Signatures of A_4 symmetry in the charged lepton flavour violating decays in a neutrino mass model

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Received: 7 December 2020 / Accepted: 21 April 2021 / Published online: 3 May 2021
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Abstract We study the charged lepton flavour violation in a popular neutrino mass model with A_4 discrete symmetry. This symmetry requires the presence of multiple Higgs doublets in the model and it also dictates the flavour violating Yukawa couplings of the additional neutral scalars of the model. Such couplings lead to the decays of the neutral mesons, the top quark and the τ lepton into charged leptons of different flavours at tree level. The A_4 symmetry of the model leads to certain characteristic signatures in these decays. We discuss these signatures and predict the rates for the most favourable charged lepton flavour violating modes.

1 Introduction

Neutrino oscillations provide the first hint of physics beyond the standard model. They also imply that the different lepton numbers, L_e , L_μ and L_τ are not conserved individually. Non-conservation of these quantum numbers opens up the possibility of flavour non-conservation in the charged lepton sector also. That is, decays such as $K_L \rightarrow \mu e$, $B_d, B_s \rightarrow \ell_1^+ \ell_2^-$ and other flavour violating decays of heavy quarks and leptons should be possible. Various experiments have been searching for signals of charged lepton flavour violation during the past two decades.

Neutrino oscillations arise because neutrino flavour eigenstates are linear combinations of mass eigenstates. It is this mismatch which leads to flavour violation in the lepton sector. To get the full picture of lepton flavour violation, we need a full-fledged theory of neutrino masses. Given such a theory, it is possible to establish connections between flavour viola-

tions in the neutrino sector and in the charged lepton sector. At present, there are many popular models of neutrino mass. Different models predict different values for the charged lepton violating decays, depending on the details of the model. For a review of charged lepton flavour violations in various popular neutrino mass model, see [1].

The relation between neutrino flavour eigenstates and mass eigenstates is described by the unitary matrix called the PMNS (Pontecorvo–Maki–Nakagawa–Sakata) matrix. This matrix is parametrized in terms of three mixing angles, θ_{12} , θ_{13} and θ_{23} and a CP violating phase δ_{CP} , in analogy to the CKM matrix of the quark sector. Neutrino oscillation data show that $\sin^2 \theta_{12} \approx \frac{1}{3}$, $\sin^2 \theta_{13} \ll 1$, and $\sin^2 \theta_{23} \approx \frac{1}{2}$. The current long baseline experiments, T2K [2] and NOvA [3], are beginning to measure δ_{CP} . However, the best fit values of the δ_{CP} preferred by the two experiments are widely different. T2K prefers δ_{CP} value close to maximal violation ($\delta_{CP} \approx -90^\circ$) whereas NOvA prefers a value close to no CP violation ($\delta_{CP} \approx 0$).

Various discrete symmetries were proposed to explain the pattern of neutrino mixings. The simplest of these is the $\mu \leftrightarrow \tau$ exchange symmetry [4, 5] which predicts $\theta_{13} = 0$ and $\theta_{23} = 45^\circ$ with θ_{12} is left as a model dependent parameter. A number of popular models are based on the group A_4 [6–9] which predict the mixing matrix to be of tri-bi-maximal (TBM) form [10]: that is, $\sin^2 \theta_{13} = 0$, $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{12} = 1/3$. A particular A_4 based model, proposed in Ref. [9], obtains the TBM form of the PMNS matrix purely from symmetry and symmetry breaking considerations without any fine tuning of parameters. In Ref. [11], it was shown that the introduction of a small perturbation in the Majorana mass matrix of the heavy right-chiral neutrinos in this model, can lead to both a realistic value of $\sin^2 \theta_{13} \approx 0.02$ and maximal CP violation.

In this article, we study the charged lepton flavour violation in the model of Ref. [9]. This model contains four $SU(2)$ Higgs doublets. These consist of an A_4 singlet ϕ_0 and an A_4

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triplet ϕ_i ($i = 1, 2, 3$). In addition, there is also an A_4 triplet of scalars χ which are singlets under $SU(2)$. These multiple Higgs representations are required to form the PMNS matrix in the TBM form purely from symmetry considerations. The presence of multiple Higgs doublets, in general, leads to flavour changing Yukawa couplings (FCYC) at tree level. Such couplings can lead to observable branching ratios for decays such as $K_L \rightarrow \mu e$, $B_d, B_s \rightarrow \ell_1^+ \ell_2^-$ and other possible charged lepton violating processes. In particular, these decays carry the characteristic signatures of A_4 symmetry. Constraints arising from the charged lepton flavour violating processes, on the extended scalar sectors due to flavour symmetries, were studied in [12]. The charged lepton flavour violation in B meson decays was studied in [13, 14] in the context of lepto-quark models and in [15] in the context of a neutrino mass model with an A_4 triplet of isospin singlet scalars.

The paper is organized as follows: We give a brief outline of the model of Ref. [9] in Sect. 2 and study its Yukawa Lagrangian in detail in Sect. 3. We first consider the fields in A_4 eigenbasis and then transform the fermion fields into mass eigenbasis and isolate the FCYC terms of interest. Later we consider the Higgs potential and obtain the transformation matrix that gives us the Higgs mass eigenstates. Finally, we write the FCYC terms in terms of the mass eigenstates of the fermions and the Higgs bosons. In Sect. 4, we consider the neutral meson decays into charged leptons of different flavours. In Sect. 5, we discuss the lepton flavour violating decays, along with their A_4 signatures, of the top quark and the τ lepton. We briefly discuss the loop induced processes, $\mu \rightarrow e + \gamma$ and muon ($g - 2$) in Sect. 6 and present our conclusions in the last section.

2 Brief description of the model

The charged fermion content of the A_4 model of Ref. [9] is the same as that of the SM with the same gauge quantum numbers. The model also contains three right-chiral neutrinos which form a triplet representation of A_4 , but have no gauge quantum numbers. The three left chiral $SU(2)$ doublets of quarks ($Q_{iL}, i = 1, 2, 3$) and leptons ($D_{iL}, i = 1, 2, 3$) are also assumed to form triplet representations of A_4 . The $SU(2)$ singlet right chiral charged fermions have non-trivial transformation properties under A_4 . The gauge and the A_4 quantum numbers of all the fermions are shown below:

$$Q_{iL} = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix} \sim (3, 2, \frac{1}{3}) (\underline{3})$$

$$d_{1R} \oplus d_{2R} \oplus d_{3R} \sim (3, 1, -\frac{2}{3}) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'')$$

$$u_{1R} \oplus u_{2R} \oplus u_{3R} \sim (3, 1, \frac{4}{3}) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'')$$

