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Heavy quark radiation in the quark–gluon plasma in the Moliere theory: angular distribution of the radiation

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Abstract We study the effects of adding the Coulomb interactions to the harmonic oscillator (HO) approximation of the heavy parton propagating through the quark–gluon plasma (the extension to QCD of the Molliere theory). We explicitly find the expression for the transverse momentum distribution of the gluon radiation of the heavy quark propagating in the quark gluon plasma in the framework of the Moliere theory, taking into account the BDMPSZ radiation in the HO approximation, and the Coulomb logarithms described by the additional logarithmic terms in the effective potential. We show that these Coulomb logarithms significantly influence the HO distribution, derived in the BDMPSZ works, especially for the small transverse momenta, filling the dead cone, and reducing the dead cone suppression of the heavy quark radiation (dead cone effect). In addition we study the effect of the phase space constraints on the heavy quark energy loss, and argue that taking into account of both the phase space constraints and of the Coulomb gluons reduces the dependence of the heavy quark energy loss on its mass in the HO approximation.

1 Introduction

The energy loss of a quark propagating in the quark–gluon plasma (QGP) was extensively studied in recent years in different approaches. in particular in the harmonic oscillator approximation, developed by BDMPSZ [1–7], and in the GLV opacity expansion formalism [8–10]. The harmonic oscillator approximation (HO) enables taking control of the coherence effects in the QGP media, while the GLV expansion, although it includes N hard scatterings (the order N opacity expansion) includes also the potentially large Coulomb effects.

The study of the heavy quark energy loss has been long recognized as an important phenomenological tool to diagnose the medium created in heavy ion collisions. Moreover, the heavy quark energy loss can be studied in different formalisms, like opacity expansion, harmonic oscillator approximation (mean field BDMPSZ), ADS/CFT. As a result the detailed understanding of heavy quark energy loss and diffusion is of a paramount importance for understanding the properties of quark–gluon plasma.

The heavy quark energy loss was first explicitly studied in [11] in the harmonic oscillator approximation, whose authors predicted significant decrease of the heavy quark energy loss and of the heavy quark quenching weights due to the dead cone effect, similar to the dead cone effect in vacuum:

$$\omega\frac{dI^{\rm vac}}{d\omega dk_t^2}\sim\frac{\alpha_s C_F}{\pi^2}\frac{k_t^2}{(k_t^2+\theta^2\omega^2)^2}, \tag{1}$$

where m is the mass of the radiating heavy quark, E is the heavy quark energy, $\theta = m/E$ and k_t, ω are the transverse momenta and the energy of the radiated gluon.

This effect however was found to be in a disagreement with the experimental data that shows that quenching weights for heavy and light quark are very close up to rather small jet energies of 25–35 GeV [12,13]. This contradiction led to an extensive research on the heavy quark radiation in the quark gluon plasma.

One interesting question is whether the heavy quark energy loss mechanism can be studied in the pQCD framework. There were two approaches to this problem. First the studies based on N = 1 opacity expansion, second based on the mean field BDMPSZ mean field approach.

The studies based on the N = 1 opacity expansion, starting from the pioneering works [14,15] eventually led to a number of realistic models [16–32], see also [33–36] for recent reviews.

The N = 1 opacity expansion approach, although it accounts for some of the coherence and Coulombic interactions includes only single hard scattering [37] . Thus we have another approach, based on the extension of coherence effects

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connected with Landau–Pomeranchuk–Migdal (LPM) effect to the case of QCD media. This just the mean field BDSMPZ formalism. The connection between BDMPSZ approach and the N = 1 opacity expansion was studied in detail in [38]. This approach was extended to heavy quark case in [11] and later heavy quark effects were studied in this framework in [15,39,40]. In particular, in [15] the important role of correct phase space restrictions was found, and in [40] the higher order corrections to heavy quark jet energy loss were studied.

Note that up to recently only harmonic oscillator approximation for the BDMPSZ approach was developed. The inclusion of Coulombic Logarithms in the BDMPSZ formalism (called the Moliere theory in the framework of conventional LPM effect in QED, (see i.e. [41] for a review) was done only recently in [42–44] for light quarks and gluons. The Moliere theory was extended to the case of heavy quarks in [45]. The latter works however did not include the study of the transverse momenta distributions, and the energy loss calculated without taking into account the proper phase constraints on the radiated gluons. Thus it is of great interest to include the effects of Coulomb Logarithms in the transverse distributions in the BDMPSZ approach. This problem arises the special interest in the case of heavy quarks where the important role of phase constraints was first stressed in [15].

In this paper we extend the BDMPSZ approach for parton propagation in the QGP to include the Coulomb logarithms in the angular/transverse momentum distributions for both massless and massive quarks. We shall explicitly calculate the form and the effects of Coulomb distributions in angular distributions of heavy (and light) quarks and estimate the combined influence of Coulomb effects and phase constraints on the heavy quark energy loss in the ASW framework. We shall see that Coulombic contribution is always positive and tends to increase the angular HO BDMPSZ distributions in the N = 1 GLV direction.

Along the previous papers on the subject [42–44] we shall make only rather qualitative comparison to the experimental data, and concentrate on model independent calculations.

The detailed comparison to the experimental data needs additional model dependent inputs like inclusion of the expansion, correct phase constraints (including realistic phase space constraints, taking into account on the use of soft gluon approximation [46], general for BDMPSZ approach. Thus the detailed calculation of v_2 and R_A for heavy quarks will be done elsewhere. will be done elsewhere.

The paper is organized in the following way. In Sect. 2 we consider the basic formalism for calculation the angular distribution of the radiation. In Sect. 3 we review the calculation in the BDMPS approach in the harmonic oscillator approximation [15,46], in Sect. 4 we have the Moliere theory calculation, in Sect. 5 we present numerical results for angular distributions, for corresponding energy loss and quenching weights in the soft gluon approximation and integrating in transverse momenta $k_t \le \omega$, where ω is the energy of the radiated gluon. In Sect. 6 we take into account the energy conservation in the Leading Logarithmic Approximation, leading to improved phase space constraints for gluon radiation. We see that the inclusion of these constraints leads to further decrease of the dependence of heavy quark energy loss on its mass. Our results are summarised in conclusion. Some useful mathematical formulae are given in the Appendix.

2 Basic formalism

2.1 Basic formulae

The heavy quark angular distribution in the media is given by [15]

$$\omega\frac{dI}{d\omega d^2k} = \frac{C_F\alpha_s}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^\infty dt_1\int_0^{t_1} dte^{-i\vec{k}_t\vec{y}} \times e^{-\int_{t_1}^\infty dsn(s)V(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t) -K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}. \quad (2)$$

Here K is the propagator of the particle in the media with the two dimensional effective potential due. to the scattering centres, and K_0 is the corresponding propagator of the free particle in the vacuum. The effective two dimensional potential is given by

$$V(\vec{\rho}) = i\int\frac{d^2q_t}{(2\pi)^2}(1-\exp(i\vec{q}_t\vec{\rho}))\frac{d^2\sigma_{el}}{d^2q_t}. \quad (3)$$

Here $d^2\sigma_{el}/d^2q_t$ is the cross section of elastic scattering of high energy particle on the media centre. We assume the static media of the form

$$n(s) = U(L-s)U(s) \quad (4)$$

where $U = 1$ if $s \ge 0$ and 0 if $s < 0$ is a conventional step function.

The media is described by Gyulassy–Wang model [47]. The effective potential in the momentum space is given by

$$\frac{d\sigma(\vec{q}_t)}{d^2q_t} = \frac{4\pi\alpha_s m_D^2 T}{(q_t^2+\mu^2)^2} \equiv \frac{g^4n}{(q_t^2+\mu^2)^2}, \quad (5)$$

where the parameter $\mu \sim m_D$, and the Debye mass m_D is given by

$$m_D \sim 4\pi\alpha_s T^2(1+N_f/6) = \frac{3}{2}g^2T^6 \quad (6)$$

for $N_f = 3$ light quarks, T is the media/QGP temperature. The density of the scattering centres in the GW model is given by $n = \frac{3}{2}T^3$, and the strong coupling is $\alpha_s = \frac{g^2}{4\pi}$. The effective potential in the coordinate space is

$$V(\rho) = \frac{\hat{q}}{4N_c}(1 - \mu\rho K_1(\mu\rho)$$
$$= \frac{\hat{q}\rho^2}{4N_c}\left(\log\left(\frac{4}{\mu^2\rho^2}\right) + 1 - 2\gamma_E\right), \quad (7)$$

where $\gamma_E = 0.577$ is the Euler constant, and the bare quenching coefficient is

$$\hat{q} = 4\pi\alpha_s^2 N_c n. \quad (8)$$

Note that $\hat{q}$ is fully determined by media properties, and does not depend on the quark mass.

