A tomographic test of cosmic anisotropy with the recently-released quasar sample

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Abstract We test the cosmic anisotropy in the dipole-modulated Λ CDM model and Finslerian cosmological model with the recently-released quasar sample. Based on the redshift tomographic method, the quasar sample is divided into two subsets $z \leq z_{cut}$ and $z > z_{cut}$ by different cutting redshifts. The dipole amplitudes of the two cosmological models from the subsets $z \leq z_{cut}$ are very weak. We find that quasars at a higher redshift range may provide more detailed information about the dipole amplitude A_D . The dipole directions of each cosmological model from the subsets $z \leq 1.1$ and $z > 1.1$ are deviated by 1σ level. The Pantheon sample is combined with the two subsets. The dipole amplitude from the two combined datasets is also very weak. In the dipole-modulated Λ CDM model, the dipole direction from the combined dataset quasar at $z \leq 1.1$ and Pantheon sample is far away from the one given by the Pantheon sample. In the Finslerian cosmological model, the dipole directions from the two combined datasets are close to the one in the Pantheon sample.

1 Introduction

As one of the foundations in modern cosmology, the cosmological principle assumes that the Universe is homogeneous and isotropic at large scales [1]. The cosmological principle is well consistent with current cosmological observations, such as the Cosmic Microwave Background (CMB) radiation from the Wilkinson Microwave Anisotropy Probe (WMAP) [2, 3] and Planck satellites [4–7]. Though the validity of the cosmological principle has been verified by the WMAP and Planck with high precision, there are still exist some phenomena which are inconsistent with the cosmological principle. An incomplete list includes the spatial variation of the electromagnetic fine-structure constant [8–11], the anisotropic accelerating expansion of the Universe [12–17], the MOND

acceleration scale [18–20], the alignment of quasar polarization vectors [21–23], the alignment of quadrupole and octopole in the CMB temperature anisotropies [5, 24–26], the hemispherical power asymmetry in CMB [27–30], the parity asymmetry in CMB [27, 31–35], the large dipoles obtained in the number counts of radio and mid-IR sources [36–50], the anisotropies found in the $L_X - T$ relation in X-ray galaxy clusters [51–53], and the large-scale asymmetry of galaxy spin directions [54]. These phenomena hint that the existence of cosmic anisotropy.

The low redshift observational datasets used to test cosmological models are mostly in the redshift range $0 \leq z \leq 2.3$, such as the Type Ia supernova (SNe Ia) sample, the Hubble parameter observations, and the baryon acoustic oscillation measurements. The observational dataset at the higher redshift $z \sim 1100$ is cosmic microwave background anisotropy data from Planck and WMAP. Cosmological models are rarely tested in the intermediate redshift range $2.3 \leq z \leq 1100$. An incomplete list of the datasets and related works to test cosmological models in the intermediate redshift range include the gamma-ray burst [55–66], the HIIIG starburst galaxy [59, 67–73], the quasar angular size [59, 69, 74–76], and the X-ray and UV flux of quasars [77–87].

In recent years, SNe Ia sample, i.e., the Union2&2.1 sample [88, 89], Joint Light-Curve Analysis (JLA) sample [90], and Pantheon sample [91] are widely used to test the anisotropy of the Universe. Certain preferred directions were found by the hemisphere comparison (HC) method with the Union2&2.1 sample [15–17, 92–98]. The Λ CDM model is barely consistent with the SNe Ia data at the redshift range $z < 0.5$ in the Union2 sample [99]. Mariano and Perivolaropoulos found a dipole direction at 2σ level with the Union2 sample [10]. Zhao et al. found a dipole of cosmic acceleration at more than 2σ level with the Union2 sample [100]. Unlike the Union2&2.1 sample, there is no anisotropic signal in the JLA sample [101–103] and Pantheon sample [104–106]. Zhao and Zhou et al. [107] found that the SDSS

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subsample has a decisive impact on the dipole anisotropy in the Pantheon sample.

Besides the SNe Ia sample, galaxy [18–20,54], galaxy cluster [51–53], number counts of radio and mid-IR sources [36–50], gamma-ray burst [13,66,108], gravitational wave [109], fast radio burst [109,110], and quasar [83,87] are also used to investigate the cosmic anisotropy. The UV and X-ray luminosity of quasars are used to test the cosmic anisotropy by Hu et al. [87]. They combined the 2019 compilation of quasars [78] with the Pantheon sample [91] to test the dipole-modulated Λ CDM model. They found that there is no evidence of cosmic anisotropy in the SN-Q sample. Zhao and Xia [83] tested the Finslerian cosmological model with the 2015 compilation of quasars. They found the cosmic anisotropy is very weak from the combination of the Pantheon sample and 2015 quasar compilation.

As the most luminous and persistent energy source, quasars have a lot of potential in exploring our Universe. Risaliti and Lusso (et al.) have been attempting to test cosmological models with the non-linear relation between the UV and X-ray luminosity of quasars since 2015 [77–79]. More recently, Lusso and Risaliti (et al.) released a new catalogue of optically selected quasars [79]. The 2020 compilation of quasars contains 2421 quasars that over the redshift range $0.009 \leq z \leq 7.5413$. Due to the host-galaxy contamination, the 2500 Å UV fluxes of quasars at the redshift range $z < 0.7$ are less reliable. So the 2023 quasars at the redshift range $z > 0.7$ and 15 local quasars with higher quality at the redshift range $z < 0.7$ are used in the test of cosmic anisotropy.

In this work, we will analyze the quasar sample by using a redshift tomographic method. The quasar sample is divided into two subsets by different cutting redshifts. We will use the subsets to constrain the dipole-modulated Λ CDM model and Finslerian cosmological model. The Pantheon sample is combined with the subsets in our analysis. The rest of this paper is organized as follows. In Sect. 2, we briefly introduce the quasar sample and the two cosmological models. We show our results in Sect. 3. Finally, discussions and conclusions are given in Sect. 4.

2 The quasar sample and methodology

2.1 The quasar sample

The recently-released quasar compilation used in our work was presented by Lusso et al. [79]. The 2020 compilation consists of 2421 quasars over the redshift range $0.009 \leq z \leq 7.5413$. Because of the host-galaxy contamination at low redshift, the 2500 Å UV fluxes of quasars at the redshift range $z < 0.7$ are less reliable. We choose the quasars at the redshift range $z > 0.7$ and 15 local quasars with higher quality at the

Fig. 1 The redshift distribution of the quasar sample

redshift range $z < 0.7$ as the full quasar sample. We show the redshift distribution of the full quasar sample in Fig. 1.

