Holographic RG flows and Janus solutions from matter-coupled $N = 4$ gauged supergravity

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Abstract We study an $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation of four-dimensional $N = 4$ gauged supergravity coupled to six vector multiplets with $SO(4) \times SO(4)$ gauge group and find a new class of holographic RG flows and supersymmetric Janus solutions. In this truncation, there is a unique $N = 4$ supersymmetric AdS_4 vacuum dual to an $N = 4$ SCFT in three dimensions. In the presence of the axion, the RG flows generally preserve $N = 2$ supersymmetry while the supersymmetry is enhanced to $N = 4$ for vanishing axion. We find solutions interpolating between the AdS_4 vacuum and singular geometries with different residual symmetries. We also show that all the singularities are physically acceptable within the framework of four-dimensional gauged supergravity. Accordingly, the solutions are holographically dual to RG flows from the $N = 4$ SCFT to a number of non-conformal phases in the IR. We also find $N = 4$ and $N = 2$ Janus solutions with $SO(4) \times SO(4)$ and $SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetries, respectively. The former is obtained from a truncation of all scalars from vector multiplets and can be regarded as a solution of pure $N = 4$ gauged supergravity. On the other hand, the latter is a genuine solution of the full matter-coupled theory. These solutions describe conformal interfaces in the $N = 4$ SCFT with $N = (4, 0)$ and $N = (2, 0)$ supersymmetries.

1 Introduction

Over the past twenty years, solutions of gauged supergravities in different space-time dimensions have provided a large number of useful holographic descriptions for strongly coupled dual field theories. Among various types of these solutions, holographic RG flows and Janus solutions are of particular interest since the original proposal of the AdS/CFT correspondence [1–3]. These explain deformations of the dual

conformal field theories (CFT) and the presence of conformal interfaces or defects within the parent CFTs. The most useful results along this direction arise when there is some amount of unbroken supersymmetry in which many properties at strong coupling are controllable.

Although there are AdS/CFT dualities in many space-time dimensions, AdS_4/CFT_3 correspondence attracts much attention due to the relevance in describing world-volume dynamics of M2-branes, the fundamental degrees of freedom in M-theory. In this case, four-dimensional gauged supergravities are very useful in obtaining holographic solutions of interest which in some cases can be uplifted to ten or eleven dimensions. A number of holographic RG flows and Janus configurations have previously been found in gauged supergravities with various gauge groups and different numbers of supersymmetries [4–18, 20–25], for similar solutions with other space-time dimensions, see [26–43] for an incomplete list.

In this paper, we will give a new class of holographic RG flows and supersymmetric Janus solutions within matter-coupled $N = 4$ gauged supergravity with $SO(4) \times SO(4)$ gauge group. This gauged supergravity can be constructed by coupling pure $N = 4$ gauged supergravity to $n \geq 6$ vector multiplets. We will consider the simplest case of $n = 6$ here. In this gauged supergravity, a number of supersymmetric AdS_4 vacua and RG flows between them have recently appeared in [17]. In that case, the truncation to $SO(3) \times SO(3)$ singlet scalars have been considered and solutions with only the dilaton and the axion in the gravity multiplet or solutions with vanishing axion are consistent with supersymmetry. In addition, all solutions given in [17] preserve $N = 4$ supersymmetry. In the present work, we will find more general solutions namely solutions with both the axion and vector multiplet scalars non-vanishing and breaking the original $N = 4$ supersymmetry to $N = 2$.

We will also find a new class of supersymmetric Janus solutions in $N = 4$ gauged supergravity which has not been

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considered in [17]. To the best of the author's knowledge, only Janus solutions found in [15, 16] are obtained within the framework of $N = 4$ gauged supergravity. All the solutions given in [15] are the so-called singular Janus in the sense that they interpolate between non-conformal phases rather than between conformal phases, so these are holographically expected to describe conformal interfaces within non-conformal field theories in three dimensions. On the other hand, regular Janus solutions given in [16] only involve the dilaton and axion from the gravity multiplet. There is also a singular Janus solution found in [16] that involve only scalars from vector multiplets. Therefore, we believe that the $N = 2$ Janus solution obtained in this paper is the first regular Janus solution in the context of matter-coupled $N = 4$ gauged supergravity that involves scalars from both gravity and vector multiplets.

The paper is organized as follows. In Sect. 2, we review the general structure of $N = 4$ gauged supergravity coupled to vector multiplets in the embedding tensor formalism and at the end focus on the case of six vector multiplets with $SO(4) \times SO(4)$ gauge group. The truncation to $SO(2) \times SO(2) \times SO(2)$ singlet scalars, AdS_4 vacua and general structure of relevant BPS equations in both RG flow and Janus solutions are also given in detail. This sets up the framework for finding solutions in subsequent sections. In Sects. 3 and 4, a number of $N = 4$ and $N = 2$ holographic RG flow and Janus solutions are given, respectively. We end the paper by giving some conclusions and comments on the results in Sect. 5. Some details on the relevant field equations can be found in the appendix.

2 Matter-coupled $N = 4$ gauged supergravity

In four dimensions, $N = 4$ supersymmetry allows two types of supermultiplets, the gravity and vector multiplets. The former contains the following field content

$$(e_{\mu}^{\hat{\mu}}, \psi_{\mu}^i, A_{\mu}^m, \chi^i, \tau) \quad (1)$$

which are given by the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ . The scalar consists of the dilaton ϕ and the axion χ and can be parametrized by $SL(2, \mathbb{R})/SO(2)$ coset. Indices $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$ are respectively space-time and tangent space indices while $m, n = 1, \dots, 6$ and $i, j = 1, 2, 3, 4$ indices describe fundamental representations of $SO(6)_R$ and $SU(4)_R$ R-symmetry.

The component fields in a vector multiplet are given by a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . In general, the gravity multiplet can couple to an arbitrary number n of vector multiplets. These vector multiplets will be labeled by indices $a, b = 1, \dots, n$, so all fields in the vector multiplets

will carry an additional index in the form

$$(A_{\mu}^a, \lambda^{ia}, \phi^{ma}). \quad (2)$$

Similar to the dilaton and the axion in the gravity multiplet, the $6n$ scalar fields ϕ^{ma} can be parametrized by $SO(6, n)/SO(6) \times SO(n)$ coset.

Fermionic fields and supersymmetry parameters transforming in fundamental representation of $SU(4)_R \sim SO(6)_R$ are subject to the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i \quad (3)$$

while those transforming in anti-fundamental representation of $SU(4)_R$ satisfy

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (4)$$

In general, all possible gaugings of a supergravity theory can be described by the embedding tensor. For the matter-coupled $N = 4$ supergravity, supersymmetry requires that the embedding tensor can have only two non-vanishing components denoted by $\xi^{\alpha M}$ and $f_{\alpha MNP}$. Indices $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n+6$ describe fundamental representations of the global $SL(2, \mathbb{R})$ and $SO(6, n)$ symmetries, respectively. The electric vector fields $A^{+M} = (A_{\mu}^m, A_{\mu}^a)$, appearing in the ungauged Lagrangian with the usual Yang-Mills kinetic term, together with their magnetic dual A^{-M} form a doublet under $SL(2, \mathbb{R})$ denoted by $A^{\alpha M}$. In addition, the embedding tensor needs to satisfy the quadratic constraint in order to define a consistent gauging. This ensures that the resulting gauge generators form a closed subalgebra of $SL(2, \mathbb{R}) \times SO(6, n)$. We will not discuss all these details here but refer to the original construction in [44], see also [45–47] for an earlier construction.

Consistent gauge groups can generally be embedded in both $SL(2, \mathbb{R})$ and $SO(6, n)$ factors, and the magnetic vector fields can also participate in the gauging. However, each magnetic vector field must be accompanied by an auxiliary two-form field in order to remove the extra degrees of freedom. The analysis of the existence of maximally supersymmetric AdS_4 vacua given in [48], see also [46, 47] for an earlier result, requires the gaugings to involve both electric and magnetic vector fields with the corresponding gauge groups embedded solely in $SO(6, n)$. This implies that both electric and magnetic components of $f_{\alpha MNP}$ must be non-vanishing and $\xi^{\alpha M} = 0$. We also note that the magnetic components f_{-MNP} are related to the duality phases first introduced in [46, 47].

