




# Doubly charmed multibaryon systems

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**Abstract** We study two- and three-baryon systems with two units of charm looking for possible bound states or resonances. All two-baryon interactions are consistently derived from a constituent quark model tuned in the light-flavor hadron phenomenology: spectra and interactions. The presence of the heavy quarks makes the two-body interactions simpler than in the light-flavor sector. Our results show a narrow two-body resonance with quantum numbers  $(I, J^P) = (0, 0^+)$ . It is located 6.2 MeV below the  $\Sigma_c \Sigma_c$  threshold and has a width of 4.7 MeV. The foregoing two-body state contributes to generate a  $N \Sigma_c \Sigma_c$  resonance with quantum numbers  $(I, J^P) = (1/2, 1/2^+)$  and a separation energy of 0.2 MeV.

## 1 Introduction

The existence of molecules made of heavy baryons is a hot topic in nowadays hadronic physics [1–7]. The observation in 2017 by the LHCb Collaboration of a doubly charmed baryon [8] increased the interest in exotic states containing pairs of charmed quarks. Right now, the LHCb Collaboration has reported two structures matching the lineshape of a resonance just above twice the  $J/\Psi$  mass, that could originate from a hadron containing two charm quarks [9]. Although the existence of exotic structures containing pairs of heavy quarks is a long-term challenge [10], it has recently been noticed, for example in Refs. [11–13], that doubly charmed tetraquarks are the first to be at the edge of binding.

On general grounds, the main motivation to wonder about the existence of heavy-baryon molecules is rooted in the reduction of the kinetic energy due to larger reduced masses. However, such molecular states could be the concatenation of several effects and not just a fairly attractive interaction. The coupling between nearby channels, conflicts between differ-

ent terms of the interaction, and non-central forces often play a significant role. Some of these contributions may be reinforced by the presence of heavy quarks while others may become weaker [14, 15].

Behind all this there is the understanding of the hadron-hadron interaction governed by the dynamics of quarks and gluons, which is a topical issue. To encourage new experiments and analysis of existing data it is essential to have detailed theoretical investigations. Despite some uncertainty in contemporary interaction models, the possible existence of bound states or resonances is a key element, because their signs might be clear enough to be identified in the experimental data [9]. Thus, it is the purpose of this work to study the possible existence of hadronic molecules or resonances in two- and three-baryon systems with two units of charm, in particular,  $\Lambda_c \Lambda_c$ ,  $\Sigma_c \Sigma_c$ ,  $N \Lambda_c \Lambda_c$  and  $N \Sigma_c \Sigma_c$  states. When tackling this problem, one has to manage with an important difficulty, namely the complete lack of experimental data. Therefore, the generalization of models describing two-hadron interactions in the light-flavor sector could offer insight about the unknown interaction of hadrons with heavy flavors.

Following these ideas, we will make use of a constituent quark model (CQM) tuned on the description of the  $NN$  interaction [16] as well as the meson [17] and baryon [18, 19] spectra in all flavor sectors, to obtain parameter-free predictions that will hopefully be testable in future experiments. Let us note that the study of the interaction between charmed baryons has become an interesting subject in several contexts [9, 20–23] and it may shed light on the possible existence of exotic nuclei with heavy flavors [24–29].

The paper is organized as follows. In Sect. 2 we describe and analyze particular aspects of the  $S$  wave two-body subsystems:  $N \Lambda_c$ ,  $N \Sigma_c$ ,  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$ . Section 3 is devoted to the study of the lightest  $N \Lambda_c \Lambda_c$  and  $N \Sigma_c \Sigma_c$  three-body systems. Finally, in Sect. 4 we summarize our main conclusions.

