



Cosmic voids and filaments from quantum gravity

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Abstract Using computer simulations, we study the geometry of a typical quantum universe, i.e., the geometry one might expect before a possible period of inflation. We display it using coordinates defined by means of four classical scalar fields satisfying the Laplace equation with nontrivial boundary conditions. They are a close analogue of the harmonic coordinate condition used in the context of GR (Kuchar and Torre in Phys Rev D 43:419–441, 1991). It is highly nontrivial that these ideas can be applied to understand the structures which appear in very irregular and fluctuating geometries. The field configurations reveal cosmic web structures surprisingly similar to the ones observed in the present-day universe.

1 Introduction

One major unsolved problem in theoretical physics is how to unite the theory of general relativity and quantum mechanics. It is hoped that such a unification will allow us to understand physics at the Planck scale, where the assumed quantum nature of gravity most likely plays a dominant role. Furthermore, the idea of an inflationary period in the history of our Universe has taught us that these quantum fluctuations at or near the Planck scale can, owing to an exponential growth of the size of the Universe, freeze and be expanded into the seeds of macroscopic large-scale structures. Results presented in this article suggest that the scenario of cosmic filaments and voids that we observe in the Universe today might have its source at the very early stage of cosmic evolution of quantum geometric degrees of freedom.

2 Lattice quantum universes

To discuss universes of the size of a few Planck lengths and their fluctuating quantum geometry, one needs a non-perturbative model of quantum gravity. We will discuss here a particular model called Causal Dynamical Triangulations (abbreviated to CDT; see [2,3] for a comprehensive introduction and an explanation of its somewhat technical name), but we believe that our results are generic and will be present in any reasonable quantum model of gravity. In CDT, space-time is a triangulation built by joining together fixed-size four-dimensional simplices in a way that satisfies certain topological requirements. The edge length ε of the four-simplices acts as an ultraviolet (UV) cutoff. Its choice also fixes the geometry of a triangulation. The way in which simplicial building blocks are connected codes the information about the curvature of the particular configuration. A natural geometric way of calculating the classical Einstein-Hilbert action on such piecewise linear manifolds leads to the so-called Regge action. The lattice regularized path integral of quantum gravity is then given by

$$\mathcal{Z}_{QG} = \int \mathcal{D}_{\mathcal{M}_H}[g_L] e^{iS_{EH}[g_L]} \rightarrow \sum_{T_E \in \mathcal{T}_E} e^{-S_R[T_E]}, \quad (1)$$

where \mathcal{M}_H is a globally defined hyperbolic Lorentzian manifold and $\mathcal{D}_{\mathcal{M}_H}[g_L]$ denotes the integration over equivalence classes $[g_L]$ of Lorentzian metrics on \mathcal{M}_H . \mathcal{T}_E is a suitable set of Wick-rotated Euclidean triangulations. The Regge action $S_R[T_E]$ for a triangulation $T_E \in \mathcal{T}_E$ contains the bare couplings related to the cosmological and Newton constants. In principle, we want to adjust the bare coupling constants such that we can take the UV cutoff ε to zero while keeping the physics unchanged (see [4] for a recent review). In accordance with the imposed global hyperbolicity, CDT introduces a time foliation of the four-dimensional manifolds into three-dimensional leaves, which are three-dimensional spatial sub-manifolds with a global time t and a fixed topol-

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ogy. The explicit CDT construction permits a Wick rotation of the time coordinate t (still denoted t), whereby CDT becomes a statistical model that can be studied using Monte-Carlo simulations. Each configuration obtained in the simulations can be viewed as the quantum evolution of a three-dimensional quantum geometric state in the imaginary time. This allows us inter alia, to measure the time dependence of some global quantities such as the spatial volume. It was shown [5–12] that, for suitable choices of the bare coupling constants, both the average spatial volume and its fluctuations can, with a large degree of accuracy, be described by the Hartle–Hawking minisuperspace model, which assumes isotropy and homogeneity of the Universe such that the only dynamical variable is the scale factor $a(t)$. It should be emphasized that the isotropy and homogeneity are not put in by hand in CDT but follow from integrating out all degrees of freedom other than the scale factor, and the typical geometries encountered in the quantum path integral are not at all close to the classical homogeneous solution of GR. The approximate agreement with the classical minisuperspace solution is obtained from an average of an ensemble of highly fluctuating geometric states and is caused by a nontrivial interplay between the physical action and the entropy of configurations.

