Nonleptonic and semileptonic $\Lambda_b \rightarrow \Lambda_c$ transitions in a potential quark model

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Abstract Nonleptonic and semileptonic decay widths of $\Lambda_b \rightarrow \Lambda_c$ are analyzed within heavy quark limit and Isgur-Wise formalism. A modified QCD Cornell interaction with the additional logarithmic term in the hyperspherical coordinates is considered and the masses of heavy flavour baryons are calculated. The obtained masses are consequently employed to study the rates of $\Lambda_b \rightarrow \Lambda_c$. The achieved results are motivating.

1 Introduction

In recent years a huge number of both experimental and theoretical studies on properties of heavy baryons and the masses of ground states of charmed and bottom baryons containing a single heavy quark b or c have been published. Many decay channels of the bottom baryons have been observed due to the operation of LHC and plenty of data on heavy baryons have been collected. Weak decays of singly bottom baryons to final states involving singly charmed baryons are very interesting since they can help to understand the dynamic structure of heavy hadrons based on heavy quark effective theory (HQET) [1, 2]. HQET is an effective tool for studying weak decays of heavy baryons. Λ_b can only decay via weak interactions. It is interesting and worthy to study Λ_b because, besides study on the heavy baryon structure, it may help determine the CKM matrix element V_{cb} and also baryons containing heavy flavours b or c can play a significant role in the understanding of QCD. In the heavy quark limit, the hadronic matrix element of transition $\Lambda_b \rightarrow \Lambda_c$ can be expressed by a single Isgur-Wise function which can be defined as [3–5]

$$\langle \Lambda_c(v') | \bar{c} \Gamma b | \Lambda_b(v) \rangle = \eta(\omega) \bar{u}_{\Lambda_c}(v') \Gamma u_{\Lambda_b}(v)$$

where Γ is an arbitrary gamma matrix and ω denotes the dot product of four-velocity of initial and final baryons. There have been useful attempts to analyze the nonleptonic decays of $\Lambda_b \rightarrow \Lambda_c$ using the soft-collinear effective theory [6], in the light-front quark model [7, 8], based on HQET [9–11] and in the relativistic three-quark model [12]. Zhao obtained the transition form factors of the singly heavy baryons using the light-front approach under the diquark picture [13]. Faustov and Galkin studied semileptonic Λ_b decays in a relativistic quark model. They obtained wave functions of the ground, excited baryon states and the ratio of the Cabibbo–Kobayashi–Maskawa matrix elements [14]. Barik et al calculated static properties of baryons based on the Dirac equation with a logarithmic confining potential [15]. The hypercentral constituent quark model with color Coulomb plus power potential has been used to calculate the mass spectra of heavy flavour baryons [16–18].

In the next section, we present the Hamiltonian of a three body system including a modified Cornell interaction as effective interaction between quarks and calculate the masses of baryons. In Sect. 3 we study the two body nonleptonic decays $\Lambda_b \rightarrow \Lambda_c^+ P^-$ and $\Lambda_b \rightarrow \Lambda_c^+ V^-$ where P and V stand for pseudoscalar and vector mesons respectively. In Sect. 4 semileptonic decay of $\Lambda_b \rightarrow \Lambda_c^+$ is discussed and finally, we present a conclusion.

2 General framework

To describe the baryon as a bound state of three quarks, the configuration of three particles are defined by two Jacobi vectors, ρ and λ . These Jacobi vectors can be written as

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2), \vec{\lambda} = \sqrt{\frac{2}{3}} \left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} - \vec{r}_3 \right). \quad (1)$$

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One can employ the hyperspherical coordinates which are given by the angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$ with the hyperradius, x , and hyperangle ζ given by

$$x = \sqrt{\rho^2 + \lambda^2}, \zeta = \arctan\left(\frac{\rho}{\lambda}\right) \quad (2)$$

Therefore, the Hamiltonian of the three-body system which contains three body effects can be expressed as [16–18]

