Revisiting the $\Omega(2012)$ as a hadronic molecule and its strong decays

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Received: 20 March 2020 / Accepted: 17 April 2020 / Published online: 6 May 2020
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Abstract Recently, the Belle collaboration measured the ratios of the branching fractions of the newly observed $\Omega(2012)$ excited state. They did not observe significant signals for the $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$ decay, and reported an upper limit for the ratio of the three body decay to the two body decay mode of $\Omega(2012) \rightarrow \bar{K} \Xi$. In this work, we revisit the newly observed $\Omega(2012)$ from the molecular perspective where this resonance appears to be a dynamically generated state with spin-parity $3/2^-$ from the coupled channels interactions of the $\bar{K} \Xi^*(1530)$ and $\eta \Omega$ in s -wave and $\bar{K} \Xi$ in d -wave. With the model parameters for the d -wave interaction, we show that the ratio of these decay fractions reported recently by the Belle collaboration can be easily accommodated.

1 Introduction

In 2018, the Belle collaboration reported an Ω^* state in the $\bar{K} \Xi$ invariant mass distributions [1]. The measured mass and width of the Ω^* state are $M = 2012.4 \pm 0.7 \pm 0.6$ MeV and $\Gamma = 6.4_{-2.0}^{+2.5} \pm 1.6$ MeV. Such kind of Ω excited states have been studied before Belle collaboration publishes their results. In Refs. [2–4] using the chiral unitary approach where the coupled channels interactions of the $\bar{K} \Xi^*(1530)$ and $\eta \Omega$ were taken into account, the Ω excited states were investigated. An Ω excited state with spin-parity $J^P = 3/2^-$ and mass around 2012 MeV can be dynamically generated with a reasonable value of the subtraction constant [4]. Using a spin-flavor- $SU(6)$ extended Weinberg-Tomozawa meson-baryon interaction, the Ω resonances with $J^P = 1/2^-, 3/2^-$

and $5/2^-$ were studied in Ref. [5]. On the other hand, the Ω excited states were also investigated in classical quark models [6–10] and in the five-quark picture [11–13], in which, however, their predicted masses are always much different from the mass observed by the Belle collaboration. In Ref. [14], baryon states with strangeness -3 were predicted employing a quark model with ingredients suggested by QCD, and the mass of one predicted state with $J^P = 3/2^-$ is about 2020 MeV.

After the observation of the above mentioned $\Omega(2012)$ by the Belle collaboration [1], there were many theoretical studies on its mass, width, quantum numbers and decay modes. In Refs. [15, 16], the mass and the two-body strong decays of the $\Omega(2012)$ state were studied by the QCD sum rule method and it was found that the $\Omega(2012)$ can be interpreted as a $1P$ orbital excitation of the ground state Ω baryon with quantum numbers $J^P = 3/2^-$. In Refs. [17–19], the Ω excited spectrum and their two body strong decays were evaluated within a non-relativistic constituent quark potential model, and it was found that the $\Omega(2012)$ resonance is most likely to be a $1P$ state with $J^P = 3/2^-$. In Ref. [20], the authors performed a $SU(3)$ flavor analysis of the $\Omega(2012)$ state and discussed its $\bar{K} \Xi^*(1530)$ molecular picture. They concluded that the preferred quantum numbers of $\Omega(2012)$ are also $3/2^-$. On the other hand, the mass of the $\Omega(2012)$ is just a few MeV below the $\bar{K} \Xi^*(1530)$ mass threshold, which indicates that it could be a possible $\bar{K} \Xi^*(1530)$ molecule state [21]. Indeed, the hadronic molecule nature of the $\Omega(2012)$ were investigated in Refs. [22–25], and these calculations¹ predicted a large decay width for $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$. However, in a very recent measurement of the Belle collaboration [26], it was found that there is no significant

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¹ In Ref. [24], the partial decay width of $\Gamma[\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi] = 3$ MeV was obtained, but this calculation contained an error. The correct value is 0.8 MeV.

