Low energy pion- Λ_b interaction

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Abstract The low energy pion- Λ_b interaction is study in this work, considering effective chiral Lagrangians that include pions, baryons and the corresponding resonances. Interactions mediated by a σ meson exchange are also considered. The scattering amplitudes are calculated and then we determine the angular distributions and polarizations.

1 Introduction

The b baryons physics is a subject that recently have received special attention, as a large number of this kind of particle is being produced in many experiments, as for example, in the ATLAS [1], LHCb [2] and CMS [3]. These data in addition with previous ones [4–7] makes possible a better understanding of the b baryons and of their interactions experimentally and at the theoretical level [8–10]. Quark decays of the type $b \rightarrow s\gamma$ allow the search of physics beyond the standard model [11–13] and the experimental data impose constraints when new physics is being considered.

The polarization and the asymmetry parameter α are fundamental observables that may be used with this purpose. The basic idea to be considered is the same one that has been used in the study of hyperons in the HyperCP experiment [14, 15], where the Λ hyperon was produced in high energy reactions and its polarization and the asymmetry parameter α_Λ could be determined by the $\Lambda \rightarrow p\pi^-$ decay. A similar method has been used in the study of Λ_b considering the decay $\Lambda_b \rightarrow J/\psi \Lambda$, produced in high energy collisions, where α_b and the polarization have been measured [1–3]. If a b or a s quark are produced polarized, it is expected that a large part of this polarization should remain in the produced baryon, and in general a straightforward way to understand these results is to consider that the observed polarization is the polarization of the produced hyperon or of the b baryon.

But in fact, another hypothesis may be considered, that is that the hyperons and the bottom baryons may be polarized after their production or to have their polarizations altered by final-state interactions.

In [16–18] the polarization of hyperons and antihyperons produced in high energy collisions have been calculated taking into account the final-state interactions and was successful in explaining the polarization of antihyperons that should be produced unpolarized if this effect was not considered. In the proposed model, based on relativistic hydrodynamics, during the collision a hot expanding medium is formed, and inside this medium the hyperons are produced. These hyperons interacts with the surrounding particles and then emerges, and may be polarized by these interactions. As far as the most probable particles are pions, the dominant interaction is the pion-hyperon interaction at low energies. Despite the fact that these particles are observed with high energies at the laboratory, the relative energies of the particles inside a fluid element is small.

So, in order to understand these interactions a careful study of the low energy pion-hyperon interactions has been done [19–21], and recently, in order to improve the model, the kaon-hyperon interactions has been also studied [22] and [23]. In these models, effective nonlinear chiral Lagrangians, including hyperons, resonances and mesons have been taken into account. Then, the coupling constants, cross-sections and polarizations have been determined.

Thinking in general terms, the Λ_b polarization may be understood exactly in the same way. In processes where the Λ_b are considered to be produced unpolarized, they may become polarized if the final-state interactions are considered, or if they are produced polarized, the final polarization may be altered by these interactions. So, a first step to understand this effect is to study the low energy pion- Λ_b interactions. This study will be conducted in a way analogous to the one that it has been done in [19–23], by considering nonlinear effective Lagrangians, and then the Λ_b polarization may be studied.

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This work will have the following content: in Sect. 2 the basic formalism will be reviewed, in Sect. 3 the $\pi\text{-}\Lambda_b$ interaction will be studied, in Sect. 4 the results will be presented and in Sect. 5, the conclusions. Some expressions and integrals will be shown in the Appendix.

2 Basic formalism

In order to study the low energy $\pi\Lambda_b$ interaction we will consider a formalism that has been used to describe pion-hyperon (πY) [19–21] and recently the kaon-hyperon (KY) [22,23] interactions, which is based in nonlinear effective chiral Lagrangians that takes into account baryons, resonances and mesons as degrees of freedom. Observing the fact that the low energies pion-nucleon interactions are very well known, experimentally and theoretically, in [24,25] a model that was successful in explaining these interactions has been adapted in order to study the πY interactions [19] (and recently the KY interactions).

In [24,25] the description of the $\pi N \rightarrow \pi N$ scattering have been made considering a chiral model that takes into account vertices with spin-1/2 and spin-3/2 baryons that are represented by the effective Lagrangians

$$\mathcal{L}_{\pi NN} = \frac{g}{2m} (\bar{N} \gamma_\mu \gamma_5 \vec{\tau} N) \cdot \partial^\mu \vec{\phi}, \quad (1)$$

$$\mathcal{L}_{\pi N\Delta} = g_\Delta \left\{ \bar{\Delta}^\mu \left[g_{\mu\nu} - (Z + 1/2) \gamma_\mu \gamma_\nu \right] \vec{M} N \right\} \cdot \partial^\nu \vec{\phi}, \quad (2)$$

where N , Δ , $\vec{\phi}$ are the nucleon, delta, and pion fields with masses m , m_Δ and m_π , respectively, $\vec{M}$ and $\vec{\tau}$ are isospin matrices that combine a nucleon and a delta in a nucleon and a pion isospin 1 state, and Z is a parameter representing the possibility of the off-shell- Δ having spin 1/2. The parameters g and g_Δ are coupling constants. So, a reasonable procedure to be followed is to adapt these Lagrangians in order to study the $\pi\Lambda_b$ interactions.

