On measurement of photon polarization in radiative penguin B decays to baryons

P. Pakhlov^{1,2}, T. Uglov^{1,a}

¹ P. N. Lebedev Physical Institute of the RAS, Moscow, Russia

² Higher School of Economics, Moscow, Russia

Received: 12 May 2021 / Accepted: 6 August 2021 / Published online: 21 August 2021

© The Author(s) 2021

Abstract A measurement of the photon polarization in radiative penguin B decays provides a test of the Standard Model and a probe for New Physics, that can lead to a deviation from the Standard Model prediction of left-handed photons in $b \rightarrow s\gamma$. We propose a new method to measure the photon polarization using the baryonic decay $B^- \rightarrow \Lambda \bar{p} \gamma$. The P -violating Λ -hyperon decay allows a measurement of the Λ helicity to be performed, which can be uniquely related to the photon polarization in a model-independent way. The $B^- \rightarrow \Lambda \bar{p} \gamma$ decay was recently measured to have a large branching fraction providing a possibility to get meaningful results with the data already available at LHC and B-factory experiments. An increase of the B -meson sample at high luminosity LHC experiments and Belle II should provide a really stringent test by using this method already in the near future.

Due to the chiral structure of the W -boson interaction with fermions in the Standard Model (SM), the photon polarization in penguin $b \rightarrow s\gamma$ transition reveals a maximal parity violation: the emitted photon is polarized left-handed in the decays of B mesons (right-handed in $\bar{B}$ decays). While most theoreticians agree that admixture of “wrong” polarization is tiny in the SM, of the order of $m_s/m_b \sim 0.02$ [1–4], some of them suppose the hadronic effects can enhance the right-handed photons in exclusive channels up to $\Lambda_{\text{QCD}}/m_b \sim 0.05$ and inclusively up to $g_s/(4\pi)$ [5]. In any case, the SM prediction of photon polarization in $b \rightarrow s\gamma$ remains quite accurate to serve as the SM test.

In many SM extensions the right-handed contribution can be enhanced. Supersymmetric scenarios suggest several such mechanisms, e.g. an intermediate charged Higgs gives rise to the $b_R \rightarrow s_L$ transition in the models with R -parity violation [6], while in unconstrained MSSM the chirality flip along the gluino line in the loop involving left-right squark mixing

can inverse the SM prediction, resulting in right-handed photons [7]. In the left-right symmetric model chirality flip along the t -quark line in the loop involves $W_L - W_R$ mixing [8–10]. It was found that in certain allowed regions of the parameter space of all these models photons emitted in $b \rightarrow s\gamma$ can be largely right-handed polarized, without affecting the SM prediction for the inclusive radiative decay rate.

A clear SM prediction and a possibility to check models that have not yet been refuted by numerous tests motivate a precise measurement of the photon polarization. The only problem in performing such a test is a lack of practical methods to access it experimentally. Very few methods were proposed so far: one is to use $B^+ \rightarrow K^+ \pi^+ \pi^- \gamma$ decays, where an angular analysis of the photon direction with respect to the $K^+ \pi^+ \pi^-$ plane in their center-of-mass frame allows to distinguish between differently polarized photons through the measurement of the up-down asymmetry, which is related to the photon polarization ϵ_γ [11–13]. The experimental probe with this method performed by LHCb [14] has demonstrated non-zero photon polarization with a high significance. However, no quantitative result on the degree of polarization was obtained. Another possibility to probe for a right-handed photon component from $b \rightarrow s\gamma$ is to search for indirect CP violation in $B_{(s)}^0 \rightarrow X^0 \gamma$, which can only arise if the “wrong” component of the photon polarization allows $B_{(s)}^0$ and $\bar{B}_{(s)}^0$ interference [15, 16]. A recent LHCb analysis [17, 18] showed no significant CP violation, confirming that only one polarization dominates, but again without a quantitative limit. The third approach is based on angular analysis of the $B \rightarrow K^* e^+ e^-$ decay, where $e^+ e^-$ pair originates from the virtual photon [19]. This study limited to the region of very low q^2 ($e^+ e^-$ pair invariant mass) was performed by LHCb collaboration [20, 21] setting so far the best bounds on the virtual photon polarization.

