Constraining the anisotropy of the Universe with the X-ray and UV fluxes of quasars

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Abstract We test the anisotropy in the Finslerian cosmological model with the X-ray and ultraviolet (UV) fluxes of quasars. The 2015 and 2020 compilations of quasars are used in the cosmological constraints. We find that the dipole direction given by the 2015 quasar compilation is not far away from the one provided by the Pantheon sample and the angular differences are around 30° . The Pantheon sample is combined with quasars as the “standardized candles” to test the cosmic anisotropy. The results from two combined datasets are consistent. They show that the dipole anisotropy is weak in the Finslerian cosmological model. We investigate the Hubble constant H_0 in the Finslerian cosmological model. Though the central value of H_0 from the combination of six gravitationally lensed quasars, Pantheon sample, and 2020 quasar compilation decreases a little bit, it is consistent with the result from six gravitationally lensed quasars within statistical uncertainties.

1 Introduction

The Universe is homogeneous and isotropic on large scale, which is called the cosmological principle. Based on it, the standard Lambda cold dark matter (Λ CDM) cosmological model has been established. In the past few decades, many experiments test its validity and verify that it is consistent with most cosmological observations. The observations of Cosmic Microwave Background (CMB) temperature anisotropies and polarizations from Wilkinson Microwave Anisotropy Probe (WMAP) [1, 2] and Planck satellites [3–6] provide high-precision constraints on the six based cosmological parameters. However, there still exist several anomalies that have been reported, such as the alignment of low- ℓ multipoles in the CMB temperature anisotropies [7–10], the parity asymmetry [6, 11–15] and the hemispherical power

asymmetry [6, 16–18] in CMB, the spatial variation of the fine structure constant [19, 20], the anisotropic accelerating expansion of the Universe [21–26], the alignment of quasar polarization vectors on large scale [27], the MOND acceleration scale [28–30]. These phenomena may imply that our Universe has a preferred direction.

As the most luminous and persistent energy source, quasars have extraordinary potential in the exploration of our Universe. In recent years, quasars are tentatively used to investigate the cosmological parameters. An incomplete list includes the relation between the UV emission lines and the continuum luminosity [31], the relation between the radius of quasars and its luminosity [32–34], the relation between luminosity and mass of super Eddington accreting quasars [35], the correlation between X-ray variability and luminosity of quasars [36], the non-linear relation between UV and X-ray luminosity [37–43]. The non-linear relation between UV and X-ray luminosity was firstly discovered by the X-ray surveys [44–46]. For decades, the UV and X-ray luminosity relationship has been confirmed by observations of a few hundred quasars in the redshift range from 0 to 6.5. Since 2015, Risaliti and Lusso (et al.) [37–39] have been attempting to estimate the cosmological parameters by using quasars as standardizable candles. Hu etc. tested the cosmic anisotropy with Pantheon sample and quasars by employing the hemisphere comparison method and the dipole fitting method in the Λ CDM model, and they didn't find any distinct evidence of cosmic anisotropy [43].

In this paper, we will use two quasars datasets i.e., the 2015 [37] and 2020 [39] quasar compilation to explore the anisotropy in the Finslerian cosmological model. The 2015 quasar compilation contains 808 quasars which are thought to be standardizable candles through the relation between UV and X-ray luminosity. The 808 quasars are in the redshift range $0.061 \leq z \leq 6.28$. We will forecast the future constraints on the Finslerian cosmological model based on the 2015 quasar compilation. The 2020 quasar compilation

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contains 2421 quasars over the redshift range $0.009 \leq z \leq 7.5413$. In the 2020 quasar compilation, the $2500 \text{ \AA}$ UV fluxes for the quasars in the redshift range $z < 0.7$ are less reliable due to the host-galaxy contamination [39]. The Pantheon sample [47] is combined with quasars as the “standardized candles” to test the Finslerian cosmological model. We also attempt to investigate the Hubble constant H_0 in the Finslerian cosmological model by using six gravitationally lensed quasars with measured time delays [48].

The rest of this paper is organized as follows. In Sect. 2, we briefly introduce the UV and X-ray luminosity relationship, the time-delay strong lensing measurement, and the Finslerian cosmological model. We show our results in Sect. 3. Finally, discussions and conclusions are given in Sect. 4.

