Unifying dark matter and dark energy with non-canonical scalars

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Abstract Non-canonical scalar fields with the Lagrangian $\mathcal{L} = X^\alpha - V(\phi)$, possess the attractive property that the speed of sound, $c_s^2 = (2\alpha - 1)^{-1}$, can be exceedingly small for large values of α . This allows a non-canonical field to cluster and behave like warm/cold dark matter on small scales. We derive a general condition on the potential in order to facilitate the kinetic term X^α to play the role of dark matter, while the potential term $V(\phi)$ playing the role of dark energy at late times. We demonstrate that simple potentials including $V = V_0 \coth^2 \phi$ and a Starobinsky-type potential can unify dark matter and dark energy. *Cascading dark energy*, in which the potential cascades to lower values in a series of discrete steps, can also work as a unified model.

1 Introduction

A key feature of our universe is that 96% of its matter content is weakly interacting and non-baryonic. It is widely believed that this so-called dark sector consists of two distinct sub-components, the first of which, dark matter (DM), consists of a pressureless fluid which clusters, while the second, dark energy (DE), has large negative pressure and causes the universe to accelerate at late times.

Although numerous theoretical models have been advanced as to what may constitute dark matter, none so far has received unambiguous experimental support [1–5]. The same may also be said of dark energy. The simplest model of DE, the cosmological constant Λ , fits most observational data sets quite well [6]; see however [7, 8]. Yet the fine tuning problem associated with Λ and the cosmic coincidence issue, have motivated the development of dynamical dark energy (DDE) models in which the DE density and equation of state (EOS) evolve with time [9–16].

In view of the largely unknown nature of the dark sector several attempts have been made to describe it within a unified setting [17–35]. Perhaps the earliest prescription for a unified model of dark matter and dark energy was made in the context of the Chaplygin gas (CG) [17–19, 21–23]. CG possesses an equation of state which is pressureless ($p \simeq 0$) at early times and Λ -like ($p \simeq -\rho$) at late times. This led to the hope that CG may be able to describe both dark matter and dark energy through a unique Lagrangian. Unfortunately an analysis of density perturbations dashed these early hopes for unification [24]. In this paper we build on early attempts at unification and demonstrate that a compelling unified description of dark matter and dark energy can emerge from scalar fields with non-canonical kinetic terms.

2 Non-canonical scalar fields

The non-canonical scalar field Lagrangian [36–39]

$$\mathcal{L}(X, \phi) = X \left(\frac{X}{M^4} \right)^{\alpha-1} - V(\phi), \quad (1)$$

presents a simple generalization of the canonical scalar field Lagrangian

$$\mathcal{L}(X, \phi) = X - V(\phi), \quad X = \frac{1}{2} \dot{\phi}^2 \quad (2)$$

to which (1) reduces when $\alpha = 1$.

Two properties of non-canonical scalars make them attractive for the study of cosmology:

1. Their equation of motion

$$\ddot{\phi} + \frac{3H\dot{\phi}}{2\alpha - 1} + \left(\frac{V'(\phi)}{\alpha(2\alpha - 1)} \right) \left(\frac{2M^4}{\dot{\phi}^2} \right)^{\alpha-1} = 0, \quad (3)$$

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is of second order, as in the canonical case. Indeed, (3) reduces to the standard canonical form $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ when $\alpha = 1$.

2. The speed of sound [36]

$$c_s^2 = \frac{1}{2\alpha - 1} \quad (4)$$

can become quite small for large values of α since $c_s \rightarrow 0$ when $\alpha \gg 1$.

This latter property ensures that non-canonical scalars can, in principle, play the role of dark matter. In this paper we shall show that, for a suitable choice of the potential $V(\phi)$, non-canonical scalars can also unify dark matter with dark energy.

This paper works in the context of a spatially flat Friedmann-Robertson-Walker (FRW) universe for which the energy-momentum tensor has the form

$$T^\mu_\nu = \text{diag}(\rho_\phi, -p_\phi, -p_\phi, -p_\phi). \quad (5)$$

In the context of non-canonical scalars, the energy density, ρ_ϕ , and pressure, p_ϕ , can be written as

$$\begin{aligned} \rho_\phi &= (2\alpha - 1) X \left(\frac{X}{M^4} \right)^{\alpha-1} + V(\phi), \\ p_\phi &= X \left(\frac{X}{M^4} \right)^{\alpha-1} - V(\phi). \end{aligned} \quad (6)$$

It is easy to see that (6) reduces to the canonical form $\rho_\phi = X + V$, $p_\phi = X - V$ when $\alpha = 1$.