In this paper, we are interested in supersymmetric solutions that are asymptotic to AdS_4 vacua and involve only the metric and scalar fields. Accordingly, from now on, we will set all the other fields including $\xi^{\alpha M}$ to zero. The scalar coset $SL(2, \mathbb{R})/SO(2) \times SO(6, n)/SO(6) \times SO(n)$ can be described by the coset representative $\mathcal{V}_{\alpha}$ and $\mathcal{V}_{\alpha}^A$, respec-

tively. With the definition

$$\tau = \chi + ie^\phi, \quad (5)$$

we will choose the explicit form of $\mathcal{V}_\alpha$ to be

$$\mathcal{V}_\alpha = e^{\frac{\phi}{2}} \begin{pmatrix} \chi + ie^\phi \\ 1 \end{pmatrix}. \quad (6)$$

The coset representative $\mathcal{V}_M^A$ transforms under the global $SO(6, n)$ and local $SO(6) \times SO(n)$ by left and right multiplications, respectively. This implies the splitting of the index $A = (m, a)$. We can then write the $SO(6, n)/SO(6) \times SO(n)$ coset representative as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \quad (7)$$

Furthermore, as an element of $SO(6, n)$, the matrix $\mathcal{V}_M^A$ also satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (8)$$

with $\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, -1, 1, \dots, 1)$ being the $SO(6, n)$ invariant tensor.

With $\xi^{\alpha M} = 0$ and only the metric and scalars non-vanishing, the bosonic Lagrangian can simply be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R + \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \quad (9)$$

where $e = \sqrt{-g}$ is the vielbein determinant. The scalar potential is given in terms of the scalar coset representative and the embedding tensor by

$$V = \frac{1}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} \right]. \quad (10)$$

At this point, we should note that the components $f_{\alpha MNP}$ of the embedding tensor include the gauge coupling constants.

The $SO(6) \times SO(n)$ invariant and symmetric matrix M_{MN} is defined by

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \quad (11)$$

with its inverse denoted by M^{MN} . The tensor M^{MNPQRS} is obtained from raising indices of M_{MNPQRS} given by

$$M_{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s. \quad (12)$$

Similarly, $M^{\alpha\beta}$ is the inverse of the symmetric 2×2 matrix $M_{\alpha\beta}$ defined by

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*). \quad (13)$$

Fermionic supersymmetry transformations are given by

$$\delta \psi_\mu^i = 2D_\mu \epsilon^i - \frac{2}{3} g A_1^{ij} \gamma_\mu \epsilon_j, \quad (14)$$

$$\delta \chi^i = -\epsilon^{\alpha\beta} \mathcal{V}_\alpha D_\mu \mathcal{V}_\beta \gamma^\mu \epsilon^i - \frac{4}{3} i g A_2^{ij} \epsilon_j, \quad (15)$$

$$\delta \lambda_a^i = 2i \mathcal{V}_a^M D_\mu \mathcal{V}_M^{ij} \gamma^\mu \epsilon_j - 2ig A_{2aj}^i \epsilon^j. \quad (16)$$

The fermion shift matrices are in turn defined by

$$\begin{aligned} A_1^{ij} &= \epsilon^{\alpha\beta} (\mathcal{V}_\alpha)^* \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_2^{ij} &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_{2ai}^j &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_a^M \mathcal{V}_{ik}^N \mathcal{V}_P^{jk} f_{\beta MN}^P \end{aligned} \quad (17)$$

where $\mathcal{V}_M^{ij}$ is defined in terms of the 't Hooft symbols G_m^{ij} and $\mathcal{V}_M^m$ as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m G_m^{ij}. \quad (18)$$

Similarly, the inverse elements $\mathcal{V}_{ij}^M$ can be written as

$$\mathcal{V}_{ij}^M = -\frac{1}{2} \mathcal{V}_m^M (G_m^{ij})^*. \quad (19)$$

G_m^{ij} satisfy the relations

$$G_{mij} = (G_m^{ij})^* = \frac{1}{2} \epsilon_{ijkl} G_m^{kl}. \quad (20)$$

An explicit representation of these matrices can be chosen as

$$\begin{aligned} G_1^{ij} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, & G_2^{ij} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\ G_3^{ij} &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, & G_4^{ij} &= \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, \\ G_5^{ij} &= \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, & G_6^{ij} &= \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (21)$$

Finally, we also note that the scalar potential can be written in terms of the fermion shift matrices A_1 and A_2 as

$$V = -\frac{1}{3} A_1^{ij} A_{1ij} + \frac{1}{9} A_2^{ij} A_{2ij} + \frac{1}{2} A_{2ai}^j A_{2aj}^i. \quad (22)$$

It is useful to remark here that upper and lower $i, j, \dots$ indices are related by complex conjugation.

2.1 $SO(4) \times SO(4)$ gauge group and

$SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation

We now consider the case of $n = 6$ vector multiplets with $SO(4) \times SO(4)$ gauge group as constructed in [49]. With a slightly different notation for various gauge coupling constants, the corresponding embedding tensor has the following non-vanishing components

$$\begin{aligned} f_{+\hat{m}\hat{n}\hat{p}} &= g_1 \epsilon_{\hat{m}\hat{n}\hat{p}}, & f_{+\hat{a}\hat{b}\hat{c}} &= \tilde{g}_1 \epsilon_{\hat{a}\hat{b}\hat{c}}, \\ f_{-\hat{m}\hat{n}\hat{p}} &= g_2 \epsilon_{\hat{m}\hat{n}\hat{p}}, & f_{-\hat{a}\hat{b}\hat{c}} &= \tilde{g}_2 \epsilon_{\hat{a}\hat{b}\hat{c}}, \end{aligned} \quad (23)$$

in which we have used the convention on the $SO(6, 6)$ fundamental index as $M = (m, a) = (\hat{m}, \tilde{m}, \hat{a}, \tilde{a})$ with $\hat{m} = 1, 2, 3$, $\tilde{m} = 4, 5, 6$, $\hat{a} = 7, 8, 9$ and $\tilde{a} = 10, 11, 12$. We also note that the two $SO(4)$ factors are electrically and magnetically embedded in $SO(6, 6)$ and will be denoted by $SO(4)_+ \times SO(4)_-$. In terms of the $SO(3)$ factors given by the embedding tensor in (23), we will write the gauge group as $SO(3)_+ \times SO(3)_- \times SO(3)_+ \times SO(3)_-$ with the first two factors embedded in the $SU(4)_R \sim SO(6)_R$.

To parametrize the $SO(6, 6)/SO(6) \times SO(6)$ coset representative, we first define $SO(6, 6)$ generators in the fundamental representation by

$$(t_{MN})_P^Q = 2\delta_{[M}^Q \eta_{N]P}. \quad (24)$$

We then identify the $SO(6, 6)$ non-compact generators as

$$Y_{ma} = t_{m,a+6}. \quad (25)$$

The truncation of this gauged supergravity to $SO(4)_{\text{diag}} \sim SO(3)_{\text{diag}} \times SO(3)_{\text{diag}}$ singlet scalars has already been studied in [17] in which a number of supersymmetric AdS_4 critical points and domain walls interpolating between them have been given.

In the present case, we are interested in another truncation to $SO(2) \times SO(2) \times SO(2) \times SO(2)$ invariant scalars. For later convenience, we also note that the gauge generators are given by

$$X_{\alpha M} = f_{\alpha M}^{NP} t_{NP}. \quad (26)$$

Each $SO(2)$ factor is embedded in the $SO(3)$ factor of $SO(3)_+ \times SO(3)_- \times SO(3)_+ \times SO(3)_- \sim SO(4)_+ \times SO(4)_-$ as $\mathbf{3} \rightarrow \mathbf{2} + \mathbf{1}$. To identify these singlets, we first consider the transformation of the scalars under the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ subgroup. The 36 scalars transform under the compact subgroup $SO(6) \times SO(6)$ as (6, 6). The fundamental representation 6 of $SO(6)$ is in turn decomposed as $(\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$ under $SO(3) \times SO(3)$. With the aforementioned embedding of $SO(2)$ in $SO(3)$, we find the transformation of all scalars under $SO(2) \times SO(2) \times SO(2) \times SO(2)$

$$\begin{aligned} (\mathbf{6}, \mathbf{6}) &\rightarrow (\mathbf{2} + \mathbf{1}, \mathbf{2} + \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{2} + \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2} + \mathbf{1}) \\ &+ (\mathbf{1}, \mathbf{2} + \mathbf{1}, \mathbf{2} + \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{2} + \mathbf{1}, \mathbf{2} + \mathbf{1}) \end{aligned} \quad (27)$$

from which we can easily see that there are four singlets corresponding to the representation $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})$. With the gauge generators obtained from the embedding tensor given in (23), we can choose the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ generators to be X_{+3} , X_{-6} , X_{+9} and X_{-12} . We then identify non-compact generators corresponding to these singlets as Y_{33} , Y_{36} , Y_{63} and Y_{66} in term of which the coset representative can be written as

$$\mathcal{V} = e^{\phi_1 Y_{33}} e^{\phi_2 Y_{36}} e^{\phi_3 Y_{63}} e^{\phi_4 Y_{66}}. \quad (28)$$

Together with the dilaton and axion from the gravity multiplet which are $SO(4) \times SO(4)$ singlets, there are six scalars in the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ sector.

