Mechanics of isolated horizons in scalar-tensor theories

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Abstract Based on the first-order action for scalar-tensor theories with the Immirzi parameter, the symplectic form for the spacetimes admitting a weakly isolated horizon as internal boundary is derived by the covariant phase space approach. The first law of thermodynamics for the weakly isolated horizons with rotational symmetry is obtained. It turns out that the Immirzi parameter appears in the expression of the angular momentum of isolated horizon, and the scalar field contributes to the horizon entropy.

1 Introduction

Scalar-tensor theories (STT) are inspired by Mach's principle [1–3] and belong to a class of modified gravity theories. In some cosmological models, the non-minimally coupled scalar field in STT may cause the acceleration of the universe and hence explain the issue of dark energy [4–8]. The nonperturbative quantization of STT [9–13] and their cosmological models [14–16] has also been carried out recently by extending the method of loop quantum gravity [17–20]. The thermodynamics of black hole (BH) is an important issue in physics. While the event horizon of a BH is a global notion not suitable for local physics [21], the notion of isolated horizon is quasi-locally defined [22] and widely applied to calculate observables in numerical simulation of near horizon geometry [23, 24]. Practically, an isolated horizon satisfies the minimal requirements to derive the zeroth law and the first law of BH thermodynamics in general relativity (GR) [25–27].

The aim of this paper is to study the Hamiltonian structure and thermodynamics of isolated horizon in STT. To match the loop quantum STT, we start with the first-order action of STT with the Immirzi parameter proposed in [28]. The connection dynamical formalism derived from this action via Hamiltonian analysis is completely consistent with that

derived from the geometrical dynamics by canonical transformations [12]. It should be noted that the thermodynamics of isolated horizons with a non-minimally coupled scalar field was first studied in Ref. [29]. That theory could be regarded as a special case of the general STT that we are considering. By our general treatment, there exist new elements in both the symplectic structure and first law of isolated horizon in STT. Throughout the paper, we use the capital Latin letters $I, J, K, \dots$ to denote the internal Lorentzian indices, and the spacetime indices are denoted by $a, b, c, \dots$

2 Symplectic form

We consider the weakly isolated horizon (WIH) in STT. In comparison with the definition of WIH in GR, its definition in STT contains also the extra requirement on the non-minimally coupled scalar field ϕ [29], such that the horizon is in equilibrium. A 3-dimensional null hypersurface Δ equipped with an equivalence class $[l]$ of null normals l^a of a spacetime with metric g_{ab} and scalar field ϕ in STT is said to be a WIH if the following conditions hold.

- (i) The topology of Δ is $S^2 \times \mathbb{R}$;
- (ii) The expansion $\theta_{(l)}$ of l vanishes on Δ for any null normal l ;
- (iii) The scalar field satisfies $\mathcal{L}_l \phi \hat{=} 0$, where $\mathcal{L}_l$ denotes the Lie derivative along l and “ $\hat{=}$ ” means “equal on Δ ”;
- (iv) Equations of motion hold on Δ ;
- (v) The equivalence class $[l]$ of the future-directed l is chosen by $l \sim l'$ if and only if $l'^a = C l^a$ for a positive constant C , such that there is a connection 1-form w on Δ defined by $\nabla_a l^b \hat{=} w_a l^b$ and satisfying $\mathcal{L}_l w_a \hat{=} 0$ for all $l \in [l]$, where ∇_a denotes the pullback of the spacetime connection ∇_a compatible with g_{ab} to Δ .

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Hereafter, the pullback of a covariant index to Δ will be denoted by an arrow under that index. By the definition, one can obtain the following useful properties of WIHs in STT.

- The surface gravity $\kappa_{(l)} \hat{=} w_a l^a$ is a constant on Δ , which gives the zeroth law of its thermodynamics;
- There is a natural area 2-form ${}^2\epsilon$ on Δ satisfying $\mathcal{L}_l^2\epsilon \hat{=} 0$ and ${}^2\epsilon_{ab} l^b \hat{=} 0$;
- One can define the potential ψ of the surface gravity by $\mathcal{L}_l \psi \hat{=} \kappa_{(l)}$.
- There is a unique induced covariant derivative $\mathcal{D}$ inherited from ∇ . The actions of $\mathcal{D}$ on a vector field X^a tangent to Δ and on an 1-form Y_a intrinsic to Δ are given by $\mathcal{D}_a X^b \hat{=} \nabla_a \tilde{X}^b$ and $\mathcal{D}_a Y_b \hat{=} \nabla_a \tilde{Y}_b$ respectively, where $\tilde{X}^b$ and $\tilde{Y}_b$ are the arbitrary extensions of X^a and Y_a to the 4-dimensional spacetime.