2.2 The methodology

The non-linear relation between UV and X-ray luminosity of quasars can be parameterized as [111]

$$\log L_X = \beta + \gamma \log L_{UV}, \quad (1)$$

where $\log$ denotes $\log_{10}$. β and γ are two free parameters determined by the quasar sample. L_X is the monochromatic luminosity at 2 keV and L_{UV} is the luminosity at 2500 Å. The luminosities and fluxes of quasars are related by the luminosity distance, so we rewrite the Eq. (1) as

$$\log(F_X) = \beta + (\gamma - 1) \log(4\pi) + \gamma \log(F_{UV}) + 2(\gamma - 1) \log \frac{d_L}{\text{Mpc}}, \quad (2)$$

where F_X and F_{UV} are the X-ray and UV fluxes of quasars, respectively. The luminosity distance d_L has the form

$$d_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (3)$$

where z is the redshift, c is the speed of light, and H_0 is the Hubble constant. The expression of $E(z)$ varies with cosmological models.

In the Finslerian cosmological model, the scale factor a takes the form [112]

$$a = \frac{1 + A_D \cos \theta}{1 + z}, \quad (4)$$

where A_D is the dipole amplitude. θ is the angle between the preferred direction in the Finslerian cosmological model and the positions of quasars. Correspondingly, $E(z)$ in the Finslerian cosmological model takes the form

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3(1 + A_D \cos \theta)^{-3} + 1 - \Omega_{m0}}. \quad (5)$$

The dipole fitting (DF) method proposed by Mariano and Perivolaropoulos [10] is widely used to test the cosmic anisotropy. In this paper, we consider a dipole modulation to the theoretical distance modulus in the Λ CDM model, namely

$$\tilde{\mu}_{th} = \mu_{th} \times (1 + A_D(\hat{\mathbf{n}} \cdot \hat{\mathbf{p}})), \quad (6)$$

where A_D is the dipole amplitude. $\hat{\mathbf{n}}$ and $\hat{\mathbf{p}}$ are the dipole direction and the unit vector pointing to a quasar, respectively. In the galactic coordinate system, the dipole direction $\hat{\mathbf{n}}$ takes the form

$$\hat{\mathbf{n}} = \cos(b) \cos(l) \hat{\mathbf{i}} + \cos(b) \sin(l) \hat{\mathbf{j}} + \sin(b) \hat{\mathbf{k}}, \quad (7)$$

where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the unit vectors along the axis in the Cartesian coordinates system. l is the galactic longitude and b is the galactic latitude. The unit vector $\hat{\mathbf{p}}_i$ pointing to the position of i th quasar can be written as

$$\hat{\mathbf{p}}_i = \cos(b_i) \cos(l_i) \hat{\mathbf{i}} + \cos(b_i) \sin(l_i) \hat{\mathbf{j}} + \sin(b_i) \hat{\mathbf{k}}. \quad (8)$$

Considering the expression of distance modulus in the Λ CDM model,

$$\mu_{th} = 5 \log \frac{d_L}{\text{Mpc}} + 25, \quad (9)$$

We can write the dipole-modulated distance modulus as

$$\tilde{\mu}_{th} = 5 \log \frac{d_L}{\text{Mpc}} + 25. \quad (10)$$

Plugging Eq. (6) into Eq. (10), we obtain

$$\log \frac{d_L}{\text{Mpc}} = \left(\log \frac{d_L}{\text{Mpc}} + 5 \right) \times (1 + A_D(\hat{\mathbf{n}} \cdot \hat{\mathbf{p}})) - 5. \quad (11)$$

We combine Eqs. (11) and (2) and obtain the theoretical X-ray flux in the dipole-modulated Λ CDM model,

$$\begin{aligned} \log(F_X) = & \beta + (\gamma - 1) \log(4\pi) + \gamma \log(F_{UV}) \\ & + 2(\gamma - 1) \left(\left(\log \frac{d_L}{\text{Mpc}} + 5 \right) (1 + A_D(\hat{\mathbf{n}} \cdot \hat{\mathbf{p}})) - 5 \right). \end{aligned} \quad (12)$$

To explore the whole cosmological parameter space, we employ the likelihood function

$$\ln(\text{LF}) = -\frac{1}{2} \sum_{i=1}^N \left[\frac{\left[\log(F_{X,i}^{\text{obs}}) - \log(F_{X,i}^{\text{th}}) \right]^2}{s_i^2} + \ln(2\pi s_i^2) \right], \quad (13)$$

where $s_i^2 = \sigma_i^2 + \delta^2$. σ_i is the error of the observed flux $F_{X,i}^{\text{obs}}$ and δ is the global intrinsic dispersion. $F_{X,i}^{\text{th}}$ represents the theoretical flux at the redshift z_i in the dipole-modulated Λ CDM model or Finslerian cosmological model.

3 Results

In this paper, we use the Markov chain Monte Carlo (MCMC) method implemented in emcee¹ [113] to explore the whole parameters space. Emcee is an Affine Invariant Markov chain Monte Carlo Ensemble sampler, which is widely used to constrain parameters in astrophysics and cosmology. In the dipole-modulated Λ CDM model and Finslerian cosmological model, the free parameters are the matter density Ω_m , the dipole amplitude A_D , the dipole direction (l , b), and the three parameters β , γ , and δ related to the quasar sample. The flat priors of the free parameters are $\Omega_m \sim [0, 1]$, $\beta \sim [5, 16]$, $\gamma \sim [0.3, 0.8]$, $\delta \sim [0.1, 0.3]$, $A_D \sim [0, 0.5]$, $l \sim [0, 360]$, $b \sim [-90, 90]$.

We use a tomographic method to study the evolution of dipole parameters with redshift. The cutting redshifts are

$$z_{cut} = 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.7, 1.9, 2.1, 2.3, 2.4, 2.5, 3.0.$$

We divide the quasar sample into two subsets, i.e., $z \leq z_{cut}$ and $z > z_{cut}$ by different cutting redshifts z_{cut} . The results of the dipole-modulated Λ CDM model and Finslerian cosmological model are summarized in Tables 1 and 2.

From Tables 1 and 2, we can see that the results of the two cosmological models are similar except for the dipole amplitude A_D . The results of parameters β , γ , and δ from most subsets $z \leq z_{cut}$ are consistent within statistical uncertainties, respectively. The constraints of matter density Ω_m are very weak from all the subsets. In the Finslerian cosmological model, the results of matter density Ω_m from the subsets $z \leq 1.0, 1.1$ are consistent with ones from the Pantheon sample and CMB, but with large uncertainties.

The dipole amplitudes A_D of the two cosmological models from the subsets $z \leq z_{cut}$ are very weak. The upper limit of dipole amplitude A_D in the Finslerian cosmological model is about a hundred times larger than that in the Λ CDM model. Though the constraints of matter density Ω_m are very weak from the subsets $z > z_{cut}$, the results of the dipole amplitude A_D show that quasars at a higher redshift range may provide more detailed information about the dipole amplitude A_D .

