Forward Higgs production within high energy factorization in the heavy quark limit at next-to-leading order accuracy

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Abstract We use Lipatov's high energy effective action to determine the next-to-leading order corrections to Higgs production in the forward region within high energy factorization making use of the infinite top mass limit. Our result is based on an explicit calculation of real corrections combined with virtual corrections determined earlier by Nefedov. As a new element we provide a proper definition of the desired next-to-leading order coefficient within the high energy effective action framework, extending a previously proposed prescription. We further propose a subtraction mechanism to achieve for this coefficient a stable cancellation of real and virtual infra-red singularities in the presence of external off-shell legs. Apart from its relevance for direct phenomenological studies, such as high energy resummation of Higgs + jet configurations, our result will be further of use for the study of transverse momentum dependent factorization in the high energy limit.

1 Introduction

The observation of the Higgs boson by the ATLAS and CMS experiments [1,2] confirmed expectations that the Standard Model of particle physics is a consistent theory of strong and electro-weak interactions. The electro-weak sector of the Standard Model is being explored now in detail by measuring properties of the Higgs boson [3]. The success of its discovery is complemented with an advancement in techniques for the calculation of production cross sections and decay rates with high accuracy; for a recent review see [4]. To determine the cross section for Higgs production in the central rapidity region, which is the dominant production region, one usually uses the framework of collinear factorization, where the incoming partons are collinear with the beam axis and are approximately on shell. In this framework one can

reliably calculate the production cross section up to next-to-next-to-leading order (NNLO) accuracy [5], complemented with Monte Carlo simulations to describe complete production events including hadronization and detector simulation [6].

In this paper we use on other hand the process of Higgs boson production in the forward direction to advance further the formulation of high energy factorization [7,8] for quantum chromodynamics (QCD) to next-to-leading order (NLO) accuracy. To be precise, our discussion is based on Lipatov's high energy effective action [9,10] and the determination of NLO correction will be achieved within this framework. From a phenomenological point of view, for center of mass energies accessible at the large hadron colliders, cross-sections for forward production of Higgs bosons are most likely too small to be observed. Our result is therefore at first of formal interest and serves to explore further the proper definition of NLO coefficients within high energy factorization. We nevertheless would like to stress that there exist already studies which investigate the relevance of high energy resummation for Higgs-jet configurations [11]. We certainly expect that our result will be of relevance for the phenomenology of future colliders such as the Future Circular Collider project [12].

The formalism of high energy factorization was developed in order to resum perturbative contributions to the cross section, enhanced by logarithms in the center-of-mass energy, which are of relevance whenever the center-of-mass energy of the process is much larger than any other scale involved [13–16]. This resummation is also applicable when the configuration of the final state is such that at least one of the final state particle is produced in the forward direction, where the logarithmic dependence on the center-of-mass energy translates into large differences in rapidity. The resulting framework is then known under the term hybrid factorization [17], see also [18–20].

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With production taking place in the forward region of one of the hadrons, partons originating from this hadron are characterized by relatively large momentum fractions x with the corresponding parton distribution subject to conventional DGLAP evolution. Partons stemming from the second hadron are on the other hand characterized by a very small longitudinal momentum fraction x as well as a non-zero transverse momentum k_T . With quark exchange power-suppressed in the vacuum channel, the corresponding unintegrated gluon distribution is subject to the Balitsky–Fadin–Kuraev–Lipatov (BFKL) equation [13–16, 21, 22] and its nonlinear extensions [23–28] respectively.

The high energy factorization formalism was rather successful in describing production of final states widely separated in rapidity [29–33] and processes in the forward rapidity region [34–38] and some processes are even known at NLO accuracy both in momentum space [39–45] and coordinate space [46–48]. It is worthwhile to note that there has been progress during the recent years in the development of computational methods which allowed for a reformulation of the evaluation of high energy factorization matrix elements [49–51] as well as the subsequent automation of tree level matrix elements and the calculation of parton-level cross sections via Monte Carlo methods [52].

A very useful tool to calculate the k_T dependent matrix elements, which arise from the aforementioned factorization procedure, is then provided by the previously mentioned high energy effective action [9, 10]. It yields matrix elements for the interaction between conventional QCD fields and reggeized gluon fields, localized in rapidity. The reggeized gluon field is in this context an auxiliary degree of freedom which is used to formulate gauge invariant factorization of QCD amplitudes, see [53] for a recent review and [54] for the determination of a corresponding effective action for electroweak fields. With the present project we plan to revisit the strategy of calculations of NLO processes using Lipatov’s high energy effective action, through considering a colorless massive final state, see [55–58] for the tree level result within k_T factorization. The simplicity of the final state allows to address in a well structured manner all aspects of a complete NLO calculations and to give a compact formula for the cross section which can be presented analytically and eventually implemented in a numerical code, where the latter is left as a task for the future. As we will show in the following, the high energy effective action provides a well defined and manifestly gauge invariant setup for such calculations.

The paper is organized as follows. In Sect. 2 we provide a quick overview over the high energy factorization as formulated within the high energy effective action and how it can be used for the actual calculation of NLO corrections. In Sect. 3 we then present the results of our calculations, while in Sect. 4 we draw our conclusions. Some details of our calculations have been referred to the Appendix A.

2 The high-energy effective action

Our calculation is based on Lipatov’s high energy effective action [9]. Within this framework, QCD amplitudes are in the high energy limit decomposed into gauge invariant sub-amplitudes which are localized in rapidity space and describe the coupling of quarks (ψ), gluon (v_μ) and ghost (ϕ) fields to a new degree of freedom, the reggeized gluon field $A_\pm(x)$. The latter is introduced as a convenient tool to reconstruct the complete QCD amplitudes in the high energy limit out of the sub-amplitudes restricted to small rapidity intervals. To be explicit we consider scattering of two partons with momenta p_a and p_b which serve to define the light-cone directions of the high energy effective action

$$(n^\pm)^\mu = \frac{2}{\sqrt{\hat{s}}} p_{a,b}^\mu, \quad \hat{s} = 2p_a \cdot p_b, \quad (1)$$

which yields the following Sudakov decomposition of a generic four-momentum,

$$k = k^+ \frac{n^-}{2} + k^- \frac{n^+}{2} + k_T, \quad k^\pm = k \cdot n^\pm, \quad n^\pm \cdot k_T = 0. \quad (2)$$

Here, k_T is the embedding of the Euclidean vector $\mathbf{k}$ into Minkowski space, so $k_T^2 = -\mathbf{k}^2$. Lipatov’s effective action is then obtained by adding an induced term $S_{\text{ind.}}$ to the QCD action S_{QCD} ,

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind.}}, \quad (3)$$

where the induced term $S_{\text{ind.}}$ describes the coupling of the gluonic field $v_\mu = -it^a v_\mu^a(x)$ to the reggeized gluon field $A_\pm(x) = -it^a A_\pm^a(x)$. High energy factorized amplitudes reveal strong ordering in plus and minus components of momenta which is reflected in the following kinematic constraint obeyed by the reggeized gluon field:

$$\partial_+ A_-(x) = 0 = \partial_- A_+(x). \quad (4)$$

Even though the reggeized gluon field is charged under the QCD gauge group $SU(N_c)$, it is invariant under local gauge transformation $\delta A_\pm = 0$. Its kinetic term and the gauge invariant coupling to the QCD gluon field are contained in the induced term

$$S_{\text{ind.}} = \int d^4x \operatorname{tr} \left[(W_-[v(x)] - A_-(x)) \partial_\perp^2 A_+(x) \right] + \operatorname{tr} \left[(W_+[v(x)] - A_+(x)) \partial_\perp^2 A_-(x) \right], \quad (5)$$

with

$$W_\pm[v(x)] = v_\pm(x) \frac{1}{D_\pm} \partial_\pm, \quad D_\pm = \partial_\pm + g v_\pm(x). \quad (6)$$

(a)
(b)
(c)

Fig. 1 Feynman rules for the lowest-order effective vertices of the effective action. Wavy lines denote reggeized fields and curly lines gluons

For a more in depth discussion of the effective action we refer to the reviews [59]. Due to the induced term in Eq. (3), the Feynman rules of the effective action comprise, apart from the usual QCD Feynman rules, the propagator of the reggeized gluon and an infinite number of so-called induced vertices. Vertices and propagators needed for the current study are collected in Fig. 1.