The Friedmann equation which is solved in association with (3) is

$$H^2 = \frac{8\pi G}{3} (\rho_\phi + \rho_r + \rho_b) \quad (7)$$

where ρ_r is the radiation term and ρ_b is the contribution from baryons. Note that we do not assume a separate contribution from dark matter or dark energy which are encoded in ρ_ϕ .

As shown in [33], for sufficiently flat potentials with $V' \simeq 0$ the third term in (3) can be neglected, leading to

$$\ddot{\phi} \simeq -\frac{3H\dot{\phi}}{2\alpha - 1} \Rightarrow \dot{\phi} \propto a^{-\frac{3}{2\alpha-1}}. \quad (8)$$

Substituting (8) in

$$\rho_X = (2\alpha - 1) X \left(\frac{X}{M^4} \right)^{\alpha-1}, \quad X = \frac{1}{2}\dot{\phi}^2 \quad (9)$$

one finds

$$\rho_X \propto a^{-\frac{6\alpha}{2\alpha-1}} \quad (10)$$

which reduces to $\rho_X \propto a^{-3}$ for $\alpha \gg 1$. Comparing (10) with $\rho_X \propto a^{-3(1+w_X)}$ one finds

$$w_X = \frac{1}{2\alpha - 1}, \quad (11)$$

so that $w_X \simeq 0$ for $\alpha \gg 1$. We therefore conclude that for flat potentials and large values of α , the kinetic term, ρ_X , plays the role of dark matter while the potential term, V , plays the role of dark energy in (6).

The above argument is based on the requirement that the third term is much smaller than the first two terms in Eq. (3). In a recent paper Li and Scherrer [34] have made the interesting observation that the potential need not be flat in order that (8) be satisfied. The analysis of Li and Scherrer suggests that the third term in (3) can be neglected under more general conditions than anticipated in [33] provided the potential is ‘sufficiently rapidly decaying’ [34]. Indeed it is easy to show that the requirement of the third term in (3) being much smaller than the second translates into the inequality

$$V' \ll \left(\frac{2\alpha}{2\alpha - 1} \right) \frac{3H\rho_X}{\dot{\phi}}, \quad (12)$$

which reduces to

$$V' \ll \frac{3H\rho_X}{\dot{\phi}} \Rightarrow \dot{V} \ll 3H\rho_X, \quad (13)$$

when $\alpha \gg 1$. Equation (13) can be recast as

$$\left| \frac{dV}{dz} \right| \ll \frac{3\rho_X}{1+z}. \quad (14)$$

Equations (13) and (14) inform us that the variation in V can be quite large at early times when both H and ρ_X are large. By contrast the same equations suggest that the potential should be quite flat at late times when H and ρ_X are small. This latter property ensures that $V(\phi)$ can play the role of DE at late times and drive cosmic acceleration. The inequalities in (13), (14) can be satisfied by a number of potentials some of which are discussed below.

We first consider the inverse-power-law (IPL) family of potentials [40]

$$V(\phi) = \frac{V_0}{(\phi/m_p)^p}, \quad p > 0. \quad (15)$$

Assuming that the background density falls off as

$$\rho_B \propto a^{-m} \quad (16)$$

(where $m = 4, 3$ for radiative and matter dominated epochs respectively), one can show that (8) is a late-time attractor provided the inequality

Fig. 1 This figure illustrates the evolution of the density in radiation (red curve), baryons (dotted green curve) and a non-canonical scalar field with the potential $V \propto \phi^{-2}$. The solid green curve shows the evolution of the kinetic term $\rho_X \propto X^\alpha$ which behaves like dark matter, $\rho_X \propto a^{-3}$. The blue curve shows the evolution of the potential $V(\phi)$. Note that V scales like the background fluid so that $V \propto a^{-4}$ during the radiative regime and $V \propto a^{-3}$ during matter domination ($\alpha = 10^8$ has been assumed)

$$p \geq \frac{12\alpha}{(2\alpha - 1)m - 6} \quad (17)$$

is satisfied [34]. For the large values $\alpha \gg 1$ which interest us, (17) reduces to

$$p \geq \frac{6}{m} \quad (18)$$

for which the late-time attractor is [34]

$$V \propto (\rho_B)^{\frac{p}{2}} \quad \text{and} \quad \rho_X \propto a^{-3}. \quad (19)$$

From (19) one finds that for $p > 2$ the potential falls off faster than the background density ρ_B . The case $p = 2$ is special since $V \propto \rho_B$, i.e. the potential scales exactly like the background density. This is illustrated in Fig. 1. In this case one finds (for $\alpha \gg 1$)

$$\frac{V(\phi)}{\rho_B} \simeq \frac{V_0}{\rho_{0B}} \left(\frac{H_0 m_p}{M^2} \right)^2 \quad (20)$$

where M is a free parameter in the non-canonical Lagrangian (1).