$$D_{iL} = \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix} \sim (1, 2, -1) (\underline{3})$$

$$\ell_{1R} \oplus \ell_{2R} \oplus \ell_{3R} \sim (1, 1, -2) (\underline{1} \oplus \underline{1}' \oplus \underline{1}'') \quad (1)$$

$$\nu_R \sim (1, 1, 0) (\underline{3}).$$

The Higgs field content of the model is dictated by the requirement that the PMNS matrix should be in the TBM form. To this end, three distinct Higgs field representations are introduced: (a) an A_4 triplet of $SU(2)$ doublets ϕ_i , ($i = 1, 2, 3$), (b) an A_4 singlet of $SU(2)$ doublet ϕ_0 and (c) an A_4 triplet of $SU(2)$ singlets χ_i , ($i = 1, 2, 3$). The gauge and A_4 quantum numbers of these fields are shown below:

$$\begin{aligned} \phi_i &= \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \sim (1, 2, 1) (\underline{3}), \quad \phi_0 = \begin{pmatrix} \phi_0^+ \\ \phi_0^0 \end{pmatrix} \sim (1, 2, 1) (\underline{1}), \\ \chi_i^0 &\sim (1, 1, 0) (\underline{3}). \end{aligned} \quad (2)$$

It is possible to write the Higgs potential in such a way that the Higgs fields have the following vacuum expectation values (VEV) [9]:

$$\begin{aligned} \langle \phi_1 \rangle &= \langle \phi_2 \rangle = \langle \phi_3 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \\ \langle \phi_0 \rangle &= \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \quad \langle \chi_i^0 \rangle = (0, w_2, 0). \end{aligned} \quad (3)$$

That is, all the three members of the A_4 triplet ϕ_i have the same VEV and only the second member of the A_4 triplet χ has a non-zero VEV. This arrangement of VEVs is crucial to obtain TBM form of PMNS matrix purely from symmetry considerations.

3 Yukawa Lagrangian and charged fermions in mass eigenbasis

The Dirac mass terms for the fermions arise through the Yukawa interactions between the $SU(2)$ doublet Higgs fields and the fermion fields. Majorana masses for the neutrinos occur partly through bare mass terms and partly through Yukawa couplings of right-chiral neutrinos to the Higgs field χ_i^0 . The gauge and A_4 invariant Yukawa Lagrangian of this model, along with the bare Majorana mass terms, is given by [9, 16]

$$\begin{aligned} \mathcal{L}_{Yuk} = & - \left[h_{1d} (\bar{Q}_{1L} \phi_1 + \bar{Q}_{2L} \phi_2 + \bar{Q}_{3L} \phi_3) d_{1R} \right. \\ & + h_{2d} (\bar{Q}_{1L} \phi_1 + \omega^2 \bar{Q}_{2L} \phi_2 + \omega \bar{Q}_{3L} \phi_3) d_{2R} \\ & + h_{3d} (\bar{Q}_{1L} \phi_1 + \omega \bar{Q}_{2L} \phi_2 + \omega^2 \bar{Q}_{3L} \phi_3) d_{3R} \\ & + h_{1u} (\bar{Q}_{1L} \tilde{\phi}_1 + \bar{Q}_{2L} \tilde{\phi}_2 + \bar{Q}_{3L} \tilde{\phi}_3) u_{1R} \\ & \left. + h_{2u} (\bar{Q}_{1L} \tilde{\phi}_1 + \omega^2 \bar{Q}_{2L} \tilde{\phi}_2 + \omega \bar{Q}_{3L} \tilde{\phi}_3) u_{2R} \right] \end{aligned}$$

$$\begin{aligned}
& +h_{3u}(\bar{Q}_{1L}\tilde{\phi}_1 + \omega\bar{Q}_{2L}\tilde{\phi}_2 + \omega^2\bar{Q}_{3L}\tilde{\phi}_3)u_{3R} + h.c.] \\
& -[h_{1\ell}(\bar{D}_{1L}\phi_1 + \bar{D}_{2L}\phi_2 + \bar{D}_{3L}\phi_3)\ell_{1R} \\
& +h_{2\ell}(\bar{D}_{1L}\phi_1 + \omega^2\bar{D}_{2L}\phi_2 + \omega\bar{D}_{3L}\phi_3)\ell_{2R} \\
& +h_{3\ell}(\bar{D}_{1L}\phi_1 + \omega\bar{D}_{2L}\phi_2 + \omega^2\bar{D}_{3L}\phi_3)\ell_{3R} \\
& +h_0(\bar{D}_{1L}v_{1R} + \bar{D}_{2L}v_{2R} + \bar{D}_{3L}v_{3R})\tilde{\phi}_0 + h.c.] \\
& +\frac{1}{2}[M(v_{1R}^T C^{-1}v_{1R} + v_{2R}^T C^{-1}v_{2R} \\
& +v_{3R}^T C^{-1}v_{3R}) + h.c.] \\
& +\frac{1}{2}[h_\chi((\chi_1(v_{2R}^T C^{-1}v_{3R} + v_{3R}^T C^{-1}v_{2R}) \\
& +\chi_2(v_{3R}^T C^{-1}v_{1R} + v_{1R}^T C^{-1}v_{3R}) \\
& +\chi_3(v_{1R}^T C^{-1}v_{2R} + v_{2R}^T C^{-1}v_{1R})) + h.c.], \quad (4)
\end{aligned}$$

where $\tilde{\phi}_i = i\sigma_2\phi_i^*$ and $\tilde{\phi}_0 = i\sigma_2\phi_0^*$. When the Higgs fields acquire their VEVs, this Lagrangian leads to mass matrices for charged fermions and the neutrinos of the form

$$-\bar{f}_L M_f f_R - \bar{\nu}_L M_D \nu_R + \frac{1}{2}\nu_R^T C^{-1} M_R \nu_R + h.c. \quad (5)$$

