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On variations of G in the geometric scalar theory of gravity

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Abstract We analyze the possible variability of the effective Newtonian gravitational constant $G_{\rm N}$ in space and time in the framework of the geometric scalar theory of gravity suggested by Novello et al. (JCAP 06:014, 2013). Spatial variations of $G_{\rm N}$ in the Solar system are shown to have orders of magnitude detectable by modern instruments. As to variations of $G_{\rm N}$ with cosmological time, it is shown (at least for the particular formulation of the theory discussed in the original paper and the corresponding cosmological models) that these variations are more rapid than is allowed by observations.

1 Introduction

General relativity (GR) is well known to have a brilliant experimental status with respect to local phenomena, confirmed by observations in the Solar system and on other astrophysical objects on stellar or sub-galactic scales. The most important experimental achievements of the recent years, the detection of gravitational waves and observations with the Event Horizon Telescope, apparently confirms GR predictions for extremely strong gravitational fields, the existence and properties of black holes, see, e.g., the recent reviews [1,2].

Nevertheless, there is almost a universal opinion of the theorists that GR is not an ultimate theory of gravity and needs an improvement, and an enormous number of its extensions and modifications are discussed and analyzed in the physical literature, see, e.g., [3,4] and references therein. The reasons for such views are twofold. On one hand, beginning with the galactic length scale, and especially in cosmology, the observational status of GR is not so perfect due to the well-known Dark Matter (DM) and Dark Energy (DE) problems, the first one above all related to missing mass in galaxies and clusters of galaxies, the second one to the observed accelerated expansion of the Universe [5–10]. Another group of reasons is connected with the problems inherent to the theory itself. Thus, the most important solutions of GR contain space-time singularities with diverging values of curvature invariants, indicating situations where the theory cannot work any more. There are also long-standing problems with quantization of gravity and with its unification with other physical interactions. Such a wide set of problems has caused the advent of a great diversity of extended, or alternative theories of gravity.

Some of them differ from GR by inclusion of additional fields (scalar–tensor, Einstein-aether, bimetric, tensor–vector–scalar (TeVeS) theories, etc.; others, such as, for instance, $f(R)$ and many more complex theories contain higher-order derivatives of the metric tensor. Numerous theories involve small or large extra dimensions (Kaluza–Klein type or brane-world theories, respectively), some of the latter being related to the string concept, and many theories make use of non-Riemannian geometries, e.g., Finsler models, models with torsion and/or nonmetricity – see the vast bibliography in [3,4].

A common feature of all such models is that they introduce new dynamic degrees of freedom as compared to GR. The Geometric Scalar Theory of Gravity (GSG), recently proposed by Mario Novello and co-authors [11], makes a step in the opposite direction and shows that some opportunities of interest are not yet exhausted in attempts to simplify the description of gravity instead of adding its complexity. As said in [12], "Here we propose that it may be interesting to take a huge step backward and explore models in which the gravitational degrees of freedom are just described by the field Φ."

GSG is a metric theory of gravity in which all kinds of nongravitational matter interact with the dynamic field Φ only through the gravitational metric $q_{\mu\nu}$. In addition to the gravitational metric $q_{\mu\nu}$, GSG also employs the auxiliary Minkowski metric $\eta_{\mu\nu}$ which is not observable. It turns out

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that this kind of theory has a chance to be viable, unlike previous attempts to build a relativistic scalar theory of gravity (see [13,14] and detailed discussions in [11,12]). Thus, by properly choosing the parameters of the theory in such a way that $q_{\mu\nu}$ for a field of a gravitating center has the Schwarzschild form, it becomes possible to reproduce Newton's theory in the weak field limit and all local classical effects of GR [11]. In cosmology, GSG has been shown [15] to be able to solve the singularity, horizon and flatness problems without appeal to exotic kinds of matter, in particular, predicting a bounce instead of a singularity; it is also shown to present a basis for structure formation by gravitational instability [15]. A study of gravitational waves in GSG [16] has shown that they are described in the weak field approximation of this theory in a way similar to GR, they propagate at the same speed as light, but a characteristic longitudinal polarization mode, absent in GR, is predicted, so its possible discovery can be a strong evidence in favor of GSG. All these features seem to make GSG one of the promising alternatives to GR.