For processes that are dominated by large momentum transfer is enough to take into account only the first terms in the Taylor expansion of $V(\rho)$. The first approximation corresponds to the quadratic term in the expansion (7) and is called the HO (harmonic oscillator) approximation. In this approximation the effective potential V is given by

$$V(\rho) = \frac{1}{4}\hat{q}_{\rm eff}\rho^2. \quad (9)$$

Here $\hat{q}_{\rm eff}$ is the effective jet quenching coefficient, given by

$$\hat{q}_{\rm eff} = \hat{q}\log\left(\frac{Q^2}{\mu^2}\right), \quad (10)$$

and Q is the typical transverse momenta, accumulated by the particle on the scale of the coherence length.

The HO effectively describes the LPM bremsstrahlung [1]. More precise treatment of the energy loss includes also large Coulomb logarithms and is called in the theory of the Abelian (QED) LPM effect the Moliere theory [41]. In the QCD framework the inclusion of Coulomb interactions can be made using the perturbation theory [42,43]. Namely, instead of the usual opacity expansion [8–10], we shall consider the perturbation theory around the oscillator potential adding the Coulomb effects as a perturbation. The effective potential in Moliere theory is given by

$$V(\rho) = \frac{1}{4}\hat{q}\rho^2\log(1/\rho^2\mu^2), \quad (11)$$

and includes the short range Coulomb logarithms. In the framework of the perturbation theory this potential is split as

$$V(\rho) = V_{HO}(\rho) + V_{pert}(\rho),\ V_{HO}(\rho)$$
$$= \frac{\hat{q}\log(Q^2/\mu^2)}{4}\rho^2,\ V_{\rm pert}(\rho)$$
$$= \frac{\hat{q}}{4}\log\left(\frac{1}{Q^2\rho^2}\right), \quad (12)$$

where Q is the typical momenta, defined above, equal to $Q \sim \sqrt{\hat{q}\omega}$ in the HO approximation. We shall need sufficiently large Q, so that

$$\log(Q^2/\mu^2) \gg \log\left(\frac{1}{Q^2\rho^2}\right), \quad (13)$$

i.e. perturbation theory is applicable meaning that we probe rather small transverse distances.

Then the energy loss is given by Eq. (2), where the propagator K is calculated in perturbation theory as [42,43]

$$K(\vec{y}, t_1; \vec{x}, t) = K_{HO}(\vec{y}, t_1; \vec{x}, t)$$
$$- \int d^2z \int_t^{t_1} ds K_{HO}(\vec{y}, t_1; \vec{z}, s)V_{pert}(z)K_{HO}(\vec{z}, s; \vec{x}, t). \quad (14)$$

Here K_{HO} is the heavy quark propagator in the imaginary two dimensional potential V_{HO} [15]:

$$K_{HO}(\vec{y}, t_1; \vec{x}, t) = \frac{i\omega\Omega}{2\pi\sinh\Omega(t_1 - t)}$$
$$\times\exp\left(\frac{i\omega\Omega}{2}\{\coth\Omega(t_1 - t)(\vec{x}^2 + \vec{y}^2)\right.$$
$$\left.-\frac{2\vec{x}\vec{y}}{\sinh\Omega(t_1 - t)}\}\right)\exp(-i\theta^2\omega(t_1 - t)/2), \quad (15)$$

and

$$\Omega = \frac{(1 + i)}{2}\sqrt{\frac{\hat{q}}{\omega}}. \quad (16)$$

In the limit when there is no media this propagator reduces to free quark propagator

$$K_0(\vec{y}, t_1; \vec{x}, t) = \frac{i\omega}{2\pi}\exp\left(i\frac{\omega(\vec{x} - \vec{y})^2}{2(t_1 - t)}\right). \quad (17)$$

2.2 Qualitative dynamics of the heavy quark

The expansion written in the form (14) clearly exhibits the formation lengths described in the Introduction: the heavy quark mass leads to the oscillating exponent $\exp(i\theta^2\omega/2(t_1 - t))$ in Eq. (15), while the harmonic oscillator part of the propagator (15) oscillates with the frequency $\sqrt{\omega/\hat{q}}$. Then it is clear that when $l_c^q \ll l_c^{LPM}$ the oscillations due to heavy quark mass cut off the integral for heavy quark energy loss, the oscillating harmonic oscillator part of the propagator is approximately freezed and the LPM effect is not relevant, the energy loss is defined by the induced radiation on the scattering centres-the N = 1 GLV. On the other hand, in the opposite case, the heavy quark exponent is close to one, and the integral for energy loss is controlled by the HO multiplier. We have LPM bremsstrahlung plus corrections due to Coulomb logarithms.

We can now choose the subtraction scale Q in the momentum space. As it was explained in [38,42] this scale corresponds to the typical momentum accumulated by the quark along the coherence length propagation. Such momentum squared is $\hat{q} \times \sqrt{\omega/\hat{q}}$ for $\omega \ll \omega_{DC}$ and $\sim \theta^2\omega^2 \sim \omega/l_c^q$

for $\omega \gg \omega_{DC}$. Consequently we shall use the interpolation formula

$$Q^2 = \sqrt{\omega \hat{q}_{\text{eff}}}U(-\omega + \omega_{DC}) + \theta^2\omega^2 U(\omega - \omega_{DC}), \quad (18)$$

where $U(x)$ is a unit step function:$U(x) = 1$ if $x \geq 0$, and $U(x) = 0$ if $x \leq 0$.

Alternatively, the dynamics of the heavy quark can be approached using the arguments in [38]. Namely , in the LPM (diffusion) regime the distribution over momentum transfers in the scattering on the media centres is described by a gaussian, peaked in the $Q^2_{typ} \sim \sqrt{\hat{q}w}$. The scattering with significantly higher momentum transfers q_t is described by the tail of the distribution, which is N = 1 GLV, that essentially describes the independent scattering on the media centres. In this region the LPM gaussian is parametrically close to zero, and N = 1 GLV dominates. It was explained in [50,51] that N = 1 term in opacity expansion is a good description of large momentum transfer regime, since such scatterings in the tail occur quite rarely. Since inside dead cone the typical momenta is $k_t^2 \sim \omega/l_c^q \sim \theta^2\omega^2 \gg \sqrt{\hat{q}\omega}$, inside the dead cone we shall find ourselves in the GLV regime.

2.3 N = 1 GLV

We shall also need the explicit expression for N = 1 term in the opacity expansion for angular distribution for massive quark. The corresponding result was derived in [15], and has the form:

$$\omega\frac{dI}{d\omega d^2k_t} = \int_0^\infty dq^2 \frac{2\alpha_s C_F \hat{q}}{\pi^2\omega}\frac{LQ_1 - \sin(LQ_1)}{Q_1^2}\frac{q^2}{q^2+\theta^2\omega^2}$$
$$\times \frac{m_D^2(k^2+\theta^2\omega^2) + (k^2-\theta^2\omega^2)(k^2-q^2)}{(k^2+\theta^2\omega^2)((m^2+k^2+q^2)^2 - 4k^2q^2)^{3/2}}. \quad (19)$$

where

$$Q_1 = (q^2 + \theta^2\omega^2)/(2\omega). \quad (20)$$

Here k_t is the momentum of the radiated gluon.

3 Angular distribution in the harmonic oscillator approximation

The angular distribution of the gluon radiation was first calculated for heavy quark in [15] , and contains two contributions: The first is the bulk contribution and is given by

$$\omega\frac{dI^{HO\ Bulk}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y \int_0^L dt_1 \int_0^{t_1} dt e^{-i\vec{k}_t\vec{y}}$$
$$\times e^{-1/4\hat{q}(L-t_1)y^2}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y}, t_1, \vec{x}, t) - K_0(\vec{y}, t_1; \vec{x}, t))|_{\vec{x}=0}, \quad (21)$$

where K is the heavy quark propagator in harmonic oscillator approximation given by Eq. (15).

