THE EUROPEAN
PHYSICAL JOURNAL C

The QCD odderon in elastic (anti)proton scattering

M. A. Braun[a]

Department of High Energy Physics, Saint-Petersburg State University, 198504 Saint Petersburg, Russia

Abstract The C-odd amplitude for the elastic pp and $p\bar{p}$ scattering due to the exchange of the QCD odderon proposed by J. Bartels, L.N. Lipatov and G.P. Vacca is calculated with the Fukugita–Kwiecinski proton impact factor. The found amplitude is very small and cannot be felt in the differential cross-sections at 2.76 and 1.96 TeV respectively.

1 Introduction: the perturbative QCD odderon

The perturbative QCD odderon exists in two different states. The first one was proposed as the $C = -1$ eigenstate of the Hamiltonian for three reggeized gluons (the simplest of the series of the so-called BKP states [1–3]). Its properties and conformal invariance were discussed very long ago [4,5]. After several attempts to numerically estimate its intercept ("energy") and wave function these was finally found by Janik and Wosiek [6]. The found intercept turned out to be below unity

$$\alpha_0^{jw} = 1 - 0.24717\bar{\alpha}, \quad \bar{\alpha} = \frac{\alpha_s N_c}{\pi}.$$

Later another odderon state was constructed by J. Bartels, L.N. Lipatov and G.P. Vacca as a state composed of three reggeized gluons, which is a superposition of three states, each of them with a pair of reggeized gluons located at the same spatial point with the wave function satisfying the colorless BFKL equation with odd conformal spins [7]. Accordingly the maximal intercept of its groundstate is exactly unity. So inevitably in the limit of very high energies this BLV odderon dominates, although in view of a little difference of its intercept from the Janik–Wosiek odderon these high energies may be indeed very high.

Possible manifestations of the odderon in the experiment include first of all a series of processes which can occur exclusively by the $C = -1$ exchanges, such as transitions γ (or $\gamma*$) to η_c. Numerical estimates of the corresponding probabilities have given very small values, which practically prohibit these direct searches for the odderon. Another possibility, much discussed recently in view of the current experimental data, is to look for the odderon in the difference between pp and $p\bar{p}$ elastic cross-sections. In these reactions the odderon exchange enters with a different sign and in the cross-sections it is multiplied by the leading and big $C = +1$ exchange. So the above difference contains the odderon exchange linearly and not quadratical in contrast to reactions realizable only by the $C = -1$ exchange. This raises some hopes to see the odderon more easily. The recent experimental data seem to exhibit a definite difference between pp and $p\bar{p}$ cross-sections and so a presence of the odderon exchange [8–11], although there are certain doubts on this point [12,13].

Actually the role of the perturbative odderon in the pp and $p\bar{p}$ scattering was studied long ago in the approach in which gluon interactions inside the odderon were neglected and the odderon was considered as just the three gluon exchange [14]. Later the problem of the nonperturbative proton impact factor was discussed [15]. The conclusion of these earlier papers was quite optimistic: with a suitable choice of the QCD coupling constant use of the three gluon exchange for the $C = -1$ amplitude lead to quite good agreement with the experimental data existing at that moment: $\sqrt{s} < 62.5$ GeV for pp scattering and $\sqrt{s} = 53$ GeV for $p\bar{p}$ scattering. The authors of [15] pointed out the importance of taking account the gluon interactions inside the odderon.

In this paper we present a partial solution of this problem considering the BLV odderon exchange instead of the simple triple gluon one. We calculate the $C = -1$ pp and $p\bar{p}$ amplitudes due to the interaction with the BLV odderon and putting it together with the $C = +1$ amplitude find the final cross-section for the two elastic processes. Of course the immediate question is from where we can take the $C = +1$ amplitude. In absence of any trustful theory, as long ago, we can use only phenomenological amplitudes. In this study we

a e-mail: braun1@pobox.spbu.ru (corresponding author)

use two models which claim to successfully describe the data up to $\sqrt{s} = 7$ TeV. The first, proposed in [16], is especially convenient for our purpose, since it is based on the Regge description of different contributions to the amplitude. The second [17], although indirectly also based on the Regge approach, does not distinguish between Regge components. However with different parameterizations it describes both the pp and $p\bar{p}$ amplitudes and so allows to extract the desired $C = -1$ amplitude.

In the theoretical $C = -1$ amplitude the BLV odderon is attached to two (anti)proton impact factors, which are non-perturbative and so model-dependent We use the impact factor proposed by Fukugita and Kwiecinski based on the perturbative picture for the interaction of three quarks with three gluons [18]:

$$\Phi_p = d\left[F(\boldsymbol{q},0,0) - \sum_{i=1}^{3} F(\boldsymbol{k}_i, \boldsymbol{q} - \boldsymbol{k}_i, 0)\right.$$
$$\left. + 2F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)\right], \tag{1}$$

where

$$F(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$
$$= \frac{2a^2}{2a^2 + (\boldsymbol{k}_1 - \boldsymbol{k}_2)^2 + (\boldsymbol{k}_2 - \boldsymbol{k}_3)^2 + (\boldsymbol{k}_3 - \boldsymbol{k}_1)^2}, \tag{2}$$

with $d = 8(2\pi)^2 g_p^3$ and the scale parameter $a = m_\rho/2$. The impact factor (1) satisfies the basic requirement that it should vanish when any of the three gluon momenta goes to zero. It is proportional to g_p^3 where g_p is an effective and so unknown QCD coupling constant inside the proton. So strictly speaking the magnitude of the odderon-(anti)proton coupling is unknown and is in fact an arbitrary parameter. From the comparison with the two gluon exchange model for hadronic cross-sections the authors of [19] estimated $\alpha_p = g_p^2/4\pi \simeq 1$.