Determination of NLO corrections using this effective action approach has been addressed recently to a certain extent, through the explicit calculation of the NLO corrections to both quark [60] and gluon [44] induced forward jets (with associated radiation) as well as the determination of the gluon Regge trajectory up to 2-loops [61, 62]. These previous applications have all in common that they are, at amplitude level, restricted to a color octet projection and, therefore, single reggeized gluon exchange. Due to the particular color structure of the reggeized gluon field, which is restricted to the anti-symmetric color octet, see Fig. 1 and [9, 63], color singlet exchange requires to go beyond a single reggeized gluon exchange and to consider the two reggeized gluon exchange contribution. For a discussion of the analogous high energy effective for flavor exchange [64] at NLO see e.g. [65–67].

2.1 Factorization of partonic cross sections in the high energy limit

In the following we describe the framework to be used to determine the NLO corrections to the forward Higgs impact factor. The formulation of this framework is based on the explicit results obtained in the case of forward quark and gluon jets [44, 60] as well as the determination of the gluon Regge trajectory up to 2-loops [61, 62], where the later only addresses virtual corrections. Adapting a normalization of impact factors motivated by k_T -factorization, we factorize in the high energy limit the partonic cross section into

$$d\hat{\sigma}_{ab} = \hat{h}_a^{k_T}(\mathbf{k}) \hat{h}_b^{ugd}(\mathbf{k}) \frac{d^{2+2\epsilon} \mathbf{k}}{\pi^{1+\epsilon}}, \quad (7)$$

where we regulate both infra-red and ultra-violet corrections using dimensional regularization in $d = 4 + 2\epsilon$ dimensions.

Here $\hat{h}_a^{k_T}$ denotes the impact parameter in the fragmentation region of parton a while $\hat{h}_b^{ugd}$ is the impact factor¹ in the fragmentation of parton b . While the normalization is adapted to the asymmetric scenario where the transverse scale in the fragmentation region of parton a is significantly larger than the corresponding scale in the fragmentation region of parton b , our framework is completely general and does not assume a priori such a hierarchy. In terms of (off-shell) matrix elements of reggeized gluon fields $r^\pm$ and conventional QCD fields we have

$$\begin{aligned} \hat{h}_b^{ugd}(\mathbf{k}) &= \frac{(\pi)^{d/2}}{2p_b^-} \int dk^+ \frac{|\overline{\mathcal{M}_{br^- \rightarrow X_b^n}}|^2}{k^2} d\Phi^{(n)} \delta^{(d)} \\ &\quad \left(p_b + k - \sum_{j=1}^n p_j \right), \\ \hat{h}^{k_T}(\mathbf{k}) &= \int \frac{dk^-}{k^-} d\hat{\sigma}_{a+}, \end{aligned} \quad (8)$$

where off-shell partonic cross section and corresponding off-shell squared matrix elements $|\mathcal{A}|^2$ are in terms of effective action matrix elements obtained as:

$$\begin{aligned} d\hat{\sigma}_{a+} &= \frac{1}{2p_a^+ k^-} \frac{|\overline{\mathcal{A}_{ar^+ \rightarrow X_a^{(n)}}}|^2}{N_c^2 - 1} (2\pi)^d \delta^d \\ &\quad \left(p_a + k - \sum_{j=1}^n p_j \right) d\Phi^{(n)} \\ \overline{|\mathcal{A}_{ar^+ \rightarrow X_a^{(n)}}|^2} &= \frac{(k^-)^2}{4k^2} |\overline{\mathcal{M}_{ar^+ \rightarrow X_a^{(n)}}}|^2; \end{aligned} \quad (9)$$

$X_{a,b}^{(n)}$ denotes any n -particle system produced in the regarding fragmentation region,

$$d\Phi^{(n)} = \prod_{j=1}^n \frac{d^d p_j}{(2\pi)^{d-1}} \delta_+(p_j^2 - m_j^2), \quad (10)$$

¹ The subscript ‘ugd’ refers to unintegrated gluon density. It denotes that the normalization of this impact factor is in accordance with an unintegrated gluon density at partonic level.

the n -particle phase space and we average and sum over spin and color of incoming and produced particles respectively. Apart from production in the fragmentation region, there exists also the possibility of production at central rapidities. Within high energy factorization as provided by the high energy effective and restricting to processes with only one reggeized gluon exchange, this is described through the collision of two reggeized gluons with opposite polarizations. We have

$$d\hat{\sigma}_{ab} = \hat{h}_a^{(kT)}(\mathbf{k}_1) \hat{h}_b^{(ugd)}(-\mathbf{k}_2) V(-\mathbf{k}_1, \mathbf{k}_2) \times \frac{d^{2+2\epsilon} \mathbf{k}_1}{\pi^{1+\epsilon}} d^{2+2\epsilon} \mathbf{k}_2 d\eta, \quad (11)$$

with [60]

$$V(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_c^2 - 1}{8} \int dM^2 \times \frac{|\mathcal{M}_{r^-(k_1)r^+(k_2) \rightarrow X_c^{(n)}}|^2}{k_1^2 k_2^2} d\Phi^{(n)} \delta^{(d)} \left(k_1 + k_2 - \sum_{j=1}^n p_j \right), \quad (12)$$

where $M^2 = k_1^+ k_2^-$; to leading order in the strong coupling constant one finds

$$V(\mathbf{k}_1, \mathbf{k}_2) = \frac{\alpha_s C_A}{\pi_\epsilon \pi (\mathbf{k}_1 + \mathbf{k}_2)^2}, \quad \pi_\epsilon \equiv \pi^{1+\epsilon} \Gamma(1-\epsilon) \mu^{2\epsilon}, \quad \alpha_s = \frac{g^2 \Gamma(1-\epsilon) \mu^{2\epsilon}}{(4\pi)^{1+\epsilon}}. \quad (13)$$

For the generic inclusive process in which we are interested in, we integrate over the entire phase space of the centrally produced gluon and find that the integral over rapidity $\eta = \frac{1}{2} \ln(k_1^+/k_2^-)$ in Eq. (11) requires an appropriate regularization. The generic choice is $\rho/2 > \eta > -\rho/2$ with $\rho \rightarrow \infty$. Apart from the central rapidity production vertex, there exists also virtual corrections at central rapidities. They are obtained as self-energy corrections to the reggeized gluon fields and yield the following one-loop reggeized gluon propagator [62],

$$G(\rho; \epsilon, \mathbf{k}^2, \mu^2) = \frac{i/2}{\mathbf{k}^2} \left\{ 1 + \frac{i/2}{\mathbf{k}^2} \Sigma\left(\rho; \epsilon, \frac{\mathbf{k}^2}{\mu^2}\right) + \dots \right\}. \quad (14)$$

At the level of a partonic cross section this yields

$$d\hat{\sigma}_{ab}^c = \hat{h}_a^{(kT)}(\mathbf{k}) \hat{h}_b^{(ugd)}(\mathbf{k})$$

$$\times \left| 1 + \frac{i/2}{\mathbf{k}^2} \Sigma\left(\rho; \epsilon, \frac{\mathbf{k}^2}{\mu^2}\right) + \dots \right|^2 \frac{d^{2+2\epsilon} \mathbf{k}}{\pi^{1+\epsilon}}. \quad (15)$$

Σ has a perturbative expansion in α_s , and the first term, including this expansion parameter, is given by [44, 61, 62]

$$\frac{\Sigma^{(1)}\left(\rho; \epsilon, \frac{\mathbf{k}^2}{\mu^2}\right)}{(-2i\mathbf{k}^2)} = \frac{\alpha_s}{4\pi} \left(\frac{\mathbf{k}^2}{\mu^2}\right)^\epsilon \left[\frac{-C_A(2\rho - i\pi)}{\epsilon} - \frac{5C_A - 2n_f}{3\epsilon} + \frac{31C_A}{9} - \frac{10n_f}{9} \right] + \mathcal{O}(\epsilon). \quad (16)$$