Given the Higgs VEVs, we have $M_D = h_0 v_0 \mathbb{I}$ and

$$\begin{aligned}
M_f &= \sqrt{3}v U_\omega^\dagger \begin{pmatrix} h_{1f} & 0 & 0 \\ 0 & h_{2f} & 0 \\ 0 & 0 & h_{3f} \end{pmatrix} \mathbb{I}, \\
M_R &= \begin{pmatrix} M & 0 & h_\chi w_2 \\ 0 & M & 0 \\ h_\chi w_2 & 0 & M \end{pmatrix}, \quad (6)
\end{aligned}$$

where $f = (u, d, \ell)$. The matrix U_ω is given by

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (7)$$

where ω is the cube root of unity. From Eq. (6), we note that the matrix M_f is transformed to mass eigenbasis by making the unitary transformation U_ω on the left-chiral charged fermions f_{iL} but leaving the corresponding right-chiral fields untouched. In the case of charged fermions, we have the following relations between the Yukawa couplings and mass eigenvalues

$$h_{1f} = \frac{1}{\sqrt{3}} \frac{m_{1f}}{v}, \quad h_{2f} = \frac{1}{\sqrt{3}} \frac{m_{2f}}{v}, \quad h_{3f} = \frac{1}{\sqrt{3}} \frac{m_{3f}}{v}, \quad (8)$$

where m_{1f}, m_{2f} and m_{3f} are the masses of the first, second and third generation particles respectively. Given that $m_{3f} \gg m_{2f} \gg m_{1f}$, we have

$$h_{3f} \gg h_{2f} \gg h_{1f}. \quad (9)$$

For charged leptons, the relation between the A_4 eigenstates and the mass eigenstates is

$$\begin{aligned}
\ell_{1L} &= \frac{1}{\sqrt{3}}(e_L + \mu_L + \tau_L), \quad \ell_{2L} = \frac{1}{\sqrt{3}}(e_L + \omega^2 \mu_L + \omega \tau_L), \\
\ell_{3L} &= \frac{1}{\sqrt{3}}(e_L + \omega \mu_L + \omega^2 \tau_L). \quad (10)
\end{aligned}$$

Relations similar to Eq. (10) can be written for both up-type and down-type quarks. Since the same matrix U_ω transforms both up-type and down-type quark fields into their mass eigenstates, the CKM matrix $V_{CKM} = U_\omega^\dagger U_\omega = \mathbb{I}$. It is expected that radiative corrections can generate appropriate non-diagonal elements of this matrix [9].

The diagonalizing matrix of M_R is

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad (11)$$

which leads to the PMNS matrix

$$U = U_\omega U_\nu = \text{diag}(1, \omega, \omega^2) U_{\text{TBM}} \text{diag}(1, 1, -i), \quad (12)$$

where the TBM form is

$$U_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (13)$$

Here again, radiative corrections or other explicit A_4 breaking terms can lead to a phenomenologically viable form of the PMNS matrix with non-zero θ_{13} and δ_{CP} . For example, in ref [11], A_4 symmetry was softly broken by unequal Majorana masses for ν_{iR} . A simple adjustment of these masses leads to the correct value of θ_{13} and maximal CP violation. In terms of the fermion mass eigenstates, the Yukawa Lagrangian can be written as:

$$\mathcal{L}_{Yuk} = \mathcal{L}_{Yuk}^\ell + \mathcal{L}_{Yuk}^u + \mathcal{L}_{Yuk}^d + \mathcal{L}_{Yuk}^v$$

where

$$\begin{aligned}
\mathcal{L}_{Yuk}^\ell &= -\frac{h_{1\ell}}{\sqrt{3}} \left[(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L) \phi_1^0 \right. \\
& + (\bar{e}_L + \omega \bar{\mu}_L + \omega^2 \bar{\tau}_L) \phi_2^0 \\
& + (\bar{e}_L + \omega^2 \bar{\mu}_L + \omega \bar{\tau}_L) \phi_3^0 \left. \right] e_R \\
& - h_{1\ell} \left[\bar{\nu}_{1L} \phi_1^+ + \bar{\nu}_{2L} \phi_2^+ + \bar{\nu}_{3L} \phi_3^+ \right] e_R \\
& - \frac{h_{2\ell}}{\sqrt{3}} \left[(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L) \phi_1^0 \right. \\
& + \omega^2 (\bar{e}_L + \omega \bar{\mu}_L + \omega^2 \bar{\tau}_L) \phi_2^0 \\
& + \omega (\bar{e}_L + \omega^2 \bar{\mu}_L + \omega \bar{\tau}_L) \phi_3^0 \left. \right] \mu_R
\end{aligned}$$

$$\begin{aligned}
& -h_{2\ell} [\bar{v}_{1L}\phi_1^+ + \bar{v}_{2L}\phi_2^+ + \bar{v}_{3L}\phi_3^+] \mu_R \\
& -\frac{h_{3\ell}}{\sqrt{3}} [(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L)\phi_1^0 \\
& + \omega(\bar{e}_L + \omega\bar{\mu}_L + \omega^2\bar{\tau}_L)\phi_2^0 \\
& + \omega^2(\bar{e}_L + \omega^2\bar{\mu}_L + \omega\bar{\tau}_L)\phi_3^0] \tau_R \\
& -h_{3\ell} [\bar{v}_{1L}\phi_1^+ + \bar{v}_{2L}\phi_2^+ + \bar{v}_{3L}\phi_3^+] \tau_R \\
& + \frac{h_0}{\sqrt{3}} [(\bar{e}_L + \bar{\mu}_L + \bar{\tau}_L)v_{1R} + (\bar{e}_L + \omega\bar{\mu}_L \\
& + \omega^2\bar{\tau}_L)v_{2R} + (\bar{e}_L + \omega^2\bar{\mu}_L + \omega\bar{\tau}_L)v_{3R}] \phi_0^- \\
& h.c.
\end{aligned} \quad (14)$$

$\mathcal{L}_{Yuk}^u$ and $\mathcal{L}_{Yuk}^d$ will have a similar structure but without the terms involving h_0 . We will not explicitly discuss $\mathcal{L}_{Yuk}^v$ because it has no role to play in our calculations.

In this model, the light neutrino mass spectrum is predicted to be nearly degenerate. The present direct upper limit on light neutrino mass is 1.1 eV [17]. It is possible to satisfy this limit if the common Dirac mass $h_0 v_0 \sim m_e$ (of the order of 1 MeV) and if the heavy Majorana mass $M_R \simeq 1$ TeV. For simplicity, we further assume that v and v_0 are equal, implying $v_0 = v = v_{SM}/2 = 86$ GeV. Therefore, the Yukawa couplings h_0 and $h_{1\ell}$ are much smaller than the other two Yukawa couplings, i.e. $h_0, h_{1\ell} \ll h_{2\ell} \ll h_{3\ell}$.

$$\mathcal{M}^2 = \begin{pmatrix} \mu_2^2 + 4\lambda_3 v_0^2 + 3\lambda_4 v^2 & \lambda_4 v_0 v & \lambda_4 v_0 v & \lambda_4 v_0 v \\ \lambda_4 v_0 v & \mu_1^2 + 8\lambda_1 v^2 + \lambda_4 v_0^2 & 2\lambda_1 v^2 & 2\lambda_1 v^2 \\ \lambda_4 v_0 v & 2\lambda_1 v^2 & \mu_1^2 + 8\lambda_1 v^2 + \lambda_4 v_0^2 & 2\lambda_1 v^2 \\ \lambda_4 v_0 v & 2\lambda_1 v^2 & 2\lambda_1 v^2 & \mu_1^2 + 8\lambda_1 v^2 + \lambda_4 v_0^2 \end{pmatrix}. \quad (17)$$