Our calculations show that the gluon interactions responsible for the formation of the BLV odderon strongly diminish the odderon amplitude (around 1000 times). With $\alpha_p = 1$ the odderon exchange turns out to be far below any significant effect in the pp or $p\bar{p}$ scattering. To obtain results which more or less agree with the experimental contribution of the $C = -1$ component of the relevant amplitudes one has to augment the value of α_p from unity to ~ 14, which does not seem reasonable.

As we noted our result only partially resolves the QCD odderon problem in the pp and $p\bar{p}$ elastic scattering. The remaining task is to study the different JW odderon, which is made of three reggeons at different spatial points. This is a much more difficult question since the total spectrum of the JW odderon states and so its Green function remain

unknown and the relevant technical problems seem great. This is a problem for future studies.

2 pp and $p\bar{p}$ elastic scattering: phenomenological description

We use the normalization in which the differential cross-section for pp and $p\bar{p}$ scattering is given by the formula

$$\frac{d\sigma}{dt}(p(\bar{p}) + p \rightarrow p(\bar{p}) + p) = \frac{\pi}{s^2}|\mathcal{A}|^2. \tag{3}$$

Here $\mathcal{A}(s, t)$, is corresponding amplitude. which splits into the sum or difference of its $C = +1$ and $C = -1$ parts

$$\mathcal{A}_{pp}^{p\bar{p}} = \mathcal{A}_+ \pm \mathcal{A}_-. \tag{4}$$

. The odderon contribution included into $\mathcal{A}_-$ is given by a convolution of the two proton impact factors Φ_p and the Odderon Green function G_3

$$\mathcal{A}_O = \frac{s}{128\pi} \frac{5}{6} \frac{1}{3!} \frac{1}{(2\pi)^8} \langle \Phi_p | G_3 | \Phi_p \rangle. \tag{5}$$

Here the matrix element is

$$\langle \Phi_p | G_3 | \Phi_p \rangle$$
$$= \int d\mu(\boldsymbol{k}) \int d\mu(\boldsymbol{k}') \Phi_p^*(\{\boldsymbol{k}_i\}) G_3(y, \{\boldsymbol{k}_i\} \cdot \{\boldsymbol{k}_i'\}) \Phi_p(\{\boldsymbol{k}_i'\}) \tag{6}$$

and the measure is $d\mu(\boldsymbol{k}) = d^2k_1 d^2k_2 d^2k_3 \delta^3(k_1 + k_2 + k_3 - q)$ where $q^2 = -t$.

In the literature we have found two phenomenological descriptions of the elastic pp and $p\bar{p}$ scattering, which on the one hand successfully describe the data up to 7 TeV and on the other hand allow for the separation of the $C - 1$ amplitude.

The first one was proposed in [16] in 2018. It presented $\mathcal{A}_\pm$ as a sum of contributions from different Regge exchanges. The pomeron and odderon were taken as dipole Regge singularities. The pomeron contribution was taken as

$$\mathcal{A}_P = i\frac{a_P s}{b_P s_{0P}}\left(r_{1P}^2(s)e^{r_{1P}^2(s)(\alpha_P - 1)} - \epsilon_P r_{2P}^2(s)e^{r_{2P}^2(s)(\alpha_P - 1)}\right), \tag{7}$$

where

$$r_{1P}(s) = b_P + l_P - i\frac{\pi}{2}, \quad r_{2P} = l_P - i\frac{\pi}{2}, \quad l_P = \ln\frac{s}{s_{0P}}$$

and the pomeron trajectory α_P

$$\alpha_P(t) = 1 + \Delta_P + \alpha'_P t.$$

It contained 6 parameters: $a_P, b_P, \Delta_P, \alpha'_P, \epsilon_P$ and s_{0P}. The odderon contribution $\mathcal{A}_O$ was taken in the same form with new parameters $a_O, b_O, \Delta_O, \alpha'_O, \epsilon_O$ and s_{0O}. Apart from the pomeron the $\mathcal{A}_+$ amplitude was taken to have a contribution from the f meson

$$\mathcal{A}_f = a_f e^{-i\pi\alpha_f(t)/2 + b_f t} \left(\frac{s}{s_{0f}}\right)^{\alpha_f(t)}$$