The present study shows, however, that GSG faces a serious problem with too large variations of the effective Newtonian gravitational constant $G_{\rm N}$. Such variations are predicted by many non-Einsteinian theories of gravity, see, e.g., [17–23] and references therein. Meanwhile, there are strong observational constraints on time variations of $G_{\rm N}$ [24–26] and milder ones on its spatial dependence, mostly related to the so-called fifth force concept [17]. Our purpose here will be to estimate both temporal and spatial variations of $G_{\rm N}$ in the version of GSG presented and discussed in [11,12,15,16].

The paper is organized as follows. The next section outlines some basic relations of GSG. Section 3 is devoted to finding a general expression for the local value of $G_{\rm N}$ in terms of the fundamental scalar field Φ, and in in Sect. 4 it is used for obtaining the relevant estimates. Section 5 contains some concluding remarks.

2 Basic relations of GSG

As mentioned above, GSG contains two metrics, the observable one, $q_{\mu\nu}$, and the flat auxiliary one, $\eta_{\mu\nu}$, and both can be used in an arbitrary coordinate system since the theory is generally covariant, free of any privileged reference frame. The two metrics are connected by a disformal transformation described by the relations[1]

$$q^{\mu\nu} = \alpha\, \eta^{\mu\nu} + \frac{\beta}{w}\, \partial^\mu\Phi\, \partial^\nu\Phi, \tag{1}$$

$$q_{\mu\nu} = \frac{1}{\alpha}\, \eta_{\mu\nu} - \frac{\beta}{\alpha\,(\alpha+\beta)\, w}\, \partial_\mu\Phi\, \partial_\nu\Phi. \tag{2}$$

where $\partial^\mu\Phi \equiv \eta^{\mu\nu}\,\partial_\nu\Phi$, the parameters $\alpha > 0$ and β are certain dimensionless functions of Φ, and

$$w \equiv \eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi,$$
$$\Omega \equiv q^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi = (\alpha+\beta)\, w. \tag{3}$$

The dynamics of the theory is specified by the action $S = S_g + S_m$, where S_g and S_m are its gravitational and matter parts, respectively:

$$S_g = \frac{1}{16\pi G_0}\int \sqrt{-\eta}\, d^4x\, V(\Phi)\, w, \tag{4}$$

$$S_m = \int \sqrt{-q}\, d^4x\, L_m. \tag{5}$$

Here, $\eta = \det(\eta_{\mu\nu})$, $q = \det(q_{\mu\nu})$, $V(\Phi) > 0$ is a certain function called the potential, L_m is the Lagrangian of matter, and G_0 is an initial constant introduced similarly to the usual gravitational constant in GR, but, in general, it does not coincide with Newton's constant of gravity in GSG, as we shall see below. Furthermore, it is important that the stress-energy tensor of matter $T_{\mu\nu}$, defined in the usual way in terms of $q_{\mu\nu}$,

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-q}}\, \frac{\delta(\sqrt{-q}\, L_m)}{\delta q^{\mu\nu}}, \tag{6}$$

obeys the conservation law $\nabla_\nu T^\nu_\mu = 0$ (again in terms of $q_{\mu\nu}$).