The above analysis suggests that the IPL potential $V \propto \phi^{-p}$, $p \geq 2$ cannot give rise to cosmic acceleration at late times. Thus while being able to account for dark matter (through ρ_X) this model is unable to provide a unified description of dark matter and dark energy.

However, as we show in the rest of this paper, a unified prescription for dark matter and dark energy is eas-

ily provided by any one of the following potentials¹: (i) $V(\phi) = V_0 \coth^2 \phi$, (ii) the Starobinsky-type potential given by $V(\phi) = V_0 (1 - e^{-\phi})^2$, (iii) the step-like potential $V(\phi) = A + B \tanh \beta \phi$. It is interesting that all of these potentials belong to the α -attractor family [41,42] in the canonical case.

3 Unified models of dark matter and dark energy

3.1 Dark Matter and Dark Energy from $V = V_0 \coth^2 \phi$

The potential [43]

$$V(\phi) = V_0 \coth^p \left(\frac{\phi}{m_p} \right), \quad p > 0$$

$$\equiv V_0 \left(\frac{1 + e^{-2\phi/m_p}}{1 - e^{-2\phi/m_p}} \right)^p \quad (21)$$

can provide a compelling description of dark matter and dark energy on account of its two asymptotes:

$$V(\phi) \simeq \frac{V_0}{(\phi/m_p)^p}, \quad \text{for } \phi \ll m_p \quad (22)$$

$$V(\phi) \simeq V_0, \quad \text{for } \phi \gg m_p. \quad (23)$$

As discussed in the previous section, the IPL asymptote (22) ensures that the kinetic term behaves like dark matter $\rho_X \propto a^{-3}$, while $V(\phi)$ scales like the background density (for $p = 2$) or faster (for $p > 2$); see Eq. (19).

The late-time asymptote (23) demonstrates that the potential flattens to a constant value at late times. This feature allows $V(\phi)$ to play the role of dark energy. Indeed, a detailed numerical analysis of the coth potential, summarized in Figs. 2 and 3, demonstrates that (21) with $p = 2$ can provide a successful unified description of dark matter and dark energy in the non-canonical setting. (This is also true for $p > 2$. However for shallower potentials with $p < 2$ the third term in (3) cannot be neglected. This implies that for such potentials ρ_X does not scale as a^{-3} and therefore cannot play the role of dark matter [34].)

For non-canonical models the equation of state can be determined from (6), namely

$$w_\phi = \frac{p_\phi}{\rho_\phi} = -1 + \left(\frac{2\alpha}{2\alpha - 1} \right) \left(\frac{\rho_X}{\rho_X + V(\phi)} \right), \quad (24)$$

which simplifies to

$$w_\phi = -1 + \frac{\rho_X}{\rho_X + V(\phi)}, \quad \text{for } \alpha \gg 1. \quad (25)$$

¹ Perhaps the simplest potential for unification is $V(\phi) = V_0$ which results in a Λ CDM cosmology [33].

Fig. 2 **a** The evolution of the density (in units of $\rho_{cr,0} = 3m_p^2 H_0^2$) is shown for radiation (red), baryons (dotted green), kinetic term $\rho_X \propto X^\alpha$ (solid green) and the potential $V(\phi)$ (solid blue) for $V(\phi) \propto \coth^2 \phi$. Note that dark matter is sourced by the kinetic term which *always* drops off as $\rho_X \propto a^{-3}$, regardless of the shape of the potential. DE is sourced by $V(\phi)$ which initially scales like the background fluid, $V \propto \rho_B$, before flattening to a constant value V_0 . The dotted and dashed black curves correspond to different initial values of $V(\phi)$ and indicate that a large range of initial conditions converge onto the scaling attractor (solid blue), yielding $\Omega_{0,DE} = 0.7$ at the present epoch. **b** Same as the top panel except that results for three different values of the non-canonical parameter M are shown. Note that dark matter (solid green), sourced by the kinetic term, always drops off as $\rho_X \propto a^{-3}$, regardless of the value of M . By contrast the value of $V(\phi)$ depends quite sensitively on M , see (20). In all cases $V(\phi)$ initially scales like the background fluid, $V \propto \rho_B$, before flattening to the constant value $V(\phi) \simeq V_0$. Late time expansion in this model resembles Λ CDM