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{P_x^2}{2\mu} + V(x) \quad (3)$$

such that

$$m_\rho = \frac{2m_1m_2}{m_1+m_2}, m_\lambda = \frac{3m_3(m_1+m_2)}{2(m_1+m_2+m_3)} \quad (4)$$

where μ is the reduced mass and the kinetic energy operator takes the form

$$\frac{P_x^2}{2\mu} = \frac{-1}{2\mu} \left(\frac{\partial^2}{\partial x^2} + \frac{5}{x} \frac{\partial}{\partial x} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \xi)}{x^2} \right) \quad (5)$$

using hyperspherical coordinates. We can write the relation of the eigenvalue of L^2 as [19]

$$L^2(\Omega_\rho, \Omega_\lambda, \zeta) Y_{[\gamma], l_\rho, l_\lambda}(\Omega_\rho, \Omega_\lambda, \zeta) = \gamma(\gamma+4) Y_{[\gamma], l_\rho, l_\lambda}(\Omega_\rho, \Omega_\lambda, \zeta) \quad (6)$$

Here, the grand-angular momentum $\gamma = 2n + l_\rho + l_\lambda$ with $n = 0, 1$, and l_ρ, l_λ are the angular momenta associated with the ρ and λ Jacobi vectors.

We have chosen the following logarithmic-type potential [20] which is the effective potential in which a quark is influenced

$$V(x) = -c_F \frac{\alpha_s}{x} + \frac{5c_F n_f}{18\pi} \frac{\alpha_s^2}{x} + \frac{c_F n_f}{3\pi} \frac{\alpha_s^2}{x} \log(\gamma \mu x) + \frac{c_F n_f}{2\pi} \mu^2 \alpha_s^2 x + V_0 \quad (7)$$

where c_F is the color factor which is $\frac{2}{3}$ for the baryon, n_f is the number of flavors and γ is the Euler constant. Equation (7) has Coulomb potential and also confinement concept. We have plotted the behavior of the considered potential, Eq. (7), and Cornell potential $V(x) = -\frac{\beta}{x} + \alpha x$ by Refs. [18, 21] in Fig. 1. In fact Eq. (7) is a Cornell potential modified by a logarithmic function. The use of logarithmic potential was reported by several references [15, 22–24] to study hadron properties. The logarithmic term can guarantee quark confinement and also can generate charmonium and bottomonium bound-state masses in reasonable agreement with experiments [22]. The strong running coupling constant is [16]

$$\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \frac{33-2n_f}{12\pi} \alpha_s(\mu_0) \ln\left(\frac{\mu}{\mu_0}\right)} \quad (8)$$

Fig. 1 The behavior of the potential Eq. (7), Cornell potentials $V_2(x)$ of Ref. [21] and $V_3(x)$ of Ref. [18] $V_1(x) = -c_F \frac{\alpha_s}{x} + \frac{5c_F n_f}{18\pi} \frac{\alpha_s^2}{x} + \frac{c_F n_f}{3\pi} \frac{\alpha_s^2}{x} \log(\gamma \mu x) + \frac{c_F n_f}{2\pi} \mu^2 \alpha_s^2 x + V_0$, $V_2(x) = -\frac{2\alpha_s}{3x} + 0.03x - \frac{4\alpha_s}{3x} + 0.03x$, $V_3(x) = -\frac{6.39}{x} + 0.148x$

in terms of $\alpha_s(\mu_0 = 1 \text{ GeV}) = 0.6$. Using virial theorem, the wave function is taken as the hyper Coulomb radial wave function given by [16]

$$\psi_{n,\gamma}(g, x) = N (2gx)^\gamma e^{-gx} L_n^{2\gamma+4}(2gx) \quad (9)$$

where N is the normalization constant and $L_n^{2\gamma+4}(2gx)$ represents the Laguerre polynomial. By using

$$\frac{\partial}{\partial g} \langle \psi_{n,\gamma}(g, x) | H | \psi_{n,\gamma}(g, x) \rangle = 0, \quad (10)$$

we obtain the variation parameters, g , for baryons as shown in the second column of Table 1. The variational method is one of the ways of solving the radial Schrödinger equation and finding eigenenergies. In this method, one can choose a trial wave function which depends on one or more adjustable parameters known as variation parameter. In our work g is this parameter which can be obtained by minimizing the expectation value of the Hamiltonian and we find an approximated value for the energy. In the variational theorem because the trial energy ($E_{\text{trial}} = \langle \psi_{n,\gamma}(g, x) | H | \psi_{n,\gamma}(g, x) \rangle$) is always larger than or equal to the actual energy, we can minimize the trial energy by taking the derivative with respect to g , setting it equal to zero and solving for g as in Eq. (10). From Eqs. (9) and (10), we can see that g depends on quantum numbers and input quark masses. Thus, fixing the quantum numbers as $n = 1$ and $\gamma = 0$, g has different values for