The functions $\alpha(\Phi)$, $\beta(\Phi)$ and $V(\Phi)$, entirely fixing the formulation of the theory, are specified in [11] in such a way that the vacuum field equation for Φ has the form $\Box\Phi = 0$, where $\Box = q^{\mu\nu}\nabla_\mu\nabla_\nu$ is the d'Alembert operator corresponding to the metric $q_{\mu\nu}$, and the static, spherically symmetric vacuum solution for Φ leads to the Schwarzschild form of the metric $q_{\mu\nu}$. It is this circumstance that makes the weak-field predictions of GSG (e.g., in the Solar system) the same as those of GR. Specifically, we have [11]

$$\alpha + \beta = \alpha^3 V(\Phi),$$
$$4\alpha^3 V = (\alpha-3)^2, \qquad \alpha = {\rm e}^{-2\Phi}. \tag{7}$$

The field equation for Φ can be written in the following way for the case where L_m describes a perfect fluid with density ρ and pressure p:

$$\sqrt{V}\,\Box\,\Phi = -\frac{\kappa}{2}\left(\frac{2\alpha}{\alpha-3}\,\rho - 3p\right). \tag{8}$$

[1] We use the metric signature $(+\;-\;-\;-)$; the curvature tensor is defined via Chrisoffel symbols for $q_{\mu\nu}$ as $R^\sigma{}_{\mu\rho\nu} = \partial_\nu\Gamma^\sigma_{\mu\rho} - \ldots$, $R_{\mu\nu} = R^\sigma{}_{\mu\sigma\nu}$, so that the scalar $R = R^\mu_\mu > 0$ for de Sitter space-time or the matter-dominated cosmological epoch; the system of units $c = \hbar = 1$.

In terms of the Minkowski metric $\eta_{\mu\nu}$, using (1), we obtain

$$\Box_M \Phi + \frac{1}{2}\frac{V_\Phi}{V} w = \frac{\kappa}{2}\left(\frac{1}{\alpha\sqrt{V}}\right)^3 \left(3p - \frac{2\alpha}{\alpha-3}\rho\right), \tag{9}$$

where $\kappa = 8\pi G_0$ and $V_\Phi \equiv dV/d\Phi$. (This equation coincides with Eq. (46) of [11].) Or equivalently, due to (7),

$$\Box_M \Phi + \frac{\alpha-9}{\alpha-3} w = \frac{\kappa}{\alpha^3 V^{3/2}}\left(\frac{3}{2}p - \frac{\alpha}{\alpha-3}\rho\right). \tag{10}$$

3 The effective gravitational constant

Let us now find out the expression for the effective Newtonian gravitational constant G_N in the weak-field and low-velocity limit of GSG. In this limit, the Newtonian gravitational potential Φ_N due to a matter distribution with density ρ should, as usual, satisfy the Poisson equation

$$\Delta\Phi_N = 4\pi G_N \rho, \tag{11}$$

where Δ is the flat-space Laplace operator.

The Newtonian approximation should be valid for the gravitational interaction of any system of bodies located so closely to each other that the space-time curvature could be neglected, and moving with very small relative velocities, so that in a suitable reference frame and in properly chosen coordinates the gravitational field is dominated by the temporal component of the space-time metric written as

$$g_{00} = 1 + 2\Phi_N, \qquad |\Phi_N| \ll 1, \tag{12}$$

where Φ_N obeys Eq. (11). Such a small domain can be considered as a close neighborhood of a certain 4D point x_0^μ in which the scalar field Φ changes so slowly that $\Phi(x_0^\mu)$ can be taken as a constant background value, while Φ_N should be related to comparatively rapidly changing deflections from this constant value. In other words, Φ should be taken in the form

$$\begin{aligned} &\Phi(x^\mu) = \Phi_0 + \Phi_1(x^\mu), \\ &\Phi_0 = \Phi(x_0^\mu) \approx \text{const}. \end{aligned} \tag{13}$$

Our task is to find a relationship between Φ_1 and Φ_N and to bring Eq. (10) to the form (11) (where we put $p = 0$ since we consider nonrelativistic matter) by substituting there the expansion (13).