In this paper we propose a new promising method to measure directly the photon polarization using exclusive radiative B^- decays to $\Lambda \bar{p} \gamma$. This final state is unique due to possibil-

^a e-mail: uglov.timofey@gmail.com (corresponding author)

Fig. 1 Kinematics of the $B^- \rightarrow \Lambda \bar{p} \gamma$, $\Lambda \rightarrow p \pi^-$ decay

ity to measure the polarization of Λ through the P -violating asymmetry in its decay, e.g. into $p \pi^-$. While in general the Λ polarization does not always correspond to photon polarization, there is a region in the decay phase space, where it is possible to relate two polarizations in a model-independent way. This decay was first observed by Belle [22] with a relatively large branching fraction and a high signal purity, thus suitable for such studies.

Let us first define the observables available in the experiment for the photon polarization studies in the proposed decay chain $B^- \rightarrow \Lambda \bar{p} \gamma$, followed by $\Lambda \rightarrow p \pi^-$. For the sake of simplicity we denote the $(\Lambda \bar{p})$ combination by X throughout the paper. This does not imply that we consider X as a real state or having definite quantum numbers. The final state is characterized by the X mass and three angles (see Fig. 1): the X decay angle (the angle between photon- and Λ -momentum directions in the X rest frame), θ_1 ; the Λ decays angle (the angle between antiproton- and proton-momentum directions in the Λ frame), θ_2 ; and the angle between B and Λ decay planes (the angle between vector products of $\mathbf{p}_\gamma \times \mathbf{p}_\Lambda$ and $\mathbf{p}_p \times \mathbf{p}_\pi$ in the X rest frame), ϕ . We demonstrate that the photon polarization, defined as $\epsilon_\gamma \equiv \frac{\mathcal{B}_L - \mathcal{B}_R}{\mathcal{B}_L + \mathcal{B}_R}$, where $\mathcal{B}_{L(R)}$ are the B -decay branching fractions to the photon of helicity $-1(+1)$, can be related to the angular variables and thus extracted from the experimental fit.

The system X likely represents the S -wave ($\Lambda \bar{p}$) combination (thus having $J^P = 1^-$ quantum numbers), as the Belle studies [22, 23] demonstrated a strong $(\Lambda \bar{p})$ near-threshold enhancement while the larger waves should be suppressed near the threshold. This is supported by the measured X angular distribution consistent with being flat and the fact that the observed X -mass shape corresponds to the off-shell $K^{*-} \rightarrow \Lambda \bar{p}$ decay. Under assumption of only $(\Lambda \bar{p})$ in the S -wave contribution, the angular part of the decay matrix element has a very simple form

$$S \sim 1 + \epsilon_\gamma \alpha_\Lambda \cos \theta_1 \cdot \cos \theta_2, \quad (1)$$

where α_Λ – is the P -violating parameter in the $\Lambda \rightarrow p \pi^-$ decay.

If the dominance of the S -wave could be proved theoretically, this simple formula would serve to fit the data, since the parameter α_Λ is known here, and the only free fitting parameter is the unknown photon polarization. However, it is hardly possible today to rigorously prove using theoretical arguments or calculations, that the $\Lambda \bar{p}$ production dynamics is really dominated by the S -wave contribution only. Thus, there are two options to accomplish this task: either to find kinematic regions free of model dependencies, or to experimentally estimate the dynamics and constraints on the model uncertainty of the measured value directly from the data.

It turns out that the first option can be implemented. There is a kinematic region, where the relationship between the photon and Λ polarizations can be described unambiguously without relying on the model assumptions. This is the case of $\cos \theta_X = \pm 1$, that is, when Λ moves exactly forward or backward in the Λp rest frame, thus the momenta of all three daughter particles γ , Λ , $\bar{p}$ are collinear. Indeed, whatever the angular momenta in the $B \rightarrow \gamma X$ and $X \rightarrow \Lambda \bar{p}$ decays, their projections on the common decay axis are equal to 0. Hence the sum of the projections of the spins Λ and $\bar{p}$ on this axis is exactly equal to the projection of the photon spin. While the proton spin direction is unmeasurable, the photon transversity ensures that its spin projection to this axis is ± 1 , which should be equal to the sum of two one halves, thus the proton spin coincides with the spin of Λ .

Although the phase space of this kinematic region is vanishing, experimentally it is possible to extrapolate to the region $|\cos(\theta_X)| = \pm 1$ by studying $\lambda_\Lambda(\cos \theta_X)$ in the vicinity of $\cos \theta_X = \pm 1$. Technically, this will require a fit of the angular distribution of $\cos(\theta_\Lambda)$ with the function $1 + A \cos \theta_\Lambda + B \sin \theta_\Lambda$ in bins of $\cos \theta_X$. If one draws then the dependence of the measured $A(\cos \theta_X)$ and $B(\cos \theta_X)$ and extrapolate them into $|\cos(\theta_X)| = 1$, then the value of $A_\Lambda = \text{sign}(\cos \theta_X) A|_{\cos \theta_X = \pm 1}$ can be related to the proton polarization via $\epsilon_\gamma = \frac{A_\Lambda}{\alpha_\Lambda}$, while $B|_{\cos \theta_X = \pm 1}$ should be 0.