signals for the $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$ decay, and an upper limit was obtained, at the 90% confidence level, for the ratio of the three body decay to the two body decay mode of $\Omega(2012) \rightarrow \bar{K} \Xi$, $R = \text{Br}[\Omega(2012) \rightarrow \bar{K} \pi \Xi] / \text{Br}[\Omega(2012) \rightarrow \bar{K} \Xi]$, which is only 11.9%. There are also other experimental results for the ratios of different final charged decay modes [26], but because of large background for those decay channels these values are obtained without including the constraints of the isospin symmetry.² Later on, based on the new measurements by the Belle collaboration [26], the strong decays of the $\Omega(2012)$ were restudied in Refs. [27, 28] within the hadronic molecular approach. In Ref. [27] it concluded that the $\Omega(2012)$ can be interpreted as the p -wave $\bar{K} \Xi^*(1530)$ molecule state with $J^P = 1/2^+$ or $3/2^+$, while in Ref. [28], it was pointed out that the $\Omega(2012)$ state contains mixed $\bar{K} \Xi^*(1530)$ and $\eta\Omega$ hadronic components and the sizable $\eta\Omega$ hadronic component leads to a suppression of the $\bar{K} \pi \Xi$ decay mode.

The $\Omega(2012)$ state was investigated within a coupled channel approach in Ref. [25], in which, in addition to the interaction of $\bar{K} \Xi^*(1530)$ and $\eta\Omega$ in s -wave, the $\bar{K} \Xi$ in d -wave interaction was also taken into account. The pole position of the $\Omega(2012)$ was well reproduced in the scattering amplitude. However, the predicted value of R is about 90% [25], which is much larger than the experimental measurements [26]. Based on the new measurements of Ref. [26], we follow Ref. [25] and re-investigate the $\Omega(2012)$ state from the molecular perspective in which the resonance is dynamically generated from the interactions of $\bar{K} \Xi^*(1530)$, $\eta\Omega$ and $\bar{K} \Xi$ in coupled channels, with $\bar{K} \Xi^*(1530)$ and $\eta\Omega$ in s -wave and $\bar{K} \Xi$ in d -wave. In this work, we determine the unknown parameters α and β introduced in Ref. [25], fitting to the experimental data, and calculate the partial decay widths of the two and three body strong decays of $\Omega(2012)$, with the strong couplings obtained at the pole position of the state.

The paper is organized as follows. In Sect. 2, we present the formalism and ingredients of the chiral unitary approach for the treatment of the $\Omega(2012)$ as a dynamically generated hadronic state from the interactions of $\bar{K} \Xi^*(1530)$, $\eta\Omega$ and $\bar{K} \Xi$ in coupled channels. Numerical results for the two and three body strong decays of the $\Omega(2012)$ state and discussions are given in Sect. 3, followed by a short summary in the last section.

2 Formalism and ingredients

In this section, we briefly review the coupled channel approach to study the $\Omega(2012)$ state involving the s -wave interaction of $\bar{K} \Xi^*(1530)$, $\eta\Omega$, and d -wave interaction of

$\bar{K} \Xi$, although these interactions have been detailed in Refs. [4, 24, 25].

2.1 Scattering amplitude and the $\Omega(2012)$

Following Ref. [25], we denote $\bar{K} \Xi^*(1530)$, $\eta\Omega$, and $\bar{K} \Xi$ channels by 1, 2, and 3, respectively, and then the tree level transition amplitudes, V_{ij} ($i, j = 1, 2, 3$), between each of the two channels are given by

$$V_{11} = V_{22} = V_{33} = 0, \quad (1)$$

$$V_{12} = V_{21} = -\frac{3}{4f_\pi^2}(k_1^0 + k_2^0), \quad (2)$$

$$V_{13} = V_{31} = \alpha q_3^2, \quad (3)$$

$$V_{23} = V_{32} = \beta q_3^2, \quad (4)$$

where we take the pion decay constant $f_\pi = 93$ MeV. The k_1^0 and k_2^0 are the energies of the $\bar{K}$ meson in channel 1 and η meson in channel 2, respectively, which are,

$$k_1^0 = \frac{s + m_{\bar{K}}^2 - M_{\Xi^*}^2}{2\sqrt{s}}, \quad (5)$$

$$k_2^0 = \frac{s + m_\eta^2 - M_\Omega^2}{2\sqrt{s}}, \quad (6)$$

with $\sqrt{s}$ the invariant mass of the meson-baryon system. In addition, q_3 is the on-shell momentum of the $\bar{K}$ meson in channel 3, which reads,

$$q_3 = \frac{\sqrt{[s - (m_{\bar{K}} + M_\Xi)^2][s - (m_{\bar{K}} - M_\Xi)^2]}}{2\sqrt{s}}. \quad (7)$$