2 Methodology

2.1 The UV and X-ray luminosity relationship

The non-linear relation of the UV and X-ray luminosity is [37–42]

$$\log L_X = \beta + \gamma \log L_{UV}, \quad (1)$$

where β and γ are two free parameters. The luminosities and fluxes of quasars are connected by the luminosity distance. Now we rewrite the Eq. (1) and obtain

$$\log(F_X) = \beta + (\gamma - 1) \log(4\pi) + \gamma \log(F_{UV}) + 2(\gamma - 1) \log(D_L), \quad (2)$$

where $\log$ denotes $\log_{10}$. F_X and F_{UV} represent the X-ray and UV fluxes of quasars, respectively. The luminosity distance D_L takes the form

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (3)$$

where z denotes redshift. c is the speed of light and H_0 is the Hubble constant. The expression of $E(z)$ depends on cosmological models.

In our work, the datasets are from the 2015 [37] and 2020 [39] compilations of quasars. The 2015 quasar compilation contains 808 quasars which are in the redshift range $0.061 \leq z \leq 6.28$. The 2020 quasar compilation contains 2421 quasars over the redshift range $0.009 \leq z \leq 7.5413$. The redshift distributions of the 2015 and 2020 quasar compilations are shown in Fig. 1.

2.2 The time-delay strong lensing measurement

Strong gravitational lensing is a powerful probe of cosmological models. The time-delay strong lensing (TDSL) measurement is a fully independent method to measure the Hubble constant. Since the approach first proposed by Refsdal [49],

Fig. 1 The redshift distributions of the 2015 and 2020 quasar compilations. The orange histogram denotes the 2015 quasar compilation and the blue histogram denotes the 2020 quasar compilation

lensed quasars have generally been used to constrain H_0 by measuring the difference in arrival time of photons. TDSL provides a measurement of H_0 , which is completely independent of the CMB and the local distance ladder.

The travel time of light rays from a source to the observer depends on their path length and the gravitational potential they traverse. For a system of lenses with an image at an angular position θ and corresponding source position β , the excess time delay is

$$t(\theta, \beta) = \frac{D_{\Delta t}}{c} \left[\frac{(\theta - \beta)^2}{2} - \psi(\theta) \right], \quad (4)$$

where c is the speed of light and $\psi(\theta)$ is the lens potential. The time-delay distance $D_{\Delta t}$ is defined as [48, 49]

$$D_{\Delta t} = (1 + z_d) \frac{D_d D_s}{D_{ds}}, \quad (5)$$

where z_d denotes the lens redshift. D_d and D_s are the angular diameter distance from the observer to the lens and the angular diameter distance from the observer to the source, respectively. D_{ds} is the angular diameter distance from the lens to the source. The angular diameter distance is defined as

$$D_A = \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')}, \quad (6)$$

where z is the redshift and H_0 is the Hubble constant. The expression of $E(z)$ depends on cosmological models. The difference of excess time delays between two images A and B is

$$\Delta t_{AB} = \frac{D_{\Delta t}}{c} \left[\frac{(\theta_A - \beta)^2}{2} - \psi(\theta_A) - \frac{(\theta_B - \beta)^2}{2} + \psi(\theta_B) \right], \quad (7)$$

where θ_A and θ_B are the positions of image A and B, respectively.

Table 1 The 68% confidence level constraints on the parameters β , γ , and δ in the Finslerian cosmological model with the 2015 quasar compilation and other datasets

Data	Quasars	Quasars + Pantheon	Quasars + Pantheon + TSDL
β	$9.736^{+1.051}_{-0.968}$	$8.622^{+0.798}_{-1.020}$	$8.525^{+0.900}_{-0.949}$
γ	$0.543^{+0.040}_{-0.036}$	$0.579^{+0.033}_{-0.026}$	$0.582^{+0.032}_{-0.029}$
δ	$0.318^{+0.014}_{-0.016}$	$0.321^{+0.017}_{-0.014}$	$0.322^{+0.016}_{-0.015}$

We use six gravitationally lensed quasars with measured time delays from H0LiCOW collaboration [48] to constrain the Hubble constant and other cosmological parameters. Our work is based on the H_0 inference code¹ provided by Wong etc. [48].