3.1 Higgs potential of the model and the Higgs mass eigenstates

The most general A_4 symmetric Higgs potential, in terms of the different A_4 representations, can be written as the sum of several parts,

$$\begin{aligned}
V = & V(\phi_i) + V(\chi) + V(\phi_0) + V(\phi_i, \chi) + V(\phi_i, \phi_0) \\
& + V(\phi_0, \chi) + V(\phi_i, \chi, \phi_0).
\end{aligned} \quad (15)$$

In Ref. [9], there is a detailed discussion on the minimization of the Higgs potential in this model. The first three terms in Eq. (15) correspond to self interaction of the three Higgs multiplets while the remaining terms give the interactions between them. To identify the various Higgs mass eigen-

states, we need to diagonalize the matrix $(\partial^2 V / \partial s_i \partial s_j)_{VEV}$, where s_i, s_j are two generic Higgs fields in the model. The full calculation is algebraically cumbersome. Hence we make some simplifying assumptions. We are interested in flavour changing neutral interactions of charged leptons mediated by scalars, which arise only due to the Yukawa couplings of the $SU(2)$ Higgs doublets. The $SU(2)$ singlet Higgs χ has no role to play in such interactions. Therefore, for simplicity, we neglect the admixture of $SU(2)$ doublets and $SU(2)$ singlet in forming the mass eigenstates. Hence we drop the terms containing χ in the Higgs potential. We make a further simplification which makes the algebra easier to handle but retains all the features of charged lepton flavour violations that are the focus of our work. Among the quartic terms of the potential, we keep only the terms containing the combination $(\phi_1^2 + \phi_2^2 + \phi_3^2)$ and set all other coefficients to be zero. This approximation makes the Higgs potential CP conserving.

The simplified Higgs potential is:

$$\begin{aligned}
V(\phi_\alpha) = & \mu_1^2(\phi_1^2 + \phi_2^2 + \phi_3^2) + \lambda_1(\phi_1^2 + \phi_2^2 + \phi_3^2)^2 + \mu_2^2 \phi_0^2 \\
& + \lambda_3 \phi_0^4 + \lambda_4(\phi_1^2 + \phi_2^2 + \phi_3^2) \phi_0^2,
\end{aligned} \quad (16)$$

where $\phi_\alpha^2 = \phi_\alpha^\dagger \phi_\alpha$ ($\alpha = 0, 1, 2, 3$). The mass squared matrix is obtained from the potential by

$$\mathcal{M}_{\alpha\beta}^2 = \frac{\partial^2 V(\phi_\alpha)}{\partial \phi_\alpha^* \partial \phi_\beta} \Big|_{VEV},$$

whose explicit form is

From the assumptions we made, it follows that the $\mathcal{M}^2$ is a real symmetric matrix which is diagonalized by the following orthogonal matrix,

$$U_H = \begin{pmatrix} \frac{x}{\sqrt{3+x^2}} & \frac{y}{\sqrt{3+y^2}} & 0 & 0 \\ \frac{1}{\sqrt{3+x^2}} & \frac{1}{\sqrt{3+y^2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3+x^2}} & \frac{1}{\sqrt{3+y^2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3+x^2}} & \frac{1}{\sqrt{3+y^2}} & 0 & -\frac{2}{\sqrt{6}} \end{pmatrix}. \quad (18)$$

In Eq. (18), the parameters x and y are defined by

$$x, y = \frac{1}{b}(a \pm \sqrt{a^2 + 3b^2}), \quad (19)$$

where

$$\begin{aligned}
a = & (2\lambda_4 - 8\lambda_1)v^2 + 2\lambda_3 v_0^2, \\
b = & 2\lambda_4 v v_0.
\end{aligned} \quad (20)$$

From Eqs. (19) and (20), we find that $xy = -3$, which guarantees the orthogonality of the first two columns of U_H and

of its rows. We denote the mass eigenstates of the neutral scalars to be Φ_α^0 , ($\alpha = 0, 1, 2, 3$). It can be shown that the imaginary part of Φ_0^0 becomes the Goldstone boson coupling to Z^0 and the real part of Φ_0^0 has the same properties as the SM Higgs boson. This can be identified with the 125 GeV Higgs boson observed by ATLAS [18] and CMS [19] experiments. This model contains three heavier complex neutral scalars which are denoted by Φ_i^0 , ($i = 1, 2, 3$). The diagonalization of the $\mathcal{M}^2$ matrix in Eq. (17) leads to degenerate eigenvalues for the states Φ_2^0 and Φ_3^0 . The relationship between mass eigenbasis and A_4 eigenbasis of the $SU(2)$ doublet scalars is given by:

$$\phi_\alpha^0 = (U_H)_{\alpha\beta} \Phi_\beta^0. \quad (21)$$

3.2 Yukawa couplings in the mass eigenbasis of fermions and scalars

In this work, we are interested in tree level flavour changing couplings of charged fermions to neutral scalars. The terms in Eq. (14), proportional to h_0 , do not lead to such couplings. From now on, we concentrate on terms containing the couplings h_{1f} , h_{2f} and h_{3f} . We take the relevant terms in Eq. (14) and transform the scalars, which are in their A_4 eigenstates, into their mass eigenstates. With this transformation the Yukawa couplings are in the mass eigenbasis of both the fermions and the scalars.

$$\begin{aligned} \mathcal{L}_{Yuk}^\ell = & -\frac{h_{1\ell}}{\sqrt{3}} \left[(\bar{\ell}_L + \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \right. \\ & \left. \left. - \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \right. \\ & + (\bar{\ell}_L + \omega \bar{\mu}_L + \omega^2 \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \\ & \left. + \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \\ & + (\bar{\ell}_L + \omega^2 \bar{\mu}_L + \omega \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \\ & \left. \left. - \frac{2}{\sqrt{6}} \Phi_3^0 \right) \right] e_R \\ & - \frac{h_{2\ell}}{\sqrt{3}} \left[(\bar{\ell}_L + \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 \right. \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \frac{h_{2\ell}}{\sqrt{3}} \left[(\bar{\ell}_L + \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 \right. \right. \right. \\ & \left. \left. + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 - \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \right. \\ & \left. \left. \left. - \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \right] \right. \end{aligned}$$

$$\begin{aligned} & + (\omega^2 \bar{\ell}_L + \bar{\mu}_L + \omega \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \\ & \left. + \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \\ & + (\omega \bar{\ell}_L + \bar{\mu}_L + \omega^2 \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 \right. \\ & \left. + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 - \frac{2}{\sqrt{6}} \Phi_3^0 \right) \Big] \mu_R \\ & - \frac{h_{3\ell}}{\sqrt{3}} \left[(\bar{\ell}_L + \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \right. \\ & \left. \left. - \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \right. \\ & + (\omega \bar{\ell}_L + \omega^2 \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 \right. \\ & \left. + \frac{1}{\sqrt{2}} \Phi_2^0 + \frac{1}{\sqrt{6}} \Phi_3^0 \right) \\ & + (\omega^2 \bar{\ell}_L + \omega \bar{\mu}_L + \bar{\tau}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^0 \right. \\ & \left. + \frac{1}{\sqrt{3+y^2}} \Phi_1^0 - \frac{2}{\sqrt{6}} \Phi_3^0 \right) \Big] \tau_R + h.c. \quad (22) \end{aligned}$$

