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# Renormalization of a model for spin-1 matter fields

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**Abstract** In this work, the one-loop renormalization of a theory for fields transforming in the $(1, 0) \oplus (0, 1)$ representation of the Homogeneous Lorentz Group is studied. The model includes an arbitrary gyromagnetic factor and self-interactions of the spin 1 field, which has mass dimension one. The model is shown to be renormalizable for any value of the gyromagnetic factor.

## 1 Introduction

In the Standard Model of particle physics, only fields transforming in the $(0, 0)$, $(1/2, 0)$, $(0, 1/2)$ and $(1/2, 1/2)$ representations of the Homogeneous Lorentz Group (HLG) are needed. There is however no guiding principle restricting the possible representations, and indeed high spin fields naturally appear in Hadron physics and in Beyond the Standard Model (BSM) scenarios like supergravity and superstrings.

In an attempt to better understand the physics of fields transforming in different representations of the HLG, a series of works have been carried [1–8] based on the projection onto subspaces of the Poincaré group. In this formalism, it has been shown that the gyromagnetic factor of spin 3/2 fields is connected with their causal propagation in an electromagnetic background [1], and with the unitarity of the Compton scattering amplitude in the forward direction [2]. The formalism can also be applied to lower spins, for example, in the spin 1 case, a similar connection between the unitarity of Compton scattering in the forward direction and the gyromagnetic factor of the field exists, which is also related to the electric quadrupole moment [3].

When the Poincaré projector method is applied to spin 1/2 fields transforming in the $(1/2, 0) \oplus (0, 1/2)$ representation

[6,7], the resulting Lagrangian is a generalized version of the original second order Feynman-Gell-Mann formalism [9], enhanced with an arbitrary gyromagnetic factor and fermion self interactions. The second order fermions studied in these works are conceptually different to Dirac ones, as the former propagate 8 dynamical degrees of freedom instead of 4. As shown in [6,7], there is a consistent reduction of dynamical degrees of freedom and a direct connection between the renormalization group equations for the second order fermions and the Dirac formalism if the gyromagnetic (or chromomagnetic) factor is set to the fixed value $g = 2$.

The goal of the present work is to study the renormalization properties of spin-1 matter fields[1] transforming in the $(1, 0) \oplus (0, 1)$ representation of the HLG in a model based on the Poincaré projector formalism, as a direct generalization of the spin 1/2 case [6,7].

The difference between the pure spin 1 representation $(1, 0) \oplus (0, 1)$, described by an antisymmetric tensor field of second rank, and the more familiar $(1/2, 1/2)$ vector field is more dramatic in the massless case, as the Kalb–Ramond antisymmetric gauge field contains only one physical longitudinal degree of freedom [10], whereas the massless vector gauge field is characterized by 2 transverse ones. Switching to massive spin-1 particles, one must distinguish between gauge invariant and non-gauge invariant theories. It can be shown that a massive Stueckelberg compensated Kalb-Ramond gauge field is dual to a compensated massive gauge vector field [11]. However, for non-gauge invariant massive spin-1 theories, the properties of four-vector and antisymmetric tensor particles can differ significantly. In [12] the difference between spin-1 antisymmetric tensor mesons and the four-vector mesons has been studied in detail for composite hadrons. In the present work, we focus instead on pointlike

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[1] Here we understand matter fields as massive non-gauge fields.

massive spin-1 bosons, with emphasis on their electromagnetic properties and their possible self-interactions.

The model studied here is based on [4], where the complex antisymmetric tensor field has 6 complex degrees of freedom, making the $(1, 0) \oplus (0, 1)$ theory explicitly different to any of a massive gauge vector field. In [4] the Compton scattering of spin-1 particles described by both a massive four-vector and an antisymmetric tensor was analyzed for arbitrary values of the gyromagnetic factor, finding that the Compton scattering cross section off the parity degrees of freedom in $(1, 0) \oplus (0, 1)$ is finite in the forward direction, though it is still divergent elsewhere. Interestingly, for the antisymmetric tensor this result is independent of the gyromagnetic factor, while Compton scattering off the four-vector is only well behaved in all directions provided the gyromagnetic ratio is set to $g = 2$. Given the non-finiteness of Compton scattering in this theory, it is unclear if the renormalizable theory described here corresponds to a perturbation theory about a sensible zeroth-order Hamiltonian. However, it constitutes a unique theoretical laboratory from the point of view of the renormalization group, in the same spirit as scalar $\lambda\phi^3$ theory.

The structure of the paper is the following: In Sect. 2 we describe the model and the Feynman rules. The renormalization procedure is presented in Sect. 3 together with the cancellation of all the potentially divergent contributions to the one-loop vertices of the theory. Finally, the conclusions of the work are discussed in Sect. 4.

## 2 The model

Our model comprises a massive complex spin-1 antisymmetric tensor field $B^{\alpha\beta}$ in the $(1, 0) \oplus (0, 1)$ representation of the HLG, minimally coupled to $U(1)_{\mathrm{EM}}$ with arbitrary gyromagnetic factor and mass dimension one, allowing for self interaction terms. The Lagrangian of the model is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu B^{\alpha\beta})^\dagger (T_{\mu\nu})_{\alpha\beta\gamma\delta}(D^\nu B^{\gamma\delta})$$
$$-m^2(B^{\alpha\beta})^\dagger B_{\alpha\beta}$$
$$+\frac{\lambda_1}{2}(B^{\alpha\beta\,\dagger}1_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}1_{\mu\nu\rho\sigma}B^{\rho\sigma})$$
$$+\frac{\lambda_2}{2}(B^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma}B^{\rho\sigma})$$
$$+\frac{\lambda_3}{2}(B^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})$$
$$\times(B^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2})$$
$$+\frac{\lambda_4}{2}(B^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})$$
$$\times(B^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}), \tag{2.1}$$

where $D^\mu = \partial^\mu + ieA^\mu$ is the covariant derivative, and the tensors used are given by

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad T_{\mu\nu} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - ig(M_{\mu\nu})_{\alpha\beta\gamma\delta},$$
$$1_{\alpha\beta\gamma\delta} = \frac{1}{2}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}), \quad \chi_{\alpha\beta\gamma\delta} = \frac{i}{2}\epsilon_{\alpha\beta\gamma\delta},$$
$$(M_{\mu\nu})_{\alpha\beta\gamma\delta} = -i(g_{\mu\gamma}1_{\alpha\beta\nu\delta} + g_{\mu\delta}1_{\alpha\beta\gamma\nu}$$
$$g_{\gamma\nu}1_{\alpha\beta\mu\delta} - g_{\delta\nu}1_{\alpha\beta\gamma\mu}),$$
$$(S_{\mu\nu})_{\alpha\beta\gamma\delta} = g_{\mu\nu}1_{\alpha\beta\gamma\delta} - g_{\mu\gamma}1_{\alpha\beta\nu\delta} - g_{\mu\delta}1_{\alpha\beta\gamma\nu}$$
$$g_{\gamma\nu}1_{\alpha\beta\mu\delta} - g_{\delta\nu}1_{\alpha\beta\gamma\mu}. \tag{2.2}$$

The kinetic part of the Lagrangian is of Klein–Gordon type and spin-1 information is encoded by a Pauli-like term modulated by an arbitrary gyromagnetic factor $g$ and the four independent quartic self-interactions that can be built from the covariant basis for the $(1, 0) \oplus (0, 1)$ representation space, given by the complete set of tensors presented in [8], namely $\{1, \chi, M^{\mu\nu}, S^{\mu\nu}, \chi S^{\mu\nu}, C^{\mu\nu\alpha\beta}\}$ with

$$C_{\mu\nu\alpha\beta} = 4\{M_{\mu\nu}, M_{\alpha\beta}\} + 2\{M_{\mu\alpha}, M_{\nu\beta}\}$$
$$-2\{M_{\mu\beta}, M_{\nu\alpha}\} - 16(1_{\mu\nu\alpha\beta}). \tag{2.3}$$