The CDT model is background independent and, in the spatial directions, coordinate free. There is no background geometry in the definition of the path integral. Geometric information provided by the model is local, in the form of the neighborhood relations between the elements of the geometry. We may determine the geodesic distance between the simplices, but capturing the global properties of the system without a good choice of coordinates is difficult [13–15]. It is not a priori clear if such a choice is at all possible for a locally highly fluctuating geometry. Below we will show that such a choice is not only possible but can also give us a much better understanding of the structures present in quantum geometric configurations.

3 Boundary conditions

We will now consider a version of CDT where the piecewise linear manifolds are periodic both in time and space directions. Such a toroidal topology can be viewed as being extended to an infinite four-dimensional space, in which a four-dimensional elementary cell, bounded by a set of four independent non-contractible three-dimensional boundaries is periodically repeated in all directions. These boundaries are not physical entities, are not unique, and can be locally deformed as long as they still form an elementary cell, and yet they can serve as a reference frame for a coordinate system on a given triangulation. The nontrivial fractal structure of the encountered geometries makes it difficult to introduce

spatial coordinates in a chosen elementary cell in a constructive geometric way, using only the geodesic distance from a boundary. Below we will introduce an alternative method, where four massless classical scalar fields, which satisfy the Laplace equation with nontrivial boundary conditions will be used to fully parametrize the fractal geometry. In the case discussed in this paper, the fields depend on the geometry, but they do not modify it, providing us, for each configuration, with a system of external coordinates. One can say that they act as a microscope which uncovers the complicated four-dimensional structure of density fluctuations. Even for a configuration with a very irregular geometry, such fields allow us to define periodic pseudo-continuous coordinates and provide generalized foliations in all space-time directions. Consequently, it becomes possible to visualize and measure multidimensional correlations in all directions. As will be reported below, what we see is a quantum universe which seems surprisingly similar to our present day macroscopic universe.

3.1 Scalar fields as coordinates with values on S^1

We want to find nontrivial harmonic maps between two Riemannian manifolds $\mathcal{M}(g_{\mu\nu}) \rightarrow \mathcal{N}(h_{\alpha\beta})$, where $g_{\mu\nu}$ is an arbitrary metric and $h_{\alpha\beta}$ is a flat one. If \mathcal{N} has the topology of T^4 , then it can be defined by four scalar fields ϕ^α , $\alpha = 1, 2, 3, 4$, where $\phi^\alpha(x)$ is a map $\mathcal{M} \rightarrow S^1$, such that the following action is minimized:

$$S_M[\phi, \mathcal{M}] = \frac{1}{2} \int d^4x \sqrt{g(x)} g^{\mu\nu}(x) h_{\rho\sigma}(\phi^\nu(x)) \times \partial_\mu \phi^\rho(x) \partial_\nu \phi^\sigma(x). \quad (2)$$

Because we have chosen the trivial metric $h_{\rho\sigma}$ on \mathcal{N} , Eq. (2) splits up into four independent equations for the four scalar fields ϕ^σ . Minimizing Eq. (2) leads to the set of Laplace equations:

$$\Delta_x \phi^\sigma(x) = 0, \quad \Delta_x = \frac{1}{\sqrt{g(x)}} \partial_\mu \sqrt{g(x)} g^{\mu\nu}(x) \partial_\nu. \quad (3)$$

Thus ϕ^σ becomes a harmonic map $\mathcal{M} \rightarrow (S^1)^4$ preserving a dependence of ϕ^σ on $g^{\mu\nu}$. Let us consider a trivial one-dimensional example. In this case, let \mathcal{M} be S^1 with a unit circumference and a positive and strictly periodic density $\sqrt{g(x)}$. We want $x \rightarrow \phi(x)$ to be a non-trivial map $S^1 \rightarrow S^1$ such that ϕ can serve as a coordinate instead of x . One way to implement this is to find a solution satisfying

$$\phi(x+n) = \phi(x) + n\delta, \quad (4)$$

which maps the circle with a unit circumference to a circle with a circumference δ . The solution to the Laplace equation in this case satisfies

