THE EUROPEAN
PHYSICAL JOURNAL C

Scalar and gauge sectors in the 3-Higgs Doublet Model under the S_3 symmetry

M. Gómez-Bock[1,a], **M. Mondragón**[2,b], **A. Pérez-Martínez**[2,c]

[1] Universidad de las Américas Puebla, UDLAP, Ex-Hacienda Sta. Catarina Mártir, Cholula, Puebla, Mexico
[2] Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, 01000 Mexico, D.F., Mexico

Abstract We analyse the Higgs sector of an S_3 model with three Higgs doublets and no CP violation. After electroweak breaking there are nine physical Higgs bosons, one of which corresponds to the Standard Model one. We study the scalar and gauge sectors of this model, taking into account the conditions set by the minimisation and stability of the potential. We calculate the masses, trilinear and quartic Higgs-Higgs, and Higgs-gauge couplings. We consider two possible alignment scenarios, where only one of the three neutral scalars has couplings to the gauge bosons and corresponds to the SM Higgs, and whose trilinear and quartic couplings reduce exactly to the SM ones. We also obtain numerically the allowed parameter space for the scalar masses in each of the alignment scenarios. We use the calculated trilinear and quartic couplings to find the analytical structure of the one-loop neutral scalar mass matrix, without fermionic contributions. We show that it is possible to have a compact mass spectrum where the contribution to the oblique parameters might be small. We explore some scenarios for the loop contributions to the neutral scalar masses.

1 Introduction

The discovery the Higgs boson with a mass of 125 GeV [1,2], and the experimental study of its properties, will be relevant to gain a deeper understanding of the flavour problem and of ways to address it.

The organization of the fermions into generations or families may signal a possible underlying structure in elementary particles, although its origin or nature is not yet understood. On the other hand, the Standard Model (SM) Higgs mechanism [3,4], which is indispensable to understand the origin of the masses of gauge bosons and fermions, sheds no light on the flavor structure or the difference in the masses of the fundamental fermions.

The flavour structure of fermions has been the subject of a great amount of research throughout the years. In view of the fact that the only difference between generations in the fermionic sector are the masses of the particles, the most direct or even natural way to propose a flavour structure is through the mass generation mechanism, the Higgs sector.

One possibility to understand the flavour nature behind the SM is to construct an extended scalar sector with a flavour symmetry, where the SM is embedded. Multi-Higgs extensions of the SM, with and without extra symmetries, have been extensively studied, some diverse examples are given in [5–15] (for reviews on two Higgs Doublet Models (2HDM) and multi-Higgs models see [16,17]). Discrete symmetries have been extensively studied in this context, both at low and at high energies (for reviews of models with discrete symmetries see [18–21]). Since these models in general require the addition of more Higgs fields, the phenomenological consequences in all sectors, like allowed extra processes and couplings, have to be analysed, some examples can be found in [22–24]. Restrictions are placed on the models by confronting their phenomenology with the experimental results, in this case the ones of ATLAS [25] and CMS [26]. The Higgs sector is thus crucial to determine the viability and prospects of each model. Prime examples of this procedure are the Minimal Supersymmetric Standard Model (MSSM) and 2HDM (see for instance, [27–29] and [16,30,31], respectively).

The permutation group of three objects S_3, with three Higgs doublets, has been proposed already a long time ago [32–36] as a natural extension of the SM, even before all the quarks were discovered or the mass of the neutrinos established. Since then, the S_3 symmetry has been extensively

[a] e-mail: melina.gomez@udlap.mx

[b] e-mail: myriam@fisica.unam.mx

[c] e-mail: adrianapema7.5@gmail.com (corresponding author)

studied in different contexts, both in the quark [37–46] and lepton sectors [47–64], due to its simplicity and predictivity, as well as in the scalar sector [65–72] where more predictions arise. More recently, there have been also studies of dark matter candidates in models with S_3 symmetry [73–78].

In particular, the 3 Higgs doublet model with S_3 symmetry (which we will refer here as S3-3H) [48], has led to very interesting results in the fermionic sector. In the quark sector, it was shown that it is possible to obtain the Fritzsch and the nearest neighbour interaction (NNI) textures [42], thus fitting the CKM matrix. In the leptonic sector it was found that the S3-3H model can also reproduce the V_{PMNS} matrix and predicts a non-vanishing θ_{13} reactor mixing angle, and some flavour changing neutral currents and contributions to $g - 2$ were calculated [41,52,53,57]. In [75], a version of the S3-3H with an extra inert Higgs doublet was analysed (S3-4H), with the interesting result that it is possible to have a good dark matter (DM) candidate, coming from the inert sector and satisfying also the Higgs bounds. The indirect prospects of detection of this DM candidate have been studied in [77]. Although there has been extensive work in models with S_3 symmetry and three Higgs doublets in different contexts, the phenomenological implications in the Higgs sector have not been fully explored. Our motivation to analyse more closely the scalar sector of the S3-3H model concerns the fact that it is in this sector where novel experimental signatures can be found; also the results in the scalar sector will have an impact and allow for a deeper analysis of the fermionic sector.

The conditions for stability and symmetry breaking in the general three Higgs doublet model (3HDM) have been studied in [79]. Assuming an extra discrete symmetry reduces greatly the number of free parameters, in particular the case of the S3-3H potential was already analysed in [65], although requiring a soft breaking of the discrete symmetry. The vacuum stability of the S3-3H scalar potential, without soft breaking of S_3, was studied in [80,81], and the mass structure of the scalar bosons was analysed in [66,67]. In [66] it was found that there is a residual $\mathcal{Z}_2$ symmetry after the electroweak symmetry breaking (EWSB) in the Higgs potential, and the corresponding charges for the scalars under this symmetry were given. The conditions for having spontaneous CP violation in this potential were presented in [70].

In here, we keep the model as simple as possible, by not assuming an explicit breaking of the flavour symmetry or adding extra flavons. We calculate the scalar masses, and the trilinear and quartic Higgs self-couplings and Higgs-gauge boson couplings. We consider two possible alignment scenarios for the SM-like Higgs boson, where only one of the three neutral scalars has couplings to the vector bosons (one is always decoupled due to the $\mathcal{Z}_2$ symmetry). We use a geometrical parameterization in spherical coordinates, which allows us to express the mixing of the vacuum expectation values (vevs) of the Higgs fields in the S_3 singlet and doublet irreducible representations, in terms of one angle (θ) in our expressions. We scan the parameter space, taking into account the unitarity and stability conditions, and the SM Higgs boson mass constraints, in each of the two alignment scenarios.

Some of the trilinear scalar couplings have been obtained in [67], nevertheless we found differences with their results. Mainly in [67], the $\mathcal{Z}_2$ symmetry is not exhibited, whereas we find it explicitly in our calculations, consistent with the $\mathcal{Z}_2$ residual symmetry reported in [66]. As an additional result, we find that in each of the alignment limits, where only the SM-like Higgs couples to the vector bosons, the trilinear and quartic couplings reduce exactly to the SM ones.

We find the expression for the neutral scalar mass matrix at one-loop, where all the scalar and gauge contributions derived from the calculated trilinear and quartic couplings are taken into account. Although it reduces to a structure similar to the 2HDM mass matrix, due to the residual $\mathcal{Z}_2$ symmetry, the presence of an extra neutral scalar in our case, h_0, allows to distinguish between the models. It is possible to find values for the parameter θ and the scalar masses where the off-diagonal term of the one-loop neutral scalar mass matrix vanishes, thus minimising the radiative corrections. We give two examples of such spectra, one with light and another one with heavier masses. The latter one fulfils the conditions that can make the contributions to the oblique parameters to be small or even vanish [82,83].

The paper is organized as follows: in the next section we describe the model, and how the S_3 symmetry acts on the Higgs electroweak doublets, giving the structure and characteristics of the Higgs potential in the S3-3H model. In Sect. 3, we parameterize the vacua and rotate to the Higgs basis, to express our results in terms of physical parameters. We then calculate the tree level masses and explore numerically the two different alignment scenarios. Then, in Sect. 4, we calculate the Higgs-Higgs couplings and the Higgs-gauge bosons couplings; we present the ones involving neutral scalars in this section, and we complete with the pseudoscalars and charged scalar couplings in the Appendix. We also analyse the structure of the one-loop neutral scalar mass matrix. Finally, we present a summary and the conclusions of our work.

2 The S3-3H model scalar sector

We will discuss briefly here how the S_3 symmetry is implemented in the scalar sector of the model. The S_3 group is the smallest non-Abelian discrete group, it corresponds to the rotations and reflections that leave invariant an equilateral triangle, or equivalently, to the permutations of three objects. It has three irreducible representations (irreps): a symmetric singlet $\mathbf{1}_S$, an anti-symmetric singlet $\mathbf{1}_A$, and a doublet $\mathbf{2}$ [18].

The multiplication rules among the irreducible representations are as follows

$$1_S \otimes 1_S = 1_S, \quad 1_S \otimes 1_A = 1_A,$$
$$1_A \otimes 1_S = 1_A, \quad 1_A \otimes 1_A = 1_S,$$
$$1_S \otimes 2 = 2, \quad 1_A \otimes 2 = 2,$$
$$2 \otimes 1_S = 2, \quad 2 \otimes 1_A = 2;$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_2 \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_2 = (a_1 b_1 + a_2 b_2)_{1_S}$$
$$\oplus (a_1 b_2 - a_2 b_1)_{1_A} \oplus \begin{pmatrix} a_1 b_2 + a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix}_2. \tag{1}$$

S_3 has 6 subgroups: the trivial group, the whole group, three $\mathcal{Z}_2$ subgroups (which correspond to the reflections over the axes of symmetry of the triangle), and a $\mathcal{Z}_3$ subgroup.

We will consider here three $SU(2)$ electroweak (EW) Higgs doublets, i.e. two more than in the Standard Model. We will assign here two of the Higgs EW doublets to the **2** irrep of S_3 and the third one to the symmetric singlet 1_S, but in this work we will concentrate only on the scalar sector, so our results are general for any scalar potential of this type, irrespective of the assignment for the fermionic sector. In Ref. [48] an extension of the SM was considered, with three $SU(2)$ Higgs doublets plus three right-handed neutrinos, we refer to this model as S3-3H. In the fermionic sector of the S3-3H model, prior to EWSB, the first two generations of quarks and leptons, as well as two of the Higgs doublets, were assigned to the S_3 doublet irrep, and the third generation of fermions and one Higgs electroweak doublet, to the symmetric singlet irrep. After EWSB all the fields are mixed, giving rise to a specific texture for the mass matrices of quarks and leptons.

2.1 The S3-3H model scalar potential

The terms in the potential are the ones that preserve the discrete S_3 permutational symmetry, as reported in [35,65]. The most general Higgs potential invariant under the $SU(3)_c \times SU(2)_L \times U(1)_Y \times S_3$ in the symmetry adapted basis, according to the multiplication rules (1), is given as,

$$V = \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right) + \mu_0^2 \left(H_s^\dagger H_s \right) + \frac{a}{2} \left(H_s^\dagger H_s \right)^2$$
$$+ b \left(H_s^\dagger H_s \right) \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right)$$
$$+ \frac{c}{2} \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right)^2 + \frac{d}{2} \left(H_1^\dagger H_2 - H_2^\dagger H_1 \right)^2$$
$$+ e f_{ijk} \left(\left(H_s^\dagger H_i \right) \left(H_j^\dagger H_k \right) + h.c. \right)$$
$$+ f \left\{ \left(H_s^\dagger H_1 \right) \left(H_1^\dagger H_s \right) + \left(H_s^\dagger H_2 \right) \left(H_2^\dagger H_s \right) \right\}$$

$$+ \frac{g}{2} \left\{ \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1 \right)^2 \right\}$$
$$+ \frac{h}{2} \left\{ \left(H_s^\dagger H_1 \right) \left(H_s^\dagger H_1 \right) + \left(H_s^\dagger H_2 \right) \left(H_s^\dagger H_2 \right) \right.$$
$$\left. + \left(H_1^\dagger H_s \right) \left(H_1^\dagger H_s \right) + \left(H_2^\dagger H_s \right) \left(H_2^\dagger H_s \right) \right\}; \tag{2}$$

where $f_{112} = f_{121} = f_{211} = -f_{222} = 1$. This same potential has also been analysed in Refs. [66,69,80,81] without CP violation, and in Ref. [70] with spontaneous CP violation. We will only consider here the case without CP violation, i.e. solutions with real *vevs*.

As already mentioned, we will assign two of the Higgs doublets, H_1 and H_2 to the doublet irrep of S_3 **2**, and the third one, H_S, to the symmetric singlet irrep 1_S.

In terms of complex fields we express them as

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix},$$
$$H_s = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}. \tag{3}$$

In order to simplify the calculations, we introduce the following variables as was done in [80,81]

$$x_1 = H_1^\dagger H_1, \; x_4 = Re(H_1^\dagger H_2), \; x_7 = Im(H_1^\dagger H_2),$$
$$x_2 = H_2^\dagger H_2, \; x_5 = Re(H_1^\dagger H_s), \; x_8 = Im(H_1^\dagger H_s), \tag{4}$$
$$x_3 = H_s^\dagger H_s, \; x_6 = Re(H_2^\dagger H_s), \; x_9 = Im(H_2^\dagger H_s).$$

As an example, we show here explicitly some of the real terms of the scalar fields in the potential, with the appropriate normalization factors

$$x_1 = H_1^\dagger H_1 = \frac{1}{2}(\phi_1^2 + \phi_4^2 + \phi_7^2 + \phi_{10}^2),$$
$$x_4 = Re(H_1^\dagger H_2) = \frac{1}{2}(\phi_1\phi_2 + \phi_4\phi_5 + \phi_7\phi_8 + \phi_{10}\phi_{11}),$$
$$x_7 = Im(H_1^\dagger H_2) = \frac{1}{2}(\phi_1\phi_5 - \phi_4\phi_2 + \phi_7\phi_{11} - \phi_{10}\phi_8). \tag{5}$$

Hence, using (4) into (2), the Higgs potential is expressed as:

$$V = \mu_1^2(x_1 + x_2) + \mu_0^2 x_3 + \frac{a}{2}x_3^2 + b(x_1 + x_2)x_3 + \frac{c}{2}(x_1 + x_2)^2$$
$$- 2dx_7^2 + 2e\left[(x_1 - x_2)x_6 + 2x_4 x_5\right] + f(x_5^2 + x_6^2 + x_8^2 + x_9^2)$$
$$+ \frac{g}{2}\left[(x_1 - x_2)^2 + 4x_4^2\right] + h(x_5^2 + x_6^2 - x_8^2 - x_9^2). \tag{6}$$

From this general potential we have ten free parameters, before EWSB.

2.2 The normal minimum

In order to have a consistent Higgs potential, it is necessary to check that it is stable, i.e. bounded from below, and that it respects perturbative unitarity. These requirements impose

constraints on the potential's parameters. This analysis has already been done in [66], and we use their expressions for the unitarity and stability bounds in here.

A study of the stability of the different minima for a general Higgs potential of this kind can be found in [80], they point out the existence of three types of minima or stationary points. In here, we will consider the EWSB using the natural choices of conservation of electric and CP charges, implying that only the real part of the neutral fields will acquire *vevs*, we will refer to this as the *normal minimum*. Thus, only the real parts of each one of the doublets will acquire non-zero vacuum expectation values. Expressed in terms of the field components of H_1, H_2, H_s, Eq. (3) we have

$$\langle \phi_7 \rangle = v_1, \langle \phi_8 \rangle = v_2, \langle \phi_9 \rangle = v_3, \langle \phi_i \rangle = 0, \ i \neq 7, 8, 9, \quad (7)$$

this adds two more free parameters to the model, as they should satisfy the condition

$$\sqrt{v_1^2 + v_2^2 + v_3^2} = v = 246 \text{ GeV}. \quad (8)$$

The extreme point conditions for the potential are given by

$$\frac{\partial V}{\partial v_i} = 0 \longleftrightarrow \frac{\partial V}{\partial x_j} \frac{\partial x_j}{\partial v_i} = 0, \quad (9)$$

with $i = 1, 2, 3$; $j = 1, 2, \ldots, 9$. These conditions express the tree level tadpole equations as

$$0 = [2\mu_1^2 + (b + f + h)v_3^2 + (c + g)(v_1^2 + v_2^2)]v_1 + 6ev_1v_2v_3, \quad (10)$$

$$0 = [2\mu_1^2 + (b + f + h)v_3^2 + (c + g)(v_1^2 + v_2^2)]v_2 + 3e(v_1^2 - v_2^2)v_3, \quad (11)$$

$$0 = [2\mu_0^2 + (b + f + h)(v_1^2 + v_2^2) + av_3^2]v_3 + e(3v_1^2 - v_2^2)v_2. \quad (12)$$

These equations reduce further the original twelve free parameters relating two of them as

$$v_1^2 = 3v_2^2. \quad (13)$$

Another possible solution that satisfies these equations would be $e = 0$ [48,81], which implies the presence of a Goldstone boson due to a residual $SO(2)$ symmetry, but this scenario will not be considered for the present work. The general minima of this potential, both real and complex, have been studied in [70], with emphasis on the complex vacua. In here, we will consider only in detail the case (13), with $v_1 = +\sqrt{3}v_2$, where after EWSB there is a residual $\mathcal{Z}_2$ symmetry. This residual symmetry corresponds to one of the $\mathcal{Z}_2$ subgroups of S_3, namely, a reflection over one of the symmetry axes of the triangle. In a similar fashion, the solution $v_1 = -\sqrt{3}v_2$ has also a residual $\mathcal{Z}_2$ symmetry, which is another one of the subgroups of S_3. In this latter case the invariance is under

the reflection over the opposite axis of symmetry as the positive solution. This negative solution leads exactly to the same results for the masses and couplings as the positive solution.

3 Tree level Higgs masses and physical basis

In order to get the tree level masses of the Higgs bosons, it is necessary to diagonalize the 12×12 matrix resulting from taking the second derivatives of the potential

$$(\mathcal{M}_H^2)_{ij} = \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \bigg|_{\langle \phi_i \rangle}, \quad (14)$$

with $i, j = 1, \ldots, 12$. Due to the symmetry of the model, the mass matrix consists of four diagonal blocks, each one a 3×3 Hermitian and symmetric matrix. The Higgs mass matrices of the S3-3H model have been reported previously in [66,67], nevertheless our results differ from [67] by a factor of two in the Higgs couplings, because we have included the normalization factors $1/\sqrt{2}$ in the Higgs doublets. In this work, we will study the general case with $e \neq 0$, with a new parameterization which allows us to compare directly with the SM when we include the complete scalar couplings and scalar-gauge couplings.

Since we assume no CP violation, we obtain three 3×3 Hermitian matrices, one for the charged scalars $\mathbf{M}_C^2$, one for the neutral scalars $\mathbf{M}_S^2$, and one for the pseudoscalar bosons masses $\mathbf{M}_A^2$.