The Yukawa couplings similar to Eq. (22) can be written for down quark sector as well. The corresponding Yukawa couplings for the up quark sector are

$$\begin{aligned} \mathcal{L}_{Yuk}^u = & -\frac{h_{1u}}{\sqrt{3}} \left[(\bar{u}_L + \bar{c}_L + \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \Big) \\ & + (\bar{u}_L + \omega \bar{c}_L + \omega^2 \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} + \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \Big) \\ & + (\bar{u}_L + \omega^2 \bar{c}_L + \omega \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{2}{\sqrt{6}} \Phi_3^{0*} \Big) \Big] u_R \\ & - \frac{h_{2u}}{\sqrt{3}} \left[(\bar{u}_L + \bar{c}_L + \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \Big) \\ & + (\omega^2 \bar{u}_L + \bar{c}_L + \omega \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\ & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} + \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \Big) \end{aligned}$$

$$\begin{aligned}
 & + \left(\omega \bar{u}_L + \bar{c}_L + \omega^2 \bar{t}_L \right) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\
 & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{2}{\sqrt{6}} \Phi_3^{0*} \left. \right) c_R \\
 & - \frac{h_{3u}}{\sqrt{3}} \left[(\bar{u}_L + \bar{c}_L + \bar{t}_L) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \right. \\
 & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \left. \right) \\
 & + \left(\omega \bar{u}_L + \omega^2 \bar{c}_L + \bar{t}_L \right) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\
 & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} + \frac{1}{\sqrt{2}} \Phi_2^{0*} + \frac{1}{\sqrt{6}} \Phi_3^{0*} \left. \right) \\
 & + \left(\omega^2 \bar{u}_L + \omega \bar{c}_L + \bar{t}_L \right) \left(\frac{1}{\sqrt{3+x^2}} \Phi_0^{0*} \right. \\
 & + \frac{1}{\sqrt{3+y^2}} \Phi_1^{0*} - \frac{2}{\sqrt{6}} \Phi_3^{0*} \left. \right) \left. \right] t_R + h.c. \quad (23)
 \end{aligned}$$

Note that, in Eqs. (22) and (23), the couplings of Φ_2^0 and Φ_3^0 to charged fermions are purely flavour violating whereas those of Φ_0^0 and Φ_1^0 are purely flavour conserving. Hence, there are no tree level amplitudes for decays with flavour violation at only one vertex, such as $\mu \rightarrow e \bar{e} e$, $\tau \rightarrow \mu \bar{\mu} \mu$, $\tau \rightarrow e \bar{e} e$, $K_L \rightarrow \mu^+ \mu^-$, $B_d \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$.

Before we consider the charged lepton flavour violating phenomenology of Φ_2^0 and Φ_3^0 , let us consider the limits on the masses the heavy neutral scalars. The couplings of Φ_0^0 to fermions are expected to be flavour diagonal because the real and imaginary parts of Φ_0^0 are identified with the SM Higgs boson and the Goldstone boson coupling to Z^0 respectively. The flavour diagonal couplings of Φ_1^0 make it a SM-like heavy Higgs boson and it can be produced in proton-proton ($p-p$) collisions by the same processes which produce the SM Higgs boson. Recently, the CMS experiment has set a lower limit on the mass of such scalar $m_{\Phi_1} \geq 1870$ GeV [20]. Since, Φ_2^0 and Φ_3^0 have purely flavour violating couplings, they can not be produced via gluon-gluon fusion in $p-p$ collisions. The dominant process for their production will be vector boson fusion. At present, the lower limit on the masses of neutral heavy scalars produced via vector boson fusion is only 300 GeV [21].

4 Lepton flavour violating decays of neutral mesons

Decays of neutral mesons, made of down-type quarks, are studied in more detail compared to neutral mesons made of up-type quarks. Here we limit ourselves to the decays of neutral K , B_d and B_s mesons into charged leptons with flavour violation. Consider the decay of the meson with quark con-

Fig. 1 Tree level Feynman diagram for the $\bar{q}_i q_j \rightarrow \ell_m^+ \ell_n^-$ transition, mediated by Φ_2^0 and Φ_3^0

tent $\bar{q}_i q_j$ into the final state $\ell_m^+ \ell_n^-$, with $m \neq n$. Since Φ_2^0 and Φ_3^0 have flavour violating couplings to both quarks and to charged leptons, their exchange can mediate the above decays at tree level. The flavour changing couplings of these heavy neutral scalars can be written, in generic form, as

$$\begin{aligned}
 & g^{ij} \bar{f}_{iL} f_{jR} \Phi_2^0 + \tilde{g}^{ij} \bar{f}_{iL} f_{jR} \Phi_3^0 + (g^{ji})^* \bar{f}_{iR} f_{jL} (\Phi_2^0)^* \\
 & + (\tilde{g}^{ji})^* \bar{f}_{iR} f_{jL} (\Phi_3^0)^*. \quad (24)
 \end{aligned}$$

From Eq. (22), we find that

$$\begin{aligned}
 g^{ij} &= \frac{h_j}{\sqrt{6}} (1 - \omega) \\
 \tilde{g}^{ij} &= \frac{h_j}{\sqrt{2}} \omega^2, \quad (25)
 \end{aligned}$$

for the “odd” permutations $(ij) = (21), (32), (13)$ and

$$\begin{aligned}
 g^{ij} &= \frac{h_j}{\sqrt{6}} (1 - \omega^2) \\
 \tilde{g}^{ij} &= \frac{h_j}{\sqrt{2}} \omega, \quad (26)
 \end{aligned}$$

for the “even” permutations $(ij) = (12), (23), (31)$.

