Eur. Phys. J. C (2021) 81:467
https://doi.org/10.1140/epjc/s10052-021-09255-w

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Possible assignments of highly excited $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$

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Received: 15 April 2021 / Accepted: 16 May 2021 / Published online: 28 May 2021

Abstract Possible assignments of highly excited $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are explored in a $^3P_0$ strong decay model. Decay widths, branching fraction ratios $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ and the branching fractions of DN channels of theses assignments are computed. D^0p channel is a very important channel to provide information on the inner excitation and structure of these highly excited Λ_c. In our analysis, $\Lambda_c(2860)^+$ may be a $1D$-wave excited Λ_c with $J^P = \frac{3}{2}^+$, which has dominant DN decay channels with a branching fraction $\mathcal{B}(\Lambda_c(2860)^+ \to DN) = 75\%$ and a branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.12$. $\Lambda_c(2880)^+$ is very possibly a $1F$-wave excited Λ_c with $J^P = \frac{5}{2}^-$; In this assignment, the predicted total decay width ($\Gamma \approx 4.49$ MeV) is comparable to the measured $\Gamma = 5.6^{+0.8}_{-0.6}$ MeV, and the predicted $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.12$ is consistent with the measured $R = 0.225 \pm 0.062 \pm 0.025$; The DN channels are its dominant strong decay channels with a branching fraction $\mathcal{B}(\Lambda_c(2880)^+ \to DN) = 94\%$. $\Lambda_c(2880)^+$ seems impossibly a $1D$-wave excited Λ_c with $J^P = \frac{5}{2}^+$ once the presently measured $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ is confirmed. $\Lambda_c(2940)^+$ may be a $2P$-wave excited $\Lambda_{c1,1}^{1,0}(\frac{3}{2}^-, 2P)$. In this case, $\Lambda_c(2940)^+$ has a total decay width $\Gamma = 17.56$ MeV, a branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.89$ and the DN decay channels with a branching fraction $\mathcal{B}(\Lambda_c(2940)^+ \to DN) = 43\%$. In order to understand the inner excitation and structure of these highly excited Λ_c, measurements of those predicted quantities are required in the future.

1 Introduction

Charmed baryons with single charmed quark provide ideal windows to study the baryon structure and quark dynamics. The heavy-quark symmetry works approximately in singly-charmed baryons, and the quarks inside may correlate and exhibit their structure through their strong decays. There are now 36 established singly-charmed baryons [1], but their J^P numbers have seldom been measured in experiments.

Λ_c baryons are composed of one u quark, one d quark and one charmed quark. $\Lambda_c(2286)^+$ is believed the ground state with $J^P = \frac{1}{2}^+$, $\Lambda_c(2593)^+$ and $\Lambda_c(2625)^+$ are believed the P-wave excited states with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, respectively. $\Lambda_c(2765)^+$ or $\Sigma_c(2765)^+$ [2] was seen in $\Lambda_c^+\pi^+\pi^-$ with mass difference $m(\Lambda_c(2765)^+) - m(\Lambda_c) = 480.1 \pm 2.4$ MeV, but nothing is known about its J^P and isospin quantum numbers as indicated in PDG2020 (This state was reported as a Λ_c with zero isospin in HADRON 2019 [3]).

$\Lambda_c(2880)^+$ was first observed by CLEO collaboration [2], and its quantum numbers have been constrained by Belle and LHCb collaborations [4,5] with $J^P = \frac{5}{2}^+$. The spin hypothesis $J = \frac{5}{2}$ is favored from an angular analysis in the experiment [4] and the positive parity is assumed through the predictions of the heavy quark symmetry [4,6–8]. $\Lambda_c(2940)$ was first observed by BaBar collaboration [9] and $\Lambda_c(2860)^+$ was first observed by LHCb collaboration [5], the quantum numbers J^P of $\Lambda_c(2860)^+$ and $\Lambda_c(2940)^+$ have not been measured. The masses, decay modes and decay widths of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ were reported in these experiments.