$$d\phi(x) = \delta \cdot \sqrt{g(x)} dx. \quad (5)$$

By rescaling the field, we can always enforce $\delta = 1$. The solution $\phi(x)$ is fixed by picking x_0 where $\phi(x_0) = 0$. The map $x \rightarrow \phi(x)$ becomes a monotonically increasing invertible map in the whole domain \mathbb{R} with the geometry periodically repeated with the period δ . If we parametrize the one-dimensional manifold \mathcal{M} in terms of ϕ instead of x , we will find the volume density in the range ϕ to $\phi + d\phi$ to be proportional to $\sqrt{g(x)} dx$, so that effectively $g(\phi) = 1$. We can also consider a function $\psi(x) = \text{mod}(\phi(x) - \phi(x_1), \delta)$. This function satisfies the Laplace equation in the range between x_1 (where $\psi(x) = 0$) and $x_1 + 1$ (where $\psi(x) = \delta$). The equation satisfied by $\psi(x)$ becomes a Poisson equation with the extra inhomogeneous local term, producing jumps at boundary points $x = x_1$ and $x = x_1 + 1$. It can still be considered to be a Laplace equation with a non-trivial boundary ‘‘jump’’ condition. Generalizing this to \mathcal{M} with the topology of T^4 , we want a solution to the Laplace Eq. (3) that wraps around S^1 in a particular direction once, and, in addition, we want the points x in \mathcal{M} that satisfy $\phi^\sigma(x) = c$ to form hypersurfaces $H^\sigma(c)$ whose union for c varying in a range of length 1 covers the whole \mathcal{M} .

3.2 Classical scalar fields with a jump

In CDT, the four-dimensional manifolds are represented by regular four-dimensional triangulations constructed by gluing together four-simplices so that each face is shared by exactly two simplices. Denote the number of four-simplices in the triangulation by N_4 . Each triangulation we consider is generated by a Monte Carlo simulation, using the CDT partition function. We call such a generated triangulation a configuration. For each configuration, we keep information about the position of the four boundaries of the elementary cell. This information is non-dynamical: it does not influence the Monte Carlo process. Each boundary is a connected set of three-dimensional faces, each of which separates two simplices, for instance, i and j , belonging to two neighboring elementary cells. The connection $i \rightarrow j$ can have either a positive or negative orientation, depending on the direction in which we cross the boundary. The boundary between the neighboring elementary cells in a direction σ can be parametrized by the $N_4 \times N_4$ anti-symmetric matrix $\mathbf{B}_{ij}^\sigma = -\mathbf{B}_{ji}^\sigma$ with the elements

$$\mathbf{B}_{ij}^\sigma = \begin{cases} \pm 1 & \text{if } i \rightarrow j \text{ crosses the boundary,} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The number of directed boundary faces of a simplex i is given by $b_i^\sigma = \sum_j \mathbf{B}_{ij}^\sigma$, with the obvious constraints $-5 < b_i^\sigma < 5$ and $\sum_i b_i^\sigma = 0$. For any simplex i adjacent to a boundary, the values \mathbf{B}_{ij}^σ are all positive or zero (on one side of the boundary), or all negative or zero (on the other side). We consider four scalar fields ϕ_i^σ located in the centers of simplices and

solve the minimization problem for the following discrete version of the continuous action in Eq. (2), for each field ϕ_i^σ :

$$S_M^{CDT}[\phi^\sigma, T_E] = \frac{1}{2} \sum_{i \leftrightarrow j} (\phi_i^\sigma - \phi_j^\sigma - \delta \mathbf{B}_{ij}^\sigma)^2. \tag{7}$$