The Feynman diagram for the transition $\bar{q}_i q_j \rightarrow \ell_m^+ \ell_n^-$ is given Fig. 1. From the vertex factors given in Eqs. (25) and (26), we find the four fermion amplitudes to be

$$\begin{aligned}
 & \left[\frac{(g_q^{ij}) (g_\ell^{mn})^*}{p^2 - m_{\Phi_2}^2} + \frac{(\tilde{g}_q^{ij}) (\tilde{g}_\ell^{mn})^*}{p^2 - m_{\Phi_3}^2} \right] (\bar{q}_{iL} q_{jR}) (\bar{\ell}_{nR} \ell_{mL}) \\
 & + \left[\frac{(g_q^{ji})^* (g_\ell^{nm})}{p^2 - m_{\Phi_2}^2} + \frac{(\tilde{g}_q^{ji})^* (\tilde{g}_\ell^{nm})}{p^2 - m_{\Phi_3}^2} \right] (\bar{q}_{iR} q_{jL}) (\bar{\ell}_{nL} \ell_{mR}), \quad (27)
 \end{aligned}$$

where g ($\tilde{g}$) correspond to the generic Yukawa coupling due to Φ_2^0 (Φ_3^0) and $p^2 \ll m_{\Phi_2}^2, m_{\Phi_3}^2$ is the momentum exchanged in the process. The coefficient of the Φ_2^0 exchange amplitude is **equal** to that of the Φ_3^0 exchange amplitude if (ij) and (mn) are both even or both odd. If one is even and the other is odd, then the two coefficients are still of the same magnitude but of **opposite** sign. For such cases, the two

amplitudes exactly cancel each other in the limit $m_{\Phi_2} = m_{\Phi_3}$ that we consider here.

The net amplitude for the decay $\bar{q}_i q_j \rightarrow \ell_m^+ \ell_n^-$ has two terms, one from the first line of Eq. (27) and one from the second line. The term from the first line has the coefficient $(h_j h_n)$ and the term from the second line has coefficient $(h_i h_m)$. Depending the values of (ij) and (mn) , one of these terms will dominate the other. The amplitudes due to Φ_2^0 and Φ_3^0 exchange add for the seven decays (and their charge conjugate decays) listed below.

- $K^0(\bar{s}d) \rightarrow \mu^+ e^-$ with coefficients $(h_{1d} h_{1\ell})$ and $(h_{2d} h_{2\ell})$
- $B_d^0(\bar{b}d) \rightarrow e^+ \mu^-$ with coefficients $(h_{1d} h_{2\ell})$ and $(h_{3d} h_{1\ell})$
- $B_d^0(\bar{b}d) \rightarrow \mu^+ \tau^-$ with coefficients $(h_{1d} h_{3\ell})$ and $(h_{3d} h_{2\ell})$
- $B_d^0(\bar{b}d) \rightarrow \tau^+ e^-$ with coefficients $(h_{1d} h_{1\ell})$ and $(h_{3d} h_{3\ell})$.
- $B_s^0(\bar{b}s) \rightarrow \mu^+ e^-$ with coefficients $(h_{2d} h_{1\ell})$ and $(h_{3d} h_{2\ell})$
- $B_s^0(\bar{b}s) \rightarrow e^+ \tau^-$ with coefficients $(h_{2d} h_{3\ell})$ and $(h_{3d} h_{1\ell})$
- $B_s^0(\bar{b}s) \rightarrow \tau^+ \mu^-$ with coefficients $(h_{2d} h_{2\ell})$ and $(h_{3d} h_{3\ell})$.

In the cases of the four decays, $K^0 \rightarrow \mu^+ e^-$, $B_d^0 \rightarrow \tau^+ e^-$, $B_s^0 \rightarrow \mu^+ e^-$ and $B_s^0 \rightarrow \tau^+ \mu^-$ (and their charge conjugate modes), we have the product of the larger quark Yukawa coupling with the larger lepton Yukawa coupling. Hence these four decays are likely to have significant branching ratios in this model.

Before going into the details of the calculation, we would like to emphasize an important feature of lepton flavour violation in this model. In the case of the decays $K^0 \rightarrow \mu^+ e^-$ and $\bar{K}^0 \rightarrow \mu^- e^+$, the amplitudes due to Φ_2^0 and Φ_3^0 exchange add but in the case of the decays with charge conjugate final states, $K^0 \rightarrow \mu^- e^+$ and $\bar{K}^0 \rightarrow \mu^+ e^-$, the two amplitudes cancel. Hence a neutral meson with the given flavour quantum numbers can decay only into a particular flavour combination of charged lepton pair but not into its charge conjugate pair. This charged lepton flavour selection is a **signature** of the A_4 symmetry of the Yukawa couplings between the fermions and the scalar doublets in this model. But, such a signature will be difficult to observe experimentally in the case of neutral kaon decays because the physical decays observed are those of K_L which contains roughly equal parts of K^0 and $\bar{K}^0$. Since the model predicts equal rates for $K^0 \rightarrow \mu^+ e^-$ and $\bar{K}^0 \rightarrow \mu^- e^+$, it predicts equal branching ratios for $K_L \rightarrow \mu^+ e^-$ and $K_L \rightarrow \mu^- e^+$. It may be possible to observe the above A_4 signature of charged lepton flavour selection in the leptonic decays B_d mesons. Since $B_d^0 - \bar{B}_d^0$ are produced in pairs, it is possible to show that the final state $\tau^+ e^-$ occurred due to the decay of B_d^0 , rather than $\bar{B}_d^0$, by tagging the flavour of the B meson on the other side.

Among the four favoured decays of neutral mesons to charged leptons discussed above, the experimental upper bound on $\Gamma(K^0 \rightarrow \mu^+ e^-)$, which is easily related to the branching ratio of $(K_L \rightarrow \mu^+ e^-)$, is the strongest. We use

this mode to obtain a lower limit on m_Φ , the common mass of Φ_2^0 and Φ_3^0 . Using this value of m_Φ , we predict the branching ratios of the other three favoured decays in this model. From the expression in Eq. (27), we find the amplitude for $K^0 \rightarrow \mu^+ e^-$ to be

$$\begin{aligned} \mathcal{A}(K^0 \rightarrow \mu^+ e^-) &= -\frac{h_{2d} h_{2\ell}}{4m_\Phi^2} \langle 0 | \bar{s}(1 - \gamma_5) d | K^0 \rangle \langle \mu^+ e^- | \bar{e}(1 + \gamma_5) \mu | 0 \rangle. \end{aligned} \quad (28)$$