Table 1 Masses of some heavy baryons

Baryon (Quark content)	g	Our $J^P = \frac{1}{2}^+$	[35]	[16]	Our $J^P = \frac{3}{2}^+$	[16]
$\Lambda_b^0(udb)$	0.506	5.641	5.619	5.646	–	–
$\Lambda_c^+(udc)$	0.496	2.311	2.286	2.316	–	–
$\Omega_b^-(ssb)$	0.824	6.091	6.046	6.005	6.092	6.058
$\Omega_c^0(ssc)$	0.559	2.739	2.695	2.730	2.741	2.749
$\Xi_c^0(dsc)$	0.526	2.524	2.470	2.524	2.527	2.673
$\Xi_b^-(dsb)$	0.539	5.864	5.797	5.887	5.865	5.943
$\Xi_b^0(usb)$	0.536	5.845	5.791	5.872	5.846	5.928
$\Xi_c^+(usc)$	0.523	2.507	2.467	2.514	2.509	2.665

different baryons. For instance g will be equal to 0.506 and 0.496 for Λ_b and Λ_c respectively. The baryon mass is taken to be

$$M_B = \sum_i m_i + \langle H \rangle + \sum_{i < j} \frac{16\pi\alpha_s}{9m_i m_j} \left(\psi_0 | \delta^3(\vec{r}) | \psi_0 \right) (\vec{s}_i \cdot \vec{s}_j) \quad (11)$$

where $\vec{s}_i$ and $\vec{s}_j$ are the spin operators of the i th and j th quark respectively. For $J^P = \frac{1}{2}^+$ we have used $\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{4}$, $\vec{s}_1 \cdot \vec{s}_3 = \vec{s}_2 \cdot \vec{s}_3 = -\frac{1}{4}$ and also used $\vec{s}_1 \cdot \vec{s}_2 = \vec{s}_1 \cdot \vec{s}_3 = \vec{s}_2 \cdot \vec{s}_3 = \frac{1}{4}$ for $J^P = \frac{3}{2}^+$ [25]. ψ_0 is the baryonic wave function. Input values of quark masses are $m_u = 0.338$ GeV, $m_d = 0.350$ GeV, $m_s = 0.500$ GeV, $m_c = 1.700$ GeV and $m_b = 4.510$ GeV [16].

3 Nonleptonic decays of $\Lambda_b \rightarrow \Lambda_c$

Using HQET with factorization method [11], one can obtain the decay widths for the decay processes $\Lambda_b \rightarrow \Lambda_c^+ P^-$ and $\Lambda_b \rightarrow \Lambda_c^+ V^-$ given as [11]

$$\begin{aligned} \Gamma(\Lambda_b(v) \rightarrow \Lambda_c^+(v') P^-(p)) \\ = \frac{G_F^2}{8\pi M_{\Lambda_b}^2} |V_{UD}^* V_{cb}|^2 C_1^2(m_b) f_P^2 \eta^2(v, v') |\vec{p}| \\ \times \left((M_{\Lambda_b}^2 - M_{\Lambda_c}^2)^2 - M_P^2 (M_{\Lambda_b}^2 + M_{\Lambda_c}^2) \right) \end{aligned} \quad (12)$$

and

$$\begin{aligned} \Gamma(\Lambda_b(v) \rightarrow \Lambda_c^+(v') V^-(p)) \\ = \frac{G_F^2}{8\pi M_{\Lambda_b}^2} |V_{UD}^* V_{cb}|^2 C_1^2(m_b) f_V^2 \eta^2(v, v') |\vec{p}| \\ \times \left((M_{\Lambda_b}^2 - M_{\Lambda_c}^2)^2 + M_V^2 (M_{\Lambda_b}^2 + M_{\Lambda_c}^2 - 2M_V^2) \right) \end{aligned} \quad (13)$$

where G_F is the Fermi coupling constant and $|\vec{p}|$ is the center of mass momentum of the emitted particles that can be

obtained from $\sqrt{\left(\frac{M_{\Lambda_b}^2 - M_{\Lambda_c}^2}{2M_{\Lambda_b}}\right)^2 - M_\pi^2}$. Since U and D are either c , s or u , d quarks, for $U, D = u, d$ we have π^- and ρ^- in the final state. If $U, D = c, s$ the final P/V states are D_s/D_s^* . Moreover for $U, D = c, d$ and $U, D = u, s$ the final states will be the P mesons, D^- and K^- . The value of Wilson coefficient C_1 is $C_1(m_b) = 1.11$. We assume the IWF as [26]

$$\eta(v, v') = \exp \left(-3(v \cdot v' - 1) \frac{m_c^2}{\Lambda_B^2} \right). \quad (14)$$