Let us note that examples of physical situations where such an approximation is quite plausible include gravitational fields in various local systems (a galaxy, a stellar cluster or a planetary system) against a slowly varying cosmological or larger-scale background which can be regarded constant on the time or length scales small as compared to the corresponding scales of the background. Such examples are (i) the dynamics of a galaxy against the time scale of the Universe, (ii) the dynamics of a planetary system against the background of the galactic gravitational field, which is almost time-independent and very slowly varies in space, and even (iii) processes in the Earth-Moon system or Jupiter with its satellites against the background of the Sun's gravity.

Now, if we take an arbitrary slowly changing Φ-dependent metric $q_{\mu\nu}$, with any positive value of q_{00}, it should be rescaled to the background value $g_{00} = 1$, i.e., we must put $q_{\mu\nu} = q_{00}(\Phi_0) g_{\mu\nu}$. We thus obtain the metric $g_{\mu\nu}$ in the standard form used in general relativity to describe the Newtonian approximation. Therefore, in a neighborhood of $x^\mu = x_0^\mu$

$$Q(\Phi) = Q(\Phi_0)(1 + 2\Phi_N + \cdots), \tag{14}$$

where we have denoted for brevity $Q(\Phi) \equiv q_{00}(\Phi)$. On the other hand, we have the Taylor decomposition

$$Q(\Phi) = Q(\Phi_0 + \Phi_1) = Q(\Phi_0) + Q_\Phi(\Phi_0)\Phi_1 + \cdots, \tag{15}$$

where the subscript Φ denotes $d/d\Phi$. Comparing (14) and (15), we obtain

$$\Phi_1 = \frac{2Q(\Phi_0)}{Q_\Phi(\Phi_0)}\Phi_N. \tag{16}$$

The quantity $Q(\Phi) = q_{00}$ in Eq. (16) and in all expressions where it is multiplied by the small quantity Φ_1 or Φ_N is equal to $Q(\Phi_0)$, in other words, it is a component of the slowly changing background metric, which in turn depends on the slowly changing background field $\Phi(x^\mu)$. Thus according to (2)

$$\begin{aligned} Q(\Phi) &= \frac{1}{\alpha} - \frac{\beta}{\alpha+\beta}\frac{(\partial_t\Phi)^2}{w} \\ &= \frac{1}{\alpha}\left(1 - \frac{\beta Y}{\alpha^3 V}\right) = \frac{1}{\alpha}\left[1 - \frac{4\beta Y}{(\alpha-3)^2}\right], \end{aligned} \tag{17}$$

where

$$Y(x^\mu) \equiv \frac{(\partial_t\Phi)^2}{w} \equiv \frac{(\partial_t\Phi)^2}{\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi}. \tag{18}$$

Now we substitute the expression (13) with $\Phi_0 = $ const into (10) with $p = 0$ assuming $\Phi_1 = \Phi_1(x^i)$, that is, neglecting its possible time dependence in accordance with the slow motion approximation. Consequently, $\Box_M\Phi = -\Delta_M\Phi_1$ (where Δ_M is the flat-space Laplace operator corresponding to the metric $\eta_{\mu\nu}$). In the same approximation the second term with w in (10) is also negligible. We thus obtain

$$\Delta_M \Phi_1 = \frac{\kappa\rho}{\alpha^2(\alpha-3)V^{3/2}}, \tag{19}$$

or, with (16),

$$\Delta_M \Phi_{\rm N} = \frac{Q_\Phi}{2Q}\,\frac{\kappa\rho}{\alpha^2(\alpha-3)V^{3/2}}, \tag{20}$$

where all Φ-dependent quantities on the right-hand side are taken at $\Phi = \Phi_0$ and $Q(\Phi)$ is specified by Eq. (17).