Because of the large uncertainties, the results of dipole parameter l from all the subsets $z \leq z_{cut}$ are consistent. But the central values of parameter l from some subsets have a sudden change. For the dipole-modulated Λ CDM

¹ <https://emcee.readthedocs.io/en/stable/>.

Table 1 The 68% confidence level constraints of the parameters in the dipole-modulated Λ CDM model with different cutting redshifts. If the upper or lower limit of a parameter exists, we show the 95% confidence level of the parameter

Model	Quasar subset	N	Ω_m	β	γ	δ	$10^3 A_D$	l [°]	b [°]
Λ CDM	$z \leq 1.0$	443	> 0.189	$9.037^{+0.766}_{-0.714}$	$0.573^{+0.023}_{-0.027}$	$0.235^{+0.009}_{-0.008}$	< 1.316	$120.00^{+80.17}_{-101.84}$	$-0.91^{+27.33}_{-67.53}$
	$z \leq 1.1$	571	> 0.198	$8.521^{+0.717}_{-0.708}$	$0.591^{+0.023}_{-0.025}$	$0.243^{+0.007}_{-0.008}$	< 1.632	$112.74^{+68.24}_{-74.83}$	$-10.01^{+20.05}_{-52.58}$
	$z \leq 1.2$	719	> 0.263	$8.715^{+0.614}_{-0.617}$	$0.583^{+0.021}_{-0.020}$	$0.237^{+0.007}_{-0.006}$	< 1.160	$112.72^{+73.46}_{-99.74}$	$-28.20^{+20.07}_{-47.30}$
	$z \leq 1.3$	831	> 0.269	$8.882^{+0.663}_{-0.494}$	$0.577^{+0.018}_{-0.022}$	$0.239^{+0.006}_{-0.006}$	< 1.051	$130.92^{+91.67}_{-85.20}$	$-26.37^{+17.24}_{-55.09}$
	$z \leq 1.4$	943	> 0.312	$9.021^{+0.525}_{-0.554}$	$0.573^{+0.019}_{-0.018}$	$0.236^{+0.005}_{-0.006}$	< 0.899	$138.18^{+117.89}_{-78.17}$	$-4.55^{+22.02}_{-67.62}$
	$z \leq 1.5$	1037	> 0.339	$8.628^{+0.500}_{-0.490}$	$0.585^{+0.018}_{-0.015}$	$0.238^{+0.006}_{-0.005}$	< 0.744	$127.28^{+142.73}_{-81.31}$	$-20.91^{+30.11}_{-62.79}$
	$z \leq 1.7$	1270	> 0.445	$8.453^{+0.426}_{-0.466}$	$0.591^{+0.015}_{-0.015}$	$0.235^{+0.005}_{-0.005}$	< 0.661	$152.70^{+137.74}_{-75.27}$	$13.64^{+51.37}_{-47.64}$
	$z \leq 1.9$	1446	> 0.586	$8.100^{+0.420}_{-0.377}$	$0.603^{+0.012}_{-0.014}$	$0.235^{+0.005}_{-0.004}$	< 0.623	$293.64^{+65.28}_{-163.58}$	$13.65^{+65.38}_{-30.97}$
	$z \leq 2.1$	1593	> 0.660	$8.277^{+0.430}_{-0.311}$	$0.597^{+0.012}_{-0.013}$	$0.232^{+0.004}_{-0.005}$	< 0.622	$300.91^{+66.76}_{-155.31}$	$10.02^{+70.60}_{-21.56}$
	$z \leq 2.3$	1712	> 0.716	$8.288^{+0.331}_{-0.347}$	$0.597^{+0.011}_{-0.011}$	$0.231^{+0.004}_{-0.004}$	< 0.593	$308.19^{+154.99}_{-74.48}$	$13.64^{+61.39}_{-32.86}$
	$z \leq 2.4$	1776	> 0.736	$8.180^{+0.333}_{-0.322}$	$0.601^{+0.010}_{-0.012}$	$0.230^{+0.004}_{-0.004}$	< 0.567	$279.09^{+209.77}_{-31.50}$	$15.47^{+63.49}_{-28.68}$
	$z \leq 2.5$	1812	> 0.741	$8.107^{+0.324}_{-0.342}$	$0.603^{+0.011}_{-0.011}$	$0.229^{+0.004}_{-0.004}$	< 0.544	$145.45^{+119.22}_{-118.87}$	$33.65^{+51.70}_{-35.47}$
	$z \leq 3.0$	1943	> 0.780	$7.787^{+0.297}_{-0.312}$	$0.614^{+0.010}_{-0.010}$	$0.227^{+0.004}_{-0.004}$	< 0.615	$112.74^{+60.65}_{-178.99}$	$39.10^{+50.47}_{-22.43}$
	$1.7 < z \leq 2.4$	506	> 0.196	$10.728^{+0.602}_{-0.690}$	$0.521^{+0.022}_{-0.020}$	$0.204^{+0.007}_{-0.008}$	< 0.861	$40.01^{+100.05}_{-118.04}$	$10.01^{+64.89}_{-28.25}$
	$z > 1.0$	1595	> 0.762	$7.575^{+0.360}_{-0.343}$	$0.621^{+0.011}_{-0.012}$	$0.224^{+0.004}_{-0.005}$	< 0.752	$298.17^{+79.48}_{-127.45}$	$55.46^{+34.50}_{-31.97}$
	$z > 1.1$	1467	> 0.758	$7.792^{+0.368}_{-0.361}$	$0.615^{+0.012}_{-0.012}$	$0.218^{+0.005}_{-0.004}$	$0.435^{+0.264}_{-0.222}$	$296.36^{+68.85}_{-103.73}$	$68.19^{+21.79}_{-32.28}$
	$z > 1.2$	1319	> 0.716	$7.913^{+0.395}_{-0.368}$	$0.610^{+0.012}_{-0.013}$	$0.218^{+0.005}_{-0.005}$	$0.558^{+0.216}_{-0.255}$	$280.01^{+61.68}_{-129.05}$	$77.42^{+12.58}_{-31.44}$
	$z > 1.3$	1207	> 0.691	$8.116^{+0.384}_{-0.415}$	$0.604^{+0.014}_{-0.013}$	$0.213^{+0.005}_{-0.005}$	$0.537^{+0.251}_{-0.234}$	$305.46^{+79.54}_{-111.48}$	$77.41^{+12.58}_{-33.81}$
	$z > 1.4$	1095	> 0.647	$8.198^{+0.455}_{-0.366}$	$0.602^{+0.012}_{-0.015}$	$0.211^{+0.005}_{-0.005}$	$0.507^{+0.254}_{-0.253}$	$305.45^{+108.08}_{-100.98}$	$75.46^{+14.43}_{-32.40}$
	$z > 1.5$	1001	> 0.629	$8.647^{+0.406}_{-0.416}$	$0.588^{+0.013}_{-0.014}$	$0.206^{+0.005}_{-0.005}$	$0.534^{+0.232}_{-0.256}$	$290.90^{+46.23}_{-169.69}$	$75.57^{+14.39}_{-27.62}$
	$z > 1.7$	768	> 0.540	$9.275^{+0.448}_{-0.452}$	$0.568^{+0.015}_{-0.014}$	$0.198^{+0.006}_{-0.006}$	$0.397^{+0.290}_{-0.268}$	$350.90^{+128.57}_{-72.48}$	$66.40^{+22.68}_{-36.35}$
	$z > 1.9$	592	> 0.516	$9.303^{+0.535}_{-0.456}$	$0.567^{+0.015}_{-0.017}$	$0.187^{+0.007}_{-0.007}$	$0.450^{+0.267}_{-0.340}$	$16.37^{+114.17}_{-83.65}$	$64.55^{+25.40}_{-33.39}$
	$z > 2.1$	445	> 0.365	$9.329^{+0.547}_{-0.555}$	$0.567^{+0.018}_{-0.018}$	$0.181^{+0.009}_{-0.007}$	$0.677^{+0.433}_{-0.373}$	$89.10^{+64.21}_{-96.27}$	$48.19^{+39.05}_{-9.68}$
	$z > 2.3$	326	> 0.258	$8.706^{+0.647}_{-0.676}$	$0.589^{+0.022}_{-0.021}$	$0.169^{+0.011}_{-0.008}$	$1.121^{+0.576}_{-0.442}$	$130.91^{+72.22}_{-79.52}$	$57.63^{+25.69}_{-11.83}$
	$z > 2.4$	262	> 0.259	$9.241^{+0.786}_{-0.683}$	$0.573^{+0.023}_{-0.024}$	$0.170^{+0.012}_{-0.009}$	$1.061^{+0.465}_{-0.404}$	$329.09^{+174.40}_{-50.69}$	$79.11^{+10.89}_{-25.72}$
	$z > 2.5$	226	> 0.219	$9.701^{+0.698}_{-0.852}$	$0.560^{+0.024}_{-0.025}$	$0.173^{+0.011}_{-0.012}$	$1.061^{+0.465}_{-0.404}$	$309.09^{+170.21}_{-62.86}$	$77.30^{+12.63}_{-24.28}$
	$z > 3.0$	95	> 0.152	$10.568^{+1.605}_{-1.630}$	$0.532^{+0.052}_{-0.051}$	$0.203^{+0.020}_{-0.019}$	$1.096^{+0.456}_{-0.471}$	$309.10^{+103.05}_{-96.97}$	$19.09^{+60.52}_{-34.67}$

