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Muon anomalous magnetic moment and Higgs potential stability in the 331 model from $SU(6)$

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Abstract We consider a $SU(3)_c \times SU(3)_L \times U(1)_X$ model from a $SU(6)$ Grand Unified Theory (GUT). In order to explain the anomalous magnetic moments of muon and electron, we introduce two new scalar triplets without vacuum expectation values (VEVs) so that the leading contributions to Δa_μ and Δa_e can avoid the suppression from small muon mass. In addition, the Higgs potential stability of this 331 model is studied by giving a set of sufficient conditions to ensure the boundedness from below of the potential.

1 Introduction

The standard model (SM) has been proved to be a successful theory since the last standard model particle, Higgs, was found at the Large Hadron Collider (LHC). However, there are many problems that SM cannot explain. For instance, the gauge hierarchy problem, dark matter, neutrino masses, gauge coupling unification, etc. There are so many motivations that make people believe the existence of new physics in higher energy beyond SM. In Grand Unified Theory (GUT), it is reasonable to extend the non-Abelian gauge group $SU(2)_L$ into a larger non-Abelian gauge group $SU(3)_L$ [1–12], where the $U(1)_Y$ is substituted by $U(1)_X$. The X charges of Higgs sector are critical in generating the breaking of $SU(3)_L$ gauge symmetry [13,14]. Thus, it is necessary to find a natural way to obtain the representations of such particles instead of giving the representations by hand. In the 331 model proposed in [2,3,15], all the particle representations under the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group could be obtained from some simple representations of the $SU(6)$ group [15]. The extended Higgs sector brings exotic gauge bosons, in which the Z' is constrained to be heavier than ~ 4.5 TeV [16]. The exotic fermion sector also has rich new physical phenomenology, such as dark matter and neutrino masses and mixing that we have discussed in [3]. In this work, we will study how to explain the muon anomalous magnetic moment, $g_\mu - 2$, in this 331 model as well as the Higgs potential stability.

Although there is no doubt that the SM is consistent with most of the experimental data so far, the muon anomalous magnetic moment, $a_\mu = \frac{g_\mu - 2}{2}$, is a long-standing deviation [17–21]. Recently, the first results from the muon g-2 experiment at the Fermi-Lab has been announced. Combining with the previous results by the Brookhaven National Lab (BNL), the measured value deviates from the theoretical prediction of SM around 4.2σ and the discrepancy is given by [22]

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9}. \tag{1.1}$$

At the same time, a 2.4σ discrepancy between the experimental and theoretical values of the electron anomalous magnetic moment, $a_e = \frac{g_e - 2}{2}$, is given by [23–25]

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = (-8.7 \pm 3.6) \times 10^{-13}. \tag{1.2}$$

In order to explain these deviations, models beyond-the-SM (BSM) are proposed [21,25–41]. Basically, the deviations are accounted by loop diagrams involved the BSM fermions, scalars, and gauge bosons.

In the rest of the paper, we will first review this 331 model briefly in Sect. 2. The stability of Higgs potential is studied in Sect. 3. In Sects. 4 and 5, the anomalous magnetic moments of muon and electron are explained, respectively. Finally, we conclude in Sect. 6.

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2 The model

As the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group of the 331 model proposed in [2,3,15] comes from a $SU(6)$ gauge group, representations of the $SU(6)$ group can be decomposed into representations of the $SU(3)_C \times SU(3)_L \times U(1)_X$ group to make up of the fermions of this 331 model,

$$\bar{6} \to \left(1,\ \bar{3},\ \frac{-1}{2\sqrt{3}}\right) \bigoplus \left(\bar{3},\ 1,\ \frac{1}{2\sqrt{3}}\right) \tag{2.1}$$

$$\hookrightarrow f_i = (e_{Li},\ -\nu_{Li},\ N_i) \bigoplus d^c_{Ri}\ , \tag{2.2}$$

$$\bar{6}' \to \left(1,\ \bar{3},\ \frac{-1}{2\sqrt{3}}\right) \bigoplus \left(\bar{3},\ 1,\ \frac{1}{2\sqrt{3}}\right) \tag{2.3}$$

$$\hookrightarrow f'_i = \left(e'_{Li},\ -\nu'_{Li},\ N'_i\right) \bigoplus D^c_{Ri}\ , \tag{2.4}$$

$$5 \to (3,\ 3,\ 0) \bigoplus \left(1,\ \bar{3},\ \frac{1}{\sqrt{3}}\right) \bigoplus \left(\bar{3},\ 1,\ \frac{-1}{\sqrt{3}}\right) \tag{2.5}$$

$$\hookrightarrow F_i = (u_{Li},\ d_{Li},\ D_{Li}) \bigoplus Xf^c_i$$
$$= \left(\nu'^c_{Ri},\ e'^c_{Ri},\ e^c_{Ri}\right) \bigoplus u^c_{Ri}\ , \tag{2.6}$$

where we denote the left-handed (LH) and right-handed (RH) SM fermions respectively as $e_{Li}, \nu_{Li}, u_{Li}, d_{Li}$, and e_{Ri}, u_{Ri}, d_{Ri}, with $i = 1, 2, 3$ being a generation index. The $U(1)_X$ charges of the fermions are given according to that the $U(1)_X$ charge operator, denoted as $\hat{Q}_X$, for the 6 representation of the $SU(6)$ group is

$$\hat{Q}_X(6) = \frac{1}{2\sqrt{3}}\mathrm{diag}[-1, -1, -1,\ 1,\ 1,\ 1]\,. \tag{2.7}$$

Consequently, the $U(1)_{\mathrm{EM}}$ charge operator can be expressed as

$$\hat{Q}_{\mathrm{EM}} = \frac{1}{\sqrt{3}}\hat{T}_{8L} + \hat{T}_{3L} + \frac{2}{\sqrt{3}}\hat{Q}_X\ , \tag{2.8}$$

where $\hat{T}_{8L}$ and $\hat{T}_{3L}$ are generators of the $SU(3)_L$ gauge group. Besides, we have fermions, N_{si} and N'_{si} $(i = 1, 2, 3)$, transforming as singlets under the $SU(3)_C \times SU(3)_L \times U(1)_X$ group.