with $\alpha_f(t) = \alpha_{f0} + \alpha'_{f0} t$. It contained 5 parameters $a_f, b_f, \alpha_{0f}, \alpha'_{0f}$ and s_{0f}. The odd amplitude apart from the odderon was assumed to have a contribution from the ω meson $\mathcal{A}_\omega$ of the same form with parameters $a_\omega, b_\omega, \alpha_{0\omega}, \alpha'_{0\omega}$ and $s_{0\omega}$. From the total set of 26 parameters 7 were fixed on physical grounds and the rest were fitted to the existing experimental data on the differential elastic pp and $p\bar{p}$ cross-sections as well as to the data on the total cross-section and ρ-parameter. One can find the values of the fitted parameters in Ref. [16]. Calculation show that at $\sqrt{s} = 1$ GeV and higher the contribution to the C-odd amplitude of the ω is several orders smaller than that of the odderon. At the energy of interest $\sqrt{s} \sim 2$ GeV the ratio $|\mathcal{A}_\omega/\mathcal{A}_O|$ has its maximal value 0.6% at quite small $|t|$, of the order 0.01 (Gev/c)^2 and rapidly diminishes with the growth of $|t|$: at $|t| = 0.5, 1$ and 2 (GeV/c)^2 the ratio is $2 \cdot 10^{-5}, 2 \cdot 10^{-8}$ and 10^{-12}.

The second description was proposed in [17] in 2019. It was based on the modified Phillips–Barger model [20] in which the scattering amplitude was parameterized as follows

$$\mathcal{A}(s,t) = i \left[F_p^2 \sqrt{A} e^{Bt/2} + e^{i\phi} \sqrt{C} e^{Dt/2} \right]. \qquad (8)$$

Here F_p is the Dirac form-factor of the proton. It contains a set of only 5 parameters, different for pp and $p\bar{p}$ scattering. They all depend on the energy. In [17] an interpolation of the pp parameters was proposed for energies in the range from 25 GeV to 13 TeV as quadratic functions of $\ln s$. For the $p\bar{p}$ scattering two sets of parameters were given for energies 546 GeV and 1.8–1.96 TeV. This parametrization does not allow to separate the odderon exchange amplitude but rather the total $C = -1$ amplitude as $\mathcal{A}_- = \mathcal{A}_{p\bar{p}} - \mathcal{A}_{pp}$. However the mentioned results from [16] convince that at energies of the order 2 TeV contributions from $C = -1$ exchanges other than the odderon are insignificant, so that one can identify $\mathcal{A}_-$ with the odderon exchange.

3 The BLV odderon

The QCD BLV Odderon was found in [7]. Its properties and coupling to the proton impact factor were discussed in some details in [21]. Here we reproduce some main points necessary for understanding our calculations.

The odderon wave function in the 3-gluon momentum space is constructed from the known pomeron solutions $E^{(\nu,n)}$ [22,23], with odd $n = \pm 1, \pm 3, \dots$. Their intercept χ quickly goes down with $|n|$ and ν and is greater than unity by

$$\chi(\nu, n) = \bar{\alpha}_s \left(2\psi(1) - \psi\left(\frac{1 + |n|}{2} + i\nu\right) \right.$$
$$\left. - \psi\left(\frac{1 + |n|}{2} - i\nu\right) \right), \quad \bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}. \qquad (9)$$

It was demonstrated in [7] that

$$\Psi^{(\nu,\pm 1)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$
$$= c \sum_{(123)} \frac{(\mathbf{k}_1 + \mathbf{k}_2)^2}{\mathbf{k}_1^2 \mathbf{k}_2^2} E^{(\nu,\pm 1)}(\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_3),$$
$$c(\nu, n) = \frac{3}{(2\pi)^{3/2}} \frac{1}{\sqrt{-120\chi(\nu,n)/g_s^2}} \qquad (10)$$

satisfies the odderon equation.

Function $E^{(\nu,\pm 1)}(\mathbf{k}_1, \mathbf{k}_2)$ is the Fourier transform of the well-known BFKL eigenfunctions

$$E^{(h,\bar{h})}(\mathbf{r}_{10}, \mathbf{r}_{20}) = \left(\frac{r_{12}}{r_{10} r_{20}}\right)^h \left(\frac{\bar{r}_{12}}{\bar{r}_{10} \bar{r}_{20}}\right)^{\bar{h}}, \qquad (11)$$

where $\mathbf{r}_{10} = \mathbf{r}_1 - \mathbf{r}_0$ etc, $h = (1 + n)/2 + i\nu$, $\bar{h} = (1 - n)/2 + i\nu$, $n = \pm 1$ and the standard complex notation for two-dimensional vectors is used on the right-hand side.

The Green function G_3 corresponding to the propagation of the BLV odderon turns out to be given by

$$G_3(y|\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3|\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}'_3)$$
$$= \sum_{n=\pm 1} \int_{-\infty}^{+\infty} d\nu e^{y \chi(\nu,n)} \frac{(2\pi)^2 (\nu^2 + 1/4)}{\nu^2 (\nu^2 + 1)} \Psi^{(\nu,n)}$$
$$\times (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \Psi^{(\nu,n)*}(\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}'_3), \qquad (12)$$

where $\Psi^{(\nu,n)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ are given by (10) and $y = \ln(s/s_0)$ is the rapidity. In our calculations we take $s_0 = 100$ GeV^2, the value used in the parametrization [16]. In (12) one sees ν^2 in the denominator. However, as we shall presently find this ν^2 will be fully canceled by the ν^4 coming from the product of the proton–odderon couplings. As a result the integrand of (10) behaves as ν^2 at $\nu \to 0$.