The second contribution is a boundary term given by

$$\omega\frac{dI^{\text{HO boundary}}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y \int_L^\infty dt_1 \int_0^L dt e^{-i\vec{k}_t\vec{y}}$$
$$\times\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y}, t_1, \vec{x}, t) - K_0(\vec{y}, t_1; \vec{x}, t))|_{\vec{x}=0}, \quad (22)$$

where the propagator K is given by ($t_1 > L > t$)

$$K(\vec{y}, t_1; \vec{x}, t) = \int d^2z K_0(\vec{y}, t_1; \vec{z}, L)K_{HO}(\vec{z}, L; \vec{x}, t). \quad (23)$$

The direct calculation shows that the bulk term is given by:

$$\omega\frac{dI^{\text{HO Bulk}}}{d\omega d^2k_t} = -2\text{Re}\int_0^L dt \int_t^L dt_1 \frac{\alpha_s C_F \Omega^2}{\pi^2R^2\sinh\Omega(t_1-t)^2}$$
$$\times\left(q(L-t_1) - \frac{2i\omega\Omega\cosh\Omega(t_1-t)}{R}\right)$$
$$\times\exp(i\theta^2\omega(t-t_1)/2)\exp(-k_t^2/R), \quad (24)$$

where

$$R = q(L-t_1) - 2i\omega\Omega\coth\Omega(t_1-t). \quad (25)$$

The boundary term is given by

$$\omega\frac{dI^{\text{HO boundary}}}{d\omega d^2k_t} = \int_0^L dt \frac{-i\alpha_s C_F k_t^2}{(k_t^2+\theta^2\omega^2)(2\pi)^2\omega}$$
$$\times\frac{\exp\left(\frac{-ik_t^2\tanh\Omega(L-t)}{2\omega\Omega}\right)\exp(i\theta^2\omega(t-L)/2}{\cosh\Omega(L-t)^2} \quad (26)$$

from these expressions we subtract their $\hat{q} = 0$ limit. These expressions of course coincide with the corresponding ones in [15].

4 Coulomb corrections

Let us consider now the full expression for angular distribution:

$$\omega\frac{dI}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y \int_0^\infty dt_1 \int_0^{t_1} dt e^{-i\vec{k}_t\vec{y}}$$
$$\times e^{-\int_{t_1}^\infty ds n(s)(V_{HO}+V_{pert})(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y}, t_1, \vec{x}, t)$$
$$-K_0(\vec{y}, t_1; \vec{x}, t))|_{\vec{x}=0}, \quad (27)$$

where K is now the full propagator that is also calculated in the perturbation theory:

$$K = K_{HO} + K_{HO}V_{pert}K_{HO}. \quad (28)$$

In the Moliere theory approach we carry the perturbation theory over V_{pert} with the solution for harmonic oscillator approximation being the zero order term. Then it is clear from Eq. (27) that there are two distinct term in the perturbation theory: first term is due to the expansion of the exponent in Eq. (27) in powers of V_{pert}, while the second term is due to expansion of the propagator. The latter term is in turn a sum of two terms, first the boundary term with $t_1 > L$ and the bulk term with $t_1 < L$. We shall now move to calculation of these 3 terms: the term that comes from the exponent expansion and the two terms that come from the perturbative expansion of the propagator.

4.1 Exponent expansion

Explicitly this term is given by

$$\omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k} = -\frac{\alpha_s C_F}{(2\pi)^2\omega^2}2Re\int d^2y\int_0^L dt_1\int_0^{t_1} dt e^{-i\vec{k}_t\vec{y}} \times(L-t_1)V_{pert}(\vec{y})\partial_{\vec{x}}\partial_{\vec{y}}(K_{HO}(\vec{y},t_1,\vec{x},t) -K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}. \quad (29)$$

Substituting the known expressions for V_{pert} and the propagators we obtain

$$\omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k} = -\frac{\hat{q}}{4}\alpha_s C_F\int_0^L dt_1\int_0^{t_1} dt\frac{\Omega^2}{(2\pi)^3\sinh(\Omega(t_1-t)^2} \times\int d^2u u^2\log(1/(u^2Q^2))(2+i\omega\Omega\coth(\Omega(t_1-t))u^2) \times\exp(i\omega\Omega\coth(\Omega(t_1-t))u^2/2) \times\exp(-\frac{1}{4}\hat{q}(L-t_1)u^2-i\vec{k}_t\vec{u})\exp(i\theta^2\omega(t-t_1)/2). \quad (30)$$

The integral over the transverse momenta azimuthal angle can be easily taken using the standard integral [48]

$$\int_0^{2\pi} d\phi\exp(-i\vec{k}_t\vec{u}) = J_0(k_t u). \quad (31)$$

Let us introduce two new functions that can be expressed through elementary functions (see Appendix A):

$$F_2(p,c,Q) = \int_0^\infty x^3\log(x^2Q^2)J_0(cx)\exp(-px^2)$$
$$F_3(p,c,Q) = \int_0^\infty x^5\log(x^2Q^2)J_0(cx)\exp(-px^2). \quad (32)$$

So we finally get

$$\omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k_t} = \frac{\hat{q}}{4}2Re\frac{\alpha_s C_F\Omega^2}{(2\pi)^2}\int_0^L dt_1\int_0^{t_1} dt(2F_2(R/4,k_t,Q) +i\omega\Omega\coth(\Omega(t_1-t))F_3(R/4,k_t,Q))$$
$$R = \hat{q}(L-t_1)-2i\omega\Omega\coth\Omega(t_1-t). \quad (33)$$

4.2 Propagator expansion: the bulk term

We now consider the contribution to the angular distribution due to the perturbative expansion of the propagator in the powers of V_{pert}.. We have in the integral over t_1 two terms: the first is from 0 to L and is called a bulk term, the second corresponds to the case when $t_1 > L$ and is called a boundary term. Let us consider first the bulk term

$$\omega\frac{dI^{\text{Coulomb bulk}}}{d\omega d^2k} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}\int d^2z\int d^2u\int_0^L dt_1\int_0^{t_1} dt \times\int_t^{t_1} ds\frac{e^{-\hat{q}(L-t_1)u^2/4-i\vec{k}_t\vec{u}}}{\sinh\Omega(t_1-s)\sinh\Omega(s-t)} \times\partial_{\vec{y}}\partial_{\vec{u}}K_{HO}(u,t_1;\vec{z},s) \times\left(\frac{\hat{q}}{4}z^2\log(1/z^2Q^2)\right)K_{HO}(\vec{z},s;t,\vec{y}=0). \quad (34)$$

After integration by parts we obtain, calculating the gaussian integral over d^2u

$$\omega\frac{dI^{\text{Coulomb bulk}}}{d\omega d^2k_t} = \int d^2z\int_0^L dt_1\int_0^{t_1} dt\int_t^{t_1} ds \times\frac{-\alpha_s C_F i\omega^2\Omega^4(\hat{q}(L-t_1)z^2+2i\cosh(\Omega(t_1-s)\vec{k}_t\vec{z})}{16\pi^3\sinh(\Omega(t_1-s))^2\sinh(\Omega(s-t))^2} \times\frac{z^2\log(1/(z^2Q^2))}{R(t_1,s)^2}\exp(-k_t^2/R(t_1,s) -2\omega\Omega\vec{k}_t\vec{z}/(R(t_1,s)\sinh\Omega(t_1-s))) \times\exp\left(i\frac{\omega\Omega z^2\sinh\Omega(t_1-t)}{2\sinh(\Omega(t_1-s))\sinh(\Omega(s-t))}\frac{R(t_1,t)}{R(t_1,s)}\right) \quad (35)$$

where $R(t_1,t)$ is given by Eq. (33). The angular integral can be easily taken using the standard formulae [48]:

$$\int_0^{2\pi}\cos(x)\exp(-A\cos(x))dx = 2\pi iJ_1(iA)$$
$$\int_0^{2\pi}\exp(-A\cos(x))dx = 2\pi J_0(iA) \quad (36)$$

where J_0, J_1 are the conventional Bessel functions. Introducing an additional function

$$F_4(p,c,Q) = \int_0^\infty dz z^4 J_1(cz)\exp(-pz^2)\log(z^2Q^2) \quad (37)$$

we obtain final answer:

$$\omega\frac{dI^{\text{Coulomb bulk}}}{d\omega d^2k_t} = \alpha_s C_F\int_0^L dt_1\int_0^{t_1} dt\int_t^{t_1} ds\frac{q}{4}\frac{i\omega^2\Omega^4}{2\pi^2} \times\frac{\exp(-k_t^2/R(t_1,s))}{\sinh\Omega(t_1-s)^2\sinh\Omega(s-t)^2} \times(\hat{q}(L-t_1)F_3(p,c,Q)-2k_t\cosh\Omega \times(t_1-s)F_4(p,c,Q))\exp(i\theta^2\omega(t-t_1)/2)$$

$$p = -\frac{\omega\Omega \sinh\Omega(t_1-t)}{2\sinh\Omega(t_1-s)\sinh\Omega(s-t)}\frac{R(t_1,t)}{R(t_1,s)}$$
$$c = k_t\frac{2i\omega\Omega}{R(t_1,s)\sinh\Omega(t_1-s)}. \tag{38}$$