In our analysis, the gauge freedom is fixed by the $R_\xi$ contribution

$$\mathcal{L}_{\mathrm{G.F.}} = -\frac{1}{2\xi}(\partial^\mu A_\mu)^2 \tag{2.4}$$

with arbitrary gauge fixing parameter $\xi$, rendering the complete Lagrangian of the model as

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 + \partial^\mu B^{\alpha\beta\dagger}\partial_\mu B_{\alpha\beta}$$
$$-m^2(B^{\alpha\beta})^\dagger B_{\alpha\beta}$$
$$-ieA^\mu[B^{\alpha\beta\dagger}(T_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B^{\gamma\delta} - (\partial^\nu B^{\alpha\beta\dagger})$$
$$\times(T_{\nu\mu})_{\alpha\beta\gamma\delta}B^{\gamma\delta}] + e^2A^\mu A_\mu B^{\alpha\beta\dagger}B_{\alpha\beta}$$
$$+\frac{\lambda_1}{2}(B^{\alpha\beta\,\dagger}1_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}1_{\mu\nu\rho\sigma}B^{\rho\sigma})$$
$$+\frac{\lambda_2}{2}(B^{\alpha\beta\,\dagger}\chi_{\alpha\beta\gamma\delta}B^{\gamma\delta})(B^{\mu\nu\,\dagger}\chi_{\mu\nu\rho\sigma}B^{\rho\sigma})$$
$$+\frac{\lambda_3}{2}(B^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})$$
$$\times(B^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2})$$
$$+\frac{\lambda_4}{2}(B^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1}B^{\gamma_1\delta_1})$$
$$\times(B^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2}B^{\gamma_2\delta_2}). \tag{2.5}$$

The Feynman rules corresponding to the above Lagrangian are presented in Fig. 1, where all momenta are incoming.

The gauge invariance of the theory imposes two important Ward-Takahashi identities (see [5] for their derivation in the analogous spin 1/2 case). The first one relates the tensor–tensor–photon (TT$\gamma$) vertex function $-ie\Gamma^\mu(q, p, -p-q)$,

(I)  $\nu$ ⌇⌇⌇⌇⌇⌇⌇⌇⌇⌇ $\mu$  $\quad = -\frac{i}{q^2}[g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2}]$

$\qquad\qquad\qquad\qquad q$

(II)  $\gamma\delta$ ⟶ $\alpha\beta$  $\quad = \frac{i 1_{\alpha\beta\gamma\delta}}{p^2 - m^2 + i\varepsilon}$

$\qquad\qquad\qquad p$

(III)  $\mu$ ⌇⌇⌇⌇  $p_1, \ \alpha\beta$  $\quad = -ie[T_{\mu\nu}p_2^\nu - T_{\nu\mu}p_1^\nu]_{\alpha\beta\gamma\delta}$

$\qquad\qquad\qquad p_2, \ \gamma\delta$

(IV)  $\quad = 2ie^2 g_{\mu\nu} 1_{\alpha\beta\gamma\delta}$

(V)
$$= i\{\lambda_1(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta})$$
$$+\lambda_2(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta})$$
$$+\lambda_3[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}]$$
$$+\lambda_4[S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\}$$
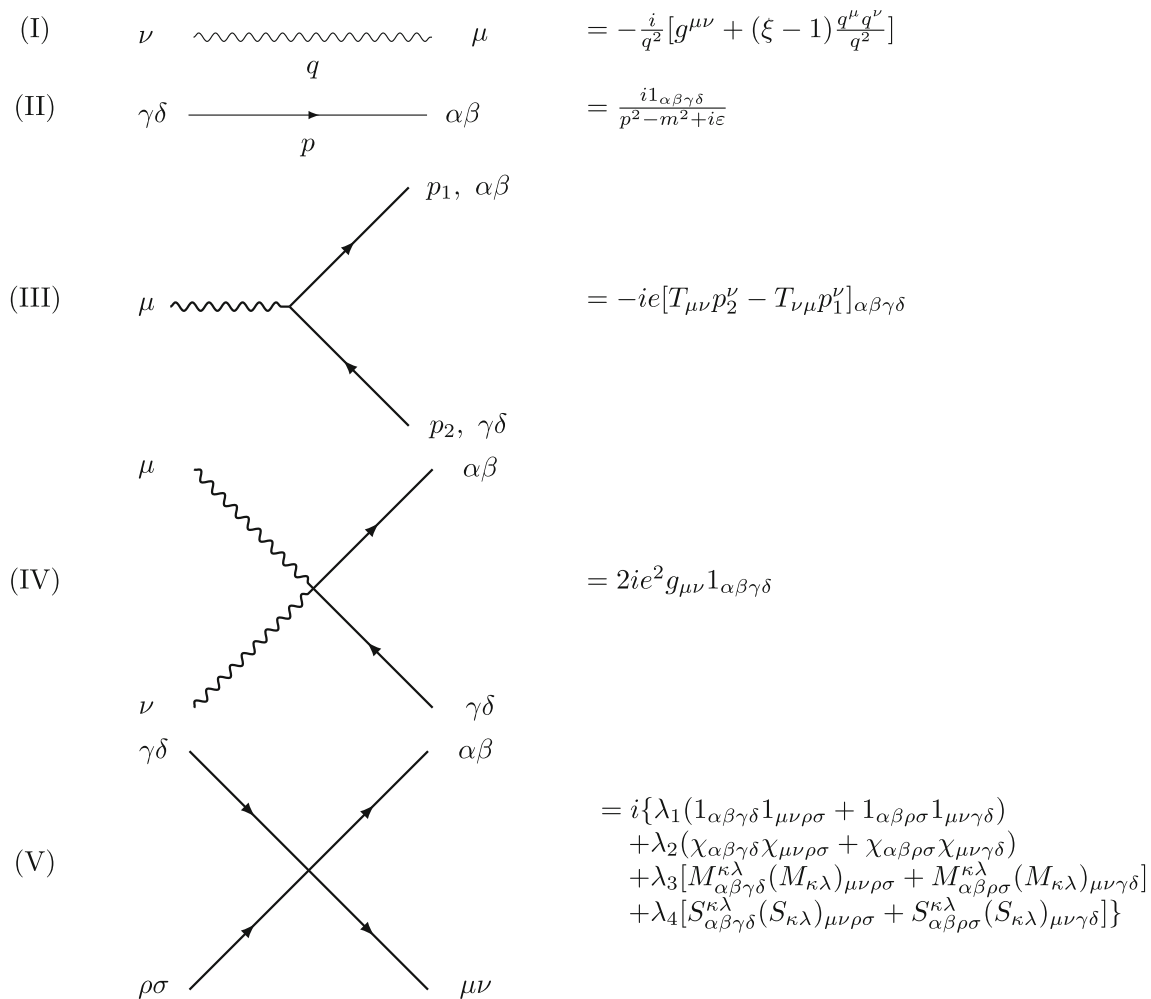
**Fig. 1** Feynman rules of the model

where $q$ is the momentum of the photon, with the tensor self-energy $-i\Sigma(p)$ according to

$$\Gamma^\mu(0, p, -p) = -\frac{\partial \Sigma(p)}{\partial p_\mu}. \tag{2.6}$$

The second one involves the tensor–tensor–photon–photon (TT$\gamma\gamma$) vertex $ie^2\Gamma^{\mu\nu}(q, q', p, p')$, with photon momenta $q$ and $q'$, and the TT$\gamma$ vertex, and reads

$$\Gamma^{\mu\nu}(0, q', p, p') = \frac{\partial \Gamma_\nu(q', p, p')}{\partial p_\mu} + \frac{\partial \Gamma_\nu(q', p, p')}{\partial p'_\mu}. \tag{2.7}$$

## 3 Renormalization

In this section, we analyze the renormalization properties of the model at one-loop level, studying the UV divergent parts of all the potentially divergent vertex functions. In this work, we use dimensional regularization with $d = 4 - 2\epsilon$ and the

naive prescription for the chirality operator $\chi$

$$[\chi, M^{\mu\nu}] = 0, \quad \{\chi, S^{\mu\nu}\} = 0. \tag{3.1}$$

This approach does not lead to inconsistencies as $\chi$ appears in pairs for all the processes involved. The subtraction scheme used in the study is the minimal subtraction (MS) one.