This 331 model contains 3 scalar triplets from two $\bar{6}$ and one 15 representations of the $SU(6)$ group, which are

$$15' \to \left(1,\ \bar{3},\ \frac{1}{\sqrt{3}}\right) : T_u = \frac{1}{\sqrt{2}}\begin{pmatrix} v_u + \rho_1 + i\sigma_1 \\ \sqrt{2}\chi_1^+ \\ \sqrt{2}\chi_2^+ \end{pmatrix},$$

$$\langle T_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_u \\ 0 \\ 0 \end{pmatrix}, \tag{2.9}$$

$$\bar{6}'' \to \left(1,\ \bar{3},\ \frac{-1}{2\sqrt{3}}\right) : T_d = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\xi_2^- \\ v_d + \rho_2 + i\sigma_2 \\ \rho_3 + i\sigma_3 \end{pmatrix},$$

$$\langle < T_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_d \\ 0 \end{pmatrix}, \tag{2.10}$$

$$\bar{6}''' \to \left(1,\ \bar{3},\ \frac{-1}{2\sqrt{3}}\right) : T = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\xi_1^- \\ \rho_4 + i\sigma_4 \\ v_t + \rho_5 + i\sigma_5 \end{pmatrix},$$

$$\langle T\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v_t \end{pmatrix}, \tag{2.11}$$

where v_t is the vacuum expectation value (VEV) that breaks the $SU(3)_L \times U(1)_X$ gauge symmetry to the $SU(2)_L \times U(1)_Y$ gauge symmetry [42–46], and v_u, v_d are the VEVs which sequently break the $SU(2)_L \times U(1)_Y$ gauge symmetry to the $U(1)_{\mathrm{EM}}$ gauge symmetry. It is known that $\sqrt{v_d^2 + v_u^2} \approx 246$ GeV is required to give masses of the SM fermions and we define

$$\tan\theta = \frac{v_u}{v_d}\,. \tag{2.12}$$

The Yukawa terms and Majorana mass terms of this 331 model are

$$\begin{aligned}
-\mathcal{L}_{qua} &= y^u_{ij} F_i u^c_{Rj} T_u + y^d_{ij} F_i d^c_{Rj} T_d \\
&\quad + y^D_{ij} F_i D^c_{Rj} T + H.c, \\
-\mathcal{L}_{lep} &= y^\nu_{ij} f_i f_j T_u + y^e_{ij} f_i X f^c_j T_d \\
&\quad + y^{L'}_{ij} f'_i X f^c_j T + y^N_{ij} f_i \bar{T} N_{sj} \\
&\quad + y^{N'}_{ij} f'_i \bar{T} N'_{sj} + H.c, \\
-\mathcal{L}^{maj}_{neu} &= \frac{1}{2}\left(N_s\ \ N'_s\right)\begin{Bmatrix} M_s & M_{ss'} \\ M^T_{ss'} & M'_s \end{Bmatrix}\begin{pmatrix} N_s \\ N'_s \end{pmatrix} + H.c,
\end{aligned} \tag{2.13}$$

where M_s, M'_s and $M_{ss'}$ are 3×3 matrix.

The most general Higgs potential in this 331 model is

$$V_{\mathrm{Higgs}} = -m_1^2|T|^2 - m_2^2|T_d|^2 - m_3^2|T_u|^2 + \left(-BT^\dagger T_d + AT_u T_d T + H.c.\right) + V^{(4)}, \tag{2.14}$$

$$\begin{aligned}
V^{(4)} &= l_1\,|T|^4 + l_2\,|T_d|^4 + l_3\,|T_u|^4 \\
&\quad + l_{13}\,|T|^2\,|T_u|^2 + l_{12}\,|T|^2\,|T_d|^2 \\
&\quad + l_{23}\,|T_u|^2\,|T_d|^2 + l'_{13}\left|T^\dagger T_u\right|^2 \\
&\quad + l'_{12}\left|T^\dagger T_d\right|^2 + l'_{23}\left|T_u^\dagger T_d\right|^2 \\
&\quad + \Big(y_1 T^\dagger T_d\,|T|^2 + y_2 T^\dagger T_d\,|T_d|^2 + y_3 T^\dagger T_d\,|T_u|^2 \\
&\quad + y_{12} T^\dagger T_d T^\dagger T_d + y_{123} T_u^\dagger T_d T_u T^\dagger + H.c\Big),
\end{aligned} \tag{2.15}$$

where $V^{(4)}$ contains all the quartic couplings of the Higgs potential. We can express B and m_i^2 $(i = 1, 2, 3)$ with other parameters of V_{Higgs} since $< \frac{\partial V_{\text{Higgs}}}{\partial \rho_j} >= 0\, (j = 1, 2, \ldots, 5)$ gives four independent relations.

The non-SM gauge bosons in this 331 model are $W'^{\pm}$, Z', and V/V^*. Details of the neutral and charged scalar mixings can be found in [3]. Eigenstates of the mixing of CP-odd scalars σ_i $(i = 1, 2, \ldots 5)$ contain three massless Goldstone bosons a_1, a_2, a_3, and massive

$$a_4 = -\frac{v_t}{\sqrt{v_d^2 + v_t^2}}\sigma_3 + \frac{v_d}{\sqrt{v_d^2 + v_t^2}}\sigma_4 \,, \tag{2.16}$$

$$a_5 = \frac{v_t v_d}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}}\sigma_1 + \frac{v_t v_u}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}}\sigma_2 + \frac{v_u v_d}{\sqrt{v_t^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}}\sigma_5 \,. \tag{2.17}$$

The mixing of CP-even scalars ρ_i $(i = 1, 2, \ldots 5)$ gives four massive eigenstates h_j $(j = 2, 3, 4, 5)$ and massless

$$h_1 = -\frac{v_d}{\sqrt{v_d^2 + v_t^2}}\rho_3 + \frac{v_t}{\sqrt{v_d^2 + v_t^2}}\rho_4 \,. \tag{2.18}$$

3 Boundedness from below

As there are three scalar triplets in this 331 model, it is necessary to find the criteria for the parameters to ensure a stable scalar potential. The relevant issues can be found in [47–54]. In this Section, we will derive a set of conditions to ensure the BFB of the potential in this 331 model.

The BFB condition is only relevant to the quartic couplings in the scalar potential, namely $V^{(4)}$ in this model. Without loss of generality, we parameterize a scalar triplet in the following form [54]

$$\Phi_i = \sqrt{r_i} e^{i\gamma_i} \begin{pmatrix} \sin a_i \cos b_i \\ e^{i\beta_i} \sin a_i \sin b_i \\ e^{i\alpha_i} \cos a_i \end{pmatrix}, \quad i = 1, 2, 3 \,, \tag{3.1}$$

where i stands for the label of three scalar triplets in this model. Each Φ_i contains five real angular parameters, a_i, b_i, α_i, β_i, γ_i, and satisfies $\Phi_i^\dagger \Phi_i = r_i > 0$.