4.3 Boundary term

Finally we consider the boundary contribution:

$$\omega\frac{dI^{\text{Coulomb boundary}}}{d\omega d^2k_t} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2}\int d^2z\int d^2r\int d^2u\int_L^\infty dt_1 \times\int_0^L dt\int_t^L ds\frac{\exp(-i\vec{k}_t\vec{u})}{\sinh\Omega(t_1-s)\sinh\Omega(s-t)} \times\partial_{\vec{y}}\partial_{\vec{u}}K_0(t_1,\vec{u};,L,\vec{r})K_{HO}(\vec{r},L;\vec{z},s) \times\frac{\hat{q}}{4}z^2\log(1/z^2Q^2)K_{HO}(\vec{z},s;t,\vec{y}=0). \tag{39}$$

We first do integral over d^2y and over t_1-L,and then gaussian integral over d^2r. Using the Eqs. (36) for angular integration and the definition (37) of the function F_4 we easily obtain a final answer for the boundary term.

$$\omega\frac{dI^{\text{Coulomb boundary}}}{d\omega d^2k_t} = \frac{\hat{q}}{4}2\alpha_s C_F Re\frac{\omega\Omega^2}{(2\pi)^2}\frac{ik_t}{k_t^2+\theta^2\omega^2} \times\exp\Big(-i\frac{k_t^2\tanh(\Omega(L-s))}{2\omega\Omega}\Big) \times F_4\Big(\frac{-i\omega\Omega\cosh(\Omega(L-t)}{\cosh(\Omega(L-s)\sinh\Omega(s-t))}, \times\frac{k_t}{\cosh\Omega(L-s)},Q\Big)\exp(i\theta^2\omega(t-L)/2). \tag{40}$$

Note that functions F_2, F_3, F_4 can be easily expressed through known special functions, the explicit expressions are given in the Appendix. In addition it is easy to perform integral over the transverse momentum k_t between 0 and some scale ω_1 analytically.

5 Numerics

Our final answer is the sum of all terms that we calculated in the previous two chapters.

$$\omega\frac{dI(\omega,\hat{q},\theta,\mu,L)}{d\omega d^2k_t} = \omega\frac{dI^{HO}(w,q_{eff},\theta,\mu,Q_{eff},L)}{d\omega d^2k_t} +\frac{\hat{q}}{4}\omega\frac{dI^{\text{Coulomb}}(\omega q_{eff},\theta,Q_{eff},\mu,L)}{d\omega d^2k_t} \tag{41}$$

$$\omega\frac{dI^{HO}}{d\omega d^2k_t} = \omega\frac{dI^{\text{HO Bulk}}}{d\omega d^2k_t}+\omega\frac{dI^{\text{HO Boundary}}}{d\omega d^2k_t} \tag{42}$$

$$\omega\frac{dI^{\text{Coulomb}}}{d\omega d^2k_t} = \omega\frac{dI^{\text{Coulomb one}}}{d\omega d^2k_t}+\omega\frac{dI^{\text{Coulomb Bulk}}}{d\omega d^2k_t} +\omega\frac{dI^{\text{Coulomb Boundary}}}{d\omega d^2k_t} \tag{43}$$

where the terms with the index HO are given by Eqs. (24), (26), while the terms with the index are given by Eqs. (33), (38) and (40) (without external multiplier $\hat{q}/4$). The effective scale Q_{eff} is given by Eq. (18), and the effective quenching coefficient is given by Eq. (10).

For our numerical estimates we shall use the same parameters for QGP as in [42,43,45]: T = 0.4 GeV, $\alpha_s = 0.3$, leading to $\mu = m_D = 0.9$ GeV and $\hat{q} = 0.3$ GeV^3.

We do numerically double and triple integrals in t, t_1, s using the Mathematica 12 software.

5.1 Angular distributions for soft gluons

We shall now present the numerical estimates for the angular distributions of the radiated gluons and compare them with the BDMPS angular spectrum [15] and N = 1 GLV angular distributions. We shall depict these angular distributions for typical values of: $\omega = 5, 10, 20$ GeV in Figs. 1, 2 and 3. The BDMPS maximum angle is $\theta_{BDMPS} = (\hat{q}/\omega^3)^{1/4} = 0.22$ for $\omega = 5$ GeV, 0.13 for $\omega = 10$ GeV, and 0.08 for 20 GeV. for $\hat{q}\sim 0.3$ that we use in our calculations.

For $\omega = 5$ GeV the BDMPS angle is outside the dead cone for all values of dead cone angle that we consider, i.e. $\theta \le 0.2$. The Coulomb corrections to BDMPS are significant and are the biggest for small k_t, although the calculations for large k_t especially of order ω are not trustworthy, since we use soft gluon approximation in the BDMPS approach. We see that the Coulomb correction is approximately constant at small k_t and starts to decrease in parallel with BDMPS contribution.

In Fig. 2 we consider $\omega = 10$ GeV. In this case two last values of $\theta = 0.15, 0.2$ correspond to the BDMPS maximum. θ_{BDMPS} inside the dead cone. We see that. the Coulomb correction together with BDMPS gluons fill the dead cone.

In Fig. 3 already three last values of $\theta = 0.1, 0.15, 0.2$ correspond to the situation when θ_{BDMPS} is inside the dead cone. In all these cases there is no dead cone effect, and BDMPS and Coulomb radiation fills the dead cone region.

We expect that the sign changing part of the distributions for large k_t is actually an artifact of the soft gluon approximation, that becomes unapplicable for large k_t. The Coulomb correction is approximately constant at small k_t and starts to decrease for k_t larger than the BDMPS maximum.

5.2 Energy loss

It is also interesting to check how the combined effect of the phase space constraints and Coulomb logarithms influence