The simplicity of the action does not mean that the geometric information about the connections between simplices is lost. Its form is purely a result of the fact that the system is built from simplices with an equal size. In (7) the sum is over all pairs of neighboring four-simplices in the triangulation T_E representing the manifold $\mathcal{M}(g_{\mu\nu})$ in Eq. (2). The parameter δ plays the same role as in the one-dimensional example considered previously, and here too, by rescaling the field, we can always set $\delta = 1$. The action (7) has two important symmetries. The first one is the invariance under a constant shift of the scalar field (the Laplacian zero mode). The second is a local invariance under a modification of the boundary \mathbf{B}_{ij}^σ and a shift by ± 1 (depending on the side of the boundary) of the field value in a simplex i adjacent to the boundary. This is equivalent to moving the simplex to the other side of the boundary and compensating for the change of the field in its center. After such a move, the number of faces belonging to the boundary will in general be changed, but the action (7) will remain constant. The classical field, henceforth denoted as ϕ_i^σ , minimizes the action (7), and thus has to satisfy the non-homogeneous Poisson-like equation

$$\mathbf{L}\phi^\sigma = b^\sigma, \tag{8}$$

where $\mathbf{L} = 5\mathbb{1} - \mathbf{A}$ is the $N_4 \times N_4$ Laplacian matrix, and \mathbf{A}_{ij} is the adjacency matrix with entries of value 1 if simplices i and j are neighbors and 0 otherwise. The Laplacian matrix \mathbf{L} has a constant zero mode but can be inverted if we fix a value of the field $\phi_{i_0}^\sigma = 0$ for an arbitrary simplex i_0 . Although \mathbf{L} is a sparse matrix, inverting it is a major numerical challenge for a system typically of size $N_4 \approx 10^6$. After we nevertheless obtain the classical solution ϕ^σ , the multi-dimensional analogue of the one-dimensional function $\psi(x)$ is given by ψ_i^σ :

$$\psi_i^\sigma = \text{mod}(\phi_i^\sigma, 1). \tag{9}$$

A new boundary is defined by $\bar{b}^\sigma = \mathbf{L}\psi^\sigma$. This allows us to reconstruct a new three-dimensional hypersurface H , separating the elementary cell from its copies in the direction σ and characterized by the fact that the field jumps from 0 to 1 when crossing H . This hypersurface can be moved to another position if we consider a family of hypersurfaces $H(\alpha^\sigma)$ obtained from

$$\psi_i^\sigma(\alpha^\sigma) = \text{mod}(\phi_i^\sigma - \alpha^\sigma, 1), \quad \bar{b}^\sigma(\alpha^\sigma) = \mathbf{L}\psi^\sigma(\alpha^\sigma). \tag{10}$$

Changing $0 \leq \alpha^\sigma < 1$, we shift the position of the hypersurface and cover the whole elementary cell defined

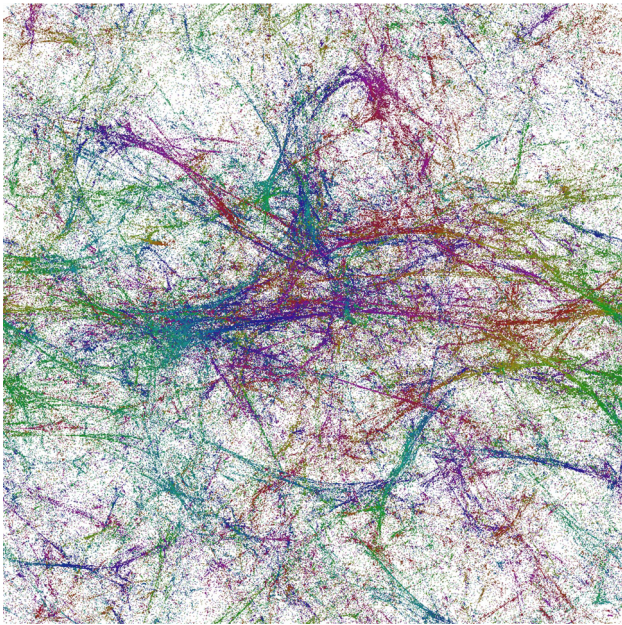


Fig. 1 The projection of four-volume on the xy -plane, as defined by (11) for a CDT configuration. Different colors correspond to different times t of the original t -foliation

