Retarded resonance Casimir–Polder interaction of a uniformly rotating two-atom system

Saptarshi Saha^a , Chiranjeeb Singha^b , Arpan Chatterjee^c

Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, WB 741 246, India

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Abstract We consider a two-atom system uniformly moving through a circular ring at an ultra-relativistic speed and weakly interacting with the common quantum fields. Two kinds of fields are introduced here: a massive free scalar field and electromagnetic (EM) vector fields. The vacuum fluctuations of the quantum fields give rise to the resonance Casimir–Polder interaction (RCPI) in the system. Using the quantum master equation formalism, we calculate the second-order energy shift of the entangled states of the system. We find two major aspects of RCPI in a circular trajectory. The first one is the presence of the centripetal acceleration, which gives rise to non-thermality in the system, and secondly, due to the interaction with the above fields, the energy shift for RCPI is retarded in comparison with the massless scalar field. The retardation effect can die out by decreasing the centripetal acceleration and increasing the Zeeman frequency of the atoms. We also show that this phenomenon can be observed via the polarization transfer technique. The coherence time for the polarization transfer is calculated, which is different for the different fields.

1 Introduction

A remarkable distinction of classical and quantum domain is the existence of the zero-point energy [1]. Feynman pointed out that the change in the zero-point energies give rise to the Lamb shift in the atomic energy levels [2]. Similarly, the Van der Waals interaction occurs due to the zero-point fluctuations. The vacuum fluctuations produce a non-vanishing dipolar moment of the atoms [3]. Hence a $1/R^6$ dependent potential appears in the system. Here R is the distance between the two atoms. In the relativistic limit, the interaction is further modified by the influence of the retardation effect

[4]. It is widely known as the Casimir–Polder interaction (CPI). The force associated with this interaction is known as the Casimir–Polder force (CPF); for two parallel conducting plates, the force is attractive, and the expression of the force is given by $\delta F = -\frac{\pi^2}{240R^4}$. In the case of two parallel plates or a flat plate and a spherical surface, various experiments can precisely measure the Casimir force [5–8]. This experimental evidence proves Casimir physics to be a hallmark in quantum field theory. CPI is also used as an essential tool to analyze local curvature effects in the presence of quantized fields [9–11]. In addition, the thermal character is connected with the vacuum fluctuations of the quantized fields. This idea is commonly known as the thermalization theorem, which tells that if a uniformly accelerated particle detector interacts with the vacuum state of an external field and spontaneous emission occurs, then the detector behaves as if it is in a thermal bath [12]. Hence the major implementation of Casimir physics is shown in thermal–non-thermal scaling of a linearly accelerating atom, interacting with a massless scalar field [13–16]. One crucial drawback of linear accelerating detectors is that the value of the acceleration is very large if one is to produce a temperature of 1 K [17]. In contrast, Bell and Leinaas pointed out that in a circular storage ring one could produce a relatively higher effective temperature within a shorter time scale [18]. Unlike the uniformly accelerated detector, the rotating detector cannot achieve a thermal state. As for the circular motion, there is no event horizon [19, 20]. However, due to the experimental efficiency of the moving electrons in the circular storage ring, exploring Casimir physics in rotating co-ordinates has been one of the developing areas in the last few decades [21–23]. Moving electrons in a storage ring can radiate due to their charge (synchrotron radiation), in addition to the spin-flip radiation that also exists in the presence of the external magnetic field [24]. As a result, the electrons are depolarized in a circular ring. This is known as the Sokolov–Ternov effect [25]. This phenomenon is quite similar to the circular Unruh effect. Hence, this depolarization effect and

^a e-mail: ss17rs021@iiserkol.ac.in

^b e-mail: cs12ip026@iiserkol.ac.in (corresponding author)

^c e-mail: ac17rs016@iiserkol.ac.in

CPI in a circular trajectory should have the same physical origin [26].

Here we consider an atomic system weakly interacting with the external quantum fields. These systems are widely known as open-quantum systems [27,28]. The reduced dynamics of the system sharply depends on the field-correlation function. Several parallel approaches exist to calculating the atomic energy level shift due to vacuum fluctuations [29]. Dalibard–Dupont–Cohen (DDC) calculated the rate of change of the mean atomic energy, which determines spontaneous excitation rates and radiative reactions under vacuum fluctuations [30]. Similar methods based on Langevin dynamics are used to calculate the fluctuation induced interactions [31]. The quantum master equation (QME) is also an important tool for deriving the reduced dynamical equation of the atoms [32,33]. Benatti was first to describe the positive time-evolution of the quantum systems weakly coupled with a scalar field in the Minkowski vacuum [34]. Later, various attempts by using the QME formalism were made to analyze the Casimir physics from scratch [15,16,35]. The system–field weak coupling Hamiltonian gives an effective second-order contribution to the QME. It consists of two parts: the real part gives dissipative dynamics, and the imaginary terms give a second-order shift term of the atomic levels. This shift is known as the Lamb shift [27]. Suppose the atoms are interacting with the same external field. In that case, there is a possibility of creating field-induced entanglement, which is known as the common-environment effect [36,37]. The individual system–field interaction cross-terms in the second order of QME also produce a shift term in the dynamics, which is particularly dependent on the distance between the two atoms. As a result, we have the formation of an inter-atomic correlation in the dynamics. One of the atoms is kept in a ground state and the other in an excited state. Due to the interaction with the vacuum of the same quantized field, the exchange of real photons occurs, and we get a non-zero expectation value of the effective Hamiltonian in the correlated state. This phenomenon gives rise to the RCPI [14,15].