The scattering amplitude to the process $\pi\Lambda_b \rightarrow \pi\Lambda_b$ may be parametrized as

$$T_{\pi\Lambda_b} = \bar{u}(\vec{p}') \left[A + \frac{1}{2} (\not{k} + \not{k}') B \right] u(\vec{p}), \quad (3)$$

where $u(\vec{p})$ and $\bar{u}(\vec{p}')$ are the spinors that represents initial and final Λ_b , p_μ and p'_μ are the initial and final Λ_b 4-momenta and k_μ k'_μ and are the pion initial and final 4-momenta. A and B are amplitudes that may be calculated.

Thus, in this work our task will be to calculate the A and B amplitudes for each considered diagram. The scattering matrix is given by

$$M = \frac{T_{\pi\Lambda_b}}{8\pi\sqrt{s}} = f(k, x) + g(k, x) \vec{\sigma} \cdot \hat{n}, \quad (4)$$

which may be decomposed into the spin-non-flip and spin-flip amplitudes $f(k, x)$ and $g(k, x)$, respectively, written in terms of the variables k and $x = \cos \theta$, determined in the center-of-mass frame, where θ is the scattering angle. These spin amplitudes may be expanded in terms of partial-waves amplitudes $a_{l\pm}$ by the expressions

$$f(k, x) = \sum_{l=0}^{\infty} \left[(l+1)a_{l+}(k) + la_{l-}(k) \right] P_l(x), \quad (5)$$

$$g(k, x) = i \sum_{l=1}^{\infty} \left[a_{l-}(k) - a_{l+}(k) \right] P_l^{(1)}(x). \quad (6)$$

Using the orthogonality relations of the Legendre polynomials, the partial-wave amplitudes may be determined

$$a_{l\pm} = \frac{1}{2} \int_{-1}^1 \left[P_l(x) f_1(k, x) + P_{l\pm 1}(x) f_2(k, x) \right] dx, \quad (7)$$

with

$$f_1(k, x) = \frac{(E+m)}{8\pi\sqrt{s}} [A + (\sqrt{s}-m)B], \quad (8)$$

$$f_2(k, x) = \frac{(E-m)}{8\pi\sqrt{s}} [-A + (\sqrt{s}+m)B], \quad (9)$$

where E is the Λ_b energy and s is a Mandelstam variable (see the Appendix). As we are interested in studying the low energy interactions ($k < 0.5$ GeV), a good approximation is to consider only the S ($l=0$) and P ($l=1$) waves, as the waves with $l \geq 2$ have amplitudes much smaller than the S and P ones at these energies and may be considered just as small corrections. In the following sections these amplitudes will be calculated.

3 $\pi\Lambda_b$ interaction

In this section we will show how to use the formalism presented in the preceding section in order to compute the amplitudes A and B of Eq. (3) considering the Feynman diagrams displayed in Fig. 1. The Table 1 shows the proprieties of the particles that will be considered in the calculations where J is the spin with parity π , and I , the isospin.

The interaction with a Σ_b spin-1/2 baryon in the intermediate state will be studied in Sect. 3.1 and the Σ_b^* spin-3/2 pole in Sect. 3.2. The amplitude for the the scalar σ meson exchange shown in Fig. 1e, will be included as a parametrization as it has been made in [19,20]. So the amplitudes to be considered are

$$A_\sigma = a + bt, \quad (10)$$

$$B_\sigma = 0, \quad (11)$$

with $a = 1.05m_\pi^{-1}$, $b = -0.8m_\pi^{-3}$ where m_π is the pion mass [26] and t is the Mandelstam variable defined in

Fig. 1 Diagrams to be considered in the $\pi \Lambda_b$ interaction

Table 1 Resonances considered in $\pi \Lambda_b$ interaction

	J^π	I	Mass (MeV)
Σ_b	$1/2^+$	1	5812
Σ_b^*	$3/2^+$	1	5832

the Appendix. More discussions about these amplitudes may be found in [21, 28–34]. From these results the partial-wave amplitudes $f_{l\pm}$ for the diagrams of Fig. 1 may be calculated.

3.1 Σ_b baryon

In order to study the $\pi \Lambda_b$ interactions, the Lagrangians (1) and (2) will be adapted considering the pion and the Λ_b as initial particles with Σ_b (spin-1/2) and Σ_b^* (spin-3/2) in the intermediate states. Thus the Lagrangian (1) will be written as

$$\mathcal{L}_{\pi \Lambda_b \Sigma_b} = \frac{g_{\pi \Lambda_b \Sigma_b}}{2m_{\Lambda_b}} (\bar{\Sigma}_b \gamma_\mu \gamma_5 \Lambda_b) \cdot \partial^\mu \vec{\phi}, \quad (12)$$

which determines the vertex

$$V_{\pi \Lambda_b \Sigma_b} = \frac{g_{\pi \Lambda_b \Sigma_b}}{2m_{\Lambda_b}} \not{k} \gamma_5, \quad (13)$$

where $g_{\pi \Lambda_b \Sigma_b}$ is the coupling constant for the $\pi \Lambda_b \Sigma_b$ interaction and m_{Λ_b} is the Λ_b mass.