The matrix elements of the charged Higgs masses in terms of the potential parameters are given as

$$\mathbf{M}_C^2 = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}, \quad (15)$$

with the elements of the symmetric mass matrix given as

$$c_{11} = -v_3[2ev_2 + \frac{v_3}{2}(f + h)] - gv_2^2,$$

$$c_{12} = \sqrt{3}v_2(ev_3 + gv_2),$$

$$c_{13} = \sqrt{3}v_2[ev_2 + \frac{v_3}{2}(f + h)],$$

$$c_{22} = -v_3[4ev_2 + \frac{v_3}{2}(f + h)] - 3gv_2^2,$$

$$c_{23} = v_2[ev_2 + \frac{v_3}{2}(f + h)],$$

$$c_{33} = -\frac{2v_2^2[2ev_2 + v_3(f + h)]}{v_3}. \quad (16)$$

The mass matrix for neutral scalars is given by

$$\mathbf{M}_S^2 = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix}, \quad (17)$$

where, the elements of the scalar symmetric mass matrix are

$$s_{11} = 3v_2^2(c+g), \quad s_{12} = \sqrt{3}v_2[v_2(c+g) + 3ev_3],$$
$$s_{13} = \sqrt{3}v_2[v_3(b+f+h) + 3ev_2],$$
$$s_{22} = v_2[v_2(c+g) - 6ev_3],$$
$$s_{23} = v_2[3ev_2 + (b+f+h)v_3],$$
$$s_{33} = \frac{(av_3^3 - 4ev_2^3)}{v_3}. \tag{18}$$

For the pseudoscalar mass matrix we find

$$\mathbf{M}_A^2 = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{19}$$

where each of the elements of the symmetric matrix are given as

$$a_{11} = -\left(v_2^2(d+g) + 2ev_2v_3 + hv_3^2\right),$$
$$a_{12} = \sqrt{3}v_2(v_2(d+g) + ev_3),$$
$$a_{13} = \sqrt{3}v_2(ev_2 + hv_3),$$
$$a_{22} = -3v_2^2(d+g) - 4ev_2v_3 - hv_3^2,$$
$$a_{23} = v_2(ev_2 + hv_3), \quad a_{33} = -\frac{4v_2^2(ev_2 + hv_3)}{v_3}. \tag{20}$$

3.1 Geometrical parameterization of the vacua and the Higgs masses

We will rewrite the *vevs* in spherical coordinates, as it was done in [75,84]

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta,$$
$$v_3 = v\cos\theta. \tag{21}$$

The use of this spherical parameterization is helpful to visualize the relation among the *vevs*. The angle θ gives the amount of mixing between the *vev* of the singlet (v_3) and the *vevs* of the doublet (v_1, v_2). We express the relations between v_1, v_2, and v_3 in terms of two angles as:

$$\tan\varphi = \frac{v_2}{v_1}, \quad \tan\theta = \frac{v_2}{v_3\sin\varphi}. \tag{22}$$

Moreover, the minimization condition of the potential (13), provides an extra constraint for the relation between v_1 and v_2 i.e. it fixes also the value of φ. We assume all the *vevs* to be real and positive (otherwise we should consider a phase between two *vevs*) implying $\varphi = \pi/6$, then $\tan\varphi = \frac{1}{\sqrt{3}}$, thus we get

$$\tan\varphi = 1/\sqrt{3} \Rightarrow \sin\varphi = \frac{1}{2}, \quad \cos\varphi = \frac{\sqrt{3}}{2}, \tag{23}$$
$$\tan\theta = \frac{2v_2}{v_3} \Rightarrow \sin\theta = \frac{2v_2}{v}, \quad \cos\theta = \frac{v_3}{v}. \tag{24}$$

The usual form for the rotation matrix R_i, to obtain the mass matrix and physical states is given as:

$$[\mathcal{M}_{diag}^2]_I = R_I^T \mathbf{M}_I^2 R_I, \quad I = S, A, C. \tag{25}$$

For the Higgs bosons we take the sub-indices $I = S, A, C$ to refer to the neutral, pseudoscalar and charged Higgs bosons respectively. The rotation matrix is the product of two rotations, i.e., $R_I = A\,B_I$, where

$$A = \begin{pmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_I = \begin{pmatrix} \cos\gamma_I & 0 & \sin\gamma_I \\ 0 & 1 & 0 \\ -\sin\gamma_I & 0 & \cos\gamma_I \end{pmatrix}, \tag{26}$$

then

$$R_I = \begin{pmatrix} \cos\gamma_I\cos\delta & -\sin\delta & \sin\gamma_I\cos\delta \\ \cos\gamma_I\sin\delta & \cos\delta & \sin\gamma_I\sin\delta \\ -\sin\gamma_I & 0 & \cos\gamma_I \end{pmatrix}. \tag{27}$$

The rotation matrix $R_{A,C}$, which diagonalizes $\mathcal{M}_A^2$ and $\mathcal{M}_C^2$, will transform the fields leading to the Goldstone states. They are given as follows:

$$R_{A,C} = \begin{pmatrix} \cos\gamma_{A,C}\cos\delta & -\sin\delta & \sin\gamma_{A,C}\cos\delta \\ \cos\gamma_{A,C}\sin\delta & \cos\delta & \sin\gamma_{A,C}\sin\delta \\ -\sin\gamma_{A,C} & 0 & \cos\gamma_{A,C} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{\sqrt{3}v_2}{v} & -\frac{1}{2} & -\frac{\sqrt{3}v_3}{2v} \\ \frac{v_2}{v} & \frac{\sqrt{3}}{2} & -\frac{v_3}{2v} \\ \frac{v_3}{v} & 0 & \frac{2v_2}{v} \end{pmatrix}. \tag{28}$$

Therefore, we can see that $\cos\gamma_{A,C} = \frac{2v_2}{v}$, $\sin\gamma_{A,C} = -\frac{v_3}{v}$, $\sin\delta = \frac{1}{2}$, and $\cos\delta = \frac{\sqrt{3}}{2}$. If we compare with (23) and (24) we can see that $\delta = \varphi$ and $\gamma_{A,C} = \frac{3\pi}{2} + \theta$. We will reparameterize the matrices in terms of the angles θ and φ, so that the matrices take the following form

$$R_{A,C} = \begin{pmatrix} \sin\theta\cos\varphi & -\sin\varphi & -\cos\theta\cos\varphi \\ \sin\theta\sin\varphi & \cos\varphi & -\cos\theta\sin\varphi \\ \cos\theta & 0 & \sin\theta \end{pmatrix}$$
$$= \begin{pmatrix} \frac{\sqrt{3}v_2}{v} & -\frac{1}{2} & -\frac{\sqrt{3}v_3}{2v} \\ \frac{v_2}{v} & \frac{\sqrt{3}}{2} & -\frac{v_3}{2v} \\ \frac{v_3}{v} & 0 & \frac{2v_2}{v} \end{pmatrix}. \tag{29}$$

Thus, we obtain the respective masses at tree level for the pseudoscalar and charged Higgs bosons as

$$m_{A_1}^2 = -v^2\left[(d+g)\sin^2\theta + \frac{5}{4}e\sin 2\theta + h\cos^2\theta\right], \tag{30}$$

$$m_{A_2}^2 = -v^2(\frac{e}{2}\tan\theta + h), \tag{31}$$

$$m_{H_1^\pm}^2 = -\frac{v^2}{4}\left[5e\sin 2\theta + 2(f+h)\cos^2\theta + 4g\sin^2\theta\right], \tag{32}$$

$$m^2_{H^\pm_2} = -\frac{v^2}{2}\left[e\tan\theta + (f+h)\right].\tag{33}$$

From these expressions it can be seen that all masses are proportional to v, with their values determined by an interplay of the self-couplings and θ.

For the diagonalization of the mass matrix of the neutral scalar bosons $\mathcal{M}^2_S$, we will have the following rotation matrix:

$$R_S = \begin{pmatrix} \cos\gamma_S\cos\delta & -\sin\delta & \sin\gamma_S\cos\delta \\ \cos\gamma_S\sin\delta & \cos\delta & \sin\gamma_S\sin\delta \\ -\sin\gamma_S & 0 & \cos\gamma_S \end{pmatrix}.\tag{34}$$

In terms of the parameters of the potential, considering also the spherical parameterization, we have

$$M^2_a = \left[(c+g)v^2\sin^2\theta + \frac{3}{4}ev^2\sin 2\theta\right],$$
$$M^2_b = \left[3ev^2\sin^2\theta + (b+f+h)v^2\sin 2\theta\right],$$
$$M^2_c = av^2\cos^2\theta - \frac{1}{2}ev^2\tan\theta\sin^2\theta,\tag{35}$$

where the mixing angle α is

$$\tan 2\alpha = -\frac{M^2_b}{M^2_a - M^2_c}.\tag{36}$$

The rotation to obtain the mass matrix directly from the interaction basis is given as

$$R_S = \begin{pmatrix} \frac{\sqrt{3}(M^2_a - M^2_c + Z_M)}{2\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} & -\frac{1}{2} & -\frac{\sqrt{3}M^2_b}{2\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} \\ \frac{M^2_a - M^2_c + Z_M}{2\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} & \frac{\sqrt{3}}{2} & -\frac{M^2_b}{2\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} \\ \frac{M^2_b}{\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} & 0 & \frac{M^2_a - M^2_c + Z_M}{\sqrt{(M^2_b)^2 + (M^2_a - M^2_c + Z_M)^2}} \end{pmatrix},\tag{37}$$

where

$$Z_M = \sqrt{(M^2_b)^2 + (M^2_a - M^2_c)^2}.$$

Using again Eqs. (23) and (36) we get

$$\tan\gamma_S = -\frac{M^2_b}{(M^2_a - M^2_c + Z_M)}, \quad \gamma_S = \frac{3\pi}{2} + \alpha,$$
$$\sin\delta = \frac{1}{2} \quad \text{and} \quad \cos\delta = \frac{\sqrt{3}}{2}.\tag{38}$$

We will work with the angles α and φ, for that reason we express the rotation matrix in the following form

$$R_S = \begin{pmatrix} \sin\alpha\cos\varphi & -\sin\varphi & -\cos\alpha\cos\varphi \\ \sin\alpha\sin\varphi & \cos\varphi & -\cos\alpha\sin\varphi \\ \cos\alpha & 0 & \sin\alpha \end{pmatrix}.\tag{39}$$

Then, we may write the scalar Higgs bosons masses as follows:

$$m^2_{h_0} = -\frac{9}{4}ev^2\sin 2\theta,\tag{40}$$
$$m^2_{H_1,H_2} = \frac{1}{2}\left[(M^2_a + M^2_c) \pm \sqrt{(M^2_a - M^2_c)^2 + (M^2_b)^2}\right],\tag{41}$$

we notice that $e < 0$. We see here that the structure of the masses is consistent with the one found in Refs. [66,81]. The expressions for $m_{H_{1,2}}$ can be written in terms of the parameters of the model as

$$m^2_{H_1} = \frac{v^2}{2}\left[2ac^2_\theta c^2_\alpha + 2(c+g)s^2_\alpha s^2_\theta + 4(b+f+h)s_\alpha s_\theta c_\alpha c_\theta\right.$$
$$\left. + et_\theta(6s_\alpha c_\alpha s_\theta c_\theta + 3c^2_\theta s^2_\alpha - s^2_\theta c^2_\alpha)\right],$$
$$= \frac{v^2}{4}\left\{a(c_{\alpha-\theta} + c_{\alpha+\theta})^2 + (c+g)(c_{\alpha-\theta} - c_{\alpha+\theta})^2\right.$$
$$+ (b+f+h)(c_{2(\alpha-\theta)} - c_{2(\alpha+\theta)})$$
$$\left. + et_\theta\left[c_{2(\alpha-\theta)} - c_{2(\alpha+\theta)} + 4s_{\alpha+\theta}s_{\alpha-\theta} + 2s^2_{\alpha+\theta}\right]\right\},\tag{42}$$
$$m^2_{H_2} = \frac{v^2}{2}\left(2ac^2_\theta s^2_\alpha + 2(c+g)c^2_\alpha s^2_\theta - 4(b+f+h)s_\alpha s_\theta c_\alpha c_\theta\right.$$
$$\left. + et_\theta(-6s_\alpha c_\alpha s_\theta c_\theta + 3c^2_\theta c^2_\alpha - s^2_\theta s^2_\alpha)\right),$$
$$= \frac{v^2}{4}\left\{a(s_{\alpha-\theta} + s_{\alpha+\theta})^2 + (c+g)(s_{\alpha+\theta} - s_{\alpha-\theta})^2\right.$$
$$- (b+f+h)(c_{2(\alpha-\theta)} - c_{2(\alpha+\theta)})$$
$$\left. + et_\theta\left[c_{2(\alpha+\theta)} - c_{2(\alpha-\theta)} + 4c_{\alpha+\theta}c_{\alpha-\theta} + 2c^2_{\alpha+\theta}\right]\right\},\tag{43}$$

here we use the reduced notation for the trigonometric functions: $s_x \equiv \sin x$, $c_x \equiv \cos x$ and $t_x \equiv \tan x$.

The SM Higgs boson has already been measured [25,26], and one of the neutral CP-even Higgs of the model should correspond to it. Thus, it is important to explore the structure of these tree level masses in terms of the self-couplings of the Higgs potential in the interaction basis, Eq. (6), and also in terms of the mixing angles relating the *vevs*, Eq. (22). The possibility of a neutral Goldstone, a massless degree of freedom has been reported previously in [48,81] when $e = 0$ is considered, and leads to $v_1 = v_2$.

From Eqs. (35), (40) and (41), we can reduce the expressions of the masses for specific cases of the parameter θ which, as we said before, gives the amount of mixing between the *vev* of the singlet and the *vevs* of the doublets. We explore a particular case, for instance $\theta = \pi/4$, where we obtain the neutral CP-even Higgs masses as

$$m^2_{h_0} = \frac{-9}{4}ev^2,\tag{44}$$
$$m^2_{H_{1,2}} = \frac{v^2}{4}\left[a+c+e+g\right.$$
$$\left. \pm \sqrt{(-a+c+2e+g)^2 + (3e+2(b+f+h))^2}\right],\tag{45}$$

where the three masses are proportional to the *vev*, $v = 246$ GeV, and combinations of the self-couplings.

Setting the *vevs* of the doublet or singlet to zero has to be considered from the beginning, to arrive at the appropriate tadpole equations. The case where $v_1 = v_2 = 0$ corresponds to one of the minima found in [70,78], which leaves μ_1 undetermined. In their solution, the three neutral scalar masses are in principle different from zero, with two of them degenerate and depending on μ_1. In our case, when we take the limit $\sin\theta \to 0$, which leads to $v_1, v_2 \to 0$, we get two almost massless scalars. This can be seen from Fig. 1, as $\tan\theta \to 0$, also $m_{h_0}, m_{H_2} \to 0$. It can also be verified from the structure of the mass matrices Eqs. (42, 43), or by noticing that in this limit also $\alpha \to 0$ (Eq. (36)) and substituting in Eqs. (42, 43). The particular choice of $2\mu_1 = -(h + b + f)v_3^2$ in the solution of refs. [70,78] implies that the two degenerate masses become zero, and the third one coincides with our $m_{H_1} = av^2$. But it is not possible to arrive to the latter condition for μ_1 from our tadpole equations, since we initially have considered $v_1, v_2 \neq 0$. Similarly, it is not possible to have exactly $\cos\theta = 0$, or equivalently $v_3 = 0$ in our case, since Eqs. (10–12) are arrived at dividing by v_3. If one assumes $v_3 = 0$ from the beginning, the tadpole equations are different, and they lead to the relationship $v_2^2 = 3v_1^2$ [70], which is the inverse of the ratio we find between v_1 and v_2.

For our particular solution, we will assume none of the *vevs* are zero, and thus there must be an admixture of the doublet and the singlet Higgs fields, which might be relevant when considering the fermionic sector.

In the following section, we will analyse the masses for the general cases of non-zero parameter values. Specifically, for the numerical analysis, we explore the tree level masses for different values of the θ parameter, in the range $0 < \theta < \pi/2$, to keep the *vevs* positive.

3.2 The Higgs basis

From the perspective of both EWSB and flavor physics, there is a basis that is particularly useful to compare with the SM or with other of its extensions, the so-called Higgs basis. It is defined as the basis in which one Higgs field carries the full *vev*, ϕ_{vev}, and the other Higgs fields ψ_1, ψ_2 are perpendicular to it [13,17,85–88]. In order to get the Goldstone bosons, which are needed for the generation of masses of the gauge bosons, we do the usual rotation.

For multi-Higgs models, the Goldstone bosons are obtained with the same rotation angle for both the pseudoscalars and charged Higgs bosons, and as we found in the previous section $\gamma_A = \gamma_C = \frac{3\pi}{2} + \theta$. The fields in the Higgs basis are then given by the transformation:

$$\begin{pmatrix} \phi_{vev} \\ \psi_1 \\ \psi_2 \end{pmatrix} = R_A^T \begin{pmatrix} H_1 \\ H_2 \\ H_s \end{pmatrix}$$
$$= \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ -\sin\varphi & \cos\varphi & 0 \\ -\cos\theta\cos\varphi & -\cos\theta\sin\varphi & \sin\theta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_s \end{pmatrix}. \tag{46}$$

In our case, the rotation matrix takes the following form

$$\begin{pmatrix} \phi_{vev} \\ \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}v_2}{v} & \frac{v_2}{v} & \frac{v_3}{v} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}v_3}{2v} & -\frac{v_3}{2v} & \frac{2v_2}{v} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_s \end{pmatrix}. \tag{47}$$

Then, the electroweak (EW) Higgs doublets in this basis are explicitly given as

$$\phi_{vev} = \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}(v + \widetilde{H} + iG_0) \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} H_1^\pm \\ \frac{1}{\sqrt{2}}(\widetilde{H}_a + iA_1) \end{pmatrix},$$
$$\psi_2 = \begin{pmatrix} H_2^\pm \\ \frac{1}{\sqrt{2}}(\widetilde{H}_b + iA_2) \end{pmatrix}. \tag{48}$$

The rotation matrix above, (47), corresponds to the matrix built for the charged and pseudoscalars Higgs bosons mass eigenstates, denoted by $H_1^\pm$, $H_2^\pm$, $G^\pm$, G_0, A_1 and A_2. Whereas the neutral part of the Higgs doublets, denoted by $\widetilde{H}$, $\widetilde{H}_a$ and $\widetilde{H}_b$ do not correspond to their mass eigenstates, but they are in the Higgs basis. Now, in order to diagonalize the neutral sector we rotate through the angle α, obtaining the relationship between the intermediate-basis states in the Higgs basis and the physical states (mass eigenstates) for the neutral scalars as

$$\begin{pmatrix} \widetilde{H} \\ \widetilde{H}_a \\ \widetilde{H}_b \end{pmatrix} = \begin{pmatrix} \cos(\alpha-\theta) & 0 & \sin(\alpha-\theta) \\ 0 & 1 & 0 \\ -\sin(\alpha-\theta) & 0 & \cos(\alpha-\theta) \end{pmatrix} \begin{pmatrix} H_1 \\ h_0 \\ H_2 \end{pmatrix}. \tag{49}$$

We can get the neutral physical states from either the direct rotation Eq. (37), which transforms the interaction basis to the physical basis (mass basis), or through a two step rotation, from the interaction basis to the Higgs basis Eq. (47), and then to the physical basis Eq. (49). Either way we obtain the scalar Higgs masses given in Eqs. (40) and (41). We can see from expression (49), that there will be two alignment scenarios: (A) When $\widetilde{H} = H_2$ corresponds to the SM-like Higgs boson; (B) when we set $\widetilde{H} = H_1$ corresponding to the SM-like Higgs boson. $\widetilde{H}_a$ already corresponds to the physical state h_0.

The $\mathcal{Z}_2$ parity assignments for the physical and intermediate-basis states are given in Table 1. Notice that in the alignment limits the intermediate-basis states become the physical states.

