



Sterile neutrinos, black hole vacuum and holographic principle

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Abstract We construct an effective field theory (EFT) model that describes matter field interactions with Schwarzschild mini-black-holes (SBH's), treated as a scalar field, $B_0(x)$. Fermion interactions with SBH's require a complex spurion field, θ_{ij} , which we interpret as the EFT description of “holographic information,” which is correlated with the SBH as a composite system. We consider Hawking's virtual black hole vacuum (VBH) as a Higgs phase, $\langle B_0 \rangle = V$. Integrating sterile neutrino loops, the information field θ_{ij} is promoted to a dynamical field, necessarily developing a tachyonic instability and acquiring a VEV of order the Planck scale. For N sterile neutrinos this breaks the vacuum to $SU(N) \times U(1)/SO(N)$ with N degenerate Majorana masses, and $\frac{1}{2}N(N+1)$ Nambu-Goldstone neutrino-Majorons. The model suggests many scalars fields, corresponding to all fermion bilinears, may exist bound nonperturbatively by gravity.

1 Introduction

In the present paper we will discuss the issue of the “black hole information paradox” in the context of an effective quantum field theory (EFT). Classical EFT is a powerful tool for summarizing low energy processes where the detailed short-distance behavior of a system is integrated out. Examples of its application to black holes can be found in [1] and references therein, where effective field theories are world-line actions and carry local operators that represent known cases of the breakdown of classical no hair theorems. We also implicitly rely on the intuition of Dvali and Gomez in their picture of threshold quantum black holes [2–4].

We assume there is a quantum limit for black holes, near the threshold of production or collisions in scattering processes, or in coherent processes such as Bose-Einstein condensation. These processes can exist in field theoretic

descriptions even through the underlying objects are complicated many body states, *e.g.*, Rubidium atoms with $Z = 37$ can be described as pointlike objects, yet form a quantum condensate which can be described as the vacuum value of a field. Hadronic boundstate production can be described by pointlike interactions despite the short distance complexity of a hadron.

Presently we will describe a black hole by a field, $B(x)$, which can create or destroy a black hole of mass M at the event x and satisfies a free field equation of motion. We ignore the warped external geometry at large distances compared to the black hole Schwarzschild radius of $B(x)$ (following [1]). Fundamental problems must then be faced in constructing the interactions of $B(x)$ with matter.

We consider the simplest case of a pair of scalars which possess a global $SO(2) = U(1)$ symmetry. How do we write down a pointlike interaction of a pair of scalars ϕ_i and ϕ_j with a real Schwarzschild black hole $B(x)$? By “no-hair” reasoning any interaction, such as $\phi_1 + \phi_1 \rightarrow B$ must have the same physical rate, or same quantum amplitude, as any other interaction such as $\phi_1 + \phi_2 \rightarrow B$. A naive “no-hair” theorem would say these must be indistinguishable processes.

However, it is not possible to mathematically write down interactions that would have this universality. Once we commit to a certain amplitude for $\phi_i + \phi_j \rightarrow B$ then we will generally not have an equivalent amplitude for some other process $\phi'_i + \phi'_j \rightarrow B$ where (ϕ'_i, ϕ'_j) is a general linear superposition of the (ϕ_i, ϕ_j) . This is only possible in special cases, such as if we maintain the symmetry $U(1)$ and restrict ourselves to $U(1)$ transformations of the fields, in which case the process $\phi_1 + \phi_1 \rightarrow B$ is forbidden.

Stemming from this it is conventional wisdom that the global symmetry is broken by quantum gravity. Then different configurations the the two scalars may have different amplitudes to interact with $B(x)$. However, given that both processes $\phi_1 + \phi_2 \rightarrow B$ and $\phi_1 + \phi_1 \rightarrow B$ must exist, since gravity is “flavor blind,” what rule dictates their amplitudes? The breaking of a symmetry is actually expressed by the form

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of the interaction. Breaking a symmetry actually *requires that we supply more information, not less.*

One might argue that the quantum effective field theory does not exist, and the S-matrix does not exist, at Planckian high energies. However, given a mini black hole, we can consider an idealized low energy experiment where the resolvable distance scales are large. We make an arbitrary initial combination of the two ϕ_i 's and allow them to collide with the mini Schwarzschild black hole, B , resulting in a final state black hole B' . We take pains to observe the exclusive process where no other particles appear in the initial or final states. Our question is, what is the effective interaction $g\theta^{ij}\phi_i\phi_jBB'$? What then dictates the couplings θ_{ij} ? Theoretically or experimentally determining θ_{ij} may be problematic, but we cannot deny its existence.

Quantum mechanics conflicts with no-hair theorems. If we believe in Dirac's formulation of the Hilbert space of superpositionable states, an S-matrix and Wilsonian effective field theory (which we do) then we must allow for the possibility that $B(x)$ carries information. To us the resolution is that the information contained in the $\theta^{ij}\phi_i\phi_j$ initial state is transferred to the B' state, and resides on its horizon. Essentially the black hole B' has acquired "information hair" and is now defined by a field $B' = \theta^{ij}B(x)$. This acquired information supplements any other information that was already present on B , though not probed in the experiment.

In Sect. 2 we will reiterate these issues in more detail for a pair of scalar fields ϕ_i , $i = (1, 2)$.

To connect to real-world fields and make these issues more concrete we consider, in addition to scalars, sterile neutrinos, possessing an $SU(N) \times U(1)$ global symmetry, and consider how they interact with a Schwarzschild mini-black hole (SBH) of mass M_{Planck} . In an effective field theory, we describe the SBH by a real quantum field, $B_0(x)$, whose excitations are minimal mass, tiny black holes. We can follow the intuition of [2–4], who view the spectroscopy of mini-blackholes as a tower of states labelled by a quantum number N where N is effectively the number of self gravitating gravitons in the core of the BH.