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**Table 1** CQM results for the  $^1S_0$  and  $^3S_1$  scattering lengths ( $a_s$  and  $a_t$ ) and effective range parameters ( $r_s$  and  $r_t$ ) in fm for the different  $S$  wave  $Y_c N$  and  $Y_c Y_c$  systems ( $Y_c = \Lambda_c$  or  $\Sigma_c$ ). The results shown in the last line, marked by a †, correspond to the uncoupled  $\Sigma_c \Sigma_c$  system

$I$	System	$a_s$	$r_s$	$a_t$	$r_t$
1/2	$\Lambda_c N$	-0.86	5.64	-2.31	2.97
	$\Sigma_c N$	$0.74 - i 0.18$	-	$-5.21 - i 1.96$	-
3/2	$\Sigma_c N$	-1.25	8.70	0.95	4.89
0	$\Lambda_c \Lambda_c$	-6.45	2.29	-	-
	$\Sigma_c \Sigma_c$	$-0.014 + i 0.26$	-	-	-
	$\Sigma_c \Sigma_c^\dagger$	1.79	0.44	-	-

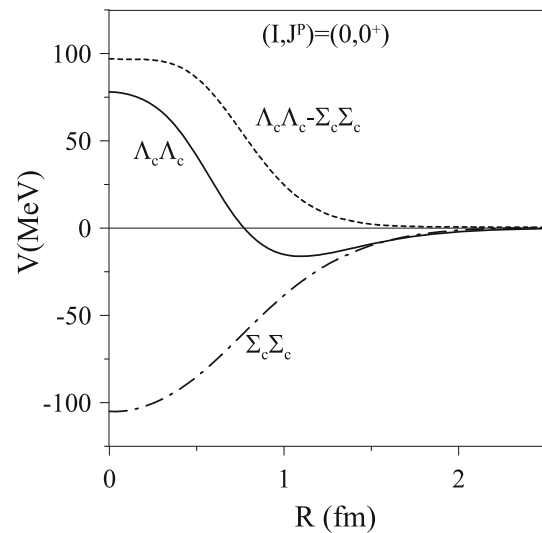
## 2 Two-baryon systems

The two-body interactions that are necessary to study the charm +2 two- and three-baryon systems have been discussed at length in the literature [30,31]. They are derived from the CQM [16–19]. The capability of the model is endorsed by the nice description of the  $NN$  phase shifts, as can be seen in figs. 2, 3 and 4 of Ref. [32]. The  $N\Lambda_c$  and  $N\Sigma_c$  interactions have been presented and discussed in detail in Ref. [30], in comparison with the other approaches available in the literature, in particular recent lattice QCD studies [29]. The  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$  interactions have been consistently derived within the CQM in Ref. [31], also in comparison with the alternative approaches available in the literature. We refer the reader to Refs. [30,31] for a thorough description of the derivation and analysis of the two-body interactions. As can be seen in table 1 of Ref. [30] and table 2 of Ref. [31] all two-body interactions are consistently derived with the same set of parameters. In the following we highlight some peculiarities of the two-body interactions that are relevant to the purpose of the present work.

We summarize in Table 1 the low-energy parameters of the two-body systems in the charm +1 and +2 sectors. The scattering length becomes complex for those two-body channels with open lower mass two-body states. The two-body interactions are in general attractive but not sufficient for having bound states, in agreement with lattice QCD estimations [29]. The singlet isospin 1/2 and triplet isospin 3/2  $\Sigma_c N$  interactions are the only repulsive ones. The last line of Table 1 presents the results for the uncoupled  $\Sigma_c \Sigma_c$  isosinglet system.<sup>1</sup> It can be seen how the scattering length is positive and larger than the range of the interaction, see Fig. 1, pointing to existence of a bound state that will be discussed further below.

Of particular interest are the results for the lightest charm +2 channel, with quantum numbers  $(I, J^P) = (0, 0^+)$ . We

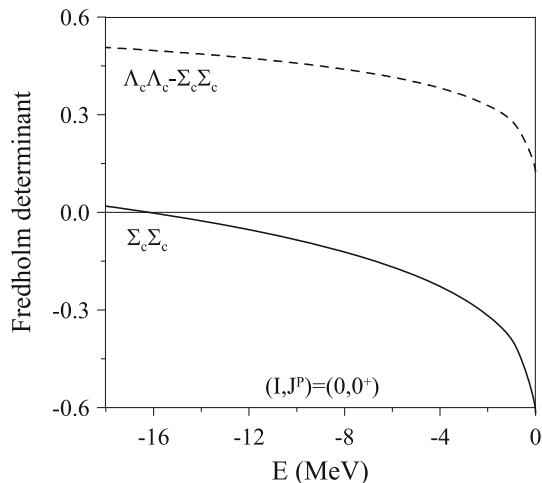
<sup>1</sup> Note that all other  $\Sigma_c \Sigma_c$   $S$  wave states are clearly unbound, see fig. 6 of Ref. [31].