The symmetry of $V^{(4)}$ includes a U(3) transformation of T, T_d, and T_u, and a U(1) transformation of T_u, which allows us to reduce 10 parameters in the three triplet fields to write

$$\frac{T}{\sqrt{r_1}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \frac{T_d}{\sqrt{r_2}} = \begin{pmatrix} 0 \\ \sin a_2 \\ e^{i\alpha_2} \cos a_2 \end{pmatrix}, \quad \frac{T_u}{\sqrt{r_3}} = \begin{pmatrix} \sin a_3 \cos b_3 \\ \sin a_3 \sin b_3 \\ e^{i\alpha_3} \cos a_3 \end{pmatrix}. \tag{3.2}$$

We assume that all the parameters in $V^{(4)}$ are real for simplicity. By using the reformulated scalar fields, it is direct to get

$$\begin{aligned} V^{(4)} =\; & l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + \left(l_{13} + l'_{13}\cos^2[a_3]\right) r_1 r_3 \\ & + \left(l_{12} + l'_{12}\cos^2[a_2] + 2y_{12}\cos^2[a_2]\cos[2\alpha_2]\right) r_1 r_2 \\ & + 2y_1 \cos[a_2]\cos[\alpha_2] r_1 \sqrt{r_1 r_2} \\ & + 2y_2 \cos[a_2]\cos[\alpha_2] r_2 \sqrt{r_1 r_2} \\ & + \Bigg(l_{23} + l'_{23}\Big(\cos[a_2]\cos[a_3]\cos[\alpha_2 - \alpha_3] \\ & + \sin[a_2]\sin[a_3]\sin[b_3]\Big)^2 \\ & + l'_{23}\left(\cos^2[a_2]\cos^2[a_3]\sin^2[\alpha_2 - \alpha_3]\right)\Bigg) r_2 r_3 \\ & + 2\,(y_{123}\cos[a_3]\,(\cos[a_2]\cos[a_3]\cos[\alpha_2 - \alpha_3] \\ & + \sin[a_2]\sin[a_3]\sin[b_3]\cos[\alpha_3]) \\ & + y_3 \cos[a_2]\cos[\alpha_2])\, r_3 \sqrt{r_1 r_2}. \end{aligned} \tag{3.3}$$

Next, we need to make $V^{(4)}$ independent of the angular parameters by setting $V^{(4)}$ as a "angular minima" , for which we define

$$\begin{aligned} k_1 =\; & \frac{\partial V^{(4)}}{\partial \alpha_3} \\ =\; & r_3 \cos[a_3]\big(r_2 l'_{23}\sin[2a_2]\sin[a_3]\sin[b_3]\sin[\alpha_2 - \alpha_3] \\ & + 2y_{123}\sqrt{r_1 r_2}\,(\cos[a_2]\cos[a_3]\sin[\alpha_2 - \alpha_3] \\ & - \sin[a_2]\sin[a_3]\sin[b_3]\sin[\alpha_3])\big)\,, \end{aligned} \tag{3.4}$$

$$\begin{aligned} k_2 =\; & \frac{\partial V^{(4)}}{\partial \alpha_2} \\ =\; & -\cos[a_2]\big(r_2 r_3 l'_{23}\sin[2a_3]\sin[a_2]\sin[b_3]\sin[\alpha_2 - \alpha_3] \\ & + 2(y_1 r_1 + y_2 r_2 + y_3 r_3)\sqrt{r_1 r_2}\sin[\alpha_2] \\ & + 4y_{12} r_1 r_2 \cos[a_2]\sin[2\alpha_2] \\ & + 2y_{123} r_3 \sqrt{r_1 r_2}\cos^2[a_3]\sin[\alpha_2 - \alpha_3]\big)\,, \end{aligned} \tag{3.5}$$

$$\begin{aligned} k_3 =\; & \frac{\partial V^{(4)}}{\partial b_3} \\ =\; & 2r_3 \sin[a_2]\sin[a_3]\cos[b_3]\big(y_{123}\sqrt{r_1 r_2}\cos[a_3]\cos[\alpha_3] \\ & + r_2 l'_{23}\,(\cos[a_2]\cos[a_3]\cos[\alpha_2 - \alpha_3] \\ & + \sin[a_2]\sin[a_3]\sin[b_3])\big)\,, \end{aligned} \tag{3.6}$$

$$
\begin{aligned}
k_4 &= \frac{\partial V^{(4)}}{\partial a_3} \\
&= -r_1 r_3 l'_{13} \sin[2a_3] \\
&\quad + 2r_3\sqrt{r_1 r_2}\, y_{123}\left(-\sin[2a_3]\cos[a_2]\cos[\alpha_2-\alpha_3]\right. \\
&\quad \left. + \sin[a_2]\sin[b_3]\cos[2a_3]\cos[\alpha_3]\right) \\
&\quad + r_2 r_3 l'_{23}\Big(-\sin[2a_3]\cos^2[a_2] + \sin[2a_2]\sin[b_3] \\
&\quad \cos[2a_3]\cos[\alpha_2-\alpha_3] + \sin^2[a_2]\sin[2a_3]\sin^2[b_3]\Big)\ ,
\end{aligned} \tag{3.7}
$$

$$
\begin{aligned}
k_5 &= \frac{\partial V^{(4)}}{\partial a_2} \\
&= -r_1 r_2\left(l'_{12} + 2y_{12}\cos[2\alpha_2]\right)\sin[2a_2] \\
&\quad - 2(y_1 r_1 + y_2 r_2 + y_3 r_3)\sqrt{r_1 r_2}\sin[a_2]\cos[\alpha_2] \\
&\quad + 2y_{123} r_3\sqrt{r_1 r_2}\left(-\sin[a_2]\cos[a_3]\cos[\alpha_2-\alpha_3]\right. \\
&\quad \left. + \sin[a_3]\sin[b_3]\cos[a_2]\cos[\alpha_3]\right) \\
&\quad + r_2 r_3 l'_{23}\Big(-\sin[2a_2]\cos^2[a_3] + \sin[b_3] \\
&\quad \times\left(\sin[2a_3]\cos[2a_2]\cos[\alpha_2-\alpha_3]\right. \\
&\quad \left. + \sin[2a_2]\sin^2[a_3]\sin[b_3]\right)\Big)\ .
\end{aligned} \tag{3.8}
$$

The "angular minima" requires that all of k_i ($i = 1, 2, \ldots, 5$) vanish, for which we find four solutions,