For the potential (21) $\rho_X \propto a^{-3}$ whereas $V(\phi)$ approaches a constant value at late times. Consequently one gets

$$w_\phi(z=0) \simeq -1 + \frac{\Omega_{0X}}{\Omega_{0X} + \Omega_{0V}}, \quad (26)$$

where $\Omega_{0V} = V_0/\rho_{cr,0}$. Assuming $\Omega_{0V} = 0.7$, $\Omega_{0b} = 0.04$, one finds $\Omega_{0X} = 0.26$ so that

Fig. 3 The evolution of density parameters ($\Omega_i = \rho_i/\rho_{cr}$) is shown for radiation (red), baryons (dotted green), kinetic term $\rho_X \propto X^\alpha$ (solid green) and the potential $V(\phi)$ (solid blue) for $V(\phi) \propto \coth^2 \phi$. This model gives rise to a matter dominated epoch (sourced by the kinetic term ρ_X) when $z > 1$ and a DE dominated epoch, sourced by the potential $V(\phi)$, when $z < 1$

$$w_\phi(z=0) \simeq -0.7292. \quad (27)$$

Our results for the deceleration parameter $q = -\frac{\ddot{a}}{aH^2}$ and w_ϕ are shown in Fig. 4 for the IPL model (15) and the coth potential (21), both with $p = 2$.

The effective equation of state of the kinetic and potential components can be determined from

$$w_X = -\frac{\dot{\rho}_X}{3H\rho_X} - 1, \quad (28)$$

$$w_V = -\frac{\dot{V}}{3HV} - 1. \quad (29)$$

Substituting for ρ_X from (9) one finds

$$w_X = -1 - \frac{2\alpha\ddot{\phi}}{3H\dot{\phi}}. \quad (30)$$

which leads to

$$w_X = c_s^2 = (2\alpha - 1)^{-1} \quad (31)$$

after substitution for $3H\dot{\phi}$ from (8).

It is instructive to rewrite (29) as

$$1 + w_V = -\frac{\dot{V}}{3H\rho_X} \left(\frac{\rho_X}{V} \right). \quad (32)$$

As noted in (13) the inequality $\dot{V} \ll 3H\rho_X$ should be satisfied in order for ρ_X to behave like dark matter. Since the densities in dark matter and dark energy are expected to be comparable at late times, one finds $\frac{\rho_X}{V} \sim \mathcal{O}(1)$ at $z \leq 1$. Substituting these results in (32) one concludes that the EOS of DE is expected to approach $w_V \simeq -1$ at late times in

Fig. 4 The equation of state w_ϕ and the deceleration parameter q are shown in the top and bottom panels respectively for the potentials $V(\phi) = V_0 \coth^2 \left(\frac{\phi}{m_p} \right)$ (solid red) and $V(\phi) = V_0 \left(\frac{m_p}{\phi} \right)^2$ (dashed green). From **a** one finds that w_ϕ in both models remains pegged at $w_\phi \approx 0$ for $z \gtrsim 10$. For $z < 10$ the EOS remains unchanged in IPL but declines to negative values in the coth potential. This is substantiated by **b** which shows that the deceleration parameter in both models remains the same until $z \sim 10$. Thereafter q declines to negative values at late times for the coth potential (red curve), reflecting the late-time acceleration of the universe. For the IPL potential on the other hand (dashed green) the universe stays matter dominated at late times ($q \approx 0.5$) and does not accelerate

unified models of the dark sector. (One should also note that since $\frac{\rho_X}{V}$ can be fairly large at early times, w_V is not restricted to being close to -1 at $z \gg 1$.)