Like the inclusive central production vertex, the virtual corrections contain a rapidity divergence which we regulate by tilting the light-cone directions of the high energy effective action against the light-cone

$$n^\pm \rightarrow n^\pm + e^{-\rho} n^\mp \quad \rho \rightarrow \infty. \quad (17)$$

2.2 Subtraction and transition function

Beyond leading order, there is an overlap between central and fragmentation region contributions, both for real and virtual corrections. Moreover both contributions are divergent and require a regulator. In [44, 60] it has been shown through the explicit calculation of NLO corrections for quark and gluon forward jet vertices, that this overlap can be removed through a subtraction procedure which removes from the NLO impact factors the corresponding matrix element which contains an internal reggeized gluon line, i.e. through subtraction of the factorized contribution. The remaining dependence on the regulator then cancels at the level of the NLO cross section, which combines NLO corrections from both fragmentation and central region, see [44, 53, 59, 60] for a detailed discussion. In the following we will slightly formalize this observation by introducing a transition function, generalizing a similar object used in [62] for the calculation of the 2-loop gluon Regge trajectory. Defining the bare one-loop 2-reggeized-gluon Green's function $G_B(\mathbf{k}_1, \mathbf{k}_2)$ through

$$G_B(\mathbf{k}_1, \mathbf{k}_2; \rho) = \delta^{(2+2\epsilon)}(\mathbf{k}_1 + \mathbf{k}_2) + G_B^{(1)}(\mathbf{k}_1, \mathbf{k}_2; \rho) + \dots \\ G_B^{(1)}(\mathbf{k}_1, \mathbf{k}_2; \rho) = \rho V(\mathbf{k}_1, \mathbf{k}_2) + \frac{\Sigma\left(\rho; \epsilon, \frac{\mathbf{k}_1^2}{\mu^2}\right) + \Sigma^*\left(\rho; \epsilon, \frac{\mathbf{k}_1^2}{\mu^2}\right)}{-2i\mathbf{k}_1^2} \delta^{(2+2\epsilon)}(\mathbf{k}_1 + \mathbf{k}_2), \quad (18)$$

we define at first the following subtracted bare NLO coefficient,

$$C_{a,B}^{(1)}(\mathbf{k}, \rho) = h_a^{(0)}(\mathbf{k}) + h_a^{(1)}(\mathbf{k}, \rho) - \left[h_a^{(0)} \otimes G_B^{(1)}(\rho) \right](\mathbf{k}) \quad (19)$$

where we implied the following expansion in α_s of the impact factors,

$$h_a(\mathbf{k}, \rho) = h_a^{(0)}(\mathbf{k}) + h_a^{(1)}(\mathbf{k}, \rho) + \dots \quad (20)$$

Rapidity divergences in the one-loop correction to the impact factors are understood to be regulated through lower cut-offs on the rapidity of all particles, $\eta_i > -\rho/2$ with $\rho \rightarrow \infty$ and $i = 1, \dots, n$ for n the number of particles produced in the fragmentation region of the initial parton a . For virtual corrections, the regularization is again implemented through tilting light-cone directions of the high energy effective action. Finally note that for the fragmentation of the parton b , the regulator would be $\eta_i < \rho/2$ with $\rho \rightarrow \infty$. We further introduced for this paragraph the following convolution convention

$$[f \otimes g](\mathbf{k}_1, \mathbf{k}_2) \equiv \int d^{2+2\epsilon} \mathbf{q} f(\mathbf{k}_1, \mathbf{q}) g(\mathbf{q}, \mathbf{k}_2). \quad (21)$$

Ignoring terms beyond NLO accuracy and combining NLO corrections in the fragmentation region of both partons as well as at the central rapidities, the partonic cross section can be compactly written as²

$$d\sigma_{ab}^{\text{NLO}} = [C_{a,B}(\rho) \otimes G_B(\rho) \otimes C_{b,B}(\rho)]. \quad (22)$$

As a next step we define a renormalized Green's function G_R through

$$G_B(\mathbf{k}_1, \mathbf{k}_2; \rho) = \left[Z^+ \left(\frac{\rho}{2} - \eta_a \right) \otimes G_R(\eta_a, \eta_b) \otimes Z^- \left(\frac{\rho}{2} + \eta_b \right) \right](\mathbf{k}_1, \mathbf{k}_2), \quad (23)$$

where the transition functions $Z^\pm$ possess the following perturbative expansion

$$Z^\pm(\hat{\rho}; \mathbf{k}, \mathbf{q}) = \delta^{(2+2\epsilon)}(\mathbf{k} - \mathbf{q}) + \hat{\rho} K_{\text{BFKL}}(\mathbf{k}, \mathbf{q}) + f^\pm(\mathbf{k}, \mathbf{q}) + \dots, \quad (24)$$

² Note that the impact factors themselves might depend on additional transverse momenta; this is however irrelevant for the following discussion of high energy factorization and we therefore suppress this dependence in the following.

and are to all orders defined through the following BFKL equation,

$$\begin{aligned} \frac{d}{d\hat{\rho}} Z^+(\hat{\rho}; \mathbf{k}, \mathbf{q}) &= [Z^+(\hat{\rho}) \otimes K_{\text{BFKL}}](\mathbf{k}, \mathbf{q}), \\ \frac{d}{d\hat{\rho}} Z^-(\hat{\rho}; \mathbf{k}, \mathbf{q}) &= [K_{\text{BFKL}} \otimes Z^-(\hat{\rho})](\mathbf{k}, \mathbf{q}), \end{aligned} \quad (25)$$

where

$$K_{\text{BFKL}}(\mathbf{k}, \mathbf{q}) = K^{(1)}(\mathbf{k}, \mathbf{q}) + K^{(2)}(\mathbf{k}, \mathbf{q}) + \dots \quad (26)$$

denotes the still undetermined BFKL kernel; $f^\pm(\mathbf{k}, \mathbf{q})$ parametrizes finite contributions and is in principle arbitrary. Symmetry of scattering amplitudes suggests $f^+(\mathbf{k}, \mathbf{q}) = f^-(\mathbf{k}, \mathbf{q})$, while Regge theory suggests to fix it in such a way that terms which are not enhanced by the parameter η are entirely transferred from the renormalized Green's function to the impact factors. Note that the factorization parameter η plays a rôle analogous to the factorization scale in *i.e.* collinear factorization and parametrizes the scale ambiguity associated with high energy factorization. Fixing the lowest order terms of G_R through

$$G_R(\eta_a, \eta_b; \mathbf{k}_1, \mathbf{k}_2) = \delta^{(2+2\epsilon)}(\mathbf{k}_1 + \mathbf{k}_2) + G_R^{(1)}(\eta_a, \eta_b; \mathbf{k}_1, \mathbf{k}_2) + \dots, \quad (27)$$

and expanding the right-hand side up to linear terms, we obtain

$$\begin{aligned} K^{(1)}(\mathbf{k}_1, \mathbf{k}_2) &= V(-\mathbf{k}_1, \mathbf{k}_2) + \delta^{(2+2\epsilon)}(\mathbf{k}_1 - \mathbf{k}_2) \omega^{(1)}(\mathbf{k}_1), \\ \omega^{(1)} \left(\epsilon, \frac{\mathbf{k}^2}{\mu^2} \right) &= -\frac{\alpha_s C_A}{\pi \epsilon} \left(\frac{\mathbf{k}}{\mu^2} \right)^\epsilon + \mathcal{O}(\epsilon), \\ f^{\pm, (1)}(\mathbf{k}_1, \mathbf{k}_2) &= \delta^{(2+2\epsilon)}(\mathbf{k}_1 - \mathbf{k}_2) \frac{\alpha_s}{4\pi} \\ &\quad \times \left[-\frac{1}{\epsilon} \left(\frac{5C_A}{3} - \frac{2n_f}{3} \right) + \frac{31C_A}{9} - \frac{10n_f}{9} \right] \\ &\quad + \mathcal{O}(\epsilon). \end{aligned} \quad (28)$$

As a consequence

$$G_R^{(1)}(\eta_a, \eta_b; \mathbf{k}_1, \mathbf{k}_2) = (\eta_a - \eta_b) K^{(1)}(\mathbf{k}_1, \mathbf{k}_2). \quad (29)$$