Solution 1: $\sin[a_3] = 0$, $\cos[a_2] = 0$, $\cos[\alpha_2] = 0$, $\sin[b_3] = 0$, and $\sin[\alpha_3] = 0$, which lead to

$$
\begin{aligned}
V^{(4)} &= V_1^{(4)} \\
&= l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + l_{23} r_2 r_3 \\
&\quad + l_{12} r_1 r_2 + (l_{13} + l'_{13}) r_1 r_3\ .
\end{aligned} \tag{3.9}
$$

Solution 2: $\cos[a_2] = 0$, $\cos[a_3] = 0$, $\cos[\alpha_2] = 0$, $\sin[2b_3] = 0$, and $\sin[b_3]\cos[\alpha_3] = 0$, which lead to

$$
\begin{aligned}
V^{(4)} &= V_2^{(4)} \\
&= l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + l_{13} r_1 r_3 \\
&\quad + l_{12} r_1 r_2 + (l_{23} + \sin^2[b_3] l'_{23}) r_2 r_3 \\
&\geq l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + l_{13} r_1 r_3 \\
&\quad + l_{12} r_1 r_2 + \left(l_{23} + \frac{l'_{23} - |l'_{23}|}{2}\right) r_2 r_3\ .
\end{aligned} \tag{3.10}
$$

Note that $\sin^2[b_3]$ can be either 0 or 1 as $\sin[2b_3] = 0$ in this solution, which makes that $\sin^2[b_3] l'_{23} \geq \frac{l'_{23} - |l'_{23}|}{2}$ is always true no matter l'_{23} is positive or negative.

Solution 3: $\sin[a_2] = 0$, $\cos[a_3] = 0$, $\sin[\alpha_2] = 0$, and $\sin[b_3]\cos[\alpha_3] = 0$, which lead to

$$
\begin{aligned}
V^{(4)} &= V_3^{(4)} \\
&= l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + l_{13} r_1 r_3 \\
&\quad + l_{23} r_2 r_3 + (l_{12} + l'_{12} + 2y_{12}) r_1 r_2 \\
&\quad + 2\cos[a_2]\cos[\alpha_2](y_1 r_1 + y_2 r_2 + y_3 r_3)\sqrt{r_1 r_2} \\
&\geq l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + l_{13} r_1 r_3 \\
&\quad + l_{23} r_2 r_3 + (l_{12} + l'_{12} + 2y_{12}) r_1 r_2 \\
&\quad - (|y_1| r_1 + |y_2| r_2 + |y_3| r_3)(r_1 + r_2)\ ,
\end{aligned} \tag{3.11}
$$

where $\cos[a_2]\cos[\alpha_2] = \pm 1$ and $2\sqrt{r_1 r_2} \leq r_1 + r_2$ have been utilized.

Solution 4: $\sin[a_2] = 0$, $\sin[a_3] = 0$, $\sin[\alpha_3] = 0$, and $\sin[\alpha_2] = 0$, which lead to

$$
\begin{aligned}
V^{(4)} &= V_4^{(4)} \\
&= l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + (l_{13} + l'_{13}) r_1 r_3 \\
&\quad + (l_{12} + l'_{12} + 2y_{12}) r_1 r_2 + (l_{23} + l'_{23}) r_2 r_3 \\
&\quad + 2\cos[a_2]\cos[\alpha_2](y_1 r_1 + y_2 r_2 + y_3 r_3)\sqrt{r_1 r_2} \\
&\quad + 2\cos[a_2]\cos[\alpha_2 - \alpha_3] y_{123} r_3 \sqrt{r_1 r_2} \\
&\geq l_1 r_1^2 + l_2 r_2^2 + l_3 r_3^2 + (l_{13} + l'_{13}) r_1 r_3 \\
&\quad + (l_{12} + l'_{12} + 2y_{12}) r_1 r_2 + (l_{23} + l'_{23}) r_2 r_3 \\
&\quad - (|y_1| r_1 + |y_2| r_2 + |y_3| r_3)(r_1 + r_2) \\
&\quad - |y_{123}| r_3 (r_1 + r_2)\ ,
\end{aligned} \tag{3.12}
$$

where $\cos[a_2]\cos[\alpha_2] = \pm 1$, $\cos[a_2]\cos[\alpha_2 - \alpha_3] = \pm 1$, and $2\sqrt{r_1 r_2} \leq r_1 + r_2$ have been utilized.

According to the four solutions given above, we obtain that

$$
V_i^{(4)} \geq (r_1,\ r_2,\ r_3) M_i^{\mathrm{BFB}} (r_1,\ r_2,\ r_3)^{\mathrm{T}}, \quad i = 1, 2, 3, 4. \tag{3.13}
$$

It is direct to get the expressions of M_i^{BFB} which we do not give here. The BFB is realized as long as that co-positivity constraints on the four M_i^{BFB} ($i = 1, 2, 3, 4$) are satisfied at the same time.

It is noted that the conditions, derived in this Section, are sufficient but not necessary to ensure the BFB of the potential in our 331 model since the left side in Eq. (3.13) could be larger than the right side.

4 g_μ-2

In this 331 model, new contributions to the muon anomalous moment a_μ arise from loop diagrams involved the BSM fermions, scalars, and gauge bosons. According to [16], $|v_t|$ is required to be larger than 10 TeV to satisfy the constraint $M_{Z'} > 4.5$ TeV, which also makes $M_V \approx M_{W'} > 3.2$ TeV. So the contributions to a_μ involving the non-SM gauge bosons are negligible due to the heavy masses of these gauge bosons. We also neglect the contributions given by the non-SM charged scalars and exotic neutrinos since in most viable parameter space they are very heavy. So, we only focus on the contributions induced by the neutral scalars.

The contributions to a_μ involving neutral scalars come from the following parts of the Lagrangian of this 331 model,

$$\mathcal{L}_{lep} \supset -y^e_{ij} f_i X f^c_j T_d - y^{L'}_{ij} f'_i X f^c_j T \,. \tag{4.1}$$

Supposing that y^e_{ij} and $y^{L'}_{ij}$ are proportional to δ_{ij} for simplicity, we define

$$y^e_{22} = y^\mu \,, \tag{4.2}$$
$$y^{L'}_{22} = y^{\mu'} \,. \tag{4.3}$$

Among the contributions involving the non-SM charged fermions, the leading ones are chirally-enhanced, which are shown in the upper left panel of Fig. 1, where the LH and RH μ' are e'_{L2} and e'_{R2}, respectively. These chirally-enhanced contributions can be expressed as

$$\Delta a_{\mu 1} = -\frac{m_\mu}{16\pi^2 m_{\mu'}} y^\mu y^{\mu'} \left(\sum_{i=2}^{5} \left(V^h_{3i} V^h_{4i} \right) f'_{\rm LR}\left(\frac{m^2_{\mu'}}{m^2_{h_i}} \right) - \sum_{j=4}^{5} \left(V^a_{3j} V^a_{4j} \right) f'_{\rm LR}\left(\frac{m^2_{\mu'}}{m^2_{a_j}} \right) \right) \tag{4.4}$$

where h_i (a_j) stands for the eigenstate of the CP-even (CP-odd) scalar mixing, V^h (V^a) is the corresponding matrix for the mixing, and

$$f'_{\rm LR}(x) = x \frac{3 - 4x + x^2 + 2\log x}{2(x-1)^3} \,. \tag{4.5}$$

Note that h_1 and a_j ($j = 1, 2, 3$) are not included in the summations in Eq. (4.4) because they are Goldstone bosons.