Figure 5 compares the behaviour of w_X and w_V in the two potentials: $\coth^2 \phi$ and ϕ^{-2} . One notices that $w_V \approx 1/3$ at early times in both potentials, which is a reflection of the scaling behaviour $V \propto \rho_B$ noted in (19). At late times w_V in the coth potential drops to negative values causing the universe to accelerate. For $V \propto \phi^{-2}$ on the other hand, w_V always tracks the dominant background fluid which results

Fig. 5 The evolution of the EOS of the kinetic term w_X (solid green curve) and the potential term w_V (solid blue curve) is shown in **a** for the IPL potential $V(\phi) = V_0 \left(\frac{m_p}{\phi} \right)^2$ and **b** for the coth potential $V(\phi) = V_0 \coth^2 \left(\frac{\phi}{m_p} \right)$. For the IPL potential, w_V scales like the background fluid and never becomes negative, indicating that the universe enters a prolonged matter dominated epoch at late times. By contrast, for the coth potential w_V behaves like a scaling solution only at early times ($w_V = w_B$) and drops to $w_V \approx -1$ at $z \sim 10^4$. For both potentials the EOS of the kinetic term stays pegged at $w_X = 0$, allowing it to play the role of dark matter

in $w_V = 0$ at late times. (Note that in this case the fluid which dominates at late times is the kinetic term, so that $V \propto \rho_X$.)

3.2 Dark matter and dark energy from a Starobinsky-type potential

A unified model of dark matter and dark energy can also arise from the potential [41,42,44]

$$V(\phi) = V_0 \left(1 - e^{-\lambda \frac{\phi}{m_p}} \right)^2, \quad \lambda > 0, \quad (33)$$

Fig. 6 This figure schematically illustrates the Starobinsky-type potential (33) with $\lambda = 1$. The main features of this potential are: the exponential wing for $\lambda|\phi| \gg m_p$ ($\phi < 0$), the flat wing for $\lambda\phi \gg m_p$, and a minimum at $\phi = 0$, near which $V \propto \phi^2$

which reduces to the Starobinsky potential in the Einstein frame [45] for $\lambda = \sqrt{\frac{2}{3}}$.

The potential in (33) is characterized by three asymptotic branches (see Fig. 6):

$$\text{Exponential branch: } V(\phi) \simeq V_0 e^{-2\lambda\phi/m_p}, \quad \phi < 0, \quad \lambda|\phi| \gg m_p, \quad (34)$$

$$\text{flat branch: } V(\phi) \simeq V_0, \quad \lambda\phi \gg m_p, \quad (35)$$

$$\text{minimum: } V(\phi) \simeq \frac{1}{2}\mu^2\phi^2, \quad \lambda|\phi| \ll m_p, \quad (36)$$

where

$$\mu^2 = \frac{2V_0\lambda^2}{m_p^2}. \quad (37)$$

Before discussing the unification of dark matter and dark energy in a Starobinsky-type potential we briefly explore the dynamics of the scalar field as it rolls down the exponential branch (34).

3.2.1 Motion along the exponential branch

The exponential branch has been extensively studied in the canonical case ($\alpha = 1$) for which the late time attractor is $w_\phi = w_B$ if $\lambda^2 > 3(1 + w_B)$. The situation radically changes for non-canonical scalars ($\alpha \neq 1$). As shown in [34], if the background density scales as $\rho_B \propto a^{-m}$, then for $(2\alpha - 1)m > 6$ the late time attractor is

$$\dot{\phi}^{2\alpha-1} \propto t^{-6/m} \Rightarrow \dot{\phi} \propto a^{-\frac{3}{2\alpha-1}}. \quad (38)$$

Fig. 7 The evolution of ϕ is shown for the exponential potential $V(\phi) = V_0 e^{-\frac{2\phi}{m_p}}$. Commencing at $z \sim 10^{15}$, different initial conditions (represented by dashed green and red curves) rapidly converge onto the attractor solution (41) (solid blue curve) which corresponds to $\phi \propto a^2$ during radiation domination and $\phi \propto a^{3/2}$ during matter domination. The rapid growth in ϕ is accompanied by a steep decline in $V(\phi)$