Now we follow the covariant phase space approach [30, 31] to derive the symplectic structure of WIH in STT. The first-order action of STT in [28] can be written in terms of differential forms as

$$S[e, \bar{\omega}, \phi] = \frac{1}{16\pi G} \int_M \phi \Sigma^{IJ} \wedge \bar{\Omega}_{IJ} - \Sigma^{IJ} \wedge \bar{\omega}_{IJ} \wedge d\phi + \Sigma^{IJ} \wedge dV_{IJ} + \frac{\omega(\phi)^*}{\phi} d\phi \wedge d\phi + \frac{1}{\gamma} \Sigma^{IJ} \wedge {}^* \bar{\Omega}_{IJ} + \xi(\phi)\epsilon, \quad (1)$$

where G is the gravitational constant, $\Sigma_{ab}^{IJ} := \frac{1}{2} \varepsilon^{IJ}{}_{KL} e_a^K \wedge e_b^L$ with e_a^I and ε_{IJKL} being the cotetrad and Levi-Civita symbol respectively, $(dV_{IJ})_{ab} = \partial_{[a} e_{b]}^I e^{cJ} \partial_c \phi$, the curvature 2-form of the Lorentzian connection $\bar{\omega}_{ab}^{IJ}$ is defined by $\bar{\Omega}_{ab}^{IJ} = d_a \bar{\omega}_b^{IJ} + \bar{\omega}_a^I{}_K \wedge \bar{\omega}_b^{KJ}$, ${}^* \bar{\Omega}_{IJ} = \frac{1}{2} \varepsilon_{IJKL} \bar{\Omega}^{KL}$, $({}^* d\phi)_{abc} = (d\phi)\epsilon^e{}_{abc}$ with ϵ being the spacetime volume element, the potential $\xi(\phi)$ and the coupling parameter $\omega(\phi)$ are general functions of ϕ , and the Immirzi parameter γ is included. It has been shown in Ref. [28] that action (1) gives the same field equations with the general second-order action of scalar-tensor theories. Our motivation to choose action (1) is that the connection dynamics can be derived from it, which is the foundation of loop quantum scalar-tensor theories [12]. It should be noted that the general scalar-tensor theories that we are considering do not include those extended theories containing the higher-order derivatives of ϕ in their actions, such as Horndeski theory [32] and GLPV theory [33].

Let us consider a spacetime region $\mathcal{M}$ which admits a WIH $(\Delta, [l])$ as its internal boundary and is bounded by two spatially partial Cauchy surfaces $M^\pm$ intersecting Δ at two-spheres $H^\pm$ and extending to spatial infinity i^0 . The asymptotic conditions on i^0 are that the metric is asymptotic flat and $\phi \rightarrow 1$. The boundary of spacetime region $\mathcal{M}$ is

$\partial\mathcal{M} = \Delta \cup M^+ \cup M^- \cup i^0$. To satisfy the variational principle, one has to add a boundary term at i^0 such that action (1) becomes

$$S[e, \bar{\omega}, \phi] = \frac{1}{16\pi G} \int_M \phi \Sigma^{IJ} \wedge \bar{\Omega}_{IJ} - \Sigma^{IJ} \wedge \bar{\omega}_{IJ} \wedge d\phi + \Sigma^{IJ} \wedge dV_{IJ} + \frac{\omega(\phi)^*}{\phi} d\phi \wedge d\phi + \frac{1}{\gamma} \Sigma^{IJ} \wedge {}^* \bar{\Omega}_{IJ} + \xi(\phi)\epsilon - \frac{1}{16\pi G} \int_{i^0} \Sigma^{IJ} \wedge \left(\phi \bar{\omega}_{IJ} + \frac{1}{\gamma} {}^* \bar{\omega}_{IJ} + e_I e_J^f \partial_f \phi \right) \quad (2)$$