In normal baryon interpretations, the quantum numbers and possible internal excitation of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ have been studied in many models. The measured mass of $\Lambda_c(2860)^+$ is consistent with the theoretical predictions of the orbital $1D$-wave Λ_c excitation with quantum numbers $\frac{3}{2}^+$ [10,11], so $\Lambda_c(2860)^+$ is supposed with the quantum numbers $J^P = \frac{3}{2}^+$. $\Lambda_c(2880)^+$ was supposed with quantum numbers $J^P = \frac{5}{2}^+$ in a framework of heavy hadron chiral perturbation theory [7,8], a constituent quark model [12,13], a relativistic flux tube model [10], and a $^3P_0$ strong decay model [14,15], etc. $\Lambda_c(2880)^+$ was also assumed with

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quantum numbers $J^P = \frac{3}{2}^+$ in a chiral quark model [16]. The assignments of $\Lambda_c(2940)^+$ is much more contradictory. $\Lambda_c(2940)^+$ was assumed with quantum numbers $J^P = \frac{1}{2}^-$, $J^P = \frac{3}{2}^+$, $J^P = \frac{3}{2}^-$, $J^P = \frac{5}{2}^-$ or $J^P = \frac{5}{2}^+$ in different models [7,8,12–16]. Obviously, the quantum numbers of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ have not been fixed, their internal structures are not clear either.

The quarks in baryons may make complex structures and have complex excitations. In order to study the internal structures and excitations, the three-quark baryons are usually described by Jacobi coordinates: a relative coordinate ρ between any two quarks, and a relative coordinate λ between the center of mass of the two quarks and the other quark. As known, a diquark may be an important correlation and cluster in hadrons with more than two quarks, and the diquark has been introduced to interpret the light scalar mesons, the missing nucleons, the charmonium-like X, Y, Z, and so on. The diquark has also been employed to describe singly-charmed baryons in many models [10,12,13,17–21]. However, there is no evidence for the existence of diquark in baryons. In this paper, the strong decay properties of Λ_c baryons with different ρ or λ mode excitations will be studied, and the relation between the excitations and the diquark correlation is explored.

In Ref. [15], all the observed Λ_c states except for the ground $\Lambda_c(2286)^+$ were systematically examined as the $1P$-wave, $1D$-wave, or $2S$-wave Λ_c baryons from their strong decay properties in the $^3P_0$ model, and their possible assignments were suggested. In this paper, we continue the examination of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ with the highly excited $1D$, $1F$ and $2P$ orbital or radial excitations assignments in detail.

The paper is organized as follows. A simple introduction of $^3P_0$ strong decay model and analyses of the strong decay properties of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are given in Sect. 2. Conclusions and discussions are reserved in Sect. 3.

2 $1D$-wave, $1F$-wave and $2P$-wave possibilities of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$

In order to fix the quantum numbers and to understand the internal structure of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$, the $^3P_0$ strong decay model is employed. As well known, the $^3P_0$ model is usually known as the quark pair creation model. It was proposed by Micu [22] and developed by Le Yaouanc et al. [23–28]. This model has been employed to compute the Okubo-Zweig-Iizuka-allowed (OZI) strong decays widths with two final states and obtained good agreements with experiments.

Following Refs. [15,29–33], the strong decay width for an initial baryon A decaying into two final hadrons B and C in the $^3P_0$ model is

$$\Gamma = \pi^2 \frac{|\mathbf{p}|}{m_A^2} \frac{1}{2J_A+1} \sum_{M_{J_A} M_{J_B} M_{J_C}} |\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}|^2 \qquad (1)$$

where $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}$ is the helicity amplitude. The explicit expression of the helicity amplitude, the flavor matrix, the space integral and some relevant notations could be found in detail in Ref. [15].

As indicated in Ref. [15], ρ is the relative coordinate between the two light quarks (quarks 1 and 2), and λ is the relative coordinate between the center of mass of the two light quarks and the charmed quark. In a constituent quark model, the internal structure of a baryon is also described by a set of quantum numbers n_ρ, n_λ, L_ρ, L_λ and S_ρ. n_ρ and n_λ denote the nodal quantum numbers of the ρ and λ coordinates, respectively. L_ρ and L_λ denote the orbital angular momentum between the two light quarks and the orbital angular momentum between the charm quark and the two-light-quark system. S_ρ denotes the total spin of the two light quarks. The total orbital angular momentum $L = L_\rho + L_\lambda$ and the total angular momentum of the baryons $J = J_l + \frac{1}{2}$ with $J_l = L + S_\rho$.