2.3 The anisotropic cosmological model

In this paper, we choose the Finslerian cosmological model as the anisotropic cosmological model. Different from the standard cosmological model, the Finslerian cosmological model has an intrinsically preferred direction that breaks the isotropy of the Universe. Many works about investigating the anisotropy of the Universe are based on the Finsler spacetime. For instance, investigating the cosmic anisotropy with supernovae of type Ia (SNe Ia) samples by the hemisphere comparison HC method [25, 50] and the dipole fitting [25, 26, 51–53], explaining the parity asymmetry and power deficit in the Finsler spacetime [54], the unified description for dipoles of the fine-structure constant and SNe Ia Hubble diagram [55].

In the Finsler spacetime, the scale factor a takes the form [55],

$$a = (1 + A_D \cos \theta) / (1 + z). \quad (8)$$

A_D is a parameter in the Finsler spacetime, which can be regarded as the dipole amplitude. When $A_D = 0$, the Finslerian cosmological model reduces to the Λ CDM model. θ is the angle between the position of quasars and the preferred direction in the Finsler spacetime. By Eq. (8), the luminosity distance in the Finsler spacetime can be written as

$$D_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (9)$$

where $E(z)$ in the Finsler spacetime takes the form

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3(1+A_D \cos \theta)^{-3} + 1 - \Omega_{m0}}. \quad (10)$$

Plugging Eq. (10) into Eqs. (3) and (6), we can get the form of the luminosity distance and angular diameter distance in the Finslerian cosmological model, respectively.

¹ https://github.com/shsuyu/H0LiCOW-public/tree/master/H0_inference_code.

Fig. 2 The marginalized posterior distribution of the parameters in the Finslerian cosmological model with the 2015 quasar compilation. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

3 Results

To constrain the dipole amplitude and the preferred direction in the Finslerian cosmological model with the X-ray and UV fluxes of quasars, we employ the likelihood function [40],

$$\ln(\text{LF}) = -\frac{1}{2} \sum_{i=1}^N \left[\frac{[\log(F_{X,i}^{\text{obs}}) - \log(F_{X,i}^{\text{th}})]^2}{s_i^2} + \ln(2\pi s_i^2) \right], \quad (11)$$

where $s_i^2 = \sigma_i^2 + \delta^2$, σ_i^2 is the error of the observed flux $F_{X,i}^{\text{obs}}$ and δ is the global intrinsic dispersion. $F_{X,i}^{\text{th}}$ is the theoretical flux at the redshift z_i .

The Markov chain Monte Carlo (MCMC) method has been used to explore the whole parameters space in our work. Emcee² [56] as the Affine Invariant Markov chain Monte Carlo Ensemble sampler is widely used to investigate the parameters in astrophysics and cosmology.

² <https://emcee.readthedocs.io/en/stable/>.

3.1 Cosmological constraints with the 2015 quasar compilation

The flat prior of each parameter in the Finslerian cosmological model is

$$\Omega_m \sim [0, 1], A_D \sim [0, 1], l \sim [-180, 180], b \sim [-90, 90],$$

and the flat priors of the parameters β , γ and δ are

$$\beta \sim [5, 12], \gamma \sim [0, 1], \delta \sim [0.2, 0.5].$$

For the 2015 quasar compilation, we find that the parameter δ is insensitive to the Λ CDM model and Finslerian cosmological model. The results of the parameter δ in the Λ CDM model [40] and Finslerian cosmological model are almost the same. For sake of brevity and clarity, we show the results of the three parameters β , γ and δ in Table 1.

The results of parameters in the Finslerian cosmological model are shown in Fig. 2 and summarized in Table 2. In Fig. 2, we show the marginalized posterior distribution of the parameters. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors. In Table 2, we show the 68% confidence level constraints on the parameters. The matter density is $\Omega_m = 0.509^{+0.453}_{-0.275}$. The dipole direction is $(l, b) = (288.92^{+23.74}_{-28.80}, 6.10^{+16.55}_{-16.40})$, which is closed to the one given by the Pantheon sample [53]. The angular difference between the two dipole directions is 31.05° . Compared with results from SNe Ia [50, 52, 53], the precision of dipole direction has a significant improvement.