Using Eq. (25), it is then straightforward to show that

$$\begin{aligned} \frac{d}{d\eta_a} G_R(\eta_a, \eta_b; \mathbf{k}_1, \mathbf{k}_2) &= \left[K^{(1)}(\mathbf{k}_1, \mathbf{k}_2) \otimes G_R(\eta_a, \eta_b) \right](\mathbf{k}_1, \mathbf{k}_2) \\ \frac{d}{d\eta_b} G_R(\eta_a, \eta_b; \mathbf{k}_1, \mathbf{k}_2) &= \left[G_R(\eta_a, \eta_b) \otimes K^{(1)}(\mathbf{k}_1, \mathbf{k}_2) \right](\mathbf{k}_1, \mathbf{k}_2). \end{aligned} \quad (30)$$

Note that through imposing,

$$G_R(\eta_b, \eta_b; \mathbf{k}_1, \mathbf{k}_2) = \delta^{(2+2\epsilon)}(\mathbf{k}_1 + \mathbf{k}_2), \quad \text{and } \eta_a > \eta_b, \quad (31)$$

no over-counting occurs. With all factors fixed, we insert now Eq. (23) into the NLO cross section Eq. (22), which then immediately leads to

$$d\sigma_{ab}^{\text{NLO}} = [C_{a,R}(\eta_a) \otimes G_R(\eta_a, \eta_b) \otimes C_{b,R}(\eta_b)], \quad (32)$$

where

$$\begin{aligned} C_{a,R}(\eta_a; \mathbf{k}_1) &\equiv \left[C_a(\rho) \otimes Z^+ \left(\frac{\rho}{2} - \eta_a \right) \right] (\mathbf{k}_1), \\ C_{b,R}(\eta_b; \mathbf{k}_2) &\equiv \left[Z^+ \left(\frac{\rho}{2} + \eta_b \right) \otimes C_b(\rho) \right] (\mathbf{k}_2). \end{aligned} \quad (33)$$

In the following paragraph we will provide an explicit verification of this procedure, through applying it to the forward Higgs impact factor. For simplicity we note that the finite coefficient is at NLO given by the following general expression,

$$\begin{aligned} C_R^{\text{NLO}}(\mathbf{k}) &= h_a^{(0)}(\mathbf{k}) + h_a^{(1)}(\mathbf{k}) + h_a^{(0)} \\ &\quad \otimes \left[\left(-\frac{\rho}{2} - \eta_a \right) K^{(1)} - f^{-(1)} \right] (\mathbf{k}) \\ &= h_a^{(0)}(\mathbf{k}) + h_a^{(1)}(\mathbf{k}, \rho) - \frac{\alpha_s N_c \left(\frac{\rho}{2} + \eta_a \right)}{\pi} \\ &\quad \times \left[\int \frac{d^{2+2\epsilon} \mathbf{r}}{r^2} h_a^{(0)} \left((\mathbf{r} + \mathbf{k})^2 \right) \right] \\ &\quad + h_a^{(0)}(\mathbf{k}) \frac{\alpha_s}{2\pi} \left(\frac{k^2}{\mu^2} \right)^\epsilon \left[\frac{C_A(\rho + 2\eta_a)}{\epsilon} \right. \\ &\quad \left. + \frac{1}{\epsilon} \left(\frac{5C_A}{6} - \frac{2n_f}{6} \right) - \frac{31C_A}{18} + \frac{10n_f}{18} \right]. \end{aligned} \quad (34)$$

3 The impact factor for forward Higgs production

We consider collisions of two hadrons A and B with momenta $p_{A,B} = p_{A,B}^\pm n^\mp / 2$ and squared center of mass energy $s = p_A^+ p_B^-$ with inclusive production of an on-shell Higgs boson in the fragmentation region of hadron A . The four momentum of the Higgs boson p and its rapidity η_H are parametrized as

$$p = x_H p_A + \frac{M_H^2 + p^2}{x_H s} p_B + p_T, \quad \eta_H = \ln \frac{x_H p_A^+}{\sqrt{M_H^2 + p^2}}, \quad (35)$$

where p_T is the embedding of the Euclidean Higgs transverse momentum $\mathbf{p}$ into Minkowski space, see also Fig. 2.

Fig. 2 Kinematics for the leading order matrix element as well as virtual corrections (left) and real next-to-leading order corrections (right). The latter implies the case where the gluon is replaced by a quark. Momenta p_a and k are in-going, while p and r are out-going

To describe the coupling of the Higgs boson to the gluonic field, we make use of the heavy top limit and employ the following effective Lagrangian [68,69],

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} g_H H F_{\mu\nu}^a F_a^{\mu\nu} \quad (36)$$

with H the scalar (Higgs) field and g_H the effective coupling [70,71]

$$g_H = -\frac{\alpha_s}{3\pi v} \left(1 + \frac{\alpha_s}{4\pi} 11 \right) + \mathcal{O}(\alpha_s^3). \quad (37)$$

Since the top quark has been integrated out, the strong coupling α_s is evaluated for $n_f = 5$ flavors and $v^2 = 1/(\sqrt{2}G_F)$ with G_F the Fermi constant. Working under the assumption that multi-reggeized gluon exchanges can be neglected, the hadronic differential cross section is factorized into

$$\begin{aligned} \frac{d^3\sigma}{d^2\mathbf{p} dx_H} &= \int_{x_H}^1 \frac{dz}{z} \sum_{a=q,g} f_a \left(\frac{x_H}{z}, \mu_F^2 \right) \\ &\quad \times \int \frac{d^2\mathbf{k}}{\pi} \frac{d\hat{C}_{ag^* \rightarrow H}(\mu_F^2, \eta_a; z, \mathbf{k})}{d^2\mathbf{p} dx_H} \mathcal{G}(\eta_a, \mathbf{k}), \end{aligned} \quad (38)$$

where $\mathcal{G}(\eta_a, \mathbf{k})$ denotes the unintegrated gluon distribution of hadron B which parametrizes non-perturbative input of hadron B and is subject to BFKL evolution; η_a is a factorization parameter associated with the highest gluon rapidity absorbed into the unintegrated gluon density. In terms of the elements defined in the previous section we have

$$\mathcal{G}(\eta_a, \mathbf{k}, Q_0) = \int d^2\mathbf{q} G_R(\eta_a, \mathbf{k}, \mathbf{q}) h^{ugd}(\mathbf{q}, Q_0), \quad (39)$$

where h^{ugd} is obtained as the convolution of partonic impact factor and parton distribution functions. In particular, collinear singularities, which arise from the infra-red region of transverse momentum integration are assumed to be absorbed into the parton distribution function of hadron B following the general procedure outlined in [72], see also [22,73]. The dependence on the scale Q_0 is understood to

Fig. 3 Feynman diagrams for real correction in the gluon (left) and quark (right) channels respectively. Here the wavy line indicates the off-shell reggeized gluon state. The gray vertex indicates the gluon-gluon-reggeized gluon which is obtained as a combination of a first

order induced vertex and the three-gluon vertex, see second line. The quark channel includes both contributions due to quarks and anti-quarks

arise as a consequence of such a factorization of collinearly enhanced contributions. For the partonic differential coefficient, we assume the following perturbative expansion

$$\begin{aligned} \frac{d^3 \hat{C}_{ag^* \rightarrow H}^{NLO}}{dx_H d^2 \mathbf{p}} &= \sigma_0 \left(\frac{d^3 \hat{C}_{ag^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} + \frac{\alpha_s}{2\pi} \cdot \frac{d^3 \hat{C}_{pg^* \rightarrow H}^{(1)}}{dx_H d^2 \mathbf{p}} + \dots \right). \\ a &= q, g, \end{aligned} \quad (40)$$

With

$$\begin{aligned} \frac{d^3 h_{ag^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} &= \sigma_0 \int \frac{dk^-}{k^-} \delta^{(2)}(\mathbf{p} - \mathbf{k}) \delta(1 - z) \\ &\quad \times \delta \left(1 - \frac{M_H^2 + \mathbf{k}^2}{p_a^+ k^-} \right), \\ \sigma_0 &= \frac{g_H^2 \pi}{8(N_c^2 - 1)}, \end{aligned} \quad (41)$$

we have

$$\frac{d\hat{C}_{gg^* \rightarrow H}^{(0)}(\mu_F^2, \eta_a; z, \mathbf{k})}{d^2 \mathbf{p} dx_H} = \delta^{(2)}(\mathbf{p} - \mathbf{k}) \delta(1 - z), \quad (42)$$

at leading order, while the corresponding contribution from the quark-channel vanishes. In the following we will determine the next-to-leading order corrections to this impact factor. This will be the main result of this paper.