By a straightforward computation, we find a very simple scalar potential

$$V = -\frac{1}{2}e^{-\phi}(g_1^2 + e^{2\phi}g_2^2 + g_2^2\chi^2) - 2g_1g_2 \cosh \phi_1 \cosh \phi_2 \cosh \phi_3 \cosh \phi_4. \quad (29)$$

It can be readily verified that this potential admits a unique AdS_4 critical point at

$$\phi = \ln \left[\frac{g_1}{g_2} \right] \quad \text{and} \quad \phi_1 = \phi_2 = \phi_3 = \phi_4 = \chi = 0. \quad (30)$$

The cosmological constant and AdS_4 radius are given by

$$V_0 = -3g_1g_2 \quad \text{and} \quad L = \sqrt{-\frac{3}{V_0}} = \frac{1}{\sqrt{g_1g_2}}. \quad (31)$$

This is the maximally $N = 4$ supersymmetric AdS_4 vacuum preserving the full $SO(4) \times SO(4)$ gauge symmetry. By shifting the dilaton ϕ by a constant, we can choose $g_2 = g_1 = g$ which make the dilaton vanish at the AdS_4 critical point. It is useful to note that all scalars have mass $m^2L^2 = -2$ at this vacuum and are dual to operators of dimensions $\Delta = 1, 2$ in the dual $N = 4$ SCFT, see more detail in [17].

2.2 Superpotential and BPS equations

We are interested in two types of supersymmetric solutions in the forms of holographic RG flows and Janus configurations. The former is described by the standard (flat) domain wall with the metric

$$ds^2 = e^{2A(r)} dx_{1,2}^2 + dr^2 \quad (32)$$

while the latter takes the form of a curved domain wall with an AdS_3 -sliced world-volume corresponding to the metric

$$ds^2 = e^{2A(r)} (e^{\frac{2\xi}{\ell}} dx_{1,1}^2 + d\xi^2) + dr^2. \quad (33)$$

It should be noted that the Janus ansatz (33) is obtained from the domain wall metric (32) by replacing the three-dimensional Minkowski metric $dx_{1,2}^2$ by the metric on AdS_3 with radius ℓ . The flat domain wall is in turn recovered from the Janus metric (33) in the limit $\ell \rightarrow \infty$.

To preserve the isometry of $dx_{1,2}^2$ or the AdS_3 metric, scalar fields can depend only on the radial coordinate r . In the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation, the kinetic term for all the six scalars $\Phi^r = (\phi, \chi, \phi_1, \phi_2, \phi_3, \phi_4)$, with $r, s = 1, 2, \dots, 6$, takes the form

$$\begin{aligned} \mathcal{L}_{\text{kin}} &= \frac{1}{2} G_{rs} \Phi^{r'} \Phi^{s'} \\ &= -\frac{1}{4} (\phi'^2 + e^{-2\phi} \chi'^2) - \frac{1}{16} [6 + \cosh 2(\phi_2 - \phi_3) \\ &\quad + \cosh 2(\phi_2 + \phi_3) + 2 \cosh 2\phi_4] \end{aligned}$$

$$\begin{aligned}
& \times (\cosh 2\phi_2 \cosh 2\phi_3 - 1)] \phi_1'^2 \\
& - \cosh \phi_2 \cosh \phi_4 \sinh \phi_3 \sinh \phi_4 \phi_1' \phi_2' \\
& - \cosh \phi_3 \cosh \phi_4 \sinh \phi_2 \sinh \phi_4 \phi_1' \phi_3' \\
& + \sinh \phi_2 \sinh \phi_3 \phi_1' \phi_4' - \frac{1}{2} \cosh^2 \phi_4 \phi_2'^2 \\
& - \frac{1}{2} \cosh^2 \phi_4 \phi_3'^2 - \frac{1}{2} \phi_4'^2
\end{aligned} \quad (34)$$

in which we have introduced a symmetric matrix G_{rs} . Throughout the paper, we will denote the r -derivative by $'$.

We now analyze the BPS equations obtained by setting fermionic supersymmetry transformations to zero. We use Majorana representation for space-time gamma matrices with all $\gamma^{\hat{\mu}}$ real and γ_5 purely imaginary. This implies that left and right chiralities of fermions are simply related to each other by complex conjugation. To proceed, we first note that for both types of solutions, the variations $\delta\chi^i$ and $\delta\lambda_a^i$ contain $\gamma^{\hat{r}}$ matrix due to the r -dependence of scalar fields. We then impose the following projector

$$\gamma^{\hat{r}} \epsilon^i = e^{i\Lambda} \epsilon_i \quad (35)$$

with an r -dependent phase Λ . This projector relates the two chiralities of ϵ^i and breaks half of the supersymmetry.

To complete the analysis, we consider the variation of the gravitini $\delta\psi_\mu^i$. With the coset representative (28), we obtain the A_1^{ij} tensor of the form

$$A_1^{ij} = \text{diag}(\alpha_+, \alpha_-, \alpha_-, \alpha_+) \quad (36)$$

with

$$\begin{aligned}
\alpha_\pm = \frac{3}{4} e^{-\frac{\phi}{2}} & [(g_2 e^\phi \cosh \phi_3 \pm i g_1 \sinh \phi_1 \sinh \phi_3) \cosh \phi_4 \\
& + g_1 \cosh \phi_1 (\cosh \phi_2 \pm i \sinh \phi_2 \sinh \phi_4) \\
& + i g_2 \chi \cosh \phi_3 \cosh \phi_4].
\end{aligned} \quad (37)$$

For $\chi = 0$, α_+ and α_- are complex conjugate of each other. The eigenvalue of A_1^{ij} tensor corresponding to unbroken supersymmetry will give a superpotential in term of which the scalar potential can be written. From the structure of A_1^{ij} given above, we find that in general, solutions within the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation will preserve at most $N = 2$ supersymmetry corresponding to $\epsilon^{1,4}$ or $\epsilon^{2,3}$ depending on which eigenvalue among α_+ and α_- is chosen to be the superpotential.

By defining the superpotential $\mathcal{W}$ as

$$\mathcal{W} = \frac{2}{3} \alpha, \quad (38)$$

for α being α_+ or α_- , and taking into account the projector (35), we can write the variations $\delta\psi_\mu^i$ for $\mu = 0, 1, 2$ in the RG flow case as

$$A' e^{i\Lambda} - \mathcal{W} = 0. \quad (39)$$

This condition is satisfied along $\epsilon^{1,4}$ and $\epsilon^{2,3}$ for $\mathcal{W} = \mathcal{W}_\pm = \frac{2}{3} \alpha_\pm$, respectively. In the following analysis, we will consider only $\mathcal{W} = \mathcal{W}_+$. The final results show that the other choice $\mathcal{W} = \mathcal{W}_-$ is related to this by a sign change in χ , so the two options are equivalent. Equation (39) implies that

$$A' = \pm W = \pm |\mathcal{W}| \quad \text{and} \quad e^{i\Lambda} = \pm \frac{\mathcal{W}}{W}. \quad (40)$$

Using the explicit phase factor $e^{i\Lambda}$ in term of $\mathcal{W}$ in $\delta\chi^i$ and $\delta\lambda_a^i$ equations, we obtain the BPS equations for scalar fields. In what follows, we will choose the upper sign choice in order to relate the AdS_4 vacuum to the limit $r \rightarrow \infty$.

Finally, the variation $\delta\psi_r^i$ leads to

$$2\epsilon^{\hat{i}'} - \mathcal{W} \gamma_{\hat{r}} \epsilon_{\hat{i}} = 0, \quad \hat{i} = 1, 4 \quad (41)$$

which, by the projector (35) and Eq. (39), gives the Killing spinors of the form

$$\epsilon^{\hat{i}} = e^{\frac{A}{2}} \epsilon_0^{\hat{i}} \quad (42)$$

for constant spinors $\epsilon_0^{\hat{i}}$ satisfying (35).

For Janus solutions, we can perform a similar analysis with some modifications. The variations $\delta\psi_{0,1}^{\hat{i}}$ and $\delta\psi_\xi^{\hat{i}}$ lead to the following conditions

$$\left(A' \gamma^{\hat{r}} + \frac{e^{-A}}{\ell} \gamma^{\hat{\xi}} \right) \epsilon^{\hat{i}} - \mathcal{W} \epsilon_{\hat{i}} = 0, \quad (43)$$

$$2\partial_\xi \epsilon^{\hat{i}} + A' e^A \gamma^{\hat{\xi}} \gamma^{\hat{r}} \epsilon^{\hat{i}} - e^A \mathcal{W} \gamma^{\hat{\xi}} \epsilon_{\hat{i}} = 0. \quad (44)$$

The analysis closely follows that given in [19] to which we refer for more detail.

Equation (43) implies the integrability condition

$$A'^2 + \frac{e^{-2A}}{\ell^2} = W^2. \quad (45)$$

Using Eq. (43) in (44), we find

$$2\partial_\xi \epsilon^{\hat{i}} = \frac{1}{\ell} \epsilon^{\hat{i}} \quad (46)$$

which gives

$$\epsilon^{\hat{i}} = e^{\frac{\xi}{2\ell}} \tilde{\epsilon}^{\hat{i}} \quad (47)$$

with $\tilde{\epsilon}^{\hat{i}}$ being ξ -independent spinors.