model, the central values of parameter l from the subsets $z \leq 1.9, 2.1, 2.3, 2.4$ and $z > 1.9, 2.1, 2.3$ have a sudden change. For the Finslerian cosmological model, the central values of parameter l from the subsets $z \leq 1.9, 2.1, 2.3$ and $z > 1.9, 2.1, 2.3$ have a sudden change.

We analyze the subset $1.7 < z \leq 2.4$ and show the results in Tables 1 and 2. The central value of parameter l from subsets $1.7 < z \leq 2.4$ is 40.01° in the dipole-modulated Λ CDM model and 83.63° in the Finslerian cosmological model. In the dipole-modulated Λ CDM model, the results of parameters β , γ , and δ are $\beta = 10.728^{+0.602}_{-0.690}$, $\gamma = 0.521^{+0.022}_{-0.020}$, and $\delta = 0.204^{+0.007}_{-0.008}$. In the Finslerian cosmological model, the results of parameters β , γ , and δ are $\beta = 10.615^{+0.706}_{-0.604}$, $\gamma = 0.526^{+0.018}_{-0.024}$, and $\delta = 0.203^{+0.009}_{-0.006}$. For parameters β , γ , and δ , the differences between the

results from subsets $1.7 < z \leq 2.4$ and the ones from the subsets $z \leq 1.9, 2.10, 2.30, 2.40$ and $z > 1.9, 2.10, 2.30$ are more than 1σ level. The central value of parameter l has a sudden change in some subsets may be due to the big deviations of parameters β , γ , and δ .

The dipole directions derived from two subsets divided by different cutting redshifts are consistent within statistical uncertainties except for the subsets $z \leq 1.1$ and $z > 1.1$. For brevity, the subset $z \leq 1.1$ is shortened to Q1 and the subset $z > 1.1$ is shortened to Q2. In Fig. 2, We show the distributions of subsets Q1 and Q2 in the galactic coordinate system. In the dipole-modulated Λ CDM model, the dipole direction from the subset Q1 is $(l, b) = (112.74^{+68.24}_{-74.83}, -10.01^{+20.05}_{-52.58})$ and the dipole direction from the subset Q2 is $(l, b) = (296.36^{+68.85}_{-103.73}, 68.19^{+21.79}_{-32.28})$. In the Finslerian cosmo-

Table 2 The 68% confidence level constraints of the parameters in the Finslerian cosmological model with different cutting redshifts. If the upper or lower limit of a parameter exists, we show the 95% confidence level of the parameter