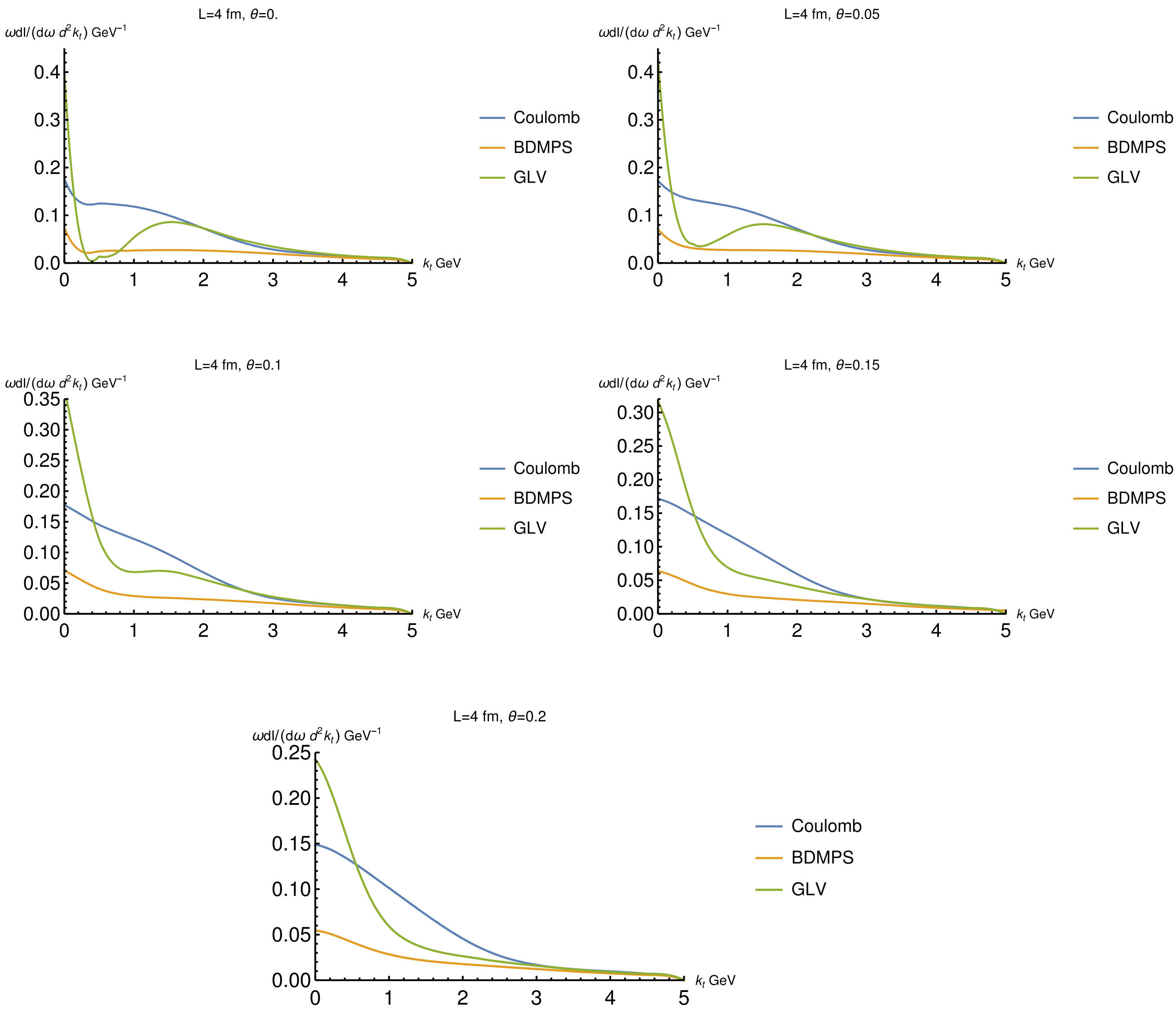

Fig. 1 The angular distribution of radiated gluons for $\omega = 5$ GeV for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$. Here and in the Figs. 1 and 2 BDMPS means the BDMPS angular distribution in the Harmonic Oscillator approximation given by Eq. (42), Coulomb means the angular distribution in the Moliere theory given by Eq. (41). All graphs here and below are presented divided by $\alpha_s C_F$

the energy loss. We used the soft gluon approximation, so introducing the explicit boundary for the k_t may be beyond the accuracy of our approach [15], but still introducing the boundary $k_t \le \omega$ will give a good indication of the effect.

It is easy to integrate over k_t in arbitrary finite limits, analytically, since the integrands in the expressions for angular distributions in the previous two chapters , since these expressions are gaussian in k_t^2. The remaining integrals are integrals in over t, t_1, s and are taken numerically using Mathematica, in the same way as the integrals for angular distributions.

We see that compared with the BDMPS spectrum calculated with the same boundary conditions, the corrections increase the energy loss and is rather close to GLV energy loss. Note however that this is just the qualitative estimate since, as we remarked above, the angular distributions were calculated in soft gluon approximation, and precise phase space constraints are beyond the accuracy of this approximation [15,46].

5.3 Quenching weights

It will be also interesting to estimate the quenching weights in soft gluon approximation [37,49] but including the finite integration region in the transverse momentum $k_t \le \omega$ and the gluons.

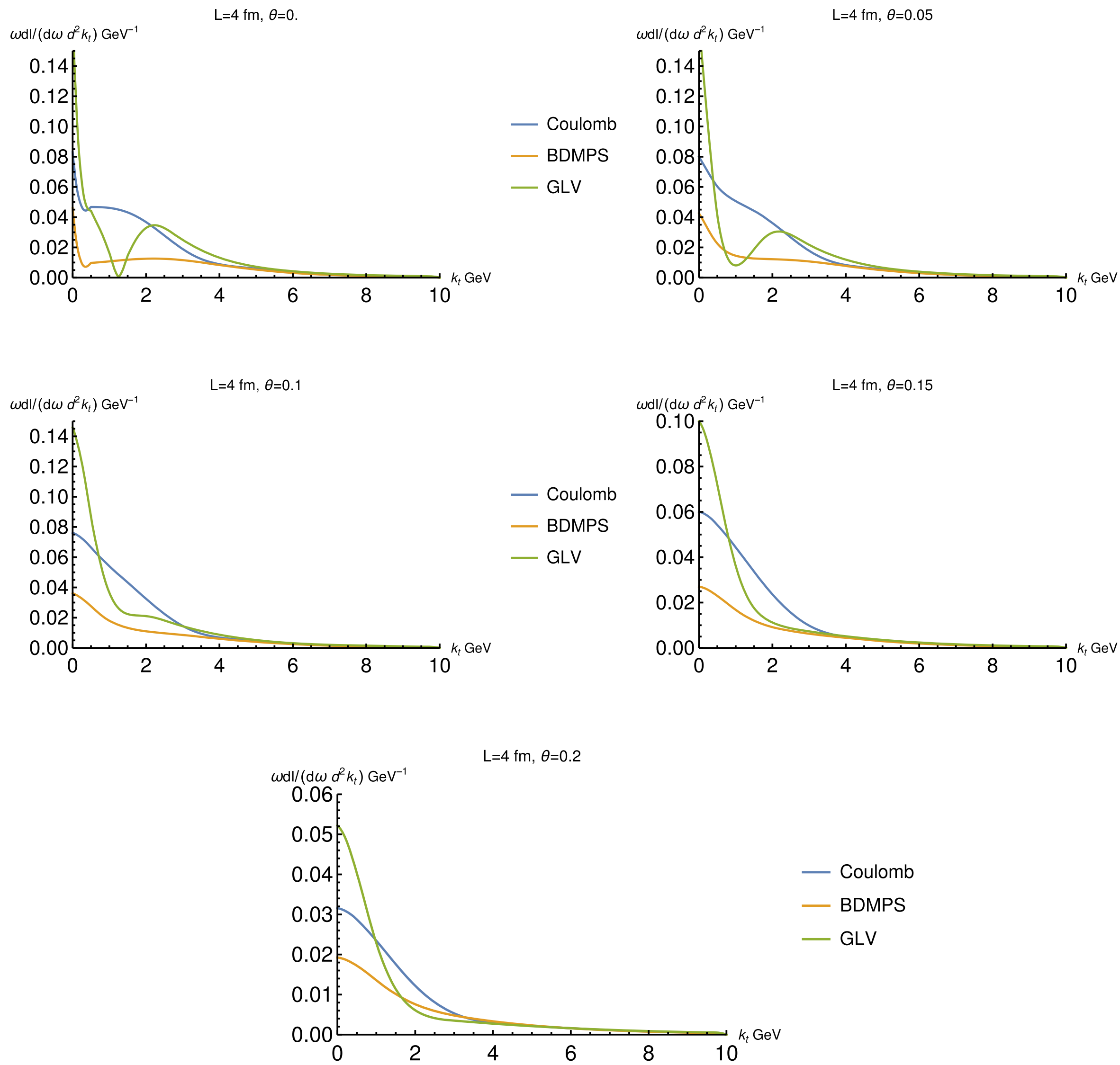

Fig. 2 The angular distribution of the radiated gluons for $\omega = 10$ GeV for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$

As it is known the jet quenching factor describes the energy loss due to the arbitrary number of Poisson distributed gluons. Indeed, in the previous chapters we calculated the energy loss probability $\omega dI d\omega$ in the first order in α_s. Then we can calculate the quenching factor

$$Q(E) = \exp\left(-\int_0^E \left(1 - \exp\left(-\frac{R}{E}\omega\right)\right)\frac{dI}{d\omega}\right), \tag{44}$$

where

$$R = \frac{d\sigma^0}{dp_t^2} \tag{45}$$

is determined from the experimental data, $R \sim 5$. Here σ^0 is the radiation cross section in the vacuum, outside of the media.

