4700)$ are possible to be a compact tetraquark resonance state with $J^{PC} = 0^{++}$ in present calculation. For the $J^{PC} = 1^{++}$ system, only one resonance state is obtained, the energy range of which is 4307–4341 MeV. According to the above analysis, we tend to use this resonance state to explain the $X(4274)$ in this work. However, for the $J^{PC} = 1^{+-}$ system, no any resonance state is found. Lastly, for the $J^{PC} = 2^{++}$ system, two new exotic states are available with the range of energy 4394–4448 MeV and 4526–4536 MeV, respectively. Those results may be valuable for further revealing the fine internal structure of some unknown exotic states. So we looking forward to more experimental data to help us check the existence of all these possible resonance states.

4 Summary

The $c\bar{c}s\bar{s}$ tetraquark systems with $J^{PC} = 0^{++}$, 1^{++} , 1^{+-} and 2^{++} have been systemically investigated by using the RGM in the framework of QDCSM with three different sets of parameters. Our goal is to search for any bound state or resonance state to explain the exotic states, which have been recently observed in the invariant mass distribution of $J/\psi\phi$ and another exotic state $\chi_{c0}(3930)$ observed by the LHCb collaboration. In this work, two structures: the meson–meson and diquark–antidiquark structures are taken into account. Both single-channel and channel-coupling calculations are performed. Besides, to search for any resonance state, a sta-

bilization method is applied to the coupling calculation of all channels of both two configurations.

The numerical results show that we obtain a bound molecular state $D_s^+ D_s^-$ with the quantum number $J^{PC} = 0^{++}$ and the energy 3930 MeV, which can be used to explain the observed $\chi_{c0}(3930)$. Moreover, several resonant states are obtained in this work, which are four $J^{PC} = 0^{++}$ states with the resonance masses around 4028–4033 MeV, 4378–4401 MeV, 4500–4530 MeV, and 4630–4648 MeV, respectively; one $J^{PC} = 1^{++}$ state with the resonance mass around 4307–4341 MeV; and two $J^{PC} = 2^{++}$ states with the resonance masses around 4394–4448 MeV and 4526–4536 MeV, respectively. All of them are obtained by coupling all channels of both the meson–meson and diquark–antidiquark structures, so they are compact tetraquarks in present quark model calculations. By comparing with the experimental data, we are inclined to explain the exotic states $X(4350)$, $X(4500)$ and $X(4700)$ as the compact tetraquark state with $J^{PC} = 0^{++}$. The $X(4274)$ is possible to be a candidate of the compact tetraquark state with $J^{PC} = 1^{++}$. Besides, the $J^{PC} = 0^{++}$ resonance state with energy range 4028–4033 MeV, the two $J^{PC} = 2^{++}$ resonance states with energy range of 4394–4448 MeV and 4526–4536 MeV are possible to be new exotic states, which are indeed worthy of attention.

All these resonance states are worth searching by experiments. We suggest more experimental tests to check the existence of all these possible resonance states. In addition, to confirm the existence of these $c\bar{s}c\bar{s}$ tetraquark, the study of the scattering process of the corresponding open channels is needed in future work.

Acknowledgements This work is supported partly by the National Natural Science Foundation of China under Contract no. 11675080, no. 11775118, no. 11535005, and no. 11775050; the Fundamental Research Funds for the Central Universities no. 1107022104; and the China Postdoctoral Science Foundation no. 1107020201.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The data have been illustrated in the figures and tables, so they are not necessary to be deposited. Data may be made available upon request.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>. Funded by SCOAP³.