The decay rate for $K_L \rightarrow \mu^+ e^-$, due to this amplitude, is

$$\begin{aligned} \Gamma(K_L \rightarrow \mu^+ e^-) &= \frac{1}{16\pi} \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 \left(\frac{m_s}{m_s + m_d}\right)^2 \frac{m_\mu^2 f_K^2 m_K^5}{(3v^2)^2 (2m_\Phi)^4} \\ &\approx \frac{m_K}{16\pi} \left(\frac{m_\mu}{\sqrt{3}v}\right)^2 \left(\frac{f_K}{\sqrt{3}v}\right)^2 \left(\frac{m_K}{2m_\Phi}\right)^4 \end{aligned} \quad (29)$$

where we set $(1 - m_\mu^2/m_K^2 \approx 1)$ and $m_s/(m_s + m_d) \approx 1$. In Eq. (29), m_K (m_μ) is the mass of the kaon (muon), f_K is the kaon decay constant and $v = v_{\text{SM}}/2$ is the common VEV of the four Higgs doublets (the A_4 singlet ϕ_0 and the A_4 triplet ϕ_i). Substituting the appropriate values for the parameters in Eq. (29) and comparing the resultant expression to the experimental upper bound $BR(K_L \rightarrow \mu^+ e^-) < 4.7 \times 10^{-12}$ [22], we obtain the lower bound on m_Φ to be

$$m_\Phi \geq 750 \text{ GeV}. \quad (30)$$

This bound satisfies the present experimental lower limit of 300 GeV [21]. For the lowest allowed value of m_Φ , the branching ratios of the other favoured modes are predicted to be

$$\begin{aligned} BR(B_d^0 \rightarrow \tau^+ e^-) &= 8 \times 10^{-9}, \\ BR(B_s^0 \rightarrow \mu^+ e^-) &= 3.5 \times 10^{-11}, \\ BR(B_s^0 \rightarrow \tau^+ \mu^-) &= 8 \times 10^{-9}. \end{aligned} \quad (31)$$

The respective present experimental upper bounds on these branching ratios are (3×10^{-5}) [23], (5.4×10^{-9}) [24] and (4.2×10^{-5}) [25].

5 Signatures of A_4 symmetry in the decays of the τ lepton and the top quark

5.1 Decays of τ lepton

Important signatures of the A_4 symmetry occur in the decay of τ leptons into three charged leptons. From the Yukawa couplings in Eq. (22), we can show that in the case of the decays with the same charge dileptons $\tau^- \rightarrow e^- e^- \mu^+$ and $\tau^- \rightarrow \mu^- \mu^- e^+$, the amplitudes to Φ_2^0 and Φ_3^0 exchange add

but they cancel for the decays with opposite charge dileptons $\tau^- \rightarrow \mu^+ \mu^- e^-$ and $\tau^- \rightarrow \mu^- e^+ e^-$ (which, in principle, can occur with flavour violation at both vertices). This occurrence of same sign dileptons in τ^- decays is a very distinctive signature of the A_4 symmetry of the Yukawa couplings.

From the form of the vertices given in Eqs. (25) and (26), we find that the amplitude for the decay $\tau^- \rightarrow \mu^- \mu^- e^+$ is proportional to $h_{2\ell} h_{3\ell} / m_\Phi^2$. Since the Yukawa couplings to μ and τ are rather small, we find that the decay rate into this mode is quite small. We calculate the branching ratio of this mode to be $\simeq 10^{-14}$ for $m_\Phi = 750$ GeV.

The branching ratio for $\tau^- \rightarrow e^- e^- \mu^+$ will be smaller by four more orders of magnitude because the corresponding amplitude is proportional to $h_{1\ell} h_{3\ell} / m_\Phi^2$.

5.2 Decays of top quark

In this model, a number of flavour changing couplings of the top quark have amplitudes proportional to the large top Yukawa coupling. Hence the branching ratios of the decays of the top quark, mediated by Φ_2^0 and Φ_3^0 can be measurably large. The A_4 structure of the top quark couplings to Φ_2^0 and Φ_3^0 implies that the amplitudes for the decays $t \rightarrow (u, c) \ell_1^+ \ell_2^-$ have the following forms:

- $\mathcal{A}(t \rightarrow c \mu^+ e^-) \propto \frac{h_{3u} h_{2\ell}}{m_\Phi^2}$
- $\mathcal{A}(t \rightarrow c \tau^+ \mu^-) \propto \frac{h_{3u} h_{3\ell}}{m_\Phi^2}$
- $\mathcal{A}(t \rightarrow u \mu^+ \tau^-) \propto \frac{h_{3u} h_{2\ell}}{m_\Phi^2}$
- $\mathcal{A}(t \rightarrow u \tau^+ e^-) \propto \frac{h_{3u} h_{3\ell}}{m_\Phi^2}$

In the above list, we omitted the two decays, whose amplitude is proportional to the electron Yukawa coupling, which makes the branching ratio much smaller than those of the above modes. Also, note that the A_4 symmetry of the model is dictating the flavours and charges of the final state leptons. Cancellation of the amplitudes due to Φ_2^0 and Φ_3^0 exchange means that the decays into final states with the lepton charges reversed can not occur. Since the dominant production of the top quark is in the form of $t\bar{t}$ pairs, it is possible to identify the flavour of the quark decaying into charged leptons of different flavours by tagging the flavour of the quark on the opposite side. Thus, establishing the A_4 signature of the charged lepton flavour selection in top decays is straight forward.

The two decays $t \rightarrow c \tau^+ \mu^-$ and $t \rightarrow u \tau^+ e^-$ have the largest couplings possible and their branching ratios are $\simeq 10^{-9}$ for $m_\Phi = 750$ GeV. The branching ratios for the other two modes are $\simeq 5 \times 10^{-12}$.

At present, the upper bound on the branching ratio of the decays of top quark into charged leptons of different flavours is 2×10^{-5} [26]. Thus there is a possibility that the favourable

two decays listed above can be observed in the next run of the Large Hadron Collider (LHC).

6 Other flavour violating processes

Neutral meson mixing usually provides the strongest possible constraints on tree level scalars with flavour violating couplings. Such mixing involves quark flavour transitions $\bar{q}_i q_j \rightarrow \bar{q}_i q_j$. Comparing it to $\bar{\ell}_m^+ \ell_n^-$ transition, we note the permutation structure involves the product $(ij) * (ji)$, which is **always odd**. Hence the term due to Φ_3 exchange exactly cancels the term due to Φ_2 exchange and the neutral mixing is absent at tree level.