It should be noted that condition (43) takes the form of a projector on $\epsilon^{\hat{i}}$ which must be compatible with the projector (35) used in the scalar flow equations. Following [19], we impose the projector

$$\gamma^{\hat{\xi}} \epsilon^{\hat{i}} = i\kappa e^{i\Lambda} \epsilon_{\hat{i}} \quad (48)$$

with $\kappa^2 = 1$. Using Eqs. (35) and (48) in (43), we arrive at

$$\left(A' + \frac{i\kappa}{\ell} e^{-A} \right) e^{i\Lambda} = \mathcal{W} \quad (49)$$

which can be used to determine the phase $e^{i\Lambda}$ in terms of $\mathcal{W}$ and A . We note again that in the limit $\ell \rightarrow \infty$, the phase factor reduces to that given in (40).

Finally, with all the previous results, the condition $\delta\psi_r^{\hat{i}} = 0$ gives

$$2\partial_r \epsilon^{\hat{i}} = A' \epsilon^{\hat{i}} + \frac{i\kappa}{\ell} e^{-A} \epsilon^{\hat{i}}. \quad (50)$$

By redefining the Killing spinors with the phase factor $e^{\frac{i\Lambda}{2}}$, this condition implies the following form of the Killing spinors

$$\epsilon^{\hat{i}} = e^{\frac{A}{2} + \frac{r}{2\ell} + \frac{i\Lambda}{2}} \epsilon_0^{\hat{i}} \quad (51)$$

in which $\epsilon_0^{\hat{i}}$ can include an r -dependent phase and satisfy the $\gamma^{\hat{r}}$ and $\gamma^{\hat{z}}$ projectors without the phase $e^{i\Lambda}$

$$\gamma^{\hat{r}} \epsilon_0^{\hat{i}} = \epsilon_{0\hat{i}} \quad \text{and} \quad \gamma^{\hat{z}} \epsilon_0^{\hat{i}} = i\kappa \epsilon_{0\hat{i}}. \quad (52)$$

We also note that the constant $\kappa = \pm 1$ is related to the chirality of the Killing spinors on the two-dimensional interface dual to the AdS_3 slice.

With all these results, we will look for supersymmetric RG flow and Janus solutions within the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation of the $SO(4) \times SO(4)$ $N = 4$ gauged supergravity.

3 Holographic RG flows

We are now in a position to present an explicit form of the BPS equations given in the previous section and look for possible solutions. In this section, we consider RG flow solutions. Since there is only one supersymmetric AdS_4 critical point dual to a single conformal fixed point, the solutions will describe RG flows from the dual $N = 4$ SCFT to non-conformal phases in the IR. This type of solutions in the case of pure $N = 4$ gauged supergravity, with all scalars but the dilaton and axion vanishing, has already been studied in [17]. In this paper, we will find solutions in the matter-coupled $N = 4$ gauged supergravity.

As previously mentioned, the flow equations for scalars can be obtained by using the projector (35) and the phase $e^{i\Lambda}$ given in (40). Within the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation, the variation $\delta\chi^i$ reduces to a single complex equation which can be straightforwardly solved for ϕ' and χ' . However, it turns out that consistency of the equations from the variation $\delta\lambda_a^i$ requires some scalars to vanish. This is similar to the $SO(3) \times SO(3)$ truncation studied in [17] in which the BPS equations from $\delta\lambda_a^i$ imply $\chi = 0$, and non-vanishing χ is possible only when all scalars from the vector multiplets are set to zero. In that case, the axion and vector multiplet scalars cannot be turned on simultaneously.

In the present case, the constraint is somewhat weaker. We find that a consistent set of BPS equations that are compatible with the second-order field equations can be obtained only for two of the scalars ϕ_i , $i = 1, 2, 3, 4$ vanishing. This leads to $N = 2$ supersymmetric solutions. In addition, if the axion χ vanishes, supersymmetry is enhanced to $N = 4$. As expected, truncating out all scalars from the vector multiplets also leads to $N = 4$ supersymmetry. We then see that in the presence of both axion and vector multiplet scalars, the solutions break some supersymmetry. We also point out that this is not possible in the $SO(3) \times SO(3)$ truncation considered in [17] since in that case the A_1^{ij} tensor is proportional to the identity matrix. Therefore, partial supersymmetry breaking is not possible there.

3.1 $N = 4$ holographic RG flow with

$SO(2) \times SO(3) \times SO(2) \times SO(2)$ symmetry

We first consider $N = 4$ solutions with $\chi = 0$ and $SO(2) \times SO(3) \times SO(2) \times SO(2)$ symmetry. In this case, the $SO(4)$ subgroup of the $SO(6)$ R-symmetry is broken to $SO(2) \times SO(3)$ while the remaining $SO(2) \times SO(2)$ factor is a subgroup of $SO(3) \times SO(3) \sim SO(4)$ symmetry of the vector multiplets. There are two possibilities namely setting $\phi_1 = \phi_2 = 0$ or $\phi_3 = \phi_4 = 0$. The two choices are related to each other by interchanging e^ϕ with $e^{-\phi}$ factors. Therefore, we will consider only the solution with $\phi_3 = \phi_4 = 0$.

The superpotential is real and given by

$$W = W_+ = W_- = \frac{1}{2}(g_2 e^{\frac{\phi}{2}} + g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 \cosh \phi_2). \quad (53)$$

The resulting BPS equations read

$$\phi' = -4 \frac{\partial W}{\partial \phi} = g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 \cosh \phi_2 - g_2 e^{\frac{\phi}{2}}, \quad (54)$$

$$\phi'_1 = -2 \operatorname{sech}^2 \phi_2 \frac{\partial W}{\partial \phi_1} = -g_1 e^{-\frac{\phi}{2}} \operatorname{sech} \phi_2 \sinh \phi_1, \quad (55)$$

$$\phi'_2 = -2 \frac{\partial W}{\partial \phi_2} = -g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 \sinh \phi_2, \quad (56)$$

$$A' = W = \frac{1}{2}(g_2 e^{\frac{\phi}{2}} + g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 \cosh \phi_2). \quad (57)$$

To find the solution, we first take the following combination

$$\frac{d\phi_1}{d\phi_2} = \operatorname{csch} \phi_2 \operatorname{sech} \phi_2 \tanh \phi_1 \quad (58)$$

with the solution

$$\sinh \phi_1 = \tan \varphi_0 \tanh \phi_2 \quad (59)$$

for a constant φ_0 . Similarly, taking the combination between ϕ' and ϕ'_2 equations together with this result leads to

$$\frac{d\phi}{d\phi_2} = \frac{g_2 e^{\frac{\phi}{2}} \operatorname{csch} \phi_2}{g_1 \sqrt{1 + \tan^2 \varphi_0 \tanh^2 \phi_2}} - \coth \phi_2. \quad (60)$$

The solution to this equation is given by

$$\phi = \ln \left[\frac{2\sqrt{2}g_1 \cos^2 \varphi_0 \operatorname{csch} \phi_2 (\cos \varphi_0 \operatorname{csch} \phi_2 \sqrt{\cos 2\varphi_0 + \cosh 2\phi_2} - C)}{g_2 [1 - 2C^2 + \cos 2\varphi_0 + 2 \cos^2 \varphi_0 (\coth^2 \phi_2 + \cos 2\varphi_0 \operatorname{csch}^2 \phi_2)]} \right]. \quad (61)$$

Using these results and redefining the radial coordinate to ρ given by $\frac{d\rho}{dr} = e^{-\frac{\phi}{2}}$, we obtain the solution for $\phi_2(\rho)$ of the form

$$\cosh 2\phi_2 = 2 \cos^2 \varphi_0 \tanh [g_1 \sec \varphi_0 (\rho - \rho_0)] - \cos 2\varphi_0 \quad (62)$$

with an integration constant ρ_0 .

Finally, taking a linear combination $2A' + \phi'$, we find

$$2 \frac{dA}{d\phi_2} + \frac{d\phi}{d\phi_2} = - \frac{2(e^{2\phi_2} + 1)}{e^{2\phi_2} - 1} \quad (63)$$

with the solution

$$A = \phi_2 - \frac{1}{2}\phi - \ln(1 - e^{2\phi_2}). \quad (64)$$

We have neglected an additive integration constant in A since this can be removed by rescaling coordinates on $dx_{1,2}^2$.

We now look at asymptotic behaviors of the solution. For convenience, we first set $g_2 = g_1$ to bring the AdS_4 critical point to the origin of the scalar manifold. As $r \rightarrow \infty$, we find

$$\phi \sim \phi_1 \sim \phi_2 \sim e^{-g_1 r} \sim e^{-\frac{r}{L}}, \quad A \sim g_1 r \sim \frac{r}{L}. \quad (65)$$

This gives the supersymmetric AdS_4 vacuum and indicates that all scalars are dual to relevant operators of dimensions $\Delta = 1, 2$.