The variation of action (2) reads

$$\delta S[\Psi] = \int_{\mathcal{M}} E[\Psi] \delta\Psi + \int_{\partial\mathcal{M}-i^0} \mathcal{J}[\Psi, \delta\Psi],$$

where Ψ denotes the fields e , $\bar{\omega}$ and ϕ , $E[\Psi] = 0$ is the equation of motion for Ψ , and the current 3-form J reads

$$\mathcal{J}[\Psi, \delta\Psi] = \frac{1}{16\pi G} \left[\Sigma^{IJ} \wedge \delta \left(\phi \bar{\omega}_{IJ} + \frac{1}{\gamma} {}^* \bar{\omega}_{IJ} + e_I e_J^f \partial_f \phi \right) - \left(\frac{\omega(\phi)^*}{\phi} d\phi - \frac{1}{3} \varepsilon_{IJKL} (\nabla_d e^{dI}) e^J \wedge e^K \wedge e^L \right) \delta\phi \right]. \quad (3)$$

Note that the potential $\xi(\phi)$ does not contribute to J . Comparing with the current 3-form in [29], the boundary term (3) contains also the scalar field part. We will see that the scalar part in the second term in the right hand side of Eq. (3) also contributes to the boundary symplectic structure. Note that, different from that in [29], the connection $\bar{\omega}$ is compatible with the tetrad e in our covariant phase space Γ .

The (pre-)symplectic form can be obtained by the second variation of the action [25]. First, the anti-symmetrized second variation gives the symplectic current J on a point p in the covariant phase space Γ . J is a closed 3-form on $\mathcal{M}$, since the fields Ψ satisfy the equations of motion and the variations $\delta\Psi$ satisfy the linearized equations off p . Second, the asymptotic conditions guarantee that the integral of symplectic current over i^0 vanishes. Therefore the integral of J over a partial Cauchy surface M and its intersection with Δ is conserved, which can be defined as a pre-symplectic form. The symplectic current J in our situation satisfies

$$8\pi G J(p, \delta_1, \delta_2) = \delta_{[1} \Sigma^{IJ} \wedge \delta_{2]} \left(\phi \bar{\omega}_{IJ} + \frac{1}{\gamma} {}^* \bar{\omega}_{IJ} + e_I e_J^f \partial_f \phi \right) - \delta_{[1} \left(\frac{\omega(\phi)^*}{\phi} d\phi - \frac{1}{3} \varepsilon_{IJKL} (\nabla_d e^{dI}) e^J \wedge e^K \wedge e^L \right) \delta_{2]} \phi. \quad (4)$$

For the spacetime region $\mathcal{M}$, the closed condition of J leads to

$$\left(\int_{M^+} - \int_{M^-} + \int_{\Delta} \right) J(p, \delta_1, \delta_2) = 0. \quad (5)$$

Note that $\mathcal{M}$ is topologically $M \times \mathbb{R}$, where M is an oriented spatial manifold with an internal 2-sphere boundary H . It is convenient to fix an internal null tetrad $(l^I, n^I, m^I, \bar{m}^I)$ on Δ such that each element of the tetrad is annihilated by a fiducial flat connection ∂ . The conditions of WIH ensure that one could choose the tetrads e^a_I such that their contraction with the internal null tetrad could give the spacetime null tetrad $(l^a, n^a, m^a, \bar{m}^a)$ adapted to Δ and M [25]. This means that l^a belongs to $[l]$ fixed on Δ , the complex null vector m^a is tangential to H , n^a is future directed and transverse to Δ , and they satisfy $n^a l^b g_{ab} = -1$, $m^a \bar{m}^b g_{ab} = 1$ and all other scalar products vanishing. To calculate the symplectic current on Δ , it is useful to express J by the null tetrads. The pull-back of the two-forms Σ^{IJ} to Δ can be expressed as [25]

$$\underline{\Sigma}^{IJ} \cong 2l^{[I} n^{J]} \epsilon + 2n \wedge (iml^{[I} \bar{m}^{J]} - i\bar{m}l^{[I} m^{J]}), \quad (6)$$