The product is determined by considering the momentum conservation of two body decays [11]

$$v \cdot v' = \frac{M_{\Lambda_b}^2 + M_{\Lambda_c}^2 - M_{P/V}^2}{2M_{\Lambda_b} M_{\Lambda_c}} \quad (15)$$

Different parameterizations have been proposed for the baryonic IWF $\eta(v, v')$ such as linear, pole-type and exponential-type [27–33]. To parametrize the baryonic IW function, Ref. [3] also considered an exponential form as

$$\eta(\omega) = \eta(1) \exp \left(-\rho^2(\omega - 1) \right) \quad (16)$$

where $\eta(1) = 1$ due to the normalization of IWF at zero recoil point and $\rho^2 = 1.35$ [3]. We report our values in Table 2 for nonleptonic decays of $\Lambda_b \rightarrow \Lambda_c^+$.

4 Semileptonic decays of $\Lambda_b \rightarrow \Lambda_c$

The differential semileptonic decay rate of $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ is given as [6]

$$\begin{aligned} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu})}{d\omega} = \frac{G_F^2 M_{\Lambda_b}^5 |V_{cb}|^2}{24\pi^3} r_\Lambda^3 \sqrt{\omega^2 - 1} [6\omega \\ + 6\omega r_\Lambda^2 - 4r_\Lambda - 8r_\Lambda \omega^2] \eta^2(\omega) \end{aligned} \quad (17)$$

in the physical and kinematical ω range (about 1–1.42) where $r_\Lambda = \frac{M_{\Lambda_c}}{M_{\Lambda_b}}$. With the calculated masses of Λ_b and Λ_c by Eq.

(11) and IWF as Eq. (14), the decay rate then can be obtained to be $\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}) = 2.88 \times 10^{-14} \text{ GeV}$. By considering IWF in MIT bag model as [30]

$$\eta(\omega) = \left(\frac{2}{\omega + 1} \right)^{3.5 + \frac{1.2}{\omega}} \quad (18)$$

we find the semileptonic decay width of $\Lambda_b \rightarrow \Lambda_c$ as $2.70 \times 10^{-14} \text{ GeV}$ and branching ratio of this decay as 6.03 %. We have also reported the partial semileptonic decay rates of $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ in Table 4. It involves three cases for IWF, Eqs. (14), (16) and (18).

5 Results and discussion

In conclusion, we have presented a model based on Cornell potential plus logarithmic term and calculated semileptonic and nonleptonic decay properties of $\Lambda_b \rightarrow \Lambda_c$ using HQET. $V_0 = -0.244 \text{ GeV}$ was fitted by experimental masses of bottom baryons and we get $V_0 = -0.746 \text{ GeV}$ by fitting experimental masses of charmed ones. Input parameters are listed as follows. The decay constants are from Ref. [34]. The masses of mesons, CKM matrix elements and lifetime of Λ_b

are taken from PDG [35].

$$\begin{aligned} V_{ud} &= 0.97420, V_{cb} = 0.0422, V_{cs} = 0.997, V_{cd} = 0.218, \\ V_{us} &= 0.2243, f_\pi = 0.130 \text{ GeV}, \\ f_{D_s} &= f_{D_s^*} = 0.257 \text{ GeV}, f_{D^\pm} = 0.203 \text{ GeV}, \\ f_K &= 0.156 \text{ GeV}, f_\rho = 0.210 \text{ GeV}, \tau_{\Lambda_b} = 1.471 \text{ ps}, \\ M_{D_s} &= 1.968 \text{ GeV}, M_{D^\pm} = 1.869 \text{ GeV}, \\ M_{K^\pm} &= 0.493 \text{ GeV}, M_\rho = 0.775 \text{ GeV}, \\ M_{D_s^*} &= 2.112 \text{ GeV} \text{ and } M_{\pi^\pm} = 0.139 \text{ GeV}. \end{aligned}$$