As usual for a flow to non-conformal phases, the solution is singular for a finite value of ρ . We find that, for $C < 0$, the above solution is singular at $\rho = \rho_0 + \rho_*$ with

$$\rho_* = \frac{1}{g_1} \coth^{-1} \left[\sec \varphi_0 \sqrt{\frac{\cos 2\varphi_0 + \cosh \left(2 \coth^{-1} \sqrt{1 - \sec^2 \varphi_0 + \frac{1}{2} C^2 \sec^4 \varphi_0} \right)}{2}} \right]. \quad (66)$$

Near the singularity, we find

$$\begin{aligned} \phi_2 &= \phi_* = \coth^{-1} \sqrt{1 - \sec^2 \varphi_0 + \frac{1}{2} C^2 \sec^4 \varphi_0}, \\ \phi_1 &= \sinh^{-1} (\tan \varphi_0 \tanh \phi_*), \quad \phi \sim -\ln(\phi_2 - \phi_*), \\ A &\sim -\frac{\phi}{2} \sim \frac{1}{2} \ln(\phi_2 - \phi_*) \end{aligned} \quad (67)$$

and

$$V \sim -g_2^2 e^\phi \rightarrow -\infty. \quad (68)$$

We see that near the singularity both ϕ_1 and ϕ_2 are constant, and the singularity is physical by the criterion given in [50]. Therefore, the solution describes an RG flow from

$N = 4$ SCFT to a non-conformal phase in the IR with the $N = 4$ superconformal symmetry broken to $N = 4$ Poincaré supersymmetry in three dimensions. The $SO(4) \times SO(4)$ symmetry is also broken to $SO(2) \times SO(3) \times SO(2) \times SO(2)$ along the flows.

3.2 $N = 4$ holographic RG flow with

$SO(2) \times SO(2) \times SO(2) \times SO(2)$ symmetry

Unlike the previous case, if we set $\phi_2 = \phi_3 = 0$ or $\phi_1 = \phi_4 = 0$ along with $\chi = 0$, we obtain $N = 4$ solutions with only $SO(2) \times SO(2) \times SO(2) \times SO(2)$ symmetry. The two choices give rise to equivalent results with scalars (ϕ_1, ϕ_4) and (ϕ_2, ϕ_3) interchanged. We will consider only the $\phi_2 = \phi_3 = 0$ case for definiteness.

Setting $\chi = \phi_2 = \phi_3 = 0$, we again find a real superpotential

$$\mathcal{W} = \frac{1}{2} (g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 + g_2 e^{\frac{\phi}{2}} \cosh \phi_4). \quad (69)$$

By repeating the same procedure as in the previous case, we find the following BPS equations

$$A' = W = \frac{1}{2} (g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 + g_2 e^{\frac{\phi}{2}} \cosh \phi_4), \quad (70)$$

$$\phi' = -\frac{\partial W}{\partial \phi} = g_1 e^{-\frac{\phi}{2}} \cosh \phi_1 - g_2 e^{\frac{\phi}{2}} \cosh \phi_4, \quad (71)$$

$$\phi_1' = -2 \frac{\partial W}{\partial \phi_1} = -g_1 e^{-\frac{\phi}{2}} \sinh \phi_1, \quad (72)$$

$$\phi_4' = -2 \frac{\partial W}{\partial \phi_4} = -g_2 e^{\frac{\phi}{2}} \sinh \phi_4. \quad (73)$$

Unlike the previous case, solving these equations is slightly more complicated due to the different dilaton prefactors in ϕ_1' and ϕ_4' equations. However, with some effort, we can find an analytic solution to these equations.

We first treat ϕ_4 as an independent variable and take a combination

$$\frac{d\phi_1}{d\phi_4} = \frac{g_1}{g_2} e^{-\phi} \operatorname{csch} \phi_4 \sinh \phi_1. \quad (74)$$

By changing to a new variable φ defined by

$$\phi_1 = \ln \sinh \frac{\varphi}{2} - \ln \cosh \frac{\varphi}{2}, \quad (75)$$

we can rewrite the above equation as

$$\frac{d\varphi}{d\phi_4} = \frac{g_1}{g_2} e^{-\phi} \operatorname{csch} \phi_4. \quad (76)$$

We then combine ϕ' and ϕ_4' equations with the $e^{-\phi}$ factor obtained from (76) and arrive at

$$\frac{d\phi}{d\phi_4} = \coth \phi_4 + \coth \varphi \frac{d\varphi}{d\phi_4} \quad (77)$$

with the solution

$$\phi = \phi_0 + \ln \sinh \phi_4 + \ln \sinh \varphi. \quad (78)$$

We can now use this solution in (76) and find the solution

$$\cosh \varphi = \frac{g_1}{g_2} e^{-\phi_0} \coth \phi_4. \quad (79)$$

Taking a linear combination $2A' - \phi'$ gives

$$2 \frac{dA}{d\phi_4} - \frac{d\phi}{d\phi_4} = -2 \coth \phi_4 \quad (80)$$

with the solution

$$A = \frac{1}{2} \phi - \ln \sinh \phi_4. \quad (81)$$

Finally, changing r to a new coordinate ρ given by $\frac{d\rho}{dr} = e^{\frac{\phi}{2}}$, we can readily find $\phi_4(\rho)$ solution

$$\phi_4 = \ln(1 + e^{-g_2(\rho - \rho_0)}) - \ln(1 - e^{-g_2(\rho - \rho_0)}). \quad (82)$$

As in the previous case, for $r \rightarrow \infty$, we find

$$\phi \sim \phi_1 \sim \phi_4 \sim e^{-g_1 r} \sim e^{-\frac{r}{L}} \quad (83)$$

in which we have taken $g_2 = g_1$ for convenience. This is an asymptotic AdS_4 geometry.

As $\rho \rightarrow \rho_0$, the solution is singular with

$$\begin{aligned} \phi_1 &\sim \text{constant}, \quad \phi_4 \sim \ln(\rho - \rho_0), \\ \phi &\sim -\ln(\rho - \rho_0), \quad A \sim \frac{1}{2} \ln(\rho - \rho_0). \end{aligned} \quad (84)$$

Unlike the previous case, the scalars from vector multiplets are not all constant near the singularity. Using the scalar potential given in (29), we find that, as $\rho \rightarrow \rho_0$,

$$V \sim -\frac{g_1^2}{(\rho - \rho_0)} \rightarrow -\infty. \quad (85)$$

The singularity is again physically acceptable, and the solution could be interpreted as an RG flow from the $N = 4$ SCFT to a non-conformal phase in the IR driven by relevant operators of dimensions $\Delta = 1, 2$.

3.3 $N = 2$ holographic RG flow with

$SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetry

We now consider the most general solutions in the $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation with non-vanishing χ and $\phi_1 = \phi_3 = 0$. In this case, the $SO(4) \subset SO(6)_R$ is broken to $SO(2) \times SO(2)$ while the $SO(4)$ subgroup of the $SO(6)$ symmetry of the vector multiplets is broken to $SO(3) \times SO(2)$, or $SO(2) \times SO(3)$ if we choose $\phi_2 = \phi_4 = 0$ rather than $\phi_1 = \phi_3 = 0$.

With $\phi_1 = \phi_3 = 0$, the superpotential is given by

$$\begin{aligned} \mathcal{W} = \mathcal{W}_+ = & \frac{1}{2} \left[g_1 e^{-\frac{\phi}{2}} (\cosh \phi_2 + i \sinh \phi_2 \sinh \phi_4) \right. \\ & \left. + g_2 e^{\frac{\phi}{2}} \cosh \phi_4 + i g_2 e^{-\frac{\phi}{2}} \chi \cosh \phi_4 \right] \end{aligned} \quad (86)$$

in which for definiteness, we have chosen $\mathcal{W}_+$ to be the superpotential with ϵ^1 and ϵ^4 being the corresponding Killing spinors. If we choose $\mathcal{W} = \mathcal{W}_-$, the unbroken supersymmetry will correspond to ϵ^2 and ϵ^3 .