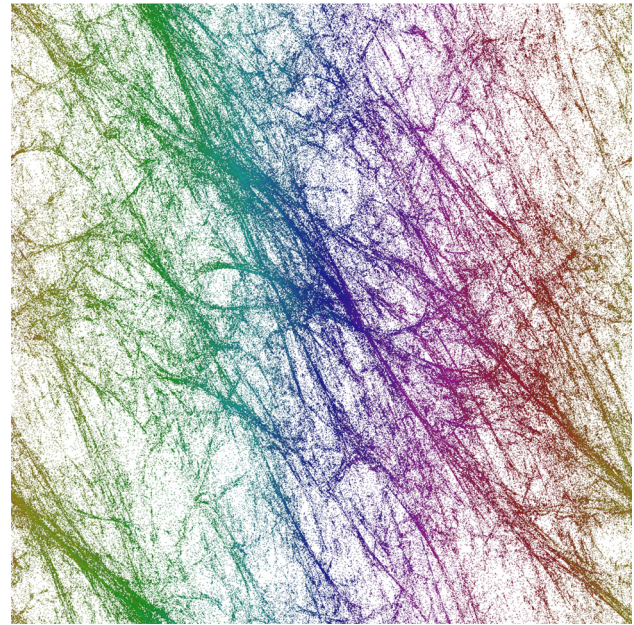


Fig. 2 The projection of four-volume on the tx -plane for a CDT configuration. There is a strong correlation between the original t -foliation (color) and new time coordinate ψ^t (horizontal axis)

by the boundary (10), and in this way we obtain a foliation in the direction σ . We may now use $\psi_i^\sigma = \psi_i^\sigma(0)$ as a coordinate in the σ -direction. The same construction can be repeated in all directions $\sigma \in \{x, y, z, t\}$ for any configuration obtained in the numerical simulations, and in this way every simplex i will be assigned a unique set of coordinates $\{\psi_i^x, \psi_i^y, \psi_i^z, \psi_i^t\}$, all in the range between 0 and 1. A solution to the Laplace equation has the property that the coordinates of each simplex are equal to the mean value of the coordinates of its neighbors (up to the shift of the field at the boundary), i.e., it preserves the triangulation structure. This is the required map from our configuration with a topology of T^4 to $(S^1)^4$ (which of course also has the topology T^4). Note that the coordinate ψ_i^t is not the same as the one coming from the original foliation of the CDT model. The parametrization defined above permits to analyze the distribution of the four-volume (the number of simplices) contained in hypercubic blocks with sizes $\{\Delta\psi_i^x, \Delta\psi_i^y, \Delta\psi_i^z, \Delta\psi_i^t\}$, which is equivalent to measuring the integrated $\sqrt{g(\psi)}$:

$$\Delta N(\psi) = \sqrt{g(\psi)} \prod_{\sigma} \Delta\psi^\sigma = N(\psi) \prod_{\sigma} \Delta\psi^\sigma. \quad (11)$$

We can measure the full four-dimensional distribution $N(\psi)$. In Figs. 1 and 2 we show projections of the volume density distribution of a typical configuration on two-dimensional parameter subspaces, the xy -plane and the tx -plane respectively, integrating over the remaining two directions. One observes a remarkable pattern of voids and filaments, which qualitatively looks quite similar to the pic-

tures of voids and filaments observed in our real Universe (see e.g. [16]; the plots can be found on the web [17, 18]). Using the new coordinates, we observe a pattern of volume concentrations in the spatial directions. The higher-density domains tend to attract each other, forming denser clouds, which survive in the imaginary time evolution in a quantum trajectory (see Fig. 2). There seems to appear a sequence of scales, characterizing a gradual condensation of gravitationally interacting “objects”, but one should remember that there is no matter in this system, only pure geometry, which behaves as if quantum fluctuations could produce massive interacting objects, somewhat analogous to dark matter. This indicates that quantum geometric fluctuations are highly correlated, an effect which could not be easily analyzed without introducing a global set of coordinates. Of course, we are talking about quantum objects of Planckian size, but if a more extended model exhibited inflation, one could imagine that aspects of these objects would be frozen when entering the horizon, like the standard Gaussian fluctuations in simple inflationary models, and then would re-enter the horizon at a later stage, after reheating, as classical densities.

4 Discussion

CDT presents us with a model of what we believe are generic fluctuations of geometry at the Planck scale. We hope that measurements of space-time correlations will allow us to determine “experimentally” (i.e., using Monte Carlo data) the

effective continuum action that governs our lattice model not only in time, but also in spatial directions. The construction of such an effective action will help us to understand if CDT is an UV-complete quantum field theory of gravity, as imagined in the so-called asymptotic safety scenario, or only an effective quantum theory of geometries.

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Declarations

Conflict of interest Not applicable.

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