Table 1 $\mathcal{Z}_2$ parity assignment for the physical states h_0, $A_{1,2}$ and $H_{1,2}^{\pm}$, and the intermediate-basis states $\tilde{H}$, and $\tilde{H}_b$. In the alignment limit the last two will correspond also to the physical states

Neutral scalars		Pseudoscalars		Charged scalars	
h_0	Odd	A_1	Odd	$H_1^{\pm}$	Odd
$\tilde{H}$	Even	A_2	Even	$H_2^{\pm}$	Even
$\tilde{H}_b$	Even				

3.2.1 Gauge-Higgs sector

In here we examine the scalar kinetic structure of the Lagrangian through the covariant derivative of the scalar fields. It is important to analyse the covariant derivative for scalar doublets in order to verify not only the electroweak symmetry breaking mechanism (EWSB), *i.e.* the contributions of the *vevs* to the gauge boson masses, but also to find the possible couplings among the Higgs and gauge bosons. The kinetic terms are taken as usual

$$\mathcal{L}_{kin} = (\mathcal{D}_\mu H_1)^\dagger (\mathcal{D}_\mu H_1) + (\mathcal{D}_\mu H_2)^\dagger (\mathcal{D}_\mu H_2)$$
$$+ (\mathcal{D}_\mu H_s)^\dagger (\mathcal{D}_\mu H_s). \tag{50}$$

Using the Higgs basis and Eq. (49) in order to get the physical states, we obtain the electroweak gauge bosons masses, $W^\pm$ and Z^0, as well as their couplings with the Higgs bosons, including the ones with A_μ, after performing the canonical rotation with the weak angle. Using the covariant derivative for the kinetic term, Eq. (50), in the Higgs basis and expanding the Higgs field about the vacuum, we get the Lagrangian in the physical basis. We show here some of the terms for illustration, the complete set of explicit couplings are given in next section and in Appendix A,

$$\mathcal{L}_{kin} \approx \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v \sin(\alpha - \theta)}{2} H_2 W_\mu^+ W^{-\mu}$$
$$+ \frac{g^2}{4} H_2 H_2 W_\mu^+ W^{-\mu}$$
$$+ \frac{g^2 v \cos(\alpha - \theta)}{2} H_1 W_\mu^+ W^{-\mu} + \frac{g^2}{4} H_1 H_1 W_\mu^+ W^{-\mu}$$
$$+ \frac{g^2}{4} h_0 h_0 W_\mu^+ W^{-\mu} + \cdots +$$
$$+ \frac{(g^2 + g'^2) v^2}{8} Z_\mu Z^\mu$$
$$+ \frac{(g^2 + g'^2) v \sin(\alpha - \theta)}{4} H_2 Z_\mu Z^\mu$$
$$+ \frac{(g^2 + g'^2)}{8} H_2 H_2 Z_\mu Z^\mu$$
$$+ \frac{(g^2 + g'^2) v \cos(\alpha - \theta)}{4} H_1 Z_\mu Z^\mu$$
$$+ \frac{(g^2 + g'^2)}{8} H_1 H_1 Z_\mu Z^\mu$$

$$+ \frac{(g^2 + g'^2)}{8} h_0 h_0 Z_\mu Z^\mu + \cdots \tag{51}$$

where H_1, H_2 and h_0 are the physical states. Then, the masses for the EW gauge bosons $W^\pm$ and Z^0 are obtained in the usual form

$$m_{W^\pm}^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = \frac{v^2}{4}(g^2 + g'^2) . \tag{52}$$

Furthermore, as the model has two different charged Higgs bosons $H_1^\pm$, $H_2^\pm$, we explicitly verified that mixed charged Higgs and gauge bosons couplings do not appear (e.g. $H_1^+ H_2^- \gamma$) as it should be in order to preserve the $\mathcal{Z}_2$ symmetry. We show it by calculating explicitly the part of the Lagrangian for the photon (this is exhibited implicitly in [66] as they calculate $H_{SM} \to \gamma\gamma$ through a loop of charged Higgs bosons)

$$\mathcal{L}_{H^+ H^- \gamma} = \frac{i g g'}{\sqrt{g^2 + g'^2}} \Big(H_1^+ \partial_\mu H_1^- - H_1^- \partial_\mu H_1^+$$
$$+ H_2^+ \partial_\mu H_2^- - H_2^- \partial_\mu H_2^+ \Big) A^\mu. \tag{53}$$

This is the usual expression that appears in other multi-Higgs models with a specific symmetry [89]; it coincides exactly with the one for some two Higgs doublet models [27], where a $\mathcal{Z}_2$ was assumed. In our case, the $\mathcal{Z}_2$ is a subgroup of the original S_3, a residual symmetry left over after EWSB, and not imposed separately.

In Sect. 4.1 we give the explicit form of the gauge-scalar couplings of the H_1 and H_2 neutral Higgs bosons of the model, in order to compare with SM couplings scenarios. The rest of the couplings, for the extended scalar sector, are given in Appendix 5.3, where we can see the manifestation of the $\mathcal{Z}_2$ symmetry as it allows only certain couplings.

3.3 Higgs masses and scenarios

The neutral Higgs boson h_0 is decoupled from the other two, due to the residual symmetry $\mathcal{Z}_2$, as was reported already in [66] (see Table 1). In the next section, we explicitly calculate all possible tree level couplings among the scalars and also, between the scalars and the gauge bosons. We show that, as expected, due to the $\mathcal{Z}_2$ symmetry, h_0 couples only in even numbers to the gauge bosons Eq. (51). Also, its trilinear scalar coupling is absent, as we will see in Sect. 4.2, Eqs. (56) and (71), so it is immediately excluded as the SM-like Higgs boson. Nevertheless, this neutral Higgs boson could be interesting as a possible dark matter candidate, provided it is the lightest particle in the $\mathcal{Z}_2$ odd sector, and has no couplings to the SM fermions.

The above discussion leaves us with two possible scenarios for which, either H_1 or H_2 is aligned to have the mass and couplings of the SM Higgs boson, and the other one would be

practically decoupled from the gauge bosons. We consider both scenarios for the numerical analysis.

Scenario A is defined by setting H_2, which has the lower mass among H_1 and H_2, as the SM-like Higgs boson. We further restrict the tree level value of its mass to be in the range 120–130 GeV, taking into account that it will receive radiative corrections. On the other hand, in scenario B the heavier Higgs boson H_1 is taken as the SM-like one, with its mass restricted to the interval 120–130 GeV, as in the previous scenario. For these two scenarios, the alignment means that the SM-like Higgs boson is maximally coupled to the gauge bosons, while the other one is practically decoupled. The alignment of the neutral scalar bosons in models with extended scalar sectors, in what would be the equivalent to our scenario A, is discussed in [90], as can be derived from Eq. (49).

A third, less natural case, would be a non-alignment scenario, where both Higgs bosons would couple equally or similarly to the gauge bosons. This analysis would be more complex, and a way to establish the non-observation of the second neutral Higgs would be needed, we will not consider that possibility here.

3.3.1 Higgs masses: scenario A

As already mentioned, in scenario A, from Eq. (49) we get $\tilde{H} = H_2$, and we set H_2 to be the SM-like Higgs boson with mass $\sim$ 125 GeV. The alignment limit, given in Ref. [90], can be seen explicitly in our case from Eq. (49) and corresponds to

$$\sin(\alpha - \theta) = 1 \quad \text{and} \quad \cos(\alpha - \theta) = 0. \tag{54}$$

In this scenario, H_2 couples maximally to the gauge bosons, and H_1 is decoupled from the gauge bosons. The third neutral scalar, h_0, is always decoupled from the gauge bosons due to the $\mathcal{Z}_2$ symmetry. A study of the masses in this scenario has been performed in [66], but with slightly different considerations as the ones taken here, as we explain below.

3.3.2 Higgs boson: scenario B

In scenario B, we take H_1 as the SM-like Higgs boson, coupled maximally to the gauge bosons (here from Eq. (49) we have $\tilde{H} = H_1$). The alignment limit in this scenario is expressed as

$$\sin(\alpha - \theta) = 0 \quad \text{and} \quad \cos(\alpha - \theta) = 1. \tag{55}$$

Although H_2 is always lighter than H_1, as can be seen from the expressions for the masses Eqs. (42) and (43), it does not couple to the gauge sector in this scenario, thus it could escape experimental detection. This scenario has not been analysed in the S3-3H model before. This could be interesting in the context of a possible Higgs decay of an exotic scalar with

mass $m_\Phi = 96$ GeV, as reported by CMS [91] and discussed later.

Applying these alignment limits to the Higgs neutral masses H_1 and H_2, Eqs. (42) and (43), the reduced expressions in each scenario can be obtained.

3.4 Numerical analysis and results

From the tree level Higgs mass expressions Eqs. (30–33), (42), and (43), we can calculate the masses in terms of the Higgs self-couplings $(a, \ldots, h)$ and θ, where $\tan \theta = 2v_2/v_3$. We perform a scan on the eight self-couplings and $\tan \theta$ (see Eqs. (2) and (24)). We produce $\mathcal{O}(10^{11})$ points with a pseudo-random generator, on these we first apply the stability and unitarity constrains as given in Ref. [66], to calculate the masses, and then we take out all points where the charged Higgs scalar masses are below 80 GeV [92,93]. On the surviving points, we apply the alignment constraints in both A and B scenarios. Finally, we impose a restriction on the mass of the respective SM-like Higgs boson. This gives us the mass range at tree level for the scalars in this model, with the above restrictions.

A similar analysis on scenario A has been performed in ZDas:2014fea, but in their analysis they restricted the mass of h_0 to be always heavier than m_{H_2} (the SM-like Higgs). Another difference is that we have applied the alignment limit within an approximation, to allow for the possibility of a minimal coupling to the non-SM Higgs, and we also allowed for a range of masses for the SM-like Higgs boson. On the other hand, scenario B has not been analysed before.

In Fig. 1 we show the dependence of the three neutral scalars masses m_{h_0}, m_{H_1}, and m_{H_2} on $\tan \theta$, for both scenarios, A in the left panel and B in the right one. The upper two graphs correspond to the mass of h_0, the two graphs in the middle correspond to the mass of H_1 and the two bottom graphs correspond to the mass of the H_2. The magenta points correspond to the unitarity and stability constraints only (also excluding charged Higgs boson masses below 80 GeV), the maroon points, are a subset of the magenta ones, which also satisfy the respective alignment limit in each scenario, with a 10% uncertainty on the $(\alpha - \theta)$ values, i.e., ± 0.1. Finally, the green points are a subset of the maroon ones, in which the SM-like Higgs boson mass has been restricted to the 120–130 GeV range.

As can be seen from the green points in Fig. 1, the restriction of the H_1 mass to the 120–130 GeV range constrains the allowed values for h_0 in scenario B, much more strongly than the equivalent restriction in scenario A. In the case of h_0 the allowed upper bound for $m_{h_0} \sim 600$ GeV in scenario B, is lower than in scenario A, where $m_{h_0} \lesssim 900$ GeV. It can also be seen from Fig. 1, that the Higgs neutral bosons could be degenerate in mass, nevertheless once we restrict to the SM value for one of them, this possibility gets drastically reduced
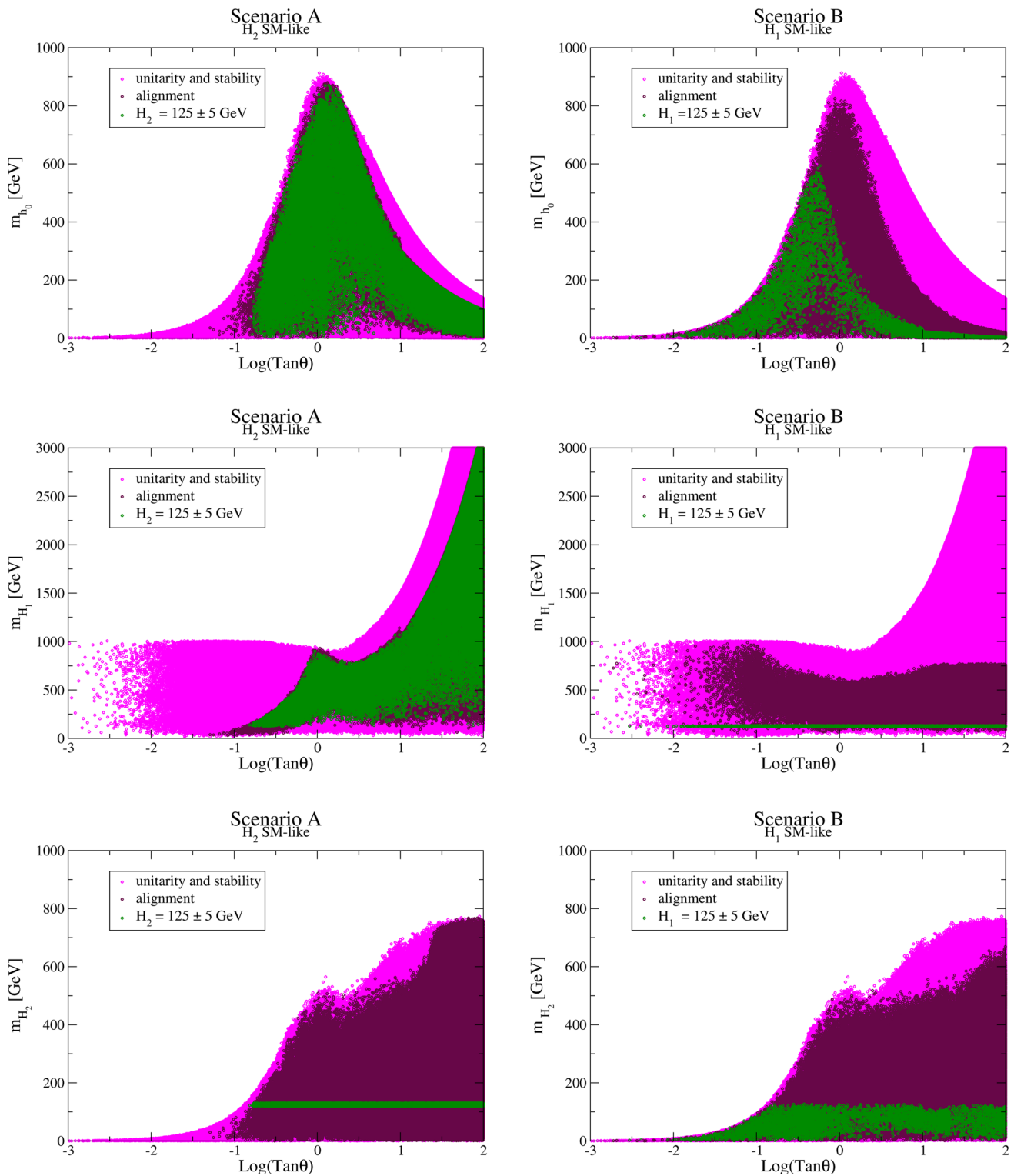
Fig. 1 Dependence of the neutral scalar masses, m_{h_0} and $m_{H_{1,2}}$, on $\tan\theta$ for scenario A (left) and B (right). The magenta points comply with the unitarity and stability conditions, the maroon points comply further with the alignment conditions in each scenario. Finally, the green ones have the SM-like mass restricted to $m_{H_{2,1}} = 125 \pm 5$ GeV, respectively
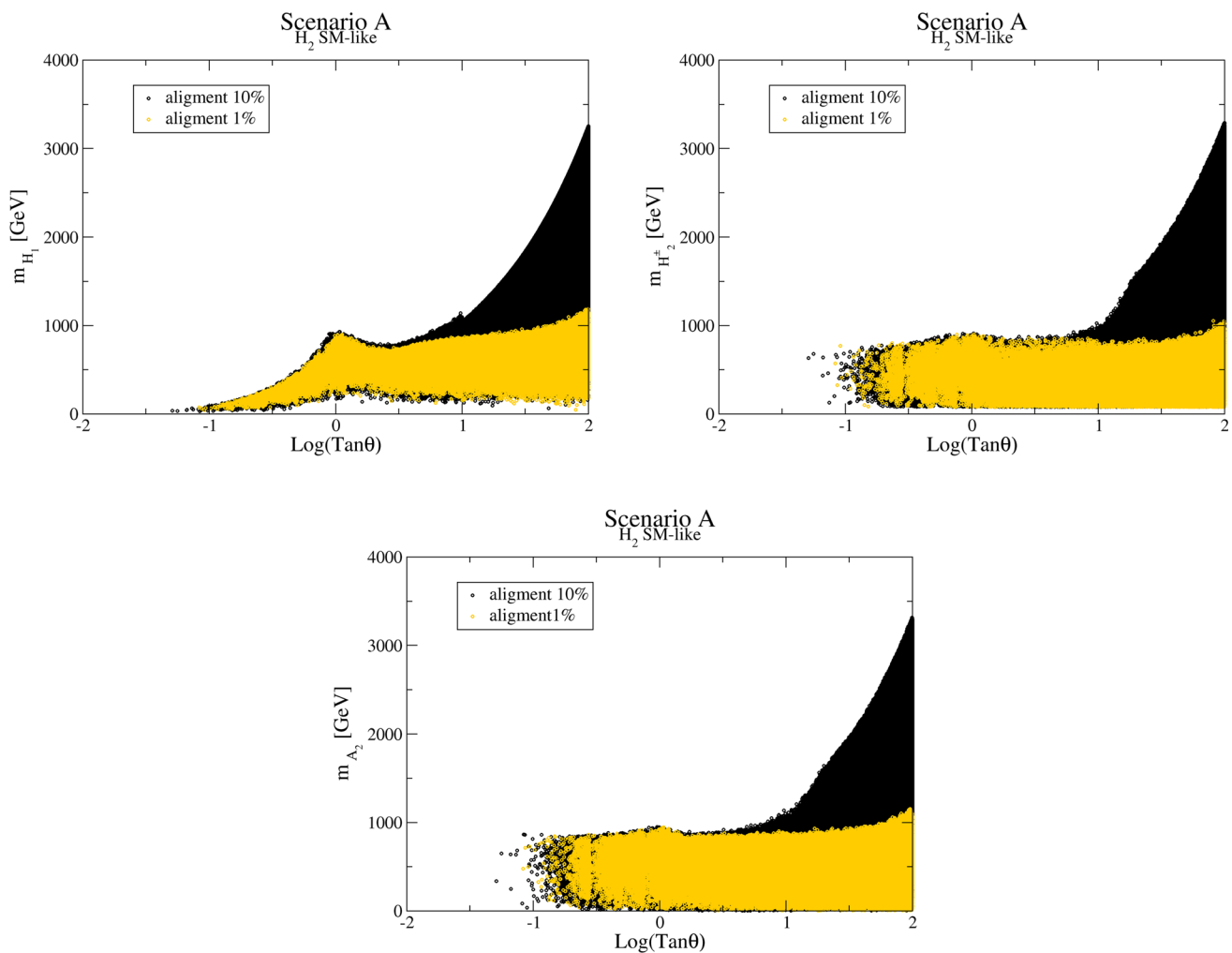
Fig. 2 Dependence of the masses m_{H_1}, $m_{H_2^\pm}$, and m_{A_2} on $\tan\theta$ for scenario A, applied with a 10% uncertainty (black points) and 1% uncertainty (yellow points) on $(\alpha - \theta)$. The points shown comply with the unitarity and stability conditions, and the restriction of $m_{H_2} = 125 \pm 5$ GeV

among H_1 and H_2, although h_0 could be still degenerate in mass with the other two (at tree level).

Notice that in both scenarios there exists the prospect of a lighter neutral Higgs boson to explain the possible decay of a scalar with $m_\phi \sim 96$ GeV reported by CMS [91,94]. This was reported as a $\gamma\gamma$ excess signal that could be due to a lighter neutral Higgs boson decay via a fermionic loop. This exotic Higgs boson role could be played by the lighter H_2 in scenario B, since it is always lighter than the SM Higgs, or by h_0 in both scenarios if it has couplings to fermions. This possibility, of a second lighter Higgs scalar consistent with this signal, has been explored in SUSY models in [95]. There are also recent analyses along these lines in 2HDM and N2HDM [96–98]. Experimental bounds on possible decays of this type of Higgs bosons will constrain further the parameter space.

In Fig. 2 we present the masses of H_1, $H_2^\pm$ and A_2 in scenario A which satisfy the alignment limit, Eq. (54), applied

with a 10% and 1% uncertainty on the $(\alpha - \theta)$ values. In the graphs we show only the masses which are affected by the precision of the values in $(\alpha - \theta)$. The black points are within 10% of the alignment limit and the yellow ones within 1%. The restriction to an alignment limit with 1% precision only appears as a noticeable difference for values of $\log(\tan\theta) > 1$, where the values of the masses are constrained to be below ~ 1 TeV. The rest of the masses in scenario A and the masses in scenario B are affected only very slightly by changing the precision in the alignment limit.