A more mathematically rigorous approach can be applied, but it requires a simple model assumption, namely, a constraint on the maximum total orbital angular momentum of the system X : to fit the angular distributions with a general function, where the QCD dynamics is represented by free parameters. This approach has the advantage that it uses the entire kinematic decay region, rather than its small part, and the output will not only give a measurement of the photon polarization, but also the parameters describing the QCD decay dynamics, which are of additional interest. The angular part of the matrix elements for the studied process is conve-

nient to be written in the helicity formalism as follows:

$$\mathcal{S} = \sum_{\substack{\lambda_\gamma = \pm 1 \\ \lambda_p, \lambda_{\bar{p}} = \pm 1/2}} \left| \sum_{J_X=1 \dots J_X^{\max}} \sum_{\substack{\lambda_X = -J_X \dots J_X \\ \lambda_\Lambda = \pm 1/2}} \left(D_{0, \lambda_X - \lambda_\gamma}^0 \cdot a_{\lambda_X, \lambda_\gamma} \right) \cdot \left(D_{\lambda_X, \lambda_\Lambda - \lambda_{\bar{p}}}^{J_X} \cdot b_{\lambda_\Lambda, \lambda_{\bar{p}}}^{J_X} \right) \cdot \left(D_{\lambda_\Lambda, \lambda_p - \lambda_\pi}^{1/2} \cdot c_{\lambda_p, \lambda_\pi} \right) \right|^2, \quad (2)$$

where the first, second and third Wigner D -functions describe B decay to photon and X , the X transition to Λ and antiproton, and $\Lambda \rightarrow p\pi^-$ decay, respectively. Complex constants a , b and c define contributions of various helicity states to the total decay amplitude. The value of the X total orbital momentum, J_X , should be limited by some number, $J_X^{\max}$ to avoid infinite numbers of parameters, while the minimal value is equal to 1, as 0 is forbidden by momentum conservation. Since the values of J_X and the polarization of the Λ baryon in contrast to the polarization of the photon, proton and antiproton, can not be measured, corresponding amplitudes must be summed coherently before squared.

Taking into account that pions and B mesons are scalars and that $D_{0, \lambda_X - \lambda_\gamma}^0$ not vanishes only if $\lambda_X = \lambda_\gamma$, one can simplify the Eq. (2) to:

$$\mathcal{S} \sim \sum_{\substack{\lambda_\gamma = \pm 1 \\ \lambda_p, \lambda_{\bar{p}} = \pm 1/2}} \left| a_{\lambda_\gamma} \sum_{\substack{J_X=1 \dots J_X^{\max} \\ \lambda_\Lambda = \pm 1/2}} \left(D_{\lambda_\gamma, \lambda_\Lambda - \lambda_{\bar{p}}}^{J_X} \cdot b_{\lambda_\Lambda, \lambda_{\bar{p}}}^{J_X} \right) \cdot \left(D_{\lambda_\Lambda, \lambda_p}^{1/2} \cdot c_{\lambda_p} \right) \right|^2. \quad (3)$$

The matrix element (3) depends on $4 \times J_X$ complex amplitudes $b_{\pm 1/2, \pm 1/2}^{J_X}$ and the other two amplitudes $c_{\pm 1/2}$. The magnitudes of the latter two are, in turn, known from the measured P asymmetry in Λ decays [24]:

$$\alpha_\Lambda = \frac{|c_{+1/2}|^2 - |c_{-1/2}|^2}{|c_{+1/2}|^2 + |c_{-1/2}|^2} = 0.732 \pm 0.014. \quad (4)$$

The phases of the c are unobservable, and hence cancel out.

The expressions (3) and (4) explicitly define the ready-made function that can be directly used in the fit to the angular variables in the experimental data to extract ϵ_γ , which in this parameterization is defined as $\epsilon_\gamma = \frac{a_{+1}^2 - a_{-1}^2}{a_{+1}^2 + a_{-1}^2}$. The number of free parameters in addition to the sought ϵ_γ depends on the assumption about the maximum orbital momentum of the X system. Up to now only $K^{(*)}$ with spin 1 or 2 were observed in the B radiative decays [24], thus it is justified to assume $J_X^{\max} = 2$. Moreover, high J_X suppression by relativistic phase space (much reduced in case of the baryon-antibaryon final state and low X mass tendency) supports this assumption. Limiting $J_X^{\max} \leq 2$ there are seven complex

variables in addition to the overall normalization and ϵ_γ that describe $X \rightarrow \Lambda \bar{p}$ dynamics in the fit function.