With the same analysis as in the previous cases, we find the following BPS equations

$$\begin{aligned} A' &= W \\ &= \frac{1}{2} \sqrt{(g_1 e^{-\frac{\phi}{2}} \cosh \phi_2 + g_2 e^{\frac{\phi}{2}} \cosh \phi_4)^2 + e^{-\phi} (g_1 \sinh \phi_2 \sinh \phi_4 + g_2 \cosh \phi_4 \chi)^2}, \end{aligned} \quad (87)$$

$$\begin{aligned} \phi' &= -4 \frac{\partial W}{\partial \phi} \\ &= \frac{1}{8W} e^{-\phi} \left[g_1^2 (3 + \cosh 2\phi_2 + 2 \cosh 2\phi_4 \sinh^2 \phi_2) \right. \\ &\quad \left. - 4g_2^2 e^{2\phi} \cosh^2 \phi_4 \right. \\ &\quad \left. + 4g_2 \chi (g_1 \sinh \phi_2 \sinh 2\phi_4 + g_2 \chi \cosh^2 \phi_4) \right], \end{aligned} \quad (88)$$

$$\begin{aligned} \chi' &= -4e^{2\phi} \frac{\partial W}{\partial \chi} = -\frac{e^\phi}{W} g_2 \cosh \phi_4 (g_1 \sinh \phi_2 \sinh \phi_4 \\ &\quad + g_2 \cosh \phi_4 \chi), \end{aligned} \quad (89)$$

$$\begin{aligned} \phi_2' &= -\frac{2}{\cosh^2 \phi_4} \frac{\partial W}{\partial \phi_2} \\ &= -\frac{1}{2W} e^{-\phi} g_1 [\sinh \phi_2 (g_1 \cosh \phi_2 + g_2 e^\phi \text{sech} \phi_4) \\ &\quad + g_2 \chi \cosh \phi_2 \tanh \phi_4], \end{aligned} \quad (90)$$

$$\begin{aligned} \phi_4' &= -2 \frac{\partial W}{\partial \phi_4} \\ &= -\frac{1}{4W} e^{-\phi} \left[\sinh 2\phi_4 (g_2^2 e^{2\phi} + g_2^2 \chi^2 + g_1^2 \sinh^2 \phi_2) \right. \\ &\quad \left. + 2g_1 g_2 e^\phi \cosh \phi_2 \sinh \phi_4 + 2g_1 g_2 \chi \cosh 2\phi_4 \sinh \phi_2 \right]. \end{aligned} \quad (91)$$

In this case, we are not able to find analytic solutions, so we will instead perform a numerical analysis for solving these equations. By choosing the numerical values of the coupling constants $g_1 = g_2 = \sqrt{2}g_0$, we obtain examples of numerical RG flow solutions for suitable boundary conditions and different values of the parameter g_0 as shown in Figs. 1, 2 and 3. From these figures, we see some patterns namely, in all of these solutions, ϕ_2 is constant near the singularities as in the previous cases, and all the singularities are physically acceptable. The former indeed seems to be a general feature at least among all the solutions we have found. The latter follows directly from the scalar potential (29) for $g_2 = g_1$,

$$V = -\frac{1}{2} g_1^2 e^{-\phi} (1 + e^{2\phi} + \chi^2 + 4e^\phi \cosh \phi_2 \cosh \phi_4) \quad (92)$$

from which we immediately see that $V \rightarrow -\infty$ for any diverging behaviors of all the scalar fields. This implies that all possible singularities in this $SO(2) \times SO(2) \times SO(3) \times SO(2)$ truncation are physical. Accordingly, the corresponding solutions describe RG flows from the dual $N = 4$ SCFT to various non-conformal phases in the IR.

4 Supersymmetric Janus solutions

In this section, we look at supersymmetric Janus solutions with $N = 4$ and $N = 2$ supersymmetries. In this case, the resulting BPS equations are much more complicated than those in the RG flow case, and we are not able to obtain any analytic solutions.

4.1 $N = 4$ Janus solutions with $SO(4) \times SO(4)$ symmetry

We first consider solutions with all $\phi_i = 0$ that can be regarded as solutions of pure $N = 4$ gauged supergravity with $SO(4) \sim SO(3) \times SO(3)$ gauge group. In this case, we find a simple superpotential

$$\mathcal{W} = \frac{1}{2}e^{-\frac{\phi}{2}}[g_1 + g_2(e^\phi + i\chi)]. \quad (93)$$

Using the prescription given in Sect. 2, we obtain the following BPS equations

$$\begin{aligned} \phi' &= -4\frac{A'}{W}\frac{\partial W}{\partial \phi} - 4\kappa\frac{e^{-A}}{\ell W}e^\phi\frac{\partial W}{\partial \chi} \\ &= \frac{2\ell A'(g_1^2 - g_2^2e^{2\phi} + g_2^2\chi^2) - 4\kappa g_2^2\chi e^{\phi-A}}{\ell[(g_1 + g_2e^\phi)^2 + g_2^2\chi^2]}, \end{aligned} \quad (94)$$

$$\begin{aligned} \chi' &= -4e^{2\phi}\frac{A'}{W}\frac{\partial W}{\partial \chi} + 4\kappa\frac{e^{-A}}{\ell W}e^\phi\frac{\partial W}{\partial \phi} \\ &= -\frac{4\ell g_2^2A'\chi e^{2\phi} - 2\kappa e^{\phi-A}(g_1^2 - g_2^2e^{2\phi} + g_2^2\chi^2)}{\ell[(g_1 + g_2e^\phi)^2 + g_2^2\chi^2]}, \end{aligned} \quad (95)$$

$$A'^2 = \frac{1}{4}e^{-\phi}[(g_1 + g_2e^\phi)^2 + g_2^2\chi^2] - \frac{e^{-2A}}{\ell^2}. \quad (96)$$

As expected, in the limit $\ell \rightarrow \infty$, these equations reduce to those of the RG flows studied in [17] after a redefinition of the scalars and coupling constants. It should also be noted that setting $\chi = 0$ is not consistent unless $\ell \rightarrow \infty$. Therefore, Janus solutions are possible only for non-vanishing pseudoscalars needed to support the curvature of the AdS_3 -sliced world-volume. This has also been pointed out in [19].

We now perform a numerical analysis and give some examples of supersymmetric Janus solutions as shown in Fig. 4. These solutions preserve half of the original $N = 4$ supersymmetry, or eight supercharges, with all ϵ^i non-vanishing and subject to the projector (35). On the conformal interfaces, the Killing spinors can have left or right chiralities depending on whether the values of $\kappa = 1$ or $\kappa = -1$.

The two-dimensional interfaces then preserve $(4, 0)$ or $(0, 4)$ supersymmetries. In the numerical analysis, we have conventionally set $\kappa = 1$.

4.2 $N = 2$ Janus solutions with

$SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetry

We now look at more complicated solutions in matter-coupled $N = 4$ gauged supergravity. We first note that Janus solutions are not possible for all the subtruncations with $\chi = 0$. We then consider the case of $N = 2$ supersymmetry and $SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetry with $\phi_1 = \phi_3 = 0$ and $\epsilon^2 = \epsilon^3 = 0$. The other equivalent cases can be obtained by replacing (ϕ_2, ϕ_4) and/or (ϵ^1, ϵ^4) by (ϕ_1, ϕ_3) and/or (ϵ^2, ϵ^3) .

With the superpotential (86) and the same procedure as in the previous case, we find the following BPS equations

$$\phi'_2 = -2\text{sech}^2\phi_4\frac{A'}{W}\frac{\partial W}{\partial \phi_2} - 2\text{sech}\phi_4\frac{\kappa e^{-A}}{\ell W}\frac{\partial W}{\partial \phi_4}, \quad (97)$$

$$\phi'_4 = -\frac{A'}{W}\frac{\partial W}{\partial \phi_4} + 2\text{sech}\phi_4\frac{\kappa e^{-A}}{\ell W}\frac{\partial W}{\partial \phi_2}, \quad (98)$$

$$\phi' = -4\frac{A'}{W}\frac{\partial W}{\partial \phi} - 4e^\phi\frac{\kappa e^{-A}}{\ell W}\frac{\partial W}{\partial \chi}, \quad (99)$$

$$\chi' = -4e^{2\phi}\frac{A'}{W}\frac{\partial W}{\partial \chi} + 4e^\phi\frac{\kappa e^{-A}}{\ell W}\frac{\partial W}{\partial \phi}, \quad (100)$$

$$A'^2 = W^2 - \frac{e^{-2A}}{\ell^2} \quad (101)$$

with

$$\begin{aligned} \frac{1}{W}\frac{\partial W}{\partial \phi_2} &= \frac{1}{4W^2}g_1e^{-\phi}\cosh\phi_4[\sinh\phi_2(g_2e^\phi \\ &\quad + g_1\cosh\phi_2\cosh\phi_4) + g_2\chi\cosh\phi_2\sinh\phi_4], \end{aligned} \quad (102)$$

$$\begin{aligned} \frac{1}{W}\frac{\partial W}{\partial \phi_4} &= \frac{1}{4W^2}[e^{-\phi}(g_1\sinh\phi_2\sinh\phi_4 \\ &\quad + g_2\chi\cosh\phi_4)(g_1\cosh\phi_4\sinh\phi_2 + g_2\chi\sinh\phi_4) \\ &\quad + g_2\sinh\phi_4(g_1\cosh\phi_2 + g_2e^\phi\cosh\phi_4)], \end{aligned} \quad (103)$$

$$\begin{aligned} \frac{1}{W}\frac{\partial W}{\partial \phi} &= \frac{1}{8W^2}[2g_2\cosh\phi_4(g_1\cosh\phi_2 + g_2e^\phi\cosh\phi_4) \\ &\quad - e^{-\phi}(g_1\sinh\phi_2\sinh\phi_4 + g_2\chi\cosh\phi_4)^2 \\ &\quad - e^{-\phi}(g_1\cosh\phi_2 + g_2e^\phi\cosh\phi_4)^2], \end{aligned} \quad (104)$$

$$\begin{aligned} \frac{1}{W}\frac{\partial W}{\partial \chi} &= \frac{1}{4W^2}g_2e^{-\phi}\cosh\phi_4(g_1\sinh\phi_2\sinh\phi_4 \\ &\quad + g_2\chi\cosh\phi_4). \end{aligned} \quad (105)$$