where ${}^2\epsilon = im \wedge \bar{m}$ is the area 2-form on the 2-sphere H . The pull-back of the connection $\bar{\omega}^{IJ}_a$ to Δ can be expressed by [34]

$$\begin{aligned} \bar{\omega}^{IJ} = & -2l^{[I} n^{J]} w + 2m^{[I} \bar{m}^{J]} V + 2l^{[I} \bar{m}^{J]} U \\ & + 2l^{[I} m^{J]} \bar{U}, \end{aligned} \quad (7)$$

where the components of 1-forms ω , V and U can be expressed by the Newman–Penrose spin coefficients as

$$w = -\kappa_{(I)} n + (\alpha + \bar{\beta}) m + (\bar{\alpha} + \beta) \bar{m}, \quad (8a)$$

$$V = -(\epsilon - \bar{\epsilon}) n + (\alpha - \bar{\beta}) m + (\beta - \bar{\alpha}) \bar{m}, \quad (8b)$$

$$U = -\bar{\pi} n + \bar{\mu} m + \bar{\lambda} \bar{m}. \quad (8c)$$

Now we consider the terms containing the scalar field in Eq. (4). Because the pull-back of the covariant index annihilates the l_a direction and the scalar field satisfies the condition (iii) of WIH, the pull-back of the one-form $e_{cI} e^f_J \partial_f \phi$ can be written as

$$\begin{aligned} e_{cI} e^f_J \partial_f \phi = & (-l_I n_c + \bar{m}_I m_c + m_I \bar{m}_c) \\ & \times (-l_J n^f + \bar{m}_J m^f + m_J \bar{m}^f) \partial_f \phi. \end{aligned} \quad (9)$$

Since the right hand side of Eq. (9) does not contain n_I or n_J components, its contraction with Eq. (6) is zero. The pull-

back of $*d\phi$ to Δ reads

$$\begin{aligned} (*d\phi)_{\underline{abc}} &= (d\phi) \epsilon^d_{\underline{abc}} \\ &= -\epsilon_{IJKL} e^I_{\underline{a}} \wedge e^J_{\underline{b}} \wedge e^K_{\underline{c}} e^{dL} d\phi. \end{aligned} \quad (10)$$

Since the tetrads appeared in the right hand side of Eq. (10) do not contain the n^I components, their contraction with ϵ_{IJKL} is zero. Taking account of the identities $\epsilon_{IJKL} l^I n^J m^K \bar{m}^L = i$ and $\nabla_a l^a = \kappa_{(I)}$, the pull-back of the three-form $\frac{1}{3} \epsilon_{IJKL} (\nabla_d e^{dI}) \cdot e^J \wedge e^K \wedge e^L$ reads

$$\begin{aligned} & \frac{1}{3} \epsilon_{IJKL} (\nabla_d e^{dI}) e^J_{\underline{a}} \wedge e^K_{\underline{b}} \wedge e^L_{\underline{c}} \\ &= 2(\nabla_d l^d) \epsilon_{IJKL} n^I l^J m^K \bar{m}^L n_a \wedge \bar{m}_b \wedge m_c \\ &= 2\kappa_{(I)} n_a \wedge i m_b \wedge \bar{m}_c \\ &= 2\kappa_{(I)}^3 \epsilon, \end{aligned} \quad (11)$$

where ${}^3\epsilon$ denotes the three-volume element on Δ . It should be noted that the variations in Eq. (4) do not change the WIH $(\Delta, [I])$. Therefore the restriction to Δ and the variation of Ψ commute with each other. For instance, one has

$$\delta(\Sigma^{IJ}_{\underline{ab}}) = (\delta\Sigma)^{IJ}_{\underline{ab}}.$$

Thus, by Eqs. (6), (7), (8), (9), (10) and (11), the symplectic current on Δ reduces to

$$\begin{aligned} J_{\underline{a}}(p, \delta_1, \delta_2) \cong & \frac{1}{4\pi G} \delta_{[1}^2 \epsilon \wedge \delta_{2]} (\phi w + \frac{i}{\gamma} V) \\ & + \frac{1}{4\pi G} \delta_{[1} (\kappa_{(I)}^3 \epsilon) \delta_{2]} \phi. \end{aligned} \quad (12)$$