The $f'_{\rm LR}(x)$ increases with x and satisfies

$$f'_{\rm LR}(x) < \frac{1}{2} \,. \tag{4.6}$$

It is know that $y^{\mu'}$ is related to the mass of μ' by

$$m_{\mu'} = y^{\mu'} \frac{v_t}{\sqrt{2}} \,. \tag{4.7}$$

According to Eqs. (2.16), (2.17), and (2.18), we obtain that

$$\left| V^h_{3i} V^h_{4i} \right| = \left(V^h_{3i} \right)^2 \left| \frac{v_d}{v_t} \right| < \left| \frac{v_d}{v_t} \right| \,, \qquad i = 2, 3, 4, 5, \tag{4.8}$$

$$\left| V^a_{34} V^a_{44} \right| = \frac{|v_t v_d|}{v_t^2 + v_d^2} < \left| \frac{v_d}{v_t} \right| \,, \tag{4.9}$$

$$V^a_{35} V^a_{45} = 0 \,. \tag{4.10}$$

Considering Eq. (4.6) to Eq. (4.10), we can estimate that

$$\left| \Delta a_{\mu 1} \right| \sim \frac{1}{16\pi^2} \left(\frac{m_\mu}{v_t} \right)^2 \sim 10^{-12} \,, \tag{4.11}$$

where $|v_t| > 10$ TeV has been used.

The contribution involving muon and the non-SM scalar is shown in the upper right panel of Fig. 1, which can be expressed as

$$\Delta a_{\mu 2} = \frac{y^{\mu 2}}{16\pi^2} \left(\sum_{i=3}^{5} \left(V^h_{2i} \right)^2 \left(f'_{\rm LR}\left(\frac{m^2_\mu}{m^2_{h_i}} \right) + f'_{\rm LL}\left(\frac{m^2_\mu}{m^2_{h_i}} \right) \right) + \sum_{j=4}^{5} \left(V^a_{2j} \right)^2 \left(-f'_{\rm LR}\left(\frac{m^2_\mu}{m^2_{a_j}} \right) + f'_{\rm LL}\left(\frac{m^2_\mu}{m^2_{a_j}} \right) \right) \right) \,, \tag{4.12}$$

where h_2 (the Higgs boson) is not included because the relevant contribution is not BSM, and

$$f'_{\rm LL}(x) = x \frac{2 + 3x - 6x^2 + x^3 + 6x\log x}{6(1-x)^4} \,. \tag{4.13}$$

Since $f'_{\rm LL}(x) \ll f'_{\rm LR}(x)$ when $x = \frac{m^2_\mu}{m^2_{h_i/a_j}} \ll 1$, the magnitude of $\Delta a_{\mu 2}$ can be estimated as

$$\left| \Delta a_{\mu 2} \right| \sim \frac{y^{\mu 2}}{16\pi^2} f'_{\rm LR}\left(\frac{m^2_\mu}{m^2_{h_i/a_j}} \right) \,, \tag{4.14}$$

which will be about $\sim 0.4 \times 10^{-13}$ if we choose that $\tan\theta = 6$ and $m_{h_i/a_j} = 600$ GeV.

We conclude that it is impossible to account for the muon anomalous moment with the scalars now available. To account for the muon anomalous moment in this 331 model, we introduce another two scalar triplets, T' and T'_d, which are in the same representation with T (T_d) but have no VEVs

$$T'_d = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\xi^-_3 \\ \rho_6 + i\sigma_6 \\ \rho_7 + i\sigma_7 \end{pmatrix} \,, \tag{4.15}$$

$$T' = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\xi^-_4 \\ \rho_8 + i\sigma_8 \\ \rho_9 + i\sigma_9 \end{pmatrix} \,. \tag{4.16}$$

Very similar to [55], the following terms can be added in the Lagrangian of this 331 model

$$\Delta\mathcal{L} = \left(-\lambda^e_{ij} f_i X f^c_j T'_d - \lambda^{L'}_{ij} f'_i X f^c_j T' + A' T_u T'_d T' + H.c. \right) - m_4^2 \left| T' \right|^2 - m_5^2 \left| T'_d \right|^2 \,. \tag{4.17}$$

It should be emphasized that we do not add all the gauge invariant terms involving T' and T'_d for simplicity.