### 3.1 Counterterms

Taking Eq. (2.5) as the bare Lagrangian, with all bare quantities denoted by a 0 subscript, its parameters are the tensor mass $m_0$, the tensor charge $e_0$ and the gyromagnetic factor $g_0$. The renormalized fields are defined in terms of the bare ones through

$$A_r^\mu = Z_1^{-\frac{1}{2}} A_0^\mu, \qquad B_r^{\alpha\beta} = Z_2^{-\frac{1}{2}} B_0^{\alpha\beta}. \tag{3.2}$$

It is convenient to split the Lagrangian as the sum of two terms

$$\mathcal{L}_0 = \mathcal{L}_r + \mathcal{L}_{ct}, \tag{3.3}$$

where the first piece is the renormalized Lagrangian, and has the same structure as Eq. (2.5)

$$
\begin{aligned}
\mathcal{L}_r = &-\frac{1}{4} F_r^{\mu\nu} F_{r\,\mu\nu} - \frac{1}{2\xi_r}(\partial_\mu A_r^\mu)^2 + \partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} \\
&-m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
&-i e_r A_r^\mu [B_r^{\alpha\beta\dagger}(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} \\
&-(\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\
&+e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} \\
&+\frac{\lambda_{r1}}{2}(B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
&+\frac{\lambda_{r2}}{2}(B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
&+\frac{\lambda_{r3}}{2}(B_r^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1}) \\
&\times (B_r^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\
&+\frac{\lambda_{r4}}{2}(B_r^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1}) \\
&\times (B_r^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),
\end{aligned}
\tag{3.4}
$$

and the second one contains the relevant counterterms

$$
\begin{aligned}
\mathcal{L}_{ct} = &-\frac{1}{4}\delta_1 F_r^{\mu\nu} F_{r\,\mu\nu} + \delta_2[\partial^\mu B_r^{\alpha\beta\dagger}\partial_\mu B_{r\,\alpha\beta} \\
&-m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta}] - \delta_m m_r^2 (B_r^{\alpha\beta})^\dagger B_{r\,\alpha\beta} \\
&-i e_r \delta_e A_r^\mu [B_r^{\alpha\beta\dagger}(T_{r\,\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} \\
&-(\partial^\nu B_r^{\alpha\beta\dagger})(T_{r\,\nu\mu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\
&-i e_r \delta_{eg} A_r^\mu [B_r^{\alpha\beta\dagger}(-ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}\partial^\nu B_r^{\gamma\delta} \\
&-(\partial^\nu B_r^{\alpha\beta\dagger})(ig_r)(M_{\mu\nu})_{\alpha\beta\gamma\delta}B_r^{\gamma\delta}] \\
&+\delta_{e2}e_r^2 A_r^\mu A_{r\,\mu} B_r^{\alpha\beta\dagger} B_{r\,\alpha\beta} + \frac{\lambda_{r1}}{2}\delta_{\lambda1}(B_r^{\alpha\beta\,\dagger} 1_{\alpha\beta\gamma\delta} B_r^{\gamma\delta}) \\
&\times (B_r^{\mu\nu\,\dagger} 1_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
&+\frac{\lambda_{r2}}{2}\delta_{\lambda2}(B_r^{\alpha\beta\,\dagger} \chi_{\alpha\beta\gamma\delta} B_r^{\gamma\delta})(B_r^{\mu\nu\,\dagger} \chi_{\mu\nu\rho\sigma} B_r^{\rho\sigma}) \\
&+\frac{\lambda_{r3}}{2}\delta_{\lambda3}(B_r^{\alpha_1\beta_1\,\dagger}(M^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1}) \\
&\times (B_r^{\alpha_2\beta_2\,\dagger}(M_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}) \\
&+\frac{\lambda_{r4}}{2}\delta_{\lambda4}(B_r^{\alpha_1\beta_1\,\dagger}(S^{\mu\nu})_{\alpha_1\beta_1\gamma_1\delta_1} B_r^{\gamma_1\delta_1}) \\
&\times (B_r^{\alpha_2\beta_2\,\dagger}(S_{\mu\nu})_{\alpha_2\beta_2\gamma_2\delta_2} B_r^{\gamma_2\delta_2}),
\end{aligned}
\tag{3.5}
$$

with the following definitions

$$
\begin{aligned}
&\delta_1 \equiv Z_1 - 1, \quad \delta_2 \equiv Z_2 - 1, \quad \delta_m \equiv Z_m - Z_2, \quad \delta_e \equiv Z_e - 1, \\
&\delta_{eg} \equiv Z_{eg} - Z_e, \quad \delta_{e2} \equiv Z_{e2} - 1, \quad \delta_{\lambda j} \equiv Z_{\lambda j} - 1, \quad \xi_r \equiv Z_1^{-1}\xi_0,
\end{aligned}
\tag{3.6}
$$

and

$$
Z_m \equiv \frac{m_0^2}{m_r^2} Z_2, \quad Z_e \equiv \frac{e_0}{e_r} Z_1^{\frac{1}{2}} Z_2, \quad Z_{eg} \equiv \frac{g_0}{g_r} Z_e,
$$

$$
Z_{e2} \equiv \frac{e_0^2}{e_r^2} Z_1 Z_2 \quad Z_{\lambda j} \equiv \frac{\lambda_{0j}}{\lambda_{rj}} Z_2^2.
\tag{3.7}
$$

In $d = 4 - 2\epsilon$ dimensions, the renormalized parameters must be scaled according to

$$
e_r \to \mu^\epsilon e_r, \quad g_r \to g_r, \quad \lambda_{r\,i} \to \mu^{2\epsilon}\lambda_{r\,i}, \quad m_r \to m_r,
\tag{3.8}
$$

where $\mu$ is the arbitrary scale introduced by dimensional regularization. In what follows, we will omit the $r$ subscript for the renormalized parameters. In this notation, the Feynman rules for counterterms are given in Fig. 2.

In the following subsections, we will compile the results obtained for the calculation of all the divergent processes showing that all the divergencies can be absorbed successfully into the given set of counterterms provided by the theory.

### 3.2 Vacuum polarization

There are two diagrams contributing to the vacuum polarization, depicted in Fig. 3. The divergent piece, denoted by $-i\Pi^{\mu\nu}(q)^*$ is given by

$$
-i\Pi^{\mu\nu}(q)^* = i\frac{e^2(2g^2 - 1)}{8\pi^2\epsilon}(q^2 g^{\mu\nu} - q^\mu q^\nu),
\tag{3.9}
$$

and can be removed in the MS scheme by fixing the counterterm $\delta_1$ as

$$
\delta_1 = \frac{e^2(2g^2 - 1)}{8\pi^2\epsilon}.
\tag{3.10}
$$

### 3.3 Tensor self-energy

In Fig. 4 are shown the three diagrams contributing to the Tensor self-energy. The divergent part of this amplitude is

$$
\begin{aligned}
-i\Sigma^*_{\alpha\beta\gamma\delta}(p) = \frac{-i}{32\pi^2\epsilon}\Big\{ &m^2\left(2e^2 g^2 + e^2\xi + 7\lambda_1 + \lambda_2 \right. \\
&\left. +8\lambda_3 + 12\lambda_4\right) - e^2(\xi - 3)p^2 \Big\} 1_{\alpha\beta\gamma\delta},
\end{aligned}
\tag{3.11}
$$