References

1. M. Gell-Mann, Phys. Lett. **8**, 214 (1964)
2. G. Zweig, in Developments in the quark theory of hadrons. Volume 1. 1964–1978, edited by D. Lichtenberg and S. P. Rosen (1964), p. 22–101
3. S. Choi et al. (Belle), Phys. Rev. Lett. **91**, 262001 (2003)
4. M. Ablikim et al. (BESIII), Phys. Rev. Lett. **110**, 252001 (2013)
5. Z. Liu et al. (Belle), Phys. Rev. Lett. **110**, 252002 (2013)
6. M. Ablikim et al., (BESIII), Phys. Rev. Lett. **112**, 022001 (2014)
7. M. Ablikim et al., (BESIII), Phys. Rev. Lett. **112**, 132001 (2014)
8. M. Ablikim et al., (BESIII), Phys. Rev. Lett. **111**, 242001 (2013)
9. M. Ablikim et al., (BESIII), Phys. Rev. Lett. **126**, 102001 (2021)
10. R. Aaij et al., (LHCb), Phys. Rev. Lett. **125**, 242001 (2020)
11. R. Aaij et al., (LHCb), Phys. Rev. D **102**, 112003 (2020)
12. T. Aaltonen et al., (CDF Collaboration), Phys. Rev. Lett. **102**, 242002 (2009)
13. R. Aaij et al., (LHCb Collaboration), Phys. Rev. D **85**, 091103 (2012)
14. S. Chatrchyan et al., (CMS Collaboration), Phys. Lett. B **734**, 261 (2014)
15. V.M. Abazov et al., (D0 Collaboration), Phys. Rev. D **89**, 012004 (2014)
16. J.P. Lees et al., (BABAR Collaboration), Phys. Rev. D **91**, 012003 (2015)
17. C.P. Shen et al., Belle (Collaboration), Phys. Rev. Lett. **104**, 112004 (2010)
18. T. Aaltonen et al., (CDF Collaboration), Mod. Phys. Lett. A **32**, 1750139 (2017)
19. R. Aaij et al., (LHCb Collaboration), Phys. Rev. Lett. **118**, 022003 (2017)
20. R. Aaij et al., (LHCb Collaboration), Phys. Rev. D **95**, 012002 (2017)
21. R. Aaij et al. (LHCb Collaboration), [arXiv:2103.01803v1](https://arxiv.org/abs/2103.01803v1) (2021)
22. P.G. Ortega, J. Segovia, D.R. Entem, F. Fernandez, Phys. Rev. D **94**, 114018 (2016)
23. Q.-F. Lv, Y.-B. Dong, Phys. Rev. D **94**, 074007 (2016)
24. E.S. Swanson, Phys. Rev. D **91**, 034009 (2015)
25. J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Phys. Rev. D **94**, 094031 (2016)
26. L. Maiani, A.D. Polosa, V. Riquer, Phys. Rev. D **94**, 054026 (2016)
27. F. Stancu, J. Phys. G **37**, 075017 (2010)
28. H.X. Chen, E.L. Cui, W. Chen, X. Liu, S.L. Zhu, Eur. Phys. J. C **77**, 160 (2017)
29. C.R. Deng, J.L. Ping, H.X. Huang, F. Wang, Phys. Rev. D **98**, 014026 (2018)
30. Y.F. Yang, J.L. Ping, Phys. Rev. D **99**, 094032 (2018)
31. Z.G. Wang, Eur. Phys. J. C **77**, 78 (2017)
32. Z.G. Wang, Eur. Phys. J. C **76**, 657 (2016)
33. X.H. Liu, Phys. Lett. B **766**, 117 (2017)
34. D. Ebert, R.N. Faustov, V.O. Galkin, Eur. Phys. J. C **58**, 399 (2008)
35. F. Wang, G.H. Wu, L.J. Teng, J.T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992)
36. L.Z. Chen, H.R. Pang, H.X. Huang, J.L. Ping, F. Wang, Phys. Rev. C **76**, 014001 (2007)
37. M. Chen, H.X. Huang, J.L. Ping, F. Wang, Phys. Rev. C **83**, 015202 (2011)
38. A. De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D **12**, 147 (1975)
39. N. Isgur, G. Karl, Phys. Rev. D **18**, 4187 (1978)
40. N. Isgur, G. Karl, Phys. Rev. D **19**, 2653 (1979)
41. N. Isgur, G. Karl, Phys. Rev. D **20**, 1191 (1979)
42. G.H. Wu, L.J. Teng, J.L. Ping, F. Wang, T. Goldman, Phys. Rev. C **53**, 1161 (1996)
43. H.X. Huang, P. Xu, J.L. Ping, F. Wang, Phys. Rev. C **84**, 064001 (2011)

44. H.X. Huang, C.R. Deng, J.L. Ping, F. Wang, Eur. Phys. J. C **76**, 624 (2016)
45. M. Tanabashi et al., (Particle Data Group), Phys. Rev. D **98**, 030001 (2018)
46. G.H. Wu, J.L. Ping, L.J. Teng, F. Wang, T. Goldman, Nucl. Phys. A **673**, 279 (2000)
47. J.L. Ping, F. Wang, T. Goldman, Nucl. Phys. A **657**, 95 (1999)
48. H.R. Pang, J.L. Ping, F. Wang, T. Goldman, Phys. Rev. C **65**, 014003 (2001)
49. M.M. Xu, M. Yu, L.S. Liu, Phys. Rev. Lett. **100**, 092301 (2008)
50. M. Kamimura, Suppl. Prog. Theor. Phys. **62**, 236 (1977)
51. S. Prelovsek, S. Collins, D. Mohler, M. Padmanath, S. Piemonte, (2020) [arXiv:2011.02542](https://arxiv.org/abs/2011.02542)
52. X.-K. Dong, F.-K. Guo, B.-S. Zou, Prog. Phys. **41**, 65–93 (2021)
53. J. Simon, J. Chen, Physics **75**, 2465 (1981)
54. E. Hiyama, M. Kamimura, A. Hosaka, H. Toki, M. Yahiro, Phys. Lett. B **633**, 237–244 (2006)
55. E. Hiyama, A. Hosaka, M. Oka, J.M. Richard, Phys. Rev. C **98**(4), 045208 (2018)
56. X. Jin, Y. Xue, H. Huang, J. Ping, Eur. Phys. J. C **80**, 1083 (2020)
57. E. Braaten, C. Langmack, D.H. Smith, Phys. Rev. D **90**, 014044 (2014)
58. W. Chen, S.L. Zhu, Phys. Rev. D **83**, 034010 (2011)