Model	Quasar subset	N	Ω_m	β	γ	δ	A_D	$l [^\circ]$	$b [^\circ]$
Finslerian	$z \leq 1.0$	443	$0.505^{+0.347}_{-0.257}$	$9.066^{+0.751}_{-0.753}$	$0.572^{+0.026}_{-0.025}$	$0.234^{+0.008}_{-0.007}$	< 0.230	$116.37^{+80.08}_{-94.95}$	$-2.73^{+27.27}_{-63.21}$
	$z \leq 1.1$	571	$0.596^{+0.218}_{-0.376}$	$8.473^{+0.710}_{-0.700}$	$0.593^{+0.022}_{-0.025}$	$0.242^{+0.008}_{-0.007}$	< 0.273	$109.09^{+68.75}_{-66.59}$	$-13.64^{+20.58}_{-49.61}$
	$z \leq 1.2$	719	$0.648^{+0.276}_{-0.266}$	$8.741^{+0.596}_{-0.654}$	$0.583^{+0.022}_{-0.020}$	$0.236^{+0.007}_{-0.006}$	< 0.163	$109.07^{+79.76}_{-98.57}$	$-51.83^{+38.00}_{-29.27}$
	$z \leq 1.3$	831	$0.566^{+0.408}_{-0.107}$	$8.945^{+0.592}_{-0.595}$	$0.575^{+0.020}_{-0.020}$	$0.239^{+0.006}_{-0.006}$	< 0.148	$127.29^{+86.59}_{-86.48}$	$-20.92^{+16.83}_{-54.70}$
	$z \leq 1.4$	943	> 0.264	$9.017^{+0.554}_{-0.506}$	$0.573^{+0.019}_{-0.017}$	$0.236^{+0.006}_{-0.005}$	< 0.113	$138.18^{+110.78}_{-83.28}$	$-6.36^{+24.16}_{-60.95}$
	$z \leq 1.5$	1037	> 0.287	$8.665^{+0.443}_{-0.557}$	$0.586^{+0.018}_{-0.016}$	$0.238^{+0.006}_{-0.005}$	< 0.086	$130.91^{+115.84}_{-110.89}$	$-17.27^{+36.14}_{-57.85}$
	$z \leq 1.7$	1270	> 0.402	$8.409^{+0.449}_{-0.424}$	$0.594^{+0.014}_{-0.016}$	$0.235^{+0.005}_{-0.005}$	< 0.074	$149.09^{+139.04}_{-76.65}$	$11.82^{+51.48}_{-43.42}$
	$z \leq 1.9$	1446	> 0.564	$8.111^{+0.375}_{-0.420}$	$0.604^{+0.013}_{-0.013}$	$0.236^{+0.004}_{-0.005}$	< 0.062	$275.46^{+87.04}_{-144.50}$	$13.64^{+61.47}_{-34.10}$
	$z \leq 2.1$	1593	> 0.629	$8.285^{+0.358}_{-0.363}$	$0.597^{+0.012}_{-0.012}$	$0.233^{+0.004}_{-0.004}$	< 0.061	$297.27^{+72.71}_{-144.37}$	$15.45^{+61.70}_{-30.36}$
	$z \leq 2.3$	1712	> 0.706	$8.317^{+0.317}_{-0.366}$	$0.597^{+0.012}_{-0.011}$	$0.232^{+0.004}_{-0.004}$	< 0.053	$297.27^{+123.55}_{-104.95}$	$15.46^{+62.73}_{-30.10}$
	$z \leq 2.4$	1776	> 0.725	$8.171^{+0.350}_{-0.318}$	$0.601^{+0.010}_{-0.012}$	$0.230^{+0.004}_{-0.004}$	< 0.052	$100.91^{+28.75}_{-217.06}$	$22.73^{+58.39}_{-33.06}$
	$z \leq 2.5$	1812	> 0.729	$8.138^{+0.289}_{-0.366}$	$0.602^{+0.012}_{-0.009}$	$0.230^{+0.004}_{-0.004}$	< 0.054	$116.36^{+47.79}_{-191.76}$	$19.10^{+62.70}_{-27.16}$
	$z \leq 3.0$	1943	> 0.781	$7.818^{+0.266}_{-0.341}$	$0.613^{+0.011}_{-0.009}$	$0.227^{+0.004}_{-0.004}$	< 0.057	$130.91^{+50.25}_{-188.28}$	$37.29^{+49.54}_{-24.21}$
	$1.7 < z \leq 2.4$	506	> 0.119	$10.615^{+0.706}_{-0.604}$	$0.526^{+0.018}_{-0.024}$	$0.203^{+0.009}_{-0.006}$	< 0.102	$83.63^{+68.07}_{-141.55}$	$11.82^{+51.99}_{-44.72}$
	$z > 1.0$	1595	> 0.745	$7.605^{+0.344}_{-0.369}$	$0.620^{+0.012}_{-0.011}$	$0.224^{+0.004}_{-0.005}$	< 0.069	$294.54^{+109.15}_{-107.58}$	$62.73^{+25.02}_{-41.31}$
	$z > 1.1$	1467	> 0.751	$7.794^{+0.369}_{-0.354}$	$0.614^{+0.012}_{-0.012}$	$0.219^{+0.004}_{-0.005}$	$0.040^{+0.024}_{-0.020}$	$296.36^{+79.05}_{-103.80}$	$70.00^{+19.94}_{-32.58}$
	$z > 1.2$	1319	> 0.722	$7.978^{+0.318}_{-0.424}$	$0.609^{+0.014}_{-0.010}$	$0.218^{+0.005}_{-0.004}$	$0.050^{+0.024}_{-0.022}$	$280.00^{+63.19}_{-130.81}$	$71.85^{+18.07}_{-26.66}$
	$z > 1.3$	1207	> 0.681	$8.124^{+0.366}_{-0.425}$	$0.604^{+0.014}_{-0.012}$	$0.214^{+0.005}_{-0.005}$	$0.051^{+0.022}_{-0.023}$	$305.45^{+91.33}_{-97.87}$	$77.51^{+12.48}_{-33.54}$
	$z > 1.4$	1095	> 0.623	$8.199^{+0.428}_{-0.390}$	$0.602^{+0.013}_{-0.014}$	$0.211^{+0.006}_{-0.004}$	$0.047^{+0.024}_{-0.024}$	$305.46^{+73.35}_{-137.00}$	$75.47^{+14.53}_{-32.05}$
	$z > 1.5$	1001	> 0.637	$8.632^{+0.407}_{-0.416}$	$0.588^{+0.014}_{-0.013}$	$0.207^{+0.005}_{-0.006}$	$0.052^{+0.023}_{-0.026}$	$294.55^{+43.67}_{-172.14}$	$73.66^{+16.28}_{-27.65}$
	$z > 1.7$	768	> 0.515	$9.264^{+0.424}_{-0.501}$	$0.568^{+0.017}_{-0.014}$	$0.199^{+0.006}_{-0.007}$	$0.036^{+0.025}_{-0.027}$	$358.18^{+117.00}_{-82.80}$	$50.00^{+39.06}_{-20.77}$
	$z > 1.9$	592	> 0.499	$9.320^{+0.487}_{-0.507}$	$0.566^{+0.017}_{-0.015}$	$0.188^{+0.007}_{-0.007}$	< 0.102	$20.00^{+112.70}_{-90.66}$	$64.55^{+25.23}_{-32.58}$
	$z > 2.1$	445	> 0.317	$9.280^{+0.546}_{-0.579}$	$0.569^{+0.019}_{-0.017}$	$0.181^{+0.008}_{-0.008}$	$0.067^{+0.048}_{-0.040}$	$92.73^{+56.14}_{-99.19}$	$46.41^{+30.81}_{-17.88}$
	$z > 2.3$	326	> 0.278	$8.653^{+0.685}_{-0.640}$	$0.590^{+0.022}_{-0.021}$	$0.171^{+0.009}_{-0.009}$	$0.113^{+0.060}_{-0.050}$	$127.30^{+83.42}_{-75.08}$	$59.09^{+25.22}_{-13.21}$
	$z > 2.4$	262	> 0.250	$9.226^{+0.688}_{-0.764}$	$0.573^{+0.025}_{-0.022}$	$0.172^{+0.011}_{-0.011}$	$0.109^{+0.048}_{-0.046}$	$321.81^{+201.46}_{-28.95}$	$77.32^{+12.68}_{-24.11}$
	$z > 2.5$	226	> 0.230	$9.547^{+0.846}_{-0.757}$	$0.561^{+0.025}_{-0.026}$	$0.172^{+0.011}_{-0.012}$	$0.109^{+0.054}_{-0.048}$	$319.99^{+138.43}_{-93.81}$	$75.51^{+14.45}_{-23.92}$
	$z > 3.0$	95	> 0.128	$10.504^{+1.540}_{-1.678}$	$0.530^{+0.056}_{-0.047}$	$0.204^{+0.021}_{-0.019}$	< 0.237	$305.45^{+100.78}_{-119.73}$	$17.26^{+66.24}_{-27.84}$