We study the RCPI between two atoms rotating synchronically and maintaining a vertical separation among each other. The atoms are assumed to be interacting with their environments, which in our case is either a massive scalar field or an electromagnetic field. The reason for choosing such fields is the incorporation of several non-thermal features which in the case of a massless scalar field (conventionally chosen as environment) do not appear [14,21,35,38,39]. We find that the interaction energy acquires a phase depending on the nature of the environment and this has no thermal analog (compared to the massless case). In this paper, we analyze the nature and the characteristics of the phase shift and study its dependence on certain atomic and environmental parameters, such as the atomic transition frequency, the atomic separation, and the

mass of the interacting field. We also discuss here the different regimes of various parameters, where the phase lag is more effective. The energy shift can be encoded by the polarization transfer technique. For two stationary qubits, quantum information can be transferred between different polarization modes within the coherence time [40]. That information transfer can be envisaged using any kind of spectroscopy [41–44]. Finally, in Sect. 6 we describe a scenario according to which such a phase shift can be detected. For that, we calculate the instantaneous polarization transfer time between different atomic modes. A particular example of the transfer of magnetization modes between spin-1/2 detectors is discussed. We find that the transfer time is different depending on the several parameters of RCPI, as discussed above. We quantify the transfer time for the above-mentioned fields and thus propose a possible way for detecting the phase shift.

Throughout the paper, we will write the following notation: the four vector is denoted by ‘ x ’, the time is represented by ‘ t ’ and the proper time by ‘ τ ’. We denote the space coordinate by ‘ X_p ’, ($X_p = X, Y, Z$).

2 Dynamics of a two-atom system

We assume here that two atoms are rotating synchronically in a circular orbit and the perpendicular distance in the rotating plane is defined as L , which is fixed [22]. The angular speed is λ_o , R is the radius of the circular orbit. The tangential velocity is $\mathcal{V} = \lambda_o R$. Now, we focus on the dynamics in the ultra-relativistic regime because the high acceleration needed for experimental realization can only be achieved in that regime [20]. The positions of the atoms in terms of proper time are written by

$$\begin{aligned} t_1(\tau) &= \gamma\tau, & X_1(\tau) &= R \cos \frac{\gamma\tau\mathcal{V}}{R}, \\ Y_1(\tau) &= R \sin \frac{\gamma\tau\mathcal{V}}{R}, & Z_1(\tau) &= 0 \\ t_2(\tau) &= \gamma\tau, & X_2(\tau) &= R \cos \frac{\gamma\tau\mathcal{V}}{R}, \\ Y_2(\tau) &= R \sin \frac{\gamma\tau\mathcal{V}}{R}, & Z_2(\tau) &= L. \end{aligned} \quad (1)$$

Here $\gamma = 1/\sqrt{1 - \mathcal{V}^2}$ is the Lorentz factor. The centripetal acceleration is given by $a = \mathcal{V}^2\gamma^2/R$, which will provide a length scale in the dynamics. The atoms are weakly coupled with the external quantum fields. The distance between the atoms is taken to be smaller than the correlation length of the field. The total Hamiltonian of the system plus field is written as [34]

$$\mathcal{H} = \mathcal{H}_s + \mathcal{H}_f + \mathcal{H}_{sf}.$$

$\mathcal{H}_s$ is the free Hamiltonian of the system, it is expressed as $\mathcal{H}_s = \omega_o/2(\sigma_z \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_z)$. The atoms are chosen to be spin-

1/2 particles. Hence the suitable operators are defined using Pauli spin matrices, $(\sigma_i, i = x, y, z)$. The system is taken to be isotropic (the frequencies of the two atoms are the same). $\mathcal{H}_f$ is the normal ordered field Hamiltonian. Following the second quantization technique, it can be expressed as

$$\mathcal{H}_f = \int \frac{d^3k}{(2\pi)^3} E_k a^\dagger(k) a(k). \quad (2)$$

E_k is the frequency of the field and $a^\dagger, a$ are the creation and annihilation operators. The exact expression of the operators depends on the characteristics of the field. The system–field coupling Hamiltonian is expressed as [34],

$$\mathcal{H}_{sf} = \alpha \sum_{\mu=0}^3 [\sigma_\mu^{(1)} \otimes \phi_\mu(x_1) + \sigma_\mu^{(2)} \otimes \phi_\mu(x_2)], \quad (3)$$

ϕ_μ is the external field, which satisfies the Klein–Gordon equation. x_1, x_2 are the individual trajectories of two atoms which we have defined earlier. For $\mu = 0$, we define $\sigma_0 = \mathbb{I}$ (identity matrix). Here,

$$\phi_\mu(x) = \sum_{a=1}^N [\chi_\mu^a \phi^-(x) + (\chi_\mu^a)^\dagger \phi^+(x)], \quad (4)$$

$\phi^\mp(x)$ is the negative and positive field operator of the field and $\alpha \chi_\mu^a$ is the generalized coupling amplitude. The χ_μ^a also denote the different field modes, where μ represents the spacetime indices. The symbol a is used for denoting the number of the harmonic oscillators. It runs from 1 to N ; here we assume that the system is coupled with N independent harmonic oscillators. The system plus field forms a closed system. To find the dynamical equation of the system, the starting point is the “Von Neumann–Liouville” equation [27]. It is given by

$$\frac{d\rho(t)}{dt} = -i[\mathcal{H}_s + \mathcal{H}_f + \mathcal{H}_{sf}, \rho(t)]. \quad (5)$$

The time-evolution operator is defined as $U(t_1, t_2) = \mathcal{T} \exp \left[-\int_{t_2}^{t_1} dt \mathcal{H} \right]$, here $\mathcal{T}$ is the time ordering operator. Using the perturbation expansion of the Dyson series, collecting the first-order and second-order contribution of the system–field weak coupling Hamiltonian and taking the trace over the field variables (denoted by Tr_f in the equation given below), we get the following equation for the two atoms [27]:

$$\begin{aligned} \frac{d\rho_s}{dt} = & -i[\mathcal{H}_s, \rho_s(t)] \\ & - \int_0^\infty d\bar{\tau} \times \mathcal{J}(\bar{\tau}) \text{Tr}_f[\mathcal{H}_{sf}, [\mathcal{H}_{sf}, \rho_s(t) \otimes |0\rangle\langle 0|]]. \end{aligned} \quad (6)$$