We know that an s -wave pair of sterile neutrinos can fall into the SBH, or scatter close to it. We can write couplings of the neutrinos to B_0 that preserves or violates the global symmetry. This then raises the question of how the neutrino number current is carried (or destroyed) by the black hole?

A conserved global-charge current cannot be carried by a real scalar field alone. To engineer a coupling of a Weyl neutrino pair, $\sim \epsilon^{\alpha\beta}v_\alpha^i(x)v_\beta^j(x)$, to $B_0(x)$ we need a complex "information" that is either intrinsic to the black hole or is a field that can be attached to the SBH. We will designate this as $\theta_{ij}(x)$. Physically, the neutrino pair falls into the SBH, but never appears to cross the horizon. However, at some point we can no longer distinguish the neutrino pair and black hole from a pure black hole, though there may be some

information record in the EFT black hole that is absorbing them.

In the effective field theory we describe the information by a "spurion," a complex field that transforms under $SU(N)$ identically to the neutrino pair and allows us to tie all indices together in the interaction vertex. This is co-localized with the SBH and the neutrinos at the interaction vertex. The spurion represents information on the horizon of the SBH. The EFT is describing a mini-black hole and the region immediately surrounding it, external to the horizon, which includes anything orbiting or in the process of infall as seen by the Schwarzschild observers and this includes $\theta_{ij}(x)$.

We distinguish three possible logical cases for a dynamical θ_{ij} . (I) Information is explicitly lost; (II) Information is conserved but does not propagate; (III) Information is conserved and is carried by the black hole.

Case (I) evidently implies an explicit, arbitrary fixed value of the spurion permeating all of space with a fixed orientation in the group space, hence it breaks $SU(N) \times U(1)$ transformation, and the associated Noether current is violated. It seems that this case makes no sense fundamentally, since there is no procedure to specify $\theta_{ij}(x)$, even as a random variable, however it may arise spontaneously through the formation of a condensate of underlying black holes. Note that just naively summing indices, $\sum_{ij}\epsilon^{\alpha\beta}v_\alpha^i v_\beta^j B_0(x)$, implies a particular basis choice and therefore a particular choice of $\theta_{ij}(x) \sim \sum_{nm}\delta_{in}\delta_{jm}$, a matrix with all elements = 1. This is logically equivalent to case I and seems unreasonable unless it arises by spontaneous symmetry breaking.

Case (II) implies that we introduce a random field variable $\theta_{ij}(x)$, that has no correlation with the black hole itself and, minimally, has no derivatives. This implies that there is no current associated with $\theta_{ij}(x)$, and information is not carried by the SBH, yet the neutrino global current is conserved. We can treat $\theta_{ij}(x)$ as a random field, and average over $\theta_{ij}(x)$ configurations through which the black hole and the neutrinos propagate. This is somewhat akin to a "spin-glass" in condensed matter physics [5,6]. It simply ends up promoting $\theta_{ij}(x)$ to being a propagating dynamical field.

Case (III) implies that the information is attached to the effective SBH and is dynamically transported by it. This requires a "conjoined kinetic term" where the $\theta_{ij}(x)$ moves with $B_0(x)$ as a composite field, $\sim \theta_{ij}(x)B_0(x)$. Here $\theta_{ij}(x)$ is positionally correlated with $B_0(x)$. Case (III) is the most sensible to us, and we interpret it tentatively as an *effective field theory description* of the holographic principle [7–9]. Once we engineer the interaction described in Case (III) above, we find that there are potential dynamical consequences.

As an application, we consider the effect of this on the neutrino vacuum. The vacuum of the underlying $B_0(x)$ field may be nontrivial. Indeed, we know of three classical and important (effective) scalar fields in nature, the Higgs field of the

standard model (SM), the σ -meson of QCD chiral dynamics, and the Ginzburg-Landau field of a superconductor (an effective description of a Cooper pair). In each of these examples the vacuum is a ‘‘Higgs phase.’’ In our present scenario we consider the possibility that $B_0(x) \rightarrow V + B$ where V is a nontrivial vacuum expectation value (VEV).

A black hole Higgs phase will, in our model, have implications for the dynamical behavior of information encoded in θ_{ij} : owing to the effects of neutrino loops external to the horizon (in the EFT) *global information becomes a propagating massless field in a Higgs phase of $B_0(x)$* . Moreover, this back reaction of the neutrino fields induces an instability, causing a tachyonic potential for $\theta_{ij}(x)$ to develop, and in turn, $\theta_{ij}(x)$ acquires a VEV. This happens in both cases (II) and (III) (though there are slight differences) and we obtain an effective potential that leads to a VEV for θ_{ij} , and sterile neutrinos develop Planck scale masses. The $SU(N) \times U(1)$ global symmetry is broken to $SO(N)$.

2 Quantum inconsistency with the ‘‘no-hair’’ theorem

Consider the ‘‘no-hair’’ theorem of classical black holes. This allows for a black-hole, B , to have gauge charges, or quantum numbers. For example, black holes with the quantum numbers of the standard model Higgs boson could in principle be produced in collisions, such as a pair of electrons $e_L + \bar{e}_R \rightarrow B$. However we could equally well consider a collision $c_L + \bar{t}_R \rightarrow B$ of a lefthanded-charm quark with an anti-righthanded-top quark. These initial states have the same gauge charges, but involve different combinations of flavors of initial state fermions. Gravity, being flavor blind, supposedly cannot make a flavored black hole. Hence, B must be universally coupled to any and all fermion bilinears.