logical model, the dipole direction from the subset Q1 is $(l, b) = (109.09^{+68.75^\circ}_{-66.59^\circ}, -13.64^{+20.58^\circ}_{-49.61^\circ})$ and the dipole direction from the subset Q2 is $(l, b) = (296.36^{+79.05^\circ}_{-103.80^\circ}, 70.00^{+19.94^\circ}_{-32.58^\circ})$. In each cosmological model, the difference between the two directions from subsets Q1 and Q2 is about 1σ level. We summarize the results from the subsets Q1 and Q2 in Table 3 for clarity. We show the marginalized posterior distribution for each cosmological model in Figs. 3 and 4.

From Table 3, we can see that the results of the two cosmological models are similar except for the parameter A_D . The constraints of matter density Ω_m in the two cosmological models are very weak. For the parameters β and γ in each cosmological model, the results from subsets Q1 and Q2 are consistent within 1σ uncertainties, respectively. For the parameter δ in each cosmological model, the differences

between the results from subset Q1 and Q2 are about 2σ level.

In the dipole-modulated Λ CDM model, the dipole amplitude A_D is constrained as $A_D < 1.632 \times 10^{-3}$ at 95% CL with subset Q1, and the dipole amplitude A_D is $0.435^{+0.264}_{-0.222} \times 10^{-3}$ with subset Q2. In the Finslerian cosmological model, the dipole amplitude A_D is constrained as $A_D < 0.273$ at 95% CL with subsets Q1, and the dipole amplitude A_D is $0.040^{+0.024}_{-0.020}$ with subsets Q2. The dipole amplitude A_D in the Finslerian cosmological model is about a hundred times larger than that in the Λ CDM model.

The Pantheon sample is combined with the two subsets Q1 and Q2 to test the cosmological models. In Fig. 2, We show the distributions of Pantheon sample in the galactic coordinate system. For brevity, the dataset Q1 + Pantheon is shortened to Q1-P and the dataset Q2 + Pantheon is short-

Fig. 2 The distribution of subsets Q1, Q2, and Pantheon sample in the galactic coordinate system. The pseudo-colors represent the redshift of quasars or SNe Ia

ened to Q2-P. The results from the combined datasets are shown in Figs. 3 and 4 and summarized in Table 3. In Figs. 3 and 4, we show the marginalized posterior distribution for each cosmological model. In Table 3, we show the 68% CL constraints on the parameters in each cosmological model. If the upper or lower limit of a parameter exists, we show the 95% CL limit of the parameter.

In the dipole-modulated Λ CDM model, the dataset Q1-P gives a dipole direction points to $(l, b) = (121.82^{+66.19}_{-148.87}, -73.64^{+40.15}_{-16.36})$. The dipole direction is consistent with the one from the subset Q1 within statistical uncertainties, but the central values of parameter b change a lot. The dipole amplitude is constrained as $A_D < 0.832 \times 10^{-3}$ at 95% CL. The results of parameters β , γ , and δ are $\beta = 8.378^{+0.683}_{-0.674}$, $\gamma = 0.596^{+0.022}_{-0.023}$, and $\delta = 0.242^{+0.008}_{-0.007}$. The matter density is $\Omega_m = 0.297^{+0.023}_{-0.020}$, which is deviated from the result in the subset $z \leq 1.10$ by 1σ level. It is worth noting that the dipole direction in the dataset Q1-P is far away from the one $((l, b) = (306.00^{+91.94}_{-125.98}, -23.41^{+22.97}_{-54.71}))$ in the Pantheon sample [106].

For the dataset Q2-P, the dipole direction is $(l, b) = (305.47^{+72.54}_{-78.58}, 11.82^{+56.80}_{-17.39})$, which is consistent with the one from the subset Q2. The central value of parameters b in the two dipole directions also change a lot. The dipole amplitude is constrained as $A_D < 0.818 \times 10^{-3}$ at 95% CL.

The results of parameters β , γ , and δ are $\beta = 7.545^{+0.330}_{-0.382}$, $\gamma = 0.626^{+0.012}_{-0.011}$, and $\delta = 0.221^{+0.004}_{-0.005}$. The result of parameter δ in the dataset Q2-P deviates from that in the dataset Q1-P by 1σ level. The matter density is $\Omega_m = 0.319^{+0.024}_{-0.022}$, which is against the result in the subset Q2.

In the Finslerian cosmological model, the dataset Q1-P gives a dipole direction points to $(l, b) = (301.81^{+56.80}_{-152.99}, -39.09^{+17.47}_{-50.73})$. The dipole direction is consistent with the one from the subset Q1 within statistical uncertainties, but the central values of parameter l change a lot. The dipole amplitude is constrained as $A_D < 0.018$ at 95% CL. The results of parameters β , γ , and δ are $\beta = 8.319^{+0.654}_{-0.698}$, $\gamma = 0.598^{+0.023}_{-0.022}$, and $\delta = 0.243^{+0.007}_{-0.008}$ and the matter density is $\Omega_m = 0.297^{+0.023}_{-0.020}$.

For the dataset Q2-P, the dipole direction points to $(l, b) = (309.09^{+72.06}_{-119.77}, -19.09^{+22.96}_{-57.63})$, which has 1σ level deviation from the one in the subset Q2. The dipole amplitude is constrained as $A_D < 0.019$ at 95% CL. The results of parameters β , γ , and δ are $\beta = 7.605^{+0.315}_{-0.397}$, $\gamma = 0.624^{+0.012}_{-0.011}$, and $\delta = 0.221^{+0.004}_{-0.005}$. The result of parameter δ deviates from the one in the dataset Q1-P by 1σ level. The matter density is $\Omega_m = 0.319^{+0.024}_{-0.022}$, which is against the result in the subset Q2.