3.1 Virtual next-to-leading order corrections

Virtual NLO corrections to the operator $-\frac{1}{2} \text{tr}[G_{\mu\nu} G^{\mu\nu}]$ have been calculated in [67]. Adapting the conventions of that

paper to the ones used here we find:

$$\begin{aligned} \frac{dh_{gg^* \rightarrow H}^{(1)}(z, \mathbf{k})}{d^2 \mathbf{p} dx_H} &= \frac{dh_{gg^* \rightarrow H}^{(0)}(z, \mathbf{k})}{d^2 \mathbf{p} dx_H} \frac{\alpha_s}{2\pi} \cdot \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\epsilon \\ &\quad \times \left\{ -\frac{C_A}{\epsilon^2} - \frac{1}{\epsilon} \left(\frac{8C_A}{3} - \frac{2n_f}{3} \right) \right. \\ &\quad + \frac{C_A}{\epsilon} \left[-\rho + \ln \frac{\mathbf{k}^2}{(p_a^+)^2} \right] \\ &\quad + C_A \left[2\text{Li}_2 \left(1 + \frac{M_H^2}{\mathbf{k}^2} \right) + \frac{\pi^2}{6} + \frac{49}{9} \right] \\ &\quad \left. + 11 - \frac{10}{9} n_f \right\} \end{aligned} \quad (43)$$

where we also added the contribution due to the 1-loop corrections of the Higgs-gluon-gluon coupling in the heavy quark limit, Eq. (37).

3.2 Real next-to-leading order corrections

Real NLO corrections contain both contributions from the gluon and the quark channel. The relevant Feynman diagrams are depicted in Fig. 3. Our convention for momenta is as follows

$$g(p_a) + r_+(k) \rightarrow H(p) + g(r), \quad (44)$$

where we replace $g \leftrightarrow q, \bar{q}$ for the contributions with initial and final (anti-) quark states. We further use $0 < z < 1$ to parametrize the initial parton momentum fraction, carried on by the Higgs particle. For the gluon channel we find

$$\frac{d^3 h_{gg^* \rightarrow H_g}^{(0)}(z, \mathbf{k})}{dx_H d^2 \mathbf{p}} = \frac{\alpha_s C_A \sigma_0}{2\pi \epsilon k^2} H_{ggH}(z, \mathbf{p}, \mathbf{k}) \theta \left(\eta_g + \frac{\rho}{2} \right) \quad (45)$$

where $\eta_g = \ln \frac{(1-z)x_H p_A^+}{z\sqrt{r^2}}$ is the gluon rapidity and ρ the regulator for the high energy divergence, which we take in the limit $\rho \rightarrow \infty$. We further kept the dependence the dimensional regularization parameter ϵ explicit:

$$\begin{aligned}
 H_{ggH}(z, \mathbf{p}, \mathbf{k}) = & \frac{2}{z(1-z)} \left\{ 2z^2 \right. \\
 & + \frac{(1-z)zM_H^2(\mathbf{k} \cdot \mathbf{r})[z^2 + (1-z) \cdot 2\epsilon] - 2z^3(\mathbf{p} \cdot \mathbf{r})(\mathbf{p} \cdot \mathbf{k})}{r^2(p^2 + (1-z)M_H^2)} \\
 & + \frac{(1+\epsilon)(1-z)^2z^2M_H^4}{2} \left(\frac{1}{\Delta^2 + (1-z)M_H^2} \right. \\
 & \left. \left. + \frac{1}{p^2 + (1-z)M_H^2} \right)^2 \right. \\
 & - \frac{2z^2(\mathbf{p} \cdot \Delta)^2 + 2\epsilon \cdot (1-z)^2z^2M_H^4}{(p^2 + (1-z)M_H^2)(\Delta^2 + (1-z)M_H^2)} \\
 & - \frac{2z(1-z)^2M_H^2}{\Delta^2 + (1-z)M_H^2} - \frac{2z(1-z)^2M_H^2}{p^2 + (1-z)M_H^2} \\
 & - \frac{(1-z)zM_H^2(\mathbf{k} \cdot \mathbf{r})[z^2 + (1-z) \cdot 2\epsilon] - 2z^3(\Delta \cdot \mathbf{r})(\Delta \cdot \mathbf{k})}{r^2(\Delta^2 + (1-z)M_H^2)} \left. \right\} \\
 & + \frac{2k^2}{r^2} \left\{ \frac{z}{1-z} + z(1-z) \right. \\
 & \left. + 2(1+\epsilon) \frac{(1-z)}{z} \frac{(\mathbf{k} \cdot \mathbf{r})^2}{k^2 r^2} \right\}; \quad (46)
 \end{aligned}$$

$\mathbf{r} = \mathbf{k} - \mathbf{p}$ denotes the transverse momentum of the real final state parton and

$$\Delta = z\mathbf{r} - (1-z)\mathbf{p}. \quad (47)$$

For the case of an initial quark we obtain instead

$$\frac{d^3 h_{qg^* \rightarrow Hq}^{(0)}(z, \mathbf{k})}{dx_H d^2 \mathbf{p}} = \frac{\alpha_s C_F \sigma_0}{2\pi_\epsilon k^2} H_{qqH}(z, \mathbf{p}, \mathbf{k}), \quad (48)$$

with

$$H_{qqH}(z, \mathbf{p}, \mathbf{k}) = \frac{1+\epsilon}{z} \left[z^2 + 4(1-z) \frac{(\mathbf{k} \cdot \mathbf{r})^2}{k^2 r^2} \right]. \quad (49)$$

Both Eqs. (46) and (49) were evaluated directly using Lipatov's high energy effective action as well as using the conventional k_T factorization procedure, where the sum over polarization of the incoming off-shell gluon is given by eikonal projectors. We furthermore cross-checked the result numerically using KATIE [52].

Note that the quark channel is free of high energy divergences and we therefore took already the limit $\rho \rightarrow \infty$. To address the high energy divergence at $z = 1$ of the gluonic

real corrections, we note at first that

$$\lim_{z \rightarrow 1} \frac{\alpha_s C_A \sigma_0}{2\pi_\epsilon} \frac{H_{ggH}(z, \mathbf{p}, \mathbf{k})}{k^2} = \frac{\alpha_s C_A \sigma_0}{\pi_\epsilon \cdot r^2} \frac{1}{1-z} + \text{finite}, \quad (50)$$

where 'finite' indicates all the terms which do not require a regulator. Making use of the following identity, where $f(z)$ is a generic test function,

$$\begin{aligned}
 \lim_{\rho \rightarrow \infty} \int_{x_H}^1 dz \frac{f(z)\theta(\eta_g + \rho/2)}{1-z} \\
 = \int_{x_H}^1 dz \frac{f(z)}{(1-z)_+} \\
 + f(1) \left[\ln \frac{x_H p_A^+}{\sqrt{r^2}} + \frac{\rho}{2} \right], \quad (51)
 \end{aligned}$$

with

$$\int_{x_H}^1 dz \frac{f(z)}{(1-z)_+} = \int_{x_H}^1 dz \frac{f(z) - f(1)}{1-z} - \int_0^{x_H} dz \frac{f(1)}{1-z}, \quad (52)$$

we identify the high energy singularity of the real corrections as

$$\frac{\alpha_s C_A \sigma_0}{2\pi_\epsilon} \frac{H_{ggH}(z, \mathbf{p}, \mathbf{k})}{k^2} = \delta(1-z) \frac{\rho}{2} \sigma_0 \cdot \frac{\alpha_s C_A}{\pi_\epsilon r^2} + \dots, \quad (53)$$

while the $z \rightarrow 1$ singularity in H_{ggH} is now regulated through a plus-prescription and the dots indicate terms finite in the limit $\rho \rightarrow \infty$.