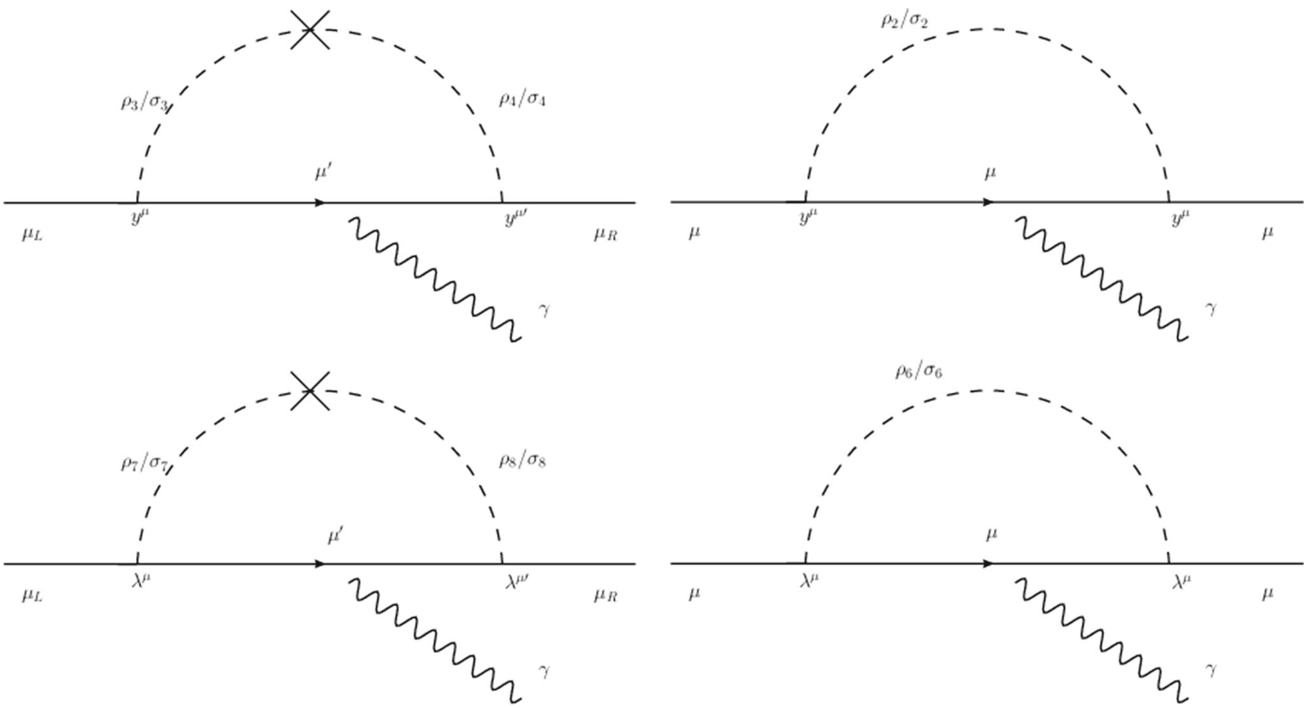

Fig. 1 Leading contributions to Δa_μ

New mixing of neutral scalars arises. The mass matrix and the mixing matrix satisfy

$$U^{1,2\dagger}\begin{pmatrix} m_4^2 & \pm A'v_u/\sqrt{2} \\ \pm A'v_u/\sqrt{2} & m_5^2 \end{pmatrix} U^{1,2} = \begin{pmatrix} M_1^2 & \\ & M_2^2 \end{pmatrix}, \tag{4.18}$$

where $+A'v_u/\sqrt{2}$ in the mass matrix in company with U^1 is for the mixing of ρ_7 and ρ_8 (also for the mixing of σ_6 and σ_9), and $-A'v_u/\sqrt{2}$ in the mass matrix together with U^2 is for the mixing of ρ_6 and ρ_9 (also for the mixing of σ_7 and σ_8). It is direct to obtain the physical masses and mixing, which are

$$M_{1,2}^2 = \left(m_4^2 + m_5^2 \mp \Delta M^2\right)/2,$$
$$\Delta M^2 \equiv \sqrt{\left(m_4^2 - m_5^2\right)^2 + 2A'^2 v_u^2}\,, \tag{4.19}$$

$$U^{1,2} = \begin{pmatrix} \frac{\pm\sqrt{2}A'v_u}{\sqrt{\left(m_5^2-m_4^2-\Delta M^2\right)^2+2A'^2v_u^2}} & -\frac{m_5^2-m_4^2-\Delta M^2}{\sqrt{\left(m_5^2-m_4^2-\Delta M^2\right)^2+2A'^2v^2}} \\ \frac{m_5^2-m_4^2-\Delta M^2}{\sqrt{\left(m_5^2-m_4^2-\Delta M^2\right)^2+2A'^2v_u^2}} & \frac{\pm\sqrt{2}A'v_u}{\sqrt{\left(m_5^2-m_4^2-\Delta M^2\right)^2+2A'^2v_u^2}} \end{pmatrix}. \tag{4.20}$$

Supposing that λ_{ij}^e and $\lambda_{ij}^{L'}$ are proportional to δ_{ij} for simplicity, we define

$$\lambda_{22}^e = \lambda^\mu\,, \tag{4.21}$$
$$\lambda_{22}^{L'} = \lambda^{\mu'}\,. \tag{4.22}$$

Similarly, the leading contributions involving μ' and neutral scalars from T' and T'_d are the chirally-enhanced ones shown in the lower left panel of Fig. 1, which can be expressed as

$$\begin{aligned}\Delta a_{\mu 3} &= -\frac{m_\mu}{16\pi^2 m_{\mu'}}\lambda^\mu\lambda^{\mu'}\sum_{i=1}^{2}\left(\left(U_{1i}^1U_{2i}^1\right)f'_{\rm LR}\left(\frac{m_{\mu'}^2}{M_i^2}\right)\right.\\ &\quad\left.-\left(U_{1i}^2U_{2i}^2\right)f'_{\rm LR}\left(\frac{m_{\mu'}^2}{M_i^2}\right)\right)\\ &= -\frac{m_\mu}{8\pi^2 m_{\mu'}}\lambda^\mu\lambda^{\mu'}\left(\sum_{i=1}^{2}\left(U_{1i}^1U_{2i}^1\right)f'_{\rm LR}\left(\frac{m_{\mu'}^2}{M_i^2}\right)\right),\end{aligned} \tag{4.23}$$

where $U_{1i}^1U_{2i}^1 = -U_{1i}^2U_{2i}^2$ $(i=1,2)$ is used.

The contribution involving muon and neutral scalars from T'_d is shown in the lower right panel of Fig. 1, which can be expressed as

$$\begin{aligned}\Delta a_{\mu 4} &= \frac{\lambda^{\mu 2}}{16\pi^2}\sum_{i=1}^{2}\left(\left(U_{1i}^2\right)^2\left(f'_{\rm LR}\left(\frac{m_\mu^2}{M_i^2}\right)+f'_{\rm LL}\left(\frac{m_\mu^2}{M_i^2}\right)\right)\right.\\ &\quad\left.+\left(U_{1i}^1\right)^2\left(-f'_{\rm LR}\left(\frac{m_\mu^2}{M_i^2}\right)+f'_{\rm LL}\left(\frac{m_\mu^2}{M_i^2}\right)\right)\right)\\ &= \frac{\lambda^{\mu 2}}{8\pi^2}\left(\sum_{i=1}^{2}\left(U_{1i}^1\right)^2 f'_{\rm LL}\left(\frac{m_\mu^2}{M_i^2}\right)\right),\end{aligned} \tag{4.24}$$

where $\left(U^1_{1i}\right)^2 = \left(U^2_{1i}\right)^2$ $(i = 1, 2)$ is used.

Based on the above discussions, we only need to consider contributions $\Delta a_{\mu 3}$ and $\Delta a_{\mu 4}$, and thus have

$$\Delta a_\mu \approx \Delta a_{\mu 3} + \Delta a_{\mu 4} \ . \tag{4.25}$$

To study the dependences of Δa_μ on λ^μ, $\lambda^{\mu'}$, and $m_{\mu'}$, we set other parameters to that $m_4 = 300$ GeV, $m_5 = 600$ GeV, $A' = 10$ GeV, and $\tan\theta = 6$. In Fig. 2, points in the orange (blue) region on the $m_{\mu'} - \lambda^\mu$ plane give values of Δa_μ within 2σ (1σ) deviation from the measured value.