This is still not the end of the story. The point is that the derivatives involved in Δ_M are taken with respect to the coordinates x^i in which the auxiliary flat metric has the form $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, while the physical metric $q_{\mu\nu}$ in the flat-space approximation (actually, the tangent space metric) has the form $q_{\mu\nu} = Q(\Phi_0)\eta_{\mu\nu}$, whereas the Poisson equation (11) expressing Newton's law should be obtained in terms of the rescaled coordinates $y^i = \sqrt{Q}x^i$, i.e., the spatial coordinates corresponding to the physical metric of the form $\eta_{\mu\nu}$. Since in our approximation $Q = \text{const}$, we can write $\partial/\partial x^i = \sqrt{Q}\partial/\partial y^i$, hence $\Delta_M = Q\Delta \equiv Q\Delta[y^i]$, and Eq. (20) in terms of y^i takes the form

$$\Delta\Phi_{\rm N} = \frac{Q_\Phi}{2Q^2}\,\frac{\kappa\rho}{\alpha^2(\alpha-3)V^{3/2}}. \tag{21}$$

Comparing (21) with (11), we find the following final expression for the effective Newtonian gravitational constant $G_{\rm N}$:

$$G_{\rm N} = \frac{G_0 Q_\Phi}{Q^2\alpha^2(\alpha-3)V^{3/2}} = \frac{8\,G_0 Q_\Phi \alpha^{5/2}}{Q^2(\alpha-3)^4}\,\mathrm{sign}(\alpha-3), \tag{22}$$

where V is taken from (7), and Q should be taken from (17). The factor $\mathrm{sign}(\alpha-3)$ in (22) appears because according to (7) $\sqrt{V} = |\alpha-3|/(2\alpha^{3/2})$.

The expression (22) shows the following features of $G_{\rm N}$ in GSG:

- $G_{\rm N}$ is variable in space and time, depending on the behavior of the fundamental scalar field Φ;
- $G_{\rm N} > 0$ only if $Q_\Phi(\alpha-3) > 0$, otherwise we obtain antigravity;
- Due to the factor Y in (17), $G_{\rm N}$ is different in the cases of cosmological and spatial dependences of Φ. More specifically, if $\Phi = \Phi(x^0)$, we have $Y = 1$ in (17) whereas if $\Phi = \Phi(x^i)$, we obtain $Y = 0$.

4 Variations of $G_{\rm N}$

4.1 Cosmological variations

Assuming $\Phi = \Phi(x^0)$ and substituting (7), we obtain

$$Q = \frac{4}{(\alpha-3)^2} \quad\Rightarrow$$
$$\frac{Q_\Phi}{Q^2} = -\frac{1}{2}(\alpha-3)\alpha_\Phi = \alpha(\alpha-3), \tag{23}$$

and Eq. (22) leads to

$$G_{\rm N} = \frac{8\,G_0\alpha^{7/2}}{(\alpha-3)^3}\mathrm{sign}(\alpha-3) = \frac{8\,G_0\alpha^{7/2}}{|\alpha-3|^3}. \tag{24}$$

Evidently, in this cosmological setting the effective gravitational constant $G_{\rm N}$ is always positive.

A logarithmic derivative of $G_{\rm N}$ with respect to physical time t (with $\alpha = e^{-2\Phi}$ according to (7)) gives its cosmological variation

$$\frac{\dot{G_{\rm N}}}{G_{\rm N}} = \dot\Phi\,\frac{21-\alpha}{\alpha-3}, \tag{25}$$

where the dot denotes d/dt. Furthermore, in the cosmological models of GSG considered briefly in [11] and in detail in [15], $\dot\Phi$ coincides with the Hubble parameter H:

$$\dot\Phi = H(t) = \frac{\dot a}{a} \quad\Rightarrow\quad \frac{\dot{G_{\rm N}}}{G_{\rm N}} = H\,\frac{21-\alpha}{\alpha-3}, \tag{26}$$

where $a(t)$ is the cosmological scale factor.