Though the dipole anisotropy is well-constrained by the 2015 quasar compilation, the matter density Ω_m and its uncertainty are very large compared to the estimates given by the CMB and SNe Ia. We combine the Pantheon sample with the 2015 quasar compilation as the “standardized candles” to constrain the Finslerian cosmological model. The results are shown in Fig. 3 and summarized in Table 2. In Fig. 3, we show the marginalized posterior distribution of the parameters. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors. In Table 2, we show the 68% confidence level constraints on the parameters. For the combined datasets,

Fig. 3 The marginalized posterior distribution of the parameters in the Finslerian cosmological model from the 2015 quasar compilation and Pantheon sample. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

the 95% confidence level upper limit of the dipole amplitude A_D is 0.024 and the dipole direction points towards $(l, b) = (298.80^{+209.95}_{-118.30}, -30.60^{+74.50}_{-59.38})$. The matter density is $\Omega_m = 0.295^{+0.040}_{-0.042}$. The result is consistent with the one from Pantheon sample in the Finslerian cosmological model [53].

We use six gravitationally lensed quasars with measured time delays from H0LiCOW collaboration [48] to investigate the Hubble constant H_0 in the Finslerian cosmological model. The six gravitationally lensed quasars are combined with the 2015 quasar compilation and Pantheon sample. The flat prior on the Hubble constant is $H_0 \sim [60, 100]$. We show the results in Fig. 4 and Table 2. We find that the 95% confidence level upper limit of the dipole amplitude A_D is 0.021 and the dipole direction is $(l, b) = (298.80^{+215.08}_{-115.79}, -23.42^{+61.75}_{-66.56})$. The results of dipole parameters are consistent with the ones from Pan-

Table 2 The 68% confidence level constraints on the parameters in the Finslerian cosmological model with the 2015 quasar compilation and other datasets

Data	Quasars	Quasars + Pantheon	Quasars + Pantheon + TSDL
Ω_m	$0.509^{+0.453}_{-0.275}$	$0.295^{+0.040}_{-0.042}$	$0.295^{+0.040}_{-0.042}$
A_D	$0.302^{+0.185}_{-0.124}$	—	—
l	$288.92^{+23.74}_{-28.80}$	$298.80^{+209.95}_{-118.30}$	$298.80^{+215.08}_{-115.79}$
b	$6.10^{+16.55}_{-16.40}$	$-30.60^{+74.50}_{-59.38}$	$-23.42^{+61.75}_{-66.56}$
H_0^a	—	—	73.4 ± 3.2

^a $\text{km s}^{-1} \text{Mpc}^{-1}$

Fig. 4 The marginalized posterior distribution of the parameters in the Finslerian cosmological model with the combination of 2015 quasar compilation, Pantheon sample, and six gravitationally lensed quasars. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

theon sample [53]. The Hubble constant is $H_0 = 73.4 \pm 3.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is consistent with the result from six gravitationally lensed quasars within statistical uncertainties.

We construct simulations from the 2015 quasar compilation to investigate the future constraints of quasars on the dipole parameters in the Finslerian cosmological model. We assume a fiducial Finslerian cosmological model with $\Omega_m = 0.509$, $A_D = 0.302$, $(l, b) = (288.92^\circ, 6.10^\circ)$, $\beta = 9.736$, $\gamma = 0.543$, and $\delta = 0.318$. In the simulation, the positions and redshifts of simulated quasars are generated from ones of the 808 quasars. The X-ray fluxes of simulated quasars are calculated from the fiducial Finslerian cosmological model. The statistical errors associated with the simulated F_X are the ones from the 808 quasars. We construct 2000, 5000, and 10000 simulations for comparison, and the results of the dipole parameters from the simulated quasar dataset are shown in Fig. 5 and summarized in Table 3. In Fig. 5, the blue, red, and dark lines denote 2000, 5000, and 10,000 simulations, respectively. As can be seen, the inferred errors of the dipole parameters become smaller as the number of simulations increases. For the dipole parameters A_D , l , and b , the precisions increase by 50%–60% when the results of 10,000 simulations are compared with the ones of 2000 simulations. The results show that the X-ray and UV fluxes of quasars have a promising future as a probe of the Finslerian cosmological model.