The matrix element in (5) becomes

$$\langle \Phi_p | G_3 | \Phi_p \rangle = \sum_{n=\pm 1} \int_{-\infty}^{+\infty} d\nu e^{y \chi(\nu,n)} \frac{(2\pi)^2 (\nu^2 + 1/4)}{\nu^2 (\nu^2 + 1)}$$
$$\times \left| \left\langle \Phi_p | \Psi^{(\nu,n)} \right\rangle \right|^2, \qquad (13)$$

where

$$\left\langle \Phi_p | \Psi^{(\nu,n)} \right\rangle = \int d\mu(\mathbf{k}) \Phi_p(\{\mathbf{k}_i\}) \Psi(\nu, n)(\{\mathbf{k}_i\}), \qquad (14)$$

or using (10)

$$\left\langle \Phi_p | \Psi^{(\nu,\pm 1)} \right\rangle = 3c \int d^2 l f(\mathbf{l}) E^{(\nu,\pm 1)}(\mathbf{l}, \mathbf{q} - \mathbf{l}), \qquad (15)$$

where

$$f(\mathbf{l}) = \int d^2 k \frac{l^2}{k^2 (\mathbf{l} - \mathbf{k})^2} \Phi_p(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \qquad (16)$$

with $\mathbf{k}_1 = \mathbf{k}$, $\mathbf{k}_2 = \mathbf{l} - \mathbf{k}$ and $\mathbf{k}_3 = \mathbf{q} - \mathbf{l}$.

Performing Fourier transformation one obtains the explicit expression for $E^{(h\bar{h})}(\mathbf{k}_1, \mathbf{k}_2)$ [21]

$$E^{(h\bar{h})}(\mathbf{k}_1, \mathbf{k}_2) = C(X(\mathbf{k}_1, \mathbf{k}_2) + -X(\mathbf{k}_2, \mathbf{k}_1)),$$
$$C = \frac{\nu}{(4\pi)^2}(1 + i\nu)\Gamma(1 - i\nu)\Gamma(2 - i\nu) \qquad (17)$$

where X is expressed via hypergeometric functions. For $n = 1$

$$X(\mathbf{k}_1, \mathbf{k}_2) = \left(\frac{k_1}{2}\right)^{i\nu-2} \left(\frac{\bar{k}_2}{2}\right)^{i\nu-1}$$
$$F\left(-i\nu, 1 - i\nu; 2; -\frac{\bar{k}_1}{\bar{k}_2}\right) F\left(1 - i\nu, 2 - i\nu; 2; -\frac{k_2}{k_1}\right). \qquad (18)$$

The behavior of $\chi(\nu)$ indicates that at large y the contribution comes from the region of small ν. This means that in the limit of large y it is sufficient to know function $E^{(\nu,\pm 1)}(\mathbf{k}_1, \mathbf{k}_2)$ at small values of ν. One obtains in the first order in ν

$$E_1(\mathbf{k}_1, \mathbf{k}_2) = \frac{\nu}{2\pi^2 q} \left(\frac{1}{k_1 \bar{k}_2} - \frac{1}{k_2 \bar{k}_1}\right). \qquad (19)$$

This function is antisymmetric in the azimuthal angle. So it is orthogonal to the two impact factors which are azimuthal symmetric. For this reason a non-zero contribution only comes from the terms quadratic in ν. Omitting those of them which have the same structure as (19) we find

$$E_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{i\nu^2}{2\pi^2 q} \left[\frac{1}{k_2^2} \ln k_1^2 - \frac{1}{k_1^2} \ln k_2^2\right.$$
$$\left. + \left(\frac{1}{k_1^2} - \frac{1}{k_2^2}\right) \ln q^2\right] \equiv \nu^2 g(\mathbf{k}_1, \mathbf{k}_2) \qquad (20)$$

As a result the matrix element (22) behaves as ν^2 at small ν. Its square gives ν^4, which converts ν^2 in the denominator of (13) to ν^2 in the numerator.

Leaving in the integrand of (13) only the exponential factor multiplied by ν^2, performing integration over ν and summation over $n = \pm 1$ and absorbing their result into factor b one finally obtains

$$\langle \Phi_p | G_3 | \Phi_p \rangle = b y^{-3/2} J^2(q), \quad b = \frac{9}{320\sqrt{2\pi}} \bar{\alpha}^{-3/2} \zeta(3)^{-5/2}. \qquad (21)$$

Here

$$J(q) = \int d^2 l f(\mathbf{l}) g(\mathbf{l}, \mathbf{q} - \mathbf{l}). \qquad (22)$$

The matrix element (21) diminishes with energy as $y^{-3/2}$.

4 The impact factor and the final odderon amplitude

The proton impact factor is nonperturbative. We use the Fukugita–Kwiecinski impact factor (1), (2) proposed in [18] and used in [19,21] with $\alpha_p = 1$. As mentioned the impact factor (1) vanishes when any of the three gluon momenta goes to zero. This guarantees that calculation of $f(\mathbf{l})$ given by (16) is infrared convergent.