Note that only the components along n_a in the expressions (8a) and (8b) of ω_a and V_a contribute in the right hand side of Eq. (12). To describe the contribution of V_a , one can define a potential μ via $\mathcal{L}_I \mu \cong i(\epsilon - \bar{\epsilon})$. Using μ and the potential ψ defined in the property (c) of WIH, Eq. (12) can be expressed as

$$\begin{aligned} J_{\underline{a}}(p, \delta_1, \delta_2) \cong & \frac{1}{4\pi G} \delta_{[1}^2 \epsilon \wedge \delta_{2]} (\phi d\psi + \frac{1}{\gamma} d\mu) \\ & - \frac{1}{4\pi G} \delta_{[1} (\kappa_{(I)} (dv) \wedge^2 \epsilon) \delta_{2]} \phi. \end{aligned} \quad (13)$$

where v is the parameter for the integral curve of l^a such that $n_a = -(dv)_a$. Taking account of the fact that $d({}^2\epsilon) = 0$ on Δ and the condition (iii) of WIH, the symplectic current (13) can be written as

$$J_{\underline{a}}(p, \delta_1, \delta_2) \cong dj(p, \delta_1, \delta_2), \quad (14)$$

where the 2-form j on Δ is given by

$$j(p, \delta_1, \delta_2) = \frac{1}{4\pi G} \delta_{[1}^2 \epsilon \delta_{2]} (\phi \psi + \frac{1}{\gamma} \mu) - \frac{1}{4\pi G} \delta_{[1} (\kappa_{(l)} v^2 \epsilon) \delta_{2]} \phi. \quad (15)$$

Together with Eqs. (4), (14) and (15), Eq. (5) gives the conserved symplectic form

$$\begin{aligned} \Omega(\delta_1, \delta_2) &= \frac{1}{8\pi G} \int_M \delta_{[1} \Sigma^{IJ} \wedge \delta_{2]} (\phi \bar{\omega}_{IJ} + \frac{1}{\gamma} \bar{\omega}_{IJ} + e_I e_J^f \partial_f \phi) \\ &+ \frac{1}{8\pi G} \int_M \delta_{[1} \phi \delta_{2]} \left(\frac{\omega(\phi)^*}{\phi} d\phi \right. \\ &- \left. \frac{1}{3} \epsilon_{IJKL} (\nabla_d e^{dI}) e^J \wedge e^K \wedge e^L \right) \\ &+ \frac{1}{4\pi G} \int_H \delta_{[1}^2 \epsilon \delta_{2]} (\phi \psi + \frac{1}{\gamma} \mu) \\ &+ \frac{1}{4\pi G} \int_H \delta_{[1} \phi \delta_{2]} (\kappa_{(l)} v^2 \epsilon). \end{aligned} \quad (16)$$

Note that $\Omega(\delta_1, \delta_2)$ is actually a pre-symplectic form on the covariant phase space, since it has degenerate directions. However, it can be corresponded to the symplectic form on the phase space. Eq. (16) shows that, for the spacetimes admitting a WIH as internal boundary, the symplectic form contains the bulk part and boundary part. The conjugate pairs for the geometry part and the scalar field part can be easily read from Eq. (16) respectively. Let $\psi = \kappa_{(l)} v$, which can always be satisfied by adjusting the initial values of ψ and v on H^- . Then except for the term containing γ , the other terms in the boundary symplectic form in Eq. (16) can be combined into that in [29]. However, our expression indicates that the scalar field has its own degrees of freedom. Moreover a term of Immirzi parameter is introduced.