Taking cut off parameter $\Lambda_B = 2.12 \text{ GeV}$ in IWF leads to successful results. In particular, a good description of the experimental data for $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$ and $\Lambda_b \rightarrow \Lambda_c^+ D^-$ are obtained. We have also computed and plotted the root mean square deviations of the predicted branching ratios of $\Lambda_b \rightarrow \Lambda_c$ nonleptonic decays of Table 2 considering experimental data. As we have shown in Fig. 2. the standard deviation drops to $\sigma = 0.138$ at $\Lambda_B = 2.12 \text{ GeV}$. This parameter, Λ_B , has been taken different values such as 2.408 GeV in Ref. [36], 2.5 GeV in Ref. [4], 3.037 and 2.408 GeV [12] and in the case of double heavy baryons 2.5 and 3.5 GeV [26].

We have shown the ground states of mass spectrum for some heavy baryons in Table 1 and compared our results with Refs. [16, 32]. In the second column of Table 2 we have shown our calculated branching ratios of nonleptonic decays of $\Lambda_b \rightarrow \Lambda_c$ and compared them with Refs. [9–11, 35, 37]. The second and the third columns of Table 3 show our results for Λ_b semileptonic decay parameters regarding to the exponential type of IWF and IWF in MIT bag model respectively which are compared with Refs. [3, 7, 14, 31, 37]. In Table 4, the values of partial decay width of $\Lambda_b \rightarrow \Lambda_c$ are well compatible with Refs. [38, 39].

Ke et al in Ref. [7] calculated the widths of two body nonleptonic $\Lambda_b \rightarrow \Lambda_c M$ as $0.261 \times 10^{10} \text{ s}^{-1}$ for $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$, $0.769 \times 10^{10} \text{ s}^{-1}$ for $\Lambda_b \rightarrow \Lambda_c^+ \rho^-$, $0.0209 \times 10^{10} \text{ s}^{-1}$ for $\Lambda_b \rightarrow \Lambda_c^+ K^-$. We have obtained $0.290 \times 10^{10} \text{ s}^{-1}$, $0.805 \times 10^{10} \text{ s}^{-1}$ and $0.0222 \times 10^{10} \text{ s}^{-1}$ for the mentioned decays respectively which are in good agreement with their results. Also, we have obtained $\Lambda_b \rightarrow \Lambda_c^+ D_s^- = 0.912 \times 10^{10} \text{ s}^{-1}$, $\Lambda_b \rightarrow \Lambda_c^+ D_s^{*-} = 1.182 \times 10^{10} \text{ s}^{-1}$ and $\Lambda_b \rightarrow \Lambda_c^+ D^- = 0.0293 \times 10^{10} \text{ s}^{-1}$ which are also in good agreement with

Fig. 2 Variation of standard deviation with parameter Λ_B

Table 2 Branching ratios of nonleptonic decays of $\Lambda_b \rightarrow \Lambda_c$

Decay	Ours (with Eq. 14)	Ours (with Eq. 16)	Ours (with Eq. 18)	[11]	[9,10]	[35]	[37]
$\Lambda_b \rightarrow \Lambda_c^+ \pi^-$	0.4268	0.6978	0.4126	0.342	2.0	0.49	0.375
$\Lambda_b \rightarrow \Lambda_c^+ D_s^-$	1.3417	1.8485	1.2347	1.156	6.5	1.10	1.140
$\Lambda_b \rightarrow \Lambda_c^+ D^-$	0.0431	0.0603	0.0397	–	–	0.046	0.050
$\Lambda_b \rightarrow \Lambda_c^+ K^-$	0.0326	0.0528	0.0314	–	–	0.0359	0.030
$\Lambda_b \rightarrow \Lambda_c^+ \rho^-$	1.1843	1.8868	1.1326	0.954	2.5	–	0.673
$\Lambda_b \rightarrow \Lambda_c^+ D_s^{*-}$	1.7390	2.3341	1.5961	1.769	4.7	–	0.996

Table 3 Comparison of theoretical predictions for the Λ_b semileptonic decay parameters