Examples of numerical solutions are given in Fig. 5. In the figure, the solutions for the warped factor A are very close to each other. In this case, the solutions describe conformal interfaces with $N = (2, 0)$ or $N = (0, 2)$ supersymme-

Fig. 1 An $N = 2$ RG flow from the $N = 4$ SCFT in the UV to a non-conformal phase in the IR for $g_1 = g_2 = \frac{1}{\sqrt{2}}$

Fig. 2 An $N = 2$ RG flow from the $N = 4$ SCFT in the UV to a non-conformal phase in the IR for $g_1 = g_2 = \frac{3\sqrt{2}}{2}$

Fig. 3 An $N = 2$ RG flow from the $N = 4$ SCFT in the UV to a non-conformal phase in the IR for $g_1 = g_2 = \frac{5\sqrt{2}}{2}$

Fig. 4 $N = 4$ supersymmetric Janus solutions from $N = 4$ $SO(4) \times SO(4)$ gauged supergravity for $g_1 = g_2 = 2\sqrt{2}$ and $\ell = \kappa = 1$ with different boundary conditions

tries corresponding respectively to $\kappa = 1$ or $\kappa = -1$ and could provide a holographic description of deformations by position-dependent operators within the dual $N = 4$ SCFT.

5 Conclusions and discussions

In this paper, we have studied a number of holographic RG flows from matter-coupled $N = 4$ gauged supergravity with $SO(4) \times SO(4)$ gauge group by truncating to $SO(2) \times SO(2) \times SO(2) \times SO(2)$ singlet scalars. For vanishing axion, there exist $N = 4$ supersymmetric flow solutions with $SO(2) \times SO(3) \times SO(2) \times SO(2)$ and $SO(2) \times SO(2) \times SO(2) \times SO(2)$ symmetries. In the presence of axion, the solutions preserve only $N = 2$ supersymmetry and $SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetry. All of these solutions are singular at finite values of the radial coordinate, but all the singularities turn out to be physically acceptable according to the criterion given in [50]. The solutions can be interpreted as RG flows from the $N = 4$ SCFT, dual to the unique $N = 4$ supersymmetric AdS_4 vacuum within this truncation, to non-conformal phases in the IR with different global symmetries. Although the $SO(4) \times SO(4)$ gauged supergravity under consideration here has currently no known higher dimensional origin, all of these solutions could hopefully be useful in studying various mass deformations of strongly coupled $N = 4$ SCFT via AdS/CFT holography.

We have also found $N = 4$ and $N = 2$ supersymmetric Janus solutions with $SO(4) \times SO(4)$ and $SO(2) \times SO(2) \times SO(3) \times SO(2)$ symmetries. The former is obtained by truncating out all scalars from the vector multiplets resulting in the solutions of pure $N = 4$ gauged supergravity while the latter is a genuine solution of the matter-coupled theory. These solutions holographically describe two-dimensional conformal interfaces in the dual $N = 4$ SCFT with unbroken $N = (4, 0)$ or $N = (2, 0)$ supersymmetries on the interfaces. Generally, these configurations are identified with deformations of the dual SCFT by operators or vacuum expectation values that depend on the coordinate transverse to the interfaces.

It would be interesting to explicitly identify the dual $N = 4$ SCFT together with relevant deformations dual to the solutions found here. Uplifting the $SO(4) \times SO(4)$ gauged supergravity considered here to higher dimensions will allow to embed these solutions in ten or eleven dimensions giving rise to new examples of AdS/CFT duality in the context of string/M-theory. In particular, it could be interesting to check whether the singularities allowed by the criterion of [50] are physical in string/M-theory by using the criterion of [51]. If this is indeed the case, identifying relevant M-brane or D-brane configurations related to these four-dimensional solutions also deserves further study. Finally, finding more general holographic solutions of $N = 4$ gauged supergravities in other truncations or in other gauge groups could be interesting as well. These solutions might include holographic solutions describing spatially varying mass terms given in [52, 53]. We leave all these issues for future investigation.

Fig. 5 $N = 2$ supersymmetric Janus solutions from $N = 4$ $SO(4) \times SO(4)$ gauged supergravity for $g_1 = g_2 = 2\sqrt{2}$, $\ell = 2$ and $\kappa = 1$ with different boundary conditions

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Appendix A: Field equations

In this appendix, we give some details on the field equations for the metric and scalar fields in $SO(2) \times SO(2) \times SO(2) \times SO(2)$ truncation of the $N = 4$ gauged supergravity with $SO(4) \times SO(4)$ gauge group. As pointed out in Sect. 2.2, the corresponding equations for RG flows can be obtained from those of Janus solutions by taking the limit $\ell \rightarrow \infty$, we will only give the field equations for Janus solutions.

We first note the relevant Lagrangian

$$e^{-1}\mathcal{L} = \frac{1}{2}R + \mathcal{L}_{\text{kin}} - V \quad (106)$$

with the scalar kinetic term $\mathcal{L}_{\text{kin}}$ and the scalar potential given in (34) and (29), respectively. With the metric ansatz (33), we find the following equations from Einstein's equation

$$2A'' + 3A'^2 + \frac{1}{\ell^2}e^{-2A} - \mathcal{L}_{\text{kin}} + V = 0, \quad (107)$$

$$3A'^2 + \frac{3}{\ell^2}e^{-2A} + \mathcal{L}_{\text{kin}} + V = 0. \quad (108)$$

The first equation arises from the components along the AdS_3 -slice while the second one is obtained from the (rr) -component. As in other cases, these two equations are not independent.

It is straightforward to derive the scalar equations which are given by

$$\frac{d}{dr} \left(\frac{\partial \mathcal{L}}{\partial \Phi^{s'}} \right) - \frac{\partial \mathcal{L}}{\partial \Phi^s} = 0 \quad \text{with } e = e^{3A + \frac{2\xi}{\ell}}. \quad (109)$$

We recall that $\Phi^s = (\phi, \chi, \phi_1, \phi_2, \phi_3, \phi_4)$, $s = 1, 2, \dots, 6$. We will not give the explicit form of these equations here due to their complexity. It should also be noted that since the scalars do not depend on the coordinate ξ , the scalar

field equations for both the RG flow and Janus cases are the same. The resulting BPS equations need to satisfy these field equations. This imposes some restrictions on the BPS equations as given in the main text.