Explicitly one finds

$$f(\mathbf{l}) = \int d^2 k \frac{l^2}{k^2 (\mathbf{l} - \mathbf{k})^2}$$
$$\times \left[F(\mathbf{q}, 0, 0) - \sum_{j=1}^{3} F(\mathbf{k}_j, \mathbf{q} - \mathbf{k}_j, 0) + 2F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \right] \qquad (23)$$

where $\mathbf{k}_1 = \mathbf{k}$, $\mathbf{k}_2 = \mathbf{l} - \mathbf{k}$ and $\mathbf{k}_3 = \mathbf{q} - \mathbf{l}$. The integral (23) is infrared finite, since the square bracket vanishes if any of the gluon momenta go to zero. However, individual terms inside the square bracket are infrared divergent. So at intermediate stages it is convenient to introduce an auxiliary infrared regularization. The integral $J(q)$ given by (22) contains 4 integrations. One of them in $f(\mathbf{l})$ can be done analytically due to the simple form of Φ_p (see [21] for details). The other three require numerical integration.

The final odderon amplitude $\mathcal{A}_O$ depends on the two coupling constants $\bar{\alpha}$ and α_p, the latter referring to the nonperturbative coupling inside the proton. In fact only the overall magnitude of the amplitude depends on them: it is proportional to $\alpha_p^3 \bar{\alpha}^{-3/2}$. In our calculations we fixed $\bar{\alpha} = 0.2$.

With the original value $\alpha_p = 1$ our results are presented in Fig. 1. To avoid energy dependence of the plot we actually show $y^{3/2} \mathcal{A}_O$ which is energy independent. The two panels in Fig. 1 illustrate on the one hand the t dependence in the whole region $0 < -t < 100$ GeV^2 with particular attention to the behavior at very small $|t|$ and on the other the t dependence in the region $0.2 < -t < 4$ GeV^2 relevant for the experimental setup. As we observe the odderon amplitude exhibits a rather whimsical behavior in t. At $t = 0$ it goes to zero as $\propto |t|$. So it does not contribute to the ratio
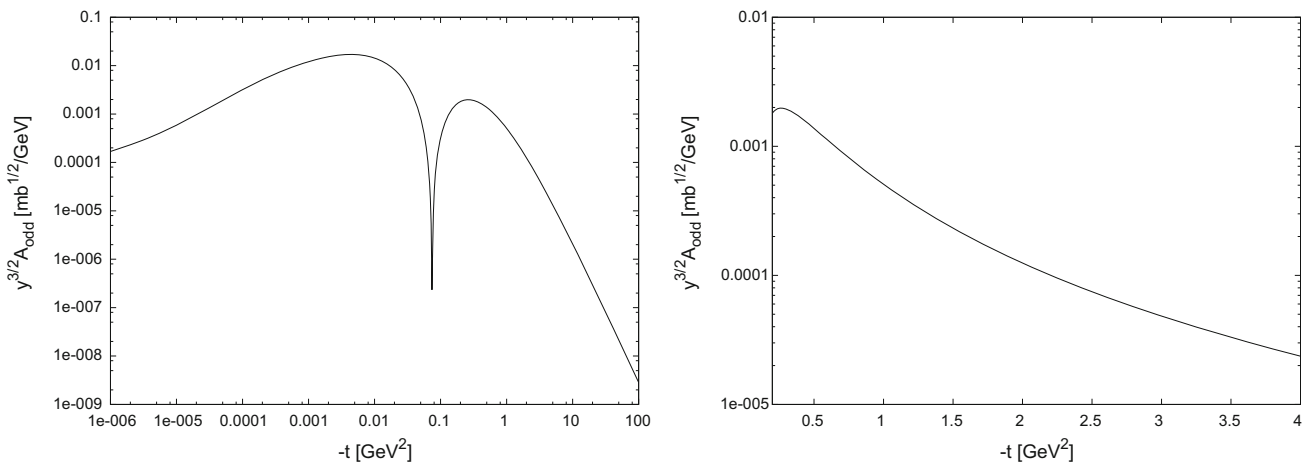
Fig. 1 The calculated odderon amplitude $\mathcal{A}_O$, Eq. (5), with the proton coupling $\alpha_p = 1$ multiplied by $y^{3/2}$ for $0 < -t < 100$ GeV^2 in the logarithmic scale (left panel) and for $0.2 < -t < 4$ GeV^2 in the natural scale (right panel)

$\rho = \text{Re}\mathcal{A}/\text{Im}\mathcal{A}$ at $t = 0$, which is important in relation to experimental observations (see [9,12]).

To study the influence of the gluon interaction in the 3-gluon exchange we also calculated the $C = -1$ amplitude corresponding to the non-interacting three-gluon exchange, used in the old paper [15] for low energies with very optimistic conclusions. The three gluon exchange $\mathcal{A}_{3g}$ is given by the same formula (5) in which the matrix element is just

$$\langle \Phi_p | G_3 | \Phi_p \rangle_{3g} = \int d\mu(\boldsymbol{k}) \left| \Phi_p(\{\boldsymbol{k}_i\}) \right|^2 \frac{1}{k_1^2 k_2^2 k_3^2}. \qquad (24)$$