and the counterterms that cancel the UV divergence are then given by

$$
\delta_2 = -\frac{e^2(\xi - 3)}{16\pi^2\epsilon},
\tag{3.12}
$$

$$
\delta_m = -\frac{e^2\left(2g^2 + 3\right) + 7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4}{16\pi^2\epsilon}.
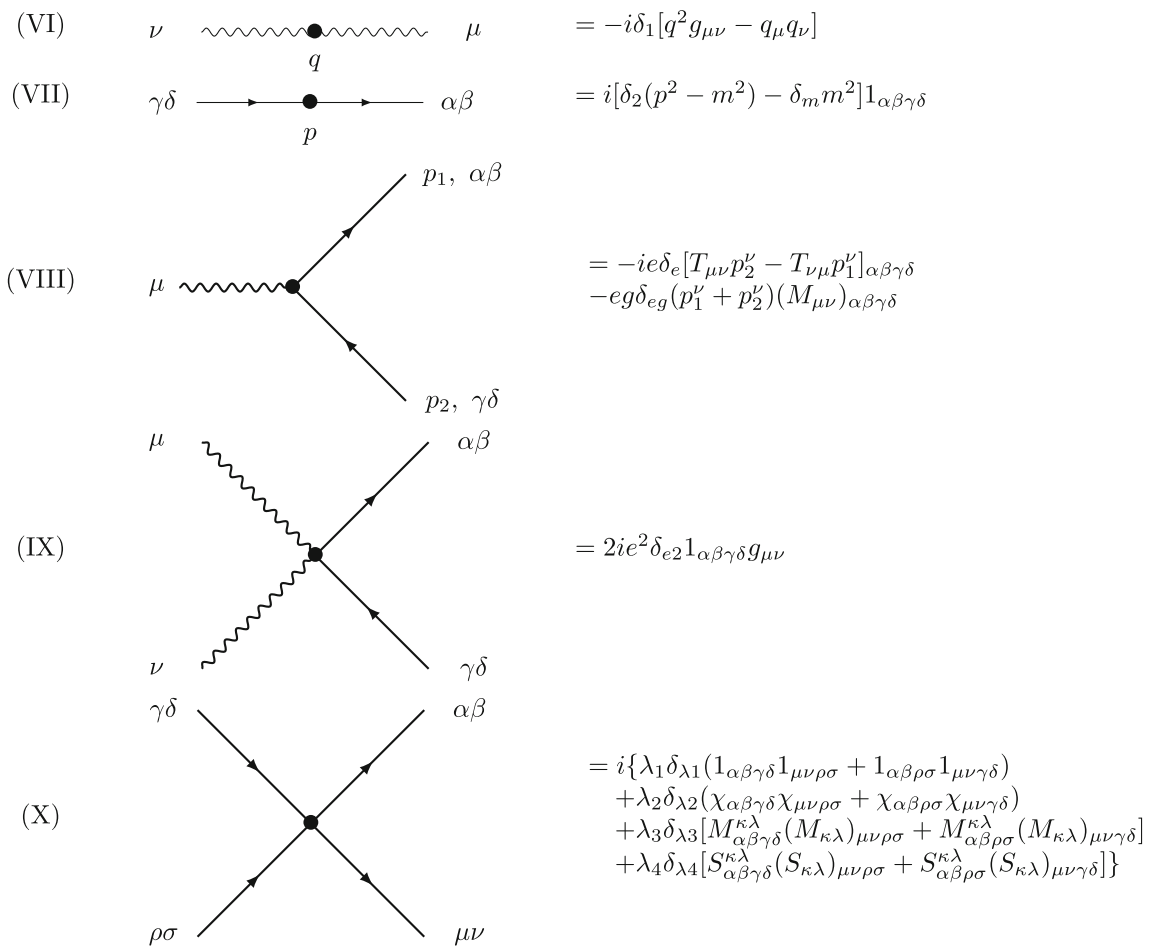\tag{3.13}
$$

(VI)  $\nu$ 〜〜〜〜$\bullet$〜〜〜〜 $\mu$

$q$

$$= -i\delta_1[q^2 g_{\mu\nu} - q_\mu q_\nu]$$

(VII)  $\gamma\delta$ ———$\rightarrow$——$\bullet$——$\rightarrow$——— $\alpha\beta$

$p$

$$= i[\delta_2(p^2 - m^2) - \delta_m m^2]1_{\alpha\beta\gamma\delta}$$

(VIII)  $\mu$ 〜〜〜$\bullet$

$p_1,\ \alpha\beta$

$p_2,\ \gamma\delta$

$$= -ie\delta_e[T_{\mu\nu}p_2^\nu - T_{\nu\mu}p_1^\nu]_{\alpha\beta\gamma\delta}$$
$$-eg\delta_{eg}(p_1^\nu + p_2^\nu)(M_{\mu\nu})_{\alpha\beta\gamma\delta}$$

(IX)

$\mu$    $\alpha\beta$

$\nu$    $\gamma\delta$

$$= 2ie^2\delta_{e2}1_{\alpha\beta\gamma\delta}g_{\mu\nu}$$

(X)

$\gamma\delta$    $\alpha\beta$

$\rho\sigma$    $\mu\nu$

$$= i\{\lambda_1\delta_{\lambda1}(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta})$$
$$+\lambda_2\delta_{\lambda2}(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta})$$
$$+\lambda_3\delta_{\lambda3}[M_{\alpha\beta\gamma\delta}^{\kappa\lambda}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M_{\alpha\beta\rho\sigma}^{\kappa\lambda}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}]$$
$$+\lambda_4\delta_{\lambda4}[S_{\alpha\beta\gamma\delta}^{\kappa\lambda}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S_{\alpha\beta\rho\sigma}^{\kappa\lambda}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}]\}$$

**Fig. 2** Feynman rules for the counterterms

**Fig. 3** Feynman diagrams for the vacuum polarization at one-loop

$q,\nu$ 〜〜〜〜 $q,\mu$      $q,\nu$ 〜〜〜〜 $q,\mu$

1                                    2
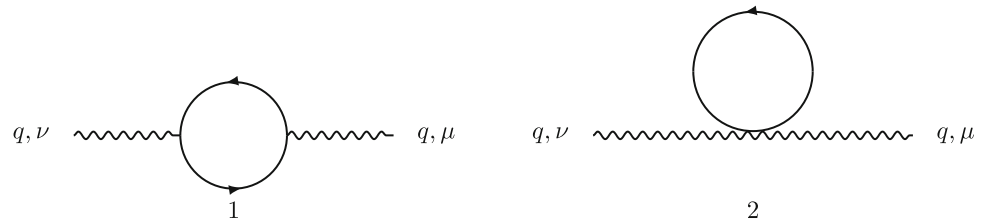
**Fig. 4** Feynman diagrams for the Tensor self-energy at one-loop

$p,\gamma\delta$ ———$\rightarrow$——— $p,\alpha\beta$      $p,\gamma\delta$ ———$\rightarrow$——— $p,\alpha\beta$

1                                    2

$p,\gamma\delta$ ———$\rightarrow$——— $p,\alpha\beta$

3
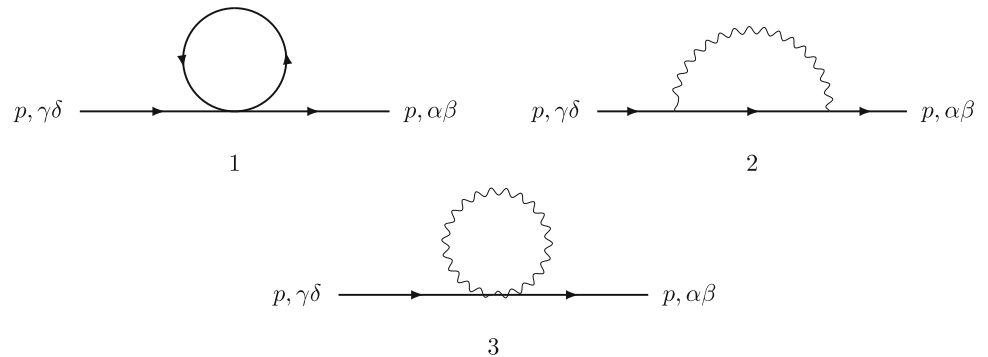
**Fig. 5** Feynman diagrams for the $\gamma\gamma\gamma$ vertex at one-loop
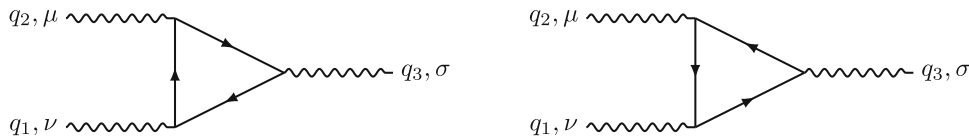


### 3.4 $\gamma\gamma\gamma$ vertex

As expected, the contribution to the $\gamma\gamma\gamma$ vertex from the diagrams in Fig. 5 vanishes identically from the charge conjugation invariance of the theory.