This is known as the Markovian master equation [27]. Initially the field is in the vacuum state. The system–field coupling is chosen to be isotropic, so the first-order contribution of $\mathcal{H}_{sf}$ vanishes. The general form of the integrand is

expressed as $\mathcal{J}(\bar{\tau}) = e^{\pm i(\omega_0 \mp E_k)\bar{\tau}}$, where $\bar{\tau}$ represents the time difference over which the above Hamiltonians act on the density matrix in the second order [28]. The result of the integration is given by

$$\int_0^\infty d\bar{\tau} e^{\pm i(\omega_0 \mp E_k)\bar{\tau}} = \delta(\omega_0 \mp E_k) \pm i\mathcal{P} \frac{1}{\omega_0 \mp E_k}. \quad (7)$$

$\mathcal{P}$ is the Cauchy principal value. The real part contributes in the dissipative features of the dynamics and the imaginary part gives a shift term. We will work in the continuum limit, where the summation in Eq. (4) turns into an integral over the whole frequency range of the external field. Equation (6) is represented in the famous Lindblad form by

$$\frac{d\rho_s(\tau)}{d\tau} = -i[\mathcal{H}_s + \mathcal{H}_{\text{lamb}}, \rho_s(\tau)] + \mathcal{L}(\rho_s(\tau)). \quad (8)$$

The above equation is called the Lindblad equation or the quantum master equation [32,33]. The time-evolution operator is a one-parameter semi-group and the completely positivity and trace preservation holds. The initial correlation between the system and field is ignored; it is called the “Born approximation” [28]. $\mathcal{H}_{\text{lamb}}$ is the second-order effective Hamiltonian. It produces the shift term in the dynamics. $\mathcal{L}$ is called the dissipator, which generates the irreversibility in the reduced dynamics of the system. The exact mathematical expression of these two quantities is written by

$$\mathcal{L}(\rho_s) = \sum_{a,b=1}^2 \sum_{j,k=1}^3 \gamma_{jk}^{ab} \left(\sigma_b^k \rho_s \sigma_a^j - \frac{1}{2} \{ \sigma_a^j \sigma_b^k, \rho_s \} \right), \quad (9)$$

$$\mathcal{H}_{\text{lamb}} = -\frac{i}{2} \sum_{a,b=1}^2 \sum_{j,k=1}^3 \mathcal{S}_{jk}^{ab} \sigma_a^j \sigma_b^k. \quad (10)$$

Here $\mathcal{S}_{jk}^{ab}$ and γ_{jk}^{ab} originate from the Fourier transform of the two-point correlation function. They are Kramers–Kronig pairs. Due to the common-environment effect, the atoms become entangled through the field-correlation function. This entanglement has an initial value dependency [34]. The field-induced shift term is calculated from the Hilbert transforms of the response function of the field. We have

$$\mathcal{K}^{ab}(\omega_0) = \frac{\mathcal{P}}{\pi i} \int_{-\infty}^{\infty} dE_k \frac{\mathcal{G}^{ab}(E_k, |X_a - X_b|)}{E_k - \omega_0}. \quad (11)$$

$\mathcal{P}$ is the Cauchy principal value. The summation of Eq. (4) is replaced by the integration in the continuum limit. $\mathcal{G}^{ab}(E_k)$ is the response function of the field and it is defined as

$$\begin{aligned} \mathcal{G}^{ab}(E_k, |X_a - X_b|) \\ = \int_{-\infty}^{\infty} d\Delta\tau e^{iE_k\Delta\tau} G^{ab}(\Delta\tau, |X_a - X_b|). \end{aligned} \quad (12)$$

Here we assume that χ_μ^a satisfies $\sum_{a=1}^N \chi_\mu^a (\chi_\nu^a)^\dagger = \delta_{\mu\nu}$. So the field-correlation functions are diagonal i.e. $G_{ij}^{ab}(x - y) = \delta_{ij} G^{ab}(x - y)$, which is given by [34]

$$\begin{aligned} \langle \Phi_i(\tau, X_a) \Phi_j(\tau', X_b) \rangle &= \sum_{q=1}^N \chi_i^q (\chi_j^q)^\dagger G^{ab}(\Delta\tau, |X_r|) \\ &= G_{ij}^{ab}(\Delta\tau, |X_r|). \end{aligned} \quad (13)$$

Here $\Delta\tau = (\tau - \tau')$ and $X_r = X_a - X_b$. In the following sections we represent $G^{ab}(\Delta\tau, |X_r|)$ as $G^{ab}(\Delta\tau)$. The Planck factor of the detector response function for the linear acceleration case ensures that the particle detector perceives a thermal bath of temperature T . Here, $T = a/2\pi$, a is the linear acceleration [45]. For a circularly rotating observer, the Planck spectrum is replaced by several ad hoc factors which impose non-thermality in the system [20]. In the RCPI, only the spontaneous radiative process occurs; hence there is no use of the number densities in the calculation of the response function [15, 16, 46]. Here we can write $\mathcal{S}_{jk}^{ab}$ in the form

$$\mathcal{S}_{jk}^{ab} = A^{ab} \delta_{jk} - i B^{ab} \epsilon_{jkl} \delta_{3l} - A^{ab} \delta_{3j} \delta_{3k}, \quad (14)$$

where the terms A^{ab} and B^{ab} are given by

$$A^{ab} = \frac{\lambda^2}{4} [\mathcal{K}^{ab}(\omega_o) + \mathcal{K}^{ab}(-\omega_o)], \quad (15)$$

$$B^{ab} = \frac{\lambda^2}{4} [\mathcal{K}^{ab}(\omega_o) - \mathcal{K}^{ab}(-\omega_o)]. \quad (16)$$