We check the capability of the near future experiments (Belle II [25] and LHC experiments under high luminosity) to extract ϵ_γ . It is easy to estimate that Belle II will be able to reconstruct approximately 8000 signal events with its full data set, corresponding to the 50 ab^{-1} integrated luminosity. We generate 10,000 samples with random parameters $b_{\pm 1/2, \pm 1/2}^{J_X}$ using toy Monte Carlo simulation. The parameters $b_{\pm 1/2, \pm 1/2}^{J_X}$ for $J_X = 1, 2$ are uniformly distributed within the circle of radius 10, thus representing all possible models of the studied decay. Each sample consists of 8000 “signal” events generated according to (3). For all samples we perform an unbinned likelihood fit with the fitting function defined as:

$$PDF(\theta_1, \theta_2, \varphi) \propto \epsilon_\gamma \mathcal{S}_{-1} + (1 - \epsilon_\gamma) \mathcal{S}_{+1}, \quad (5)$$

where $\mathcal{S}_{\lambda_\gamma = \mp 1}$ are normalized matrix elements for two photon polarizations (3) that include all $(b_{\pm 1/2, \pm 1/2}^{J_X})$ as free parameters. It was found that almost all fits converged during automatic fitting, and the procedure is able to extract the true (generated) values of helicity parameters, while the desired result of ϵ_γ extraction is unbiased by the fit. The scatter plot of generated and reconstructed ϵ_γ values shown in Fig. 2a demonstrates linear dependence and scattering compatible with the expected statistical fluctuations. In Fig. 2b the spread of the returned values at $\epsilon_{\gamma \text{ seed}} = 0.95$ is shown. The mean value of the resulting ϵ_γ distribution is 0.948 ± 0.013 which is compatible with the seed value of 0.95 while the width is 0.02 and is described by the fit error.

The presently available data is likely insufficient for both model-independent methods due to a large number of free parameters in the fit. Belle has doubled the data sample since the last publication of the $B^- \rightarrow \Lambda \bar{p} \gamma$ measurements, and can thus reconstruct up to 200 signal events. Though LHCb has a higher production rate, due to its much smaller Λ reconstruction efficiency, the number of reconstructed events at full current integrated LHCb luminosity can be estimated comparable to the Belle one [26]. While use of the model-independent methods is not feasible now both at Belle/BaBar and LHCb, the meaningful measurements can still be done. The main concern of the model-dependent study is to keep the model uncertainties under control.

We thus propose to use the assumption about the S -wave and perform a fit to the data using the formula 1. The main task will be to estimate the model uncertainties. This can be done at the level of existing statistical errors if the same data is used to restrict the parameters of the models. It is necessary to scan all models, discard those of them that contradict the angular distribution of the data (for example, by 90% CL), and of the remaining ones, check the spread of the

Fig. 2 **a** Distribution of ϵ_γ , returned by fits to the Monte Carlo samples, depending on the seed value. **b** ϵ_γ spectrum with the ϵ_γ seed value fixed to 0.95. Vertical lines show $\epsilon_{\gamma\text{seed}} = 0.95$

extracted parameter ϵ_γ from the generated one. We estimated that with 200 signal events, with a signal-to-background ratio of 1:1, one can expect a statistical accuracy of ϵ_γ around 0.4 (while the value itself can vary from -1 to $+1$) using toy MC. Although the model uncertainties depend on how well the data will match the S -wave distribution, we encourage to perform such an analysis for the first measurement of the ϵ_γ .

Due to low statistic of the currently available data samples, it could also be useful to check, whether ϵ_γ is zero or not. Belle attempted to search for P -parity violation in $B^- \rightarrow \Lambda \bar{p} \gamma$ decay by study of Λ polarization [23]. However, the answer obtained with this method is not informative, as Λ polarization could vanish after integration over all $\cos(\theta_X)$ region even in presence of P violation. The correct approach is to search for the correlation between $\cos(\theta_X)$ and Λ polarization, which is obviously absent in case of $\epsilon_\gamma = 0$. A presence of significant correlation would prove non-zero photon polarization without any assumption on decay dynamics. As a formal correlation function covariation $\text{cov}(\cos(\theta_X), \cos(\theta_\Lambda)) = \sum \cos(\theta_X) \cdot \cos(\theta_\Lambda)$ can be used. We estimate the ability of Belle to exclude zero photon polarization using toy Monte Carlo simulation. For 200 events, corresponding to the current Belle statistics and generated in S -wave assumption, the mean covariation value is 0.081 and its distribution has a width of 0.028, which means that $\sim 3\sigma$ indication for non-zero photon polarization can be obtained by Belle.