Table 1 The estimate for quenching coefficients S(E) for light and heavy quarks, for $L = 4$ fm width. The jet quenching factor $Q(E) = \exp(S(E))$ Here $\alpha_s = 0.3$. The BDMPS means quenching weight calculated in the harmonic oscillator approximation with constraint $k_t \le \omega$, $\omega \le E$. BDMPS+Coulomb means quenching weights calculated in the Moliere Theory (BDMPS plus Coulomb logarithms) with the same limitation $k_t \le \omega$, $\omega \le E$

BDMPS	$E = 25$ GeV − S(E)	$E = 35$ GeV − S(E)	$E = 50$ GeV − S(E)
Light quark m = 0	0.66	0.61	0.53
Heavy quark $m_b = 5$ GeV	0.33	0.36	0.4
BDMPS+Coulomb	− S(E)	− S(E)	− S(E)
Light quark m = 0	1.8	1.6	1.32
Heavy quark $m_b = 5$ GeV	1.1	1.13	1.1

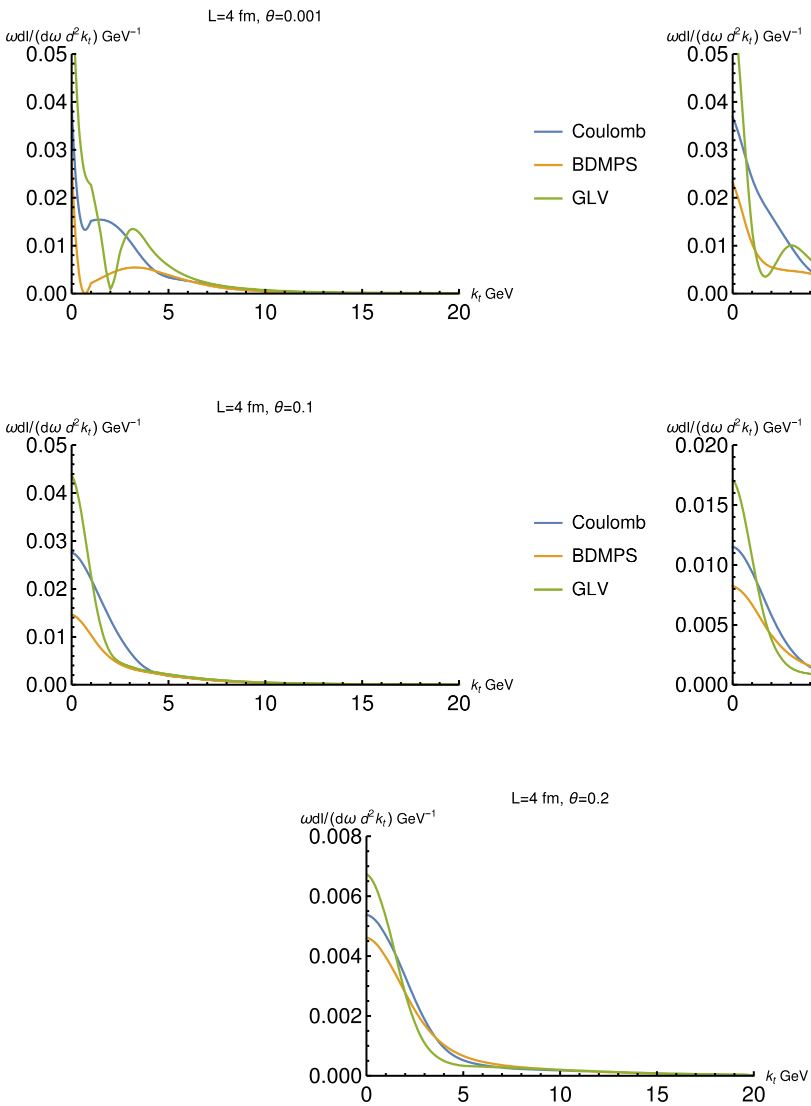

Fig. 3 The angular distribution of radiated gluons for $\omega = 20$ GeV for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$

The quenching weights for given energy do not change between $\theta = 0$ and $\theta \sim 0.06$. We see that the inclusion of Coulomb gluons improves the agreement with experimental data, leading to ratio of quenching weights of massless and heavy quarks with $m_Q = 5$ GeV (bottom quark). We have $Q = \exp(-S(E)$ of order 1 at 100 GeV, 0.8 at 50 GeV and 0.65–0.5 for 35 and 25 GeV, There is no difference between massless and charm quarks, at least for jet energies above 25 GeV. These quenching weights are depicted in Table 1.

6 Longitudinal phase space constraints

In the previous section we studied the heavy quark radiation in the Moliere theory in the soft gluon approximation $k_t \ll \omega$, $\omega \ll E$. It was shown in [5–7,38,50,52], that one can take into account the finite gluon energy.

This means that for a parton with the energy $Ez, 0 < z < 1$ whose propagator we calculate the effective mass in the propagator is substituted from $\omega = Ez$ to $Ez(1-z)$. As it was pointed in [5] there is no sense to continue beyond $0 \le z \le 1/2$. Then the effective potential which was without phase constraints

$$V = \frac{1}{4}\hat{q}\rho^2 \log(1/(\mu^2\rho^2) = \frac{1}{4}\hat{q}\rho^2 \log(Q^2/\mu^2)$$
$$= \frac{1}{4}\hat{q}\rho^2 \log(1/(Q^2\rho^2)) \tag{46}$$

becomes

$$V(\rho) = \frac{1}{8}\hat{q}(\rho^2 \log(1/(\rho^2\mu^2)$$
$$+((1-z)^2\rho^2 \log 1/((1-z)^2\rho^2\mu^2) - z^2/9x^2 \log$$
$$\times(1/(z^2\rho^2\mu^2) \tag{47}$$