Finally in Fig. 3, we show the pseudoscalar masses $m_{A_{1,2}}$, and charged Higgs masses $m_{H_{1,2}^\pm}$ dependence on $\tan\theta$, after all constraints have been applied, including when one of the neutral scalars is restricted to be the SM-like Higgs boson. As already mentioned, points where $m_{H_{1,2}^\pm} < 80$ GeV have already been excluded in every figure. The figures are shown with a precision of 10% in the alignment limits. The first
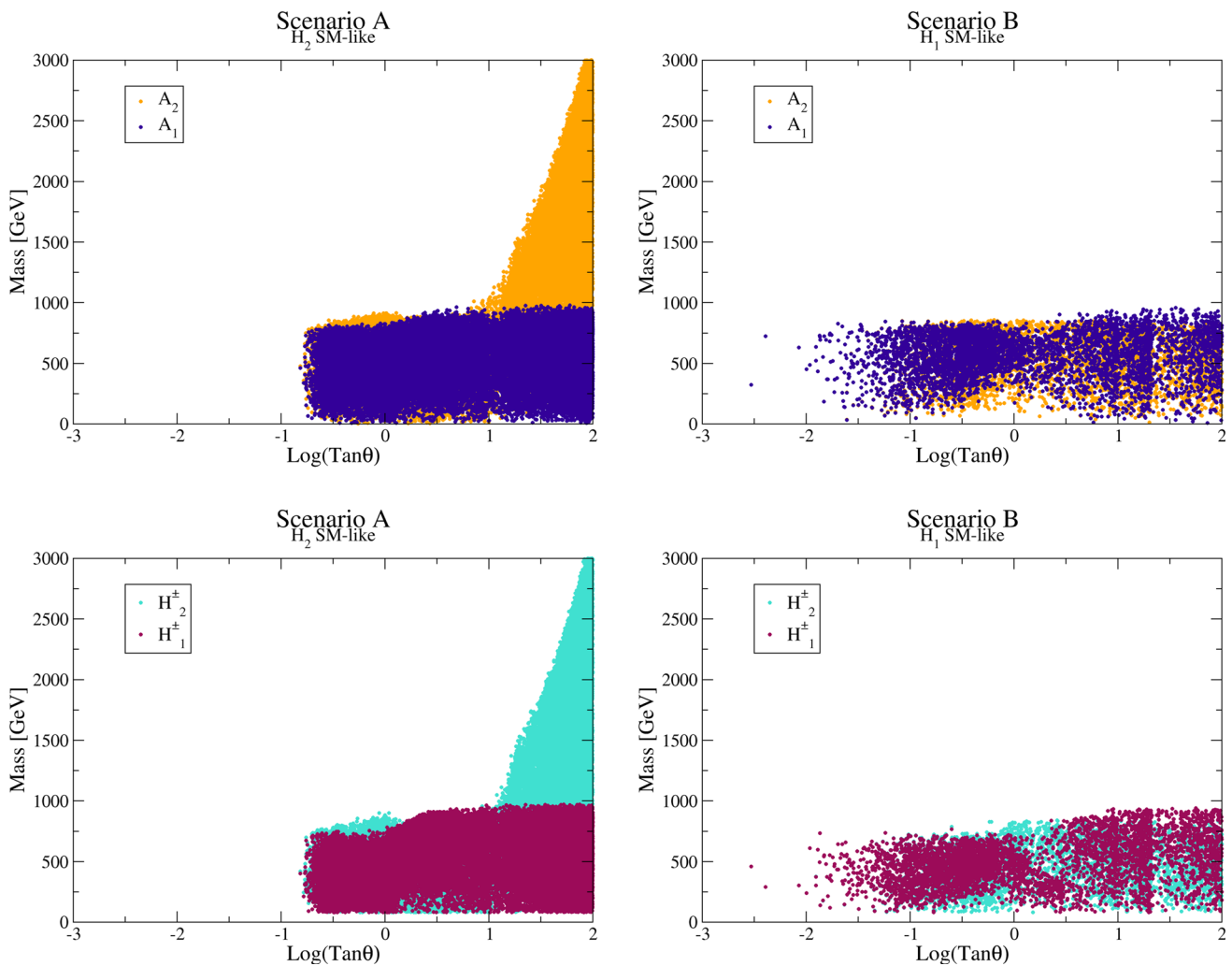
Fig. 3 Dependence of the two pseudoscalar masses $m_{A_{1,2}}$ (upper panel) and charged scalars $m_{H^{\pm}_{1,2}}$ (lower panel) on $\tan\theta$. The points shown comply with the constraints of previous figures plus the bounds on the SM-like Higgs boson mass for each scenario

two graphs correspond to the masses of the pseudoscalars in each scenario (A in the left, B in the right), the orange points represent the mass of A_2 and the purple ones represent the mass of A_1. The last two graphs correspond to the masses of the charged scalars in each scenario, the cyan points represent the mass of $H^{\pm}_2$ and the pink ones represent the mass of $H^{\pm}_1$. From this figure we can see what are the upper bounds for these masses, and that for small values of $\tan\theta$ they will be constrained to be below ~ 1 TeV. It can also be seen that there are regions in parameter space where the masses can be very close in value. This is relevant when calculating the values of the trilinear and quartic couplings, as well as the possible contributions to the oblique parameters, as will be discussed in next section.

We want to highlight here that these are tree level masses, radiative corrections will change their actual theoretical value, which in the case of the one identified as the SM Higgs,

we have taken into account with a conservative uncertainty of ± 5 GeV. A next-to-leading order (NLO) calculation of the masses should be done in order to give more accurate theoretical predictions, that could be tested at the LHC or future colliders. Work in this direction is proposed in [99]. NLO analytical expressions for the scalar contributions are given in 4.4. In order to perform a numerical calculation for these loop corrections, we need to establish the parameter dependence for the trilinear and quartic Higgs couplings, which we have calculated and whose expressions are given in Sect. 4.2 and in Appendix A.

An analysis of the scalar sector of a similar model with 4 Higgs doublets (S3-4H) [75], where the fourth doublet is inert and with some considerations in the Yukawa sector, shows that the region that satisfies the bounds on extra scalar searches [100], prefers values of $\tan\theta \lesssim 5$. As we will see in next section, this might also apply here.

4 Higgs couplings

In this section we calculate the trilinear and quartic couplings among the Higgs bosons, as well as with the gauge bosons, and give the analytical expressions in terms of the physical parameters. We analyse the contributions of these couplings to the neutral one-loop scalar mass matrix. We give the explicit expressions for scenario A.

4.1 Gauge-Higgs couplings

We expand the scalar kinetic Lagrangian term, Eq. (51), to calculate the couplings to the EW gauge bosons, performing the usual EW rotation on the gauge fields W_μ^3 and B_μ.

The residual $\mathcal{Z}_2$ symmetry manifests itself also in the gauge-Higgs couplings. We show here the couplings of gauge bosons with the three neutral scalars, the rest of the couplings are given in Appendix A, in this expression we have not taken into account the combinatorial factor from two identical particles in the Lagrangian term. Notice that h_0 does not couple in a single scalar coupling with the gauge bosons but it does in pairs with gauge bosons:

$$g_{h_0 W^\pm W^\mp} = 0, \quad g_{h_0 ZZ} = 0; \tag{56}$$

$$g_{H_1 W^\pm W^\mp} = \frac{2M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v},$$

$$g_{H_2 W^\pm W^\mp} = \frac{2M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v}; \tag{57}$$

$$g_{H_1 ZZ} = \frac{M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v},$$

$$g_{H_2 ZZ} = \frac{M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v}; \tag{58}$$

$$g_{h_0 h_0 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{h_0 h_0 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}; \tag{59}$$

$$g_{H_1 H_1 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{H_2 H_2 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}; \tag{60}$$

$$g_{H_1 H_1 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}, \quad g_{H_2 H_2 ZZ} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}. \tag{61}$$

The form of the cubic couplings reflects the residual $\mathcal{Z}_2$ symmetry. The couplings of the gauge bosons with h_0 vanish, as expected. The expressions for the gauge couplings to the other two neutral scalars $H_{1,2}$ are similar to the ones in the 2HDM [30], reflecting the fact that these two scalars decouple from h_0 due to the $\mathcal{Z}_2$ symmetry.

For the trilinear couplings, in the exact alignment limit, only the ones corresponding to the SM-like Higgs boson in each scenario will be different from zero.

4.2 Higgs-Higgs couplings

The trilinear and quartic Higgs couplings will be important to estimate radiative corrections, in particular for the SM Higgs, as well as possible loop contributions to physical processes. Previously, the trilinear couplings for the neutral scalars in the 3HDM with S_3 symmetry were reported in [67], nevertheless our results differ from the ones calculated there. On the other hand, our expressions for the trilinear couplings do coincide with the presence of a residual $\mathcal{Z}_2$, as reported in [66]. Besides the confirmation of this residual symmetry, we additionally show that the couplings reduce to the SM ones for the particular alignment limits.

The self-couplings given in the Higgs potential, Eq. (2) can be obtained in terms of physical parameters using the rotation matrices. The angle α given in Eq. (36) can be rewritten using the relations we obtained in Eq. (41). Thus, we can write Eq. (35) in terms of the physical Higgs masses and rotation angle α. Moreover, also using Eqs. (30)–(33) and Eqs. (40)–(41), we obtain expressions for the self-couplings in the scalar potential, Eq. (2), given in terms of the physical parameters *i.e.* masses, *vevs*, and rotation angles $(v, m_{h_0}^2, m_{H_1}^2, m_{H_2}^2, m_{A_1}^2, m_{A_2}^2, m_{H_1^\pm}^2, m_{H_2^\pm}^2, \tan\alpha, \tan\theta)$, as

$$a = \frac{1}{v^2 \cos^2\theta} \left[m_{H_1}^2 \cos^2\alpha + m_{H_2}^2 \sin^2\alpha - \frac{1}{9} m_{h_0}^2 \tan^2\theta \right], \tag{62}$$

$$b = \frac{1}{v^2} \left[\frac{\sin 2\alpha}{\sin 2\theta} (m_{H_1}^2 - m_{H_2}^2) + \frac{m_{h_0}^2}{9\cos^2\theta} + 2m_{H_2^\pm}^2 \right], \tag{63}$$

$$c = \frac{1}{v^2 \sin^2\theta} \left[m_{H_1}^2 \sin^2\alpha + m_{H_2}^2 \cos^2\alpha - \frac{1}{9} m_{h_0}^2 - m_{H_2^\pm}^2 \cos^2\theta + m_{H_1^\pm}^2 \right], \tag{64}$$

$$d = \frac{1}{v^2 \sin^2\theta} \left[(m_{H_1^\pm}^2 - m_{A_1}^2) - (m_{H_2^\pm}^2 - m_{A_2}^2) \cos^2\theta \right], \tag{65}$$

$$e = -\frac{4m_{h_0}^2}{9v^2 \sin 2\theta}, \tag{66}$$

$$f = \frac{1}{v^2} \left[\frac{m_{h_0}^2}{9\cos^2\theta} + m_{A_2}^2 - 2m_{H_2^\pm}^2 \right], \tag{67}$$

$$g = \frac{1}{v^2 \sin^2\theta} \left[\frac{4}{9} m_{h_0}^2 + m_{H_2^\pm}^2 \cos^2\theta - m_{H_1^\pm}^2 \right], \tag{68}$$

$$h = \frac{1}{v^2} \left[\frac{m_{h_0}^2}{9\cos^2\theta} - m_{A_2}^2 \right]. \tag{69}$$

This parameterization of the scalar potential self-couplings differs slightly from the ones presented in other works, as in [66,67], due to our normalization of the couplings in the scalar potential.

From the scalar potential we can get the trilinear scalar couplings as usual, where all possible combinations are given from the terms considered in the potential.

$$-i\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}. \qquad (70)$$

As we already mentioned, the $\mathcal{Z}_2$ residual symmetry present will imply a vanishing trilinear $h_0 h_0 h_0$ due to its odd charge under $\mathcal{Z}_2$, we explicitly confirmed this and also obtain the other trilinear and quartic scalar couplings. These couplings are essential in order to determine experimentally the shape of the actual Higgs potential. To this end, a one-loop calculation of the self-energy corrections and vertices should be performed. Moreover, it is possible to restrict parameters from the SM Higgs boson mass corrections, as we are going to consider in next section.

The following analytical expressions are the scalar-scalar couplings written in the physical basis and in terms of the physical parameters:[1]

$$g_{h_0 h_0 h_0} = 0, \qquad (71)$$

$$g_{H_2 H_2 H_2} = -\frac{1}{v\, s_{2\theta}} \left[m_{h_0}^2 \frac{c_{\alpha-\theta}^3}{9c_\theta^2} + m_{H_2}^2 \left(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta \right) \right], \qquad (72)$$

$$g_{H_1 H_1 H_1} = \frac{1}{v\, s_{2\theta}} \left[m_{h_0}^2 \frac{s_{\alpha-\theta}^3}{9c_\theta^2} - m_{H_1}^2 \left(c_\alpha^2 s_{\alpha-\theta} - s_\alpha c_\theta \right) \right], \qquad (73)$$

$$g_{h_0 h_0 H_1} = \frac{1}{v\, s_{2\theta}} (m_{h_0}^2 s_{\alpha+\theta} + m_{H_1}^2 s_\alpha c_\theta), \qquad (74)$$

$$g_{h_0 h_0 H_2} = -\frac{1}{v\, s_{2\theta}} (m_{h_0}^2 c_{\alpha+\theta} + m_{H_2}^2 c_\alpha c_\theta), \qquad (75)$$

$$g_{H_1 H_1 H_2} = -\frac{s_{\alpha-\theta}}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2} \right) + m_{H_1}^2 s_{2\alpha} + \frac{m_{H_2}^2 s_{2\alpha}}{2} \right), \qquad (76)$$

$$g_{H_1 H_2 H_2} = \frac{c_{\alpha-\theta}}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2} \right) + \frac{m_{H_1}^2 s_{2\alpha}}{2} + m_{H_2}^2 s_{2\alpha} \right), \qquad (77)$$

here we use the reduced notation $s_x \equiv \sin x$, $c_x \equiv \cos x$ and $t_x \equiv \tan x$.

In the following we show the analytical expressions for the trilinear couplings between scalars and pseudoscalars, as well as with the Goldstone boson. The residual $\mathcal{Z}_2$ symmetry is also evident in the allowed couplings with the pseudoscalars

(the forbidden ones are not present), which are given as:[2]

$$g_{A_1 A_1 H_1} = \frac{1}{v s_\theta} \left(-m_{h_0}^2 \frac{s_{\alpha-\theta}}{6c_\theta} + \frac{1}{2} m_{H_1}^2 s_\alpha + m_{A_1}^2 s_\alpha - m_{A_2}^2 c_\theta s_{\alpha-\theta} \right), \qquad (78)$$

$$g_{A_1 A_1 H_2} = \frac{1}{v s_\theta} \left(\frac{m_{h_0}^2 c_{\alpha-\theta}}{6c_\theta} - \frac{m_{H_2}^2 c_\alpha}{2} - m_{A_1}^2 c_\alpha + m_{A_2}^2 c_\theta c_{\alpha-\theta} \right), \qquad (79)$$

$$g_{A_2 A_2 H_1} = \frac{1}{v s_{2\theta}} \left(\frac{m_{h_0}^2 s_{\alpha-\theta}}{9c_\theta^2} + m_{H_1}^2 \left(s_\alpha c_\theta^3 - c_\alpha s_\theta^3 \right) + m_{A_2}^2 s_{2\theta} c_{\alpha-\theta} \right), \qquad (80)$$

$$g_{A_2 A_2 H_2} = \frac{1}{v s_{2\theta}} \left(-\frac{m_{h_0}^2 c_{\alpha-\theta}}{9c_\theta^2} + m_{H_2}^2 \left(s_\alpha s_\theta^3 - c_\alpha c_\theta^3 \right) + m_{A_2}^2 s_{2\theta} s_{\alpha-\theta} \right), \qquad (81)$$

$$g_{A_1 A_2 h_0} = \frac{2}{3 v s_{2\theta}} \left(-m_{h_0}^2 (c_{2\theta} + c_\theta^2) + 3 m_{A_1}^2 c_\theta^2 - 3 m_{A_2}^2 c_\theta^2 \right), \qquad (82)$$

$$g_{H_2 G_0 G_0} = \frac{m_{H_2}^2 s_{\alpha-\theta}}{2v},$$

$$g_{H_1 G_0 G_0} = \frac{m_{H_1}^2 c_{\alpha-\theta}}{2v}, \qquad (83)$$

$$g_{H_2 G_0 A_2} = \frac{c_{\alpha-\theta}}{v} (m_{H_2}^2 - m_{A_2}^2),$$

$$g_{H_1 G_0 A_2} = \frac{s_{\alpha-\theta}}{v} (-m_{H_1}^2 + m_{A_2}^2), \qquad (84)$$

$$g_{h_0 G_0 A_1} = \frac{1}{v} (m_{h_0}^2 - m_{A_1}^2). \qquad (85)$$

The couplings involving charged Higgses can be found in the Appendix. The diagonalization of the mass matrices leaves a structure which is similar to a 2HDM in the 2×2 block, which is $\mathcal{Z}_2$ even (H_2, H_1, $H_2^\pm$, A_2), plus decoupled particles which are $\mathcal{Z}_2$ odd ($H_1^\pm$, A_1, h_0). Notice though, that the allowed parameter spaces of both models might be different due to the extra $\mathcal{Z}_2$ odd scalar particles and their associated couplings in our model as compared to the 2HDM. For instance, we have possible extra channels for SM Higgs boson production and di-Higgs production, which in the LHC could be $pp \rightarrow Hqq$ and $pp \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$, $b\bar{b}b\bar{b}$, $b\bar{b}\tau^+\tau^-$ respectively, which will happen via the exchange of odd $\mathcal{Z}_2$ scalars, and will depend on the scalar trilinear couplings and on the Yukawa couplings of the model. There will also be loop corrections to the SM Higgs mass due to the $\mathcal{Z}_2$ odd particles, which are absent in the 2HDM. These processes may provide a possible way to differentiate

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[1] As we mentioned in the previous section the symmetry factor n! has to be added in front of the couplings for n identical particles in the vertex.

[2] The couplings with the Goldstone boson may be important depending on the renormalization procedure used.

this model from the 2HDM in the alignment limit [30]. These type of analyses should be performed in order to have more precise predictions and bounds on the parameter space of the model.

From expressions (82, 104) it can be seen that the trilinear couplings among h_0 and the other $\mathcal{Z}_2$ odd particles are allowed. As already mentioned, h_0 may be a dark matter candidate, provided it has no couplings to SM fermions and it is the lightest particle in the $\mathcal{Z}_2$ odd sector, thus $m_{H^\pm}, m_{A_1} > m_{h_0}$.