The cross-terms of the individual system–field Hamiltonian contribute in the off-diagonal elements of $\mathcal{H}_{\text{lamb}}$ and the expectation values of the symmetric ($|E\rangle$) and anti-symmetric states ($|A\rangle$) are non-zero, which tells that the atoms become entangled in the intermediate time regime due to the Lamb shift. The expectation values of these two states are given by

$$\begin{aligned} \delta E_{S_{LS}} &= \langle E | \mathcal{H}_{\text{lamb}} | E \rangle \\ &= -2i [A_2 + A_1], \\ \delta E_{A_{LS}} &= \langle A | \mathcal{H}_{\text{lamb}} | A \rangle \\ &= 2i [A_2 - A_1]. \end{aligned} \quad (17)$$

Here $A^{11} = A^{22} = A_1$, $A^{12} = A^{21} = A_2$. A_1 denotes the self-term in the interaction and A_2 corresponds to cross-terms. The resonance Casimir–Polder force (RCPF) is defined as $\delta f = -\delta E/\delta L$. Hence we consider only the length (L) dependent terms for the derivation of RCPI [35]. As a result, the contribution from A_1 in Eq. (17) is neglected. Finally, we define $\delta E_{S_{LS}} = -\delta E_{A_{LS}} = \delta E$. In the following section, we will find the expressions for the Green function and the response function [Eq. (13)] for a particular setup and also calculate $\mathcal{S}_{jk}^{ab}$ to compute the energy shift due to the

resonance interaction for massive and EM field by solving the QME [Eqs. (8) and (10)].

3 Two atoms in a massive scalar field

Here we consider two atoms moving in a circular orbit and weakly coupled with the massive scalar field. The frequency of the field in Eq. (2) is given by $E_k = \sqrt{k^2 + m^2}$. The two-point correlation function of the massive scalar field is [47]

$$\begin{aligned} G(x, x') &\equiv \langle 0 | \hat{\Phi}(t, X) \hat{\Phi}(t', X') | 0 \rangle \\ &= \int \frac{d^4 k}{(2\pi)^{\frac{3}{2}}} \delta(k^2 - m^2) e^{-ik(x-x')}, \\ &= -\frac{m}{4\pi^2} \frac{K_1\left(m\sqrt{(t-t'-i\epsilon)^2 - (X_p - X'_p)^2}\right)}{\sqrt{(t-t')^2 - (X_p - X'_p)^2}}. \end{aligned} \quad (18)$$

Here we define $x = \{t, X_p\}$, where the subscript ‘ p ’ stands for all the space indices. We follow the Einstein summation convention rule in the above equation for the space components. The Green function in Eq. (18) is known as the positive-frequency Wightman function with parameter $i\epsilon$ which is chosen to be small. K_1 is a Bessel function of the second kind. For the small mass limit, the expression in Eq. (18) is reduced to the case of a massless scalar field. On the other hand, in the high mass limit, the correlation function has an exponential decay factor, so the RCPI has a similarity with the Yukawa potential in that limit. We are working in the ultra-relativistic limit ($\gamma \gg 1$) and in this limit, using the co-ordinates given in Eq. (2), we get the expression for the two-point correlation function:

$$\begin{aligned} G^{11}(\Delta\tau) &= G^{22}(\Delta\tau) \\ &= \frac{m}{4\pi^2} \frac{K_1\left(m\Delta\tau\sqrt{1+a^2\Delta\tau^2/12}\right)}{\Delta\tau\sqrt{1+a^2\Delta\tau^2/12}}, \\ G^{12}(\Delta\tau) &= G^{21}(\Delta\tau) \\ &= \frac{m}{4\pi^2} \frac{K_1\left(m\sqrt{\Delta\tau^2(1+a^2\Delta\tau^2/12)-L^2}\right)}{\sqrt{\Delta\tau^2(1+a^2\Delta\tau^2/12)-L^2}}. \end{aligned} \quad (19)$$

The spontaneous emission rates for the massless and the massive case generally do not coincide. The differences lie in the mass gap of the energy spectra, which is independent of the particle trajectory [48]. In the case of circular motion in a massless scalar field the expressions for the response function were calculated in [22, 23]. For the massive case, the expression for the response function should merge with the massless case in the small mass limit. For the

massive scalar field, the lowest energy value is shifted from $E_k = 0$ to $\sqrt{E_k^2 - m^2} = 0$. So in the leading order approximation, all the terms containing $E_k \theta(E_k)$ in the expression of the response function in Ref. [22] are replaced by terms with $\sqrt{E_k^2 - m^2} \theta(E_k - m)$. Following the description, the response function for the massive scalar field is given by

$$\begin{aligned} \mathcal{G}^{11}(E_k) &= \mathcal{G}^{22}(E_k) \\ &= \frac{a}{8\sqrt{3}\pi} e^{-2\sqrt{3}\frac{\Omega(E_k, m)}{a}} + \frac{\Omega(E_k, m)}{2\pi}; \quad (E_k > m), \\ \mathcal{G}^{12}(E_k) &= \mathcal{G}^{21}(E_k) \\ &= \frac{a}{4\pi} \frac{e^{-\frac{\Omega(E_k, m)}{a}} \sqrt{6\sqrt{1+a^2L^2/3}+6}}{\sqrt{6(\sqrt{1+a^2L^2/3}+6)(1+a^2L^2/3)}} \\ &\quad + \frac{a}{2\pi} \frac{\sin\left(\frac{\Omega(E_k, m)}{a} \sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{\sqrt{6(\sqrt{1+a^2L^2/3}-6)(1+a^2L^2/3)}}; \\ &\quad \times (E_k > m). \end{aligned} \quad (20)$$