3.3 Counter-terms

Our result requires a number of counter-terms both to ultra-violet renormalization, collinear factorization (initial parton) and high energy factorization (reggeized gluon field). For the former two we will employ the $\overline{\text{MS}}$ -scheme, while for the latter we will make use of the scheme presented in Sect. 2.2. The ultra-violet counter-term is identical to the one used in the determination of collinear NLO corrections in [70] and can be entirely expressed through renormalization of the QCD strong coupling in the Higgs-gluon-gluon coupling constant g_H . With

$$\begin{aligned}
 \alpha_s = \alpha_s(\mu_R) \left[1 + \frac{\alpha_s(\mu_R)\beta_0}{(4\pi)} \left(\frac{1}{\epsilon} + \ln \frac{\mu_R^2}{\mu^2} \right) \right], \\
 \beta_0 = \frac{11C_A}{3} - \frac{2n_f}{3}, \quad (54)
 \end{aligned}$$

where in the following we set $\mu_R = \mu$. The remaining singularities both in the limit $\epsilon \rightarrow 0$ and $\rho \rightarrow \infty$ can be factored into process independent functions associated with the external legs (collinear parton and off-shell reggeized gluon state). With

$$\frac{d^3 h_{ag^* \rightarrow Ha}}{dx_H d^2 \mathbf{p}} = \frac{d^3 h_{ag^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} + \frac{d^3 h_{ag^* \rightarrow H}^{(1)}}{dx_H d^2 \mathbf{p}} + \frac{d^3 h_{ag^* \rightarrow Ha}^{(0)}}{dx_H d^2 \mathbf{p}}, \quad (55)$$

the physical coefficient is then implicitly defined through the relation

$$\begin{aligned} \frac{d^3 h_{ag^* \rightarrow Ha}^{(0)}}{dx_H d^2 \mathbf{p}} &= \sum_{b=q,g} \int_{x_H}^1 d\xi \\ &\times \int d^{2+2\epsilon} \tilde{\mathbf{k}} \frac{d^3 \hat{C}_{bg^* \rightarrow Hb}(z, \tilde{\mathbf{k}}, \mathbf{p})}{dx_H d^2 \mathbf{p}} \Gamma_{ba} \\ &\left(\xi, \frac{\mu_F^2}{\mu^2} \right) \tilde{\Gamma}_{g^*g^*}(\xi, \tilde{\mathbf{k}}, \mathbf{k}), \end{aligned} \quad (56)$$

where

$$\begin{aligned} \Gamma_{ba}(z) &= \delta_{ba} \delta(1-z) - \frac{\alpha_s}{2\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu_F^2}{\mu^2} \right) P_{ga}(z) \\ &+ \mathcal{O}(\alpha_s^2), \quad a = q, g \end{aligned} \quad (57)$$

are the 1-loop partonic parton distribution with gluon splitting functions,

$$\begin{aligned} P_{gq}(z) &= C_F \frac{1 + (1-z)^2}{z}, \\ P_{gg}(z) &= 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] \\ &+ \frac{\beta_0}{2} \delta(1-z), \end{aligned} \quad (58)$$

where corresponding quark splitting functions are absent since the leading order quark coefficient vanishes. The 1-loop unintegrated gluon distribution is, following Sect. 2.2, given by

$$\begin{aligned} \tilde{\Gamma}_{g^*g^*}(\xi, \tilde{\mathbf{k}}, \mathbf{k}) &= \delta(1-\xi) \delta^{(2)}(\mathbf{k} - \tilde{\mathbf{k}}) \left[1 - \frac{\alpha_s}{2\pi} \right. \\ &\times \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\epsilon \left(\frac{5C_A - 2n_f}{6\epsilon} - \frac{31C_A - 10n_f}{18} \right) \\ &+ \delta(1-\xi) (\rho + 2\eta_a) \\ &\times \left[\frac{\alpha_s C_A}{2\pi\epsilon (\tilde{\mathbf{k}} - \mathbf{k})^2} - \delta^{(2)}(\mathbf{k} - \tilde{\mathbf{k}}) \frac{\alpha_s}{2\pi\epsilon} \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\epsilon \right], \end{aligned} \quad (59)$$

which contains apart from the 1-loop BFKL kernel in the second term, also terms due to the gluon self-energy. In the current setup we preferred to formulate this distribution in $d = 4 + 2\epsilon$, while it is in principle straightforward to remove the remaining UV divergence through an appropriate counter term associated with the gluon self-energy.

3.4 Subtraction mechanism to achieve numerical stability

Given the above counter terms, it is a relatively straightforward task to verify finiteness of the resulting coefficient in the limit $\rho \rightarrow \infty$ and $\epsilon \rightarrow 0$. While the subtraction of high energy and ultraviolet singularities is straightforward, extracting of infrared singularities is more cumbersome and requires the use of phase space slicing parameters, see [41–43] for instance. While this is sufficient to demonstrate finiteness at a formal level, the use of such phase space slicing parameters is in general complicated for numerical studies at NLO accuracy. For the case of collinear NLO calculation, the by now conventional tool to overcome this difficulty is provided by subtraction methods, in particular the dipole subtraction as formulated in [74]. Within the current setup, collinear and soft singularities are directly associated with the convolution integral over transverse momenta and the formulas of [74] cannot be directly translated to the present case. In the following we therefore present a subtraction mechanism which closely follows the spirit of [74], but which is adapted to the current setup. In particular a generalization to other partonic high energy coefficients appears to be possible. Following [74], the basic idea is to subtract a certain auxiliary term from the real NLO corrections which (a) renders the latter finite and (b) can be easily integrated analytically and added to the virtual NLO corrections. We therefore propose the following decomposition:

$$\begin{aligned} &\int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon}} \frac{\kappa(\mathbf{r})}{\mathbf{r}^2} G((\mathbf{p} + \mathbf{r})^2) \\ &= \int \frac{d^2 \mathbf{r}}{\pi} \left[\frac{\kappa(\mathbf{r})}{\mathbf{r}^2} \right]_+ G((\mathbf{p} + \mathbf{r})^2) + \mathcal{O}(\epsilon) \\ &+ \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon}} \frac{\kappa(\mathbf{r})}{\mathbf{r}^2} \frac{\mathbf{p}^2 G(\mathbf{p}^2)}{\mathbf{r}^2 + (\mathbf{p} + \mathbf{r})^2}, \end{aligned} \quad (60)$$

with

$$\begin{aligned} &\int \frac{d^2 \mathbf{r}}{\pi} \left[\frac{\kappa(\mathbf{r})}{\mathbf{r}^2} \right]_+ G((\mathbf{p} + \mathbf{r})^2) \\ &\equiv \int \frac{d^2 \mathbf{r}}{\pi} \frac{\kappa(\mathbf{r})}{\mathbf{r}^2} \left[G((\mathbf{p} + \mathbf{r})^2) - \frac{\mathbf{p}^2 G(\mathbf{p}^2)}{\mathbf{r}^2 + (\mathbf{p} + \mathbf{r})^2} \right]. \end{aligned} \quad (61)$$