The quartic scalar couplings written in the physical basis and in terms of the physical parameters are calculated from

$$-i\lambda_{ijkl} = \frac{-i\,\partial^4 V}{\partial H_i \partial H_j \partial H_k \partial H_l}. \tag{86}$$

We also give the analytical expressions for the quartic self-scalar couplings written in the physical basis and in terms of the physical parameters. We are interested in particular in the couplings with $H_{1,2}$, in order to compare with the SM ones in the alignment limits. The quartic couplings also necessary to calculate the one-loop corrections to the Higgs bosons masses. We give here some explicit four scalar couplings examples, the rest of the couplings can be found at the Appendix A.2:

$$g_{h_0 h_0 h_0 h_0} = \frac{1}{24 v^2 s_\theta^2}\left(m_{h_0}^2 + 3m_{H_1}^2 s_\alpha^2 + 3m_{H_2}^2 c_\alpha^2\right), \tag{87}$$

$$g_{H_1 H_1 H_1 H_1} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2 s_{\alpha-\theta}^3 \frac{(s_{\alpha-\theta} + 2s_{\alpha+\theta})}{9c_\theta^2}\right. \\ \left. + m_{H_1}^2(s_\alpha^2 s_{\alpha-\theta} + c_\alpha s_\theta)^2 + m_{H_2}^2\frac{s_{2\alpha}^2 s_{\alpha-\theta}^2}{4}\right), \tag{88}$$

$$g_{H_2 H_2 H_2 H_2} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2 c_{\alpha-\theta}^3 \frac{(c_{\alpha-\theta} + 2c_{\alpha+\theta})}{9c_\theta^2}\right. \\ \left. + m_{H_1}^2\frac{s_{2\alpha}^2 c_{\alpha-\theta}^2}{4} + m_{H_2}^2(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta)^2\right). \tag{89}$$

4.3 Couplings in scenario A

We show here how the scalar couplings are reduced in the alignment limit of scenario A. Recalling that the alignment limit is given as, $\sin(\alpha - \theta) = 1$, $\cos(\alpha - \theta) = 0$, the trigonometric functions for α and θ satisfy the following relations

$$\sin\alpha = \cos\theta; \ \cos\alpha = -\sin\theta; \ \sin 2(\alpha - \theta) = 0;$$
$$\cos(3\alpha - \theta) = \sin 2\theta; \ \sin(\alpha + \theta) = \cos 2\theta;$$
$$\cos(\alpha + \theta) = -\sin 2\theta. \tag{90}$$

In scenario A in the alignment limit, the Higgs boson H_2 trilinear coupling coincides exactly with the trilinear cou-

pling of the SM Higgs boson λ_{SM},

$$g_{H_2 H_2 H_2} = \frac{1}{v\,s_{2\theta}}\left[m_{H_2}^2 s_\alpha s_\theta\right]$$
$$= \frac{1}{2v}\frac{s_\alpha}{c_\theta}m_{H_2}^2 = \frac{m_{H_2}^2}{2v} \equiv \lambda_{SM}. \tag{91}$$

And the H_1 trilineal couplings reduces to

$$g_{H_1 H_1 H_1} = \frac{1}{v\,s_{2\theta}}\left[\frac{1}{9c_\theta^2}m_{h_0}^2 - s_\theta^2 m_{H_1}^2\right]$$
$$= \frac{1}{v\,s_{2\theta}c_\theta^2}\left[\frac{1}{9}m_{h_0}^2 - \frac{1}{2}s_{2\theta}m_{H_1}^2\right]. \tag{92}$$

The H_2 quartic coupling (88) also reduces exactly to the SM one in the alignment limit,

$$g_{H_2 H_2 H_2 H_2} = \frac{1}{2v^2 s_{2\theta}^2}m_{H_2}^2(-s_\theta^3 c_\theta - c_\theta^3 s_\theta)^2$$
$$= \frac{m_{H_2}^2}{8v^2}. \tag{93}$$

The $H_2 - h_0$ quartic coupling reduces in this limit to

$$g_{H_2 H_2 h_0 h_0} = \frac{1}{v^2 s_{2\theta}}\left(\frac{1}{6}m_{h_0}^2 3s_{2\theta} + \frac{1}{4}m_{H_2}^2 s_{2\theta}\right)$$
$$= \frac{1}{4v^2}(2m_{h_0}^2 + m_{H_2}^2). \tag{94}$$

Some of the reduced scalar couplings for scenario A depend only on the masses involved, and are given as

$$g_{H_2 h_0 h_0} = \frac{1}{2v}(m_{H_2}^2 + 2m_{h_0}^2),$$
$$g_{H_2 A_1 A_1} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_1}^2),$$
$$g_{H_2 A_2 A_2} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_2}^2),$$
$$g_{H_2 H_1^\pm H_1^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_1^\pm}^2),$$
$$g_{H_2 H_2^\pm H_2^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_2^\pm}^2),$$
$$g_{H_2 H_2 H_2 H_1} = g_{H_1 H_1 H_1 H_2} = 0. \tag{95}$$

From these expressions, a lower bound for all the scalar masses (other than H_1, which is always heavier than H_2 in this scenario), can be set at $\gtrsim 63$ GeV, since there is no experimental observation of decays of the SM-Higgs boson to other scalars. This is in natural agreement with the current bounds for charged scalars, which set their masses above ~ 80 GeV [92,93]. The recent signal for the rare three-body decay of the SM Higgs boson to photon and dileptons [101], will put extra constraints in the values of the allowed trilinear couplings.

The couplings of the gauge bosons to the SM Higgs have been determined with a $\sim 5\%$ precision [25,26,102]. From our tree level expressions for the gauge-Higgs couplings,

Eqs. (57, 58) we can parameterize a deviation of the SM value by

$$\cos(\alpha - \theta) = \cos(\frac{\pi}{2} - \epsilon) = \sin \epsilon \equiv \delta, \tag{96}$$

where in the exact alignment limit $\delta = 0 = \epsilon$. A value of $\delta \lesssim 0.1$ is compatible with the current experimental measurements and is consistent with our assumption of a 10% deviation of the alignment limit in $(\alpha - \theta)$ in Fig. 1.

On the other hand, a deviation in the SM trilinear self-coupling λ_{SM} will have an impact in di-Higgs production at tree-level [103,104], single Higgs boson production and decays at one-loop level [105], as well as in electroweak precision observables at two-loop level [106]. In our case, we can describe a small deviation of the alignment limit at tree level in terms of δ, θ and m_{h_0} as

$$g_{H_2 H_2 H_2} \equiv \lambda_{SM} \kappa_\lambda = \frac{m_{H_2}^2}{2v} \left[(1 + 2\delta^2)\sqrt{1 - \delta^2} \right.$$
$$\left. + \delta^3 (\tan\theta - \cot\theta) - \frac{m_{h_0}^2}{m_{H_2}^2} \frac{\delta^3}{9 s_\theta c_\theta^3} \right], \tag{97}$$

where the term in square brackets κ_λ, is the scaling factor that parameterizes the deviation of the SM Higgs trilinear self-coupling, in this case at tree level. The value of the trilinear self coupling has already been constrained experimentally [102,107]. In here, we will make use of the modifier or κ framework [108] and the results in [109], where they set limits to κ_λ, assuming the rest of the SM Higgs couplings to fermions and gauge bosons are the same or very close to the SM. In our case, the value of κ_λ will depend on δ, m_{h_0} and θ. From Fig. 1 we can see the dependence on m_{h_0} on $\tan\theta$, which for a given δ allows us to determine the value of κ_λ. As an example we take $\delta \sim 0.1$ and we fix m_{h_0} to its possible maximum value for a given $\tan\theta$. In order to satisfy the bounds $-1.8 < \kappa_\lambda < 9.2$, as determined in [109], $\tan\theta \lesssim 15$. For smaller values of δ larger values of $\tan\theta$ are allowed.

In case the alignment limit is exact, λ_{SM} will still get corrections, but at loop level. In that case the factor κ_λ will have a different expression, and depending on how complicated it is and what other restrictions are taken into account it might be possible to restrict the parameter space through it.

Analogous expressions for the couplings can be found for scenario B. In this case, the SM-like Higgs boson would be H_1 and the other neutral Higgs, H_2, would be lighter than the SM-like, at tree level. As we already discussed, we cannot fully discard this possibility since in this alignment scenario, H_2 would not have couplings to the gauge bosons, and it could escape experimental detection.

We do not consider the most general case, without any alignment, since it implies that both neutral Higgs bosons couple to the gauge bosons, which is highly restricted from the experimental data.

4.4 Higgs one-loop self energy

As we said before, the importance of having explicitly the Higgs couplings is relevant to calculate the radiative corrections or the possible loop contributions to different processes where the Higgs bosons are involved, including the radiative corrections to the SM Higgs mass and its renormalization [110]. For any BSM model the extra contributions to the oblique parameters [111], should fit the experimental data. There is work done in this direction for multi-Higgs models in [82,83], where they explore the parameter space for N-Higgs doublet models. Their results imply that the Higgs masses should be almost degenerate, in a compact scalar spectrum. In their work, the assumption is that the new Higgs bosons mass scale should be above the EWSB scale, and that all scalars couple to the gauge bosons. In our case, from the explicit form of the couplings, as e.g. in Eq. (56), it can be seen that some Higgs loop contributions will not be present, so the relevant loop contribution calculated in [83] will not appear for h_0, meaning that the restriction of $m_{h_0} > m_V$ (with $V = W, Z$) considered there is not required in our case. The same applies for the other Higgs bosons, (not the SM-like) considered in the two alignment scenarios that we explore in this work, where some of the couplings to gauge bosons are null.

Considering CP-invariance, the renormalized neutral Higgs masses would be written as two diagonal 3×3 block matrices, one for the CP-even neutral states of the Higgs doublets (h_0, H_1, H_2), and the second for the CP-odd Higgs states (A_1, A_2, G^0)

$$\mathcal{M}_{\phi^0}^2(s) = \mathcal{M}_H^{(0)2} + \begin{pmatrix} \hat{\Pi}^S(s) & 0 \\ 0 & \hat{\Pi}^P(s) \end{pmatrix}, \tag{98}$$

where $\mathcal{M}_H^{(0)2}$ is the Higgs mass matrix at tree level given in Sect. 3, the neutral part of expression (25), with explicit tree-level neutral masses given in (40), (41), (30) and (31). The complete renormalized neutral Higgs self-energies at one-loop level, $\hat{\Pi}^S(s)$ should be taken with the usual prescription, given for example in [112], adapting it to the S3-3H model.

In our model, the one-loop contributions to the unrenormalized mass corrections $\Pi^{S,(\bar{q})}(s)$, that come only from the scalar sector self-energy, denoted as Σ^ϕ, would indicate corrections due to scalar bosons on the loop. In general we would have corrections to the Higgs masses considering scalar and gauge bosons, as well as fermions; in particular to the neutral scalar mass matrix we will have:

$$\Pi^{S,(\bar{q})}(s) = \Sigma^\phi(s) + \Sigma^V(s) + \Sigma^f(s). \tag{99}$$

Due to the $\mathcal{Z}_2$ residual symmetry, the only trilinear coupling that involves a single h_0, which would give rise to a one-loop mass correction, is the one with two different charged scalars $h_0 H_1^\pm H_2^\mp$ (Eq. (104) in the Appendix). This mixed

charged Higgs coupling is not present for the other two neutral Higgs bosons, avoiding the mixing of h_0 with the other neutral scalars at one-loop level, in this case via charged Higgs loops.

For the quartic couplings, there is no coupling that involves a single h_0 with a pair of identical Higgs bosons (including with three identical ones), see Appendix A.2. This implies that there are no possible mass one-loop corrections that could mix h_0 with the other neutral scalars, H_1 and H_2. Moreover, we can see from the gauge couplings with h_0 given in Sect. 4.1, that there are only corrections to the h_0 mass but no mixing with other neutral Higgs bosons. Thus, the decoupling is kept at one-loop level, as expected, with the consequence that the one-loop neutral scalar mass matrix will attain a block diagonal form

$$\Sigma^\phi(s) + \Sigma^V(s) = \begin{pmatrix} \Sigma_{h_0}^{\phi,V}(s) & 0 & 0 \\ 0 & \Sigma_{H_1}^{\phi,V}(s) & \Sigma_{H_1 H_2}^{\phi,V}(s) \\ 0 & \Sigma_{H_2 H_1}^{\phi,V}(s) & \Sigma_{H_2}^{\phi,V}(s) \end{pmatrix}. \tag{100}$$

We see from the above expression, that even at one-loop the h_0 scalar is decoupled from the other two, so the mass matrix structure of the other two neutral scalars is similar to the 2HDM. Nevertheless, there will be loop corrections to the $H_{1,2}$ masses due to h_0, as can be seen from the couplings (74) and (75). On the other hand, h_0 will also receive corrections to its mass via the gauge boson loop, due to the allowed couplings (59).

The general scalar and gauge bosons contributions to the square mass terms for H_2 and H_1 are given as:

$$\Sigma_{H_n}^{\phi,V} = \sum_i \frac{g_{H_n H_n \phi_i^0 \phi_i^0}}{16\pi^2} A0(m_{\phi_i^0}^2)$$

$$+ \sum_{i,j} \frac{g_{H_n \phi_i^0 \phi_j^0}^2}{8\pi^2} B0(p^2, m_{\phi_i^0}^2, m_{\phi_j^0}^2)$$

$$+ \sum_k \frac{g_{H_n \phi_k^\pm \phi_k^\mp}^2}{8\pi^2} B0(p^2, m_{\phi_k^\pm}^2, m_{\phi_k^\pm}^2)$$

$$+ \sum_i \frac{g_{H_n H_n V_i V_i}}{16\pi^2} A0(m_{V_i}^2)$$

$$+ \sum_i \frac{g_{H_n V_i V_i}^2}{8\pi^2} B0(p^2, m_{V_i}^2, m_{V_i}^2), \tag{101}$$

with $n = 1, 2$.[3] For the mixing term H_{12} we get

$$\Sigma_{H_1 H_2}^{\phi,V} = \sum_i \frac{g_{H_1 H_2 \phi_i^0 \phi_i^0}}{16\pi^2} A0(m_{\phi_i^0}^2)$$

$$+ \sum_{i,j} \frac{g_{H_1 \phi_i^0 \phi_j^0} g_{H_2 \phi_i^0 \phi_j^0}}{8\pi^2} B0(p^2, m_{\phi_i^0}^2, m_{\phi_j^0}^2)$$

$$+ \sum_k \frac{g_{H_1 \phi_k^\pm \phi_k^\mp} g_{H_2 \phi_k^\pm \phi_k^\mp}}{8\pi^2} B0(p^2, m_{\phi_k^\pm}^2, m_{\phi_k^\pm}^2)$$

$$+ \sum_i \frac{g_{H_1 V_i V_i} g_{H_2 V_i V_i}}{8\pi^2} B0(p^2, m_{V_i}^2, m_{V_i}^2)$$

$$+ \sum_k \frac{g_{H_1 \phi_k^\pm W^\mp} g_{H_2 \phi_l^\pm W^\mp}}{8\pi^2} B0(p^2, m_{\phi_l^\pm}^2, m_W^2), \tag{102}$$

where $\phi_{i(j)}^0 = h_0, H_1, H_2, A_1, A_2, G^0$, $\phi_k^\pm = H_{1,2}^\pm, G^\pm$ and $V_i = W^\pm, Z^0$. In these expressions, $A0$ and $B0$ are the Passarino-Veltman functions of the masses involved [113]. The radiative contributions to the mixing of $\Sigma_{H_1 H_2}^{\phi,V}(s)$ reduce when we apply the alignment limit. For scenario A, the couplings reduce such that the one-loop corrections to the mixing term are given as follows

$$\Sigma_{H_1 H_2}^\phi = \sum_i \frac{g_{H_1 H_2 \phi_i^0 \phi_i^0}}{16\pi^2} A0(m_{\phi_i^0}^2)$$

$$+ \sum_i \frac{g_{H_1 \phi_i^0 \phi_i^0} g_{H_2 \phi_i^0 \phi_i^0}}{8\pi^2} B0(p^2, m_{\phi_i^0}^2, m_{\phi_j^0}^2)$$

$$+ \sum_k \frac{g_{H_1 \phi_k^\pm \phi_k^\mp} g_{H_2 \phi_k^\pm \phi_k^\mp}}{8\pi^2} B0(p^2, m_{\phi_k^\pm}^2, m_{\phi_k^\pm}^2), \tag{103}$$

in this case we will only have $\phi_i^0 = h_0, A_1, A_2, \phi_k^\pm = H_{1,2}^\pm$, since all the terms involving gauge and Goldstone bosons vanish, so we simplify the notation to $\Sigma_{H_1 H_2}^\phi$. This is taking into account only the scalar and gauge contributions to the one-loop corrections. An equivalent expression can be found for scenario B.

We explore the structure of the loop contributions to the h_0, H_1 and H_2 masses coming from the gauge bosons and all scalars (neutral scalars and pseudoscalars, and charged scalars) by fixing the tree level masses and varying the θ parameter.

A mixing term for H_1 and H_2 in the mass matrix, Eq. (100), would imply that they are not the true eigenstates. At one-loop level, we would expect this mixing parameter to be small, as the tree level should be the dominant order. Formally, we should take the poles of the propagator of the mass matrix at the order we are calculating to obtain the masses of the particles, in order to define two different states the mass matrix should be diagonalized at n-loop order [110]. Keeping this corrections small, is another condition we could consider to constrain the free parameters of the model. Although complete NLO corrections should be taken into account (including fermions), our goal here is to show the importance of having the explicit form of the cubic and quartic couplings

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[3] The terms where gauge bosons are involved show only the coupling contributions, the actual calculation will have to involve the gauge fixing.

in order to be able to calculate loop corrections, which are functions of the model's parameters.

In Table 2 we present two example sets of parameters for these corrections under scenario A. The choice of the benchmark points where these mixing parameters vanish, is meant to exemplify that there are regions of parameter space where indeed these one-loop corrections may be small. These examples correspond to points in parameter space where the mixing term in the mass matrix Eq. (100) vanishes.

For the light scalar spectrum we achieve $\Sigma^{\phi}_{H_1 H_2}(s) = 0$ with $\tan\theta = 1$, but the scalar masses are different, so the condition to keep the contributions to the oblique parameters S, T, small might not be met. On the other hand, we find a spectrum with heavier scalar masses, where $\Sigma^{\phi}_{H_{1,2}}(s) = 0$ and $\tan\theta \sim 2$. A complete numerical exploration of these corrections could restrict more the parameter space, as they should be kept small. Our goal here is only to show the possible loop corrections that will be present in their general form. The small value of $\tan\theta$ found in these two examples indicates a maximal mixing between the S_3 singlet and doublet (see Eqs. (44, 45)). It is also consistent with a numerical study of the S3-4H model (basically the S3-3H with one extra inert Higgs doublet), where compliance with the experimental Higgs bounds was found for small values of $\tan\theta$, assuming certain conditions on the Yukawa couplings [75].

Results in Table 2 are not conclusive, since we should take into account the fermionic contributions to have a more accurate estimation of the radiative corrections to scalar masses. In particular, the top quark contribution is expected to be sizeable, due to its large Yukawa coupling. Work along these lines is in progress.

5 Summary and conclusions

The S3-3H model is an interesting and promising extension of the SM, that can accommodate well the masses and mixing of the quarks, leading to the NNI matrices [42], as well as leptons, where it naturally gives a non-zero reactor mixing angle [52,57]. In this paper we studied the gauge and scalar sectors of this model. We chose a geometrical parameterization in spherical coordinates, which allowed us to express our results and analytical expressions in terms of the mixing angle, $\tan\theta$, between fields in the doublet and the singlet irreps. From here it became clear that, in order to have realistic physical scenarios, without massless scalars, this mixing must be always different from zero. As previously found [66], there is a residual $\mathcal{Z}_2$ symmetry, which decouples one of the neutral scalars h_0 from the gauge bosons. This raises the interesting possibility of treating this decoupled scalar as a dark matter candidate, although we still have to probe its fermionic couplings. This possibility will be explored in a forthcoming publication.

We performed a numerical analysis on the parameter space, taking into account unitarity and stability bounds, as well as the current experimental bounds on the charged masses. We studied two possible alignment scenarios, A and B, in which one of the two $\mathcal{Z}_2$ even neutral scalars $H_{1,2}$, is maximally coupled to the gauge bosons, and is thus taken to be as the SM Higgs. In scenario A, the lighter of the two neutral scalars, H_2, is the SM-like Higgs boson. The other possibility, scenario B, where the heavier H_1 is the SM-like Higgs boson, cannot be a priori excluded, since H_2 could have escaped detection due to the absence of couplings to the vector bosons.

We found the allowed ranges for the scalar masses, in terms of $\tan\theta$ in each alignment scenario, with a 10% and 1% precision on the $(\alpha - \theta)$ values. Our results show a clear restriction for all the scalar masses, which are mostly below TeV. This corroborates similar analysis in this direction for scenario A, given in [66] (although we allowed for a small deviation of the alignment limit, and for some uncertainty in the SM Higgs mass). Scenario B in this model has not been analysed before. The light H_2 scalar in this scenario opens the possibility that it might be regarded as the 96 GeV scalar, which was suggested as a diphoton signal reported by CMS [91], and discussed in the literature in the context of SUSY and 2HDM models [95,96,98]. The same is possible for h_0 in both alignment limits.