The exact cancellation of the neutral meson mixing due to the exchange of Φ_2^0 and Φ_3^0 occurs only for the case when their masses are exactly equal. If these masses are unequal, then the neutral meson mixing leads to a very strong constraint on the difference of the two masses. The effective four fermion operator for $K^0 \rightarrow \bar{K}^0$ transition has the form

$$H_{\text{eff}}(K^0 \rightarrow \bar{K}^0) = 2\bar{s}_L d_R \left[\frac{(g^{sd})(g^{ds})^*}{p^2 - m_{\Phi_2}^2} + \frac{(\tilde{g}^{sd})(\tilde{g}^{ds})^*}{p^2 - m_{\Phi_3}^2} \right] \bar{s}_R d_L. \quad (32)$$

If $m_{\Phi_2} = m_{\Phi_3} = m_\Phi$, the coefficient of this operator is

$$\frac{(g^{sd})(g^{ds})^* + (\tilde{g}^{sd})(\tilde{g}^{ds})^*}{p^2 - m_\Phi^2} = 0.$$

For unequal masses, we write $m_{\Phi_2} = m_\Phi - \delta m$ and $m_{\Phi_3} = m_\Phi + \delta m$. so that Φ_2^0 and Φ_3^0 are no longer degenerate. We assume that $\delta m \ll m_\Phi$ and keep terms which are first order in $\delta m / m_\Phi$. In this approximation, the four fermion operator becomes

$$H_{\text{eff}}(K^0 \rightarrow \bar{K}^0) = -4 \frac{1}{m_\Phi^2} \frac{\delta m}{m_\Phi} \left[(g^{sd})(g^{ds})^* - (\tilde{g}^{sd})(\tilde{g}^{ds})^* \right] \bar{s}_L d_R \bar{s}_R d_L. \quad (33)$$

From the g^{ij} and $\tilde{g}^{ij}$ couplings, we can show that

$$\left[(g^{sd})(g^{ds})^* - (\tilde{g}^{sd})(\tilde{g}^{ds})^* \right] = -h_{1d} h_{2d} \omega.$$

We find the neutral kaon mass difference to be

$$\begin{aligned} \frac{\Delta M_K}{m_K} &= \frac{1}{m_K^2} \text{Re}(\langle \bar{K}^0 | H_{\text{eff}} | K^0 \rangle) \\ &= \frac{1}{2} (h_{1d} h_{2d}) \frac{f_K^2}{m_\Phi^2} \frac{m_K^2}{(m_s + m_d)^2} \frac{\delta m}{m_\Phi}. \end{aligned} \quad (34)$$

For $m_\Phi = 750$ GeV, we get

$$\frac{\Delta M_K}{m_K} (NP) = 10^{-14} \frac{\delta m}{m_\Phi}.$$

Fig. 2 Feynman diagram for muon ($g - 2$) with Φ_2^0 and Φ_3^0 exchange

Demanding that this should be less than the experimental value 7×10^{-15} **does not** give a useful constraint on $(\delta m/m_\Phi)$.

We do, however, get a very strong constraint when we consider ϵ_K , which characterizes the CP violation in $K^0 - \bar{K}^0$ mixing. Since $Im(\omega) = \sqrt{3}Re(\omega)$, we get

$$\begin{aligned} \epsilon_K(NP) &\approx \frac{Im(M_{12})(NP)}{\Delta m_K(exp)} = \frac{Im(M_{12})(NP)}{m_K} \left(\frac{\Delta m_K(exp)}{m_K} \right)^{-1} \\ &= \frac{\sqrt{3} \times 10^{-14}(\delta m/m_\Phi)}{7 \times 10^{-15}} \leq \epsilon_K(exp) = 2 \times 10^{-3}, \end{aligned}$$

which leads to the very strong constraint $(\delta m/m_\Phi) \leq 10^{-3}$. Even if Φ_2^0 and Φ_3^0 have unequal masses, their splitting can not be more than a GeV or so. The amplitudes for the **cancelling processes** are suppressed by $(\delta m/m_\Phi) \sim 10^{-3}$ relative to the **adding processes**. Hence, the rates for the unfavoured processes, which are predicted to have zero rate in the case of exactly equal masses for Φ_2 and Φ_3 , are smaller by a factor of 10^{-6} compared to the rates of processes which were discussed earlier in the paper.

Looking at the loop process $\mu \rightarrow e + \gamma$, we find that its amplitude also is a product of even-odd permutations and hence vanishes in this model. On the other hand the muon ($g - 2$) operator has the product of odd-odd permutations for which the amplitudes due to Φ_2^0 and Φ_3^0 exchange add up. The corresponding Feynman diagram is shown in Fig. 2 and the amplitude is proportional to $h_{3\ell}^2$, which is the largest leptonic Yukawa coupling. However, the contribution of Fig. 2 to muon ($g - 2$) is about 2×10^{-14} for $m_\Phi = 750$ GeV [27]. This value is much smaller than the present discrepancy between the experiment and the theory.

7 Conclusions

In this paper, we studied the charged lepton flavour violation in a neutrino mass model with A_4 symmetry. This model

has the attractive feature that it predicts the tri-bi-maximal form of the neutrino mixing matrix purely from the symmetry considerations. The Yukawa couplings of the fermions to the multiple Higgs doublets of this model are guided by the A_4 symmetry. The flavour violating decays, mediated by heavy neutral scalars of this model, carry signatures of the A_4 symmetry of the Yukawa couplings.

We used a simplified form of the Higgs potential to obtain the Higgs mass eigenstates. In this simplified form, two neutral scalars have only flavour conserving couplings and two degenerate neutral scalars have only flavour violating couplings. In this situation, the amplitudes for certain flavour violating transitions add and for certain other transitions exactly cancel. Usually the neutral meson mixing and radiative charged lepton flavour violating decays provide the strongest constraints on the heavy neutral scalar masses. But there is **no** contribution to these processes when the flavour violating neutral scalars of this model have exactly equal masses. If we assume a small splitting between the two masses, the CP violation in kaon mixing constrains the splitting to be one part in a thousand. Thus, even if the neutral scalars have unequal masses, the rates of the unfavoured processes are about a million times smaller than the rates of the favoured processes.

We calculated the rates for a number favoured charged lepton flavour violating decays of neutral mesons, the top quark and the τ lepton in this work. Comparing the prediction of this model for the branching ratio of $K_L \rightarrow \mu^+ e^-$ to the present upper bound, we derived a lower bound $m_\Phi \geq 750$ GeV, on the mass of the heavy neutral scalars which have flavour changing couplings. We found a number of charged lepton flavour violating decay modes of neutral B mesons and the top quark which can be measured in the near future at LHC experiments or at Belle-II [28].

Acknowledgements JM would like to thank the Department of Science and Technology (DST), Government of India, for financial support through Grant No. SR/WOS-A/PM-6/2019(G).

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: There is no new data generated in our work. We derived constraints on a popular model using data already published by different experiments. The data that we used was appropriately cited.]

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Funded by SCOAP³.

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