The expression in the squared brackets on the right-hand side vanish in the limit $|\mathbf{r}| \rightarrow 0$, and $G(\mathbf{k})$ is a function which parametrizes the transverse momentum dependence

of the reggeized gluon state. The function $\kappa(\mathbf{r})$ is such that the integral on the right hand side of Eq. (61) is well-defined, which in practice means that it does not behave worse than $\ln|\mathbf{r}|$ for $|\mathbf{r}| \rightarrow 0$ and $|\mathbf{r}| \rightarrow \infty$. Furthermore, it should be such that the integral in the last term in Eq. (60) can be calculated analytically. Note that the factor $\mathbf{p}^2/[\mathbf{r}^2+(\mathbf{p}+\mathbf{r})^2]$ is needed to achieve convergence in the ultraviolet. We have the following results for the choices $\kappa(\mathbf{r}) = 1$, for $\kappa(\mathbf{r}) = -\ln \mathbf{r}^2$, and for $\kappa(\mathbf{r}) = (\mathbf{p} \cdot \mathbf{r})^2/\mathbf{r}^2$ which we will need:

$$\begin{aligned} \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon} \mathbf{r}^2 [\mathbf{r}^2 + (\mathbf{p} + \mathbf{r})^2]} &= \frac{\Gamma(1-\epsilon)\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)} (\mathbf{p}^2)^{\epsilon-1} \\ &= \frac{\Gamma(1-\epsilon)}{\epsilon} (\mathbf{p}^2)^{\epsilon-1} + \mathcal{O}(\epsilon) \\ \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon} \mathbf{r}^2 [\mathbf{r}^2 + (\mathbf{p} + \mathbf{r})^2]} &= \frac{\Gamma(1-\epsilon)}{(\mathbf{p}^2)^{1-\epsilon}} \left(\frac{1}{\epsilon^2} - \frac{\ln \mathbf{p}^2}{\epsilon} - \frac{\pi^2}{4} \right) + \mathcal{O}(\epsilon) \\ \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon}} \frac{(\mathbf{p} \cdot \mathbf{r})^2}{(\mathbf{r}^2)^2 [\mathbf{r}^2 + (\mathbf{p} + \mathbf{r})^2]} &= \frac{\Gamma(1-\epsilon)}{(\mathbf{p}^2)^{-\epsilon}} \left(\frac{1}{2\epsilon} - 1 + \ln 2 \right) + \mathcal{O}(\epsilon). \end{aligned} \quad (62)$$

Note that through representing the function $G(\mathbf{k}^2)$ through its Mellin transform, it is further possible to verify that the subtraction does not generate any divergent left-overs, see Appendix A for an explicit calculation. Following [74], we further use in the subtraction terms the splitting functions which are not averaged over the regarding azimuthal angle. They are given by

$$\begin{aligned} \hat{P}_{gg}^r(z, \mathbf{p}, \mathbf{r}) &= 2C_A \left(\frac{z}{(1-z)_+} + (1-z)z + 2(1+\epsilon) \frac{1-z}{z} \frac{(\mathbf{p} \cdot \mathbf{r})^2}{\mathbf{p}^2 \mathbf{r}^2} \right) \\ \hat{P}_{qg}^r(z, \mathbf{p}, \mathbf{r}) &= \frac{C_F}{z} \left(z^2 + 4(1-z) \frac{(\mathbf{p} \cdot \mathbf{r})^2}{\mathbf{p}^2 \mathbf{r}^2} \right), \end{aligned} \quad (63)$$

and yield after averaging over the azimuthal angle the real part of the regarding splitting functions in $d = 4 + 2\epsilon$ dimensions, i.e.

$$\int \frac{d^{1+\epsilon} \Omega}{\pi^{1+\epsilon}} \hat{P}_{ag}^r(z, \mathbf{p}, \mathbf{r}) = P_{ag}^r(z, \epsilon) \quad a = q, g, \quad (64)$$

where

$$\begin{aligned} P_{qg}^r(z, \epsilon) &= C_F \frac{1 + (1-z)^2 + \epsilon z^2}{z} \\ P_{gg}^r(z, \epsilon) &= 2C_A \left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right) \end{aligned} \quad (65)$$

We finally have

$$\begin{aligned} &\int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon} \mathbf{r}^2} G((\mathbf{p} + \mathbf{r})^2) \hat{P}_{gg}^r(z, \mathbf{r}, \mathbf{k}) \\ &= \left[P_{gg}^r(z, 0) \frac{\Gamma(1-\epsilon)}{\epsilon(\mathbf{p}^2)^{-\epsilon}} + \frac{1-z}{z} 2 \left(\ln 2 - \frac{1}{2} \right) \right] G(\mathbf{p}^2) \\ &\quad + \int \frac{d^2 \mathbf{r}}{\pi} \left[\frac{\hat{P}_{gg}^r(z, \mathbf{r}, \mathbf{k})}{\mathbf{r}^2} \right]_+ G((\mathbf{p} + \mathbf{r})^2), \\ &\int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon} \mathbf{r}^2} G((\mathbf{p} + \mathbf{r})^2) \hat{P}_{qg}^r(z, \mathbf{r}, \mathbf{k}) \\ &= \left[P_{qg}^r(z, 0) \frac{\Gamma(1-\epsilon)}{\epsilon(\mathbf{p}^2)^{-\epsilon}} + \frac{1-z}{z} 2 (\ln 2 - 1) \right] G(\mathbf{p}^2) \\ &\quad + \int \frac{d^2 \mathbf{r}}{\pi} \left[\frac{\hat{P}_{qg}^r(z, \mathbf{r}, \mathbf{k})}{\mathbf{r}^2} \right]_+ G((\mathbf{p} + \mathbf{r})^2). \end{aligned} \quad (66)$$

While for the term associated with the high energy divergence we further need

$$\begin{aligned} &\int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon} \mathbf{r}^2} \ln \frac{x_H p_A^+}{|\mathbf{r}|} G((\mathbf{p} + \mathbf{r})^2) \\ &= \frac{\Gamma(1-\epsilon)}{(\mathbf{p}^2)^{-\epsilon}} \left(\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \ln \frac{x_H p_A^+}{|\mathbf{p}|} - \frac{\pi^2}{8} \right) \\ &\quad + \int \frac{d^2 \mathbf{r}}{\pi} \left[\frac{\ln(x_H p_A^+) - \ln |\mathbf{r}|}{\mathbf{r}^2} \right]_+ G((\mathbf{p} + \mathbf{r})^2), \end{aligned} \quad (67)$$

where we systematically neglect terms of the order ϵ and higher.

3.5 The NLO coefficient for forward Higgs production

We present our result for the differential cross section,

$$\begin{aligned} \frac{d^3 \sigma}{dx_H d^2 \mathbf{p}} &= \int \frac{d^2 \mathbf{r}}{\pi} \int_{x_H}^1 \frac{dz}{z} \sum_{a=q,g} f_a \left(\frac{x_H}{z}, \mu_F^2 \right) \\ &\quad \frac{d^3 \hat{C}_{ag^* \rightarrow H}^{NLO}}{dx_H d^2 \mathbf{p}} \mathcal{G}(\eta_a, (\mathbf{r} + \mathbf{p})^2) \end{aligned} \quad (68)$$

where $\mathcal{G}(\eta_a, \mathbf{k})$ is the transverse momentum dependent unintegrated gluon density with $\mathbf{k}$ the transverse momentum and η_a the evolution parameter. We have

$$\begin{aligned} \frac{d^3 \hat{C}_{ag^* \rightarrow H}^{NLO}}{dx_H d^2 \mathbf{p}} &= \sigma_0 \left(\frac{d^3 \hat{C}_{ag^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} \right. \\ &\quad \left. + \frac{\alpha_s}{2\pi} \frac{d^3 \hat{C}_{g^* \rightarrow H}^{(1)}}{dx_H d^2 \mathbf{p}} + \dots \right), \quad a = q, g \end{aligned} \quad (69)$$

with

$$\frac{d^3 \hat{C}_{gg^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} = \delta^{(2)}(\mathbf{r}) \delta(1-z), \quad \frac{d^3 \hat{C}_{qg^* \rightarrow H}^{(0)}}{dx_H d^2 \mathbf{p}} = 0, \quad (70)$$

and

$$\begin{aligned} \frac{d^3 \hat{C}_{gg^* \rightarrow H}^{(1)}(z, \mathbf{r}, \mathbf{p}; \eta_a, \mu_F, \mu)}{dx_H d^2 \mathbf{p}} = & \\ & \delta(1-z) \left\{ 2C_A \left[\ln \frac{(1-x_H)p_A^+}{\sqrt{r^2}} \right. \right. \\ & \left. \left. - \eta_a \frac{1}{r^2} \right]_+ + \delta^{(2)}(\mathbf{r}) \left[11 - \frac{\beta_0}{2} \ln \frac{p^2}{\mu^2} - \frac{5n_f}{9} \right. \right. \\ & \left. \left. + C_A \left(\frac{67}{18} + 2\text{Li}_2 \left(1 + \frac{M_H^2}{p^2} \right) - \frac{\pi^2}{12} \right) \right] \right\} \\ & + \frac{H_{ggH}^{fin.}(z, \mathbf{p}, \mathbf{r})}{k^2} + \left[\frac{\hat{P}_{gg}^r(z, \mathbf{p}, \mathbf{r})}{r^2} \right]_+ \\ & + \delta^{(2)}(\mathbf{r}) \left[C_A \frac{1-z}{z} (4 \ln 2 - 2) - \ln \frac{\mu_F^2}{\mu^2} P_{gg}(z) \right] \end{aligned} \quad (71)$$

while

$$\begin{aligned} \frac{d^3 \hat{C}_{qg^* \rightarrow H}^{(1)}(z, \mathbf{r}, \mathbf{p}; \eta_a, \mu_F, \mu)}{dx_H d^2 \mathbf{p}} = & \\ & \delta^{(2)}(\mathbf{r}) \left[C_F \frac{1-z}{z} (4 \ln 2 - 4) - \ln \frac{\mu_F^2}{\mu^2} P_{qq}(z) \right] \\ & + \frac{H_{qqH}^{fin.}(z, \mathbf{p}, \mathbf{r})}{k^2} + \left[\frac{\hat{P}_{qg}^r(z, \mathbf{p}, \mathbf{r})}{r^2} \right]_+. \end{aligned} \quad (72)$$