Then we solve the Bethe–Salpeter equation with the V_{ij} given above, and obtain the unitarized scattering amplitude T :

$$T = V + VGT = [1 - VG]^{-1}V \quad (8)$$

where G is the loop function for each channel and it is a diagonal matrix containing the meson and baryon propagators. Explicitly

$$G = \begin{pmatrix} G_{11}(\sqrt{s}) & 0 & 0 \\ 0 & G_{22}(\sqrt{s}) & 0 \\ 0 & 0 & G_{33}(\sqrt{s}) \end{pmatrix}, \quad (9)$$

² Private communications with Prof. Cheng-Ping Shen.

where $G_{ii}(\sqrt{s})$ can be regularized with a cutoff prescription and the explicit results are:³

$$\begin{aligned} G_{11} &= \int_0^{\Lambda_1} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_1} \frac{M_{\Xi^*}}{E_1} \frac{1}{\sqrt{s} - \omega_1 - E_1 + i\epsilon} \\ G_{22} &= \int_0^{\Lambda_2} \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_2} \frac{M_{\Omega}}{E_2} \frac{1}{\sqrt{s} - \omega_2 - E_2 + i\epsilon} \\ G_{33} &= \int_0^{\Lambda_3} \frac{d^3q}{(2\pi)^3} \frac{(q/q_3)^4}{2\omega_3} \frac{M_{\Xi}}{E_3} \frac{1}{\sqrt{s} - \omega_3 - E_3 + i\epsilon} \end{aligned} \quad (10)$$

where E_i and ω_i are the baryon and meson energies for i -th channel. In general, Λ_1 , Λ_2 and Λ_3 are different. Yet, to minimize the model parameters, $\Lambda_1 = \Lambda_2 = 726$ MeV are used in Ref. [24] and $\Lambda_1 = \Lambda_2 = \Lambda_3 = 735$ MeV were used in Ref. [25]. In this work, we will determine them with the experimental data of the Belle collaboration [1, 26], and discuss them in the following.

However, since the $\Xi^*(1530)$ resonance has a sizable decay width and the $\bar{K}\Xi^*(1530)$ mass threshold is close to the mass of $\Omega(2012)$, the width of $\Xi^*(1530)$ should be considered. For this purpose, we need to perform a convolution with the spectral function [31]

$$\bar{G}_{33} = \frac{1}{N} \int_{M_{\Xi^*}-6\Gamma_{\Xi^*}}^{M_{\Xi^*}+6\Gamma_{\Xi^*}} d\tilde{M} \frac{G_{33}(\sqrt{s}, \tilde{M}) \tilde{\Gamma}_{\Xi^*}}{(\tilde{M} - M_{\Xi^*})^2 + \tilde{\Gamma}_{\Xi^*}^2/4}, \quad (11)$$

with

$$N = \int_{M_{\Xi^*}-6\Gamma_{\Xi^*}}^{M_{\Xi^*}+6\Gamma_{\Xi^*}} d\tilde{M} \frac{\tilde{\Gamma}_{\Xi^*}}{(\tilde{M} - M_{\Xi^*})^2 + \tilde{\Gamma}_{\Xi^*}^2/4}. \quad (12)$$

Note that the range of $(M_{\Xi^*} - 6\Gamma_{\Xi^*}, M_{\Xi^*} + 6\Gamma_{\Xi^*})$ includes most of the distribution. Here, the $\tilde{\Gamma}_{\Xi^*}$ is energy dependent, and its explicit form is given by

$$\tilde{\Gamma}_{\Xi^*}(\tilde{M}) = \Gamma_{\Xi^*} \frac{M_{\Xi^*}}{\tilde{M}} \left(\frac{\tilde{p}_\pi}{p_\pi^{\text{on}}} \right)^3, \quad (13)$$

with

$$\begin{aligned} \tilde{p}_\pi &= \frac{\sqrt{[\tilde{M}^2 - (m_\pi + M_{\Xi^*})^2][\tilde{M}^2 - (m_\pi - M_{\Xi^*})^2]}}{2\tilde{M}}, \\ p_\pi^{\text{on}} &= \frac{\sqrt{[M_{\Xi^*}^2 - (m_\pi + M_{\Xi^*})^2][M_{\Xi^*}^2 - (m_\pi - M_{\Xi^*})^2]}}{2M_{\Xi^*}}. \end{aligned}$$

In this work, the physical masses and spin-parities of the involved particles are taken from PDG [32], and tabulated in Table 1. Note that we take the isospin averaged values for m_K , M_{Ξ^*} , M_{Ξ} and Γ_{Ξ^*} , where we take $\Gamma_{\Xi^*} = 9.5$ MeV.