It is necessary to check whether the inhomogeneous distributions of quasars and SNe Ia have some influence on the

Table 3 The 68% confidence level constraints on the parameters in the dipole-modulated Λ CDM model and Finslerian cosmological model. The datasets are Q1, Q2, Q1-P, and Q2-P. If the upper or lower limit of a parameter exists, we show the 95% confidence level of the parameter

Model	Dataset	Ω_m	β	γ	δ	$10^3 A_D$	$l [^\circ]$	$b [^\circ]$
Λ CDM	Q1	> 0.198	$8.521^{+0.717}_{-0.708}$	$0.591^{+0.023}_{-0.025}$	$0.243^{+0.007}_{-0.008}$	< 1.632	$112.74^{+68.24}_{-74.83}$	$-10.01^{+20.05}_{-52.58}$
	Q2	> 0.758	$7.792^{+0.368}_{-0.361}$	$0.615^{+0.012}_{-0.012}$	$0.218^{+0.005}_{-0.004}$	$0.435^{+0.264}_{-0.222}$	$296.36^{+68.85}_{-103.73}$	$68.19^{+21.79}_{-32.28}$
	Q1-P	$0.297^{+0.023}_{-0.020}$	$8.378^{+0.683}_{-0.674}$	$0.596^{+0.022}_{-0.023}$	$0.242^{+0.008}_{-0.007}$	< 0.832	$121.82^{+66.19}_{-148.87}$	$-73.64^{+40.15}_{-16.36}$
	Q2-P	$0.322^{+0.021}_{-0.022}$	$7.545^{+0.330}_{-0.382}$	$0.626^{+0.012}_{-0.011}$	$0.221^{+0.004}_{-0.005}$	< 0.818	$305.47^{+72.54}_{-78.58}$	$11.82^{+56.80}_{-17.39}$
Model	Dataset	Ω_m	β	γ	δ	A_D	$l [^\circ]$	$b [^\circ]$
Finslerian	Q1	$0.596^{+0.218}_{-0.376}$	$8.473^{+0.710}_{-0.700}$	$0.593^{+0.022}_{-0.025}$	$0.242^{+0.008}_{-0.007}$	< 0.273	$109.09^{+68.75}_{-66.59}$	$-13.64^{+20.58}_{-49.61}$
	Q2	> 0.751	$7.794^{+0.369}_{-0.354}$	$0.614^{+0.012}_{-0.012}$	$0.219^{+0.004}_{-0.005}$	$0.040^{+0.024}_{-0.020}$	$296.36^{+79.05}_{-103.80}$	$70.00^{+19.94}_{-32.58}$
	Q1-P	$0.297^{+0.023}_{-0.020}$	$8.319^{+0.654}_{-0.698}$	$0.598^{+0.023}_{-0.022}$	$0.243^{+0.007}_{-0.008}$	< 0.018	$301.81^{+56.80}_{-152.99}$	$-39.09^{+17.47}_{-50.73}$
	Q2-P	$0.319^{+0.024}_{-0.022}$	$7.605^{+0.315}_{-0.397}$	$0.624^{+0.012}_{-0.011}$	$0.221^{+0.004}_{-0.005}$	< 0.019	$309.09^{+72.06}_{-119.77}$	$-19.09^{+22.96}_{-57.63}$

Fig. 3 The marginalized posterior distribution of the parameters in the dipole-modulated Λ CDM model. The blue lines denote the results from dataset Q1, the orange lines denote the results from dataset Q2, the red lines represent the results from dataset Q1-P, and the green lines represent the results from dataset Q2-P

results. We construct 200 simulations based on the dataset Q1-P to test the dipole parameters in the dipole-modulated Λ CDM model. In the simulation, the positions of quasars and SNe Ia are homogeneously distributed in the sky and other observational data are the same as ones of the dataset Q1-P. Because the dataset Q1-P has weak constraints on the dipole amplitude A_D and only can give the upper limit of

A_D , we are interested in the dipole directions from the simulations. The results of dipole directions from the simulations are shown in Fig. 5. We can see that the dipole directions derived from the simulations are randomly distributed in the sky. It may imply that the distribution of quasars and SNe Ia have significant effects on the dipole direction derived from the dataset Q1-P. We also construct 200 simulations based on

Fig. 4 The marginalized posterior distribution of the parameters in the Finslerian cosmological model. The blue lines denote the results from dataset Q1, the orange lines denote the results from dataset Q2, the red lines represent the results from dataset Q1-P, and the green lines represent the results from dataset Q2-P

Fig. 5 The distributions of dipole directions derived from the 200 simulations in the galactic coordinate system

the dataset Q2-P and the combination of quasar and Pantheon samples, respectively. The results are similar to ones of the simulations based on the dataset Q1-P.

In the end, we make some comparisons between the dipole directions shown in Table 3 with preferred directions derived from other samples. The preferred directions derived from different samples are summarized in Table 4 and shown in Fig. 6. Most of the preferred directions derived from different samples are located in a relatively small part of the south galactic hemisphere and they are consistent within sta-

tistical uncertainties. The dipole direction derived from the dataset Q2-P in the dipole-modulated Λ CDM model and the dipole directions derived from the datasets Q1-P and Q2-P in the Finslerian cosmological model are consistent with most preferred directions within statistical uncertainties. The dipole directions derived from the dataset Q2 in the dipole-modulated Λ CDM model and Finslerian cosmological model are close to each other. The two directions are also close to the preferred directions derived from CMB octopole, quasar alignment, and the combination of Pantheon sample and 1421 quasars. The dipole directions derived from the dataset Q1 in the dipole-modulated Λ CDM model and Finslerian cosmological model are also close to each other. There are only two preferred directions around them. One is the preferred direction derived from the Pantheon sample with the HC method and the other is the preferred direction derived from the combination of Union2 and 67 GRBs with the DF method. The dipole direction derived from the dataset Q1-P in the dipole-modulated Λ CDM model is far away from the one derived from the dataset Q1-P in the Finslerian cosmological model. This may be due to the difference between the two cosmological models.