In summary, we conclude that the measurement of the photon polarization in rare radiative B decays, which is very sensitive to possible NP contributions, is feasible using $B^- \rightarrow \Lambda \bar{p} \gamma$ decay. The access to the photon polarization is provided by a measurement of the Λ polarization. We propose a method which allows one to accurately measure the polarization of a photon in a model-independent way using the large statistics of the near-future B experiments. We also checked that it is possible to perform the first measurement of the polarization of a photon using the existing relatively small statistics, while the model dependence of the measurement will remain under control.

The work of P. Pakhlov was conducted within the framework of the Basic Research Program at the National Research University Higher School of Economics (HSE). T. Uglov acknowledges the support of the Russian Ministry of Science and Higher Education under contract 14.W03.31.0026.

Data Availability Statement This manuscript has no associated data or the data will not be deposited [Authors' comment: The manuscript has no associated data due to its theoretical nature.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>. Funded by SCOAP³.

References

1. B. Grinstein, D. Pirjol, Phys. Rev. D **62**, 093002 (2000)
2. M. Beneke, T. Feldmann, D. Seidel, Nucl. Phys. B **612**, 25 (2001)
3. S.W. Bosch, G. Buchalla, Nucl. Phys. B **621**, 459 (2002)
4. A. Ali, A.Y. Parkhomenko, Eur. Phys. J. C **23**, 89 (2002)
5. B. Grinstein, Y. Grossman, Z. Ligeti, D. Pirjol, Phys. Rev. D **71**, 011504 (2005)
6. O.C.W. Kong, R.D. Vaidya, Phys. Rev. D **71**, 055003 (2005)
7. L. Everett, G.L. Kane, S. Rigolin, L.T. Wang, T.T. Wang, JHEP **0201**, 022 (2002) or a review of B physics in SUSY models, see S. Bertolini, F. Borzumati, A. Masiero, B Decays, second edition, ed. S. Stone, World Scientific, 1994, p. 620
8. K. Fujikawa, A. Yamada, Phys. Rev. D **49**, 5890 (1994)
9. K.S. Babu, K. Fujikawa, A. Yamada, Phys. Lett. B **333**, 196 (1994)
10. P. Cho, M. Misiak, Phys. Rev. D **49**, 5894 (1994). For a review of B physics in LR models, see M. Gronau, B Decays, second edition, ed. S. Stone, World Scientific, 1994, p. 644
11. M. Gronau, Y. Grossman, D. Pirjol, A. Ryd, Phys. Rev. Lett. **88**, 051802 (2002)
12. E. Kou, A. Le Yaouanc, A. Tayduganov, Phys. Rev. D **83**, 094007 (2011)

13. M. Gronau, D. Pirjol, Phys. Rev. D **96**, 013002 (2017)
14. R. Aaij et al., LHCb Collaboration, Phys. Rev. Lett. **112**, 161801 (2014)
15. D. Atwood, M. Gronau, A. Soni, Phys. Rev. Lett. **79**, 185 (1997)
16. F. Muheim, Y. Xie, R. Zwicky, Phys. Lett. B **664**, 174 (2008)
17. R. Aaij et al., [LHCb Collaboration], Phys. Rev. Lett. **118**, 021801 (2017)
18. Addendum: Phys. Rev. Lett. **118**, 109901 (2017)
19. Y. Grossman, D. Pirjol, JHEP **06**, 029 (2000)
20. R. Aaij et al., LHCb Collaboration, JHEP **04**, 064 (2015)
21. R. Aaij et al., LHCb Collaboration, JHEP **12**, 081 (2020)
22. Y.-J. Lee et al., Belle Collaboration, Phys. Rev. Lett. **95**, 061802 (2005)
23. M.-Z. Wang et al., Belle Collaboration, Phys. Rev. D **76**, 052004 (2007)
24. P.A. Zyla et al., (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)
25. T. Abe et al., [Belle II Collaboration], [arXiv:1011.0352](https://arxiv.org/abs/1011.0352) [physics.ins-det]
26. R. Aaij et al., LHCb Collaboration, JHEP **1704**, 162 (2017)