Here the momentum is defined as $\Omega(E_k, m) = \sqrt{E_k^2 - m^2}$. The presence of a mass gap is quite similar to the linear case in a massive field [35]. The non-Planck exponential terms in Eq. (20) are neglected. The RCPI for the two atoms in a massive scalar field can be computed from Eq. (17), where the quantities A_1 , A_2 are given by Eq. (15). The expression of RCPI is given by

$$\delta E = \frac{\alpha^2 \mathcal{P}}{2\pi} \int_m^\infty dE_k \left(\frac{\mathcal{G}^{12}(E_k)}{E_k - \omega_o} + \frac{\mathcal{G}^{12}(E_k)}{E_k + \omega_o} \right). \quad (21)$$

For $m > \omega_o$ the interaction behaves like a Yukawa potential. The interaction becomes of short range and it decays beyond a characteristic time scale $1/m$ [35]. We are interested in the other limit, $\omega_o > m$. In this limit the RCPI is given by

$$\delta E = \alpha_1^2 a \frac{\cos\left(\frac{\sqrt{\omega_o^2 - m^2}}{a} \sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{\sqrt{6(\sqrt{1+a^2L^2/3}-6)(1+a^2L^2/3)}}. \quad (22)$$

All the numerical factors are absorbed in α , so the modified interaction strength is defined as α_1 .

3.1 The length dependence and retarded response of RCPI

The expression in Eq. (22) shows a dependency on aL , ($aL = \frac{\gamma^2 L}{(1-\gamma^2)R}$). We analyze the resonance interaction in two limiting cases. The condition $aL \ll 1$ can be obeyed when the inter-atomic separation is very small in comparison with the radius of the circular path, ($L \ll R$). Basically this setup is nearly equal to the linear acceleration case, where the inertial approximation is valid. In this limit, the expression

can be written as

$$\delta E = \alpha_1^2 \frac{\cos\left(\sqrt{\omega_o^2 - m^2} L\right)}{L}. \quad (23)$$

For the limit $aL \ll 1$, the result exactly matches with the linear acceleration case when the inertial approximation is valid. In another sense, in the local inertial frame, the atoms behave like the static ones in the flat spacetime as there is no curvature in this spacetime. In the small mass limit ($m \ll \omega_o$), the phase lag ($\delta\eta$) in RCPI with respect to the massless case is given by

$$\begin{aligned} \delta E &= \alpha_1^2 \frac{\cos(\omega_o L - \delta\eta)}{L}, \\ \delta\eta &= \frac{m^2 L}{2\omega_o}. \end{aligned} \quad (24)$$

In this limit, the length dependence of RCPI exactly matches with the linear case and for $m = 0$. It matches with the thermal response. For the other case, $aL \gg 1$, which is experimentally easier to achieve in the ultra-relativistic limits. The RCPI is given by

$$\delta E = 3^{1/4} \alpha_1^2 \frac{\cos\left(12^{1/4} \sqrt{\omega_o^2 - m^2} \sqrt{\frac{L}{a}}\right)}{\sqrt{2} \sqrt{aL^3}}. \quad (25)$$

Similarly in the small mass limit ($m \ll \omega_o$), the phase lag w.r.t. massless one is given by

$$\begin{aligned} \delta E &= 3^{1/4} \alpha_1^2 \frac{\cos\left(12^{1/4} \omega_o \sqrt{\frac{L}{a}} - \delta\eta\right)}{\sqrt{2} \sqrt{aL^3}}, \\ \delta\eta &= \frac{12^{1/4} m^2}{2\omega_o} \sqrt{\frac{L}{a}}. \end{aligned} \quad (26)$$

In this limit the local inertial approximation is violated so non-thermality arises. The length dependency is also not equal to that in the linear acceleration case [35]. The l/L^2 dependence is replaced by $1/\sqrt{aL^3}$. Here $1/l$ is the linear acceleration. The presence of a centripetal acceleration in the system gives a different result from the linear case. For $m^2 > \omega_o^2$, in Eqs. (23) and (25) the $\cos(\sqrt{\omega_o^2 - m^2})$ term is replaced by $\exp(-\sqrt{m^2 - \omega_o^2})$, so the response for the Yukawa like potential is also exponentially decaying with the mass [35].

4 Two atoms in electromagnetic vector field

In this section we consider the two-atom system to be rotating in a circular path and to be weakly coupled with the electromagnetic field. The Lagrangian of the EM field is obtained

by [49]

$$\mathcal{L} = -\frac{1}{4}F_{uv}F^{uv} - \frac{1}{2}(\partial_v A^v)^2. \quad (27)$$

Here $F_{uv} = \partial_u A_v - \partial_v A_u$, A_u is the electromagnetic vector potential, $A_u = (\phi, \vec{A})$. Here we use the Feynman gauge [49]. Hence the photon operator can be written as

$$A_v(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2|\vec{p}|}} \sum_{\lambda=0}^3 \epsilon_v^\lambda(\vec{p}) [a_p^\lambda e^{i\vec{p}\cdot\vec{x}} + \text{h.c.}]. \quad (28)$$

Here ϵ_v^λ is the polarization vector. The normalization is defined as $\epsilon_v^\lambda \epsilon_u^{\lambda'} \eta_{\lambda\lambda'} = \eta_{uv}$. The two-point correlation function in the Feynman gauge is written as [21, 49],

$$\langle 0|A_i(x)A_j(x')|0\rangle = \frac{\eta_{ij}}{4\pi^2\{(t-t')^2 - (X_p - X'_p)^2\}}. \quad (29)$$

In Eq. (29) ‘ p ’ is a dummy index used to express the sum over all space components.