### 3.5 $TT\gamma$ vertex

The one-loop contribution to the $TT\gamma$ vertex comes from the four diagrams in Fig. 6. Its divergent piece can be written as

$$
\begin{aligned}
&-ie\Gamma^{*\mu}_{\alpha\beta\gamma\delta}(-p_1-p_2, p_1, p_2) \\
&= -i\left[\frac{e^3(\xi-3)}{16\pi^2\epsilon}\right][T_{\mu\rho}p_2^\rho - T_{\rho\mu}p_1^\rho]_{\alpha\beta\gamma\delta} \\
&\quad -eg\left[\frac{e^2(g^2+2)+\lambda_1+\lambda_2+12\lambda_3}{16\pi^2\epsilon}\right] \\
&\quad \times (p_1^\rho + p_2^\rho)(M_{\mu\rho})_{\alpha\beta\gamma\delta},
\end{aligned}
\tag{3.14}
$$

and is canceled by the corresponding counterterm with

$$
\delta_e = -\frac{e^2(\xi-3)}{16\pi^2\epsilon}, \tag{3.15}
$$

$$
\delta_{eg} = -\frac{e^2(g^2+2)+\lambda_1+\lambda_2+12\lambda_3}{16\pi^2\epsilon}. \tag{3.16}
$$

Notice that this result is consistent with the Ward identity

$$
\Gamma^{*\mu}(0, p, -p) = -\frac{\partial\Sigma^*(p)}{\partial p_\mu}. \tag{3.17}
$$

as $\delta_e = \delta_2$. Gauge invariance also fixes the counterterm involved in the finiteness of the $TT\gamma\gamma$ vertex, as Eq. (2.7) dictates that

$$
\delta_{e2} = \delta_e = -\frac{e^2(\xi-3)}{16\pi^2\epsilon}. \tag{3.18}
$$

### 3.6 $TT\gamma\gamma$ vertex

There are 12 diagrams contributing to the $TT\gamma\gamma$ vertex at one-loop, as shown in Fig. 7. The corresponding divergent piece is

$$
ie^2\Gamma^{*\mu\nu}_{\alpha\beta\gamma\delta} = ie^2\left[\frac{e^2(\xi-3)}{8\pi^2\epsilon}\right]1_{\alpha\beta\gamma\delta}g^{\mu\nu}, \tag{3.19}
$$

and, as anticipated from the Ward identities, the full $TT\gamma\gamma$ vertex becomes finite with $\delta_{e2}$ given by Eq. (3.18).

### 3.7 $\gamma\gamma\gamma\gamma$ vertex

The one-loop correction to the $\gamma\gamma\gamma\gamma$ vertex involves 21 diagrams, shown in Fig. 8, and there is no counterterm available to cancel a potential divergence in this case. By an explicit calculation, we have found that the divergent piece of the total amplitude vanishes exactly.

### 3.8 $TTTT$ vertex

The last potentially divergent function is the $TTTT$ vertex and there are 19 diagrams contributing to the total amplitude, as shown in Fig. 9. The divergent part of the $TTTT$ vertex is

$$
\begin{aligned}
i\Lambda_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} &= \frac{1}{16\pi^2\epsilon}\Big\{e^4(3g^4 - 8g^2 + 6) + 2\lambda_1 \\
&\times (e^2(2g^2+\xi)+\lambda_2+8\lambda_3+12\lambda_4) \\
&\quad +11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4\Big\}(1_{\alpha\beta\gamma\delta}1_{\mu\nu\rho\sigma} + 1_{\alpha\beta\rho\sigma}1_{\mu\nu\gamma\delta}) \\
&\quad + \frac{1}{8\pi^2\epsilon}\Big\{\lambda_2(e^2(2g^2+\xi)+4\lambda_1+8\lambda_3-8\lambda_4) \\
&\quad +8(\lambda_3-\lambda_4)(e^2g^2+3\lambda_3-3\lambda_4) \\
&\quad +4\lambda_2^2\Big\}(\chi_{\alpha\beta\gamma\delta}\chi_{\mu\nu\rho\sigma} + \chi_{\alpha\beta\rho\sigma}\chi_{\mu\nu\gamma\delta}) \\
&\quad -\frac{1}{16\pi^2\epsilon}\Big\{e^2g^2(\lambda_1+\lambda_2) + 2\lambda_3(e^2\xi+4\lambda_1+4\lambda_2) \\
&\quad +8\lambda_3^2 - 24\lambda_4^2\Big\}[M^{\kappa\lambda}_{\alpha\beta\gamma\delta}(M_{\kappa\lambda})_{\mu\nu\rho\sigma} + M^{\kappa\lambda}_{\alpha\beta\rho\sigma}(M_{\kappa\lambda})_{\mu\nu\gamma\delta}] \\
&\quad -\frac{1}{64\pi^2\epsilon}\Big\{e^4g^4 + 8\lambda_4(e^2(2g^2-\xi)-4\lambda_1+16\lambda_3-8\lambda_4)\Big\} \\
&\quad \times [S^{\kappa\lambda}_{\alpha\beta\gamma\delta}(S_{\kappa\lambda})_{\mu\nu\rho\sigma} + S^{\kappa\lambda}_{\alpha\beta\rho\sigma}(S_{\kappa\lambda})_{\mu\nu\gamma\delta}],
\end{aligned}
\tag{3.20}
$$

and the corresponding counterterms that render the total amplitude finite are given in the MS scheme by

$$
\begin{aligned}
\delta_{\lambda 1} &= -\frac{1}{16\pi^2\lambda_1\epsilon}\Big\{e^4(3g^4 - 8g^2 + 6) \\
&\quad +2\lambda_1(e^2(2g^2+\xi)+\lambda_2+8\lambda_3+12\lambda_4) \\
&\quad +16(\lambda_4(e^2g^2+6\lambda_3)+\lambda_3(e^2g^2+3\lambda_3)+6\lambda_4^2) \\
&\quad +11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4\Big\},
\end{aligned}
\tag{3.21}
$$

$$
\begin{aligned}
\times\delta_{\lambda 2} &= -\frac{1}{8\pi^2\lambda_2\epsilon}\Big\{\lambda_2(e^2(2g^2+\xi)+4\lambda_1+8\lambda_3-8\lambda_4) \\
&\quad +8(\lambda_3-\lambda_4)(e^2g^2+3\lambda_3-3\lambda_4)+4\lambda_2^2\Big\},
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
\delta_{\lambda 3} &= -\frac{1}{16\pi^2\lambda_3\epsilon}\Big\{e^2g^2(\lambda_1+\lambda_2) \\
&\quad +2\lambda_3(e^2\xi+4\lambda_1+4\lambda_2)
\end{aligned}
$$