Calculating the contribution of the diagrams *a* and *b* of Fig. 1 to the $T_{\pi \Lambda_b}$ amplitude, relative to the process $\pi \Lambda_b \rightarrow$

$\pi \Lambda_b$ and writing them in the form of Eq. (3) we have

$$A_{\Sigma_b} = \frac{g_{\pi \Lambda_b \Sigma_b}^2}{4m_{\Lambda_b}^2} (m_{\Sigma_b} + m_{\Lambda_b}) \left(\frac{s - m_{\Lambda_b}^2}{s - m_{\Sigma_b}^2} + \frac{u - m_{\Lambda_b}^2}{u - m_{\Sigma_b}^2} \right), \quad (14)$$

$$B_{\Sigma_b} = \frac{g_{\pi \Lambda_b \Sigma_b}^2}{4m_{\Lambda_b}^2} \left[\frac{2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b}) + u - m_{\Lambda_b}^2}{u - m_{\Sigma_b}^2} - \frac{2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b}) + s - m_{\Lambda_b}^2}{s - m_{\Sigma_b}^2} \right], \quad (15)$$

where u is a Mandelstam variable and m_{Σ_b} is the Σ_b mass.

Using Eqs. (14) and (15) in Eq. (8) we have

$$f_1 = \frac{g_{\pi \Lambda_b \Sigma_b}^2 (E + m_{\Lambda_b})}{32\pi m_{\Lambda_b}^2 \sqrt{s}} \left\{ \frac{1}{s - m_{\Sigma_b}^2} \left[(m_{\Sigma_b} + 2m_{\Lambda_b} - \sqrt{s})(s - m_{\Lambda_b}^2) + 2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} - \sqrt{s}) \right] + \frac{1}{u - m_{\Sigma_b}^2} \left[2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(\sqrt{s} - m_{\Lambda_b}) + (m_{\Sigma_b} + \sqrt{s})(u - m_{\Lambda_b}^2) \right] \right\}. \quad (16)$$

Considering Eqs. (44) and (45) defined in Appendix, is convenient write the f_1 amplitude as function of $x = \cos \theta$

$$f_1 = \frac{g_{\pi \Lambda_b \Sigma_b}^2 (E + m_{\Lambda_b})}{32\pi m_{\Lambda_b}^2 \sqrt{s}} \left\{ a_1 + \frac{b_1 + c_1 x}{\gamma_{\Sigma_b} + 2|\vec{k}|^2 x} \right\}, \quad (17)$$

where

$$a_1 = \frac{(m_{\Sigma_b} + 2m_{\Lambda_b} - \sqrt{s})(s - m_{\Lambda_b}^2) + 2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} - \sqrt{s})}{s - m_{\Sigma_b}^2};$$

$$b_1 = 2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} - \sqrt{s}) + (m_{\Sigma_b} + \sqrt{s})(2Ek_0 - m_\pi^2);$$

$$c_1 = 2|\vec{k}|^2(\sqrt{s} + m_{\Sigma_b});$$

$$\gamma_{\Sigma_b} = m_{\Sigma_b}^2 - m_{\Lambda_b}^2 - m_\pi^2 + 2Ek_0.$$

and k_0 is the pion energy.

From (14), (15) and (9), we have

$$f_2 = \frac{g_{\pi \Lambda_b \Sigma_b}^2 (E - m_{\Lambda_b})}{32\pi m_{\Lambda_b}^2 \sqrt{s}} \left\{ \frac{1}{s - m_{\Sigma_b}^2} \left[(m_{\Sigma_b} + 2m_{\Lambda_b} + \sqrt{s})(m_{\Lambda_b}^2 - s) - 2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} + \sqrt{s}) \right] + \frac{1}{u - m_{\Sigma_b}^2} \left[2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} + \sqrt{s}) + (\sqrt{s} - m_{\Sigma_b})(u - m_{\Lambda_b}^2) \right] \right\}, \quad (18)$$

and following the same procedure that has been followed in the calculation of f_1 we obtain

$$f_2 = \frac{g_{\pi\Lambda_b\Sigma_b}^2(E - m_{\Lambda_b})}{32\pi m_{\Lambda_b}^2\sqrt{s}} \left\{ a_2 + \frac{b_2 + c_2x}{\gamma_{\Sigma_b} + 2|\vec{k}|^2x} \right\}, \quad (19)$$

where

$$a_2 = \frac{(m_{\Sigma_b} + 2m_{\Lambda_b} + \sqrt{s})(m_{\Lambda_b}^2 - s) - 2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(m_{\Lambda_b} + \sqrt{s})}{s - m_{\Sigma_b}^2};$$

$$b_2 = -2m_{\Lambda_b}(m_{\Lambda_b} + m_{\Sigma_b})(\sqrt{s} + m_{\Lambda_b}) + (\sqrt{s} - m_{\Sigma_b})(2Ek_0 - m_{\pi}^2);$$