We calculated all trilinear and quartic couplings between the Higgs bosons, and also among the Higgs and gauge bosons, giving analytical expressions in terms of the physical parameters of the model. We found discrepancies with previously reported trilinear scalar couplings in this model given in [67], where the $\mathcal{Z}_2$ symmetry is not reported nor explicitly present. On the contrary, our expressions do confirm the existence of the residual $\mathcal{Z}_2$ symmetry, as only $\mathcal{Z}_2$ preserving couplings are present, consistent with the model structure reported in [66], although they do not give the couplings explicitly. In our expressions, both the trilinear and quartic couplings for the SM-like Higgs boson reduce to the SM ones in the exact alignment limits.

From our numerical analysis we found that the scalar masses could be very close or even degenerate, but since we performed a random scan of the Higgs self-couplings, the allowed masses shown do not imply they are necessarily degenerate for the same set of parameters, although we do find specific examples where this is the case. One such example is the heavy spectrum of Table 2, where all the scalar masses are degenerate, as is required to keep the contributions to the oblique parameters small.

The small deviation δ we considered of the alignment limit at tree level, is compatible with the latest experimental results on Higgs-gauge boson couplings. This deviation can also be used to parameterize the contribution of the extra scalars of our model to the SM trilinear coupling λ_{SM}. The current

Table 2 Parameter values in scenario A that make the one-loop mixing parameter vanish, $\Sigma^{\phi}_{H_1 H_2} = 0$, taking into account only the scalar and gauge contributions

Scalar benchmarks	Masses (GeV)	$\tan\theta$
light spectrum	$m_{h_0} = 80,\, m_{H_1} = 200,\, m_{A_{1,2}} = 80,\, m_{H^{\pm}_{1,2}} = 100$	1
heavy spectrum	$m_{h_0} = 800,\, m_{H_1} = 800,\, m_{A_{1,2}} = 800,\, m_{H^{\pm}_{1,2}} = 800$	2.1

fits on the λ_{SM} value, and our assumption of a deviation of the alignment limit of $\delta \sim 0.1$, sets an upper bound on $\tan\theta \lesssim 15$.

In the exact alignment limit, some of the trilinear couplings depend only on the scalar masses, which sets a natural lower bound for all masses (other than $H_{1,2}$) to $\gtrsim 63$ GeV, since no SM Higgs decay into two lighter scalars has been observed experimentally. The inclusion of radiative corrections might change these bounds.

We obtained the analytical expressions for the one-loop corrections to the SM-like Higgs, due to scalar and gauge bosons in the loop, and found that the decoupling of h_0 remains at one-loop level, as expected from a symmetry of the Lagrangian. From the reduced expressions for the couplings in scenario A, we calculated the value of $\tan\theta$ for which the one-loop mixing of H_1 and H_2 vanishes, $\Sigma^{\phi}_{H_1 H_2} = 0$, for two benchmark mass values. These results point to a value of $\tan\theta \approx \mathcal{O}(1)$, indicating a large mixing between the S_3 doublet and the singlet, consistent with what was reported in [75].

The model has different one-loop couplings to the Higgs bosons through the extra $\mathcal{Z}_2$ odd particles (A_1, $H^{\pm}_1$, h_0), as compared to the 2HDM. Thus, although it reduces to a form similar to the 2HDM due to the presence of the residual $\mathcal{Z}_2$ symmetry, the extra $\mathcal{Z}_2$ odd particles will change the possible channels for decay and production of particles, and also the structure of radiative corrections. In particular they will have an impact on SM Higgs production, di-Higgs production, and loop corrections to the SM Higgs boson mass.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The results on this article are based upon the authors' computations, as shown in the graphs. There is no further associated data.]

Appendix A: Higgs couplings

5.1 A.1 Scalar trilinear couplings

Here we explicitly write down the trilinear couplings of neutral with charged Higgs bosons and the rest of allowed quartic couplings of Higgs bosons. Here we are able to see the residual $\mathcal{Z}_2$ symmetry, the rest of the couplings are absent, as can be found from the direct calculation given in (70) and (86)

$$g_{h_0 H^{\pm}_1 H^{\pm}_2} = \frac{1}{3 v s_\theta}\left(-m^2_{h_0}\frac{c_{2\theta} + c^2_\theta}{c_\theta}\right.$$
$$\left. + 3 m^2_{H^{\pm}_1} c_\theta - 3 m^2_{H^{\pm}_2} c_\theta\right), \tag{104}$$

$$g_{H_1 H^{\pm}_1 H^{\pm}_1} = \frac{2}{v s_{2\theta}}\left(-\frac{m^2_{h_0}}{3} s_{\alpha-\theta} + m^2_{H_1} c_\theta s_\alpha\right.$$
$$\left. + 2 m^2_{H^{\pm}_1} c_\theta s_\alpha - 2 m^2_{H^{\pm}_2} c^2_\theta s_{\alpha-\theta}\right), \tag{105}$$

$$g_{H_2 H^{\pm}_1 H^{\pm}_1} = \frac{2}{v s_{2\theta}}\left(\frac{m^2_{h_0}}{3} c_{\alpha-\theta} - m^2_{H_2} c_\theta c_\alpha\right.$$
$$\left. - 2 m^2_{H^{\pm}_1} c_\theta c_\alpha + 2 m^2_{H^{\pm}_2} c^2_\theta c_{\alpha-\theta}\right), \tag{106}$$

$$g_{H_1 H^{\pm}_2 H^{\pm}_2} = \frac{2}{v s_{2\theta} c^2_\theta}\left(\frac{m^2_{h_0}}{9} s_{\alpha-\theta} + m^2_{H_1} c^2_\theta\left(s_\alpha c^3_\theta + c_\alpha s^3_\theta\right)\right.$$
$$\left. + m^2_{H^{\pm}_2} s_{2\theta} c^2_\theta c_{\alpha-\theta}\right), \tag{107}$$

$$g_{H_2 H^{\pm}_2 H^{\pm}_2} = \frac{2}{v s_{2\theta}}\left(-m^2_{h_0}\frac{c_{\alpha-\theta}}{9 c^2_\theta} - m^2_{H_2}(c_\alpha c^3_\theta - s_\alpha s^3_\theta)\right.$$
$$\left. + m^2_{H^{\pm}_2} s_{\alpha-\theta} s_{2\theta}\right), \tag{108}$$

$$g_{H_2 G^{\pm} G^{\pm}} = \frac{m^2_{H_2} s_{\alpha-\theta}}{v}, \tag{109}$$

$$g_{H_1 G^{\pm} G^{\pm}} = \frac{m^2_{H_1} c_{\alpha-\theta}}{v}, \tag{110}$$

$$g_{h_0 H^{\pm}_1 G^{\pm}} = \frac{1}{v}(m^2_{h_0} - m^2_{H^{\pm}_1}), \tag{111}$$

$$g_{H_2 H_2^\pm G^\pm} = \frac{c_{\alpha-\theta}}{v}(m_{H_2}^2 - m_{H_2^\pm}^2), \qquad (112)$$

$$g_{H_1 H_2^\pm G^\pm} = \frac{s_{\alpha-\theta}}{v}(-m_{H_1}^2 + m_{H_2^\pm}^2), \qquad (113)$$

$$g_{A_1 H_1^\pm G^\mp} = \mp\frac{1}{v}(m_{H_1^\pm}^2 - m_{A_1}^2 + 2c_\theta^2(m_{A_2}^2 - m_{H_2^\pm}^2)), \quad (114)$$

$$g_{A_2 H_2^\pm G^\mp} = \pm\frac{1}{v}(m_{H_2^\pm}^2 - m_{A_2}^2), \qquad (115)$$

$$g_{A_1 H_1^\pm H_2^\mp} = \pm\frac{1}{v t_\theta}(m_{H_1^\pm}^2 - m_{A_1}^2 + c_{2\theta}(m_{A_2}^2 - m_{H_2^\pm}^2)). \qquad (116)$$

5.2 A.2 Quartic scalar couplings

$$g_{H_2 H_2 H_1 H_1} = \frac{1}{16v^2 s_{2\theta}^2}\left(\frac{4m_{h_0}^2 s_{2(\alpha-\theta)}}{3c_\theta^2}(2s_{2\alpha} + s_{2(\alpha-\theta)})\right.$$
$$-2m_{H_1}^2 s_{2\alpha}(3c_{2\alpha}s_{2(\alpha-\theta)}$$
$$-3s_{2\alpha} + s_{2\theta}) + 2m_{H_2}^2 s_{2\alpha}(3c_{2\alpha}s_{2(\alpha-\theta)}$$
$$\left.+3s_{2\alpha} + s_{2\theta})\right), \qquad (117)$$

$$g_{H_2 H_2 h_0 h_0} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2\left(c_{\alpha+\theta}^2 - \frac{1}{3}c_{\alpha-\theta}^2\right) + m_{H_1}^2 c_\theta s_\alpha s_{2\alpha} c_{\alpha-\theta}\right.$$
$$\left.+m_{H_2}^2 c_\theta c_\alpha\left(c_{\alpha+\theta} + c_{2\alpha}c_{\alpha-\theta}\right)\right), \qquad (118)$$

$$g_{H_1 H_1 h_0 h_0} = \frac{1}{2v^2 s_{2\theta}^2}\left(m_{h_0}^2\left(s_{\alpha+\theta}^2 - \frac{1}{3}s_{\alpha-\theta}^2\right)\right.$$
$$\left.+m_{H_1}^2 c_\theta s_\alpha\left(s_{\alpha+\theta} - c_{2\alpha}s_{\alpha-\theta}\right) + m_{H_2}^2 c_\theta c_\alpha s_{2\alpha}s_{\alpha-\theta}\right), \qquad (119)$$

$$g_{H_1 H_2 H_2 H_2} = -\frac{1}{v^2 s_{2\theta}^2}\left(\frac{2m_{h_0}^2 c_\alpha c_{\alpha-\theta}^2(s_\alpha + 2c_\theta s_{\alpha-\theta})}{9c_\theta^2}\right.$$
$$+\frac{m_{H_1}^2 s_{2\alpha}^2 s_{2(\alpha-\theta)}}{4}$$
$$\left.-\frac{m_{H_2}^2 s_{2\alpha}c_{\alpha-\theta}(c_{\alpha-\theta}c_{2\alpha} + c_{\alpha+\theta})}{2}\right), \qquad (120)$$

$$g_{H_1 H_1 H_1 H_2} = -\frac{1}{v^2 s_{2\theta}^2}\left(\frac{2m_{h_0}^2 s_\alpha s_{\alpha-\theta}^2(c_\alpha + 2c_\theta c_{\alpha-\theta})}{9c_\theta^2}\right.$$
$$+\frac{m_{H_2}^2 s_{2\alpha}^2 s_{2(\alpha-\theta)}}{4}$$
$$\left.+\frac{m_{H_1}^2 s_{2\alpha}s_{\alpha-\theta}(s_{\alpha-\theta}c_{2\alpha} - s_{\alpha+\theta})}{2}\right), \qquad (121)$$

$$g_{H_1 H_2 h_0 h_0} = -\frac{1}{v^2 s_{2\theta}^2}\left(\frac{m_{h_0}^2}{3}(2c_{2\alpha}s_{2\theta} + s_{2\alpha}c_{2\theta})\right.$$
$$\left.+m_{H_1}^2 s_{2\alpha}s_\alpha c_\theta s_{\alpha-\theta} + m_{H_2}^2 s_{2\alpha}c_\alpha c_\theta c_{\alpha-\theta}\right), \qquad (122)$$

$$g_{H_2 H_2 A_1 A_1} = \frac{1}{2v^2 s_\theta^2}\left(-\frac{m_{h_0}^2}{36c_\theta^2}(3c_{2\alpha} + 3c_{2\theta} + 4c_{\alpha-\theta}^2)\right.$$
$$+\frac{m_{H_1}^2 s_\alpha^2 c_\alpha c_{\alpha-\theta}}{2c_\theta}$$
$$+\frac{m_{H_2}^2 c_\alpha}{2c_\theta}\left(c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta\right)$$

$$+m_{A_1}^2 c_\alpha^2 - m_{A_2}^2(c_\alpha^2 - s_\theta^2)\Big), \qquad (123)$$

$$g_{H_1 H_1 A_1 A_1} = \frac{1}{2v^2 s_\theta^2}\left(\frac{m_{h_0}^2}{36c_\theta^2}(3c_{2\alpha} - 3c_{2\theta} - 4s_{\alpha-\theta}^2)\right.$$
$$+\frac{m_{H_2}^2 c_\alpha^2 s_\alpha s_{\alpha-\theta}}{2c_\theta}$$
$$+\frac{m_{H_1}^2 s_\alpha}{2c_\theta}(s_\alpha^2 s_{\alpha-\theta} + c_\alpha s_\theta)$$
$$\left.+m_{A_1}^2 s_\alpha^2 + m_{A_2}^2(s_\theta^2 - s_\alpha^2)\right), \qquad (124)$$

$$g_{H_2 H_2 A_2 A_2} = \frac{1}{2v^2 s_{2\theta}^2}\left(\frac{m_{h_0}^2}{9c_\theta^2}(2c_{2\alpha} + s_{2\theta}s_{2(\alpha-\theta)}\right.$$
$$+2c_{(\alpha-\theta)}^2 + 2c_{2\theta})$$
$$+\frac{m_{H_1}^2 s_{2\alpha}}{4}\left(s_{2\alpha} + s_{2\theta} + c_{2\theta}s_{2(\alpha-\theta)}\right)$$
$$+m_{A_2}^2 s_{\alpha-\theta}^2 s_{2\theta}^2$$
$$+\frac{m_{H_2}^2}{8}(3c_{2\alpha}^2 + 3c_{2\theta}^2 + 4c_{(\alpha-\theta)}^4$$
$$\left.+c_{2(\alpha-\theta)} + 5c_{2(\alpha+\theta)})\right), \qquad (125)$$

$$g_{H_1 H_1 A_2 A_2} = \frac{1}{2v^2 s_{2\theta}^2}\left(-\frac{m_{h_0}^2}{9c_\theta^2}(2c_{2\alpha} + s_{2\theta}s_{2(\alpha-\theta)}\right.$$
$$-2s_{\alpha-\theta}^2 - 2c_{2\theta})$$
$$+\frac{m_{H_2}^2 s_{2\alpha}}{4}(s_{2\alpha} - s_{2\theta} + c_{2\theta}s_{2(\alpha-\theta)})$$
$$+m_{A_2}^2 c_{\alpha-\theta}^2 s_{2\theta}^2$$
$$+\frac{m_{H_1}^2}{8}(3c_{2\alpha}^2 + 3c_{2\theta}^2 - 4s_{\alpha-\theta}^4$$
$$\left.-c_{2(\alpha-\theta)} - 5c_{2(\alpha+\theta)})\right), \qquad (126)$$

$$g_{h_0 h_0 A_1 A_1} = \frac{m_{h_0}^2 + 3m_{H_1}^2 s_\alpha^2 + 3m_{H_2}^2 c_\alpha^2}{12v^2 s_\theta^2}, \qquad (127)$$

$$g_{h_0 h_0 A_2 A_2} = \frac{1}{v^2 s_{2\theta}^2}\left(-\frac{m_{h_0}^2}{9}(4c_\theta^2 + c_{2\theta})\right.$$
$$+m_{H_1}^2 c_\theta s_\alpha(s_{\alpha+\theta} - s_\theta c_\theta c_{\alpha-\theta})$$
$$+m_{H_2}^2 c_\theta c_\alpha(c_{\alpha+\theta} + s_\theta c_\theta s_{\alpha-\theta}) + 2m_{A_1}^2 c_\theta^4$$
$$\left.+2m_{A_2}^2 c_\theta^2 c_{2\theta}\right), \qquad (128)$$

$$g_{H_1 H_2 A_1 A_1} = \frac{1}{v^2 s_{2\theta}^2}\left(\frac{m_{h_0}^2}{9}(3s_{2\alpha} + 2s_{2(\alpha-\theta)})\right.$$
$$-m_{H_1}^2 s_{2\alpha}s_\alpha c_\theta s_{\alpha-\theta} - m_{H_2}^2 s_{2\alpha}c_\alpha c_\theta c_{\alpha-\theta}$$
$$\left.+2(m_{A_2}^2 - m_{A_1}^2)s_{2\alpha}c_\theta^2\right), \qquad (129)$$

$$g_{H_1 H_2 A_2 A_2} = \frac{1}{2v^2 s_{2\theta}^2} \Big(-\frac{2m_{h_0}^2}{9c_\theta^2}(s_{2\alpha} + 2c_\theta^2 s_{2(\alpha-\theta)})$$
$$+ \frac{m_{H_1}^2}{2} s_{2\alpha}(2c_{2\alpha} + s_{2\theta} s_{2(\alpha-\theta)} - 2c_{2\theta})$$
$$- \frac{m_{H_2}^2}{2} s_{2\alpha}(2c_{2\alpha} + s_{2\theta} s_{2(\alpha-\theta)} + 2c_{2\theta})$$
$$+ m_{A_2}^2 s_{2(\alpha-\theta)} s_{2\theta}^2 \Big), \tag{130}$$

$$g_{H_1 h_0 A_1 A_2} = \frac{1}{v^2 s_\theta^2} \Big(-\frac{m_{h_0}^2}{9c_\theta^2}(s_{\alpha+\theta} + c_\theta s_\alpha)(c_{2\theta} + c_\theta^2)$$
$$+ m_{H_1}^2 s_\alpha c_\theta - m_{H_2}^2 (c_\alpha s_\theta + c_\theta^2 s_{\alpha-\theta}) \Big), \tag{131}$$

$$g_{H_2 h_0 A_1 A_2} = \frac{1}{v^2 s_\theta^2} \Big(\frac{m_{h_0}^2}{9c_\theta^2}(c_{\alpha+\theta} + c_\theta c_\alpha)(c_{2\theta} + c_\theta^2)$$
$$- m_{H_1}^2 c_\alpha c_\theta - m_{H_2}^2 (s_\theta s_\alpha - c_\theta^2 c_{\alpha-\theta}) \Big), \tag{132}$$

$$g_{A_1 A_1 A_1 A_1} = \frac{m_{h_0}^2 + 3m_{H_1}^2 s_\alpha^2 + 3m_{H_2}^2 c_\alpha^2}{24v^2 s_\theta^2}, \tag{133}$$

$$g_{A_2 A_2 A_2 A_2} = \frac{1}{2v^2 s_{2\theta}^2} \Big(\frac{m_{h_0}^2}{9c_\theta^2}(2c_\theta^2 + c_{2\theta})$$
$$+ m_{H_1}^2 (s_\alpha c_\theta - s_\theta^2 s_{\alpha-\theta})^2$$
$$+ m_{H_2}^2 (c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta)^2 \Big), \tag{134}$$

$$g_{A_1 A_1 A_2 A_2} = \frac{1}{v^2 s_{2\theta}^2} \Big(\frac{m_{h_0}^2}{3}(c_{2\theta}^2 - 2c_\theta^2 s_\theta^2)$$
$$+ m_{H_1}^2 c_\theta s_\alpha(s_{\alpha+\theta} - s_\theta c_\theta c_{\alpha-\theta})$$
$$+ m_{H_2}^2 c_\theta c_\alpha(c_{\alpha+\theta} + s_\theta c_\theta s_{\alpha-\theta}) \Big), \tag{135}$$

$$g_{H_1 H_1 G_0 G_0} = \frac{1}{2v^2} \Big(\frac{m_{H_1}^2}{4s_{2\theta}}(s_{2\theta} + s_{2\alpha} - c_{2\alpha} s_{2(\alpha-\theta)})$$
$$- \frac{m_{H_2}^2 s_{2\alpha} s_{\alpha-\theta}^2}{2s_{2\theta}} + m_{A_2}^2 s_{\alpha-\theta}^2 \Big), \tag{136}$$