The spins are coupled to the electric field via a dipolar coupling. The dipolar moment of the individual atoms is given by $\vec{d} = e\vec{r}$. e is the charge of the atoms. The dipole moment in terms of Pauli matrices is given by

$$\vec{d} = -\sigma^-\vec{Y}^* - \sigma^+\vec{Y}. \quad (30)$$

Here $\vec{Y} = e\langle E|\vec{r}|A\rangle$. The self-terms produce a very small shift of the Zeeman energy levels, so $\langle E|\vec{r}|E\rangle = \langle A|\vec{r}|A\rangle \approx 0$. We only consider the off-diagonal elements, which cause the transition in the atomic energy levels. The coupling Hamiltonian is written as [28]

$$\mathcal{H}_{sf} = \vec{d}_1 \cdot \vec{E}(x_1) + \vec{d}_2 \cdot \vec{E}(x_2). \quad (31)$$

The two spins are identical, so $\vec{d}_1 = \vec{d}_2 = \vec{d}_0$. Here $E_i = -\partial A_i/\partial t$. Therefore the two-point function of the electric field is given by

$$\langle 0|E_i(x)E_j(x')|0\rangle = (\partial_0\partial'_0\delta_{ij} - \partial_i\partial_j)\langle 0|A_i(x)A_j(x')|0\rangle. \quad (32)$$

The two-point correlation function in the ultra-relativistic regime is given by

$$\begin{aligned} G^{11}(\Delta\tau) &= G^{22}(\Delta\tau) \\ &= \frac{1}{\pi^2} \frac{1}{\Delta\tau^4(1+a^2\Delta\tau^2/12)^2}, \\ G^{12}(\Delta\tau) &= G^{21}(\Delta\tau) \\ &= \frac{1}{\pi^2} \frac{1}{(\Delta\tau^2(1+a^2\Delta\tau^2/12) - L^2)^2}. \end{aligned} \quad (33)$$

The Fourier transform of Eq. (33) is given by

$$\begin{aligned} \mathcal{G}^{11}(E_k) &= \mathcal{G}^{22}(E_k) \\ &= \frac{2a^2 E_k}{2\pi} + \frac{2E_k^3}{\pi}, \\ \mathcal{G}^{12}(E_k) &= \mathcal{G}^{21}(E_k) \\ &= \frac{a^2 E_k \cos\left(\frac{E_k}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{3\pi(\sqrt{1+a^2L^2/3}-1)(1+a^2L^2/3)} \\ &\quad + \frac{4a^3 \sin\left(\frac{E_k}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{6^{3/2}\pi(\sqrt{1+a^2L^2/3}-1)^{3/2}(1+a^2L^2/3)} \\ &\quad + \frac{8a^3 \sin\left(\frac{E_k}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{3\sqrt{6}\pi\sqrt{(\sqrt{1+a^2L^2/3}-1)(1+a^2L^2/3)^{3/2}}}. \end{aligned} \quad (34)$$

The contour is chosen in the upper half of the complex plane. The contributions from the imaginary poles are neglected as they give an exponential decay. Similarly, following the massive case, in Eq. (21) if we use the form of $\mathcal{G}^{12}(E_k)$ from Eq. (34), the RCPI of the atom in the electromagnetic field is then given by

$$\begin{aligned} \delta E &= d_{o'}^2 \left(\frac{a^2 \omega_o \sin\left(\frac{\omega_o}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{3(\sqrt{1+a^2L^2/3}-1)(1+a^2L^2/3)} \right. \\ &\quad + \frac{4a^3 \cos\left(\frac{\omega_o}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{6^{3/2}(\sqrt{1+a^2L^2/3}-1)^{3/2}(1+a^2L^2/3)} \\ &\quad \left. + \frac{8a^3 \cos\left(\frac{\omega_o}{a}\sqrt{6\sqrt{1+a^2L^2/3}-6}\right)}{3\sqrt{6}\sqrt{(\sqrt{1+a^2L^2/3}-1)(1+a^2L^2/3)^{3/2}}} \right). \end{aligned} \quad (35)$$

All the numerical factors are absorbed in the dipolar coupling strength. The modified dipolar coupling constant is defined as $d_{o'}$.

4.1 The length dependence and retarded response of RCPI

In the same manner, if we analyze the resonance interaction in the two limits, for $aL \ll 1$, it can be expressed as

$$\delta E = d_{o'}^2 \left(\frac{2\omega_o \sin(\omega_o L)}{L^2} + \frac{4 \cos(\omega_o L)}{L^3} + \frac{8a^2 \cos(\omega_o L)}{3L} \right). \quad (36)$$

The length scale dependence is quite ambiguous. It consists of three terms. The terms are also simultaneously propor-

tional to the system energy levels and the square of the centripetal acceleration. It clearly denotes non-thermality in the region where a local inertial approximation is valid. In the opposite limit, $aL \gg 1$, the expression is

$$\delta E = d_o^2 \left(\frac{\sqrt{3} \omega_o \sin \left(12^{1/4} \omega_o \sqrt{\frac{L}{a}} \right)}{aL^3} + \frac{\cos \left(12^{1/4} \omega_o \sqrt{\frac{L}{a}} \right)}{\sqrt{aL^7}} \left(\frac{25 \times 3^{1/4}}{3\sqrt{2}} \right) \right). \quad (37)$$

In this limit the RCPI also shows a strong non-thermality. The expressions for the RCPI in the two limits can be expressed as

$$\delta E = d_o^2 \sqrt{\alpha^2 + \beta^2} \cos(\gamma - \delta\eta). \quad (38)$$