Since $\rho_X \propto \dot{\phi}^{2\alpha}$ one finds

$$\rho_X \propto a^{-\frac{6\alpha}{2\alpha-1}} \quad (39)$$

which reduces to

$$\rho_X \propto a^{-3} \quad \text{for } \alpha \gg 1. \quad (40)$$

We therefore find that, as in the IPL case, for large values of the non-canonical parameter α the density of the kinetic term scales just like pressureless (dark) matter. From (38) one also finds that $\dot{\phi} \sim \text{constant}$ when $\alpha \gg 1$. From this it is easy to show that for $\alpha \gg 1$ the amplitude of the scalar field grows as

$$\frac{\phi}{m_p} \propto a^{m/2}, \quad (41)$$

so that $\phi \propto a^2$ during the radiative regime and $\phi \propto a^{3/2}$ during matter domination. This behaviour is illustrated in Fig. 7. Substituting (41) in (34) one finds $V \propto \exp[-2\lambda\phi] \sim \exp[-2\lambda a^{m/2}]$, which implies an exponentially rapid decline in the value of the potential as the universe expands and $a(t)$ increases. One therefore concludes that like the IPL potential, an exponential potential too can never dominate the energy density of the universe and source cosmic acceleration.

3.2.2 Accelerating cosmology from a Starobinsky-type potential

In the context of the Starobinsky-type potential in (33), the rapid growth of ϕ in (41) enables the scalar field to pass

Fig. 8 **a** The density of radiation (red curve), baryons (dotted green curve) and the non-canonical scalar field are shown for the Starobinsky-type potential (33) with $\lambda = 1$. The solid green curve shows the evolution of the kinetic term $\rho_x \propto X^\alpha$ which behaves like dark matter throughout the history of the universe. The solid blue curve shows the evolution of the potential $V(\phi)$. Note that $V(\phi)$ rapidly decreases as ϕ rolls down the exponentially steep left wing (A) of the potential. This results in $V(\phi)$ falling to zero (at $\phi \simeq 0$) and then increasing to V_0 , as ϕ commences to roll along the flat right wing (B); see Fig. 6. Since $\rho_x \propto a^{-3}$, the late time expansion in this model resembles Λ CDM. **b** The equation of state of the kinetic term (w_x) and the potential term (w_v) are shown for the transition region when the scalar field rolls from the steep left wing (A) onto the flat right wing (B) of the potential (33). The change in slope of the potential at $\phi \simeq 0$ leads to a pole-like singularity in w_v . (Note that w_v as defined in (29) is an effective quantity hence its value can exceed unity.) By contrast the EOS of state of the kinetic term remains pegged at $w_x = 0$

from the steep left wing to the flat right wing of $V(\phi)$. In other words the scalar field rolls from A to B in Fig. 6. Since $V \simeq V_0$ on the flat right wing, cosmological expansion in this model mimicks Λ CDM at late times. This is illustrated in Fig. 8. (Note that $\rho_x \propto a^{-3}$ on both wings of the potential.)

Figure 8 shows the behaviour of w_x and w_v as ϕ moves under the influence of the potential (33). A key feature to be noted is that w_v encounters a pole as ϕ rolls from the steep

left wing to the flat right wing of $V(\phi)$ (i.e. from A to B in Fig. 6).

3.3 Cascading dark energy

A question occasionally directed towards dark energy is whether cosmic acceleration will continue forever (as in Λ CDM) or whether, like the earlier transient epochs (inflation, radiative/matter dominated) dark energy will also will be a fleeting phenomenon. In this section we investigate a transient model of DE in which the potential $V(\phi)$ is piecewise flat and resembles a staircase; see Fig. 9. Such a potential might mimick a model in which an initially large vacuum energy cascades to lower values through a series of waterfalls – discrete steps [46].

Locally the i -th step of this potential may be described by [33,47]

$$V(\phi) = A + B \tanh \beta \phi \quad (42)$$

where $A + B = V_i$ and $A - B = V_{i+1}$. If $V_{i+1} \simeq 10^{-47} \text{ GeV}^4$ then this potential could account for cosmic acceleration. (One might imagine yet another step at which $V_{i+2} < 0$. In this case the universe would stop expanding and begin to contract at some point in the future.) Motion along the staircase potential leads to a cascading model of dark energy. Remarkably, the inequality in (13) holds even as ϕ cascades from higher to lower values of V . This ensures that the kinetic term scales as $\rho_x \propto a^{-3}$ and behaves like dark matter while $V(\phi)$ behaves like dark energy, as shown in Fig. 9.