Fig. 5 The marginalized posterior distribution of the dipole parameters in the Finslerian cosmological model with the quasar dataset simulated from the 2015 quasar compilation. The horizontal and vertical solid lines denote the maximum of 1-dimensional marginalized posteriors. The blue, red, and dark lines denote 2000, 5000 and, 10,000 simulations, respectively

Table 3 The 68% confidence level constraints on the dipole parameters in the Finslerian cosmological model with the quasar dataset simulated from the 2015 quasar compilation

Simulations	2000	5000	10,000
A_D	$0.280^{+0.105}_{-0.078}$	$0.289^{+0.065}_{-0.054}$	$0.290^{+0.046}_{-0.037}$
l	$288.35^{+16.15^\circ}_{-19.92^\circ}$	$289.70^{+9.81^\circ}_{-11.78^\circ}$	$289.00^{+7.73^\circ}_{-7.57^\circ}$
b	$6.66^{+11.45^\circ}_{-12.17^\circ}$	$5.81^{+7.36^\circ}_{-6.47^\circ}$	$6.40^{+4.64^\circ}_{-4.87^\circ}$

3.2 Cosmological constraints with the 2020 quasar compilation

Due to the host-galaxy contamination, the 2500 Å UV fluxes determined by extrapolation from the optical are less reliable at $z < 0.7$ in the 2020 quasar compilation [39]. In our work, we use 2023 quasars at $z > 0.7$ and 15 local quasars with higher quality at $z < 0.7$ to constrain the cosmological model. The flat prior of each parameter in the Finslerian cosmological model is

$$\Omega_m \sim [0, 1], A_D \sim [0, 1], l \sim [-180, 180], b \sim [-90, 90],$$

and the flat priors of the parameters β , γ and δ are

$$\beta \sim [5, 12], \gamma \sim [0, 1], \delta \sim [0.2, 0.5].$$

The results of parameters Ω_m , A_D , l , and b are shown in Fig. 6 and summarized in Table 4. In Fig. 6, we show the marginalized posterior distribution of the param-

Fig. 6 The marginalized posterior distribution of the parameters in the Finslerian cosmological model with the 2020 quasar compilation. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

eters. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors. In Table 4, we show the 68% confidence level constraints on the parameters. From Fig. 6, we can see the constraints of the parameters Ω_m , A_D , l , and b are very weak. The 95% confidence level upper limit of the dipole amplitude A_D is 0.060. The dipole direction points towards $(l, b) = (298.80^{+220.58^\circ}_{-117.76^\circ}, 34.20^{+55.80^\circ}_{-81.38^\circ})$. Though the constraints of parameters with the 2020 quasar compilation is very weak, the two dipole directions from the 2015 and 2020 quasar compilations are not far away and the angular difference between

Fig. 7 The marginalized posterior distribution of the parameters in the Finslerian cosmological model from the 2020 quasar compilation and Pantheon sample. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

the two dipole directions is 29.5° . The results of parameters β , γ , and δ are summarized in Table 5.

We combine the Pantheon sample with the 2020 quasar compilation as the “standardized candles” to constrain the Finslerian cosmological model. The results are shown in Fig. 7 and summarized in Table 4. In Fig. 7, we show the marginalized posterior distribution of the parameters. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors. In Table 4, we show the 68% confidence level constraints on the parameters. The results of parameters β , γ and δ are summa-

Table 4 The 68% confidence level constraints on the parameters in the Finslerian cosmological model with the 2020 quasar compilation and other datasets

Data	Quasars	Quasars + Pantheon	Quasars + Pantheon + TSDL
Ω_m	—	$0.323^{+0.045}_{-0.037}$	$0.325^{+0.044}_{-0.038}$
A_D	—	—	—
l	$298.80^{+220.58^\circ}_{-117.76^\circ}$	$313.19^{+196.56^\circ}_{-133.14^\circ}$	$313.22^{+226.09^\circ}_{-106.81^\circ}$
b	$34.20^{+55.80^\circ}_{-81.38^\circ}$	$-23.40^{+77.64}_{-66.60}$	$-19.81^{+70.93^\circ}_{-70.19^\circ}$
H_0^a	—	—	$72.9^{+3.5}_{-2.8}$

^a km s⁻¹ Mpc⁻¹

Table 5 The 68% confidence level constraints on the parameters β , γ , and δ in the Finslerian cosmological model with the 2020 quasar compilation and other datasets