Therefore, in the constituent quark model with the heavy-quark symmetry [15,34], there are one $1S$-wave, seven $1P$-wave, seventeen $1D$-wave, and thirty-one $1F$-wave Λ_c baryons. For the first radial excitations, the corresponding states doubled. That is to say, there are two $2S$-wave, fourteen $2P$-wave, thirty-four $2D$-wave, and sixty-two $2F$-wave Λ_c baryons. Internal quantum numbers of the $1D$-wave excited Λ_c were given in Ref. [15], quantum numbers of the $1F$-wave and $2P$-wave excited Λ_c are given in the appendix.

Some parameters are chosen as those in Refs. [15,32,35]. The dimensionless pair-creation $\gamma = 13.4$. The $\beta_{\lambda,\rho} = 600$ MeV in $1S$-wave baryon wave function, $\beta = 400$ MeV in the wave function of π and K mesons, $\beta = 600$ MeV for the D meson [15,35]. $\beta_{\rho,\lambda} = 400$ MeV is for the excited $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$. The masses of relevant hadrons are chosen from Particle Data Group [1].

D^0p mode is an important channel for Λ_c baryons in the $^3P_0$ model. In this channel, the heavy charmed quark in initial baryon enters the final D meson and other two light quarks enter the final p baryon. Therefore, this channel may provide some information on the inner excitation and structure of Λ_c.

In theory, the helicity amplitudes of many high-lying Λ_c decaying into D^0p channel vanish. Therefore, many possible assignments of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ can be excluded through the observed D^0p final states. So does the D^+n channel. Possible high-lying Λ_c which can decay into D^0p channel are given in Table 1. In these Λ_c excita-

Table 1 Possible Λ_c decaying into DN final states

$\Lambda^{L,L_\rho}_{cJ_L,n_\rho}(J^P,nL)$	n_ρ	n_λ	L_ρ	L_λ	L	S_ρ	J_L	J^P
$\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$	0	0	0	2	2	0	2	$\frac{3}{2}^+$
$\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$	0	0	0	2	2	0	2	$\frac{5}{2}^+$
$\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-,1F)$	0	0	0	3	3	0	3	$\frac{5}{2}^-$
$\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-,1F)$	0	0	0	3	3	0	3	$\frac{7}{2}^-$
$\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-,2P)$	0	1	0	1	1	0	1	$\frac{1}{2}^-$
$\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-,2P)$	0	1	0	1	1	0	1	$\frac{3}{2}^-$
$\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-,2P)$	1	0	0	1	1	0	1	$\frac{1}{2}^-$
$\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-,2P)$	1	0	0	1	1	0	1	$\frac{3}{2}^-$

tions, $\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-,2P)$ and $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-,2P)$ have radial ρ mode excitation, while others have only λ mode excitation.

2.1 $1D$-wave excitations

Among the seventeen $1D$-wave Λ_c states, there are only two λ mode excited states $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$ and $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$ with D^0p decay channel. The masses of $\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$ are comparable to the predicted spectrum of D-wave excited Λ_c [10,11], and they could be the $1D$-wave excitations. These two $1D$-wave excitations are examined through their strong decay properties in this section.

In the framework of $^3P_0$ model, the decay widths of possible assignments of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are computed. The numerical results of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are presented in Tables 2, 3 and 4, respectively. Their total strong decay widths are also given. The results in these three tables are similar except for a new $\Sigma_c(2800)\pi$ channel for $\Lambda_c(2940)^+$. In our computation, $\Sigma_c(2800)$ is regarded as a $1P$-wave Σ_c with $J^P=\frac{3}{2}^-$ [36].

In each table, the main differences for the assignments of $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$ and $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$ are the total decay widths, branching fraction of DN channels and the branching fraction ratios $R=\frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$. $\Lambda_c(2860)^+$ or $\Lambda_c(2880)^+$ in the $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$ assignment has a larger total width, a smaller R and a dominant DN channel in comparison with the $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$ assignment.