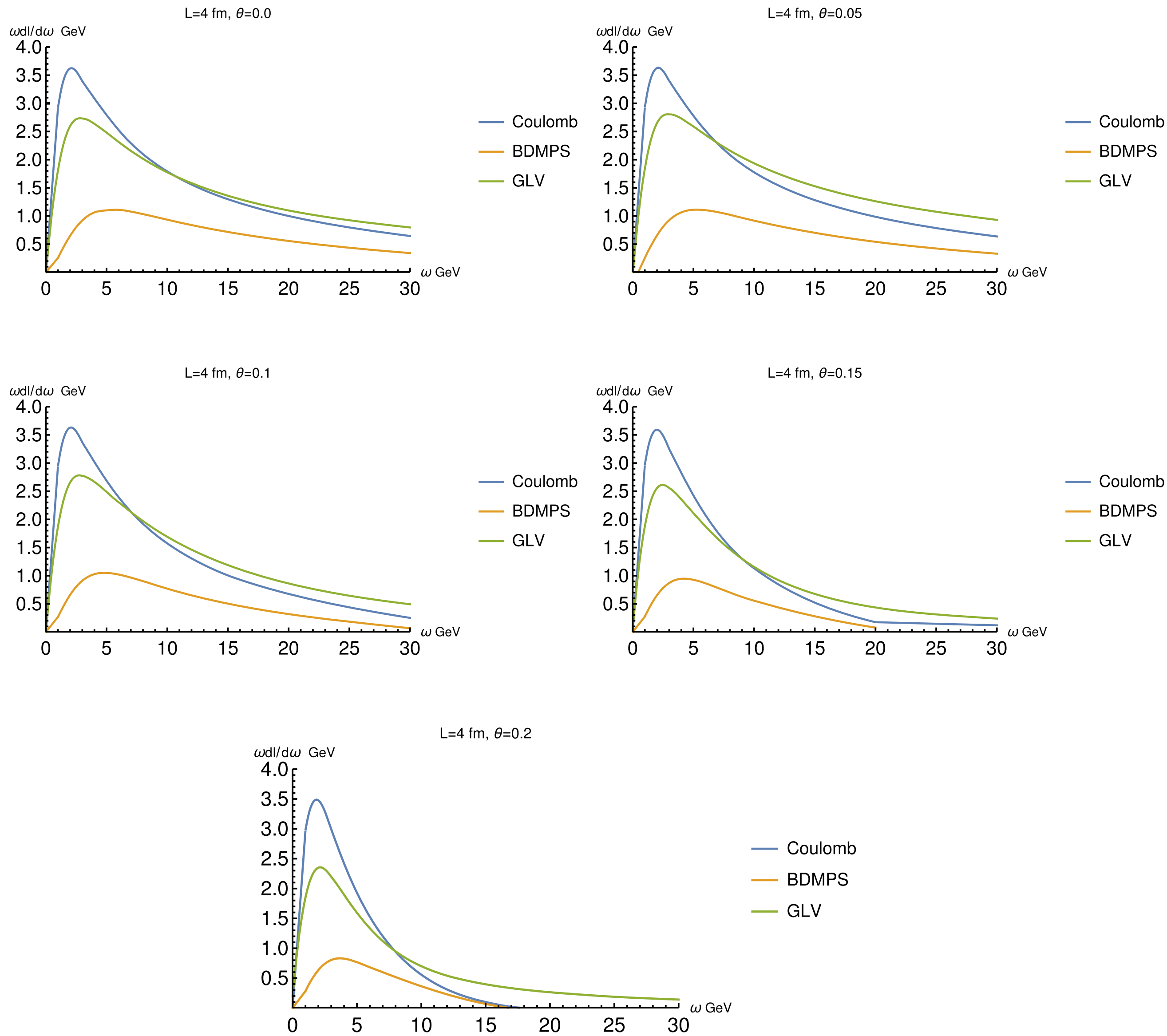

Fig. 4 The energy loss $\omega dI/d\omega$ with Coulomb gluons for different $\theta = 0, 0.05, 0.1, 0.15, 0.2$. The energy loss $\omega dI/d\omega$ was calculated by integrating the corresponding angular distribution over k_t in the finite interval of k_t from $k_t = 0$ to the kinematical bound $k_t \leq \omega$. BDMPS means the expression for soft gluon emission in harmonic oscillator approximation, and Coulomb means the full result including Coulomb logarithms (Moliere Theory)

note that in the $z \to 0$ limit the potential will be given by Eq. (46). The expression for angular distribution is now

$$z\frac{dI}{dzd^2k_t} = \frac{(1+(1-z)^2)}{2}\frac{C_F\alpha_s}{(2\pi)^2(z(1-z))^2} \times 2Re\int d^2y\int_0^\infty dt_1\int_0^{t_1} dte^{-i\vec{k}_t\vec{y}} \times e^{-\int_{t_1}^\infty dsn(s)V(\vec{y}(s))}\partial_{\vec{x}}\partial_{\vec{y}}(K(\vec{y},t_1,\vec{x},t) -K_0(\vec{y},t_1;\vec{x},t))|_{\vec{x}=0}, \tag{48}$$

where the propagator for massive quark is now calculated with substitution $\omega \to Ez(1-z)$ and satisfies

$$\left(i\frac{\partial}{\partial t} + \frac{\vec{\partial}^2}{2z(1-z)E} + iV(x) + m^2\right)K(\vec{y},t_1;\vec{x},t)) = i\delta(\vec{x}-\vec{y})\delta(t-t_1). \tag{49}$$

The function n(s) = U(L − s)U(s) is a QGP density profile for the propagating heavy quark. We split the potential into a sum

$$V = V_0 + V_{pert}. \tag{50}$$

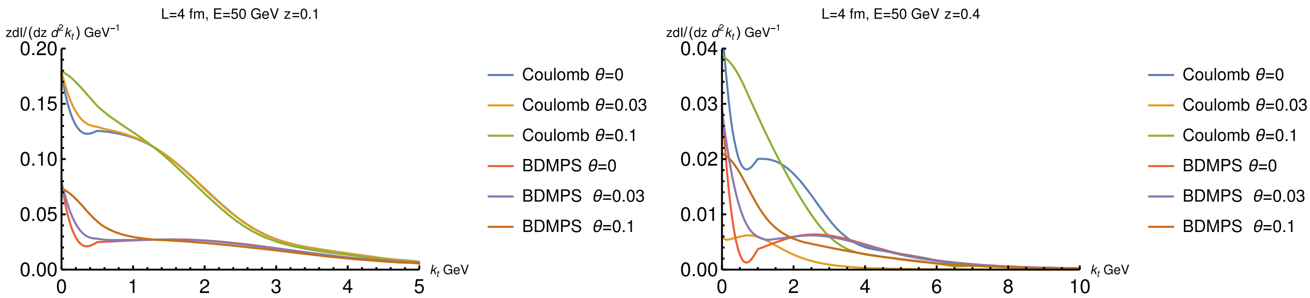

Fig. 5 The angular distributions $z\frac{dI}{dzd^2k_t}$ for different $\theta = 0, 0.03, 0.1$ for E = 50 GeV (corresponding to massless, charmed and bottom quarks). As above BDMPS means the angular gluon distribution calculated in the harmonic oscillator approximation but including finite gluon energy, Coulomb means the angular distribution in the Moliere theory (i.e. BDMPS+Coulomb Logarithms) calculated taking into account finite gluon energy.

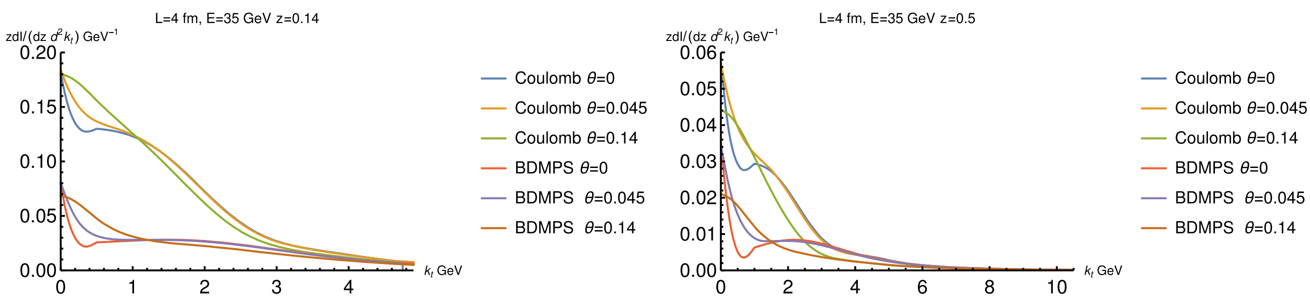

Fig. 6 The angular distributions $z\frac{dI}{dzd^2k_t}$ for different $\theta = 0, 0.045, 0.14$ (corresponding to massless, charmed and bottom quarks) for E = 35 GeV and z = 0.14 (left), z = 0.5 (right)

The potential V_0 is now given by

$$V_0(\vec{\rho}) = \frac{1}{8}\hat{q}\rho^2(\log(Q^2/\mu^2) \\ +((1-z)^2 \log Q^2/((1-z)^2\mu^2) \\ -z^2/9\log(Q^2/(z^2\mu^2) \tag{51}$$

meaning that the effective coefficient $\hat{q}$ is given by

$$\hat{q}_{eff} = \hat{q}\frac{1}{2}(\log(Q^2/\mu^2) \\ + ((1-z)^2 \log Q^2/((1-z)^2\mu^2) - z^2/9\log(Q^2/(z^2\mu^2) \tag{52}$$

while the perturbation is now as before given by

$$V_{pert}(\vec{\rho}) = \frac{\hat{q}}{8}(1 + (1-z)^2 - z^2/9)\rho^2 \log 1/(Q^2\rho^2). \tag{53}$$

To obtain the numerical results we just need to substitute $\omega- > Ez(1-z)$ in the results of the previous section, including the choice of the effective momentum scales [42,43]. In addition the coefficient in front of ic term is given by

$$\frac{\hat{q}}{8}(1 + (1-z)^2 - z^2/9). \tag{54}$$

6.1 Angular distributions

We have depicted the corresponding angular distributions in Fig. 4 for several values of z and for energies $E = 50$ and 35 GeV. We chose z = 0.1, 0.4 for E = 50 GeV and z = 0.14, 0.5 for E = 35 GeV (Figs. 5, 6).