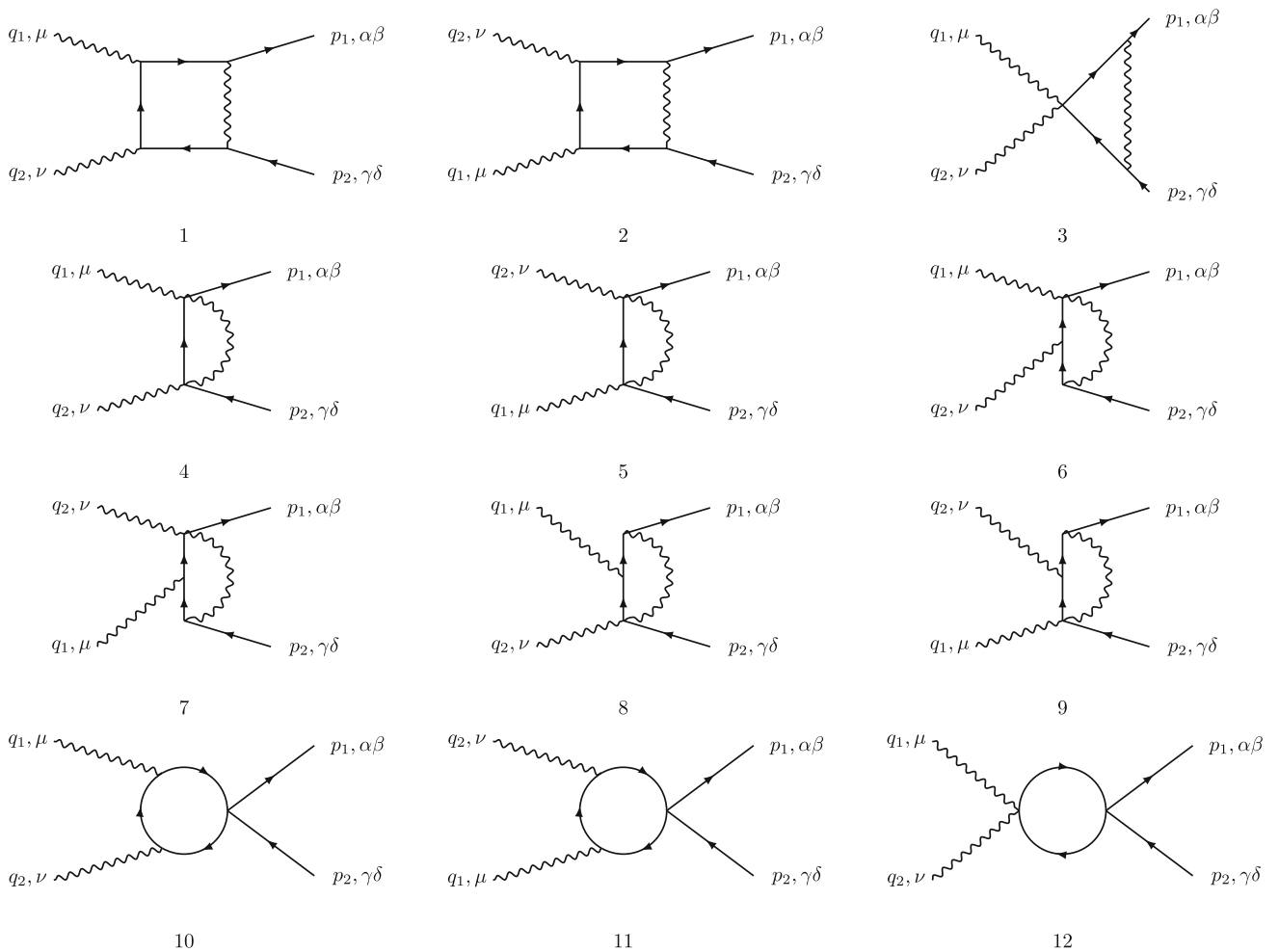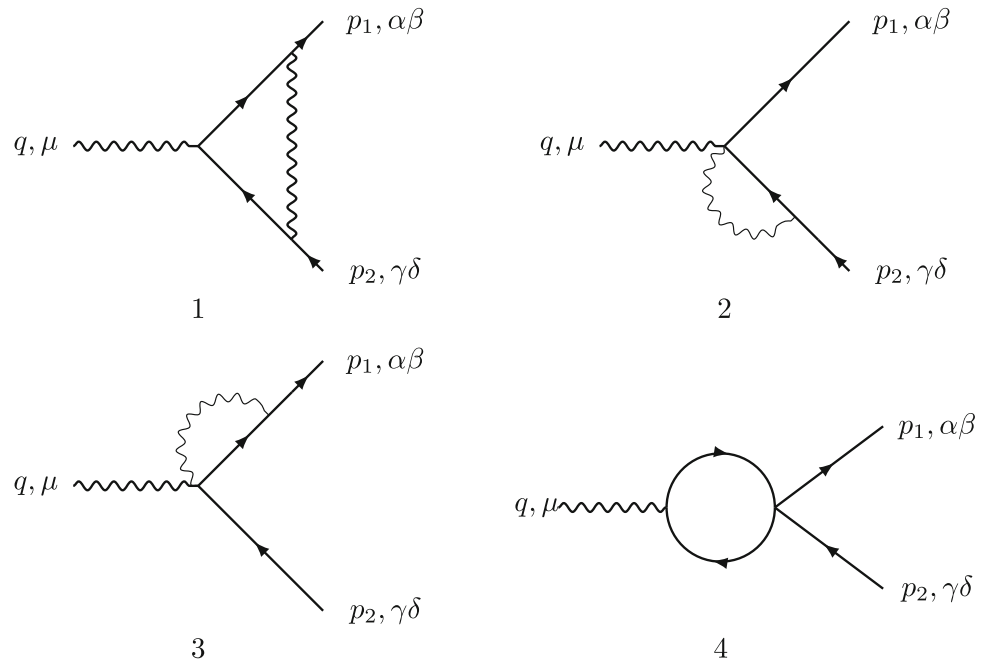**Fig. 6** Feynman diagrams for the TT$\gamma$ vertex at one-loop



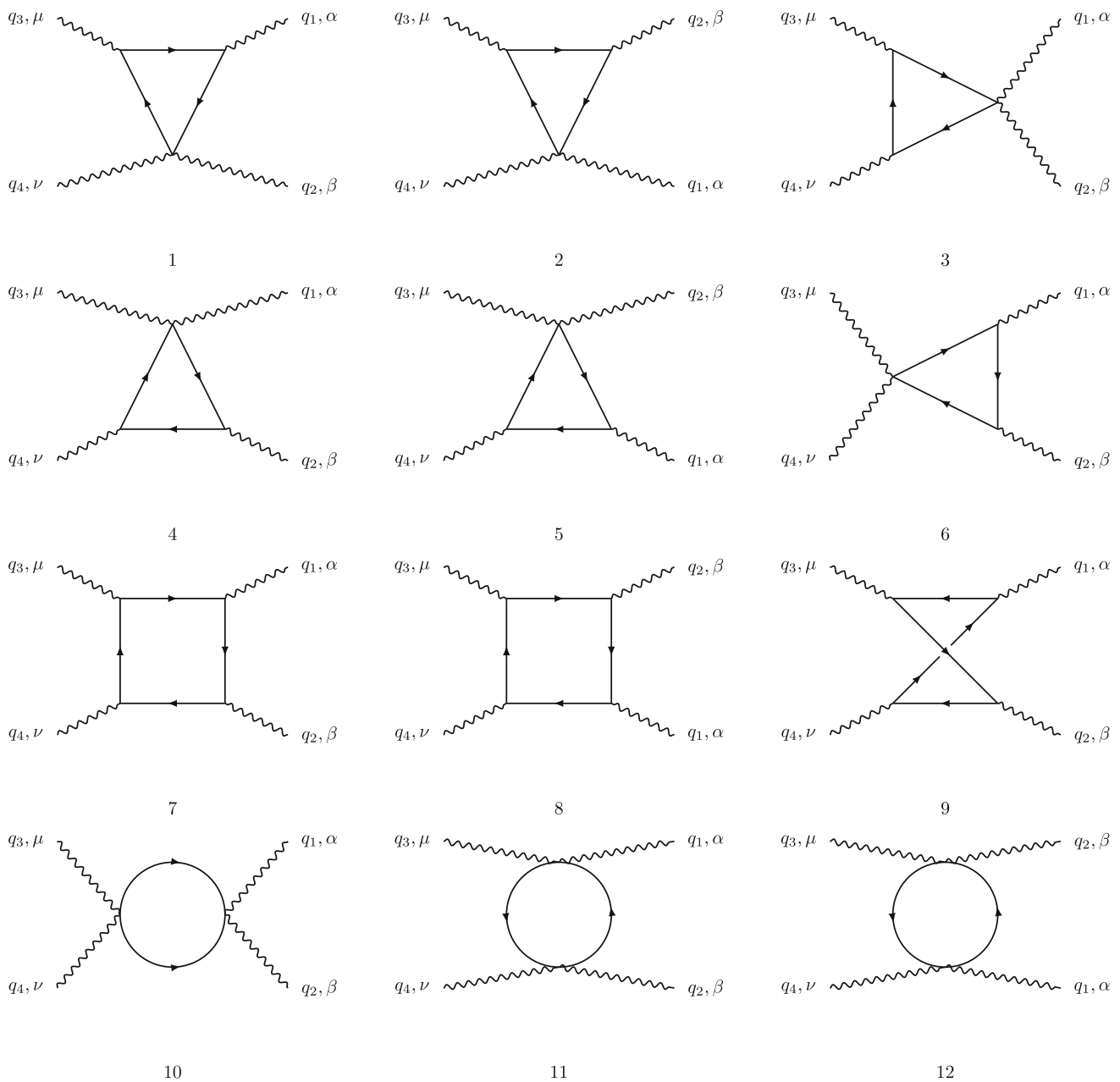**Fig. 7** Feynman diagrams for the TT$\gamma\gamma$ vertex at one-loop