Note that the cascading DE model runs into trouble in the canonical context since the kinetic energy of a canonical scalar field moving along a flat potential declines as $\frac{1}{2} \dot{\phi}^2 \propto a^{-6}$. This puts the brakes on $\phi(t)$ which soon approaches its asymptotic value ϕ_* , resulting in inflation sourced by $V(\phi_*)$. By contrast $\dot{\phi} \sim \text{constant}$ in non-canonical models with $\alpha \gg 1$, see (38). This allows $\phi(t)$ to cross each successive step on the DE staircase in a finite amount of time², $\Delta t \simeq \frac{\Delta \phi}{\dot{\phi}}$, and drop to a lower value of $V(\phi)$; see the top panel (a) of Fig. 9.

4 Discussion

In this paper we have demonstrated that a scalar field with a non-canonical kinetic term can play the dual role of dark matter and dark energy. The key criterion which must be satisfied by unified models of the dark sector is (13). This inequality ensures that the third term in the equation of motion (3) is small and can be neglected, resulting in $\rho_x \propto a^{-3}$ and

² Note that a step potential in the non-canonical framework can behave like early dark energy and may help alleviating the H_0 tension, which we will explore in a future project.

Fig. 9 A schematic view of the cascading potential is shown in (a), while (b) shows the evolution in the density of the kinetic term $\rho_X \propto a^{-3}$ (solid green line) and the cascading potential $V(\phi)$ (blue). Also shown are the densities in radiation (red) and baryons (dotted green). (Note that $V_{\text{initial}} = 10^{25} \times V_{\text{final}}$.)

$w_X \simeq c_s \simeq 0$. In other words if (13) is satisfied the kinetic term behaves like dark matter with vanishing pressure and sound speed. Of equal importance is the fact that if (13) holds then Eq. (32) implies $w_V \simeq -1$ at late times. This ensures that the potential $V(\phi)$ can dominate over ρ_X and source cosmic acceleration at late times.

The following unified models of the dark sector have been discussed in this paper: (i) Models with exactly flat potentials $V' = 0$, $V = V_0$. As shown in [33] the entire expansion history of this model resembles Λ CDM. (ii) Successful unification of the dark sector can also arise from potentials which are steep at early times and flatten out at late times. Both the coth potential (21) and the Starobinsky-type potential (33) provide us with examples of this category. (iii) The step-like potential (42) also leads to unification³. In this case the

motion of ϕ resembles a series of waterfalls as $V(\phi)$ cascades to lower and lower values. It is interesting to note that for all of the above potentials⁴ the kinetic term scales as $\rho_X \propto a^{-3}$ throughout the expansion history of the universe, even as the shape of the potential continuously changes. This property allows the kinetic term to play the role of dark matter while the potential term $V(\phi)$ plays the role of dark energy and leads to cosmic acceleration at late times.

In our analysis, initial conditions $\{\phi_i, \dot{\phi}_i\}$ for the evolution of dark matter and dark energy in the universe are set in the following way. In the context of initial conditions for the kinetic term in Lagrangian (1), we note that the kinetic term evolves as dark matter at all cosmic epochs. Hence after fixing the values of the non-canonical parameters α and M , commencing at an initial epoch with a scale factor a_i , the initial value of the kinetic term is given by $\rho_X(X_i) \equiv (2\alpha - 1) X \left(\frac{X}{M^4} \right)^{\alpha-1} \Big|_{X_i} \simeq \rho_{0m} (a_0/a_i)^3$, which

determines the value of X_i (and hence $\dot{\phi}_i$). Similarly the initial field value ϕ_i is determined from the initial conditions for the potential term $V(\phi)$. Commencing the evolution at an initial epoch a_i for a given functional form of the potential, the initial field value ϕ_i must ensure that the criterion (13) is satisfied. Furthermore the value of the normalisation parameter V_0 appearing in the potential term is determined by demanding the dark energy density to be $\Omega_{0,DE} = 0.7$ at the present epoch. Note that for the coth potential (21) while the dark matter density always drops off as $\rho_X \propto a^{-3}$, the potential term, which is a scaling tracker, is sensitive to the value of M . In fact smaller the value of M , larger is the potential energy at early times, which is reflected in equation (20), and hence later is the approach to Λ -like behaviour, as can be seen from the lower panel of Fig. 2.

The following distinct features of the Lagrangian (1) differentiates our present model of unified dark matter and dark energy from several other models existing in the literature (for example [17–19, 25]).