**Fig. 1** Charm +2  $(I, J^P) = (0, 0^+)$  two-body interactions

show in Fig. 1 the two-body potentials involved in this channel. The  $\Lambda_c \Lambda_c$  interaction is slightly attractive at intermediate distances but, however, repulsive at short range. It is decoupled from the closest two-baryon threshold, the  $N\Xi_{cc}$  state [31], which is relevant for the possible existence of a resonance in the strange sector [33,34]. There is a general agreement on the overall attractive character of the  $\Sigma_c \Sigma_c$  interaction [1,2,5]. Finally, the CQM coupling between the  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$  channels is a bit stronger than in hadronic theories, based solely on a one-pion exchange potential [2], due to quark-exchange effects [31]. All of this fits the scenario of the strange sector, as can be seen by comparing with fig. 1b of Ref. [35], but the absence of the one-kaon exchange potential gives rise to a less attractive interaction.

In Fig. 2 we present the Fredholm determinant [36] for the two-body  $(I, J^P) = (0, 0^+)$  charm +2 channel in two different cases. The dashed line corresponds to the result considering the full coupling between the  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$  states, whereas the solid line considers only the  $\Sigma_c \Sigma_c$  channel. The coupled channel calculation shows an attractive character but not sufficient to generate a bound state, the Fredholm determinant does not become negative for energies below threshold. This result is in agreement with other estimations in the literature [1,2,5] in which in spite of the attractive character of the  $\Lambda_c \Lambda_c$  interaction, the central potential alone is not enough to generate a bound state. The coupling to larger mass channels could be important for the existence of a bound state or a resonance. However, due to the large mass difference between the two coupled channels in the  $(I, J^P) = (0, 0^+)$  partial wave, 338 MeV, the coupled channel effect is weakened. Let us just note that, for example, in the strange sector the coupling to the  $N\Sigma$  state is relevant for the  $N\Lambda$  system [37] due to a smaller mass difference,  $M(\Sigma) - M(\Lambda) = 77$  MeV. Heavier mass channels play a minor role, such as the



**Fig. 2** Fredholm determinant of the two-body  $(I, J^P) = (0, 0^+)$  charm +2 channel. The dashed line corresponds to the  $\Lambda_c \Lambda_c - \Sigma_c \Sigma_c$  coupled system, whereas the solid line considers only the  $\Sigma_c \Sigma_c$  channel. The zero energy represents the mass of the lowest threshold,  $2m_{\Lambda_c}$  for the dashed line and  $2m_{\Sigma_c}$  for the solid line

$\Delta\Delta$  channel (584 MeV above the  $NN$  threshold) in the  $NN$  system [38]. Thus, one does not expect higher channels, as it could be  $\Sigma_c^* \Sigma_c^*$  (468 MeV above the  $\Lambda_c \Lambda_c$  threshold) to play a relevant role, as it has been explicitly checked in the literature [5].