$$c_2 = 2|\vec{k}|^2(\sqrt{s} - m_{\Sigma_b}).$$

Thus, using Eqs. (17) and (62) in Eq. (7) the S , P_1 and P_3 partial-waves amplitudes may be calculated

$$a_S^{(\Sigma_b)} = \frac{g_{\pi\Lambda_b\Sigma_b}^2}{64\pi m_{\Lambda_b}^2\sqrt{s}} \left\{ 2(E + m_{\Lambda_b})a_1 + (E + m_{\Lambda_b})b_1I_0 + [(E + m_{\Lambda_b})c_1 + (E - m_{\Lambda_b})b_2]I_1 + (E - m_{\Lambda_b})c_2I_2 \right\}, \quad (20)$$

$$a_{P_1}^{(\Sigma_b)} = \frac{g_{\pi\Lambda_b\Sigma_b}^2}{64\pi m_{\Lambda_b}^2\sqrt{s}} \left\{ 2(E - m_{\Lambda_b})a_2 + (E - m_{\Lambda_b})b_2I_0 + [(E + m_{\Lambda_b})b_1 + (E - m_{\Lambda_b})c_2]I_1 + (E + m_{\Lambda_b})c_1I_2 \right\}, \quad (21)$$

$$a_{P_3}^{(\Sigma_b)} = \frac{g_{\pi\Lambda_b\Sigma_b}^2}{64\pi m_{\Lambda_b}^2\sqrt{s}} \left\{ \left(\frac{m_{\Lambda_b} - E}{2}\right)b_2I_0 + [(E + m_{\Lambda_b})b_1 + \left(\frac{m_{\Lambda_b} - E}{2}\right)c_2]I_1 + \left[\frac{3}{2}(E - m_{\Lambda_b})b_2 + (E + m_{\Lambda_b})c_1\right]I_2 + \frac{3}{2}(E - m_{\Lambda_b})c_2I_3 \right\}, \quad (22)$$

where I_0 , I_1 , I_2 and I_3 are integrals defined in the Appendix.

These amplitudes will be used in Sect. 4 in order to calculate the observables of interest.

3.2 Σ_b^* baryon

Now, adapting the Lagrangian (2) in order to describe a the spin-3/2 Σ_b^* resonance in the intermediate state, represented in the diagrams *c* and *d* of Fig. 1, we have

$$\mathcal{L}_{\pi\Lambda_b\Sigma_b^*} = g_{\pi\Lambda_b\Sigma_b^*} \left\{ \bar{\Sigma}_b^{*\mu} [g_{\mu\nu} - (Z + 1/2)\gamma_\mu\gamma_\nu] \Lambda_b \right\} \cdot \partial^\nu \vec{\phi}, \quad (23)$$

as the Λ_b hyperon has isospin 0, the isospin matrix $\vec{M}$ in Eq. (2) is not needed. So, the Feynman vertex is

$$V_{\pi\Lambda_b\Sigma_b^*} = g_{\pi\Lambda_b\Sigma_b^*} [k^\mu - (Z + 1/2)\gamma^\mu \not{k}], \quad (24)$$

and then the amplitudes are

$$A_{\Sigma_b^*} = \frac{2g_{\pi\Lambda_b\Sigma_b^*}^2}{9m_{\Lambda_b}} \left[\frac{\nu_{\Sigma_b^*}}{\nu_{\Sigma_b^*}^2 - \nu^2} \hat{A} + m_{\Lambda_b}(-a_0 + a_z k \cdot k') \right], \quad (25)$$

$$B_{\Sigma_b^*} = \frac{2g_{\pi\Lambda_b\Sigma_b^*}^2}{9m_{\Lambda_b}} \left[\frac{\nu}{\nu_{\Sigma_b^*}^2 - \nu^2} \hat{B} - (2m_{\Lambda_b}^2 b_0)\nu \right], \quad (26)$$

where $g_{\pi\Lambda_b\Sigma_b^*}$ is the coupling constant and ν , $\nu_{\Sigma_b^*}$ and $k \cdot k'$ are defined in the Appendix. The other terms that appear in Eqs. (25) and (26) are defined as

$$\hat{A} = \frac{(m_{\Sigma_b^*} + m_{\Lambda_b})^2 - m_{\pi}^2}{2m_{\Sigma_b^*}^2} \left[2m_{\Sigma_b^*}^3 - 2m_{\Lambda_b}^3 - 2m_{\Lambda_b}m_{\Sigma_b^*}^2 - 2m_{\Lambda_b}^2m_{\Sigma_b^*} + m_{\pi}^2(2m_{\Lambda_b} - m_{\Sigma_b^*}) \right] + \frac{3}{2}(m_{\Lambda_b} + m_{\Sigma_b^*})t, \quad (27)$$