However, a universal coupling to any and all fermion bilinears cannot exist in quantum mechanics. In the above example if we allow the latter process then there will be some mixed state collision, such as a Cabibbo rotated charm-strange combination, $(c_L \cos \theta + s_L \sin \theta) + \bar{t}_R \rightarrow B$, where we did not Cabibbo rotate the top quark. This process will have a θ -dependent rate and thus does not respect the flavor-blind universality of gravity.

To simplify, consider the production of a black hole in a collision of two scalar particles ϕ_i , with two global flavors, $i = 1, 2$. We want to describe the process $\phi_i + \phi_j \rightarrow B$ in an effective field theory. We could describe this process directly by an S -matrix element, $\langle ij|B \rangle$ where B can be a Schwarzschild black hole if the collision of the scalars is s -wave. The existence of the S -matrix implicitly assumes that $|B \rangle$ is a quantum state.

Since gravity is flavor blind, a no-hair theorem would evidently imply that the S -matrix element, $\langle ij|B \rangle = S$, must be a constant independent of the choice of normalized in-states

for the ϕ_i particles, for arbitrary i and j . (A weaker statement might be that the probability is the same for each such initial state). In a quantum field theory description the S -matrix must exist and we can introduce a field for the black hole which we designate as $B(x)$, and we also have the two fields $\phi_i(x)$. Our equivalent field theory problem is then, how do we write the local interaction vertex for the process $\langle ij|B \rangle$?

Let us introduce a complex field $\Phi = \phi_1 + i\phi_2$ Consider the interaction:

$$g|\Phi|^2 B = g \sum_i \phi_i^2 B \tag{1}$$

This would forbid $\phi_1 + \phi_2 \rightarrow B$ i.e., $i \neq j$. It equivalently forces the S -matrix to be diagonal in flavor, $\langle ij|B \rangle = \delta_{ij} S$ hence $\langle 1, 2|B \rangle = 0$. Therefore an $U(1) = SO(2)$ singlet cannot describe the process since we certainly expect that $\phi_1 + \phi_2$ can collide to make a Schwarzschild black hole and should have the same S -matrix.

However, the interaction vertex also cannot be

$$ig(\Phi^2 - \Phi^{*2})B = -2g2\phi_1\phi_2 B \tag{2}$$

This indeed describes the process $\langle 1, 2|B \rangle = 2g$. However, suppose we consider the collision $\langle 1', 2'|B \rangle = 2g$ where:

$$\begin{aligned} |1'\rangle &= \cos(\theta)|1\rangle + \sin(\theta)|2\rangle \\ |2'\rangle &= -\sin(\theta)|1\rangle + \cos(\theta)|2\rangle \end{aligned} \tag{3}$$

These are simple mixed states, and can readily be produced in a laboratory, such as with neutral K-mesons, or oscillating neutrinos. In this case we see that the amplitude is theta dependent:

$$\langle 1', 2'|B \rangle = 2g \cos(2\theta) \tag{4}$$

Hence we see that we cannot write a local field description of $\phi_1 + \phi_2 \rightarrow B$ that yields an S -matrix, $\langle ij|B \rangle = S$ that is constant independent of i and j . The ‘‘no-hair’’ theorem, in this sense, is incompatible with quantum theory. Note that in the $U(1)$ language, this is an interaction of the form $\Phi^2 e^{i\chi} + hc$, and the ambiguity comes from the non-invariance of the interaction under $U(1)$ owing to the phase χ .

To us, the only sensible resolution is that the black hole does have memory of the initial state, with global quantum numbers. The black hole field is then a B^{ij} which is a BH that has flavor information, hence ‘‘hair’’. The vertex takes the form

$$g \sum_{ij} \phi_i \phi_j B^{ij} \tag{5}$$

B^{ij} remembers the initial states that made it and the resulting interaction is $SO(2)$ invariant. Equivalently, the black hole must be a complex field in the interaction $\Phi^2 B + hc$ and the phase transformation $\Phi \rightarrow e^{i\theta} \Phi$ together with $B \rightarrow e^{-2i\theta} B$ must be a symmetry. This implies the BH carries a global $U(1)$ charge.

By maintaining the no-hair theorem we would conclude that global charges cannot exist. However, it has been recently understood that classically time dependent hair exists. For example, Ref. [1] nicely treats and reviews the situation. A BH immersed in a time dependent scalar field will absorb and reemit this field producing a time dependent, coulombic “halo.” If, for example, the time dependent field is a Nambu-Goldstone boson (eg, axion) then the associated charge is given by the chiral current $f\partial_t\phi$ so indeed, the BH has acquired a global chiral charge density.

Moreover, by claiming that the $U(1)$ symmetry is broken by gravity is, by itself, a vacuous statement in no way resolves this issue. Even if global symmetries do not exist at short distances, but if quantum mechanics applies at all distances, then we hface these issues. The Dirac superposition of Hilbert space states is the underling issue here.¹

3 Effective field theory

We introduce a real scalar field $B_0(x)$ to decribe Schwarzschild mini-black holes as pointlike “particles” of the minimal mass M_P , which become the excitations of $B_0(x)$. In what follows we will neglect Hawking radiation. We choose the Lagrangian of $B_0(x)$ to be

$$L = \frac{1}{2}\partial B_0\partial B_0 - \frac{1}{2}M^2B_0^2 + JB_0 + \Lambda \tag{6}$$

Hawking proposed a virtual black hole vacuum (VBH) [12], in which the vacuum is viewed as consisting of Euclidean (instanton-like) loops of tiny black holes, appearing and disappearing on time scales of order M_P^{-1} (for a review, see [13]).² This is connected in the literature to various ideas in AdS hologaphy, gravitational instantons and string theory [14–18]. The consequences of topological instanton fluctuations at the Plank scale, associated with anomalous neutrino currents, have also been considered recently [19]. Other authors have discussed possible gravitational effects in neutrino physics [20–23].