In experiments, $\Lambda_c(2880)^+$ has a small total decay width ($\Gamma=5.6^{+0.8}_{-0.6}$ MeV) and a branching ratio $R=0.225\pm0.062\pm0.025$ [1], which was doubted by an influence from its nearby state $\Lambda_c(2860)^+$ in Ref. [37]. $\Lambda_c(2860)^+$ and $\Lambda_c(2940)^+$ have total decay widths $\Gamma=67.6^{+10.1}_{-8.1}\pm1.4^{+5.9}_{-20.0}$ MeV and $\Gamma=20^{+6}_{-5}$ MeV, respectively, but no branching fraction has been measured.

Table 2 Possible decay widths (MeV), branching fraction of DN channels and $R=\frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ of $\Lambda_c(2860)^+$ as $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$ and $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$

channel	$\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$	$\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$
$\Sigma_c^{++}\pi^-$	4.03	0.05
$\Sigma_c^{+}\pi^0$	4.10	0.05
$\Sigma_c^{0}\pi^+$	4.04	0.05
$\Sigma_c^{++}(2520)\pi^-$	0.47	2.72
$\Sigma_c^{+}(2520)\pi^0$	0.48	2.79
$\Sigma_c^{0}(2520)\pi^+$	0.47	2.72
D^0p	22.33	0.06
D^+n	19.19	0.04
Total width	55.11	8.48
R	0.12	54.87
$\mathcal{B}(\Lambda_c(2860)^+\to DN)$	75%	1%

From the total strong decay width, $\Lambda_c(2860)^+$ is very possibly the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$ while impossibly the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$. In this assignment, DN are the main two body strong decay channels with branching fraction $\mathcal{B}(\Lambda_c(2860)^+\to DN)=75\%$, and the ratio $R=\frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}=0.12$. Their measurement in the future will provide more information on $\Lambda_c(2860)^+$.

From the total strong decay width, $\Lambda_c(2880)^+$ is impossible the the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+,1D)$, but may be a $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$. In the $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+,1D)$ assignment, $\Sigma_c(2520)\pi$ are the dominant two body strong decay channels with branching fraction $\mathcal{B}(\Lambda_c(2880)^+\to\Sigma_c(2520)\pi)=94\%$. DN channels have branching fraction $\mathcal{B}(\Lambda_c(2880)^+\to DN)=3\%$. However, the predicted branching ratio $R=\frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}=44.29$ is much larger than the observed $R=0.225\pm0.062\pm0.025$. Similarly large R was predicted in Refs. [37,38]. Even though the theoretical uncertainties in the $^3P_0$ model have been taken into account, it is difficult to assign the $\Lambda_c(2880)^+$ with the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$ through its strong decay properties.

Through the strong decay widths only, $\Lambda_c(2940)^+$ could be the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$ and is impossibly the $1D$-wave excitation $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$. Taking into account the fact that $\Lambda_c(2940)^+$ has a higher mass than the predicted $1D$-wave excited Λ_c, it can not be the $1D$-wave excitation.

2.2 $1F$-wave excitations

Among the thirty-one $1F$-wave Λ_c states, there are also only two λ mode excited states $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$ and $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$ with D^0p decay channel. As a $1F$-wave excitation candidate, $\Lambda_c(2940)^+$ has numerical results similar to $\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$ except for the $\Sigma_c(2800)\pi$ channel. The decay widths of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are computed. To avoid tedious duplicate tables, only the results of $\Lambda_c(2880)^+$ are presented in Table 5.

Obviously, as a $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$, the predicted total decay widths $\Gamma = 4.49$ MeV and the branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.12$ of $\Lambda_c(2880)^+$ are consistent with the observed total decay width $\Gamma = 5.6^{+0.8}_{-0.6}$ MeV and branching ratio $R = 0.225 \pm 0.062 \pm 0.025$. $\Lambda_c(2880)^+$ is very possibly the $1F$-wave excitation $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$. In this assignment, the channels DN are the dominant two-body decay channels with branching fraction $\mathcal{B}(\Lambda_c(2880)^+ \to DN) = 94\%$.

$\Lambda_c(2860)^+$ seems impossibly the $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$ for its much larger decay width in comparison to the predicted one. Since no branching ratio has been measured, it is difficult to assign $\Lambda_c(2940)^+$ with the $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$ only from its total decay width.