With this formalism and the former ingredients, one can easily obtain the scattering matrix T . Then one can also look for poles of the scattering amplitude T_{ij} on the complex plane

³ More details about the d -wave $\bar{K}\Xi$ loop function can be found in Ref. [25] and in Refs. [29, 30] for the case of the $\Lambda(1520)$ resonance where the interactions of $\bar{K}N$ and $\pi\Sigma$ in d -wave are included.

Table 1 Masses and spin-parities of the particles involved in the present work

Particle	Spin-parity (J^P)	Mass (MeV)
$\bar{K}$	0^-	495.644
η	0^-	547.862
Ξ^*	$\frac{3}{2}^+$	1533.4
Ω	$\frac{3}{2}^+$	1672.45
Ξ	$\frac{1}{2}^+$	1318.285

of $\sqrt{s}$. The pole, z_R , on the second Riemann sheet could be associated with the $\Omega(2012)$ resonance. The real part of z_R is associated with the mass (M) of the state, and the minus imaginary part of z_R is associated with one half of its width (Γ). Close to the pole at $z_R = M_R - i\Gamma_R/2$, T_{ij} can be written as⁴

$$T_{ij} = \frac{g_{ii}g_{jj}}{\sqrt{s} - z_R}, \quad (14)$$

where g_{kk} is the coupling constant of the resonance to the k -th channel. Thus, by determining the residues of the scattering amplitude T at the pole, one can obtain the couplings of the resonance to different channels, which are complex in general.

2.2 The strong decays of $\Omega(2012)$

Since we consider the s -wave interactions of the $\bar{K}\Xi^*(1530)$ and the $\eta\Omega$ channels, the quantum numbers of the $\Omega(2012)$ should be $J^P = 3/2^-$, and it decays into $\bar{K}\Xi$ in d -wave as shown in Fig. 1, where the effective interactions are obtained from the s -wave $\Omega(2012)\bar{K}\Xi^*(1530)$ and $\Omega(2012)\eta\Omega$ decays and the re-scattering of the $\bar{K}\Xi^*(1530)$ and $\eta\Omega$ pairs, which proceed as shown in Fig. 2. In this respect, the final state interactions of $\bar{K}\Xi \rightarrow \bar{K}\Xi$ is already taken into account in the effective coupling of the $\Omega(2012)\bar{K}\Xi$ vertex as shown in Fig. 1, since the coupling constant $g_{\Omega^*\bar{K}\Xi}$ ⁵ is extracted from the unitarized scattering amplitudes T of Eq. (8), where the re-scattering of all the three coupled channels are already included, as shown in Fig. 2.

⁴ It should be noted that in the present work the final state interactions of $\bar{K}\Xi^*(1530)$, $\eta\Omega$, and $\bar{K}\Xi$ are taken into account by solving the Bethe–Salpeter equation, while they can also be determined more directly from data (if available), e.g., see, Refs. [33, 34].

⁵ In the following, we will use $g_{\Omega^*\bar{K}\Xi} = g_{11}$, $g_{\Omega^*\eta\Omega} = g_{22}$ and $g_{\Omega^*\bar{K}\Xi} = g_{33}$ for convenience.

Fig. 1 The effective $\Omega(2012) \rightarrow \bar{K} \Xi$ decay in d -wave

Fig. 2 The $\Omega(2012) \rightarrow \bar{K} \Xi$ decay through the re-scattering of the $\bar{K} \Xi^*(1530)$ and $\eta \Omega$ channels

Then the partial decay width of the $\Omega(2012) \rightarrow \bar{K} \Xi$ is easily obtained as⁶

$$\Gamma_{\Omega(2012) \rightarrow \bar{K} \Xi} = \frac{|g_{\Omega^* \bar{K} \Xi}|^2}{2\pi} \frac{M_{\Xi}}{M} q_{\bar{K}}, \quad (15)$$

where $g_{\Omega^* \bar{K} \Xi}$ is the effective coupling constant of $\Omega(2012) \bar{K} \Xi$ vertex obtained as explained above, and M is the mass of the obtained $\Omega(2012)$ state, and

$$q_{\bar{K}} = \frac{\sqrt{[M^2 - (m_{\bar{K}} + M_{\Xi})^2][M^2 - (m_{\bar{K}} - M_{\Xi})^2]}}{2M}. \quad (16)$$