¹ t Hooft has reformulated quantum mechanics in an underlying cellular automaton structure [10, 11] that seems to provide an alternative resolution. There the Diracian superposition of states is only effective, but not fundamental. Presumably superpositions arise by rapidly fluctuating discrete alternatives (on Planckian time scales). It is argued that there is no conflict with quantum mechanics at larger distances and time scales. We think this is alternative path to resolving these issues.

² Hawking’s rationale behind considering Euclidean spacetime may in part have been that the black hole loops are simply finite action instantonic field configurations, and Hawking radiation does not arise in Euclidean space. Given the source J there is no decay of the static VEV, V in our Higgs phase. In any case, the decay width Γ can be significantly less than M_P of a physical Planckian SBH of mass M_P [24].

For a simple model of a VBH, we have added a source term, J . The vacuum value of B_0 is therefore shifted and we obtain the field $B_0 = B + V$ where $V = J/M^2$ in a Higgs phase:

$$L \rightarrow \frac{1}{2}\partial B\partial B - \frac{1}{2}M^2B^2 - \frac{J^2}{2M^2} + \Lambda \tag{7}$$

We can always choose the source term to cancel an anti-DeSitter cosmological constant $\Lambda = J^2/2M$. When particles of the standard model propagate through a VBH they interact with the $B + V$ field. Our main objective presently is to provide a field theory description of physical processes that involve matter interactions with the SBH’s. This is immediately related to the issue of information loss as described in the introduction.

In the original view of Hawking, global charges would simply be swallowed by mini-black holes which subsequently evaporate, and large violations of global charge conservation would be expected. This has been implemented to conjecture, e.g., gravitationally induced violation of $B + L$ in the standard model [25] (see also an exception, [26]). In the modern prevailing view global charges are conserved, with the global charge “information” holographically painted onto the horizon of the black hole, to be recovered upon evaporation [7–9].³

For N Weyl fermions we have the global $SU(N) \times U(1)$ kinetic term $\bar{\psi}_i^\alpha i\partial_\mu\sigma_{\alpha\beta}^\mu\psi^{i\beta}$. For the Weyl fermion pair, $\epsilon^{\alpha\beta}\psi_\alpha^i\psi_\beta^j \sim [\psi^i\psi^j]$, we require a complex spurion, $\theta_{ij}(x)$, to tie indices of fermions onto $B(x)$, as

$$[\psi^i(x)\psi^j(x)]\theta_{ij}(x)\frac{B_0^2(x)}{M_P^2} + h.c. \tag{8}$$

For the case of ψ representing neutrinos this is depicted in Fig. 1. Note, we include here $B_0^2(x)$ since the fermion pair is colliding with an existing black hole in the initial state and producing one in the final state.⁴ In order to make an $SU(N)$ invariant interaction we require that θ lies in the symmetric representation of $SU(N)$ of dimension $\frac{1}{2}N(N + 1)$.

Naturally, $\theta_{ij}(x)$ refers to the associated horizon information of $B_0(x)$. However, in the context of field theory we must face the issue of how to treat the spurion dynamically. In the following we consider two possibilities.

In case (II) we argue that the spurion is simply a random complex valued variable at the point of interaction. We therefore average path integrals over $\theta_{ij}(x)$. In this sense, the

³ We are not considering anomalies, as in [19], which more definitively break the global symmetries by violating the conservation of the global current and with an instanton allows a mechanism to exchange visible charge with the vacuum.

⁴ One could extend this to a model of production with vertex $\sim g[\psi\psi]\theta B_0$ but this is complicated by a large energy dependence in a form factor which suppresses $g(\mu)$ for $\mu \ll M_P$ [27, 28].

theory is analogous to a “spin-glass” in which spins propagate through a random potential, and the partition function is averaged over the potentials [5, 6]. This promotes $\theta_{ij}(x)$ to a physical field. Since gravity loses all memory of the information, there is no further autocorrelation between $\theta_{ij}(x)$ and itself, e.g., no mass or kinetic terms, such as $\sim \theta_{ij}(x)\theta^{\dagger ij}(x)$. Since there is no current associated with $\theta_{ij}(x)$, we interpret this as “lost information.”

In case (III), following [7–9] we implement conservation of information, i.e., the holographic principle which implies a conserved global current that involves the information. To do this we must view the SBH as analogous to a very heavy atom, $H(x)$ (such as Uranium) that is bound to a light particle $\phi(x)$ (such as a neutron in its nucleus) to make a composite system, $\phi(x)H(x)$. The physical properties of the composite state are similar to those of the unbound $H(x)$, but the location in space-time of the light field $\phi(x)$ must be correlated with the heavy $H(x)$. However, the heavy particle carries the bulk mass of the system. This implies both a conventional kinetic term for $H(x)$, e.g., $\frac{1}{2}\partial H\partial H$, and a conjoined kinetic term for the composite system,

$$\frac{1}{2M_P^2}\partial_\mu(\phi(x)H(x))\partial^\mu(\phi(x)H(x)). \tag{9}$$

However, the mass term involves only $H(x)$, as $M^2H(x)^2$.