$$\hat{B} = \frac{1}{2m_{\Sigma_b^*}^2} \left[(m_{\Sigma_b^*}^2 - m_{\Lambda_b}^2)^2 - 2m_{\Lambda_b}m_{\Sigma_b^*}(m_{\Lambda_b} + m_{\Sigma_b^*})^2 - 2m_{\pi}^2(m_{\Lambda_b} + m_{\Sigma_b^*})^2 + 6m_{\pi}^2m_{\Sigma_b^*}(m_{\Lambda_b} + m_{\Sigma_b^*}) + m_{\pi}^4 \right] + \frac{3}{2}t, \quad (28)$$

$$a_0 = \frac{(m_{\Lambda_b} + m_{\Sigma_b^*})}{m_{\Sigma_b^*}^2} (2m_{\Sigma_b^*}^2 + m_{\Lambda_b}m_{\Sigma_b^*} - m_{\Lambda_b}^2 + 2m_{\pi}^2), \quad (29)$$

$$a_z = \frac{4}{m_{\Sigma_b^*}^2} \left[(m_{\Sigma_b^*} + m_{\Lambda_b})Z + (2m_{\Sigma_b^*} + m_{\Lambda_b})Z^2 \right], \quad (30)$$

$$b_0 = \frac{4Z^2}{m_{\Sigma_b^*}^2}, \quad (31)$$

where $m_{\Sigma_b^*}$ is the resonance mass.

Then, substituting (25) and (26) in the Eq. (8), we find

$$f_1 = \frac{g_{\pi\Lambda_b\Sigma_b^*}^2(E + m_{\Lambda_b})}{9 \cdot 4\pi\sqrt{s}} \left\{ \frac{\hat{A} + (\sqrt{s} - m_{\Lambda_b})\hat{B}}{m_{\Sigma_b^*}^2 - s} + \frac{\hat{A} + (m_{\Lambda_b} - \sqrt{s})\hat{B}}{m_{\Sigma_b^*}^2 - u} - a_0 + a_z k \cdot k' - 2m_{\Lambda_b}(\sqrt{s} - m_{\Lambda_b})b_0\nu \right\}, \quad (32)$$

that in terms of x may be written as

$$f_1 = \frac{g_{\pi\Lambda_b\Sigma_b}^2(E+m_{\Lambda_b})}{36\pi\sqrt{s}} \left\{ \alpha_1 + \beta_1 x + \frac{a_1 + b_1 x}{\gamma_{\Sigma_b}^* + 2|\vec{k}|^2 x} \right\}, \quad (33)$$

where

$$\begin{aligned} \alpha_1 &= \frac{3(\sqrt{s} + m_{\Sigma_b}^*)}{4m_{\Sigma_b}^{*2}s} \left[m_{\Sigma_b}^{*2}s - (m_{\Lambda_b}^2 - m_{\pi}^2)^2 \right] \\ &\quad + \frac{(E_{\Sigma_b}^* + m_{\Lambda_b})^2}{\sqrt{s} + m_{\Sigma_b}^*} \\ &\quad - a_0 + a_z k_0^2 + b_0(m_{\Lambda_b} - \sqrt{s})(2Ek_0 + |\vec{k}|^2); \\ \beta_1 &= \frac{3|\vec{k}|^2}{m_{\Sigma_b}^* - \sqrt{s}} - \left[a_z - (m_{\Lambda_b} - \sqrt{s})b_0 \right] |\vec{k}|^2; \\ a_1 &= 3(2m_{\Lambda_b} + m_{\Sigma_b}^* - \sqrt{s})(|\vec{q}_{\Sigma_b}^*|^2 - |\vec{k}|^2) \\ &\quad + (m_{\Sigma_b}^* - 2m_{\Lambda_b} + \sqrt{s})(E_{\Sigma_b}^* + m_{\Lambda_b})^2; \\ b_1 &= 3|\vec{k}|^2(2m_{\Lambda_b} + m_{\Sigma_b}^* - \sqrt{s}); \\ \gamma_{\Sigma_b}^* &= m_{\Sigma_b}^{*2} - m_{\Lambda_b}^2 - m_{\pi}^2 + 2Ek_0. \end{aligned}$$

For the f_2 amplitude we have

$$\begin{aligned} f_2 &= \frac{g_{\pi\Lambda_b\Sigma_b}^2(E-m_{\Lambda_b})}{9} \frac{1}{4\pi\sqrt{s}} \left\{ \frac{-\hat{A} + (\sqrt{s} + m_{\Lambda_b})\hat{B}}{m_{\Sigma_b}^{*2} - s} \right. \\ &\quad - \frac{\hat{A} + (\sqrt{s} + m_{\Lambda_b})\hat{B}}{m_{\Sigma_b}^{*2} - u} + a_0 - a_z k k' \\ &\quad \left. - 2m_{\Lambda_b}(\sqrt{s} + m_{\Lambda_b})b_0 v \right\}, \quad (34) \end{aligned}$$

then,

$$f_2 = \frac{g_{\pi\Lambda_b\Sigma_b}^2(E-m_{\Lambda_b})}{36\pi\sqrt{s}} \left\{ \alpha_2 + \beta_2 x - \frac{a_2 + b_2 x}{\gamma_{\Sigma_b}^* + 2|\vec{k}|^2 x} \right\}, \quad (35)$$