Table 4 The preferred directions derived from different samples

Model	Data	Direction (l [°], b [°])	References
Λ CDM	Q1 (DF)	$112.74^{+68.24}_{-74.83}, -10.01^{+20.05}_{-52.58}$	This paper
	Q2 (DF)	$296.36^{+68.85}_{-103.73}, 68.19^{+21.79}_{-32.28}$	This paper
	Q1 + Pantheon (DF)	$121.82^{+66.19}_{-148.87}, -73.64^{+40.15}_{-16.36}$	This paper
	Q2 + Pantheon (DF)	$305.47^{+72.54}_{-78.58}, 11.82^{+56.80}_{-17.39}$	This paper
	Union2 (HC)	$309.4 \pm 18.0, -15.1 \pm 11.5$	[92]
	Union2 (DF)	$309.0 \pm 22.4, -19.3 \pm 12.9$	[10]
	Union2.1 (HC)	$241.9, -19.5$	[17]
	Union2.1 (DF)	$307.1 \pm 16.2, -14.3 \pm 10.1$	[96]
	JLA (HC) max	$23.49^{+21.62}_{-12.83}, -2.25^{+3.80}_{-22.68}$	[114]
	JLA (HC) submax	$299.47^{+46.13}_{-23.39}, -28.39^{+6.52}_{-17.01}$	[114]
	JLA (DF)	$316^{+107}_{-100}, -5^{+41}_{-60}$	[101]
	Pantheon (HC)	$123.05^{+11.25}_{-4.22}, 4.78^{+1.80}_{-8.36}$	[107]
	Pantheon (DF)	$306.00^{+82.95}_{-125.01}, -34.20^{+16.82}_{-54.93}$	[107]
	Pantheon + 1421Quasars (HC)	$316.08^{+27.41}_{-129.48}, 4.53^{+26.29}_{-64.06}$	[87]
	Pantheon + 1421Quasars (DF)	$327.55 \pm 32.45, 51.01 \pm 26.50$	[87]
	67 GRBs (DF)	$336^{+33}_{-233}, -5^{+34}_{-26}$	[13]
	Union2 + 67 GRBs (DF)	$129^{+16}_{-23}, 16^{+17}_{-10}$	[13]
	Union2.1 + 116 GRBs (DF)	$309.2 \pm 15.8, -8.6 \pm 10.5$	[108]
	Keck + VLT (DF)	$320.5 \pm 11.8, -11.7 \pm 7.5$	[10]
	Galaxy rotation curves (HC)	$175.5^{+6}_{-10}, -6.5^{+8}_{-3}$	[18]
	Galaxy rotation curves (DF)	$171.30 \pm 7.18, -15.41 \pm 4.87$	[19]
	Galaxy cluster	$303, -27$	[53]
-	CMB dipole	$263.99 \pm 0.14, 48.26 \pm 0.03$	[115]
	CMB quadrupole	$240, 63$	[24, 116]
	CMB octopole	$308, 63$	[24]
	$\Delta\alpha/\alpha$	$330, -13$	[8]
	Quasar alignment	$267, 69$	[21, 117]
Finslerian	Velocity flows	$282, 6$	[118, 119]
	Q1 (DF)	$109.09^{+68.75}_{-66.59}, -13.64^{+20.58}_{-49.61}$	This paper
	Q2 (DF)	$296.36^{+79.05}_{-103.80}, 70.00^{+19.94}_{-32.58}$	This paper
	Q1 + Pantheon (DF)	$301.81^{+56.80}_{-152.99}, -39.09^{+17.47}_{-50.73}$	This paper
	Q2 + Pantheon (DF)	$309.09^{+72.06}_{-119.77}, -19.09^{+22.96}_{-57.63}$	This paper
	Pantheon (DF)	$298.80^{+75.31}_{-118.69}, -23.41^{+19.26}_{-57.41}$	[106]
	808Quasars (DF)	$288.92^{+23.74}_{-28.80}, 6.10^{+16.55}_{-16.40}$	[83]

4 Discussions and conclusions

In this paper, we tested the cosmic anisotropy in the dipole-modulated Λ CDM model and Finslerian cosmological model with the recently-released quasar sample by using a redshift tomographic method. The quasar sample is divided into two subsets by different cutting redshifts. The results of the two cosmological models are similar except for the dipole amplitude A_D . The upper limit of dipole amplitude in the Finslerian cosmological model is about a hundred

times larger than that in the Λ CDM model. The constraints of matter density Ω_m are very weak from all the subsets. In the Finslerian cosmological model, the results of matter density Ω_m from the subsets $z \leq 1.0, 1.1$ are consistent with ones from the Pantheon sample and CMB, but with large uncertainties. The results of parameters β , γ , and δ from most subsets $z \leq z_{cut}$ are consistent within statistical uncertainties, respectively. Though the constraints of matter density Ω_m are very weak from the subsets $z > z_{cut}$, the results of the dipole amplitude A_D show that quasars

Fig. 6 The distributions of preferred directions derived from different samples in the galactic coordinate system. The blue markers indicate preferred directions obtained by the DF method. The red markers indicate preferred directions obtained by the HC method. The green markers indicate the other preferred directions. A indicates the directions in the dipole-modulated Λ CDM model and F indicates the directions in the Finslerian cosmological model

at a higher redshift range may provide more detailed information about the dipole amplitude A_D . The dipole directions derived from two subsets divided by different cutting redshifts are consistent within statistical uncertainties except for the subsets Q1 and Q2. In each cosmological model, the dipole directions from the subsets Q1 and Q2 are deviated by 1σ level. The Pantheon sample is combined with the two subsets Q1 and Q2 to test the cosmological models. In each cosmological model, the dipole amplitude A_D from the combined datasets is very weak. In the dipole-modulated Λ CDM model, the dipole directions from the two combined datasets are far away from each other. For the dataset Q1-P, the dipole direction is $(l, b) = (121.82^{+66.19^\circ}_{-148.87^\circ}, -73.64^{+40.15^\circ}_{-16.36^\circ})$. For the dataset Q2-P, the dipole direction points to $(l, b) = (305.47^{+72.54^\circ}_{-78.58^\circ}, 11.82^{+56.80^\circ}_{-17.39^\circ})$. It is worth noting that the dipole direction in the dataset Q1-P is far away from the one $((l, b) = (306.00^{+91.94^\circ}_{-125.98^\circ}, -23.41^{+22.97^\circ}_{-54.71^\circ}))$ in the Pantheon sample [106]. In the Finslerian cosmological model, the dipole directions from the two combined datasets are close to each other. The two directions are also close to the one given by the Pantheon sample in the Finslerian cosmological model.

Our results also show that the inhomogeneous distributions of quasar and Pantheon samples in the sky have significant effects on dipole directions. We need more homogeneous distribution of datasets to search for a convincing cosmic anisotropy. In the future, the surveys such as J-PAS, Euclid, SKA, LSST, and eROSITA, will provide us with larger and more precise observational data of SNe Ia and quasars. Therefore the test of cosmological principle will be more rigorous by employing more precise datasets that have homogeneous distribution in the sky.

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