3 The first law

To set up the thermodynamics of WIH, one needs to define the quasilocal notions of energy, entropy, angular momentum and so on for the horizon. It turns out that these notions can be obtained naturally by asking for the existence of a consistent Hamiltonian evolution in the covariant phase space Γ associate to $\mathcal{M}$ [30, 31]. Given a vector field t^a satisfying proper boundary condition on $\mathcal{M}$, it can define a vector field δ_t on Γ by $\delta_t := (\mathcal{L}_t e, \mathcal{L}_t \bar{\omega}, \mathcal{L}_t \phi)$, which satisfies the linearized equations of motion. δ_t will be a Hamiltonian vector field, if it preserves the symplectic form, i.e., $\mathcal{L}_{\delta_t} \Omega = 0$ on Γ . The necessary and sufficient condition for this requirement is that there exists a function H_t such that $\delta H_t = \Omega(\delta, \delta_t)$ for all

vector field δ in Γ [25]. This condition is equivalent to that the one-form X_t on Γ defined by

$$X_t(\delta) := \Omega(\delta, \delta_t) \quad (17)$$

is closed, i.e., $\mathbf{d}X_t = 0$, where $\mathbf{d}$ denotes the exterior derivative on Γ . If this condition is satisfied, up to an additive constant, the Hamiltonian function is given by

$$\mathbf{d}H_t = X_t. \quad (18)$$

Suppose that the internal boundary of a spacetime region $\mathcal{M}$ is a WIH $(\Delta, [l])$ with a rotational symmetry. It is convenient to introduce a fixed rotational vector field φ^a on Δ and admit only those spacetimes in Γ which have this φ^a as the horizon symmetry. Thus the geometry restricted to Δ and the scalar field ϕ are both Lie dragged by φ^a . Moreover, we ask φ^a to be tangent to H and have closed circular orbits of affine length 2π . Consider a vector field t^a on $\mathcal{M}$, which approaches an asymptotic time translation at infinity and reduces to a symmetry on Δ such that

$$t^a \cong B_{(l,t)} l^a - \Omega_{(t)} \varphi^a \quad (19)$$

for some constants $\Omega_{(t)}$ and $B_{(l,t)}$, where $B_{(l,t)} l^a$ is unchanged under the rescalings of $l \in [l]$, and $\Omega_{(t)}$ will be referred as the angular velocity of Δ related to t^a . Note that, while the variations δ_t of the fields $(e, \bar{\omega}, \phi)$ induced by t^a are Lie derivatives along t^a , the actions of δ_t on the parameter v and the potentials ψ and μ are not the same case. To preserve the initial assignment of (ψ, μ, v) , the variations in Δ should satisfy $\delta_t \psi = \delta_t \mu = \delta_t v = 0$ [27]. To analyze whether δ_t is a Hamiltonian vector field on Γ , we need to use (16) to calculate $X_t(\delta) = \Omega(\delta, \delta_t)$. The boundary part in (16) gives

$$\begin{aligned} \Omega|_H(\delta, \delta_t) &= \frac{1}{8\pi G} \int_H \delta^2 \epsilon (\mathcal{L}_t \phi) \psi - (\mathcal{L}_t^2 \epsilon) \delta \left(\phi \psi + \frac{1}{\gamma} \mu \right) \\ &+ \frac{1}{8\pi G} \int_H \delta \phi \mathcal{L}_t (\kappa_{(l)}^2 \epsilon) v - (\mathcal{L}_t \phi) \delta (\kappa_{(l)} v^2 \epsilon) \\ &= 0, \end{aligned}$$

where we used the above requirement for the variations of (ψ, μ, v) on Δ in the first step, and we replaced t by Eq. (19) and used the condition (iii), the properties (a), (b), and the geometric symmetry of the WIH in the last step. Therefore, the boundary symplectic form in Eq. (16) does not contribute to $X_t(\delta)$. To calculate the bulk integral, we need use the Stokes' theorem and the identity $\mathcal{L}_t \underline{u} = t \cdot d\underline{u} + d(t \cdot \underline{u})$ for a form $\underline{u}$, where d is the exterior derivative on spacetime.