From this we can easily extract the expression for the length dependence ($\sqrt{\alpha^2 + \beta^2}$) and the phase lag ($\delta\eta$). For $aL \ll 1$, γ has the same form as in the massless case ($\gamma = \omega_o L$). The amplitude and phase lag are given by

$$\sqrt{\alpha^2 + \beta^2} = \sqrt{\frac{4\omega_o^2}{L^4} + \left(\frac{4}{L^3} + \frac{8a^2}{3L} \right)^2},$$

$$\delta\eta = \tan^{-1} \left(\frac{\omega_o L}{2} \right), \quad (39)$$

and $aL \gg 1$ ($\gamma = 12^{1/4} \omega_o \sqrt{\frac{L}{a}}$). The phase lag and amplitude are given by

$$\sqrt{\alpha^2 + \beta^2} = \sqrt{\frac{3\omega_o^2}{a^2 L^6} + \frac{1}{aL^7} \left(\frac{25 \times 3^{1/4}}{3\sqrt{2}} \right)^2},$$

$$\delta\eta = \tan^{-1} \left(\frac{3 \times 12^{1/4} \omega_o}{25} \sqrt{\frac{L}{a}} \right). \quad (40)$$

In both cases, the resonance interaction exhibits non-thermal signatures. The amplitude has a crucial dependency on the Zeeman frequency of the atoms. The response also shows a phase lag in the dynamics.

5 Orders of magnitude

It is well known that, for a relativistic uniform acceleration ‘ a ’, the corresponding temperature is given by [12],

$$kT = \frac{\hbar a}{2\pi c}. \quad (41)$$

Here k is the Boltzmann constant, c is the velocity of light, and $\hbar$ is the Planck constant. The above equation tells that, to produce a temperature of 1 K, the value of the acceleration should be $a = 2.4 \times 10^{20} \text{ m/s}^2$ [18]. Experimentally achieving this value for a linear case is impossible. Hence,

Fig. 1 Filled contours show the ratio of phase lag and frequency of oscillation due to EM field ($\frac{\delta\eta}{\gamma}$). The distance between the atoms, $L = 6\mu\text{m}$, is chosen to be fixed. The acceleration (a) is varied from 10^{23} – 10^{25} m/s^2 and the Zeeman frequency (ω_o) is varied from 10^{14} – 10^{16} Hz . We note that, for lower ω_o and higher a , the effect of the phase lag is strong, and in the other limit, for higher ω_o and lower a , the effect of the phase lag is vanishingly small

one was interested in the rotating case. Although for the rotating detectors one cannot find the event horizon, still, it was shown that in a circular storage ring, the depolarization effect of the electron gives rise to an effective temperature, which is similar to the Unruh effect. Bell and Leinaas pointed out that in the 3.1 km accelerator ring of SPEAR at Stanford, the ultra-relativistic electron can rotate at an acceleration of $a = 2.9 \times 10^{23} \text{ m/s}^2$, which is equivalent to a temperature of 1200 K [18].

Now we consider the motion of two spin-1/2 particles (the perpendicular distance between them is fixed) in a circular ring in the presence of an EM field. Bressi et al. measured the Casimir–Polder force between two parallel conducting plates at a distance range of 0.5–3.0 μm [6]. So we choose our perpendicular distance between the atoms in that range. We try to show the change in the ratio of the phase lag and the corresponding phase $\frac{\delta\eta}{\gamma}$ with respect to the centripetal acceleration and a proper choice of the Zeeman frequency graphically. The other important aspect, the amplitude of the force, was observed earlier in different scenarios [5–8]. So we will skip that part. We are interested particularly in the limit $aL \gg 1$ ($aL \gg c^2$) as the other limit, $aL \ll 1$, has no a dependence. The latter limit is known as the inertial limit. For circular ring storage, the inertial limit arises when the acceleration has a comparatively low value, and the associated temp is of the milli-kelvin order. It is expected that one needs atoms with higher energies (10^{15} Hz) to perform this kind of experiment (Fig. 1).

The plot clearly tells that, on increasing the Zeeman frequency and decreasing the acceleration, the effect of the phase lag is weak. Hence it will neutralize the effect of the phase lag too. For the higher acceleration value, the effect

is strong, so the periodic behavior (in L) of RCPI is gone. In this case, the effective interaction shows the behavior of a two-atom system whose frequency of Zeeman levels is zero in a massless scalar field.

The massive case can be visualized by using the well-known experiment performed by Klaers et al. [50]. A 2D photon gas can act as a massive boson in a dye-filled optical micro-cavity. If we use the setup in a circular storage ring, the retarded effect due to the effective mass can be visible. For the small mass limit ($m \ll \omega_o$), the ratio $\frac{\delta\eta}{\gamma}$ is defined as $\frac{m^2}{\omega_o^2}$. The ratio does not depend on the acceleration or the distance between the atoms. For a fixed mass, increasing the ω_o will decrease the effect of the phase lag. Increasing the Zeeman frequency, the effect of the phase lag also becomes weaker for the massive case.

6 Instantaneous polarization transfer using RCPI

In this section, we give a computation protocol to inspect the retardation effect using the polarization transfer technique. In particular, as an example, we will discuss the transfer of different magnetization modes in a system of two spin-1/2 magnetic dipoles. For single spin system, there exist three possible magnetization modes, namely σ_x , σ_y , σ_z . Following the same logic, for the two-spin system the number of possible magnetization modes is 15. The trace preservation is a constraint in the system. As the atoms have the same energy levels, the number of the independent magnetization modes is reduced to nine. The modes are given by

$$\begin{aligned}\mathcal{F}_i &= \text{Tr}_s\{(\sigma_i \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_i)\rho_s\}, \\ \mathcal{F}_{ii} &= \text{Tr}_s\{(\sigma_i \otimes \sigma_i)\rho_s\}, \\ \mathcal{F}_{ij} &= \text{Tr}_s\{(\sigma_i \otimes \sigma_j + \sigma_j \otimes \sigma_i)\rho_s\}.\end{aligned}\quad (42)$$