In our case $H(x) \sim B_0(x)$ and $\phi(x) \sim \theta_{ij}(x)$. The conjoined kinetic term $\partial(\theta^\dagger B_0)\partial(\theta B_0)$, and no stand-alone $\partial(\theta^\dagger)\partial(\theta)$ term implies that $\theta(x)$ can never escape $B_0(x)$ unless the system decays through a fermion pair. This leads to the following Lagrangian:

$$L = \frac{1}{2}\partial B_0\partial B_0 - \frac{1}{2}M^2B_0^2 + JB_0 + \Lambda + \frac{1}{M_P^2}\partial(\theta^{*ij}B_0)\partial(\theta_{ij}B_0) \tag{10}$$

The fact that θ is always accompanied by a factor of $B_0(x)$ implies that it cannot escape the SBH, other than by a neutrino interaction. In the limit $B_0 \rightarrow V$ the kinetic term becomes:

$$\langle B_0\theta|L|B_0\theta \rangle \sim L_0 + \frac{V^2}{M_P^2}Tr(\partial_\mu\theta^\dagger\partial^\mu\theta) \tag{11}$$

Remarkably, in the VBH vacuum upon replacing $B_0 \rightarrow V$ constant, the information field θ freely propagates through the medium. We can then absorb a factor of V/M_P into θ to write $(V^2/M^2)\partial_\mu(\theta^\dagger)\partial^\mu(\theta) \rightarrow \partial_\mu\theta^\dagger\partial^\mu\theta$ and θ is then canonical. Mainly, there is now a conserved global current that involves both the fermions and the field θ .

The interaction vertex with the fermions then becomes, again, that of Eq. (8), in the VBH and suitably renormalized

$$[\psi^i(x)\psi^j(x)]\theta_{ij}(x) + h.c. \tag{12}$$

The interaction annihilates an information-less SBH B_0 from the vacuum, and creates a composite SBH in the vacuum with

information, $\theta_{ij}(x)B_0(x)$. θ is thus created (annihilated) by absorbing (producing) a fermion pair, always in coincidence with B_0 through its VEV. We now apply this to sterile neutrinos.

4 Sterile neutrinos

4.1 If information is lost

Assume we have N sterile neutrino flavors. This will imply an $SU(N) \times U(1)$ invariant kinetic term, and we assume this is a valid global symmetry at the Planck scale. Consider an s -wave pair of massless right-handed neutrinos scattering off of an SBH. Here we have a unique situation in the SM that an s -wave combination of two massless right-handed fermions, of flavors i and j can have zero local gauge coupling constant but nonzero global flavor.

We assume the neutrinos interact with the SBH B_0 field as in Fig. 1:

$$v_\alpha^i(x)v_\beta^j(x)\epsilon^{\alpha\beta}\theta_{ij}(x)\frac{B_0^2(x)}{M_P^2} + h.c. \tag{13}$$

where $\theta_{ij}(x)$ is a constant dimensionless spurion (note we given θ dimensions of a mass). This is case(I) alluded to in the Introduction and will break the conservation of the global currents of the neutrinos explicitly. There is nothing intrinsic to gravity that can dictate the flavor structure or phase of θ_{ij} in $SU(N) \times U(1)$ and we conclude that this is not a sensible theory.

4.2 Random field as a spin-glass

A more reasonable possibility is (II) that $\theta_{ij}(x)$ is a complex random variable. We have the interaction of Eq. (13), but $\theta_{ij}(x)$ can have no autocorrelation due to gravity, since the SBH has lost all knowledge of the ij indices, hence no term like $\mu^2\theta_{ij}^*\theta^{ij}$ is induced. This is the “information lost” scenario.

A given choice of $\theta_{ij}(x)$ describes a particular subprocess. However we then have to average over this field. The system is analogous to “spin-glasses” which have Hamiltonians that involve random variables (such as the Edwards-Anderson model [5, 6]).

For spin-glasses the averaging is done over the partition functions and not in the action itself. In our case, we average over the path integrals, and this has the effect of promoting θ_{ij} to a random quantum field:

$$\rightarrow \int D\theta \exp\left(i \int d^4x \theta_{ij}v_\alpha^i v_\beta^j \epsilon^{\alpha\beta}(B_0^2/M_P^2) + h.c.\right) \tag{14}$$

While with a fixed $\theta_{ij}(x)$ the neutrino flavor current conservation would be violated, it isn't hard to see that upon averaging over θ matrix elements yield a conserved global current $(\partial^\mu \bar{\nu} T^A \sigma_{\mu\nu}) = 0$.

The theory is singular, however, since the equation of motion of θ would enforce the vanishing of the vertex. To see this, we pass to the VBH vacuum upon replacing $B_0 \rightarrow V$ constant, and absorb the factor of $\sqrt{Z} = V/M_P$ into θ to canonically normalize θ . If we then introduce a small $\mu^2 \theta_{ij}^* \theta^{ij}$ term in the action, the fermion current is manifestly conserved upon use of the equation of motion of the neutrinos and θ . In the VBH vacuum we have

$$v_\alpha^i v_\beta^j \epsilon^{\alpha\beta} \theta_{ij} + h.c. - \mu^2 \theta_{ij} \theta^{\dagger ij} \tag{15}$$

where $Z = V^2/M_P^2$. The corresponding 4-fermion interaction of Eq. (15) is

$$\frac{[v^i v^j][\bar{\nu}_i \bar{\nu}_j]}{\mu^2} \tag{16}$$

and is singular as $\mu \rightarrow 0$. Likewise, as $\mu^2 \rightarrow 0$ the equation of motion of θ enforces $v_\alpha^i v_\beta^j \epsilon^{\alpha\beta} B_0(x) \rightarrow 0$.