Due to the large mass difference between the  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$  channels, we have studied the uncoupled  $\Sigma_c \Sigma_c$  system. The dynamics could be dominated by the attraction in the  $\Sigma_c \Sigma_c$  channel in a way that the  $\Lambda_c \Lambda_c$  channel would be mainly a tool for detection. This mechanism is somewhat related to the ‘synchronization of resonances’ proposed by D. Bugg [39]. A similar situation could be the case of the  $d^*(2380)$  resonance in the  $\Delta\Delta$  system, see Ref. [40] for a recent review. The result is depicted in Fig. 2 by the solid line, showing a bound state 16.2 MeV below the  $\Sigma_c \Sigma_c$  threshold, corresponding to the scattering length given in the last line of Table 1. However, since the  $\Lambda_c \Lambda_c$  channel is open, the  $\Sigma_c \Sigma_c$  state would decay showing a resonance behavior. This scenario, a two-body coupled channel problem showing a bound state in the upper channel but not in the coupled channel calculation has been studied in detail in Ref. [41]. It was demonstrated how the width of the resonance does not come only determined by the available phase space for its decay to the detection channel, but it greatly depends on the relative position of the mass of the resonance with respect to the masses of the coupled-channels generating the state.<sup>2</sup>

<sup>2</sup> The equivalence of the results obtained using a two-cluster interaction or a variational approach for the multiquark problem has been recently shown, see for example in Ref. [42]. Dealing with resonances, the two-cluster interaction allows to look for the poles of the propagator without resorting to numerical extensions of the variational approach, like the

Thus, making use of the interactions given in Fig. 1, we have studied the width of the resonance produced in between the two thresholds,  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$ . The Lippmann–Schwinger equation in the case of  $S$ -wave interactions is written as,

$$t^{ij}(p, p'; E) = v^{ij}(p, p') + \sum_{k=1,2} \int_0^\infty p''^2 dp'' \times V^{ik}(p, p'') \frac{1}{E - \Delta E \delta_{2,k} - p''^2/2\mu_k + i\epsilon} \times t^{kj}(p'', p'; E), \quad i, j = 1, 2, \tag{1}$$

where  $\mu_1 = m_{\Lambda_c}/2$ ,  $\mu_2 = m_{\Sigma_c}/2$ , and  $\Delta E = 2m_{\Sigma_c} - 2m_{\Lambda_c}$ . The interactions in momentum space are given by,

$$V^{ij}(p, p') = \frac{2}{\pi} \int_0^\infty r^2 dr j_0(pr) V^{ij}(r) j_0(p'r), \tag{2}$$

where  $V^{ij}(r)$  are the two-body potentials in Fig. 1. The resonance exists at an energy  $E = E_R$  such that the phase shift  $\delta(E_R) = 90^\circ$ , for energies between the  $\Lambda_c \Lambda_c$  and  $\Sigma_c \Sigma_c$  thresholds, i.e.,  $0 < E_R < \Delta E$ . The mass of the resonance is given by  $W_R = E_R + 2m_{\Lambda_c}$ . The width of the resonance is calculated using the Breit–Wigner formula as [43–45],

$$\Gamma(E) = \lim_{E \rightarrow E_R} \frac{2(E_R - E)}{\cotg[\delta(E)]}. \tag{3}$$

Although the Breit–Wigner formula is not very accurate close to threshold, however, we have explicitly checked by analytic continuation of the  $S$  matrix on the second Riemann sheet that at low energy the width follows the expected  $\Gamma \sim E^{1/2}$  behavior.

Using the formalism described above we have calculated the width of the  $\Sigma_c \Sigma_c$  state. We found a resonance 331.8 MeV above the  $\Lambda_c \Lambda_c$  threshold, 6.2 MeV below the  $\Sigma_c \Sigma_c$  threshold, with a width of 4.7 MeV. As a consequence of the coupling to the lower  $\Lambda_c \Lambda_c$  channel, the pole approaches the threshold moving from  $-16.2$  MeV in the real axis to the complex plane,  $(-6.2 - i 4.7/2)$  MeV. The mechanism we have discussed helps in understanding the narrow width of experimental resonances found in the heavy hadron spectra with a large phase space in the decay channel. The observation of a small width for the decay to a low-lying channel could thus point to a dominant contribution of some upper channel to the formation of the resonance.