Here $i = x, y, z$. We consider here only those elements of the RCPI Hamiltonian which are responsible for the creation of the inter-atomic correlation. The form of the effective Hamiltonian in the Zeeman basis is given by

$$\mathcal{H}_{ia} = \frac{\delta E}{2}(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y). \quad (43)$$

$\mathcal{H}_{ia}$ is the effective Hamiltonian of the resonance interaction. In the intermediate time scale, the effect of dissipation can be ignored. Hence the dynamics is purely unitary. So the magnetization modes are changing with time. The dynamical equation in the interaction frame is written as

$$\frac{d\rho_s^I}{d\tau} = -i[\mathcal{H}_{ia}^I, \rho_s^I]. \quad (44)$$

ρ_s^I is for the density matrices in the interaction frame. For simplicity we neglect the I in the superscript of the density

matrices. The RCPI Hamiltonian is unchanged in the interaction frame. In terms of the magnetization modes, it is given as

$$\begin{aligned}\frac{d}{d\tau}(\mathcal{F}_x - \mathcal{F}_y) &= \frac{\delta E}{2}(\mathcal{F}_{xz} + \mathcal{F}_{yz}), \\ \frac{d}{d\tau}(\mathcal{F}_{xz} + \mathcal{F}_{yz}) &= -\frac{\delta E}{2}(\mathcal{F}_x - \mathcal{F}_y).\end{aligned}\quad (45)$$

The other modes are unchanged under the interaction. For the initial magnetization, $\mathcal{F}_x(0) - \mathcal{F}_y(0) = N_1$ and $\mathcal{F}_{xz}(0) + \mathcal{F}_{yz}(0) = 0$, and the solutions of Eq. (45) are expressed as

$$\begin{aligned}\mathcal{F}_x - \mathcal{F}_y &= N_1 \cos \frac{\delta E \tau}{2}, \\ \mathcal{F}_{xz} + \mathcal{F}_{yz} &= N_1 \sin \frac{\delta E \tau}{2}.\end{aligned}\quad (46)$$

We can distinguish the energy shift due to RCPI for massive and EM field cases in the massless case using the polarization transfer technique. The magnetization transfer time is different for the interaction of the atoms with various quantum fields. For $\tau_{in} = \frac{\pi}{\delta E}$, the magnetization switches to $\mathcal{F}_{xz}(\tau) + \mathcal{F}_{yz}(\tau)$. Here τ_{in} is the instantaneous time scale. We assume $\tau_c \ll \tau_{in} \ll T_1$. τ_c is the field-correlation time, and T_1 is the system relaxation time in the proper frame. In the lab frame the expression for the instantaneous coherence time is

$$t_{in} = \frac{\pi}{\delta E \sqrt{1 - \lambda_o^2 R^2}}. \quad (47)$$

An initial magnetization mode $\mathcal{F}_x(0) - \mathcal{F}_y(0)$ can be created by using a different pulse sequence which is routinely used in nuclear magnetic resonance spectroscopy [51, 52]. Similarly, the frequency spectrum in the Fourier domain gives a clear picture of the different types of responses for a fixed L . The Fourier transform of the temporal response of RCPI is a delta function [$f(\omega) = \delta(\omega - \delta E)$]. So, the various peaks correspond to different δE . In the presence of relaxation, the delta function is modified by a Lorentzian distribution.

7 Discussion

As a summary, in this paper, we explore the RCPI of a synchronically rotating two-atom system in a circular storage ring. The atoms move through a quantized field. They are kept in the ground state and an excited state and become entangled due to the interaction with the common external field, which is called the resonance interaction. The quantum master equation is the essential tool to calculate the energy shift due to the resonance interaction. In the presence of a massless free scalar field, the thermalization theorem holds for a uniformly accelerating particle detector. The thermal nature can be observed when the inertial approximation is

maintained. The non-thermal characters arise when a centripetal acceleration is present in the system. For the circular motion in the presence of centripetal acceleration, one cannot find an event horizon. The absence of a local inertial approximation eventually leads to a non-thermal spectrum. We can get back the thermal limit when the radius of the circle is much larger than the inter-atomic distance. For the interaction with the massive scalar field, the length scale dependency is similar to the massless case. For $aL \ll 1$, it has a $1/L$ dependence and, for $aL \gg 1$, it has a $1/\sqrt{aL^3}$ dependence. The major aspect of the massive scalar field case is the retarded response. It is expected that RCPI shows a periodic response. In the presence of mass, the retardation effect is basically governed by a m^2 factor. When the inertial approximation is not valid, then the phase lag is also modified by the centripetal acceleration. If we consider an electromagnetic vector field, the response function is also corrected by a a^2 term, and the non-Planck factor is also present. As a result, the RCPI exhibits a non-thermal behavior, which is also independent of the inertial approximation. In both limits ($aL \gg 1$, $aL \ll 1$), the amplitude of RCPI decays much faster than the massless case. The amplitude also depends on the Zeeman energy of the atoms. A retarded response was also noticed in this case. RCPI for the massless case shows an oscillatory behavior in length. Hence, the nature of RCPF changes in each period of oscillations. The presence of a massive field and an EM field can alter the characteristics of RCPI in comparison to the massless case. It seems that an attractive RCPF for the massless case may behave like a repulsive force for massive or EM field case and vice versa. We also found that, on increasing the acceleration, the effect of the phase lag is strong for the EM field and, decreasing the Zeeman frequency, the effect of the phase lag also becomes stronger for both EM and massive case. The characteristics of the interaction also can be verified by using the polarization-transfer technique. The expectation value of the energy is different for massless, massive, and EM field case. The magnetization transfer time should be different for them, as $t_{in} = \pi/(\gamma \delta E)$. Similarly, in the frequency domain, the energy peaks appear at different points, which is also used as an important tool to distinguish the different responses. Future experimental protocols can be designed by using NMR spectroscopy to justify the Casimir effect in a circular storage ring.

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