$\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$	Ours (with Eq. 14)	Ours (with Eq. 16)	Ours (with Eq. 18)	[7]	[31]	[3]	[14]	[37]
$\Gamma (\times 10^{10} s^{-1})$	4.38	5.59	4.10	4.22	4.88			
$\Gamma (\times 10^{-11} V_{cb} ^2 \text{ GeV})$	1.62	2.06	1.51			2.12		
Br	6.45	8.22	6.03		7.1		6.48	6.3

Table 4 Partial decay rates for semileptonic decay of $\Lambda_b \rightarrow \Lambda_c, \Gamma$ (in $10^{10} s^{-1}$)

$\omega_{\max}$	Γ (with Eq. 14)	Γ (with Eq. 16)	Γ (with Eq. 18)	Γ [38]	Γ [39]
1.1	0.99	1.05	0.95	$0.91^{+14.4}_{-11.2}$	1.09
1.15	1.63	1.80	1.55	$1.56^{+40}_{-28.8}$	1.83
1.20	2.26	2.57	2.13	$2.24^{+8}_{-6.4}$	2.57
1.25	2.84	3.33	2.66	$2.88^{+14.4}_{-11.2}$	3.26
1.30	3.37	4.05	3.14	$3.52^{+2.2}_{-1.9}$	3.87

the values of Ref. [7] which reported as $\Lambda_b \rightarrow \Lambda_c^+ D_s^- = 0.927 \times 10^{10} s^{-1}$, $\Lambda_b \rightarrow \Lambda_c^+ D_s^{*-} = 1.403 \times 10^{10} s^{-1}$ and $\Lambda_b \rightarrow \Lambda_c^+ D^- = 0.0355 \times 10^{10} s^{-1}$. Furthermore, our results are in agreement with the branching ratios of Chua [8] $\Lambda_b \rightarrow \Lambda_c^+ \pi^-: 4.19^{+1.94}_{-1.44} \times 10^{-3}$, $\Lambda_b \rightarrow \Lambda_c^+ K^-: 0.32^{+0.15}_{-0.11} \times 10^{-3}$, $\Lambda_b \rightarrow \Lambda_c^+ D^-: 0.53^{+0.32}_{-0.22} \times 10^{-3}$, $\Lambda_b \rightarrow \Lambda_c^+ D_s^-: 13.58^{+8.15}_{-5.63} \times 10^{-3}$, $\Lambda_b \rightarrow \Lambda_c^+ \rho^-: 12.39^{+5.79}_{-4.28} \times 10^{-3}$, $\Lambda_b \rightarrow \Lambda_c^+ D_s^{*-}: 16.33^{+7.94}_{-5.81} \times 10^{-3}$.

It is interesting to be pointed out that the mass-difference of $M_{\Omega_b^-} - M_{\Lambda_b^0}$ is $426.4 \pm 2.2 \text{ MeV}$ [35] and we have calculated this mass-difference of bottom baryons as 449.94 MeV . For the case of $M_{\Xi_b} - M_{\Lambda_b}$ our calculation shows 203.77 MeV and Ref. [40] reported $M_{\Xi_b} - M_{\Lambda_b} = 182.7 \text{ MeV}$. The mass-difference of $\Omega_b(\frac{3}{2}^+) - \Xi_b(\frac{3}{2}^+) = 124 \text{ MeV}$ stated by Garcilazo et al [41]. We have reported this spacing as 246.31 MeV .

In the scheme adopted in this work, we evaluate the ratio $\frac{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l)}{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)}$ to be 15.10 (using Eq. 14) and 11.78 (using Eq. 16) by considering exponential-type as IWF. By using Eq. (18), we have also obtained $\frac{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l)}{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)} = 14.62$. These theoretical predictions are consistent with experimental measurement $\frac{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l)}{BR(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)} = 20.0 \pm 3.0 \pm 1.2$ [42] and theoretical prediction 16.8 of Ref. [34]. Moreover, the PDG [35] reported the ratio $\frac{\Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l)}{\Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-)} = 12.7^{+3.1}_{-2.7}$ and our results are in consistent with them. We have obtained $\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}) = 3.67 \times 10^{-14} \text{ GeV}$ and $\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}) = 2.70 \times 10^{-14} \text{ GeV}$ in the case of Eq. (16) and MIT bag model as IWFs respectively. By the presented model, we can investigate B mesons, other heavy flavor baryons and their inclusive decays both nonleptonic and semileptonic.

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