### 3 The three-baryon system

The  $\Lambda_c \Lambda_c - \Sigma_c \Sigma_c$  system in a pure  $S$  wave configuration has quantum numbers  $(I, J^P) = (0, 0^+)$  so that adding

complex scaling method, that would just give an indication about the possible existence of a resonance.

one more nucleon, the  $N\Lambda_c\Lambda_c$  system has necessarily quantum numbers  $(I, J^P) = (1/2, 1/2^+)$ . It is coupled to the  $N\Sigma_c\Sigma_c$  channel. A detailed description of the Faddeev equations of the three-body system can be found in Ref. [46]. It has been explained how to deal with coupled channels containing identical particles of various types in the upper and lower channels. We show in Table 2 the different two-body channels that contribute to the  $N\Lambda_c\Lambda_c - N\Sigma_c\Sigma_c$   $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$  three-body system. Notice that the charm +2  $S$  wave channels  $\Lambda_c\Sigma_c$  and  $\Sigma_c\Sigma_c$  with isospin 1 are not considered since they do not couple to the isosinglet  $\Lambda_c\Lambda_c$  two-body subsystem. Therefore, the Faddeev equations of the  $(I, J^P) = (1/2, 1/2^+)$  three-body system are of the form,

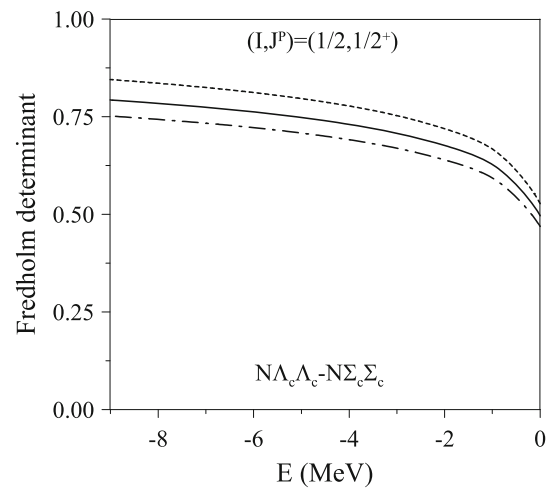
$$\begin{aligned}
 T_{N\Lambda_c}^{\Lambda_c} &= -t_{N\Lambda_c}^{\Lambda_c} G_0 T_{N\Lambda_c}^{\Lambda_c} \\
 &\quad + 2t_{N\Lambda_c}^{\Lambda_c} G_0 T_{\Lambda_c\Lambda_c}^N \\
 &\quad + t_{N\Lambda_c-N\Sigma_c}^{\Lambda_c} G_0 T_{N\Lambda_c}^{\Sigma_c} \\
 T_{N\Lambda_c}^{\Sigma_c} &= t_{N\Lambda_c}^{\Sigma_c} G_0 T_{N\Lambda_c}^{\Lambda_c} \\
 &\quad + 2t_{N\Lambda_c-N\Sigma_c}^{\Sigma_c} G_0 T_{\Sigma_c\Sigma_c}^N \\
 &\quad + t_{N\Lambda_c-N\Sigma_c}^{\Sigma_c} G_0 T_{N\Lambda_c}^{\Sigma_c} \\
 T_{N\Sigma_c}^{\Lambda_c} &= t_{N\Sigma_c}^{\Lambda_c} G_0 T_{N\Lambda_c}^{\Sigma_c} \\
 &\quad + 2t_{N\Sigma_c-N\Lambda_c}^{\Lambda_c} G_0 T_{\Lambda_c\Lambda_c}^N \\
 &\quad + t_{N\Sigma_c-N\Lambda_c}^{\Lambda_c} G_0 T_{N\Lambda_c}^{\Lambda_c} \\
 T_{N\Sigma_c}^{\Sigma_c} &= -t_{N\Sigma_c}^{\Sigma_c} G_0 T_{N\Sigma_c}^{\Sigma_c} \\
 &\quad + 2t_{N\Sigma_c}^{\Sigma_c} G_0 T_{\Sigma_c\Sigma_c}^N \\
 &\quad + t_{N\Sigma_c-N\Lambda_c}^{\Sigma_c} G_0 T_{N\Lambda_c}^{\Lambda_c} \\
 T_{\Lambda_c\Lambda_c}^N &= t_{\Lambda_c\Lambda_c}^N G_0 T_{N\Lambda_c}^{\Lambda_c} \\
 &\quad + t_{\Lambda_c\Lambda_c-\Sigma_c\Sigma_c}^N G_0 T_{N\Sigma_c}^{\Sigma_c} \\
 T_{\Sigma_c\Sigma_c}^N &= t_{\Sigma_c\Sigma_c}^N G_0 T_{N\Sigma_c}^{\Sigma_c} \\
 &\quad + t_{\Sigma_c\Sigma_c-\Lambda_c\Lambda_c}^N G_0 T_{N\Lambda_c}^{\Lambda_c}, \tag{4}
 \end{aligned}$$