Data	Quasars	Quasars + Pantheon	Quasars + Pantheon + TSDL
β	$7.551^{+0.496}_{-0.560}$	$7.001^{+0.576}_{-0.486}$	$7.045^{+0.508}_{-0.518}$
γ	$0.619^{+0.022}_{-0.013}$	$0.643^{+0.016}_{-0.019}$	0.641 ± 0.017
δ	0.277 ± 0.007	0.230 ± 0.007	$0.230^{+0.007}_{-0.008}$

Fig. 8 The marginalized posterior distribution of the parameters in the Finslerian cosmological model with the combination of 2020 quasar compilation, Pantheon sample, and six gravitationally lensed quasars. The horizontal and vertical solid black lines denote the maximum of 1-dimensional marginalized posteriors

ized in Table 5. The 95% confidence level upper limit of the dipole amplitude A_D is 0.018 and the dipole direction points towards $(l, b) = (313.19^{+196.56^\circ}_{-133.14^\circ}, -23.40^{+77.64^\circ}_{-66.60^\circ})$. The result is consistent with one given by the combination of the Pantheon sample and the 2015 quasar compilation.

We use the six gravitationally lensed quasars with the 2020 quasar compilation and Pantheon sample to investigate the Hubble constant H_0 in the Finslerian cosmological model. The flat prior on the Hubble constant is $H_0 \sim [60, 100]$. The results of parameters H_0 , Ω_m , A_D , l , and b are shown in Fig. 8 and summarized in Table 4. The results of parameters β , γ , and δ are summarized in Table 5. The 95% confidence level upper limit of the dipole amplitude A_D is 0.018 and the dipole direction points towards $(l, b) = (313.22^{+226.09^\circ}_{-106.81^\circ}, -19.81^{+70.93^\circ}_{-70.19^\circ})$. The Hubble constant is $H_0 = 72.9^{+3.5}_{-2.8} \text{ km s}^{-1} \text{ Mpc}^{-1}$. Though the central value of H_0 decreases a little bit, it is consistent with the result from six gravitationally lensed quasars within statistical uncertainties.

4 Discussions and conclusions

In this paper, we tested the anisotropy in the Finslerian cosmological model with the X-ray and UV fluxes of quasars. The 2015 [37] and 2020 [39] compilation of quasars are using in the cosmological constraints. For the

2015 quasar compilation, the dipole amplitude is $A_D = 0.302^{+0.185}_{-0.124}$ and the dipole direction points towards $(l, b) = (288.92^{+23.74^\circ}_{-28.80^\circ}, 6.10^{+16.55^\circ}_{-16.40^\circ})$. We found that the dipole direction given by the 2015 quasar compilation is not far away from the one given by the Pantheon sample in the Finslerian cosmological model and the angular difference is around 30° . Though the dipole anisotropy is well-constrained by the 2015 quasar compilation, the matter density Ω_m and its uncertainty are very large compared to the estimates given by the CMB and SNe Ia. We forecasted the future constraints on the dipole parameters from the 2015 quasar compilation. We constructed 2000, 5000, and 10,000 simulations and found that the precisions of the dipole parameters have a significant improvement as the number of simulations increases. The results show that the X-ray and UV fluxes of quasars have a promising future as a probe of the cosmic anisotropy. Unlike the well-constraint from the 2015 quasar compilation, the 2020 quasar compilation gives a weak constraint on the dipole parameters. It appears that there is a tension at the 3σ – 4σ confidence level between the cosmographic fit of the 2020 quasar compilation Hubble diagram and the one obtained by assuming a flat Λ CDM model [57]. The difference between the 2015 and 2020 quasar compilation may due to the fact that the tension with the Λ CDM model is not observed in the 2015 quasar compilation, which also seems to show a much higher dispersion in the Hubble diagram compared to the 2020 quasar compilation. We combined the Pantheon sample with quasars as the “standardized candles” to constrain the Finslerian cosmological model. The 2015 and 2020 quasar compilations are combined with the Pantheon sample, respectively. Similar to results from the Pantheon sample, the results given by the two combined datasets show that the dipole anisotropy is weak in the Finslerian cosmological model. We also investigated the Hubble constant H_0 in the Finslerian cosmological model by using the six gravitationally lensed quasars from H0LiCOW collaboration. The results of H_0 are consistent with the ones from six gravitationally lensed quasars within statistical uncertainties.

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