In the upper two panels of Fig. 2 where $\lambda^\mu = \lambda^{\mu'}$, it is shown that $a_{\mu 4}/a_{\mu 3}$ increases with $m_{\mu'}$ and $\Delta a_{\mu 4}$ dominates $(a_{\mu 4}/a_{\mu 3} > 1)$ when $m_{\mu'}$ is larger than 1 TeV. When $m_{\mu'}$ is larger than 5.9 TeV, $\Delta a_{\mu 4}$ contributes more than 99% of Δa_μ and both the 1σ and 2σ regions have no dependence on $m_{\mu'}$ since $\Delta a_{\mu 4}$ is independent of $m_{\mu'}$. In the $m_{\mu'} >$5.9 TeV region, $\lambda^\mu = \lambda^{\mu'}$ within the 1σ and 2σ regions needs to be in the ranges of $(1.92, 2.43)$ and $(1.60, 2.65)$, respectively. $\Delta a_{\mu 3}$ dominates $(a_{\mu 4}/a_{\mu 3} < 1)$ when $m_{\mu'}$ is less than 1 TeV and the allowed $\lambda^\mu = \lambda^{\mu'}$ within the 1σ and 2σ regions increases with $m_{\mu'}$.

In the lower panel of Fig. 2 where $\lambda^\mu = -\lambda^{\mu'}$, $\Delta a_{\mu 4}$ is positive while $\Delta a_{\mu 3}$ is negative. Both the 1σ and 2σ regions need to ensure that $m_{\mu'} > 1$ TeV to make $\Delta a_\mu > 0$. In the 1σ region, $\lambda^\mu = -\lambda^{\mu'}$ decreases with $m_{\mu'}$ since the negative $\Delta a_{\mu 3}$ approaches zero when $m_{\mu'}$ increases. Similarly, $\Delta a_{\mu 4}$ contributes almost the whole Δa_μ $(-\Delta a_{\mu 3}/\Delta a_{\mu 4} < 0.01)$ when $m_{\mu'}$ is larger than 5.9 TeV and the 1σ and 2σ regions are approximately independent of $m_{\mu'}$.

In principle, we can add extra terms like $\left(-\lambda^e_{ij} f_i X f^c_j T - \lambda^{L'}_{ij} f'_i X f^c_j T_d\right)$ to account for the Δa_μ without introducing T' and T'_d. However, the VEV of T (T_d) will lead to mixing between μ_L and μ'_L $(\mu_R$ and $\mu'_R)$, which can bring about serious fine tuning problem. So we introduce T' and T'_d with no VEVs to account for the Δa_μ in this Section.

5 g$_e$-2

Much like the discussions for g$_\mu$-2, the non-SM particles in this 331 model (with T' and T'_d included) also induce new contributions to the electron anomalous moment a_e. Similarly, we define

$$y^e_{11} = y^e \ , \tag{5.1}$$
$$\lambda^e_{11} = \lambda^e \ , \tag{5.2}$$
$$y^{L'}_{11} = y^{e'} \ , \tag{5.3}$$
$$\lambda^{L'}_{11} = \lambda^{e'} \ . \tag{5.4}$$

There is no doubt that the leading contributions to a_e can be expressed as

$$\Delta a_e \approx \Delta a_{e3} + \Delta a_{e4} \ , \tag{5.5}$$
$$\Delta a_{e3} = -\frac{m_e}{8\pi^2 m_{e'}} \lambda^e \lambda^{e'} \left(\sum_{i=1}^{2} \left(U^1_{1i} U^1_{2i} \right) f'_{\rm LR} \left(\frac{m^2_{e'}}{M^2_i} \right) \right) \ , \tag{5.6}$$
$$\Delta a_{e4} = \frac{\lambda^{e2}}{8\pi^2} \left(\sum_{i=1}^{2} \left(U^1_{1i} \right)^2 f'_{\rm LL} \left(\frac{m^2_e}{M^2_i} \right) \right) \ . \tag{5.7}$$

We focus on the dependence of Δa_e on λ^e, $\lambda^{e'}$, and $m_{e'}$ by setting other parameters as in Fig. 2. Points in the blue, orange, and green regions on the $m_{e'} - \lambda^e$ plane in Fig. 3 give values of Δa_e within $1\sigma, 2\sigma$ and 3σ deviations from the measured value, respectively.

In the left panel of Fig. 2 where $\lambda^e = -\lambda^{e'}$, Δa_{e3} is negative while Δa_{e4} is positive. The 1σ and 2σ regions, which need to give negative value of Δa_e, must be at the left side of the line where $-\Delta a_{e4}/\Delta a_{e3} = 1$. So the allowed $m_{e'}$ within the 1σ (2σ) region has a largest value, which is 4.1 TeV (5.6 TeV) when $\lambda^e = -\lambda^{e'} = 3$.

Because Δa_{e3} and Δa_{e4} are both positive in the right panel of Fig. 2 where $\lambda^e = \lambda^{e'}$, the 1σ and 2σ regions disappear. Since $\Delta a_{e4} = 1.1 \times 10^{-13}$ when $\lambda^e = 3$, the region where $\lambda^e < 3$ is always within the 3σ region as long as Δa_{e4} dominates $(|\Delta a_{e4}/\Delta a_{e3}| > 1)$.

6 Conclusion

We derive a set of sufficient conditions to ensure the boundedness from below of the potential in the 331 model proposed in [3] which has three scalar triplets. Since the quartic couplings $V^{(4)}$ are more complex than those in [54], inequalities have to be used during the derivations which makes that the conditions obtained are sufficient but not necessary to ensure the BFB of the potential.