$$g_{H_2 H_2 G_0 G_0} = \frac{1}{2v^2} \Big(\frac{m_{H_1}^2 s_{2\alpha} c_{\alpha-\theta}^2}{2s_{2\theta}} + \frac{m_{H_2}^2}{4s_{2\theta}}$$
$$(s_{2\theta} - s_{2\alpha} - c_{2\alpha} s_{2(\alpha-\theta)}) + m_{A_2}^2 c_{\alpha-\theta}^2 \Big), \tag{137}$$

$$g_{H_1 H_2 G_0 G_0} = -\frac{s_{2(\alpha-\theta)}}{4v^2 s_{2\theta}} \Big((m_{H_1}^2 - m_{H_2}^2)s_{2\alpha} + 2m_{A_2}^2 s_{2\theta} \Big), \tag{138}$$

$$g_{G_0 G_0 A_2 A_2} = \frac{1}{8v^2 s_{2\theta}} \Big(m_{H_1}^2 (3c_{2\theta} s_{2(\alpha-\theta)} - s_{2\alpha} + 3s_{2\theta})$$
$$- m_{H_2}^2 (3c_{2\theta} s_{2(\alpha-\theta)} - s_{2\alpha} - 3s_{2\theta}) \Big), \tag{139}$$

$$g_{G_0 G_0 A_1 A_1} = \frac{1}{2v^2} \Big(m_{h_0}^2 + \frac{m_{H_1}^2}{2s_\theta} s_\alpha c_{\alpha-\theta} - \frac{m_{H_2}^2}{2s_\theta} c_\alpha s_{\alpha-\theta} \Big), \tag{140}$$

$$g_{H_1 H_1 A_2 G_0} = -\frac{1}{2v^2 s_{2\theta}} \Big(\frac{2m_{h_0}^2 s_{\alpha-\theta}^2}{9c_\theta^2} + \frac{m_{H_1}^2}{2}(2c_{2\theta}$$
$$- s_{2\alpha} s_{2(\alpha-\theta)} - 2c_{2\alpha})$$
$$+ \frac{m_{H_2}^2}{2} s_{2\alpha} s_{2(\alpha-\theta)} - m_{A_2}^2 s_{2\theta} s_{2(\alpha-\theta)} \Big), \tag{141}$$

$$g_{H_2 H_2 A_2 G_0} = -\frac{1}{2v^2 s_{2\theta}} \Big(\frac{2m_{h_0}^2 c_{\alpha-\theta}^2}{9c_\theta^2} + \frac{m_{H_1}^2}{2} s_{2(\alpha-\theta)}$$
$$+ \frac{m_{H_2}^2}{2}(2c_{2\theta} - s_{2\alpha} s_{2(\alpha-\theta)} + 2c_{2\alpha})$$
$$+ m_{A_2}^2 s_{2\theta} s_{2(\alpha-\theta)} \Big), \tag{142}$$

$$g_{H_1 H_2 A_2 G_0} = \frac{1}{v^2 s_{2\theta}} \Big(\frac{m_{h_0}^2 s_{2(\alpha-\theta)}}{9c_\theta^2} + m_{H_1}^2 s_{2\alpha} s_{\alpha-\theta}^2$$
$$+ m_{H_2}^2 s_{2\alpha} c_{\alpha-\theta}^2 - m_{A_2}^2 c_{2(\alpha-\theta)} s_{2\theta} \Big), \tag{143}$$

$$g_{H_1 h_0 A_1 G_0} = \frac{1}{v^2 s_\theta} \Big(\frac{m_{h_0}^2}{3} \Big(\frac{s_{\alpha+\theta}}{c_\theta} + s_\alpha \Big)$$
$$- m_{A_1}^2 s_\alpha + m_{A_2}^2 s_{\alpha-\theta} c_\theta \Big), \tag{144}$$

$$g_{H_2 h_0 A_1 G_0} = \frac{1}{v^2 s_\theta} \Big(-\frac{m_{h_0}^2}{3} \Big(\frac{c_{\alpha+\theta}}{c_\theta} + c_\alpha \Big)$$
$$+ m_{A_1}^2 c_\alpha - m_{A_2}^2 c_{\alpha-\theta} c_\theta \Big), \tag{145}$$

$$g_{G_0 A_2 A_2 A_2} = \frac{1}{v^2 s_{2\theta}} \Big(-\frac{m_{h_0}^2}{9c_\theta^2}$$
$$+ \frac{m_{H_1}^2}{4}(2c_{2\alpha} + s_{2\theta} s_{2(\alpha-\theta)} - 2c_{2\theta})$$
$$- \frac{m_{H_2}^2}{4}(2c_{2\alpha} + s_{2\theta} s_{2(\alpha-\theta)} + 2c_{2\theta}) \Big), \tag{146}$$

$$g_{G_0 G_0 G_0 G_0} = \frac{m_{H_1}^2 c_{\alpha-\theta}^2 + m_{H_2}^2 s_{\alpha-\theta}^2}{8v^2}, \tag{147}$$

$$g_{H_1 G^0 H_2^\pm G^\mp} = \pm \frac{1}{v^2} s_{\alpha-\theta}(m_{H_2^\pm}^2 - m_{A_2}^2), \tag{148}$$

$$g_{H_2 G^0 H_2^\pm G^\mp} = \mp \frac{1}{v^2} c_{\alpha-\theta}(m_{H_2^\pm}^2 - m_{A_2}^2), \tag{149}$$

$$g_{H_1 A_2 H_2^\pm G^\mp} = \pm \frac{1}{v^2} c_{\alpha-\theta}(m_{H_2^\pm}^2 - m_{A_2}^2), \tag{150}$$

$$g_{H_2 A_2 H_2^\pm G^\mp} = \pm \frac{1}{v^2} s_{\alpha-\theta}(m_{H_2^\pm}^2 - m_{A_2}^2), \tag{151}$$

$$g_{h_0 G^0 H_1^\pm G^\mp} = \mp \frac{1}{v^2}(m_{H_1^\pm}^2 - m_{A_1}^2 + 2c_\theta^2(m_{A_2}^2 - m_{H_2^\pm}^2)), \tag{152}$$

$$g_{h_0 A_2 H_1^\pm G^\mp} = \mp \frac{1}{v^2 t_\theta}(m_{H_1^\pm}^2 - m_{A_1}^2 + c_{2\theta}(m_{A_2}^2 - m_{H_2^\pm}^2)). \tag{153}$$

5.3 A.3 Charged scalar-vector bosons couplings

For the couplings with charged Higgs bosons we have

$$g_{H_1^\pm H_1^\pm W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2}, \tag{154}$$

$$g_{H_2^\pm H_2^\pm W^\pm W^\mp} = \frac{2M_W^2 g^{\mu\nu}}{v^2}, \tag{155}$$

$$g_{H_1^\pm H_1^\pm ZZ} = \frac{g^2 \cos^2 2\theta_W g^{\mu\nu}}{4\cos^2\theta_W}, \tag{156}$$

$$g_{H_2^\pm H_2^\pm ZZ} = \frac{g^2 \cos^2 2\theta_W g^{\mu\nu}}{4\cos^2\theta_W}, \tag{157}$$

$$g_{H_1^\pm H_1^\mp \gamma\gamma} = e^2 g^{\mu\nu}, \tag{158}$$

$$g_{H_2^\pm H_2^\mp \gamma\gamma} = e^2 g^{\mu\nu}, \tag{159}$$

$$g_{H_1^\pm H_1^\mp \gamma Z} = \frac{eg \cos 2\theta_W g^{\mu\nu}}{\cos\theta_W}, \tag{160}$$

$$g_{H_2^\pm H_2^\mp \gamma Z} = \frac{eg \cos 2\theta_W g^{\mu\nu}}{\cos\theta_W}. \tag{161}$$

And the couplings for mixed charged and neutral Higgs bosons with gauge bosons, are given as

$$g_{H_2^\mp H_1 Z W^\pm} = \frac{g'^2 \cos\theta_W \sin(\alpha - \theta) g^{\mu\nu}}{2}, \tag{162}$$

$$g_{H_2^\mp H_2 Z W^\pm} = -\frac{g'^2 \cos\theta_W \cos(\alpha - \theta) g^{\mu\nu}}{2}, \tag{163}$$

$$g_{H_2^\mp \gamma H_1 W^\pm} = -\frac{eg \sin(\alpha - \theta) g^{\mu\nu}}{2}, \tag{164}$$

$$g_{H_2^\mp \gamma H_2 W^\pm} = \frac{eg \cos(\alpha - \theta) g^{\mu\nu}}{2}, \tag{165}$$

$$g_{\gamma G^0 W^\pm G^\mp} = \pm\frac{ge}{2} g^{\mu\nu}, \tag{166}$$

$$g_{Z G^0 W^\pm G^\mp} = \mp\frac{g^2 s_{\theta_W}^2}{2c_{\theta_W}} g^{\mu\nu}, \tag{167}$$

$$g_{\gamma A_1 W^\pm H_1^\mp} = \pm\frac{ge}{2} g^{\mu\nu}, \tag{168}$$

$$g_{\gamma A_2 W^\pm H_2^\mp} = \pm\frac{ge}{2} g^{\mu\nu}, \tag{169}$$

$$g_{Z A_1 W^\pm H_1^\mp} = \mp\frac{g^2 s_{\theta_W}^2}{2c_{\theta_W}} g^{\mu\nu}, \tag{170}$$

$$g_{Z A_2 W^\pm H_2^\mp} = \mp\frac{g^2 s_{\theta_W}^2}{2c_{\theta_W}} g^{\mu\nu}. \tag{171}$$

The mixed charged Higgs boson and h_0 couplings with two gauge bosons are absent.

$$g_{\gamma H_1^+ H_1^+} = e(p + p')^\mu, \tag{172}$$

$$g_{\gamma H_2^+ H_2^+} = e(p + p')^\mu, \tag{173}$$

$$g_{H_1 W^\pm H_2^\mp} = \pm\frac{ig}{2} s_{(\alpha-\theta)}(p + p')^\mu, \tag{174}$$

$$g_{H_2 W^\pm H_2^\mp} = \mp\frac{ig}{2} c_{(\alpha-\theta)}(p + p')^\mu, \tag{175}$$

$$g_{h_0 W^\pm H_1^\mp} = \mp\frac{ig}{2}(p + p')^\mu, \tag{176}$$

$$g_{W^\pm H_2^\pm A_2} = \frac{g}{2}(p + p')^\mu, \tag{177}$$

$$g_{W^\pm H_1^\pm A_1} = \frac{g}{2}(p + p')^\mu, \tag{178}$$

$$g_{Z H_2 A_2} = \frac{g}{2\cos\theta_W} \cos(\alpha - \theta)(p + p')^\mu, \tag{179}$$

$$g_{Z H_1 A_2} = -\frac{g}{2\cos\theta_W} \sin(\alpha - \theta)(p + p')^\mu, \tag{180}$$

$$g_{H_1 W^\pm G^\mp} = \mp\frac{ig}{2} c_{(\alpha-\theta)}(p + p')^\mu, \tag{181}$$

$$g_{H_2 W^\pm G^\mp} = \mp\frac{ig}{2} s_{(\alpha-\theta)}(p + p')^\mu, \tag{182}$$

$$g_{W^\pm G^\pm G^0} = \frac{g}{2}(p + p')^\mu, \tag{183}$$

$$g_{\gamma G^+ G^+} = e(p + p')^\mu. \tag{184}$$

References

1. G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys. Lett. B **716**, 1–29 (2012). https://doi.org/10.1016/j.physletb.2012.08.020. arXiv:1207.7214 [hep-ex]
2. S. Chatrchyan et al., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. Phys. Lett. B **716**, 30–61 (2012). https://doi.org/10.1016/j.physletb.2012.08.021. arXiv:1207.7235 [hep-ex]
3. P.W. Higgs, Spontaneous symmetry breakdown without massless bosons. Phys. Rev. **145**, 1156–1163 (1966). https://doi.org/10.1103/PhysRev.145.1156
4. J.F. Gunion et al., The Higgs Hunter's Guide, vol. 80 (2000)
5. R.A. Flores, M. Sher, Higgs masses in the standard, multi-Higgs and supersymmetric models. Ann. Phys. **148**, 95 (1983). https://doi.org/10.1016/0003-4916(83)90331-7
6. I.F. Ginzburg, M. Krawczyk, Symmetries of two Higgs doublet model and CP violation. Phys. Rev. D **72**, 115013 (2005). https://doi.org/10.1103/PhysRevD.72.115013. arXiv:hep-ph/0408011
7. G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos, CP-odd invariants in models with several Higgs doublets. Phys. Lett. B **614**, 187–194 (2005). https://doi.org/10.1016/j.physletb.2005.03.075. arXiv:hep-ph/0502118
8. C.C. Nishi, The structure of potentials with N Higgs doublets. Phys. Rev. D **76**, 055013 (2007). https://doi.org/10.1103/PhysRevD.76.055013. arXiv:0706.2685 [hep-ph]
9. P. Osland, P.N. Pandita, L. Selbuz, Trilinear Higgs couplings in the two Higgs doublet model with CP violation. Phys. Rev. **D78**, 015003 (2008). https://doi.org/10.1103/PhysRevD.78.015003. arXiv:0802.0060 [hep-ph]
10. P.M. Ferreira, J.P. Silva, Discrete and continuous symmetries in multi-Higgs doublet models. Phys. Rev. D **78**, 116007 (2008). https://doi.org/10.1103/PhysRevD.78.116007. arXiv:0809.2788 [hep-ph]
11. K. Olaussen, P. Osland, M.A. Solberg, Symmetry and mass degeneration in multi-Higgs-doublet models. JHEP **07**, 020 (2011). https://doi.org/10.1007/JHEP07(2011)020. arXiv:1007.1424 [hep-ph]
12. J. Kubo, Super flavorsymmetry with multiple Higgs doublets. Fortsch. Phys. **61**, 597–621 (2013). https://doi.org/10.1002/prop.201200119. arXiv:1210.7046 [hep-ph]

13. K. Yagyu, Higgs boson couplings in multi-doublet models with natural flavour conservation. Phys. Lett. B **763**, 102–107 (2016). https://doi.org/10.1016/j.physletb.2016.10.028. arXiv:1609.04590 [hep-ph]

14. M.P. Bento et al., Multi-Higgs doublet models: the Higgs-fermion couplings and their sum rules. JHEP **10**, 143 (2018). https://doi.org/10.1007/JHEP10(2018)143. arXiv:1808.07123 [hep-ph]

15. I. de Medeiros Varzielas, I.P. Ivanov, Recognizing symmetries in a 3HDM in a basis-independent way. Phys. Rev. D **100**(1), 015008 (2019). https://doi.org/10.1103/PhysRevD.100.015008. arXiv:1903.11110 [hep-ph]

16. G.C. Branco et al., Theory and phenomenology of two-Higgs-doublet models. Phys. Rep. **516**, 1–102 (2012). https://doi.org/10.1016/j.physrep.2012.02.002. arXiv:1106.0034 [hep-ph]

17. I.P. Ivanov, Building and testing models with extended Higgs sectors. Prog. Part. Nucl. Phys. **95**, 160–208 (2017). https://doi.org/10.1016/j.ppnp.2017.03.001. arXiv:1702.03776 [hep-ph]

18. H. Ishimori et al., Non-Abelian discrete symmetries in particle physics. Prog. Theor. Phys. Suppl. **183**, 1–163 (2010). https://doi.org/10.1143/PTPS.183.1. arXiv:1003.3552 [hep-th]

19. H. Ishimori et al., An introduction to non-Abelian discrete symmetries for particle physicists. Lect. Notes Phys. **858**, 1–227 (2012). https://doi.org/10.1007/978-3-642-30805-5

20. G. Altarelli, F. Feruglio, Discrete flavor symmetries and models of neutrino mixing. Rev. Mod. Phys. **82**, 2701–2729 (2010). https://doi.org/10.1103/RevModPhys.82.2701. arXiv:1002.0211 [hep-ph]

21. S.F. King, C. Luhn, Neutrino mass and mixing with discrete symmetry. Rep. Prog. Phys. **76**, 056201 (2013). https://doi.org/10.1088/0034-4885/76/5/056201. arXiv:1301.1340 [hep-ph]

22. S. Moretti, D. Rojas, K. Yagyu, Enhancement of the $H^{\pm}W^{\mp}Z$ vertex in the three scalar doublet model. JHEP **08**, 116 (2015). https://doi.org/10.1007/JHEP08(2015)116. arXiv:1504.06432 [hep-ph]

23. J.E. Camargo-Molina et al., Heavy charged scalars from $c\bar{s}$ fusion: a generic search strategy applied to a 3HDM with U(1)×U(1) family symmetry. JHEP **03**, 024 (2018). https://doi.org/10.1007/JHEP03(2018)024. arXiv:1711.03551 [hep-ph]

24. A.G. Akeroyd, S. Moretti, M. Song, Light charged Higgs boson with dominant decay to a charm quark and a bottom quark and its search at LEP2 and future e^+e^- colliders. Phys. Rev. D **101**(3), 035021 (2020). https://doi.org/10.1103/PhysRevD.101.035021. arXiv:1908.00826 [hep-ph]

25. G. Aad et al., Combined measurements of Higgs boson production and decay using up to 80fb^{-1} of proton-proton collision data at $\sqrt{s}$ = 13 TeV collected with the ATLAS experiment. Phys. Rev. D **101**(1), 012002 (2020). https://doi.org/10.1103/PhysRevD.101.012002. arXiv:1909.02845 [hep-ex]

26. A.M. Sirunyan et al., Combined measurements of Higgs boson couplings in proton-proton collisions at $\sqrt{s}$ = 13 TeV. Eur. Phys. J. C **79**(5), 421 (2019). https://doi.org/10.1140/epjc/s10052-019-6909-y. arXiv:1809.10733 [hep-ex]

27. J.F. Gunion, H.E. Haber. Higgs bosons in supersymmetric models. 1. Nucl. Phys. B **272**, 1 (1986). [Erratum: Nucl. Phys. B 402, 567 (1993)]. https://doi.org/10.1016/0550-3213(86)90340-8. https://doi.org/10.1016/0550-3213(93)90653-7

28. M. Carena, H.E. Haber, Higgs boson theory and phenomenology. Prog. Part. Nucl. Phys. **50**, 63–152 (2003). https://doi.org/10.1016/S0146-6410(02)00177-1. arXiv:hep-ph/0208209

29. S. Heinemeyer. Higgs Physics (2014). arXiv:1405.3781 [hep-ph]

30. J.F. Gunion, H.E. Haber, The CP conserving two Higgs doublet model: the approach to the decoupling limit. Phys. Rev. D **67**, 075019 (2003). https://doi.org/10.1103/PhysRevD.67.075019. arXiv:hep-ph/0207010

31. M.P. Bento et al., Multi-Higgs doublet models: physical parametrization, sum rules and unitarity bounds. JHEP

11, 095 (2017). https://doi.org/10.1007/JHEP11(2017)095. arXiv:1708.09408 [hep-ph]

32. S. Pakvasa, H. Sugawara, Discrete symmetry and Cabibbo angle. Phys. Lett. B **73**, 61–64 (1978). https://doi.org/10.1016/0370-2693(78)90172-7

33. S. Pakvasa, H. Sugawara, Mass of the t quark in SU(2) x U(1). Phys. Lett. B **82**, 105–107 (1979). https://doi.org/10.1016/0370-2693(79)90436-2

34. H. Harari, H. Haut, J. Weyers, Quark masses and Cabibbo angles. Phys. Lett. B **78**, 459–461 (1978). https://doi.org/10.1016/0370-2693(78)90485-9

35. E. Derman, H.-S. Tsao, SU(2) X U(1) X S(n) flavor dynamics and a bound on the number of flavors. Phys. Rev. D **20**, 1207 (1979). https://doi.org/10.1103/PhysRevD.20.1207

36. V. Maurice, I.I.I. Barnhill, Generalization of S(3) mass matrix symmetry. Phys. Lett. **151B**, 257–259 (1985). https://doi.org/10.1016/0370-2693(85)90846-9