The key feature is the absence of the mass term for $\theta_{ij}(x)$ in the Planck scale effective Lagrangian. There are no derivatives of $\theta_{ij}(x)$ at this stage and no current built of $\theta_{ij}(x)$ and the theory is singular at M_P . This would be in our opinion, a realization of Hawking's information-lost hypothesis.

We will see below, however, that this situation is unstable and effects of the back-reaction of the neutrinos will lead to a nonsingular dynamics for $\theta_{ij}(x)$, on scales $\mu < M_P$ and a spontaneous breaking of $SU(N)$. The singularity at M_P share features with a Landau pole.

4.3 Information is carried by black hole

However, we can locally conserve the information (case III). We introduce a new effective field, $B_{ij} = \theta_{ij}(x) B_0(x)$, that is composite and may represent a SBH with the information of a neutrino pair encoded on its horizon holographically. The effective theory for the neutrino pair interaction with the BH's becomes:

$$v_\alpha^i(x) v_\beta^j(x) \epsilon^{\alpha\beta} \frac{B_0(x) B^{ij\dagger}(x)}{M_P} + h.c. \tag{17}$$

How do we view B^{ij} ? An analogy was given in Sect. 3 to the isotopes of Uranium. We can freely add or remove neutrons from the Uranium nucleus, and the mass is not dramatically changed, nor are the chemical properties. Therefore we can view a Uranium atom as a groundstate nucleus, U_0 , which with an additional neutron $n(x)$ becomes the effective field $U_0(x)n(x)$. Chemically (electromagnetically) we cannot easily discern which isotope we are dealing with. Yet

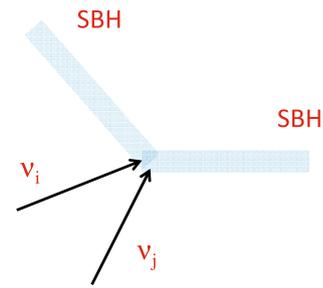


Fig. 1 Low 4-momentum sterile neutrino pair of global flavors (i, j) disappears into Schwarzschild BH (or the neutrino exchanges global charge with the SBH in a t-channel). This is described by a complex spurion field θ_{ij} , and absence of an $M^2 Tr(\theta^* \theta)$

another analogy might be bugs that end up flattened on the windshield of a car, that have little effect on the properties of the car, but become part of a conjoined kinetic term with it.

Similarly, the effective field $B^{ij}(x)$ is essentially a SBH, $B_0(x)$ with the information $\theta_{ij}(x)$ on it's horizon, but there is no experiment we can do to detect $\theta_{ij}(x)$, other than observing neutrinos emitted in Hawking radiation as the BH decays. The mass of a $\theta_{ij}(x)$ spurion field is zero.

The kinetic term becomes the ‘‘conjoined kinetic term’’ of Eq. (10). The mass term is given by that of B_0 alone, $\frac{1}{2} M_P^2 B_0^2$, and there can be no $Tr(\theta^\dagger \theta)$ term. The effective theory for the neutrino pair interaction in the VBH condensate becomes:

$$v_\alpha^i v_\beta^j \epsilon^{\alpha\beta} \theta_{ij} \frac{B_0^2}{M_P^2} + h.c. \rightarrow Z v_\alpha^i v_\beta^j \epsilon^{\alpha\beta} \theta_{ij} + h.c. \tag{18}$$

where in the second term we have replaced B_0 by the condensate and $Z = V^2/M^2$. Moreover, in the condensate the kinetic term becomes,

$$\partial B_{ij}^\dagger \partial B_{ij} \rightarrow Z Tr(\partial_\mu \theta^\dagger \partial^\mu \theta) \tag{19}$$

Therefore, if we canonically renormalize θ we can adjust $Z = 1$ and our theory in the condensate becomes:

$$v_\alpha^i v_\beta^j \epsilon^{\alpha\beta} \theta_{ij} + h.c. + \eta Tr(\partial_\mu \theta^\dagger \partial^\mu \theta) \tag{20}$$

This provides an insight into what is meant by conservation or loss of information, at least in the EFT: If we conserve information then $\eta = 1$; If information is (weakly) lost then $\eta = 0$. Note that with $\eta = 1$ there is now a formal conserved information current

$$j_\mu^A = \bar{\nu} T^A \sigma_\mu \nu + i Tr(\theta^\dagger T^A \overleftrightarrow{\partial}_\mu \theta) \tag{21}$$

Information is now dynamically transferred from neutrinos to θ and can propagate freely through the condensate.