This identity takes the following specific forms

$$\begin{aligned}\mathcal{L}_t \bar{\omega} &= t \cdot F + D(t \cdot \bar{\omega}), \\ \mathcal{L}_t e &= t \cdot De + D(t \cdot e) - (t \cdot \bar{\omega})e, \\ \mathcal{L}_t \Sigma &= t \cdot D\Sigma + D(t \cdot \Sigma) - [(t \cdot \bar{\omega}), \Sigma],\end{aligned}$$

where D is the internal exterior derivative defined by $\bar{\omega}$, and $[\cdot, \cdot]$ denotes the commutator of internal indices. Then, using the equations of motion and their linearized version, the bulk symplectic form in Eq. (16) will contribute only two surface terms to $X_t(\delta)$, i.e., the variation of the ADM energy δE_{ADM}^t at spatial infinity and the variation of the horizon energy δE_{Δ}^t at H . For the ADM energy of STT, we refer to Ref. [35]. Since our aim is the first law of WIH, let us focus on the term at the horizon,

$$\begin{aligned}\Omega|_H(\delta, \delta_t) &= \frac{1}{16\pi G} \int_H (\delta \Sigma^{IJ}) \left(\phi t \cdot \bar{\omega}_{IJ} + \frac{1}{\gamma} t \cdot \star \bar{\omega}_{IJ} + t \cdot e_I e_J^f \partial_f \phi \right) \\ &\quad - \frac{1}{16\pi G} \int_H t \cdot \Sigma^{IJ} \wedge \delta \left(\phi \bar{\omega}_{IJ} + \frac{1}{\gamma} \star \bar{\omega}_{IJ} + e_I e_J^f \partial_f \phi \right) \\ &\quad + \frac{1}{16\pi G} \int_H (\delta \phi) \left(\frac{\omega(\phi)}{\phi} t \cdot \star d\phi - 2\kappa_{(t)}(t \cdot n)^2 \epsilon \right) \\ &= \frac{1}{8\pi G} \int_H \left(\phi \kappa_{(t)} + \frac{i}{\gamma} (\epsilon - \bar{\epsilon}) \right) \delta^2 \epsilon + \frac{1}{8\pi G} \int_H \kappa_{(t)}^2 \epsilon \delta \phi \\ &\quad - \frac{\Omega_t}{8\pi G} \int_H \left(\phi \varphi \cdot w + \frac{i}{\gamma} \varphi \cdot V \right) \delta^2 \epsilon \\ &\quad + \frac{\Omega_t}{8\pi G} \int_H \varphi \cdot^2 \epsilon \wedge \delta(\phi w + \frac{i}{\gamma} V) \\ &= \int_H \frac{\kappa_{(t)}}{8\pi G} \delta(\phi^2 \epsilon) + \Omega_t \delta J_{\Delta}^{\varphi} + \int_H \frac{i}{8\pi G \gamma} (\epsilon - \bar{\epsilon}) \delta^2 \epsilon, \quad (20)\end{aligned}$$

where in the second equality we used the fact that the terms containing $t \cdot e_I e_J^f \partial_f \phi$ or $l \cdot \Sigma^{IJ}$ vanish by the contraction of internal indices and $\star d\phi = 0$, in the third equality we used the fact that the restriction of $\varphi^c w_a \wedge \epsilon_{bc}$ to H vanishes and defined

$$J_{\Delta}^{\varphi} = -\frac{1}{8\pi G} \int_H \left(\phi \varphi \cdot w + \frac{i}{\gamma} \varphi \cdot V \right)^2 \epsilon. \quad (21)$$

Note that Eq. (8b) implies $\varphi \cdot V = \varphi^a((\alpha - \bar{\beta})m_a + (\beta - \bar{\alpha})\bar{m}_a)$. Note also that we can require m^a to be Lie dragged by l , i.e., $\mathcal{L}_l m^a = (\epsilon - \bar{\epsilon})m^a \hat{=} 0$. This requirement can be realized by an appropriate spin transformation to set the spin coefficient ϵ to real numbers [25]. Thus we obtain

$$X_t(\delta) = \frac{\kappa_{(t)}}{8\pi G} \int_H \delta(\phi^2 \epsilon) + \Omega_t \delta J_{\Delta}^{\varphi} - \delta E_{\text{ADM}}^t. \quad (22)$$

The existence of a Hamiltonian H_t is equivalent to $\mathbf{d}X_t = 0$, which can be satisfied if and only if

$$\frac{1}{8\pi G} \mathbf{d}\kappa_{(t)} \lrcorner \mathbf{d} \left(\int_H (\phi^2 \epsilon) \right) + \mathbf{d}\Omega_t \lrcorner \mathbf{d}J_{\Delta}^{\varphi} = 0. \quad (23)$$