- The speed of sound for the Lagrangian (1), given by Eq. (4) to be $c_s^2 = \frac{1}{2\alpha-1}$, is a constant and does not evolve with time. This is in contrast to the Chaplygin gas model [17] (as well as its generalisation [18, 19]) where the sound speed is small at early times, but increases and becomes larger with the increase in the scale factor at late times. Similarly in the purely kinetic k essence model discussed in [25], the speed of sound is dynamic, falling off as a^{-3} .
- The speed of sound in our model can become quite small for large values of α since $c_s \rightarrow 0$ when $\alpha \gg 1$. A con-

³ Note that the width of each step is restricted by the fact that the universe does not appear to accelerate prior to $z \sim 1$ (with the exclusion of an early inflationary epoch).

⁴ We note in passing that the asymptotically flat potential [41] $V(\phi) = V_0 \tanh^2 \phi$ also leads to a unified scenario of dark matter and dark energy although we do not discuss it in this paper.

stant and small speed of sound easily evades the arguments presented in [24]. Hence our model does not suffer from the issues related to the oscillations in the matter power spectrum at scales $k \leq 1 \text{ Mpc}^{-1}$ once α is chosen suitably.⁵ Since $c_s^2 > 0$ throughout the expansion history of the universe, our model does not exhibit large exponential blow up of the matter power spectrum at small scales. This is in contrast to the behaviour of matter power spectrum in the case of generalised Chaplygin gas models where both oscillations and large exponential growth of power spectrum are possible on small length scales depending upon the sign of the parameter α in the equation of state (see [24]).

Additionally, as discussed in [33], the small (but non-vanishing) speed of sound in non-canonical models suppresses gravitational clustering on small scales. Non-canonical models with $c_s \ll 1$ can therefore have a macroscopic Jeans length which might help in resolving the cusp-core and substructure problems which afflict the standard cold dark matter scenario. In this context the dark matter content of our model shares similarities with warm dark matter [48–52] and fuzzy cold dark matter [44, 53–55] both of which are known to possess a large Jeans scale.

- As demonstrated in Sect. 2, for $\alpha \gg 1$ with a suitable choice of potential $V(\phi)$ satisfying the general condition (13), the kinetic term of the Lagrangian (1) behaves like dark matter with $\rho_\chi \propto a^{-3}$ at all epochs. Furthermore, the kinetic and potential terms in (1) evolve independently implying that the dark matter and dark energy in our model behave like two separate cosmological fluids. This is in sharp contrast to many of the unified dark matter-dark energy models in the literature. For example in the case of the purely kinetic k essence [25], the dark matter like behaviour is a transient phenomenon which requires fine tuning of the model parameters. Another important feature of the purely kinetic k essence scenario is that even though the dark energy component behaves as a cosmological constant at late times, it exhibits a negligible sound speed $c_s \ll 1$ leading to suppression of the integrated Sachs–Wolfe (ISW) effect at large angular scales (see [25]). However our model does not suffer from this problem.

It is important to note that the potential term $V(\phi)$, which plays the role of dark energy in our model, has a higher value at early times both in the case of the coth potential (21) as well as the Starobinsky-type potential (33) and hence behaves like

an extra relativistic degree of freedom in the early universe. This could result in interesting observational signature in the CMB power spectrum. For example the step potential in our non-canonical framework can behave like early dark energy and may help alleviating the H_0 tension as mentioned before.

However more important and robust observational constraints on our model, in particular on the non-canonical parameter α , can be derived from the matter power spectrum at sub-galactic scales, possibly from the observations of Lyman α forest (and lensing of halo substructure). Initial work [56] in the non-linear regime based on a simplified non-perturbative analysis of the clustering of purely k essence fields as gravitationally bound static and spherically symmetric configurations indicated that the halo mass diverges in the radial limit $r \rightarrow \infty$ for large α values. However a full treatment of the perturbations in the scenario of our unified dark matter and dark energy models within the non-canonical framework, containing both kinetic and potential terms, would require a detailed analysis of separating the dark matter and dark energy perturbations and studying their growth and clustering properties both in the linear and non-linear regime, which is outside the scope of the present work and we wish to return to it in a future work.

Finally it is interesting to note that all of the potentials discussed in this paper belong to the α -attractor family of potentials [41, 42] and lead to interesting models of inflation, dark matter and dark energy [43, 44] in the canonical case.

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⁵ Note that in order to prevent oscillations in the matter power spectrum [24] on length scales larger than 1 Mpc, speed of sound must satisfy $c_s \leq 2.25 \times 10^{-4}$, implying that $\alpha \geq 10^7$ in our model. Hence we have considered $\alpha = 10^8$ in our paper.

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