In this case all 4 integrations have to be performed numerically. Our results show that, first, $\mathcal{A}_{3g}$ shows a smooth behavior in t. At $t = 0$ it is finite and equal to $4.29 \sqrt{mb}$/GeV. With the growth of $|t|$ it monotonously diminishes. Second, $\mathcal{A}_{3g}$ is much greater than the odderon amplitude $\mathcal{A}_O$. With the same α_p three-gluon exchange amplitude is roughly 300 times greater than $y^{3/2}\mathcal{A}_O$. This is illustrated in Fig. 2 where we compare $\mathcal{A}_{3g}/300$ and $y^{3/2}\mathcal{A}_O$. Note that in [15] the pomeron coupling constant α_p was equal to 0.3, which means that their $\mathcal{A}_{3g}$ was 27 times smaller than in Fig. 2 However this still remains far above the odderon amplitude with $\alpha_p = 1$. So it turns out that gluon interactions drastically diminish the three-gluon exchange in the BLV odderon.

5 Cross-sections

In this section we study the cross-sections which are obtained with the found odderon amplitude and compare them with the existing experimental data. The pp data cover a wide energetic interval from low energies up to 13 TeV. Unfortunately the $p\bar{p}$ data are much more scarce. At high energies we shall consider both data at the closest possible energies:
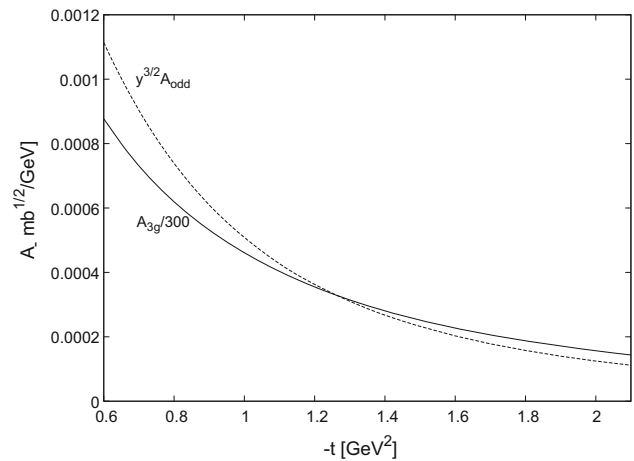

Fig. 2 The calculated odderon amplitude $\mathcal{A}_O$ with the proton coupling $\alpha_p = 1$ multiplied by $y^{3/2}$ for $0.6 < -t < 2$ GeV^2 compared with the exchange of three non-interacting gluons in the $C = -1$ state with the same α_p and divided by 300

pp at 2.76 GeV and $p\bar{p}$ at 1.96 GeV. The two models discussed in the Sect. 1 give possibilities to present the relevant odderon amplitudes (in [16] and in [17] assuming in the latter case that all other reggeon exchanges are insignificant).

Remarkably in both models the odderon amplitude is complex (in [16] because the odderon is not taken as a pole in the complex j-plane but rather as a dipole). The odderon amplitudes in both models are presented in Fig. 3. Apart from a large imaginary part both amplitudes are much greater than our calculated (real) $\mathcal{A}_O$ at these energies.

So should one try to adjust to the data the cross-sections obtained after our calculated $\mathcal{A}_O$ takes the role of the odderon amplitude, apart from the absence of the imaginary part, one is compelled to seriously increase its magnitude by taking α_p considerably higher than unity. In the two following pictures
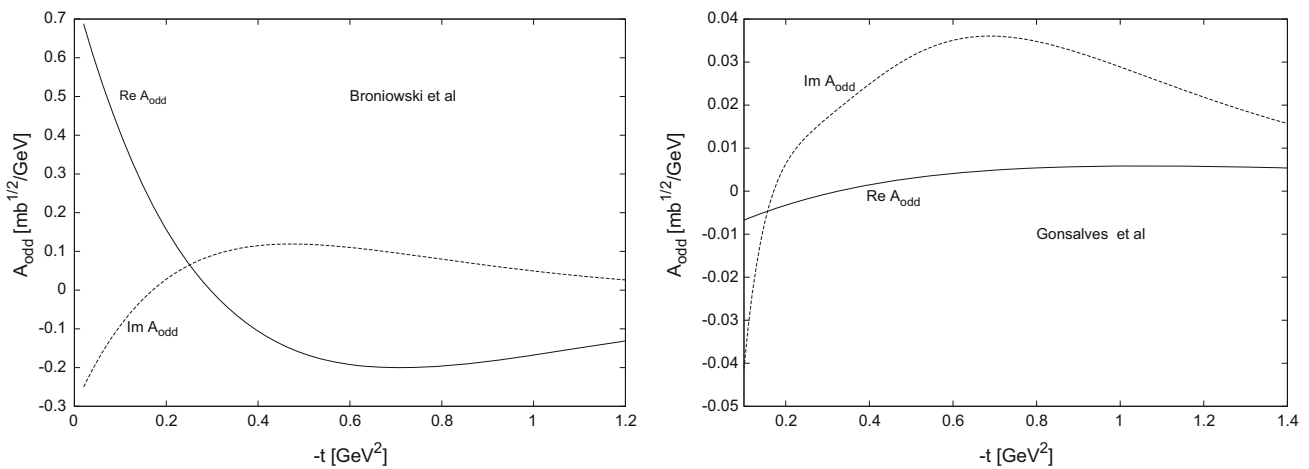
Fig. 3 The odderon amplitude extracted from the phenomenological models [16] (left panel) and [17] right panel0
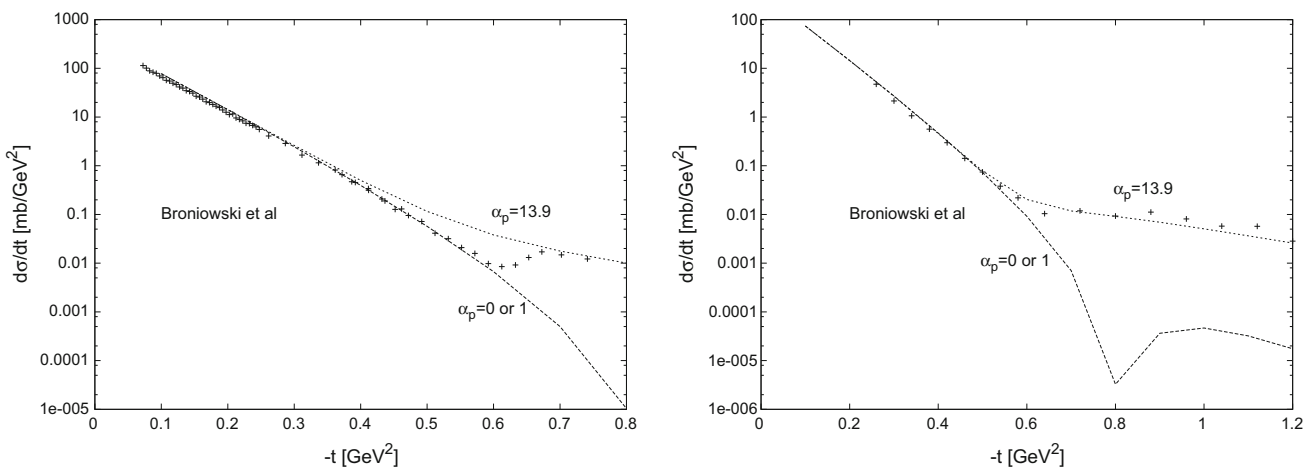