**Fig. 8** Feynman diagrams for the $\gamma\gamma\gamma\gamma$ vertex at one-loop. There are 9 additional diagrams obtained from diagrams $1-9$ reversing the arrow direction in the loop

$$+8\lambda_3^2 - 24\lambda_4^2\Big\}, \tag{3.23}$$

$$\delta_{\lambda 4} = \frac{1}{64\pi^2\lambda_4\epsilon}\Big\{e^4g^4 + 8\lambda_4\left(e^2\left(2g^2 - \xi\right) - 4\lambda_1\right.$$
$$\left. +16\lambda_3 - 8\lambda_4\right)\Big\}. \tag{3.24}$$

### 3.9 Beta functions

From the results obtained in Eqs. (3.10,3.13,3.12,3.15,3.16,3.18, 3.21,3.22,3.23,3.24) and the definitions in Eqs. (3.6,3.7), the relation between the bare and renormalized parameters of the model is

$$e_0 = Z_1^{-\frac{1}{2}}Z_2^{-1}Z_e\mu^\epsilon e, \quad e_0^2 = Z_1^{-1}Z_2^{-1}Z_{e2}\mu^{2\epsilon}e^2,$$
$$\lambda_{0j} = Z_2^{-2}Z_{\lambda_j}\mu^{2\epsilon}\lambda_j,$$
$$g_0 = Z_e^{-1}Z_{eg}g, \quad m_0^2 = Z_2^{-1}Z_m m^2, \tag{3.25}$$

The renormalization constants in the MS subtraction scheme are

$$Z_1 = 1 + \frac{e^2(2g^2 - 1)}{8\pi^2\epsilon}, \tag{3.26}$$
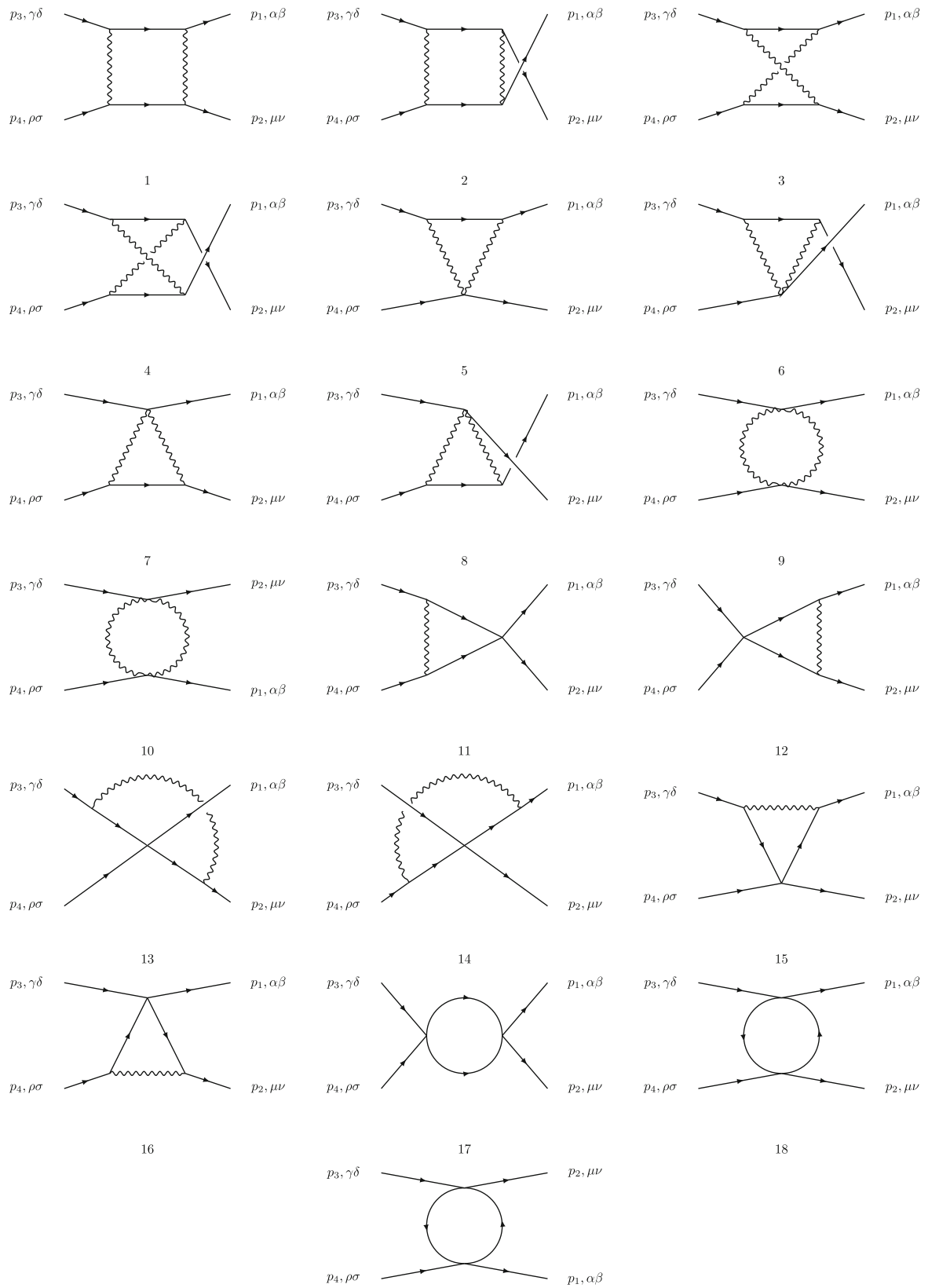
**Fig. 9** Feynman diagrams for the TTTT vertex at one-loop

$$Z_2 = Z_{e2} = Z_e = 1 - \frac{e^2(\xi - 3)}{16\pi^2\epsilon}, \tag{3.27}$$

$$Z_{\lambda_1} = 1 - \frac{1}{16\pi^2\lambda_1\epsilon}\Big\{e^4(3g^4 - 8g^2 + 6)$$
$$+ 2\lambda_1\left(e^2(2g^2 + \xi) + \lambda_2 + 8\lambda_3 + 12\lambda_4\right)$$
$$+ 16\left(\lambda_4(e^2g^2 + 6\lambda_3) + \lambda_3(e^2g^2 + 3\lambda_3) + 6\lambda_4^2\right)$$
$$+ 11\lambda_1^2 + 3\lambda_2^2 - 8\lambda_2\lambda_4\Big\}, \tag{3.28}$$