We see that the inclusions of the longitudinal phase space constraints, i.e. the finite gluon energy significantly improves the behaviour of the angular distributions, but qualitatively the situation is the same as in the soft gluon approximation: both phase space constraints and gluons lead to the filling of the dead cone, and the Coulomb gluons give a significant correction to the BDMPS distributions at small k_t. Note also that the distributions for $\theta = 0$ and $\theta = 0.05$ practically do not differ, meaning the radiation of the charmed quark is not different from the massless quark. We chose the values of θ to have a mass of heavy quark 5 GeV, corresponding to realistic case of the b quark.

6.2 Energy loss

The inclusion of longitudinal phase space constraints also has significantly influences the energy loss (Fig. 7). We consider here the spectrum up to $z = 1/2$ [5] assuming heavy quark to be the leading particle. We limit the integration over k_t up to $\omega = Ez(1-z)$. In this way we keep the whole positive

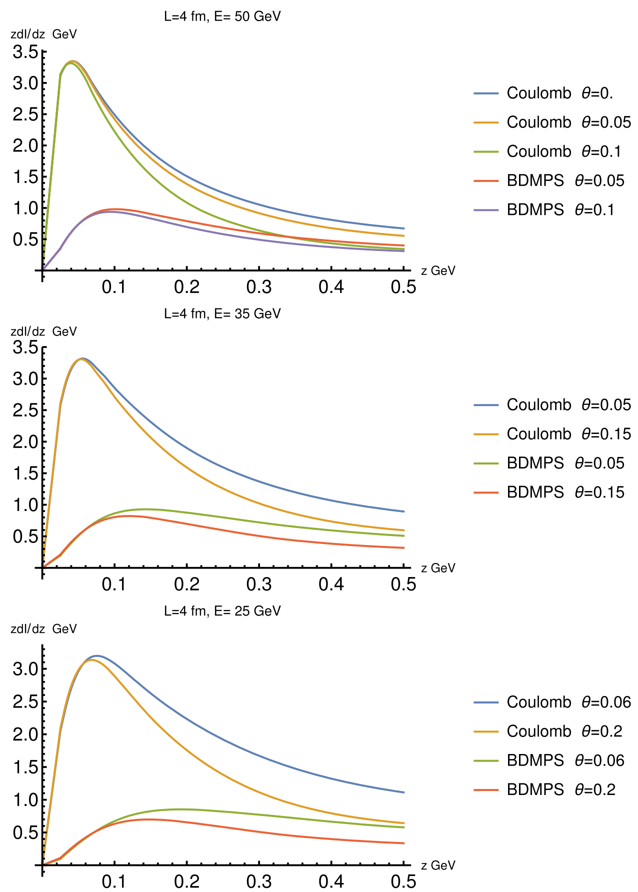

Fig. 7 The energy loss zdI/dz with ic gluons and BDMPS with phase constraints for different Energies and heavy quark masses $m = 1.5$ and 5 GeV

value region and cut off the small in magnitude tail of the distribution where we expect that approximations made in the matrix element calculations may become unreliable. We see that the longitudinal phase constraints significantly decrease the influence of the increase in the quark mass. Note that the results for energy loss obtained for $\theta = 0$ and $\theta = 0.05-0.06$ are virtually identical and thus there is no difference in the energy loss spectrum between light and charm quarks at least for energies at least above 25 GeV.

6.3 Quenching weights

We can now calculate the quenching weights and see the significant decrease of the dependence of the quenching weight on the quark mass due to imposition of the longitudinal constraints (Table 2).

Here

$$S(E) = \int_0^{1/2} zdI/dz(e^{-nz} - 1)/z \tag{55}$$

and we assume $\alpha_s = 0.3$. Here the quenching weight $Q(E) = \exp(S(E))$.

7 Conclusions

We have extended the Moliere theory to angular distributions of the radiation of the heavy quark propagating in the QGP. We have found for the first time explicit expressions for Coulomb corrections to angular gluon distributions in the harmonic oscillator approximation approach (Improved Opacity Expansion). We have shown that the Coulomb logarithms give a large contribution to harmonic oscillator approximation, with the numerical results indicating that the final answer is between N = 1 GLV and harmonic oscillator approximation. For the case of intermediate widths considered in the numerical example in this paper (L = 4 fm) the results for Moliere theory are actually rather close to GLV for both transverse distributions and energy loss.

Note that for finite quark masses the Coulomb correction is maximal at small k_t, thus enhancing the collinear contribution to the spectrum ("filling the dead cone"). This enhancement ("filling the dead cone") is already present in the Harmonic Oscillator Approximation as it was first noted in [15], and is further enhanced by Coulombic correction in Moliere theory.

We see that the inclusion of transverse phase constraints significantly decreases the dependence of energy loss on the quark mass. This was first noted in [15] for the Harmonic Oscillator Approximation, and persists in the Moliere theory.

Our results indicated that the picture will further improve if we include longitudinal DGLAP type phase constraints,

Table 2 The estimate for quenching coefficients S(E) for light and heavy quarks, for $L = 4$ fm widths. The jet quenching factor $Q(E) = \exp(S(E))$. Here $\alpha_s = 0.3$, $C_F = 4/3$

BDMPS	$E = 25$ GeV − S(E)	$E = 35$ GeV − S(E)	$E = 50$ GeV − S(E)
Light quark m = 0	0.38	0.4	0.4
Heavy quark $m_b = 5$ GeV	0.28	0.33	0.36
Coulomb	− S(E)	− S(E)	− S(E)
Light quark m = 0	1.26	1.2	1.01
Heavy quark $m_b = 5$ GeV	1.06	1.04	1.0

that take into account a finite gluon energy. In this case we see that the dead cone effect at energies above 35 GeV is rather small. Further study of the phase space constraints is needed to have quantitative agreement with the experimental data . Nevertheless we see that combining phase constraints and Moliere theory we get the results at least qualitatively agreeing with the experimental data, and making a basis of the construction of the realistic models of the heavy quark energy loss based on Moliere theory [53].

After this paper was submitted, a calculation of transverse distributions in the framework of the improved opacity expansion was presented in [54]. Our results for $\theta = 0$ look in agreement with that of [54] for transverse distributions. Some numerical differences may be related for the use of the single matching scale in the current paper, while the Ref. [54] uses different matching scales for the exponent expansion and the rest of the spectrum. The author thanks K. Tywoniuk for the discussion on this subject.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This work is a theoretical one and thus does not need any data or deposited data.]

Appendix

Here we present explicit expressions for functions F_2, F_3, F_4 used in the calculations of the angular distributions.

$$
\begin{aligned}
F_1(p,c,Q) &= \int_0^\infty dx x \log(Qx) J_0(cx) \exp(-px^2) \\
&= (1/p)\exp(-c^2/(4p))(\log(cQ/(2p)) \\
&\quad -0.5Ei(c^2/(4p)) \qquad \text{(A.1)} \\
F_2(p,c,Q) &= \int_0^\infty dx x^3 \log(Qx) J_0(cx) \exp(-px^2) \\
&= -\frac{1}{p^3}(-p\exp(-c^2/(4p)) + p/2 \\
&\quad +(-c^2/8 + p/2)Ei(c^2/(4p)) + (c^2/4 - p) \\
&\quad \times \log(0.5Qc/p))\exp(-c^2/(4p)) \qquad \text{(A.2)} \\
F_3(p,c,Q) &= \int_0^\infty dx x^5 \log(Qx) J_0(cx) \exp(-px^2) \\
&= -\frac{1}{p^5}(\exp(-c^2/(4p))(3p - c^2/2)p \\
&\quad +(c^2/8 - 3p/2)p + (\exp(-c^2/(4p)) \\
&\quad (-c^4/32 + c^2p/2 - p^2)Ei9c^2/(4p) \\
&\quad +(c^4/16 - c^2p + 2p^2)\log(Qc/(2p)) \qquad \text{(A.3)} \\
F_4(p,c,Q) &= \int_0^\infty dx x^4 \log(Qx) J_1(cx) \exp(-px^2) \\
&= \frac{1}{cp^4}((p - c^2/4)p + \exp(-c^2/(4p)) \\
&\quad \times(p(3c^2/4 - p) + c^2((c^2/16 - p/2) \\
&\quad \times Ei(c^2/(4p)) + (-c^2/8 + p) \\
&\quad \times \log(Qc/(2p))). \qquad \text{(A.4)}
\end{aligned}
$$

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