where

$$\begin{aligned} \alpha_2 &= -3 \frac{|\vec{q}_{\Sigma_b}^*|^2 - |\vec{k}|^2}{\sqrt{s} + m_{\Sigma_b}^*} + \frac{(E_{\Sigma_b}^* + m_{\Lambda_b})^2}{\sqrt{s} - m_{\Sigma_b}^*} \\ &\quad + a_0 - a_z k_0^2 - b_0(\sqrt{s} + m_{\Lambda_b})(2Ek_0 + |\vec{k}|^2); \\ \beta_2 &= \frac{-3|\vec{k}|^2}{m_{\Sigma_b}^* + \sqrt{s}} + \left[a_z - (m_{\Lambda_b} + \sqrt{s})b_0 \right] |\vec{k}|^2; \\ a_2 &= 3(2m_{\Lambda_b} + m_{\Sigma_b}^* + \sqrt{s})(|\vec{q}_{\Sigma_b}^*|^2 - |\vec{k}|^2) \\ &\quad + (m_{\Sigma_b}^* - 2m_{\Lambda_b} - \sqrt{s})(E_{\Sigma_b}^* + m_{\Lambda_b})^2; \\ b_2 &= 3|\vec{k}|^2(2m_{\Lambda_b} + m_{\Sigma_b}^* + \sqrt{s}). \end{aligned}$$

Finally, the resulting partial-waves S and P amplitudes are

$$\begin{aligned} a_S^{(\Sigma_b^*)} &= \frac{g_{\pi\Lambda_b\Sigma_b}^2}{72\pi\sqrt{s}} \left\{ 2(E+m_{\Lambda_b})\alpha_1 + \frac{2}{3}(E-m_{\Lambda_b})\beta_2 \right. \\ &\quad + (E+m_{\Lambda_b})a_1 I_0 \\ &\quad + \left[(E+m_{\Lambda_b})b_1 - (E-m_{\Lambda_b})a_2 \right] I_1 \\ &\quad \left. - (E-m_{\Lambda_b})b_2 I_2 \right\}, \quad (36) \end{aligned}$$

$$\begin{aligned} a_{P_1}^{(\Sigma_b^*)} &= \frac{g_{\pi\Lambda_b\Sigma_b}^2}{72\pi\sqrt{s}} \left\{ 2(E-m_{\Lambda_b})\alpha_2 + \frac{2}{3}(E+m_{\Lambda_b})\beta_1 \right. \\ &\quad - (E-m_{\Lambda_b})a_2 I_0 \\ &\quad + \left[(E+m_{\Lambda_b})a_1 - (E-m_{\Lambda_b})b_2 \right] I_1 \\ &\quad \left. + (E+m_{\Lambda_b})b_1 I_2 \right\}, \quad (37) \end{aligned}$$

$$\begin{aligned} a_{P_3}^{(\Sigma_b^*)} &= \frac{g_{\pi\Lambda_b\Sigma_b}^2}{72\pi\sqrt{s}} \left\{ \frac{2}{3}(E+m_{\Lambda_b})\beta_1 - \left(\frac{m_{\Lambda_b} - E}{2} \right) a_2 I_0 \right. \\ &\quad + \left[(E+m_{\Lambda_b})a_1 - \left(\frac{m_{\Lambda_b} - E}{2} \right) b_2 \right] I_1 \\ &\quad + \left[\frac{3}{2}(m_{\Lambda_b} - E)a_2 + (E+m_{\Lambda_b})b_1 \right] I_2 \\ &\quad \left. + \frac{3}{2}(m_{\Lambda_b} - E)b_2 I_3 \right\}. \quad (38) \end{aligned}$$

4 Results

Considering the amplitudes obtained before, now it is possible to calculate the observables of interest, that is the subject of this section. First of all we must observe that the calculated partial-wave amplitudes are real, and then violate the unitarity of the S matrix. These amplitudes may be unitarized [19–21] with

$$a_{l\pm}^U = \frac{a_{\pm}}{1 - i|\vec{k}|a_{\pm}}. \quad (39)$$

In the center-of-mass frame the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2, \quad (40)$$

and the total cross section may be obtained by integrating this expression over the solid angle that results in

$$\sigma_T = 4\pi \sum_l \left[(l+1)|a_{l+}^U|^2 + l|a_{l-}^U|^2 \right]. \quad (41)$$

Table 2 Constants of $\pi\Lambda_b$ interaction

m_{Λ_b}	5620 MeV
$g_{\pi\Lambda_b\Sigma_b}$	40.0
$g_{\pi\Lambda_b\Sigma_b^*}$	9.2 GeV^{-1}
Z	-0.5

Fig. 2 Breit-Wigner fit for the Σ_b and Σ_b^* resonances

The polarization is defined by

$$\vec{P} = -2 \frac{\text{Im}(f^*g)}{|f|^2 + |g|^2} \hat{n}, \quad (42)$$

where $\hat{n}$ is a unitary vector normal to the scattering plane and the phase shifts are given by

$$\delta_{l\pm} = \tan^{-1}(|\vec{k}|a_{\pm}). \quad (43)$$

The parameters used in the calculations are shown in Tables 1 and 2. The coupling constants have been determined by comparing our results for the phase-shifts with the Breit-Wigner expression (see the Appendix 6.3) [19,22], as it is shown in Fig. 2, with $\Gamma_{\pi\Lambda_b\Sigma_b} = 5\text{ MeV}$ and $\Gamma_{\pi\Lambda_b\Sigma_b^*} = 10\text{ MeV}$ [26].