References

1. J.M. Maldacena, The large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.* **2**, 231–252 (1998). [arXiv: hep-th/9711200](#)
2. S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Gauge theory correlators from non-critical string theory. *Phys. Lett. B* **428**, 105–114 (1998). [arXiv: hep-th/9802109](#)
3. E. Witten, Anti de Sitter space and holography. *Adv. Theor. Math. Phys.* **2**, 253–291 (1998). [arXiv: hep-th/9802150](#)
4. R. Corrado, K. Pilch, N.P. Warner, An $N = 2$ supersymmetric membrane flow. *Nucl. Phys. B* **629**, 74–96 (2002). [arXiv: hep-th/0107220](#)
5. C.N. Gowdigere, N.P. Warner, Flowing with eight supersymmetries in M-theory and F-theory. *JHEP* **12**, 048 (2003). [arXiv: hep-th/0212190](#)
6. K. Pilch, A. Tyukov, N.P. Warner, Flowing to higher dimensions: a new strongly-coupled phase on M2 branes. *JHEP* **11**, 170 (2015). [arXiv: 1506.01045](#)
7. C. Ahn, K. Woo, Supersymmetric domain wall and RG flow from 4-dimensional gauged $N = 8$ supergravity. *Nucl. Phys. B* **599**, 83–118 (2001). [arXiv: hep-th/0011121](#)
8. C. Ahn, T. Itoh, An $N = 1$ supersymmetric G_2 -invariant flow in M-theory. *Nucl. Phys. B* **627**, 45–65 (2002). [arXiv: hep-th/0112010](#)
9. N. Bobev, N. Halmagyi, K. Pilch, N.P. Warner, Holographic, $N = 1$ supersymmetric RG flows on M2 branes. *JHEP* **09**, 043 (2009). [arXiv: 0901.2376](#)
10. A. Guarino, On new maximal supergravity and its BPS domain-walls. *JHEP* **02**, 026 (2014). [arXiv: 1311.0785](#)
11. J. Tarrio, O. Varela, Electric/magnetic duality and RG flows in AdS_4/CFT_3 . *JHEP* **01**, 071 (2014). [arXiv: 1311.2933](#)
12. Y. Pang, C.N. Pope, J. Rong, Holographic RG flow in a new $SO(3) \times SO(3)$ sector of ω -deformed $SO(8)$ gauged $N = 8$ supergravity. *JHEP* **08**, 122 (2015). [arXiv: 1506.04270](#)
13. P. Karndumri, Holographic RG flows in $N = 3$ Chern–Simons-matter theory from $N = 3$ 4D gauged supergravity. *Phys. Rev. D* **94**, 045006 (2016). [arXiv: 1601.05703](#)
14. P. Karndumri, K. Upathambhakul, Gaugings of four-dimensional $N = 3$ supergravity and AdS_4/CFT_3 holography. *Phys. Rev. D* **93**, 125017 (2016). [arXiv: 1602.02254](#)
15. P. Karndumri, Supersymmetric deformations of 3D SCFTs from tri-Sasakian truncation. *Eur. Phys. J. C* **77**, 130 (2017). [arXiv: 1610.07983](#)
16. P. Karndumri, K. Upathambhakul, Supersymmetric RG flows and Janus from type II orbifold compactification. *Eur. Phys. J. C* **77**, 455 (2017). [arXiv: 1704.00538](#)
17. P. Karndumri, K. Upathambhakul, Holographic RG flows in $N = 4$ SCFTs from half-maximal gauged supergravity. *Eur. Phys. J. C* **78**, 626 (2018). [arXiv: hep-th/1806.01819](#)
18. P. Karndumri, C. Maneerat, Supersymmetric solutions from $N = 5$ gauged supergravity. *Phys. Rev. D* **101**, 126015 (2020). [arXiv: 2003.05889](#)
19. N. Bobev, K. Pilch, N.P. Warner, Supersymmetric Janus solutions in four dimensions. *JHEP* **1406**, 058 (2014). [arXiv: 1311.4883](#)
20. P. Karndumri, Supersymmetric Janus solutions in four-dimensional $N = 3$ gauged supergravity. *Phys. Rev. D* **93**, 125012 (2016). [arXiv: 1604.06007](#)

21. M. Suh, Supersymmetric Janus solutions of dyonic $ISO(7)$ -gauged $N = 8$ supergravity. *JHEP* **04**, 109 (2018). [arXiv:1803.00041](#)
22. N. Kim, S.J. Kim, Re-visiting supersymmetric Janus solutions: a perturbative construction. *Chin. Phys. C* **44**(7), 073104 (2020). [arXiv:2001.06789](#)
23. J. Tarrio, O. Varela, Electric/magnetic duality and RG flows in AdS_4/CFT_3 . *JHEP* **10**, 071 (2014). [arXiv:1311.2933](#)
24. P. Karndumri, J. Seeyangnok, Supersymmetric solutions from $N = 6$ gauged supergravity. *Phys. Rev. D* **103**(6), 066023 (2021). [arXiv:2012.10978](#)
25. P. Karndumri, C. Maneerat, Supersymmetric Janus solutions in ω -deformed $N = 8$ gauged supergravity. [arXiv:2012.15763](#)
26. P. Karndumri, RG flows in 6D $N = (1, 0)$ SCFT from $SO(4)$ half-maximal 7D gauged supergravity. *JHEP* **06**, 101 (2014). [arXiv:1404.0183](#)
27. P. Karndumri, Holographic RG flows in six dimensional $F(4)$ gauged supergravity. *JHEP* **01**, 134 (2013). [arXiv:1210.8064](#) [Erratum *ibid* *JHEP* **06**, 165 (2015)]
28. P. Karndumri, Gravity duals of 5D $N = 2$ SYM from $F(4)$ gauged supergravity. *Phys. Rev. D* **90**, 086009 (2014). [arXiv:1403.1150](#)
29. G. Bruno De Luca, A. Gnechhi, G. Lo Monaco and A. Tomasiello, Holographic duals of 6d RG flows, *JHEP* 03 (2019) **035**, [arXiv:1810.10013](#)
30. L. Girardello, M. Petrini, M. Porrati, A. Zaffaroni, Novel local CFT and exact results on perturbations of $N = 4$ super Yang Mills from AdS dynamics. *JHEP* **12**, 022 (1998). [arXiv:hep-th/9810126](#)
31. D.Z. Freedman, S.S. Gubser, K. Pilch, N.P. Warner, Renormalization group flows from holography supersymmetry and a c theorem. *Adv. Theor. Math. Phys.* **3**, 363–417 (1999). [arXiv:hep-th/9904017](#)
32. D. Cassani, G. Dall'Agata, A.F. Faedo, BPS domain walls in $N = 4$ supergravity and dual flows. *JHEP* **03**, 007 (2013). [arXiv:1210.8125](#)
33. N. Bobev, D. Cassani, H. Triendl, Holographic RG flows for four-dimensional $N = 2$ SCFTs. *JHEP* **06**, 086 (2018). [arXiv:1804.03276](#)
34. H.L. Dao, P. Karndumri, Holographic RG flows and AdS_5 black strings from 5D half-maximal gauged supergravity. *Eur. Phys. J. C* **79**, 137 (2019). [arXiv:1811.01608](#)
35. C. Bachas, J. de Boer, R. Dijkgraaf, H. Ooguri, Permeable conformal walls and holography. *JHEP* **06**, 027 (2002). [arXiv:hep-th/0111210](#)
36. C. Bachas, M. Petropoulos, Anti-de-Sitter D-branes. *JHEP* **02**, 025 (2001). [arXiv:hep-th/0012234](#)
37. D. Bak, M. Gutperle, S. Hirano, Three dimensional Janus and time-dependent black holes. *JHEP* **02**, 068 (2007). [arXiv:hep-th/0701108](#)
38. M. Chiodaroli, M. Gutperle, D. Krym, Half-BPS solutions locally asymptotic to $AdS_3 \times S^3$ and interface conformal field theories. *JHEP* **02**, 066 (2010). [arXiv:0910.0466](#)
39. M. Chiodaroli, E. D'Hoker, Y. Guo, M. Gutperle, Exact half-BPS string-junction solutions in six-dimensional supergravity. *JHEP* **12**, 086 (2011). [arXiv:1107.1722](#)
40. D. Bak, H. Min, Multi-faced black Janus and entanglement. *JHEP* **03**, 046 (2014). [arXiv:1311.5259](#)
41. E. D'Hoker, J. Estes, M. Gutperle, D. Krym, Janus solutions in M-theory. *JHEP* **06**, 018 (2009). [arXiv:0904.3313](#)
42. M. Gutperle, J. Kaidi, H. Raj, Janus solutions in six-dimensional gauged supergravity. *JHEP* **12**, 018 (2017). [arXiv:1709.09204](#)
43. K. Chen, M. Gutperle, Janus solutions in three-dimensional $N = 8$ gauged supergravity. *JHEP* **05**, 008 (2021). [arXiv:2011.10154](#)
44. J. Schon, M. Weidner, Gauged $N = 4$ supergravities. *JHEP* **05**, 034 (2006). [arXiv:hep-th/0602024](#)
45. E. Bergshoeff, I.G. Koh, E. Sezgin, Coupling of Yang–Mills to $N = 4$, $d = 4$ supergravity. *Phys. Lett. B* **155**, 71–75 (1985)
46. M. de Roo, P. Wagemans, Gauged matter coupling in $N = 4$ supergravity. *Nucl. Phys. B* **262**, 644–660 (1985)
47. P. Wagemans, Breaking of $N = 4$ supergravity to $N = 1$, $N = 2$ at $\Lambda = 0$. *Phys. Lett. B* **206**, 241 (1988)
48. J. Louis, H. Triendl, Maximally supersymmetric AdS_4 vacua in $N = 4$ supergravity. *JHEP* **10**, 007 (2014). [arXiv:1406.3363](#)
49. D. Roest, J. Rosseel, De Sitter in extended supergravity. *Phys. Lett. B* **685**, 201–207 (2010). [arXiv:0912.4440](#)
50. S.S. Gubser, Curvature singularities: the good, the bad and the naked. *Adv. Theor. Math. Phys.* **4**, 679–745 (2000). [arXiv:hep-th/0002160](#)
51. J. Maldacena, C. Nunez, Supergravity description of field theories on curved manifolds and a no go theorem. *Int. J. Mod. Phys. A* **16**, 822 (2001). [arXiv:hep-th/0007018](#)
52. J.P. Gauntlett, C. Rosen, Susy Q and spatially modulated deformations of ABJM theory. *JHEP* **10**, 066 (2018). [arXiv:1808.02488](#)
53. I. Arav, J.P. Gauntlett, M. Roberts, C. Rosen, Spatially modulated and supersymmetric deformations of ABJM theory. *JHEP* **04**, 099 (2019). [arXiv:1812.11159](#)