Fig. 4 The differential cross-section for elastic pp (left panel) and $p\bar{p}$ (right panel) scattering obtained after the substitution of the $C = -1$ amplitude in [16] by the calculated odderon amplitude $\mathcal{A}_O$ at $\sqrt{s} = 2.76$ and 1.96 TeV respectively. The experimental data are from [8] and [24] respectively

we show our attempts in this direction for the two models [16,17]. In both cases the original amplitude with $\alpha_p = 1$ (Fig. 1) does not practically change the cross-section without the $C = -1$ component at all, that is with $\alpha_p = 0$. In fact the difference between the curves with $\alpha_p = 0$ and $\alpha_p = 1$ is indistinguishable on the adopted scale. The results more or less in the range of the data for the parametrization of [16] require α_p in the region of ~ 14. The optimal value to simultaneously describe pp and $p\bar{p}$ data is 13.9 (shown in Fig. 4). For the parametrization of [17] the situation is worse (Fig. 5): with the same large α_p one gets a nice agreement for pp but any variation of α_p crudely fails for $p\bar{p}$, since with the growth of α_p the curve moves upwards as compared to the data.

Since values of α_p of the order 14 do not seem physically reasonable the true result of this comparison is that the BLV odderon is simply too small to be felt in pp and $p\bar{p}$ elastic scattering.

6 Conclusions

We have calculated the $C = -1$ amplitude corresponding to the exchange of the BLV odderon with the maximal intercept equal to exactly unity. This amplitude is real and shows a rather peculiar t dependance (Fig. 1). At $t = 0$ it is equal to zero. Compared to the amplitude coming from the interchange of three non-interacting gluons in the $C = -1$ state our calculated amplitude is ~ 1000 times smaller with the same coupling constant. In the existing phenomenological models the $C = -1$ amplitude is complex and about 200 times larger in magnitude. So if one believes in these models the BLV odderon with a reasonable values for the coupling constant is far smaller to manifest itself in the pp and $p\bar{p}$ elastic scattering. At $t = 0$ the BLV odderon doles not contribute to the ratio ρ.