The condition (23) implies that $\kappa_{(t)}$ and Ω_t are only the functions of $\int_H (\phi^2 \epsilon)$ and J_{Δ}^{φ} respectively. As argued in Refs. [27] and [29], there do exist the vector fields t^a such that the condition (23) is satisfied. Then, the Hamiltonian can be written as

$$H_t = E_{\Delta}^t - E_{\text{ADM}}^t, \quad (24)$$

where E_{Δ}^t is defined as the energy of WIH related to t^a and given by

$$\delta E_{\Delta}^t = \frac{\kappa_{(t)} \hbar c^3}{2\pi k_B} \delta S + \Omega^t \delta J_{\Delta}^{\varphi}, \quad (25)$$

with the entropy

$$S = \frac{k_B}{4G\hbar c^3} \oint \phi^2 \epsilon. \quad (26)$$

Here J_{Δ}^{φ} is defined as the angular momentum of WIH related to φ^a . In contrast to those expressions in Refs. [27] and [29], where the Immirzi parameter was not included in the actions, our expression (21) shows that the Immirzi term does enter the definition of J_{Δ}^{φ} . However, similar to the situation in GR [36], the total angular momentum J_{∞} at infinity will not contain the Immirzi term in STT. For the spacetime admitting a global rotational Killing vector field which reduces to φ^a on Δ , J_{Δ}^{φ} can match J_{∞} , since the Immirzi term disappears at infinity because of the asymptotically flat condition. Eq. (25) is our first law of WIH in STT. If one further required $[\mathcal{L}_l, \mathcal{D}]V = 0$, for all vector fields V tangential to Δ and all $l \in [l]$, the WIH $(\Delta, [l])$ would become isolated horizon (IH). Even the requirement for l in IH is stronger than that in WIH, IH is still a non-trivial extension of the Killing horizon [37]. For a given non-expanding horizon Δ [25], in principle infinitely many WIH can be defined by it [25], but generically only one IH can be defined [23]. Since the IH satisfies stronger conditions than those of the WIH, our first law for WIH is also valid for IH, but the l^a and hence t^a of the latter are more strict than those of the former.

4 Conclusion

To summarize, based on the first-order action (2) in STT, we derived the symplectic form (16) for the spacetimes admitting a WIH as internal boundary via the covariant phase space

approach. By asking for the existence of a consistent Hamiltonian evolution in the covariant phase space, the first law of thermodynamics for WIH with rotational symmetry was obtained as Eq. (25). It turns out that, besides the area of WIH, the scalar field of STT contributes also to the entropy. Our expression (21) for the angular momentum of WIH includes the Immirzi term in addition to the expression in Ref. [29]. In contrast to the action employed in Ref. [29], our action (2) for STT is more general and includes the Immirzi parameter. Moreover, we use the Jordan frame to express the symplectic form rather than the Einstein frame. Thus, our expressions of symplectic form and angular momentum are different from those in Ref. [29]. The appearance of the Immirzi parameter in the expression for angular momentum of WIH is an interesting issue which deserves further studying.

It should be noted that, based on the Wald formula [38] and the first-order action of GR, the first law of the thermodynamics for the Killing horizon was derived in Ref. [39] and then extended to general gauge invariant gravitational theories [40]. Thus, it would be interesting to compare our treatment for WIH to those treatments for Killing horizon in the first-order scalar-tensor theories. This issue is left for future study. Note also that the no-hair theorem still holds for the event horizon of a black hole in the general scalar-tensor theories [41, 42]. Thus, there is no scalar hair for the stationary black holes under certain conditions. However the WIH that we considered is quasi-locally defined, which does not require any global structure of spacetime. Hence our conclusion that the non-minimally coupled scalar ϕ would contribute to the IH entropy and angular momentum does not contradict the no-hair theorem.

Since the scalar field has its own degrees of freedom in the boundary symplectic form in Eq. (16) and appears in the entropy formula (26) of STT, it should also contribute to the microstate number if one calculates the entropy at quantum gravity level. The results in this paper lays a classical foundation to study the statistic origin of the black hole entropy, since the loop quantization of STT has been performed [12].

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