4.4 Back-reaction of neutrinos and spontaneous symmetry breaking

We can compute the action for the system at an energy scale μ by using the renormalization group. This follows the pro-

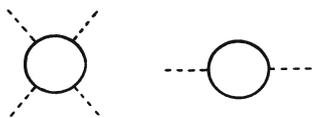


Fig. 2 The infrared theory is controlled by the neutrino loops

cedure known as the “block-spin renormalization group” for treating the Nambu–Jona-Lasinio model [29], developed in Bardeen-Hill-Lindner [30–32]. For a fixed choice of θ_{ij} we integrate out the fermions, descending from a scale M_P to a scale μ . This is obtained from the fermions loops in Fig. 2 with loop-integrals,

$$\int_{\mu}^{M_P} \frac{d^4 k}{(2\pi)^4}. \tag{22}$$

We start with Eq. (20) as the defining action at M_P . The result of integrating neutrino loops is an induced action for θ_{ij} of the form:

$$Z \partial_{\mu} \theta_{ij}^* \partial^{\mu} \theta_{ij} - M_{\mu}^2 \theta_{ij}^* \theta_{ij} - \frac{\lambda}{2} (\theta_{ij}^* \theta_{jk} \theta_{kl}^* \theta_{li}) \tag{23}$$

Using the results for Weyl spinors in Hill, Luty and Paschos, [33], we immediately obtain:

$$Z = \eta + \frac{1}{8\pi^2} \ln \left(\frac{M_P^2}{\mu^2} \right) \quad \lambda = \frac{1}{\pi^2} \ln \left(\frac{M_P^2}{\mu^2} \right) \\ M_{\mu}^2 = -\frac{1}{4\pi^2} (M_P^2 - \mu^2) \tag{24}$$

Anticipating spontaneous symmetry breaking, we denote the field VEV, $\theta_{ij} = \delta_{ij} \theta$. Then

$$\theta_{ij}^* \theta_{ij} = N \theta^2 \quad \theta_{ij}^* \theta_{jk} \theta_{kl}^* \theta_{li} = N \theta^4 \tag{25}$$

The unrenormalized potential is:

$$V_{un} = -\frac{N}{4\pi^2} (M_P^2 - \mu^2) \theta^2 + \frac{N \theta^4}{2\pi^2} \ln \left(\frac{M_P^2}{\mu^2} \right) \tag{26}$$

Note that the infrared cut-off on our loops is determined by the neutrino mass. The unrenormalized neutrino mass is $\mu = m_{\nu} = 2\theta$, and therefore the potential becomes

$$V_{un} = -\frac{N}{4\pi^2} M_P^2 \theta^2 + \frac{N \theta^4}{2\pi^2} \left(\ln \left(\frac{M_P^2}{4\theta^2} \right) + 2 \right) \tag{27}$$

To renormalize we want the kinetic term to be brought to canonical normalization.

The renormalized field VEV is therefore $\hat{\theta}^2 = Z \theta^2$. The neutrino mass becomes $\mu = m_{\nu} = 2\hat{\theta} / \sqrt{Z} = 2g_r \hat{\theta}$, where $g_r = 1/\sqrt{Z}$ is the renormalized Yukawa coupling. Note that if $\eta = 0$ we see that g_r displays the characteristic Landau pole at $\mu \rightarrow M_P$ which is the compositeness condition of

the field θ [30–32]. Thus we have the renormalized potential:

$$V_{ren} = -\frac{g_r^2 N}{4\pi^2} (M_P^2) \hat{\theta}^2 + \frac{g_r^4 N \hat{\theta}^4}{2\pi^2} \left(\ln \left(\frac{M_P^2}{4g_r^2 \hat{\theta}^2} \right) + 2 \right) \tag{28}$$

This is most conveniently rewritten in terms of the physical neutrino mass:

$$V = -\frac{N}{16\pi^2} M_P^2 m_{\nu}^2 + \frac{N m_{\nu}^4}{32\pi^2} \left(\ln \left(\frac{M_P^2}{m_{\nu}^2} \right) + 2 \right) \tag{29}$$

Here we have substituted m_{ν} for the field θ and we have a renormalization invariant potential as a function of a dynamical m_{ν} .

We now extremalize the potential with respect to m_{ν} , equivalent to extremalizing in $\hat{\theta}$. We obtain:

$$0 = \frac{16\pi^2}{N} \frac{\partial}{\partial m_{\nu}^2} V = -M_P^2 + m_{\nu}^2 \left(\ln \left(\frac{M_P^2}{m_{\nu}^2} \right) + \frac{3}{2} \right) \tag{30}$$

The physical solution for the neutrino mass is

$$m_{\nu}^2 = 0.424 M_P^2 \quad m_{\nu} = 0.651 M_P \tag{31}$$

The potential has a runaway for large values of $m_{\nu} \gg M_P$, but this is unphysical since we insist upon the cutoff M_P .

We remark that this result is sensitive to the subleading log behavior of the loops (constants), which differs from [30–32]. In that case a large hierarchy is tuned by demanding a precisely tuned cancellation between a bare mass term and the loop. Here we have no bare mass term but we find a solution in a small log limit. Hence, the boundstate field θ necessarily develop a vacuum expectation value (VEV) due to the VBH vacuum.

We note that we have neglected the production vertex, $\sim \nu \nu \theta B_0 / M_P$, since we are mainly interested in neutrino momenta below M_P . However, our loop calculation informs us that the neutrinos, in the SBH Higgs phase, form a nonzero VEV, $\langle \nu \nu \rangle \sim m_{\nu} M_P^2$ together with $\langle \theta \rangle \sim M_P$ which may be bootstrapped back to be the source term J for the BH condensate itself.

4.5 Phenomenology

The sterile neutrinos thus obtain a large common Majorana mass, $\sim M_P$. The N sterile neutrinos, coupled to gravity, have a global $SU(N) \times U(1)$ symmetry which is now broken to $SO(N)$ θ contains $N(N + 1)$ real degrees of freedom. The $SU(N)/SO(N)$ breaking implies there is one phase and there remain $\frac{1}{2} N(N + 1)$ massless Nambu-Goldstone bosons.