We focus on the BSM contributions to Δa_μ and Δa_e involving neutral scalars and charged fermions. The analysis shows that the contributions induced by the neutral scalars from T and T_d are too small to account for the muon and electron anomalous moments unless there are some serious fine tuning problems. So another two triplets T' and T'_d with non VEVs are introduced to provide new couplings to fermions. With the neutral scalars from T' and T'_d, the leading contributions to Δa_μ (Δa_e) involve both the BSM μ' (e') and μ (e) from the SM. The chirally-enhanced contributions dominate the contributions from μ' (e'), whose magnitudes decrease with $m_{\mu'}$ $(m_{e'})$, while the contributions from μ (e) are independent of $m_{\mu'}$ $(m_{e'})$. The former dominates over the latter

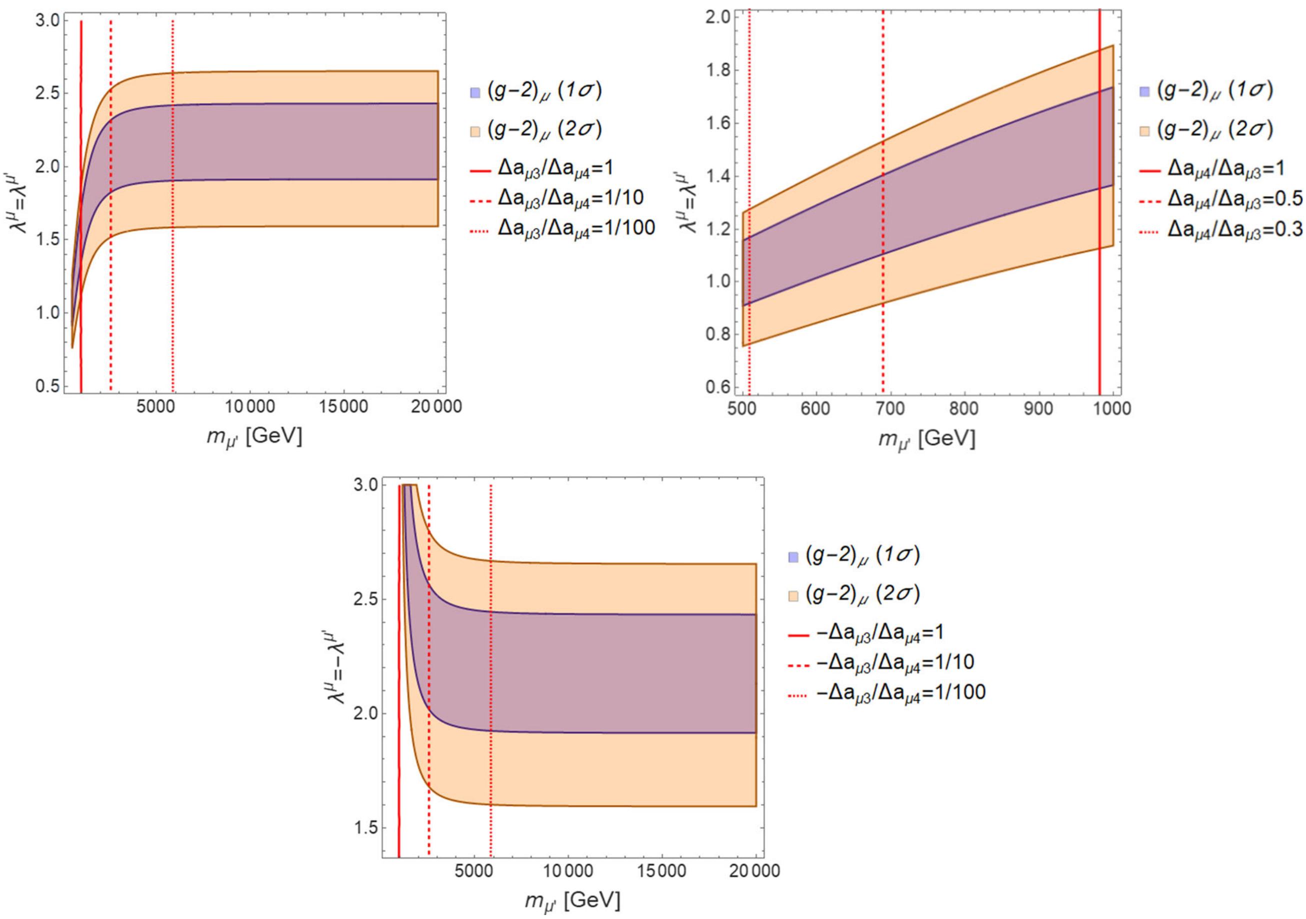

Fig. 2 Leading contributions to Δa_μ as a function of $m_{\mu'}$ and λ^μ. Here $\lambda^\mu = \pm\lambda^{\mu'}$ and other parameters are set to: $m_4 = 300$ GeV, $m_5 = 600$ GeV, $A' = 10$ GeV, $\tan\theta = 6$

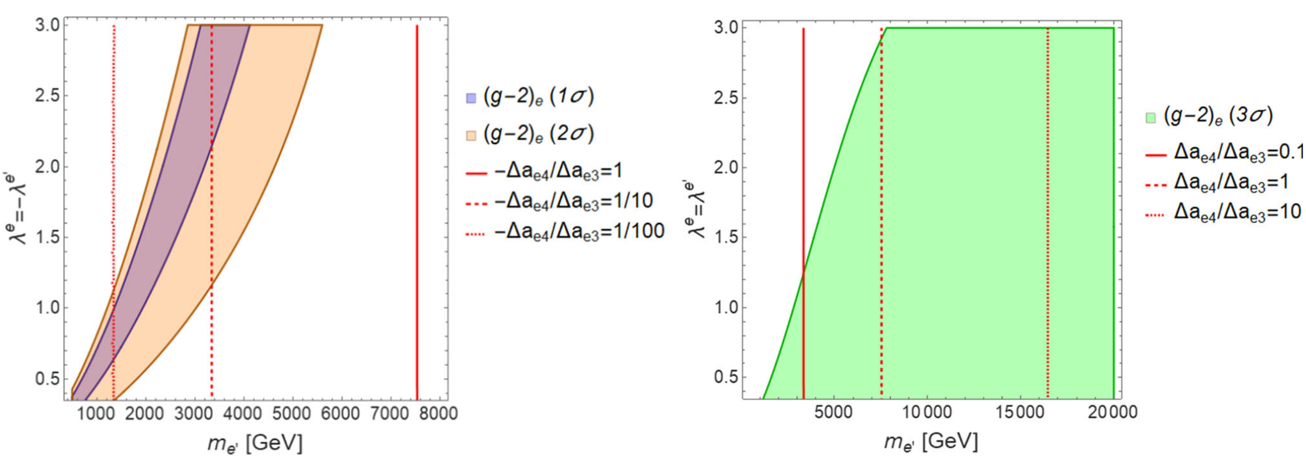

Fig. 3 Leading contributions to Δa_e as a function of $m_{e'}$ and λ^e. Here $\lambda^e = \pm\lambda^{e'}$ and other parameters are set as in Fig. 2

when $m_{\mu'} < 1$ TeV ($m_{e'} < 7.5$ TeV) if the other parameters are set as the Sect. 4.

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