where  $t_{ij}^k$  are the two-body  $t$ -matrices that already contain the coupling among all two-body channels contributing to a given three-body state, see Table 2.  $G_0$  is the propagator of three free particles. The superscript of the amplitudes indicates the spectator particle and the subscript the interacting pair.

The results are presented in Fig. 3. We have performed three different calculations. First, we have included the three-body amplitudes of the first two lines of Table 2 that do not contain the  $\Sigma_c$  baryon, the result being depicted by the dotted line. As it could have been expected, there exists attraction due to the attractive character of the  $N\Lambda_c$  and  $\Lambda_c\Lambda_c$  interactions discussed in Sect. 2. However, the attraction is not sufficient for having a bound state. Then, we have included the amplitudes containing the  $\Sigma_c\Sigma_c$  two-body subsystem, third

**Table 2**  $S$  wave two-body channels  $(i, j)$  of the various subsystems that contribute to the  $N\Lambda_c\Lambda_c - N\Sigma_c\Sigma_c$   $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$  three-body system

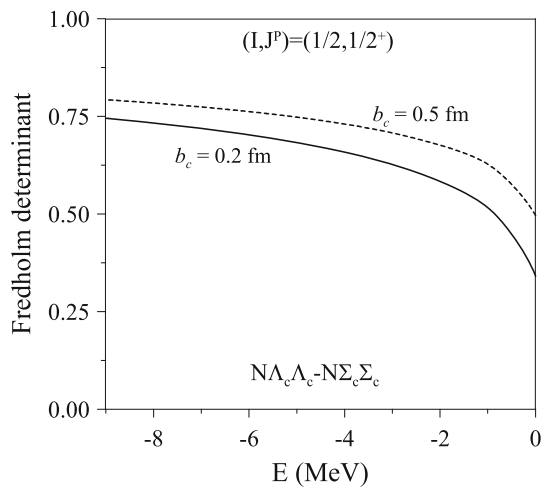
Two-body subsystem	Spectator	$(i, j)$
$\Lambda_c\Lambda_c$	$N$	$(0,0)$
$N\Lambda_c$	$\Lambda_c$	$(\frac{1}{2}, 0), (\frac{1}{2}, 1)$
$\Sigma_c\Sigma_c$	$N$	$(0,0), (1,1)$
$N\Lambda_c$	$\Sigma_c$	$(\frac{1}{2}, 0), (\frac{1}{2}, 1)$
$N\Sigma_c$	$\Lambda_c$	$(\frac{1}{2}, 0), (\frac{1}{2}, 1)$
$N\Sigma_c$	$\Sigma_c$	$(\frac{1}{2}, 0), (\frac{1}{2}, 1), (\frac{3}{2}, 0), (\frac{3}{2}, 1)$



**Fig. 3** Fredholm determinant of the  $N\Lambda_c\Lambda_c - N\Sigma_c\Sigma_c$   $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$  three-body state. See text for details

line in Table 2, and all isospin 1/2 three-body amplitudes containing a  $\Sigma_c$  baryon either as spectator or as a member of the interacting pair, lines 4 to 6 of Table 2. The result corresponds to the dashed-dotted line. The coupled channel effect induces some additional attraction, reducing the value of the Fredholm determinant. Finally, we added the isospin 3/2  $N\Sigma_c$  amplitudes indicated in the last line of Table 2, the result is depicted by the solid line. Although the singlet amplitudes increase the attraction, the repulsive triplet isospin 3/2  $N\Sigma_c$  amplitude, discussed in Sect. 2, induces an overall repulsion in the three-body system, increasing the value of the Fredholm determinant.