Table 3 Possible decay widths (MeV), branching fraction of DN channels and $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ of $\Lambda_c(2880)^+$ as $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$ and $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$

channel	$\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$	$\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$
$\Sigma_c^{++}\pi^-$	4.85	0.08
$\Sigma_c^{+}\pi^0$	4.92	0.08
$\Sigma_c^{0}\pi^+$	4.86	0.08
$\Sigma_c^{++}(2520)\pi^-$	0.62	3.52
$\Sigma_c^{+}(2520)\pi^0$	0.64	3.59
$\Sigma_c^{0}(2520)\pi^+$	0.62	3.52
D^0p	35.75	0.22
D^+n	32.71	0.17
Total width	84.97	11.26
R	0.13	44.29
$\mathcal{B}(\Lambda_c(2880)^+ \to DN)$	81%	3%

Table 4 Possible decay widths (MeV), branching fraction of DN channels and $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ of $\Lambda_c(2940)^+$ as $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$ and $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$

Channel	$\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$	$\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$
$\Sigma_c^{++}\pi^-$	6.85	0.21
$\Sigma_c^{+}\pi^0$	6.91	0.22
$\Sigma_c^{0}\pi^+$	6.85	0.21
$\Sigma_c^{++}(2520)\pi^-$	1.07	5.67
$\Sigma_c^{+}(2520)\pi^0$	1.09	5.76
$\Sigma_c^{0}(2520)\pi^+$	1.07	5.67
$\Sigma_c^{+}(2800)\pi^0$	8.71	6.38×10^{-5}
D^0p	61.62	1.36
D^+n	59.52	1.19
Total width	153.69	20.29
R	0.16	26.72
$\mathcal{B}(\Lambda_c(2880)^+ \to DN)$	79%	13%

Table 5 Possible decay widths (MeV), branching fraction of DN channels and $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ of $\Lambda_c(2880)^+$ as $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$ and $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$

Channel	$\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$	$\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$
$\Sigma_c^{++}\pi^-$	8.54×10^{-2}	6.96×10^{-4}
$\Sigma_c^{+}\pi^0$	8.78×10^{-2}	7.37×10^{-4}
$\Sigma_c^{0}\pi^+$	8.56×10^{-2}	7.00×10^{-4}
$\Sigma_c^{++}(2520)\pi^-$	1.03×10^{-2}	4.55×10^{-2}
$\Sigma_c^{+}(2520)\pi^0$	1.07×10^{-2}	4.73×10^{-2}
$\Sigma_c^{0}(2520)\pi^+$	1.03×10^{-2}	4.55×10^{-2}
D^0p	2.28	7.02×10^{-3}
D^+n	1.92	4.98×10^{-3}
Total width	4.49	0.15
R	0.12	64.84
$\mathcal{B}(\Lambda_c(2880)^+ \to DN)$	94%	8%

As a $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$, the predicted total decay widths of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are much smaller than the observed ones, so $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are impossibly the $1F$-wave excitation $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$.

2.3 $2P$-wave excitations

There are fourteen $2P$-wave excited Λ_c, among which there are four excitations with D^0p decay channels. As indicated in Table 1, these excitations are $\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-, 2P)$, $\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-, 2P)$, $\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$ and $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$. From the appendix, $\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-, 2P)$ and $\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-, 2P)$ are λ mode

Table 6 Possible decay widths (MeV), branching fraction of DN channels and $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)}$ of $\Lambda_c(2940)^+$ as four $2P$-wave excitations

Channel	$\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-, 2P)$	$\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-, 2P)$	$\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$	$\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$
$\Sigma_c^{++}\pi^-$	29.18	2.22	0.06	1.76
$\Sigma_c^{+}\pi^0$	29.14	2.28	0.05	1.79
$\Sigma_c^{0}\pi^+$	29.18	2.23	0.06	1.76
$\Sigma_c^{++}(2520)\pi^-$	2.02	28.27	1.87	1.58
$\Sigma_c^{+}(2520)\pi^0$	2.08	28.32	1.92	1.57
$\Sigma_c^{0}(2520)\pi^+$	2.02	28.26	1.87	1.58
$\Sigma_c^{+}(2800)\pi^0$	0.02	2.66×10^{-3}	0.11	0.01
D^0p	49.18	7.56×10^{-3}	51.83	3.93
D^+n	47.84	1.09×10^{-2}	52.99	3.58
Total width	190.66	91.60	110.76	17.56
R	0.07	12.61	33.29	0.89
$\mathcal{B}(\Lambda_c(2940)^+ \to DN)$	51%	≈0%	95%	43%

radial excitations, and $\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$ and $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$ are ρ mode radial excitations.