For the $\Omega(2012) \rightarrow \bar{K} \pi \Xi$ decay, it can proceed via $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$. The decay diagram is shown in Fig. 3. And the partial decay width can be calculated using

$$\frac{d\Gamma_{\Omega(2012) \rightarrow \bar{K} \pi \Xi}}{dM_{\pi \Xi}} = \frac{M_{\pi \Xi}}{\pi^2 M} \frac{|g_{\Omega^* \bar{K} \Xi^*}|^2 p_{\bar{K}} \tilde{\Gamma}_{\Xi^*}}{4(M_{\pi \Xi} - M_{\Xi^*})^2 + \tilde{\Gamma}_{\Xi^*}^2}, \quad (17)$$

where $\tilde{\Gamma}_{\Xi^*}$ is dependent on the invariant mass of π and Ξ system, $M_{\pi \Xi}$. And

$$p_{\bar{K}} = \frac{\sqrt{[M^2 - (m_{\bar{K}} + M_{\pi \Xi})^2][M^2 - (m_{\bar{K}} - M_{\pi \Xi})^2]}}{2M}.$$

With all the formulae above, one can easily work out the $\Gamma_{\Omega(2012) \rightarrow \bar{K} \Xi^* \rightarrow \bar{K} \pi \Xi}$ performing the integration over $M_{\pi \Xi}$ from $M_{\Xi} + m_{\pi}$ to $M - m_{\bar{K}}$.

3 Numerical results

To calculate the scattering amplitude T , we have to fix the unknown parameters α , β , and the cutoffs Λ_k . Since there

⁶ Note that the $\Omega(2012) \rightarrow \bar{K} \Xi$ decay is in d -wave, and it should go on $q_{\bar{K}}^5$, but the V_{13} and V_{23} potentials of Eqs. (3) and (4) incorporate the four extra powers of $q_{\bar{K}}$.

Fig. 3 Diagram for the three body decay of $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$

are very limited experimental data: the mass and the width of the $\Omega(2012)$ and the upper limit of the ratio R , we will take the same value for $\Lambda_1 = \Lambda_2 = \Lambda_3 = q_{\max}$. Even so, we still have three free parameters, and there are only two experimental data plus one more constraint, the upper limit of the ratio $R < 11.9\%$.

Varying the unknown model parameters of α , β and $q_{\max}$, we find that one can reproduce the mass and width of $\Omega(2012)$ and the upper limit $R < 11.9\%$ with the following range of the model parameters:⁷

$$\alpha < -5 \times 10^{-8} \text{ MeV}^{-3}, \quad \beta > 15 \times 10^{-8} \text{ MeV}^{-3}, \quad (18)$$

$$q_{\max} > 720 \text{ MeV}. \quad (19)$$

To minimize the number of the free parameters, we fix $q_{\max} = 735$ (Set I), 750 (Set II), 800 (Set III), 850 (Set IV), and 900 MeV (Set V), and determine α and β by fitting them to the experimental data. Since we only know the upper limit of R , it is difficult to perform a χ^2 fit to it. Technically, one can define

$$\chi^2 = \left(\frac{M^{\text{th}} - M^{\text{exp}}}{\Delta M^{\text{exp}}} \right)^2 + \left(\frac{\Gamma^{\text{th}} - \Gamma^{\text{exp}}}{\Delta \Gamma^{\text{exp}}} \right)^2, \quad (20)$$

where M^{th} and Γ^{th} are evaluated at the pole position of T , and we take $M^{\text{exp}} = 2012.4 \text{ MeV}$, $\Delta M^{\text{exp}} = 0.9 \text{ MeV}$, $\Gamma^{\text{exp}} = 6.4 \text{ MeV}$, and $\Delta \Gamma^{\text{exp}} = 3.0 \text{ MeV}$ as measured by the Belle collaboration [1]. Then we vary firstly the values of α and β in the range as in Eq. (18). If the obtained mass and width of $\Omega(2012)$, and R are in agreement with the experimental values within errors, we call that a best fit. In this way, we obtain sets of the fitted parameters (α, β) with different best χ_{best}^2 . The fitted parameters corresponding to the minimum χ_{min}^2 that we get from the best fits are: $(\alpha, \beta) = (-8.0, 17.6), (-11.1, 19.5), (-18.5, 21.2), (-21.8, 20.6),$ and $(-22.0, 18.2) \times 10^{-8} \text{ MeV}^{-3}$ for sets I, II, III, IV and V, respectively, which are listed in Table 2. We will take these values as the central values of parameters α and β . In addition, with all the fitted parameters of α and β with the χ_{best}^2 fit, we search for the minimal values of the

⁷ In fact, we find that one can only determine the relative sign between α and β , rather than their absolute signs. In this work, we take negative sign for α and positive sign for β .