Fig. 3 Total cross section of the $\pi\Lambda_b$ interaction **Fig. 4** Phase shifts of the $\pi\Lambda_b$ interaction

Adjusting the coupling constants in accord with the Lagrangians (12) and (23) we can calculate the rate $(g_{\pi\Lambda_b\Sigma_b}/2m_{\Lambda_b})/g_{\pi\Lambda_b\Sigma_b^*} = 0.39$, this result is close to the rate of Goldberger-Treiman coupling constants (66) showed in the appendix 6.4, with the value 0.58 accordingly with [27].

In Figs. 3, 4, 5 and 6 the total cross section, phase-shifts, differential cross section and polarization are plotted as functions of the pion momentum $k = |\vec{k}|$ and x . The gaps between the lines in Fig. 6, for the polarization, are 10 MeV.

5 Conclusions

In this work the low energy pion- Λ_b has been studied. We considered Lagrangians that include pions, Λ_b , Σ_b and Σ_b^* . In the interaction mediated by σ , a parametrization has been considered. The coupling constants have been determined and then the phase-shifts, cross-sections and polarizations have been calculated. As it should be expected, at low ener-

Fig. 5 Differential cross section of the $\pi\Lambda_b$ interaction

Fig. 6 Polarization in the $\pi\Lambda_b$ interaction

gies, the resonances dominate the cross sections, as it may be seen in Figs. 3 and 5. This behavior is similar to the one observed in pion-hyperon [19–21] and kaon-hyperon [22, 23] interactions and is very well determined in the pion-nucleon interactions, where the Δ particle dominates the cross section in the spin-3/2 and isospin-3/2 channel.

At the LHCb experiment [2], the transverse polarization of Λ_b at $\sqrt{s} = 7$ TeV has been measured $0.06 \pm 0.07 \pm 0.02$ and in the CMS [3], for $\sqrt{s} = 7$ and 8 TeV, the measured polarization was $0.00 \pm 0.06 \pm 0.06$, that are small values. Observing Fig. 6, we may see that in general, the polarization is not so large, except near the Σ_b and Σ_b^* masses. If this polarization is considered in mechanisms such as the ones presented in [16–18], that take into account the final-state interactions of the Λ_b , a relatively small value of the polarization 1%–2% (or even smaller) should remain when the average is computed. Obviously in order to determine the exact value, the calculations must be made, that is a task for a future work.

The results of this work are also important in the determination of nucleon- Λ_b and hyperon- Λ_b potentials, as the pion- Λ_b interaction is needed in order to calculate diagrams

Fig. 7 Examples of two pions exchange in the $Y\Lambda_b$ or $N\Lambda_b$ interactions

of the type of Fig. 7. This kind of potential is fundamental in many physical systems, as for example in order to describe exotic nuclei that contain bottom baryons. Then, the model presented in this work deals with an important aspect of physics, where still there are much to be done, that is describing and understanding the interactions and properties of the heavy-quark baryons.

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6 Appendix

6.1 Kinematics relations

Considering a process where p_μ and p'_μ are the initial and final baryon four-momenta, k_μ and k'_μ are the initial and final meson four-momenta, the Mandelstam variables may be written in terms of variables defined in the center-of-mass frame, $|\vec{k}|$ is the three-momentum absolute value of the incident particle and θ , the scattering angle

$$\begin{aligned} s &= (p + k)^2 = (p' + k')^2 = m_{\Lambda_b}^2 + m_\pi^2 + 2Ek_0 - 2\vec{k} \cdot \vec{p}, \\ s &= m_{\Lambda_b}^2 + m_\pi^2 + 2Ek_0 + 2|\vec{k}|^2, \\ u &= (p' - k)^2 = (p - k')^2 = m_{\Lambda_b}^2 + m_\pi^2 - 2Ek_0 - 2\vec{k}' \cdot \vec{p}, \\ u &= m_{\Lambda_b}^2 + m_\pi^2 - 2|\vec{k}|^2 \cos^2 \theta - 2Ek_0, \end{aligned} \quad (45)$$

$$\begin{aligned} t &= (p - p')^2 = (k - k')^2 = 2|\vec{k}|^2 x - 2|\vec{k}|^2, \\ t &= 2|\vec{k}|^2(x - 1), \end{aligned} \quad (46)$$

where

$$x = \cos \theta, \quad (47)$$

and the energies are defined as

$$k_0 = k'_0 = \sqrt{|\vec{k}|^2 + m_\pi^2}, \quad (48)$$

$$E = E' = \sqrt{|\vec{k}|^2 + m_{\Lambda_b}^2}, \quad (49)$$

also

$$E + k_0 = \sqrt{s}, \quad (50)$$

is the total energy of the system.