The decay widths of possible assignments of $\Lambda_c(2940)^+$ are computed when it is regarded as $\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-, 2P)$, $\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-, 2P)$, $\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$ or $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$. The numerical results are given in Table 6. $\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$ have similar numerical results which have not been presented explicitly.

Once the strong decay widths are taken into account only, $\Lambda_c(2860)^+$ could be the $\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$ under large uncertainty and can not be any other $2P$-wave excited Λ_c. Taking into account the fact that $\Lambda_c(2860)^+$ has a lower mass in comparison to theoretical prediction of 2P-wave, $\Lambda_c(2860)^+$ can not be a $2P$-wave excited Λ_c.

When the predicted total decay widths are compared with the observed ones of $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$, $\Lambda_c(2880)^+$ is impossibly a $2P$-wave excitation for its small total decay width, and $\Lambda_c(2940)^+$ is possibly the $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$. As a $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$ excitation, the total decay width $\Gamma = 17.56$ MeV, the branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.89$ and the branching fraction $\mathcal{B}(\Lambda_c(2940)^+ \to DN) = 43\%$ are predicted for $\Lambda_c(2940)^+$.

In comparison with the D-wave and F-wave excited Λ_c with λ mode excitation only, the $2P$-wave excited Λ_c with ρ mode excitation has a much lower branching fraction of DN channels.

3 Conclusions and discussions

The $1D$-wave, $1F$-wave and $2P$-wave assignments of the high-lying $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are examined from their strong decay properties in the $^3P_0$ model.

Based on experimental results, some possible or favored assignments of these excited Λ_c are suggested to them, and some impossible assignments are pointed out.

$\Lambda_c(2860)^+$ may be the $1D$-wave excited $\Lambda^{2,0}_{c2,0}(\frac{3}{2}^+, 1D)$, it is impossibly the $1D$-wave excited $\Lambda^{2,0}_{c2,0}(\frac{5}{2}^+, 1D)$, $1F$-wave excitation or $2P$-wave excited Λ_c. The D^0p mode is the dominant decay channel with branching fraction $\mathcal{B}(\Lambda_c(2860)^+ \to DN) = 75\%$, and the branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.12$. In experiment, only the D^0p channel has been observed, the observation of other decay channels such as $\Sigma_c\pi$ or $\Sigma_c(2520)\pi$ and measurement of their branching fractions are required to the understand its inner excitation and structure.

As reported in Ref. [4], an analysis of angular distribution in $\Lambda_c(2880)^+ \to \Sigma_c(2455)^{0,++}\pi^{+,-}$ strongly favors the $\Lambda_c(2880)^+$ with spin $\frac{5}{2}$. In their analysis, the measured $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.225 \pm 0.062 \pm 0.025$ is found around the prediction of heavy quark symmetry $R = 0.23 - 0.36$ for the $\frac{5}{2}^+$ state [6–8], so the positive parity was assumed to the $\Lambda_c(2880)^+$ [4]. In our analysis, the $\Lambda_c(2880)^+$ is very possibly the $1F$-wave excited $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$ with negative parity. As a $\Lambda^{3,0}_{c3,0}(\frac{5}{2}^-, 1F)$, our prediction of the total decay width and the branching ratio agrees well with experiments. Furthermore, the dominant decay channel D^0p with branching fraction $\mathcal{B}(\Lambda_c(2880)^+ \to DN) = 94\%$ is also predicted. $\Lambda_c(2880)^+$ is impossibly the $1D$-wave excitation, the $1F$-wave excited $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$ or the $2P$-wave excitation. Accordingly, the J, P quantum numbers of $\Lambda_c(2880)^+$ can not be $J^P = \frac{3}{2}^+$. In experiment, the channels $\Sigma_c(2455)\pi$, $\Sigma_c(2520)\pi$, $\Lambda_c\pi\pi$ and D^0p have been observed, the measurement of all the branching fractions of these channels is very important for the understanding of this state.