We consider the SM with 3 families including 3 sterile neutrinos. The $SU(3)$ symmetry of the ν_{iR} is essentially an accidental symmetry given only the gravitational couplings of the neutrinos. The left-handed lepton doublets, ψ_{Li} couple

via the Higgs boson to the right-handed neutrinos through the Higgs field H as:

$$y_{ij} \bar{\psi}_{Li} H \nu_{jR} \quad (32)$$

Generally the y_{ij} will break the $SU(3)$. Integrating out the heavy R-neutrinos yields the Weinberg operator,

$$\frac{1}{M_P} y_{ij} y^{kj} (\bar{\psi}_{Li} H) (\bar{\psi}_{Lk} H)^T + h.c. \quad (33)$$

If we now assume a typical (large) value for the $y^{kj} \sim 1$ in Eq. (33) we see that the scale of the induced Majorana mass terms of the observable L-neutrinos, with $v_{weak} \sim (175 \text{ GeV})$, is $\sim v_{weak}^2 / (10^{19} \text{ GeV}) \sim 3 \times 10^{-6} \text{ eV}$, which is rather small. According to [34], the best fit to neutrino data implies we require $\Delta m_{12}^2 \sim m_\nu^2 \sim 7 \times 10^{-5} \text{ eV}^2$ which implies $m_\nu \sim 0.8 \times 10^{-2} \text{ eV}$. Our results suggest a scale of observable neutrino masses that is small by roughly a factor of $\sim 3 \times 10^{-3}$.

Our result for the Majorana mass scale depends only upon M_P and is rather immutable. However this is the Planck mass at extremely high energies (of order M_P). It should be noted that a number of authors have argued for significant renormalization effects of the Planck scale, and that $M_P \sim 10^{16} \text{ GeV}$ may be reality in $D = 4$ [35–37]. Of course, with extra dimensions M_P can be significantly modified, but our set up requires $D = 4$ and would otherwise have to be re-explored if $D \neq 4$. However, neutrinos with the Type I seesaw may be uniquely probing gravity at M_P and offer credence to a significantly reduced Planck mass at high energies.

The $\frac{1}{2} N(N+1) = 6$ Nambu-Goldstone bosons (Majorons) will have decay constant $f \sim M_P$ but potentials governed by the explicit $SU(3)$ symmetry breaking Weinberg operator. Schematically $\sim m^4 \cos((\phi/f) + \chi)$, where χ is a CP phase and may range from $m \sim m_\nu$ to $m \sim v_{weak}$ depending upon the details of Eq. (33). This potentially offers a number of cosmological possibilities, from late time phase transitions, dark energy, to providing an inflaton [38,39]. Discussion of these is beyond the scope of the present paper.

5 Conclusions

We have given an effective field theory treatment of information loss or conservation in the dynamics of a mini-black hole interacting with fermions. In particular, we have focused upon sterile neutrinos with a global $SU(N) \times U(1)$ symmetry.

Our present model illustrates how “information” might be described in analogy to an induced effective random field θ . The weak information loss of global charge would forbid gravitationally induced auto-correlation of θ , i.e., no kinetic, mass or interaction terms. However, consistency with the

holographic view promotes θ_{ij} to a local field and θ “piggy-backs” on a Schwarzschild black hole. θ becomes dynamical in the black hole condensate.

The absence of the θ mass term for sterile neutrinos has an immediate and dramatic physical consequence. This implies that an instability driven by sterile fermion loops will always lead to a condensate of $\theta \sim M_P$. In our case the instability is provided by the neutrino loops external to the black holes, and θ becomes a sterile $\nu\nu$ boundstate.

Our present paper is introductory, but let us mention a future application. In a subsequent paper we extend these results to locally charged fermions. We find that the dynamics is more subtle. Owing to the local gauge field, the B_{ij} field essentially describes a Reissner-Nordstrom (RN) black hole. This acquires a slight mass enhancement of order αM_P above the mass of the SBH M_P . This mass enhancement is associated with the information field θ_{ij} , i.e., in the VBH the field θ freely propagates, but carries the charge of the RN hole and acquires the small RN mass term $\alpha M_P^2 \text{Tr}(\theta^\dagger \theta)$. This means that the tachyonic instability is blocked for small V/M_P .⁵ However, if an effective coupling $g^2 = V^2/M_P^2$ exceeds a critical value g_c^2 the field θ acquires a VEV and the gauge symmetry is then spontaneously broken.

A general picture that is emerging here, upon including local gauging, offers a new non-perturbative binding mechanism for fermions to produce scalar fields. In turn, this suggests a large system of composite Higgs bosons. Moreover, a near critical value of the coupling, $g \sim V/M$ implies deep scalar boundstates with (nearly) vanishing masses. Perhaps a more refined theory might lead to a conformal window with a low mass scale for the di-electron boundstate, θ . In our crude approximations this would be a coupling tuned arbitrarily near criticality $g \approx g_c$. Since there are 1176 Weyl bilinears in the standard model [40,41], there may exist a large number of composite scalars in nature that are marginally subcritical boundstates of elementary fermions due to gravity, with masses that extend down to the Higgs mass scale 125 GeV. In this picture, “scalar democracy,” the standard model Higgs is composed of $\bar{t}t$, and its nearest neighbor $\bar{b}b$ would be expected at a mass scale $\sim 5.5 \text{ TeV}$ and accessible to an upgraded LHC [40,41].

The present scheme provides a view of how a new dynamical mechanism and composite scalar bosons might arise non-perturbatively from gravity, as a “dynamically activated holographic information.”

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⁵ Given that we are not far from the strong gravity scale, the computation of such a suppression is not straightforward. However due to the difference in gauge charges, gravity must distinguish among the different symmetry breaking channels introducing a hierarchy.

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