We have studied the dependence of the results on the only free parameter of the interacting potential, the Gaussian size parameter of the charm quark wave function,  $b_c$ . It has been illustrated in fig. 4 of Ref. [30] how the central  $Y_c N$  interactions become more attractive for smaller values of  $b_c$ . However, the weakening of the non-central contributions generates a slightly less attractive systems in the presence of coupled-channel effects, see right panel of fig. 5 of Ref. [31]. It was argued in Ref. [47] that the smaller values



**Fig. 4** Fredholm determinant of the  $N\Lambda_c\Lambda_c - N\Sigma_c\Sigma_c$   $(I, J^P) = (\frac{1}{2}, \frac{1}{2}^+)$  three-body state for two different values of the Gaussian size parameter of the charm quark wave function,  $b_c$

of  $b_c$  are preferred to get consistency with calculations based on infinite expansions, as hyperspherical harmonic expansions [48], where the quark wave function is not postulated. This also agrees with simple harmonic oscillator relations  $b_c = b_n \sqrt{\frac{m_n}{m_c}}$ ,<sup>3</sup> leading to the best suited value  $b_c = 0.2$  fm for the study of the charm sector [47]. Thus, in Fig. 4 we show the Fredholm determinant for two different values of  $b_c$ , 0.5 and 0.2 fm. As can be seen, the attraction increases for smaller values of  $b_c$ , the effect not being enough to generate a bound state. Such tiny contributions might well be of importance for states at the edge of binding, as discussed in the following.

Guided by the resonance found in the  $\Sigma_c\Sigma_c$  system, we have finally studied the  $N\Sigma_c\Sigma_c$  system without coupling to  $N\Lambda_c\Lambda_c$ , looking for a three-body resonance. The results are promising if the triplet isospin 3/2 amplitude is not considered. Thus, considering only the isospin 1/2 amplitudes we obtain a bound state with a separation energy of 0.6 MeV. If the singlet isospin 3/2 amplitude is included, the separation energy increases to 0.7 MeV. If the repulsive triplet isospin 3/2 ( $N\Sigma_c$ ) $\Sigma_c$  amplitude is considered, the signal of the resonance is lost. However, adopting the best suited value for the charm sector,  $b_c = 0.2$  fm, the  $N\Sigma_c\Sigma_c$  three-body resonance is still there with a separation energy of 0.2 MeV.

## 4 Summary

In short, we have studied two- and three-baryon systems with two units of charm looking for possible bound states or reso-

nances. All two-baryon interactions are consistently derived from a constituent quark model tuned in the light-flavor hadron phenomenology. Our results show a narrow two-body resonance with quantum numbers  $(I, J^P) = (0, 0^+)$ . It is located 6.2 MeV below the  $\Sigma_c\Sigma_c$  threshold and has a width of 4.7 MeV. A detailed study of the coupled  $N\Lambda_c\Lambda_c - N\Sigma_c\Sigma_c$  three-body system as well as the uncoupled  $N\Sigma_c\Sigma_c$  system shows that the foregoing two-body state contributes to generate a  $N\Sigma_c\Sigma_c$  resonance with quantum numbers  $(I, J^P) = (1/2, 1/2^+)$  and a separation energy of 0.2 MeV.

Weakly bound states and resonances are usually very sensitive to potential details and therefore theoretical investigations with different phenomenological models are highly desirable. The existence of these states could be scrutinized in the future at the LHC, J-PARC and RHIC providing a great opportunity for extending our knowledge to some unreach part in our matter world.

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**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors' comment: There is no data since this paper deals with theoretical aspects.]

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<sup>3</sup>  $b_n$  and  $m_n$  refer to the light quarks, they are given in table 1 of Ref. [30] and table 2 of Ref. [31]



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