Furthermore

$$\begin{aligned} H_{ggH}^{fin.}(z, \mathbf{p}, \mathbf{r}) = & \frac{2C_A}{z(1-z)} \left\{ \frac{(1-z)^2 z^2 M_H^4}{2} \left(\frac{1}{\Delta^2 + (1-z)M_H^2} \right. \right. \\ & \left. \left. + \frac{1}{p^2 + (1-z)M_H^2} \right)^2 \right. \\ & - \frac{2z^2(\mathbf{p} \cdot \Delta)^2}{(p^2 + (1-z)M_H^2)(\Delta^2 + (1-z)M_H^2)} \\ & - \frac{2z(1-z)^2 M_H^2}{\Delta^2 + (1-z)M_H^2} \\ & - \frac{2z(1-z)^2 M_H^2}{p^2 + (1-z)M_H^2} \\ & - \frac{(1-z)zM_H^2(\mathbf{k} \cdot \mathbf{r})z^2 - 2z^3(\Delta \cdot \mathbf{r})(\Delta \cdot \mathbf{k})}{r^2(\Delta^2 + (1-z)M_H^2)} \\ & \left. + \frac{(1-z)zM_H^2(\mathbf{k} \cdot \mathbf{r})z^2 - 2z^3(\mathbf{p} \cdot \mathbf{r})(\mathbf{p} \cdot \mathbf{k})}{r^2(p^2 + (1-z)M_H^2)} + 2z^2 \right\} \\ & + 4C_A \frac{k^2}{r^2} \left[\frac{(1-z)}{z} \frac{(\mathbf{k} \cdot \mathbf{r})^2}{k^2 r^2} \right] \end{aligned}$$

$$\begin{aligned} H_{qqH}^{fin.}(z, \mathbf{p}, \mathbf{r}) = & 4C_F \frac{k^2}{r^2} \left[\frac{(1-z)}{z} \frac{(\mathbf{k} \cdot \mathbf{r})^2}{k^2 r^2} \right. \\ & \left. - \frac{(1-z)}{z} \frac{(\mathbf{p} \cdot \mathbf{r})^2}{p^2 r^2} \right] \end{aligned} \quad (73)$$

and

$$\Delta = z\mathbf{r} - (1-z)\mathbf{p} \quad \mathbf{k} = \mathbf{p} + \mathbf{r}. \quad (74)$$

3.6 Scale setting

As pointed out already in Sect. 2.2, our result for the NLO coefficient depends, apart from the renormalization scale μ and the collinear factorization scale μ_F on the high energy factorization parameter η_a . For the differential cross section, this η_a dependence is – at NLO accuracy – canceled against a similar dependence in the BFKL gluon Green's function. While introducing the parameters η_a and η_b is natural from the point of view of the high energy factorized matrix elements, in practical applications it is more convenient to parametrize $\eta_{a,b}$ in terms of the so-called reggeization scale s_0 , originally introduced in [75], which suggests to define

$$\eta_a = \ln \frac{p_A^+}{\sqrt{s_0}} \quad \eta_b = -\ln \frac{p_B^-}{\sqrt{s_0}} \quad (75)$$

such that

$$\eta_a - \eta_b = \ln \frac{s}{s_0} \quad s = p_A^+ p_B^-. \quad (76)$$

While the reggeization scale is in principle arbitrary, it is naturally chosen of the order of magnitude of a typical scale of one or both impact factors. Note that at NLO, an asymmetric scale choice, i.e. choosing s_0 to be of the order of a typical scale of *one* of the impact factors, leads to a modification of the NLO BFKL kernel, see e.g. [21, 76]. A discussion of this scale setting appears in principle possible using the formalism introduced in Sect. 2.2, but is clearly beyond the scope of the present paper.

4 Conclusion

We presented the NLO corrections to the impact factor for forward production of a Higgs boson. Our result can be used both for studies of forward Higgs production as well as studies of Higgs-jet configuration with jet and Higgs boson separated by a large difference in rapidity, see e.g. [11].

Apart from the actual determination of the NLO coefficient, we further provide a prescription on how to determine

such a NLO coefficient from corresponding NLO matrix elements, making use of the framework provided by Lipatov's high energy effective action. This framework turn out to be of particular use in this context for various reasons. It not only provides a manifestly gauge invariant definition of off-shell matrix elements both at tree level (which implies NLO real corrections) and 1-loop (NLO virtual corrections), but also allows for a straightforward identification of factorizing contributions, which provides then the basis for the definition of the NLO coefficient. Nevertheless it is important to stress that the same results can be always obtained through a study of exact QCD matrix elements in the high energy limit, which is actually a necessary requirement. While in some cases this might be even the preferred path to determine NLO corrections, the high energy effective action provides a natural framework to organize these results into reggeized gluon Green's function and NLO coefficient and therefore to properly define the latter.

Apart from the definition of the NLO coefficient, we further proposed a subtraction mechanism to achieve a numerically stable cancellation of soft-collinear divergences between real, virtual corrections as well as collinear counterterms, along the lines of [74]. We believe that such a subtraction formalism will be highly beneficial for the numerical implementation of this and other NLO results within high energy factorization.

From a formal point of view, the result will be useful to study further the resummation of soft-collinear logarithms within high energy factorization, see i.e. [77–80] as well as the proper definition of evolution equations for transverse momentum dependent evolution kernels along the lines of [81–83].

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Appendix A: Finiteness of the subtraction mechanism

To verify further that the subtraction does not generate any divergent left-overs, it is possible to represent the function $G(k^2)$ through its Mellin transform³:

$$G(k^2) = \frac{1}{k^2} \int \frac{d\gamma}{2\pi i} \left(\frac{k^2}{Q_0^2} \right)^\gamma \tilde{G}(\gamma), \quad (77)$$

where Q_0 is a characteristic scale of the momentum distribution. One finds:

$$\begin{aligned} & \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon}} \frac{1}{r^2 [(p+r)^2]^{1-\gamma}} \\ &= \frac{\Gamma(1-\epsilon)}{(p^2)^{\gamma-1-\epsilon}} \left[\frac{1}{\epsilon} + \chi_0(\gamma) \right], \\ & \int \frac{d^{2+2\epsilon} \mathbf{r}}{\pi^{1+\epsilon}} \frac{\ln(1/r^2)}{r^2 [(p+r)^2]^{1-\gamma}} \\ &= \frac{\Gamma(1-\epsilon)}{(p^2)^{\gamma-1-\epsilon}} \left[\frac{1}{\epsilon^2} - \frac{\ln p^2}{\epsilon} \right. \\ & \quad \left. - \left(\ln(p^2) \chi_0(\gamma) + \frac{\chi_0^2(\gamma) + \chi_0'(\gamma)}{2} + \frac{\pi^2}{6} \right) + \mathcal{O}(\epsilon) \right], \end{aligned} \quad (78)$$

with

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma), \quad (79)$$

the leading order BFKL characteristic function. We therefore find that the proposed subtraction yields indeed a finite result.

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³ This is the correct transverse momentum dependence for an unintegrated gluon distribution and/or a BFKL Green's function.

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