37. A. Mondragon, E. Rodriguez-Jauregui. A Parametrization of the CKM mixing matrix from a scheme of S(3)-L x S(3)-R symmetry breaking, in *21st Nuclear Physics Symposium* (1998). arXiv:hep-ph/9804267

38. A. Mondragon, E. Rodriguez-Jauregui, The breaking of the flavor permutational symmetry: mass textures and the CKM matrix. Phys. Rev. D **59**, 093009 (1999). https://doi.org/10.1103/PhysRevD.59.093009. arXiv:hep-ph/9807214

39. L. Lavoura, A new model for the quark mass matrices. Phys. Rev. D **61**, 077303 (2000). https://doi.org/10.1103/PhysRevD.61.077303. arXiv:hep-ph/9907538

40. A. Mondragon, E. Rodriguez-Jauregui, The CP violating phase delta(13) and the quark mixing angles theta(13), theta(23) and theta(12) from flavor permutational symmetry breaking. Phys. Rev. D **61**, 113002 (2000). https://doi.org/10.1103/PhysRevD.61.113002. arXiv:hep-ph/9906429

41. A. Mondragon, M. Mondragon, E. Peinado, S(3)-flavour symmetry as realized in lepton flavour violating processes. J. Phys. A **41**, 304035 (2008). https://doi.org/10.1088/1751-8113/41/30/304035. arXiv:0712.1799 [hep-ph]

42. F.G. Canales et al., Quark sector of S3 models: classification and comparison with experimental data. Phys. Rev. D **88**, 096004 (2013). https://doi.org/10.1103/PhysRevD.88.096004. arXiv:1304.6644 [hep-ph]

43. A.E.C. Hernández, R. Martinez, F. Ochoa, Fermion masses and mixings in the 3–3–1 model with right-handed neutrinos based on the S_3 flavor symmetry. Eur. Phys. J. C **76**(11), 634 (2016). https://doi.org/10.1140/epjc/s10052-016-4480-3. arXiv:1309.6567 [hep-ph]

44. D. Das, U.K. Dey, P.B. Pal, S_3 symmetry and the quark mixing matrix. Phys. Lett. B **753**, 315–318 (2016). https://doi.org/10.1016/j.physletb.2015.12.038. arXiv:1507.06509 [hep-ph]

45. S.-F. Ge, A. Kusenko, T.T. Yanagida, Large leptonic Dirac CP phase from broken democracy with random perturbations. Phys. Lett. B **781**, 699–705 (2018). https://doi.org/10.1016/j.physletb.2018.04.040. arXiv:1803.03888 [hep-ph]

46. J.C. Gómez-Izquierdo, M. Mondragón, B-L model with S_3 symmetry: nearest neighbor interaction textures and broken $\mu \leftrightarrow \tau$ symmetry. Eur. Phys. J. C **79**(3), 285 (2019). https://doi.org/10.1140/epjc/s10052-019-6785-5. arXiv:1804.08746 [hep-ph]

47. E. Ma, S(3) Z(3) model of lepton mass matrices. Phys. Rev. D **44**, 587–589 (1991). https://doi.org/10.1103/PhysRevD.44.R587

48. J. Kubo et al., The flavor symmetry. Prog. Theor. Phys. 109, 795–807 (2003). [Erratum: Prog. Theor. Phys. **114**, 287 (2005)]. https://doi.org/10.1143/PTP.109.795. arXiv:hep-ph/0302196

49. S.-L. Chen, M. Frigerio, E. Ma, Large neutrino mixing and normal mass hierarchy: a discrete understanding. Phys. Rev. D **70**, 073008 (2004). [Erratum: Phys. Rev. D **70**, 079905 (2004)]. https://

doi.org/10.1103/PhysRevD.70.079905. https://doi.org/10.1103/PhysRevD.70.073008. arXiv:hep-ph/0404084

50. R. Jora, S. Nasri, J. Schechter, An approach to permutation symmetry for the electroweak theory. Int. J. Mod. Phys. A **21**, 5875–5894 (2006). https://doi.org/10.1142/S0217751X0603391X. arXiv:hep-ph/0605069

51. O. Felix et al., Neutrino masses and mixings in a minimal S(3)-invariant extension of the standard model. AIP Conf. Proc. **917**(1), 383–389 (2007). https://doi.org/10.1063/1.2751980. arXiv:hep-ph/0610061

52. A. Mondragon, M. Mondragon, E. Peinado, Lepton masses, mixings and FCNC in a minimal S(3)-invariant extension of the Standard Model. Phys. Rev. D **76**, 076003 (2007). https://doi.org/10.1103/PhysRevD.76.076003. arXiv:0706.0354 [hep-ph]

53. A. Mondragon, M. Mondragon, E. Peinado, Nearly tri-bimaximal mixing in the S(3) flavour symmetry. AIP Conf. Proc. **1026**, 164–169 (2008). https://doi.org/10.1063/1.2965040. arXiv:0712.2488 [hep-ph]

54. R. Jora, J. Schechter, M. Naeem Shahid, Perturbed S(3) neutrinos. Phys. Rev. D **80**, 093007 (2009). [Erratum: Phys. Rev. D **82**, 079902 (2010)]. https://doi.org/10.1103/PhysRevD.80.093007. arXiv:0909.4414 [hep-ph]

55. D.A. Dicus, S.-F. Ge, W.W. Repko, Neutrino mixing with broken S_3 symmetry. Phys. Rev. D **82**, 033005 (2010). https://doi.org/10.1103/PhysRevD.82.033005. arXiv:1004.3266 [hep-ph]

56. D. Meloni, S. Morisi, E. Peinado, Fritzsch neutrino mass matrix from S_3 symmetry. J. Phys. **G38**, 015003 (2011). https://doi.org/10.1088/0954-3899/38/1/015003. arXiv:1005.3482 [hep-ph]

57. F.G. Canales, A. Mondragon, M. Mondragon, The S_3 flavour symmetry: neutrino masses and mixings. Fortsch. Phys. **61**, 546–570 (2013). https://doi.org/10.1002/prop.201200121. arXiv:1205.4755 [hep-ph]

58. A.G. Dias, A.C.B. Machado, C.C. Nishi, An S_3 model for lepton mass matrices with nearly minimal texture. Phys. Rev. D **86**, 093005 (2012). https://doi.org/10.1103/PhysRevD.86.093005. arXiv:1206.6362 [hep-ph]

59. A.E.C. Hernández, E.C. Mur, R. Martinez, Lepton masses and mixing in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models with a S_3 flavor symmetry. Phys. Rev. **D90**(7), 073001 (2014). https://doi.org/10.1103/PhysRevD.90.073001. arXiv:1407.5217 [hep-ph]

60. O. Felix-Beltran et al., S_3 flavour symmetry and the reactor mixing angle. J. Phys. Conf. Ser. **485**, 012046 (2014). https://doi.org/10.1088/1742-6596/485/1/012046

61. E. Ma, R. Srivastava, Dirac or inverse seesaw neutrino masses with $B - L$ gauge symmetry and S_3 flavor symmetry. Phys. Lett. B **741**, 217–222 (2015). https://doi.org/10.1016/j.physletb.2014.12.049. arXiv:1411.5042 [hep-ph]

62. A.A. Cruz, M. Mondragón, Neutrino masses, mixing, and leptogenesis in an S3 model (2017). arXiv:1701.07929 [hep-ph]

63. Z.-Z. Xing, D. Zhang, Seesaw mirroring between light and heavy Majorana neutrinos with the help of the S3 reflection symmetry. JHEP **03**, 184 (2019). https://doi.org/10.1007/JHEP03(2019)184. arXiv:1901.07912 [hep-ph]

64. J.D. García-Aguilar, J.C. Gómez-Izquierdo, Flavored multiscalar **S**₃ model with normal hierarchy neutrino mass (2020). arXiv:2010.15370 [hep-ph]

65. J. Kubo, H. Okada, F. Sakamaki, Higgs potential in minimal S(3) invariant extension of the standard model. Phys. Rev. D **70**, 036007 (2004). https://doi.org/10.1103/PhysRevD.70.036007. arXiv:hep-ph/0402089

66. D. Das, U.K. Dey, Analysis of an extended scalar sector with S_3 symmetry. Phys. Rev. D **89**(9), 095025 (2014). [Erratum: Phys. Rev. D **91**(3), 039905(2015)]. https://doi.org/10.1103/PhysRevD.91.039905. https://doi.org/10.1103/PhysRevD.89.095025. arXiv:1404.2491 [hep-ph]

67. E. Barradas-Guevara, O. Félix-Beltrán, E. Rodríguez-Jáuregui, Trilinear self-couplings in an S(3) flavored Higgs model. Phys. Rev. D **90**(9), 095001 (2014). https://doi.org/10.1103/PhysRevD.90.095001. arXiv:1402.2244 [hep-ph]

68. A.E.C. Hernández, I. de Medeiros Varzielas, E. Schumacher, Fermion and scalar phenomenology of a two-Higgs-doublet model with S_3. Phys. Rev. **D93**(1), 016003 (2016). https://doi.org/10.1103/PhysRevD.93.016003. arXiv:1509.02083 [hep-ph]

69. E. Barradas-Guevara, O. Félix-Beltrán, E. Rodríguez-Jáuregui, CP breaking in $S(3)$ flavoured Higgs model (2015). arXiv:1507.05180 [hep-ph]

70. D. Emmanuel-Costa et al., Spontaneous symmetry breaking in the S_3-symmetric scalar sector. JHEP **02**, 154 (2016). [Erratum: JHEP **08**, 169 (2016)]. https://doi.org/10.1007/JHEP08(2016)169. https://doi.org/10.1007/JHEP02(2016)154. arXiv:1601.04654 [hep-ph]

71. H.E. Haber et al., Symmetries and mass degeneracies in the scalar sector. JHEP **01**, 042 (2019). https://doi.org/10.1007/JHEP01(2019)042. arXiv:1808.08629 [hep-ph]

72. A. Kunčinas et al., S3-inspired three-Higgs-doublet models: a class with a complex vacuum. Phys. Rev. D **101**(7), 075052 (2020). https://doi.org/10.1103/PhysRevD.101.075052. arXiv:2001.01994 [hep-ph]

73. N. Chakrabarty, High-scale validity of a model with three-Higgs-doublets. Phys. Rev. D **93**(7), 075025 (2016). https://doi.org/10.1103/PhysRevD.93.075025. arXiv:1511.08137 [hep-ph]

74. A.C.B. Machado, V. Pleitez, A model with two inert scalar doublets. Ann. Phys. **364**, 53–67 (2016). https://doi.org/10.1016/j.aop.2015.10.017. arXiv:1205.0995 [hep-ph]

75. C. Espinoza et al., The S3 symmetric model with a dark scalar. Phys. Lett. B **788**, 185–191 (2019). https://doi.org/10.1016/j.physletb.2018.11.028. arXiv:1804.01879 [hep-ph]

76. S. Mishra, Majorana dark matter and neutrino mass with S_3 symmetry. Eur. Phys. J. Plus **135**(6), 485 (2020). https://doi.org/10.1140/epjp/s13360-020-00461-1. arXiv:1911.02255 [hep-ph]

77. C. Espinoza, M. Mondragón, Prospects of indirect detection for the heavy S3 dark doublet (2020). arXiv:2008.11792 [hep-ph]

78. W. Khater et al., Dark matter in three-Higgs-doublet models with S_3 symmetry (2021). arXiv:2108.07026 [hep-ph]

79. M. Maniatis, O. Nachtmann, Stability and symmetry breaking in the general three-Higgs-doublet model. JHEP **02**, 058 (2015). [Erratum: JHEP **10**, 149 (2015)]. https://doi.org/10.1007/JHEP10(2015)149. arXiv:1408.6833 [hep-ph]

80. D. Emmanuel-Costa et al., Stability of the tree-level vacuum in a minimal S(3) extension of the standard model. AIP Conf. Proc. **917**, 390–393 (2007). [390 (2007)]. https://doi.org/10.1063/1.2751981

81. O. Felix Beltran, M. Mondragon, E. Rodriguez-Jauregui, Conditions for vacuum stability in an S(3) extension of the standard model. J. Phys. Conf. Ser. **171**, 012028 (2009). https://doi.org/10.1088/1742-6596/171/1/012028

82. W. Grimus et al., The oblique parameters in multi-Higgs-doublet models. Nucl. Phys. B **801**, 81–96 (2008). https://doi.org/10.1016/j.nuclphysb.2008.04.019. arXiv:0802.4353 [hep-ph]

83. A.E.C. Hernández, S. Kovalenko, I. Schmidt, Precision measurements constraints on the number of Higgs doublets. Phys. Rev. D **91**, 095014 (2015). https://doi.org/10.1103/PhysRevD.91.095014. arXiv:1503.03026 [hep-ph]

84. C. Espinoza et al., Unitarity and stability conditions in a 4-Higgs doublet model with an S_3-family symmetry. J. Phys. Conf. Ser. **912**(1), 012022 (2017). https://doi.org/10.1088/1742-6596/912/1/012022

85. J.F. Donoghue, L.F. Li, Properties of charged Higgs bosons. Phys. Rev. D **19**, 945 (1979). https://doi.org/10.1103/PhysRevD.19.945

86. H. Georgi, D.V. Nanopoulos, Ordinary predictions from grand principles: T quark mass in O(10). Nucl. Phys. B **155**, 52–74 (1979). https://doi.org/10.1016/0550-3213(79)90355-9

87. J.D. Wells, Lectures on Higgs boson physics in the standard model and beyond, in *39th British Universities Summer School in Theoretical Elementary Particle Physics (BUSSTEPP 2009)* Liverpool, United Kingdom, August 24–September 4, 2009 (2009). arXiv:0909.4541 [hep-ph]

88. A.G. Akeroyd et al., Light charged Higgs boson scenario in 3-Higgs doublet models. Int. J. Mod. Phys. A **32**(23n24), 1750145 (2017). https://doi.org/10.1142/S0217751X17501457. arXiv:1605.05881 [hep-ph]

89. G. Keller, D. Wyler, The couplings of Higgs bosons to two vector mesons in multi Higgs models. Nucl. Phys. B **274**, 410–428 (1986). https://doi.org/10.1016/0550-3213(86)90292-0

90. D. Das, I. Saha, Alignment limit in three Higgs-doublet models. Phys. Rev. D **100**(3), 035021 (2019). https://doi.org/10.1103/PhysRevD.100.035021. arXiv:1904.03970 [hep-ph]

91. Search for new resonances in the diphoton final state in the mass range between 80 and 110 GeV in pp collisions at $\sqrt{s} = 8$ TeV. Tech. rep. CERN, Geneva (2015). https://cds.cern.ch/record/2063739

92. K.A. Olive et al., Review of particle physics. Chin. Phys. C **38**, 090001 (2014). https://doi.org/10.1088/1674-1137/38/9/090001

93. P.A. Zyla et al., Review of particle physics. PTEP **2020**(8), 083C01 (2020). https://doi.org/10.1093/ptep/ptaa104

94. Search for new resonances in the diphoton final state in the mass range between 70 and 110 GeV in pp collisions at $\sqrt{s} = 8$ and 13 TeV. Tech. rep. CERN, Geneva (2017). https://cds.cern.ch/record/2285326

95. S. Heinemeyer, T. Stefaniak, A Higgs boson at 96 GeV? PoS **CHARGED2018**, 016 (2019). https://doi.org/10.22323/1.339.0016. arXiv:1812.05864 [hep-ph]

96. U. Haisch, A. Malinauskas, Let there be light from a second light Higgs doublet. JHEP **03**, 135 (2018). https://doi.org/10.1007/JHEP03(2018)135. arXiv:1712.06599 [hep-ph]

97. T. Biekötter, M. Chakraborti, S. Heinemeyer, A 96 GeV Higgs boson in the N2HDM. Eur. Phys. J. C **80**(1), 2 (2020). https://doi.org/10.1140/epjc/s10052-019-7561-2. arXiv:1903.11661 [hep-ph]

98. T. Biekötter, M. Chakraborti, S. Heinemeyer, An N2HDM Solution for the possible 96 GeV excess. PoS **CORFU2018**, 015 (2019). https://doi.org/10.22323/1.347.0015. arXiv:1905.03280 [hep-ph]

99. N. Chakrabarty, I. Chakraborty, On the Higgs mass fine-tuning problem with multi-Higgs doublet models. Int. J. Mod. Phys. A **34**(05), 1950025 (2019). https://doi.org/10.1142/S0217751X19500258. arXiv:1801.05272 [hep-ph]

100. P. Bechtle et al., Applying exclusion likelihoods from LHC searches to extended Higgs sectors. Eur. Phys. J. C **75**(9), 421 (2015). https://doi.org/10.1140/epjc/s10052-015-3650-z. arXiv:1507.06706 [hep-ph]

101. G. Aad et al., Evidence for Higgs boson decays to a low-mass dilepton system and a photon in pp collisions at s=13 TeV with the ATLAS detector. Phys. Lett. B **819**, 136412 (2021). https://doi.org/10.1016/j.physletb.2021.136412. arXiv:2103.10322 [hep-ex]

102. A combination of measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton–proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment. Tech. rep. All figures including auxiliary figures are available at https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF2020-027. CERN, Geneva (2020). https://cds.cern.ch/record/2725733

103. S. Dawson, S. Dittmaier, M. Spira, Neutral Higgs boson pair production at hadron colliders: QCD corrections. Phys. Rev. D **58**, 115012 (1998). https://doi.org/10.1103/PhysRevD.58.115012. arXiv:hep-ph/9805244

104. R. Frederix et al., Higgs pair production at the LHC with NLO and parton-shower effects. Phys. Lett. B **732**, 142–149 (2014). https://doi.org/10.1016/j.physletb.2014.03.026. arXiv:1401.7340 [hep-ph]

105. G. Degrassi et al., Probing the Higgs self coupling via single Higgs production at the LHC. JHEP **12**, 080 (2016). https://doi.org/10.1007/JHEP12(2016)080. arXiv:1607.04251 [hep-ph]

106. G. Degrassi, M. Fedele, P.P. Giardino, Constraints on the tri-linear Higgs self coupling from precision observables. JHEP **04**, 155 (2017). https://doi.org/10.1007/JHEP04(2017)155. arXiv:1702.01737 [hep-ph]

107. A.M. Sirunyan et al., Search for nonresonant Higgs boson pair production in final states with two bottom quarks and two photons in proton-proton collisions at $\sqrt{s} = 13$ TeV. JHEP **03**, 257 (2021). https://doi.org/10.1007/JHEP03(2021)257. arXiv:2011.12373 [hep-ex]

108. J.R. Andersen et al., Handbook of LHC Higgs cross sections: 3. Higgs properties (2013). https://doi.org/10.5170/CERN-2013-004. arXiv:1307.1347 [hep-ph]

109. G. Degrassi et al., Higgs boson self-coupling constraints from single Higgs, double Higgs and electroweak measurements. Phys. Lett. B **817**, 136307 (2021). https://doi.org/10.1016/j.physletb.2021.136307. arXiv:2102.07651 [hep-ph]

110. A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200. Fortsch. Phys. **41**, 307–420 (1993). https://doi.org/10.1002/prop.2190410402. arXiv:0709.1075 [hep-ph]

111. M.E. Peskin, T. Takeuchi, Estimation of oblique electroweak corrections. Phys. Rev. D **46**, 381–409 (1992). https://doi.org/10.1103/PhysRevD.46.381

112. M. Frank et al., The Higgs boson masses and mixings of the complex MSSM in the Feynman-diagrammatic approach. JHEP **02**, 047 (2007). https://doi.org/10.1088/1126-6708/2007/02/047. arXiv:hep-ph/0611326

113. G. Passarino, M.J.G. Veltman, One loop corrections for e+ e- annihilation into mu+ mu- in the Weinberg model. Nucl. Phys. B **160**, 151 (1979). https://doi.org/10.1016/0550-3213(79)90234-7