Other variables of interest are

$$v_{\Sigma_b^*} = \frac{m_{\Sigma_b^*}^2 - m_{\Lambda_b}^2 - k \cdot k'}{2m_{\Lambda_b}}, \quad (51)$$

$$v = \frac{s - u}{4m_{\Lambda_b}} = \frac{2Ek_0 + |\vec{k}|^2 + |\vec{k}|^2 x}{2m_{\Lambda_b}}, \quad (52)$$

$$k \cdot k' = m_\pi^2 + |\vec{k}|^2 - |\vec{k}|^2 x = k_0^2 - |\vec{k}|^2 x, \quad (53)$$

where m_{Λ_b} , $m_{\Sigma_b^*}$ and m_π are the Λ_b mass, the resonance Σ_b^* mass and the pion mass, respectively. We also define the relations

$$(E_{\Sigma_b^*} \pm m_{\Lambda_b}) = \frac{(m_{\Sigma_b^*} \pm m_{\Lambda_b})^2 - m_\pi^2}{2m_{\Sigma_b^*}}, \quad (54)$$

$$(\vec{q}_{\Sigma_b^*})^2 = E_{\Sigma_b^*}^2 - m_{\Lambda_b}^2 = (E_{\Sigma_b^*} + m_{\Lambda_b})(E_{\Sigma_b^*} - m_{\Lambda_b}), \quad (55)$$

where $E_{\Sigma_b^*}$ and $\vec{q}_{\Sigma_b^*}$ are the energy and the 3-momentum of the Σ_b^* baryon that is produced in the intermediate states.

6.2 Integrals

Some integrals that appear in the calculations has the general form

$$I_n = \int_{-1}^1 \frac{x^n dx}{\gamma + 2k^2 x}, \quad (56)$$

where γ is defined in the text, x is given by Eq. (47), $k = |\vec{k}|$ and n is an integer. Thus, we have

$$I_0 = \int_{-1}^1 \frac{dx}{\gamma + 2k^2 x} = \frac{1}{2k^2} \ln \left(\frac{\gamma + 2k^2}{\gamma - 2k^2} \right), \quad (57)$$

$$I_1 = \int_{-1}^1 \frac{x dx}{\gamma + 2k^2 x} \quad (58)$$

$$= \frac{2}{2k^2} - \frac{\gamma}{(2k^2)^2} \ln \left(\frac{\gamma + 2k^2}{\gamma - 2k^2} \right), \quad (59)$$

$$\begin{aligned} I_2 &= \int_{-1}^1 \frac{x^2 dx}{\gamma + 2k^2 x} \\ &= -\frac{2\gamma}{(2k^2)^2} + \frac{\gamma^2}{(2k^2)^3} \ln \left(\frac{\gamma + 2k^2}{\gamma - 2k^2} \right), \end{aligned} \quad (60)$$

$$\begin{aligned} I_3 &= \int_{-1}^1 \frac{x^3 dx}{\gamma + 2k^2 x} \\ &= \frac{2(2k^2)^2 + 6\gamma^2}{3(2k^2)^3} - \frac{\gamma^3}{(2k^2)^4} \ln \left(\frac{\gamma + 2k^2}{\gamma - 2k^2} \right). \end{aligned} \quad (61)$$

6.3 Breit–Wigner expression

The relativistic Breit–Wigner expression is determined in terms of experimental quantities

$$\delta_{l\pm} = \tan^{-1} \left[\frac{\Gamma \left(\frac{|\vec{k}|}{|k_0|} \right)^{2J+1}}{2(m_r - \sqrt{s})} \right], \quad (62)$$

where Γ is the width, $|k_0|$ is the momentum at the peak of the resonance in the center-of-mass system, m_r is its mass and J the total angular momentum (spin) of the resonance.

6.4 Goldberger–Treiman and coupling constants relations

The Goldberger–Treiman relation [35] is given by

$$g_{\pi\Lambda_b B_i}^{GT} = \frac{m_{\Lambda_b} + m_{B_i}}{\sqrt{2}f_\pi} g_i, \quad (63)$$

where $f_\pi = 130 \text{ MeV}$ is the pion decay constant [26], and we consider according with the reference [27]

$$B_2 = \Sigma_b, \quad g_2 = -\sqrt{\frac{2}{3}} g_A^{ud}, \quad (64)$$

$$B_4 = \Sigma_b^*, \quad g_4 = \sqrt{2} g_A^{ud}, \quad (65)$$

g_A^{ud} is the normalization factor of quarks ud .

Considering that $g_4 = -\sqrt{3}g_2$ we can find a relation between $g_{\pi\Lambda_b\Sigma_b}^{GT}$ and $g_{\pi\Lambda_b\Sigma_b^*}^{GT}$,

$$\left| \frac{g_{\pi\Lambda_b\Sigma_b}^{GT}}{g_{\pi\Lambda_b\Sigma_b^*}^{GT}} \right| = \frac{1}{\sqrt{3}} \left(\frac{m_{\Lambda_b} + m_{\Sigma_b}}{m_{\Lambda_b} + m_{\Sigma_b^*}} \right). \quad (66)$$

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