In [5], the most likely spin-parity assignment for $\Lambda_c(2940)^+$ was suggested with $J^P = \frac{3}{2}^-$. However, other solutions with spins $\frac{1}{2}$ to $\frac{7}{2}$ have not been excluded. In our analysis, $\Lambda_c(2940)^+$ could be the $2P$-wave excited $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$. It is impossibly the $1D$-wave excited Λ_c, $1F$-wave excited $\Lambda^{3,0}_{c3,0}(\frac{7}{2}^-, 1F)$ or any other $2P$-wave excitations. In the $\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$ assignment, $\Lambda_c(2940)^+$ has a total decay width $\Gamma = 17.56$ MeV, the branching ratio $R = \frac{\Gamma(\Sigma_c(2520)\pi)}{\Gamma(\Sigma_c(2455)\pi)} = 0.89$ and the DN channels with branching fraction $\mathcal{B}(\Lambda_c(2940)^+ \to DN) = 43\%$.

So far, D^0p channel has been observed in all highly excited Λ_c above their threshold, which may imply that the two light quarks in initial baryons enters the final baryon in the strong decay process. In the same time, the DN channels are dominant and the two internal light quarks in initial baryons coupling with a total spin $S_\rho = 0$ in all possible assignments of $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$, which may imply that the two light quarks in initial Λ_c make a good diquark. Furthermore, the $2P$-wave excited Λ_c with ρ mode excitation has a much lower branching fraction of DN channel in comparison with the $1D$-wave and $1F$-wave excited Λ_c with λ mode excitation only. The existence and properties of diquark require more exploration.

In addition to the normal uncertainties, three-body decay cannot be computed in the $^3P_0$ model. In our analyses, the parameters β are chosen the same for ρ mode and λ mode for simplicity though the parameters β (represent the inverse root mean square radius) of ρ and λ mode excitation may be different. More highly excited possibilities to these Λ_c have not yet analyzed. In order to identify these highly-excited Λ_c baryons and to understand their inner structure and dynamics, measurements of the J, P quantum numbers and branching fractions of the main decay channels of these highly excited Λ_c are required, more theoretical analyses in different models are also required.

Acknowledgements This work is supported by National Natural Science Foundation of China under the Grant No. 11975146.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The data can be reproduced without any difficulty through the formula in the paper.]

Funded by SCOAP^3.

Appendix

In the constituent quark model with the heavy-quark symmetry, internal quantum numbers of $1F$-wave and $2P$-wave

Table 7 $2P$-wave excited Λ_c

$\Lambda^{L,L_\rho}_{cJ_L,n_\rho}(J^P, nL)$	n_ρ	n_λ	L_ρ	L_λ	L	S_ρ	J_L	J^P
$\Lambda^{1,0}_{c1,0}(\frac{1}{2}^-, 2P)$	0	1	0	1	1	0	1	$\frac{1}{2}^-$
$\Lambda^{1,0}_{c1,0}(\frac{3}{2}^-, 2P)$	0	1	0	1	1	0	1	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c0,0}(\frac{1}{2}^-, 2P)$	0	1	1	0	1	1	0	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{1}{2}^-, 2P)$	0	1	1	0	1	1	1	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{3}{2}^-, 2P)$	0	1	1	0	1	1	1	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c2,0}(\frac{3}{2}^-, 2P)$	0	1	1	0	1	1	2	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c2,0}(\frac{5}{2}^-, 2P)$	0	1	1	0	1	1	2	$\frac{5}{2}^-$
$\Lambda^{1,0}_{c1,1}(\frac{1}{2}^-, 2P)$	1	0	0	1	1	0	1	$\frac{1}{2}^-$
$\Lambda^{1,0}_{c1,1}(\frac{3}{2}^-, 2P)$	1	0	0	1	1	0	1	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c0,1}(\frac{1}{2}^-, 2P)$	1	0	1	0	1	1	0	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c1,1}(\frac{1}{2}^-, 2P)$	1	0	1	0	1	1	1	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c1,1}(\frac{3}{2}^-, 2P)$	1	0	1	0	1	1	1	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c2,1}(\frac{3}{2}^-, 2P)$	1	0	1	0	1	1	2	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c2,1}(\frac{5}{2}^-, 2P)$	1	0	1	0	1	1	2	$\frac{5}{2}^-$