Table 2 Determined values of the unknown parameters in this work. We also give the pole positions (M_R , Γ_R) of the $\Omega(2012)$ and the couplings to different channel obtained with the central values of these fitted parameters

Set	$q_{\max}$ (MeV)	α (10^{-8} MeV $^{-3}$)	β (10^{-8} MeV $^{-3}$)	(M_R, Γ_R) (MeV)	$ g_{\Omega^* \bar{K} \Xi^*} $	$ g_{\Omega^* \eta \Omega} $	$ g_{\Omega^* \bar{K} \Xi} $
I	735	-6.6 ± 0.8	16.5 ± 0.8	$(2012.3 \pm 0.4, 8.3 \pm 0.6)$	1.83 ± 0.02	3.35 ± 0.06	0.42 ± 0.02
II	750	-9.9 ± 0.5	18.5 ± 0.5	$(2012.2 \pm 0.4, 7.8 \pm 0.8)$	1.80 ± 0.01	3.46 ± 0.06	0.41 ± 0.03
III	800	-17.5 ± 0.6	20.6 ± 0.5	$(2012.4 \pm 0.5, 6.4 \pm 1.3)$	1.58 ± 0.02	3.60 ± 0.04	0.37 ± 0.04
IV	850	-20.2 ± 1.0	19.6 ± 0.8	$(2012.4 \pm 0.5, 6.4 \pm 1.1)$	1.39 ± 0.03	3.78 ± 0.04	0.37 ± 0.03
V	900	-20.8 ± 1.7	17.5 ± 1.1	$(2012.4 \pm 0.5, 6.4 \pm 1.3)$	1.25 ± 0.04	3.85 ± 0.04	0.37 ± 0.04

Table 3 The predicted results for the two and three body strong decays of the $\Omega(2012)$ with the fitted parameters given in Table 2

Set	$\Gamma_{\Omega(2012) \rightarrow \bar{K} \pi \Xi}$ (MeV)	$\Gamma_{\Omega(2012) \rightarrow \bar{K} \Xi}$ (MeV)	$\text{Br}[\Omega(2012) \rightarrow \bar{K} \pi \Xi]$ (%)	$\text{Br}[\Omega(2012) \rightarrow \bar{K} \Xi]$ (%)	R (%)
I	0.87 ± 0.03	7.32 ± 0.64	$10.5^{+0.5}_{-0.8}$	$88.4^{+0.5}_{-1.5}$	11.88
II	0.84 ± 0.04	6.96 ± 0.63	$9.5^{+0}_{-1.0}$	$90.5^{+0}_{-2.6}$	10.50
III	0.66 ± 0.02	5.57 ± 1.37	$10.3^{+1.6}_{-1.7}$	$86.5^{+1.6}_{-2.9}$	11.90
IV	0.51 ± 0.03	5.66 ± 1.07	$7.9^{+1.9}_{-1.5}$	$88.2^{+1.9}_{-1.6}$	9.00
V	0.41 ± 0.03	5.73 ± 1.25	$6.5^{+1.7}_{-1.9}$	$90.0^{+1.7}_{-2.2}$	7.22

ratio R , which are 9%, 8%, 7%, 5% and 4% for sets I, II, III, IV and V, respectively. It is worth to mention that, in Ref. [28], the minimal value of R could be zero.

Next, we collect these sets of the fitted parameters, such that the corresponding χ^2_{best} are below $\chi^2_{\text{min}} + 1$. With these collected best fitted parameters, we obtain the standard deviations of parameters α and β , which are quoted in Table 2 as their errors. In the same table, we also show the obtained pole positions of the $\Omega(2012)$ state and the couplings with the best fitted parameters.⁸ Note that the pole position of $\Omega(2012)$ is about 17 MeV below the mass threshold of $\bar{K} \Xi^*(1530)$, and is far from the thresholds of $\eta \Omega$ and $\bar{K} \Xi$. According to the “criteria” for the inner structure of a pole as proposed in Refs. [35–37] for the scalar mesons and in Ref. [38] for the pentaquark states, we conclude that the $\Omega(2012)$ is a molecular hadronic state, dynamically generated from the interactions of $\bar{K} \Xi^*(1530)$ and $\eta \Omega$ in s -wave and $\bar{K} \Xi$ in d -wave.