Recalling the tight observational constraints on variations of $G_{\rm N}$, according to which the variation (26) in the modern epoch should be smaller than the Hubble rate H by at least a factor of 1000 [24], we can conclude that the cosmological models of GSG have a very small chance to be viable. Indeed, to fit the present-day observations, we need

$$\frac{21-\alpha}{\alpha-3} \lesssim 0.001, \tag{27}$$

which may be regarded as a kind of fine tuning. However, there are tight constraints on variations of $G_{\rm N}$ in the past: for example, at the nucleosynthesis epoch the value of $G_{\rm N}$ could not differ from the modern one by more than 20–30 % [26,27]. According to [28], the time variation of $G_{\rm N}$ between the recombination time ($G_{\rm rec}$) and the present epoch (G_0) is constrained as $G_{\rm rec}/G_0 < 1.0030$ (2σ) and $G_{\rm rec}/G_0 < 1.0067$ (4σ).

Meanwhile, as follows from (26), the field Φ evolves as $\ln(a/a_0)$, $a_0 = \text{const}$, hence $\alpha \sim a^{-2}$, and according to (24), $G_{\rm N}$ should have changed by many orders of magnitude. This contradiction shows that the GSG faces serious problems, and at least its new formulation should be sought for to fit the observations.

4.2 Spatial variations

In the case $\Phi = \Phi(x^i)$, we have $Y = 0$, and (17) gives

$$Q = \frac{1}{\alpha} \quad \Rightarrow \quad \frac{Q_\Phi}{Q^2} = -\alpha_\Phi = 2\alpha. \tag{28}$$

whence it follows

$$G_\mathrm{N} = G_0 \frac{16\alpha^{7/2}}{(\alpha-3)^4}\mathrm{sign}(\alpha-3). \tag{29}$$

It follows that $G_\mathrm{N} > 0$ only if $\alpha > 3$, otherwise there is antigravity instead of gravity. As to spatial variations of G_N, we have

$$\frac{G'_\mathrm{N}}{G_\mathrm{N}} = \Phi' \frac{\alpha+21}{\alpha-3}, \tag{30}$$

where the prime denotes a derivative in any spatial direction.

Let us look whether or not the variations (30) are in conflict with observations. To do so, let us estimate spatial variations of G_N in the Solar system considering the Sun's gravity as the background slowly changing field. Since this field itself is rather weak, it can be taken in the Newtonian approximation, with the potential

$$\Phi_\mathrm{N} = G_{\mathrm{N}0} M_\odot / r, \tag{31}$$

where $M_\odot$ is the solar mass, r is the distance from the solar center, and $G_{\mathrm{N}0}$ is the asymptotic value of the Newtonian constant sufficiently far from the Sun but still not too far, so that the gravitational field of the Galaxy could be regarded constant at this scale. We denote the corresponding background value of Φ by Φ_0. Furthermore, for $q_{00} = Q(\Phi)$ we have the expression (14), where $Q(\Phi_0) = 1/\alpha$, and $\alpha = \mathrm{e}^{-2\Phi_0}$ is an unknown constant, for which we should only require $\alpha > 3$ to provide a positive value of G_N.

To estimate G_N variations according to (30), we need $\Phi' = d\Phi/dr = \Phi'_1$, corresponding to the expansion $\Phi = \Phi_0 + \Phi_1 + \cdots$. As follows from (16) and $Q = 1/\alpha$, in our case $\Phi_1 = \Phi_\mathrm{N}$, and therefore the local Newtonian constant changes according to

$$\frac{G'_\mathrm{N}}{G_{\mathrm{N}0}} \approx \frac{G_{\mathrm{N}0} M_\odot}{r^2} \frac{\alpha+21}{\alpha-3}, \tag{32}$$

where

$$G_{\mathrm{N}0} M_\odot \approx 1.5\ \mathrm{km} \tag{33}$$

is half the Schwarzschild gravitational radius of the Sun. The α-dependent factor can be large but it tends to unity at large α. In our estimates, we will put this factor equal to unity, which corresponds to lower limits of the corresponding variations.