In fact this conclusion is not unexpected. In the processes like $\gamma^* + p \rightarrow \eta_c + p$ the cross-sections come exclu-
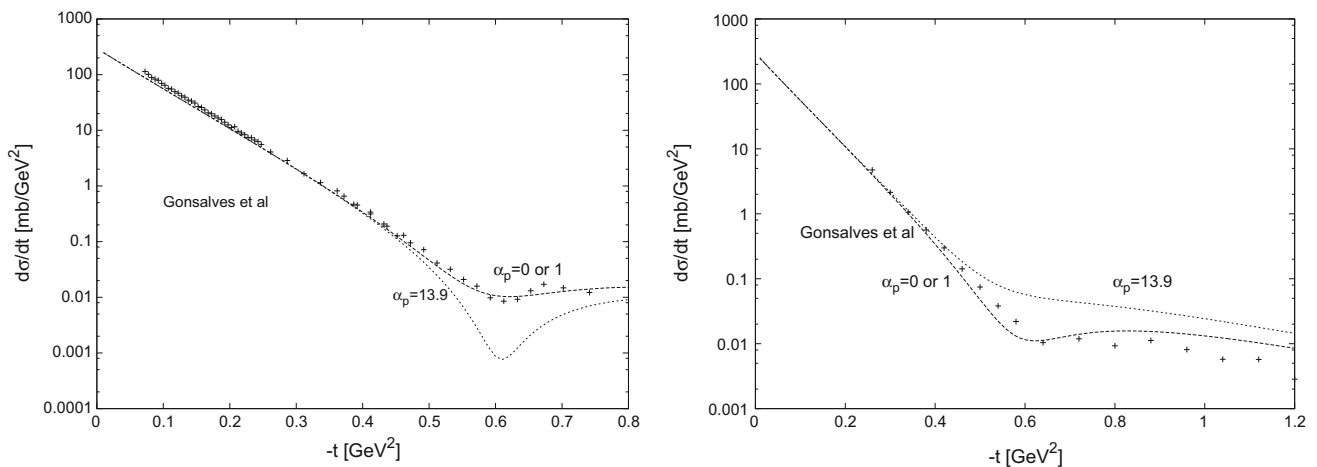
Fig. 5 The differential cross-section for elastic pp (left panel) and $p\bar{p}$ (right panel) scattering obtained after the substitution of the $C = -1$ amplitude in [17] by the calculated odderon amplitude $\mathcal{A}_O$ at $\sqrt{s} = 2.76$ and 1.96 TeV respectively. The experimental data are from [8] and [24] respectively

sively from the BLV odderon exchange. Previous calculations found that these cross-sections were extremely small, far beyond our present experimental facilities.

The remaining open question is the role of the JW odderon with all three reggeized gluons at different spatial points. It does not contribute to $\gamma^* + p \rightarrow \eta_c + p$ but certainly does in the elastic pp and $p\bar{p}$ scattering. It is possible that its contribution is much greater than of the BLV odderon. So although theoretically it diminishes with energy stronger that the BLV odderon, the dominance of the latter is not effective at presently achieved energies and the JW odderon can be discovered in (anti)proton elastic scattering. However the JW odderon is an object much more complicated than the BLV odderon. Its Green function is not known at present. So, although very important, its study is postponed for future investigations.

References

1. J. Bartels, Nucl Phys. B **151**, 293 (1979)
2. J. Bartels, Nucl Phys. B **175**, 365 (1980)
3. J. Kwiecinski, M. Praszalowicz, Phys. Lett. B **94**, 413 (1980)
4. P. Gauron, L.N. Lipatov, B. Nicolescu, Phys. Lett. B **260**, 407 (1991)
5. P. Gauron, L.N. Lipatov, B. Nicolescu, Z. Phys. C **63**, 253 (1994)
6. R.A. Janik, J. Wosiek, Phys. Rev. Lett. **82**, 1092 (1999)
7. J. Bartels, L.N. Lipatov, G.P. Vacca, Phys. Lett. B **477**, 178 (2000)
8. G. Antchev et al. (TOTEM), Eur. Phys. J. C **80**, 91 (2020)
9. E. Martynov, B. Nicolescu, Phys. Lett. B **778**, 414 (2018)
10. T. Scorgo, R. Pasechnik, A. Ster, Eur. Phys. J. **79**(1), 62 (2019)
11. T. Scorgo, T. Novak, R. Pasechnik, A. Ster, L. Szanyi, arXiv:1912.11968 [hep-ph]
12. J.R. Cudell, O.V. Selyugin, arXiv:1901.05863 [hep-ph]
13. V.A. Petrov, arXiv:2003.06280 [hep=ph]
14. A. Donnachie, P.V. Landshoff, Nucl. Phys. **31**, 189 (1984)
15. H.G. Dosch, C. Ewerz, V. Schatz, Eur. Phys. J. **24**, 561 (2002)
16. W. Broniowski, L. Jenkovszky, E.R. Arriola, I. Szanyi, Phys. Rev. D **98**, 074012 (2018)
17. V.P. Gonsalves, P.V.R.G. Silva, Eur. Phys. J. C **79**, 237 (2019)
18. M. Fukugita, J. Kwiecinski, Phys. Lett. B **83**, 119 (1979)
19. J. Czyzewski, J. Kwiecinski, L. Motyka, M. Sadzikowsky, Phys. Lett. B **398**, 400 (1997) [Erratum: Phys. Lett. B **411**, 402 (1997). 5]
20. D.A. Fagundes, G. Pancheri, A. Grau, S. Pacetti, Y.N. Srivastava, Phys. Rev. D **88**, 094019 (2013)
21. J. Bartels, M.A. Braun, D. Colferai, G.P. Vacca, Eur. Phys. J. C **20**, 323 (2001)
22. L.N. Lipatov, Pomeron in quantum chromodynamics, in *Perturbative QCD*, ed. by A.H. Mueller (World Scientific, Singapore, 1989), pp. 411–489
23. L.N. Lipatov, Phys. Rep. **286**, 131 (1997)
24. V.M. Abzov et al. (D0 Collab), Phys. Rev. D 86, 012009 (2012)