In addition, with the mass of the $\Omega(2012)$ state and coupling constants obtained from the best fit, we calculate the partial decay widths of $\Omega(2012) \rightarrow \bar{K} \pi \Xi$ and $\Omega(2012) \rightarrow \bar{K} \Xi$, and also their ratio R .⁹ We show these results in Table 3. Note that to get the uncertainties of $\text{Br}[\Omega(2012) \rightarrow \bar{K} \pi \Xi]$ and $\text{Br}[\Omega(2012) \rightarrow \bar{K} \Xi]$, we have considered that the sum of them should be less than one. From these results, one can easily find that the sum of the branching fractions of $\Omega(2012) \rightarrow \bar{K} \pi \Xi$ and $\Omega(2012) \rightarrow \bar{K} \Xi$ is more than 95%, which indicates that the other decay modes and other strong

Fig. 4 The $\pi \Xi$ invariant mass distribution of the three body decay of $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$. The numerical results are obtained with the parameters of set I

decay mechanisms of $\Omega(2012)$ are small, such as those of the triangle mechanisms of Refs. [24, 27].

Finally, we pay attention to the $\pi \Xi$ invariant mass distributions of the $\Omega(2012) \rightarrow \bar{K} \Xi^* \rightarrow \bar{K} \pi \Xi$ decay. The theoretical calculations with the parameters of Set I are shown in Fig. 4. One can see that, because of the phase space limitations, $d\Gamma_{\Omega(2012) \rightarrow \bar{K} \pi \Xi} / dM_{\pi \Xi}$ peaks around $M_{\pi \Xi} = 1515$ MeV, which is lower than the mass of $\Xi^*(1530)$. Since the width of $\Xi^*(1530)$ is narrow, it is easy to see that the $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$ decay will be much suppressed due to the highly off-shell effect of the $\Xi^*(1530)$ propagator, though the coupling of $\Omega(2012)$ to the $\bar{K} \Xi^*(1530)$ channel is strong.

⁸ The theoretical uncertainties for M_R , Γ_R and these couplings constants shown in Table 2 are obtained in the same way.

⁹ The R is obtained with the central values of $\text{Br}[\Omega(2012) \rightarrow \bar{K} \pi \Xi]$ and $\text{Br}[\Omega(2012) \rightarrow \bar{K} \Xi]$ shown in Table 3.

4 Summary

Based on the recent measurements by the Belle collaboration [26], where they did not observe significant signals for the $\Omega(2012) \rightarrow \bar{K} \Xi^*(1530) \rightarrow \bar{K} \pi \Xi$ decay, we revisit the $\Omega(2012)$ state from the molecular perspective in which this resonance appears to be dynamically generated from the coupled channels interactions of the $\bar{K} \Xi^*(1530)$ and $\eta \Omega$ in s -wave and $\bar{K} \Xi$ in d -wave. In such a scenario, the $\Omega(2012)$ is interpreted as a $3/2^-$ molecule state. We studied the two and three body strong decays of $\Omega(2012)$, within the model parameters for the d -wave interaction, it is shown that the experimental properties of the $\Omega(2012)$ reported recently by the Belle collaboration can be easily accommodated. More and precise experimental measurements on the strong decays of the $\Omega(2012)$ would be very useful to better understand its nature.

Acknowledgements XJJ and LSG would like to thank Prof. Eulogio Oset and Prof. Cheng-Ping Shen for fruitful discussions. This work is partly supported by the National Natural Science Foundation of China under Grant Nos. 11735003, 11975041, 11961141004, 11565007, 11847317, 11975083, 1191101015 and the Youth Innovation Promotion Association CAS (2016367). It is also supported by the Key Research Projects of Henan Higher Education Institutions under No. 20A140027, the Academic Improvement Project of Zhengzhou University.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: All the relevant data are already contained in the manuscript.]

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Funded by SCOAP³.