$$Z_{\lambda_2} = 1 - \frac{1}{8\pi^2\lambda_2\epsilon}$$
$$\times\Big\{\lambda_2\left(e^2\left(2g^2 + \xi\right) + 4\lambda_1 + 8\lambda_3 - 8\lambda_4\right)$$
$$+ 8(\lambda_3 - \lambda_4)\left(e^2g^2 + 3\lambda_3 - 3\lambda_4\right) + 4\lambda_2^2\Big\}, \tag{3.29}$$

$$Z_{\lambda_3} = 1 - \frac{1}{16\pi^2\lambda_3\epsilon}\Big\{e^2g^2(\lambda_1 + \lambda_2)$$
$$+ 2\lambda_3\left(e^2\xi + 4\lambda_1 + 4\lambda_2\right)$$
$$+ 8\lambda_3^2 - 24\lambda_4^2\Big\}, \tag{3.30}$$

$$Z_{\lambda_4} = 1 + \frac{1}{64\pi^2\lambda_4\epsilon}\Big\{e^4g^4 + 8\lambda_4\left(e^2\left(2g^2 - \xi\right)\right.$$
$$\left. - 4\lambda_1 + 16\lambda_3 - 8\lambda_4\right)\Big\}, \tag{3.31}$$

$$Z_{eg} = Z_e + \delta_g = 1 - \frac{1}{16\pi^2\epsilon}\Big\{e^2(g^2 + \xi - 1) + \lambda_1$$
$$+ \lambda_2 + 12\lambda_3\Big\}, \tag{3.32}$$

$$Z_m = Z_2 + \delta_m = 1 - \frac{1}{16\pi^2\epsilon}\Big\{e^2\left(2g^2 + \xi\right) + 7\lambda_1$$
$$+ \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. \tag{3.33}$$

With the above results, the two different relations between $e_0$ and $e$ in Eq. (3.25) become

$$e_0 = Z_1^{-1/2}\mu^\epsilon e. \tag{3.34}$$

From Eqs. (3.25–3.33) one can derive the relevant beta functions $\beta_\eta \equiv \mu\frac{\partial\eta}{\partial\mu}$ and anomalous dimensions $\gamma_m \equiv \frac{\mu}{m}\frac{\partial m}{\partial\mu}$ of the theory in the $\epsilon \to 0$ limit:

$$\beta_e = \frac{e^3\left(1 - 2g^2\right)}{8\pi^2}, \tag{3.35}$$

$$\beta_g = -\frac{g\left[e^2\left(g^2 + 2\right) + \lambda_1 + \lambda_2 + 12\lambda_3\right]}{8\pi^2}, \tag{3.36}$$

$$\beta_{\lambda_1} = \frac{1}{8\pi^2}\Big\{e^4\left(-3g^4 + 8g^2 - 6\right) - 2\lambda_1\left(e^2\left(2g^2 + 3\right)\right.$$
$$+ \lambda_2 + 8\lambda_3 + 12\lambda_4)$$
$$- 16\left(\lambda_4\left(e^2g^2 + 6\lambda_3\right) + \lambda_3\left(e^2g^2 + 3\lambda_3\right) + 6\lambda_4^2\right)$$
$$- 11\lambda_1^2 - 3\lambda_2^2 + 8\lambda_2\lambda_4\Big\}, \tag{3.37}$$

$$\beta_{\lambda_2} = -\frac{1}{4\pi^2}\Big\{\lambda_2\left(e^2\left(2g^2 + 3\right) + 4\lambda_1 + 8\lambda_3 - 8\lambda_4\right)$$

$$+ 8(\lambda_3 - \lambda_4)\left(e^2g^2 + 3\lambda_3 - 3\lambda_4\right) + 4\lambda_2^2\Big\}, \tag{3.38}$$

$$\beta_{\lambda_3} = -\frac{1}{8\pi^2}\Big\{e^2g^2(\lambda_1 + \lambda_2) + 2\lambda_3\left(3e^2 + 4\lambda_1 + 4\lambda_2\right)$$
$$+ 8\lambda_3^2 - 24\lambda_4^2\Big\}, \tag{3.39}$$

$$\beta_{\lambda_4} = \frac{1}{32\pi^2}\Big\{e^4g^4 + 8\lambda_4\left(e^2\left(2g^2 - 3\right)\right.$$
$$\left. - 4\lambda_1 + 16\lambda_3 - 8\lambda_4\right)\Big\}, \tag{3.40}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{e^2\left(2g^2 + 3\right) + 7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. \tag{3.41}$$

We conclude this section with a short discussion of some of the possible scenarios of the theory. There is a trivial fixed point for the beta functions of the theory when $g = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ and $\lambda_4 = 0$. This fixed point corresponds to the limit in which each component of the tensor $B^{\mu\nu}$ behaves as a complex scalar field in a $\lambda\phi^4$ theory with $\lambda_1 = -\lambda/2$. On the other hand, the $\beta_{\lambda_i}$ are all nonzero for any non-vanishing real value of the gyromagnetic factor $g$, even if all self interactions are set to $\lambda_i = 0$, $i = 1, \ldots, 4$. This means that, oppositely to the spin $1/2$ case studied in [6], pure electrodynamics for matter fields of spin 1 is not viable for $g \neq 0$, as self interactions are necessary to make the theory renormalizable. Finally, turning off the electromagnetic interactions by taking $e = 0$ and $g = 0$, the theory reduces to a renormalizable model of pure self-interacting terms for the tensor fields, with

$$\beta_{\lambda_1} = -\frac{1}{8\pi^2}\Big\{11\lambda_1^2 + 2\lambda_1\left(\lambda_2 + 8\lambda_3 + 12\lambda_4\right)$$
$$+ 3\lambda_2^2 + 48\lambda_3^2$$
$$- 8\lambda_2\lambda_4 + 96\lambda_4\left(\lambda_3 + \lambda_4\right)\Big\}, \tag{3.42}$$

$$\beta_{\lambda_2} = -\frac{1}{\pi^2}\Big\{\lambda_2^2 + \lambda_1\lambda_2 + 2\lambda_2\left(\lambda_3 - \lambda_4\right)$$
$$+ 6\left(\lambda_3 - \lambda_4\right)^2\Big\}, \tag{3.43}$$

$$\beta_{\lambda_3} = \frac{1}{\pi^2}\Big\{3\lambda_4^2 - \lambda_3\left(\lambda_1 + \lambda_2 + \lambda_3\right)\Big\}, \tag{3.44}$$

$$\beta_{\lambda_4} = -\frac{1}{\pi^2}\Big\{\lambda_4\left(\lambda_1 - 4\lambda_3 + 2\lambda_4\right)\Big\}, \tag{3.45}$$

$$\gamma_m = -\frac{1}{16\pi^2}\Big\{7\lambda_1 + \lambda_2 + 8\lambda_3 + 12\lambda_4\Big\}. \tag{3.46}$$

## 4 Summary and conclusions

In this work, we have studied the one-loop renormalization of the electrodynamics of fields transforming under the $(1, 0) \oplus (1, 0)$ representation of the HLG in the Poincaré projector formalism. The analysis has been done in an arbitrary covariant gauge, with arbitrary gyromagnetic factor and including all the independent parity conserving self-interactions. The main conclusion of the work is that the

theory is renormalizable for any value of the gyromagnetic factor, displaying a rich set of renormalization group equations. In contrast to the analogous spin 1/2 case studied in [6], there is no non-trivial finite value for the gyromagnetic factor that allows the existence of a pure electrodynamics without the inclusion of self interactions.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors' comment: . . .].

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