Table 8 $1F$-wave excited Λ_c

$\Lambda^{L,L_\rho}_{cJ_L,n_\rho}(J^P,nL)$	n_ρ	n_λ	L_ρ	L_λ	L	S_ρ	J_L	J^P
$\Lambda^{3,0}_{c2,0}(\frac{5}{2}^-,1F)$	0	0	0	3	3	0	3	$\frac{5}{2}^-$
$\Lambda^{3,0}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	0	3	3	0	3	$\frac{7}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{1}{2}^-,1F)$	0	0	1	2	1	1	0	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{1}{2}^-,1F)$	0	0	1	2	1	1	1	$\frac{1}{2}^-$
$\Lambda^{1,1}_{c0,0}(\frac{3}{2}^-,1F)$	0	0	1	2	1	1	1	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{3}{2}^-,1F)$	0	0	1	2	1	1	2	$\frac{3}{2}^-$
$\Lambda^{1,1}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	1	2	1	1	2	$\frac{5}{2}^-$
$\Lambda^{2,1}_{c2,0}(\frac{1}{2}^-,1F)$	0	0	1	2	2	1	1	$\frac{1}{2}^-$
$\Lambda^{2,1}_{c2,0}(\frac{3}{2}^-,1F)$	0	0	1	2	2	1	1	$\frac{3}{2}^-$
$\Lambda^{2,1}_{c1,0}(\frac{3}{2}^-,1F)$	0	0	1	2	2	1	2	$\frac{3}{2}^-$
$\Lambda^{2,1}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	1	2	2	1	2	$\frac{5}{2}^-$
$\Lambda^{2,1}_{c2,0}(\frac{5}{2}^-,1F)$	0	0	1	2	2	1	3	$\frac{5}{2}^-$
$\Lambda^{2,1}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	1	2	2	1	3	$\frac{7}{2}^-$
$\Lambda^{3,1}_{c3,0}(\frac{3}{2}^-,1F)$	0	0	1	2	3	1	2	$\frac{3}{2}^-$
$\Lambda^{3,1}_{c3,0}(\frac{5}{2}^-,1F)$	0	0	1	2	3	1	2	$\frac{5}{2}^-$
$\Lambda^{3,1}_{c2,0}(\frac{5}{2}^-,1F)$	0	0	1	2	3	1	3	$\frac{5}{2}^-$
$\Lambda^{3,1}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	1	2	3	1	3	$\frac{7}{2}^-$
$\Lambda^{3,1}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	1	2	3	1	4	$\frac{7}{2}^-$
$\Lambda^{3,1}_{c2,0}(\frac{9}{2}^-,1F)$	0	0	1	2	3	1	4	$\frac{9}{2}^-$
$\Lambda^{1,2}_{c1,0}(\frac{1}{2}^-,1F)$	0	0	2	1	1	0	1	$\frac{1}{2}^-$
$\Lambda^{1,2}_{c1,0}(\frac{3}{2}^-,1F)$	0	0	2	1	1	0	1	$\frac{3}{2}^-$
$\Lambda^{2,2}_{c0,0}(\frac{3}{2}^-,1F)$	0	0	2	1	2	0	2	$\frac{3}{2}^-$
$\Lambda^{2,2}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	2	1	2	0	2	$\frac{5}{2}^-$
$\Lambda^{3,2}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	2	1	3	0	3	$\frac{5}{2}^-$
$\Lambda^{3,2}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	2	1	3	0	3	$\frac{7}{2}^-$
$\Lambda^{3,3}_{c2,0}(\frac{3}{2}^-,1F)$	0	0	3	0	3	1	2	$\frac{3}{2}^-$
$\Lambda^{3,3}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	3	0	3	1	2	$\frac{5}{2}^-$
$\Lambda^{3,3}_{c1,0}(\frac{5}{2}^-,1F)$	0	0	3	0	3	1	3	$\frac{5}{2}^-$
$\Lambda^{3,3}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	3	0	3	1	3	$\frac{7}{2}^-$
$\Lambda^{3,3}_{c2,0}(\frac{7}{2}^-,1F)$	0	0	3	0	3	1	4	$\frac{7}{2}^-$
$\Lambda^{3,3}_{c3,0}(\frac{9}{2}^-,1F)$	0	0	3	0	3	1	4	$\frac{9}{2}^-$

excited Λ_c and their name are given in the following two tables (Tables 7 and 8).

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