References

1. J. Yelton et al. [Belle Collaboration], Phys. Rev. Lett. **121**, 052003 (2018)
2. E.E. Kolomeitsev, M.F.M. Lutz, Phys. Lett. B **585**, 243 (2004)
3. S. Sarkar, E. Oset, M. J. Vicente Vacas, Nucl. Phys. A **750**, 294 (2005) Erratum: [Nucl. Phys. A **780**, 90 (2006)]
4. S.Q. Xu, J.J. Xie, X.R. Chen, D.J. Jia, Commun. Theor. Phys. **65**, 53 (2016)
5. C. García-Recio, J. Nieves, L.L. Salcedo, Eur. Phys. J. A **31**, 540 (2007)
6. S. Capstick, N. Isgur, Phys. Rev. D **34**, 2809 (1986)
7. S. Capstick, N. Isgur, A.I.P. Conf. Proc. **132**, 267 (1985)
8. U. Loring, B.C. Metsch, H.R. Petry, Eur. Phys. J. A **10**, 447 (2001)
9. M. Pervin, W. Roberts, Phys. Rev. C **77**, 025202 (2008)
10. R.N. Faustov, V.O. Galkin, Phys. Rev. D **92**, 054005 (2015)
11. S.G. Yuan, C.S. An, K.W. Wei, B.S. Zou, H.S. Xu, Phys. Rev. C **87**, 025205 (2013)
12. C.S. An, B.C. Metsch, B.S. Zou, Phys. Rev. C **87**, 065207 (2013)
13. C.S. An, B.S. Zou, Phys. Rev. C **89**, 055209 (2014)
14. K.T. Chao, N. Isgur, G. Karl, Phys. Rev. D **23**, 155 (1981)
15. T.M. Aliev, K. Azizi, Y. Sarac, H. Sundu, Phys. Rev. D **98**, 014031 (2018)
16. T.M. Aliev, K. Azizi, Y. Sarac, H. Sundu, Eur. Phys. J. C **78**, 894 (2018)
17. L.Y. Xiao, X.H. Zhong, Phys. Rev. D **98**, 034004 (2018)
18. Z.Y. Wang, L.C. Gui, Q.F. Lü, L.Y. Xiao, X.H. Zhong, Phys. Rev. D **98**, 114023 (2018)
19. M.S. Liu, K.L. Wang, Q.F. Lü, X.H. Zhong, Phys. Rev. D **101**, 016002 (2020)
20. M.V. Polyakov, H.D. Son, B.D. Sun, A. Tandogan, Phys. Lett. B **792**, 315 (2019)
21. F.K. Guo, C. Hanhart, U.G. Meißner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. **90**, 015004 (2018)
22. M.P. Valderrama, Phys. Rev. D **98**, 054009 (2018)
23. Y.H. Lin, B.S. Zou, Phys. Rev. D **98**, 056013 (2018)
24. Y. Huang, M.Z. Liu, J.X. Lu, J.J. Xie, L.S. Geng, Phys. Rev. D **98**, 076012 (2018)
25. R. Pavao, E. Oset, Eur. Phys. J. C **78**, 857 (2018)
26. S. Jia et al. [Belle Collaboration], Phys. Rev. D **100**, 032006 (2019)
27. Y.H. Lin, F. Wang, B.S. Zou, [arXiv:1910.13919](https://arxiv.org/abs/1910.13919) [hep-ph]
28. T. Gutsche, V.E. Lyubovitskij, [arXiv:1912.10894](https://arxiv.org/abs/1912.10894) [hep-ph]
29. S. Sarkar, E. Oset, M. J. Vicente Vacas, Phys. Rev. C **72**, 015206 (2005)
30. L. Roca, S. Sarkar, V.K. Magas, E. Oset, Phys. Rev. C **73**, 045208 (2006)
31. D. Gamermann, C. Garcia-Recio, J. Nieves, L.L. Salcedo, Phys. Rev. D **84**, 056017 (2011)
32. M. Tanabashi et al., Particle Data Group. Phys. Rev. D **98**, 030001 (2018)
33. K.L. Au, D. Morgan, M.R. Pennington, Phys. Rev. D **35**, 1633 (1987)
34. L.Y. Dai, M.R. Pennington, Phys. Rev. D **90**, 036004 (2014)
35. D. Morgan, Nucl. Phys. A **543**, 632 (1992)
36. L.Y. Dai, X.G. Wang, H.Q. Zheng, Commun. Theor. Phys. **57**, 841 (2012)
37. L.Y. Dai, X.G. Wang, H.Q. Zheng, Commun. Theor. Phys. **58**, 410 (2012)
38. S.Q. Kuang, L.Y. Dai, X.W. Kang, D.L. Yao, [arXiv:2002.11959](https://arxiv.org/abs/2002.11959) [hep-ph]