Using (32), it is straightforward to find that a fractional variation of G_N at the Earth's orbit is about 10^{-16}/km, which makes a relative difference in G_N values of $\approx 10^{-12}$ along the Earth's diameter and $\approx 0.5 \times 10^{-10}$ at the diameter of the Moon's orbit. The corresponding displacement of the lunar orbit would be about 4 cm, which could in principle be noticed by laser ranging.

But even if the above variations of G_N are admissible from an observational viewpoint, they become much larger on the planetary scale. Indeed, it follows from (32) that the total relative variation of G_N from Mercury's orbit to (conditional) infinity, or, say, to Kuiper's belt, is about 2.7×10^{-8} whereas an analysis of ephemerides makes it possible to trace annual G_N variations up to 10^{-13} [24]. Evidently, anomalies due to variable G cannot be discovered from observations of bodies in circular or near-circular orbits (for which the product $G_\mathrm{N} M$ remains constant) but should be quite detectable for bodies in highly excentic orbits like some asteroids and comets.

For example, the asteroid Icarus has an excentric orbit located between approximately 2 and 0.2 astronomic units (3×10^{12} to 3×10^{13} cm). Its half-revolution time is about 200 days $\sim 1.8 \times 10^7$ s, while an anomalous acceleration due to changing G would contribute about 0.1 s to this time (as can be roughly estimated by considering the free-fall time from aphelion to perihelion). At typical velocities about 30 km/s this corresponds to a displacement of 3 km, easily detectable by ranging methods but making problems for optical telescopes (from a distance of, say, 60 million kilometers, a 3-km segment is seen at an angle of 0.01").

We conclude that spatial G_N variations in GSG, at least in its presently discussed formulation, do not seem to be in sharp conflict with observations, but are definitely in tension with them.

It might be tempting to compare the extra accelerations of bodies due to changing G_N with the so-called Pioneer anomaly. The latter (see the recent review [29]) consists in that the radio tracking data from Pioneer 10 and 11 spacecrafts have revealed their constant and uniform deceleration approximately directed towards the Sun,

$$A_\mathrm{Pio} = (8.74 \pm 1.33) \times 10^{-10}\ \mathrm{m\,s^{-2}} \tag{34}$$

at heliocentric distances of 20 to 70 astronomic units (a.u.), that is, $(3 \div 10.5) \times 10^{12}$ m. The anomalous acceleration Δa due to G_N variation is about 10^{-8} times the standard Newtonian acceleration a_N due to Sun's gravity, which is $1.5 \times 10^{-5}\ \mathrm{m\,s^{-2}}$ at a distance of 20 a.u., hence $\Delta a \sim 10^{-13}\ \mathrm{m\,s^{-2}}$, much smaller than the Pioneer anomaly, and decreases with distance together with a_N. Thus a variable G cannot account for the Pioneer anomaly, which is, according to [29], consistent with known physics, being almost completely explained by thermal radiation from the spacecrafts.

5 Conclusion

We have to conclude that the cosmological variations of the Newtonian gravitational constant predicted by the GSG are in striking conflict with observations whereas its spatial variations are comparatively small (at least in the Solar system) but are easily detectable by modern instruments.

A point of interest is an apparent contradiction between the geodesic nature of test body paths in the metric $q_{\mu\nu}$ in GSG and the existence of anomalous accelerations due to varying $G_{\rm N}$ in the Newtonian approximation. A possible explanation is that the Newtonian approximation deals with small velocities and gravitational potentials, and the anomalous acceleration exists in the next order of magnitude with respect to the flat-space approximation. In other words, the true geodesics are slightly non-Newtonian and are better described in the post-Newtonian approximation. The same is true for scalar-tensor and $f(R)$ theories of gravity.

The present results have been obtained for the particular version of GSG presented in [11,12,15,16]. It would be of interest to find out how they can change in its more general formulations discussed in [30–32], which can be a subject for future studies. Thus, it seems possible that some more complex action than (4), containing a function of two variables, ϕ and w, can lead to a scalar geometric theory compatible with all appropriate observational constraints.

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